Neutrinos versus the flavour puzzle

Belén Gavela

NuFact 2013, Beijing

Beyond Standard Model because

1) **Experimental evidence** for new **particle** physics:

- *** **Neutrino masses**
- *** **Dark matter**
- ** **Matter-antimatter asymmetry**

2) **Uneasiness with SM fine-tunings**

Beyond Standard Model because

1) Experimental evidence for new **particle** physics:

***** Neutrino masses**

***** Dark matter**

**** Matter-antimatter asymmetry**

2) Uneasiness with SM fine-tunings i.e. strong CP

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1) **Experimental evidence** for new **particle** physics:

*** **Neutrino masses**

*** **Dark matter**

** **Matter-antimatter asymmetry**

2) **Uneasiness with SM fine-tunings,** i.e. electroweak:

*** **Hierarchy problem**

*** **Flavour puzzle**

Beyond Standard Model because

1) **Experimental evidence** for new **particle** physics:

*** **Neutrino masses**

*** **Dark matter**

** **Matter-antimatter asymmetry**

2) **Uneasiness with SM fine-tunings,** i.e. electroweak:

*** **Hierarchy problem** → $\Lambda_{\text{electroweak}} \sim 1$ TeV ?

*** **Flavour puzzle** → $\Lambda_f \sim$ 100's TeV ???

FLAVOUR is the real issue in BSM electroweak

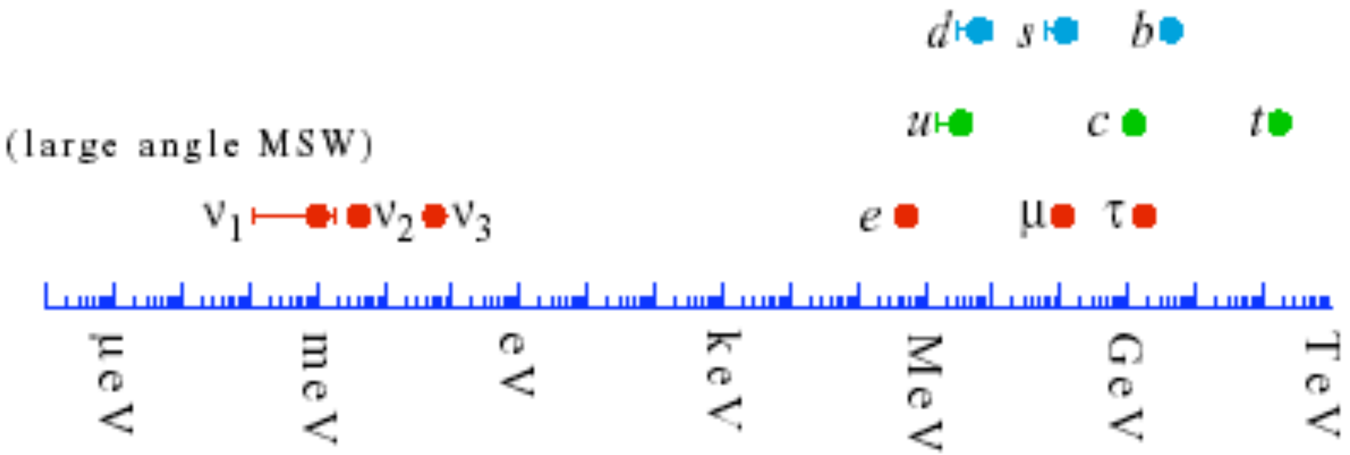
* The understanding of the physics behind is stalled since decades

* Precious data for the puzzle e.g.: B’s, neutrinos

Neutrino light on flavour ?

1) masses

2) mixing



Neutrinos lighter because Majorana?

Up to now, the only real strength in particle physics is the

gauge principle

Neutrinos are special because the SM gauge symmetry allows

to write Majorana masses for them

If new physics scale M > v

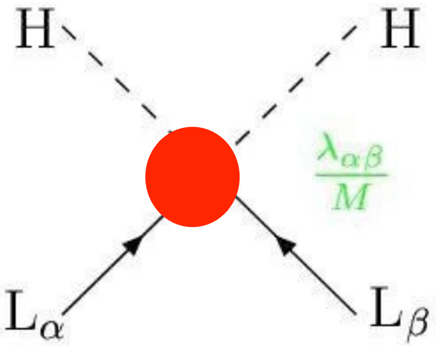
$$\mathcal{L} = \mathcal{L}_{SU(3)xSU(2)xU(1)} + \frac{O^{d=5}}{M} + \ldots\ldots$$

If Majorana masses found, this will be the

New Standard Model (νSM)

ν masses beyond the SM : tree level

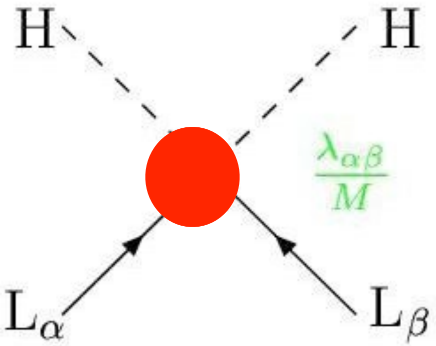
2 x 2 = 1 + 3



$SU(2)\ x U(1)_{em}$ inv.

$$(\underbrace{\overline{L^{c}}\ L\ H\ \ H}_{O^{d=5}}) \rightarrow (\overline{\nu_L^{c}}\ \nu)$$

ν masses beyond the SM : tree level



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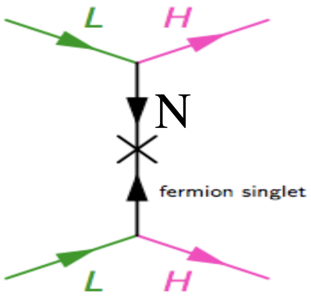
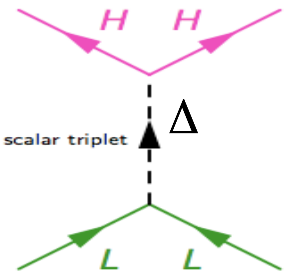
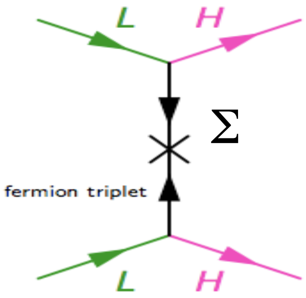


Type I



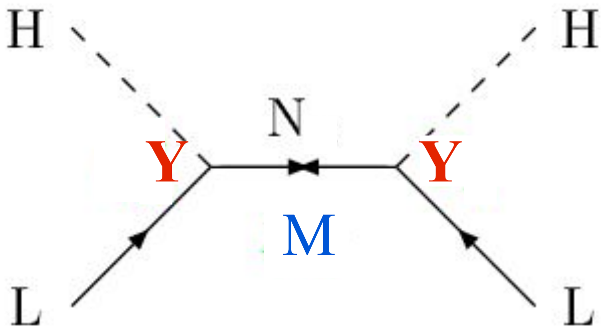
Type II



Type III

Type I seesaw

$$-\mathcal{L}_{mass} = \overline{L}\,H\,Y_E E_R + \overline{L}\,\tilde{H}\,Y N + M\,\overline{N}N^c + h.c.$$



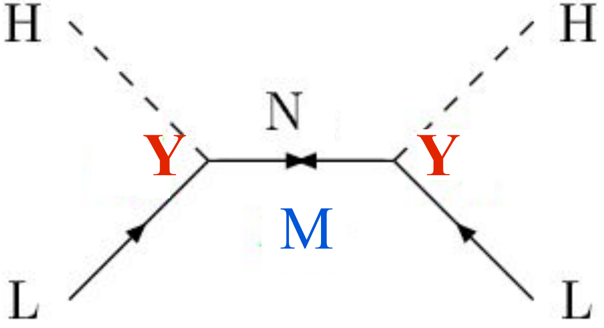
2 x 2 = ① + 3

Type I seesaw

usual Dirac term

Majorana mass allowed by gauge invariance

$$-\mathcal{L}_{mass} = \overline{L}\,H\,Y_E E_R + \boxed{\overline{L}\,\tilde{H}\,Y\,N} + \boxed{M\,\overline{N}N^c} + h.c.$$



2 x 2 = $\boxed{1}$ + 3

Type I seesaw

usual Dirac term

Majorana mass
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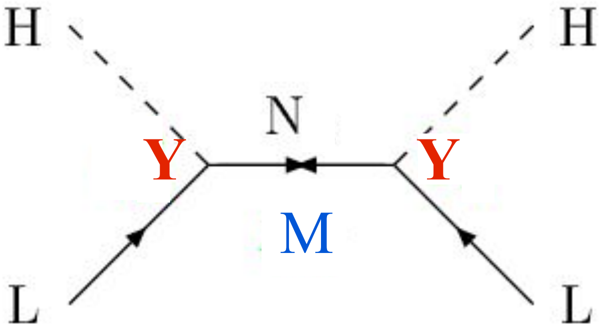
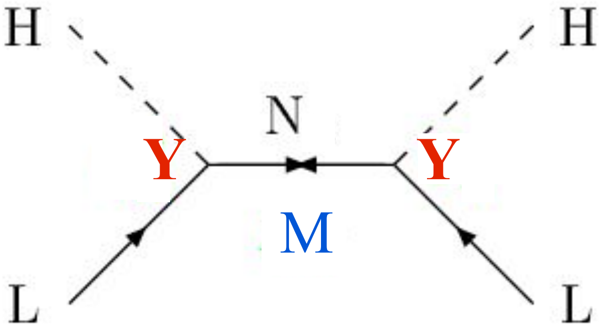


2 x 2 = ① + 3

$$m_\nu \sim \begin{pmatrix} 0 & Y_N^T\,v \\ Y_N\,v & M_N \end{pmatrix} \to Y\,\frac{v^2}{M}\,Y^T$$

Type I seesaw

$$-\mathcal{L}_{mass} = \overline{L}\,H\,Y_E E_R + \overline{L}\,\tilde{H}\,Y N + M\,\overline{N}N^c + h.c.$$



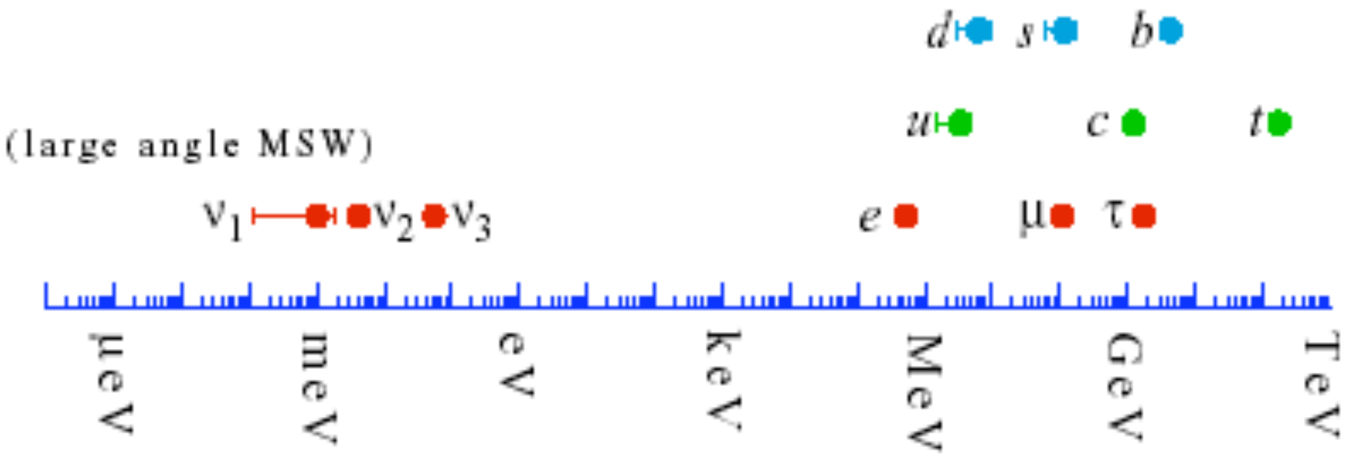
2 x 2 = ① + 3

$$m_\nu \sim Y \frac{v^2}{M} Y^T$$

$Y \sim 1$ for $M \sim M_{GUT}$

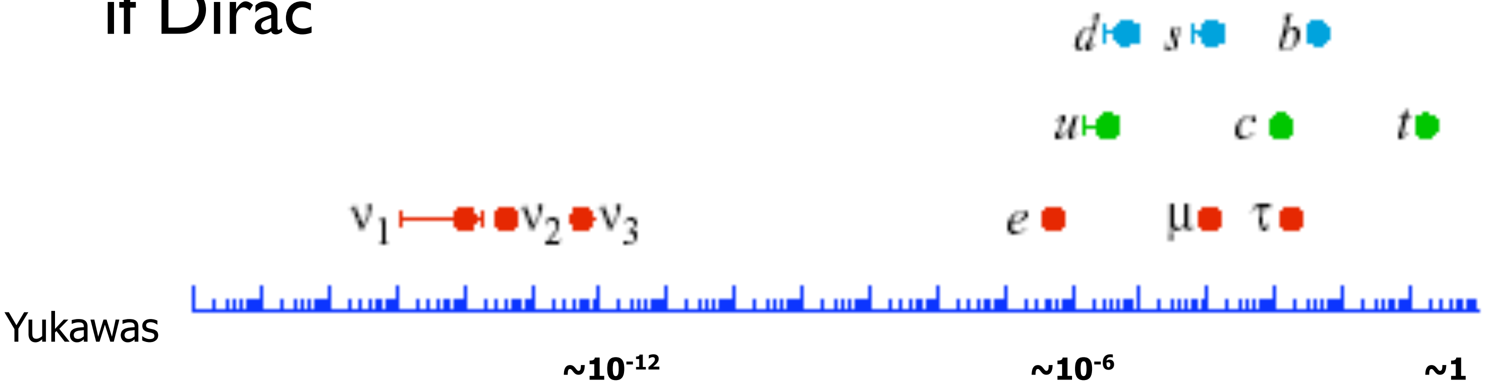
$Y \sim 10^{-6}$ for $M \sim$ TeV

In mass scale we had



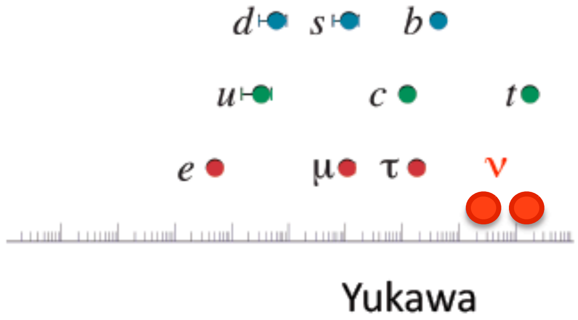
The same in Yukawa scale is

if Dirac

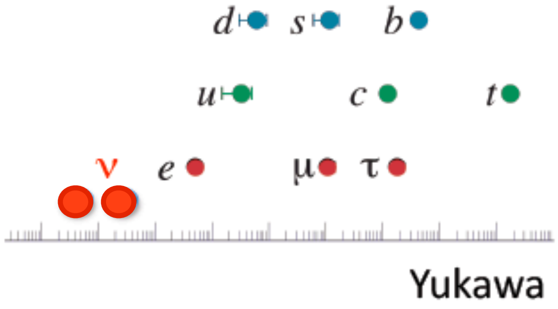


Within seesaw, the size of ν Yukawa couplings is alike to that for other fermions:

$\Lambda \lesssim$ GUT



$\Lambda =$ TeV

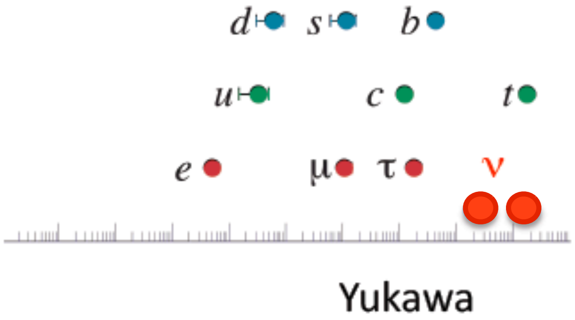


Pilar Hernandez drawings

Minkowski; Gell-Mann, Ramond Slansky; Yanagida, Glashow...

Within seesaw, the size of ν Yukawa couplings is allowed in large classes of models, e.g.:

$\Lambda \leq$ Tev



for instance in type I seesaw models with approximate $U(1)_{LN}$ symmetry

inverse, direct........

WG1 and WG2 Friday:

talk by Filipe Joaquim

Wyler+Wolfenstein 83, Mohapatra+Valle 86, Branco+Grimus+Lavoura 89, Gonzalez-Garcia+Valle 89, Ilakovac+Pilaftsis 95, Barbieri+Hambye+Romanino 03, Raidal+Strumia+Turzynski 05, Kersten+Smirnov 07, Abada+Biggio+Bonnet+Gavela+Hambye 07, Shaposhnikov 07, Asaka+Blanchet 08, Gavela+Hambye+D. Hernandez+ P. Hernandez 09 ....

Leptons

$$V_{PMNS} = \begin{pmatrix} 0.8 & 0.5 & \sim 0.1 \\ -0.4 & 0.5 & -0.7 \\ -0.4 & 0.5 & +0.7 \end{pmatrix}$$

Quarks

$$V_{CKM} = \begin{pmatrix} \sim 1 & \lambda & \lambda^3 \\ \lambda & \sim 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & \sim 1 \end{pmatrix} \quad \lambda \sim 0.2$$

Why so different?

Leptons

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Quarks

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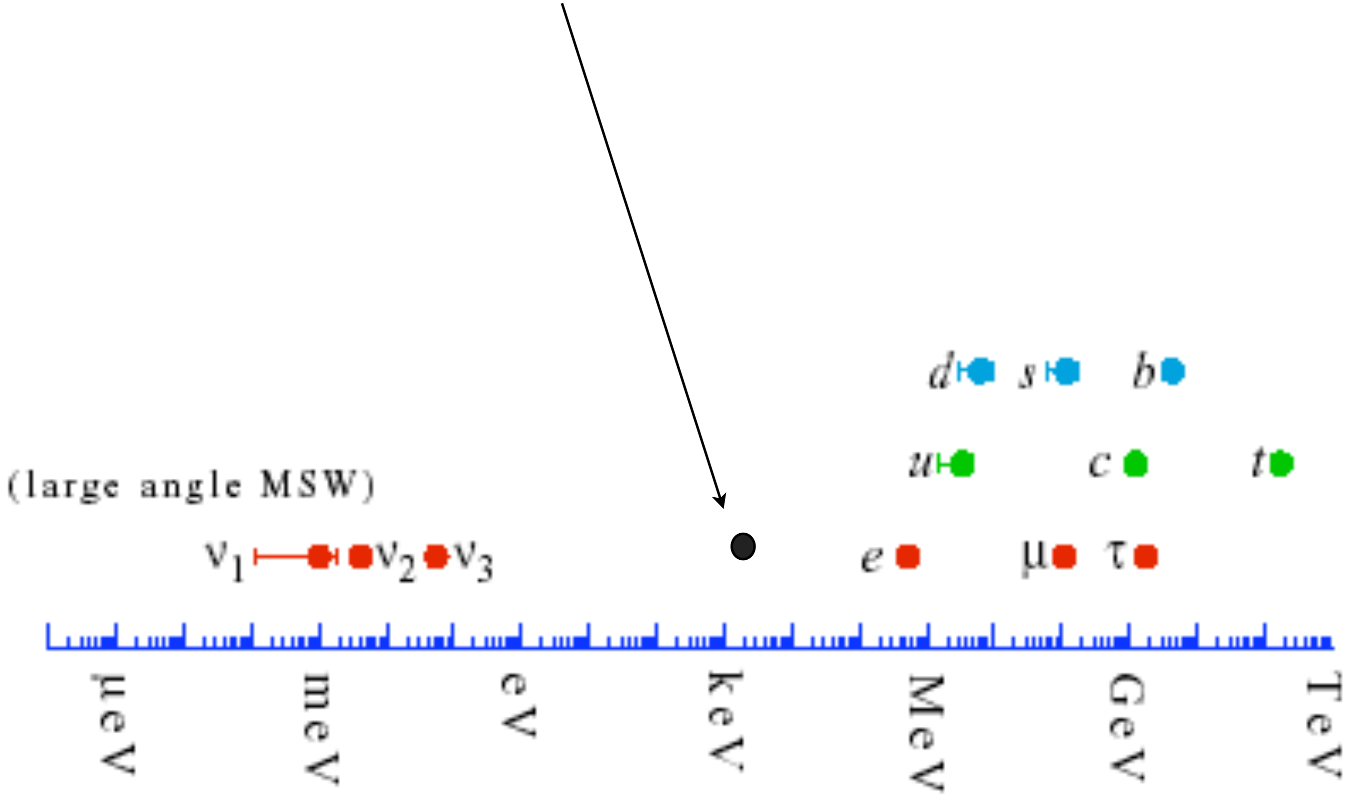
Maybe because of Majorana neutrinos ?

Neutrino are optimal windows into the exotic -dark- sectors

* Can mix with new neutral fermions, heavy or light

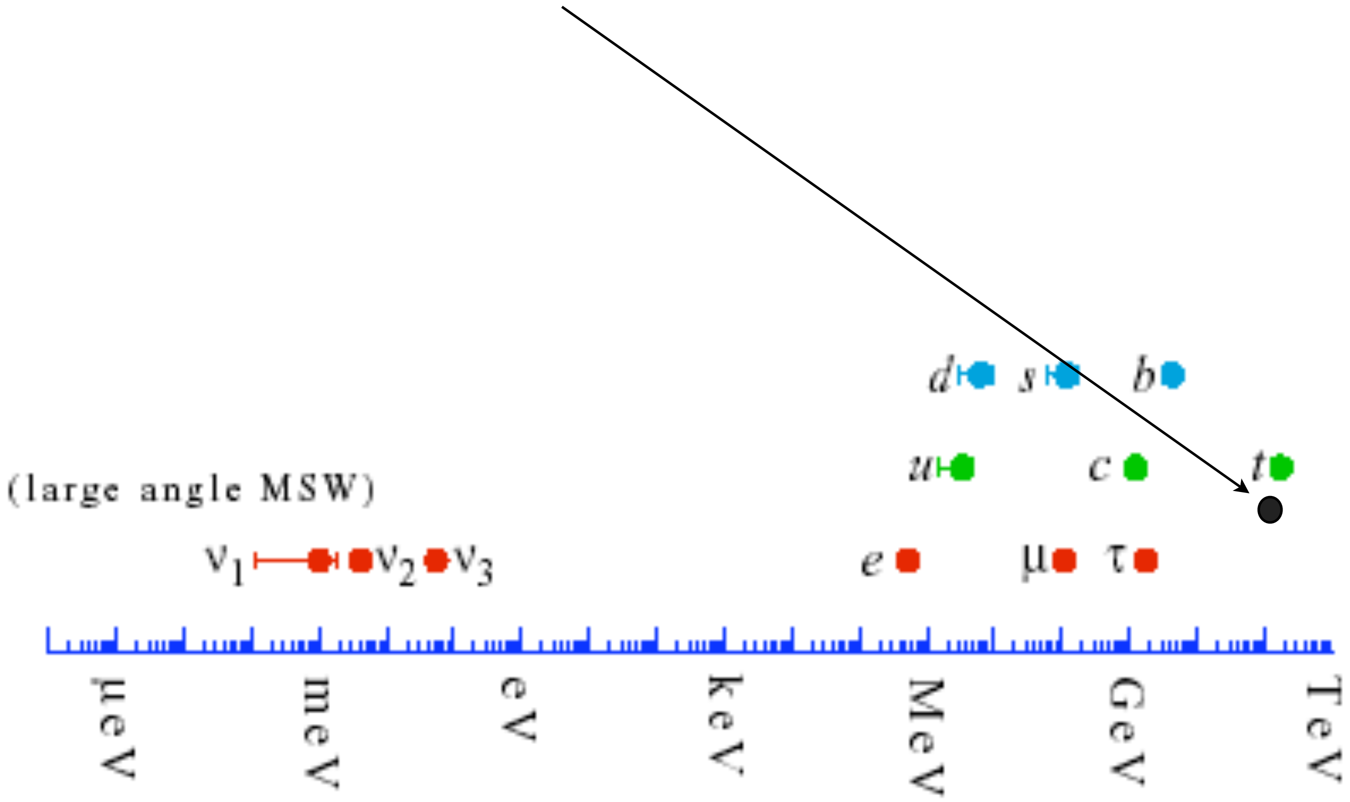
* Interactions not obscured by strong and e.m. ones

DARK FLAVOURS ?



.... they can be fermions

DARK FLAVOURS ?



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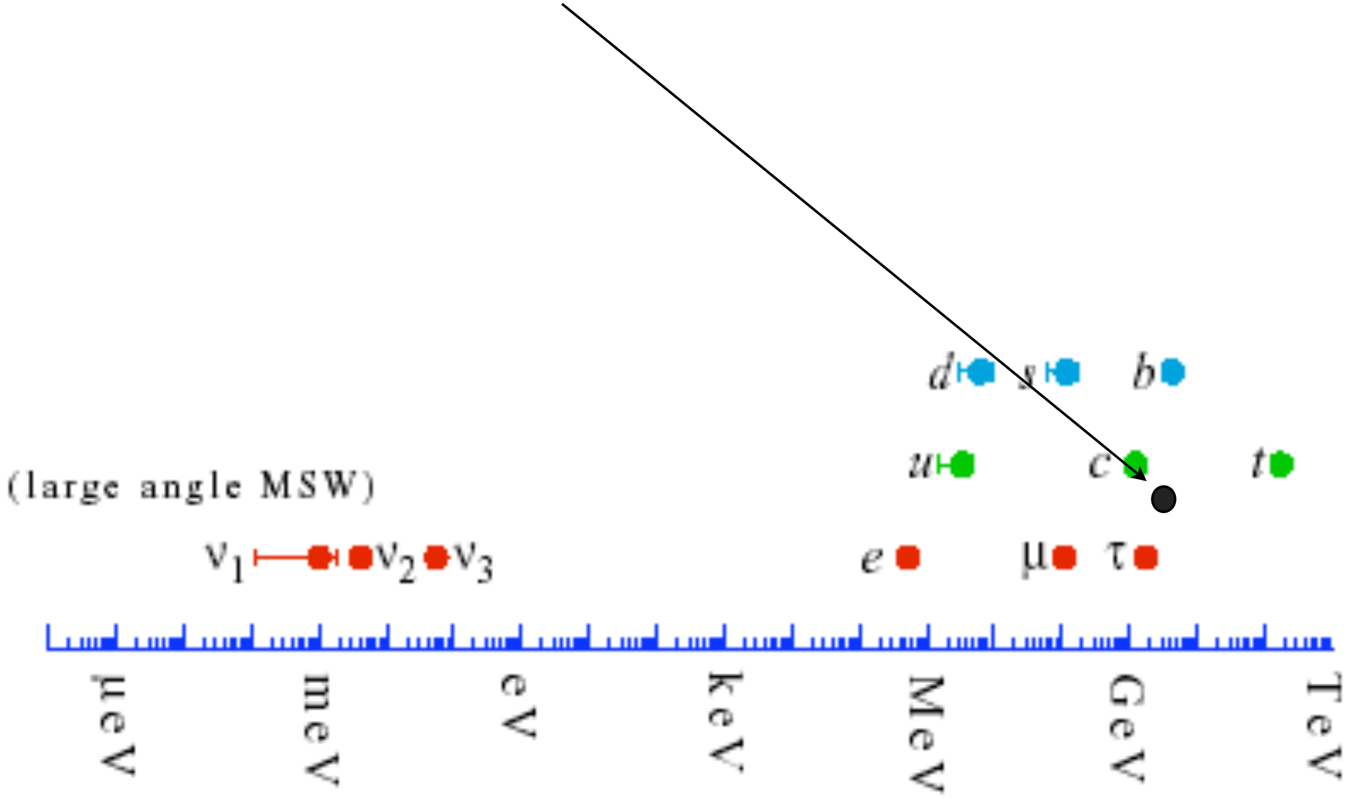

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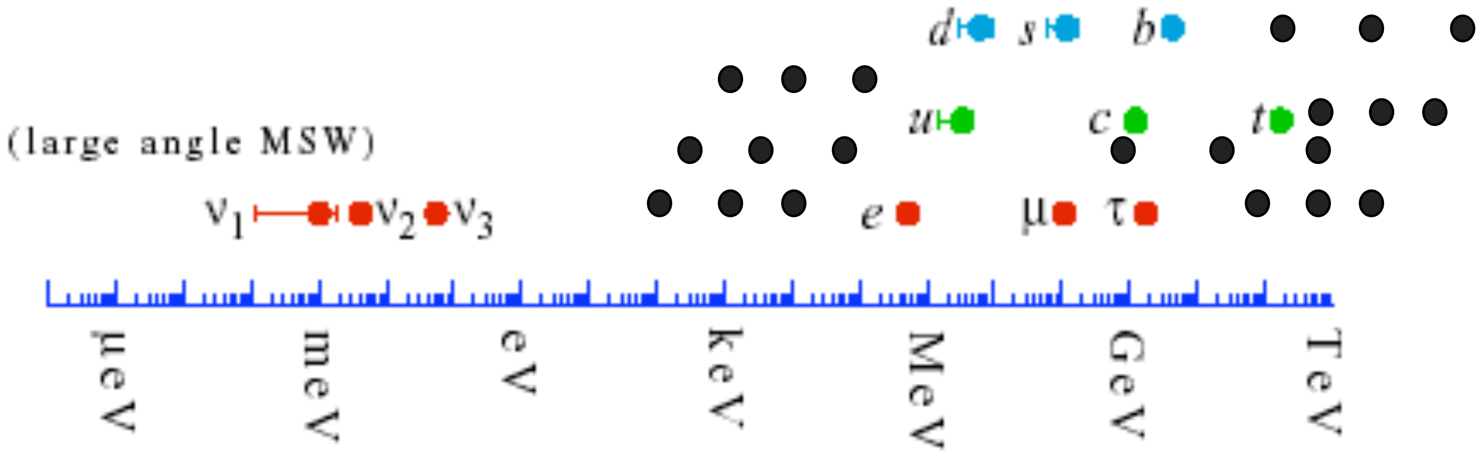

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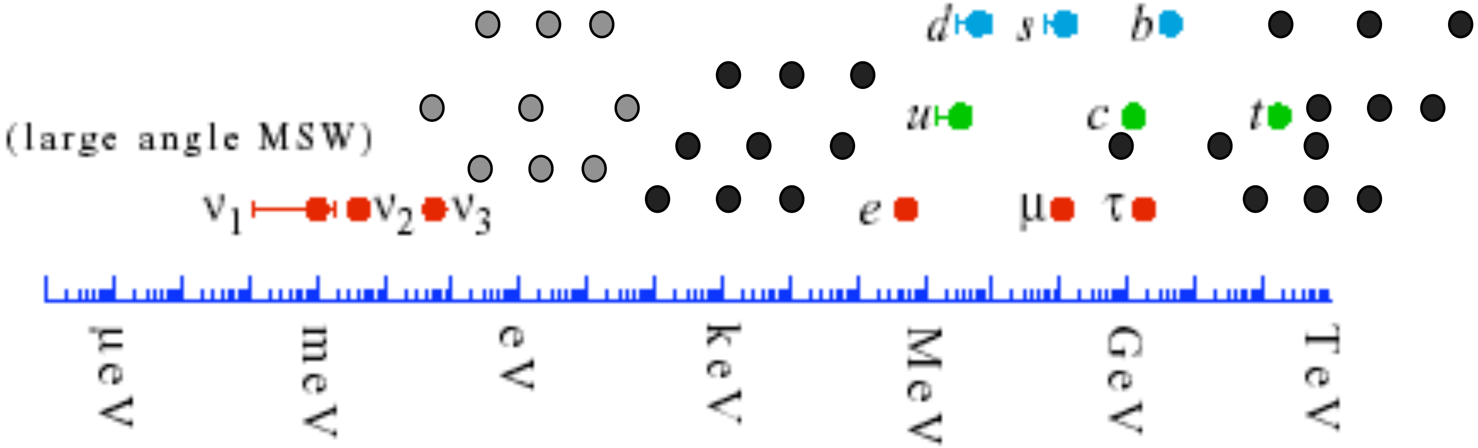


Neutrino oscillations --> talk by Patrick Huber

Flavour → Yukawas

1) Flavour violation searches with charged leptons:
   what will they tell us about the type I seesaw heavy neutrinos?

2) Theory: towards a dynamical origin of Yukawa couplings

Lepton Flavour violation (LFV) windows:

*** Neutrino oscillations** <--neutral LFV

*** μ-->e γ**

*** μ-->e e e**

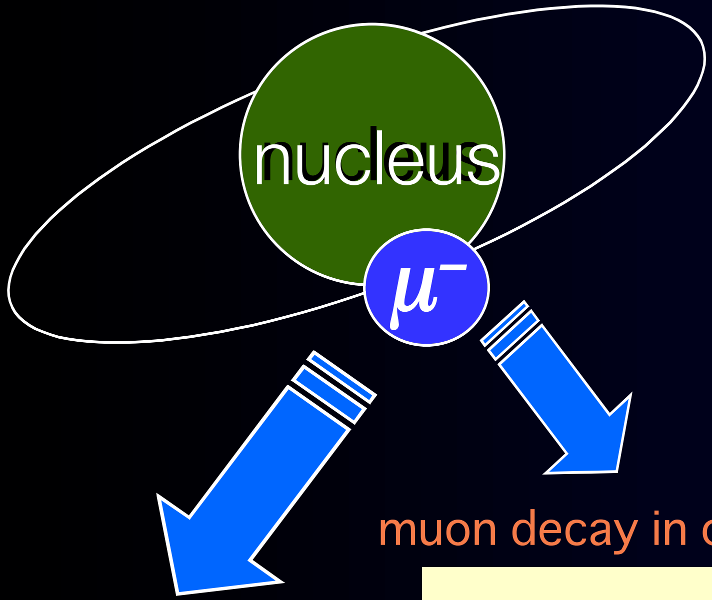
charged LFV

*** Another fantastic experimental window being opened on lepton-flavour :**

μ-e conversion in nuclei

What is Muon to Electron Conversion?

1s state in a muonic atom



muon decay in orbit

$$\mu^- \rightarrow e^- \nu_\mu \bar{\nu}_e$$

nuclear muon capture

$$\mu^- + (A,Z) \rightarrow \nu_\mu + (A,Z-1)$$

courtesy of Yoshi Kuno

Neutrino-less muon nuclear capture

$$\mu^- + (A,Z) \rightarrow e^- + (A,Z)$$

Event Signature :
a single mono-energetic electron of 100 MeV

Backgrounds:

(1) physics backgrounds
  ex. muon decay in orbit (DIO)

(2) beam-related backgrounds
  ex. radiative pion capture, muon decay in flight,

(3) cosmic rays, false tracking

Consider together

μ-->e conversion

μ-->e γ

μ-->e e e

now **expected**

μ-->e conversion

$R^{Ti}_{\mu\to e} \;<\; 4.3\times 10^{-12}$ ----------------> $\lesssim 10^{-18}$

$R^{Au}_{\mu\to e} \;<\; 7\times 10^{-13}$

$R^{Pb}_{\mu\to e} \;<\; 4.6\times 10^{-11}$

$R^{Al}_{\mu\to e} \;\lesssim 10^{-16}$

μ-->e γ

$Br(\mu \to e\gamma) < 5.7\ 10^{-13}$ ------------------> $<5\ 10^{-14}$

MEG

μ-->e e e

$Br(\mu \to eee) < 10^{-12}$ ----------------> $< 10^{-16}$

now **expected**

μ-->e conversion

several plenary talks

and

$R^{Ti}_{\mu\to e} \ < \ 4.3\times 10^{-12}$ ---------------> $\lesssim 10^{-18}$

$R^{Au}_{\mu\to e} \ < \ 7\times 10^{-13}$

$R^{Pb}_{\mu\to e} \ < \ 4.6\times 10^{-11}$

$R^{Al}_{\mu\to e} \ \lesssim 10^{-16}$

WG4 Wednesday:

Z. You, P. Kasper and

Y. Yuang

μ-->e γ

$Br(\mu \to e\gamma) < 5.7\ 10^{-13}$ ----------------> $< 5\ 10^{-14}$

MEG

WG4 tomorrow:

L.Galli and C. Cheng

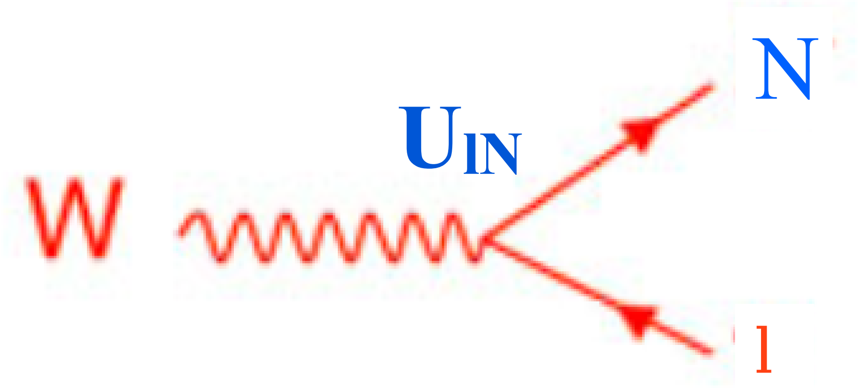
μ-->e e e

$Br(\mu \to eee) < 10^{-12}$ ---------------> $< 10^{-16}$

WG4 tomorrow:

A. Bravar

Assume that singlet fermion(s) N exists in nature

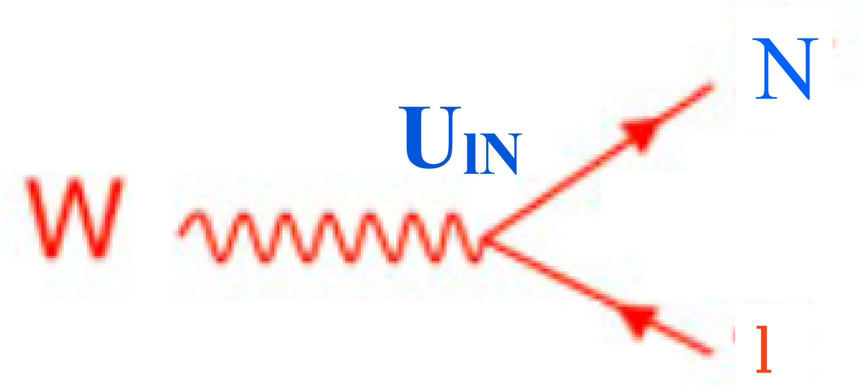


?

What are the limits on their mass m_N and mixings U_{lN} ?

Can we observe them?

Assume that singlet fermion(s) N exists in nature



?

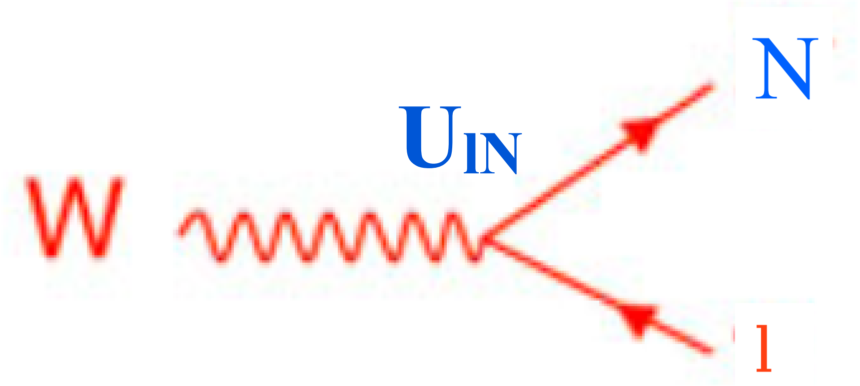
What are the limits on their mass $\mathbf{m_N}$ and mixings $\mathbf{U_{lN}}$?
Can we observe them?

The paradigm model: Seesaw type-I N_R

$$\mathcal{L} = \mathcal{L}_{SM} + i\overline{N_R}\partial\!\!\!/ N_R - \left[\overline{N_R} Y_N \tilde{\phi}^\dagger \ell_L + \frac{1}{2}\overline{N_R} M N_R{}^c + h.c.\right]$$

$$\mathbf{U_{lN}} \sim \mathbf{Y}\ \mathrm{v}/\ \mathbf{M}$$

Assume that singlet fermion(s) N exists in nature



?

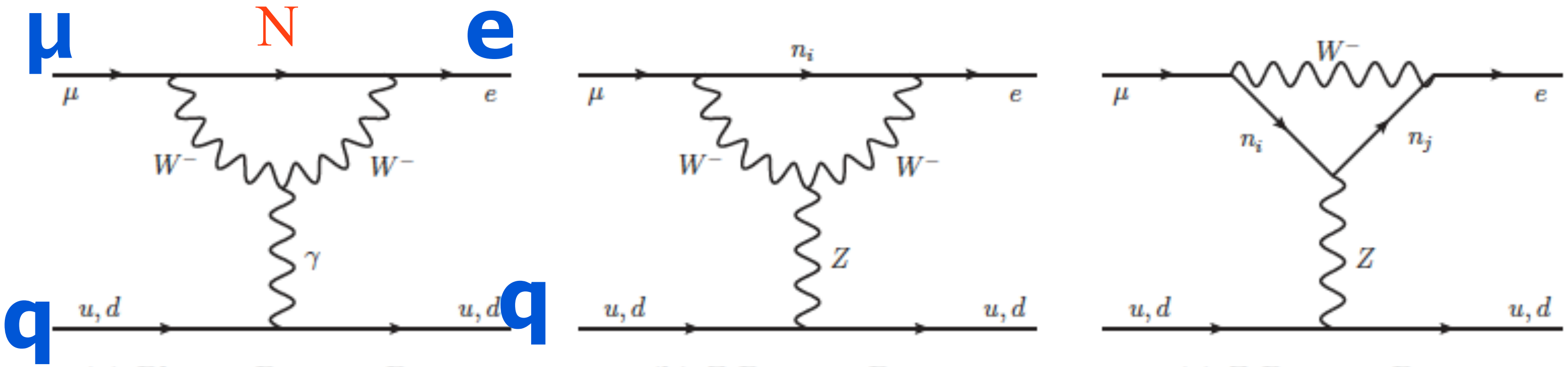
What are the limits on their mass m_N and mixings U_{lN} ?

Can we observe them?

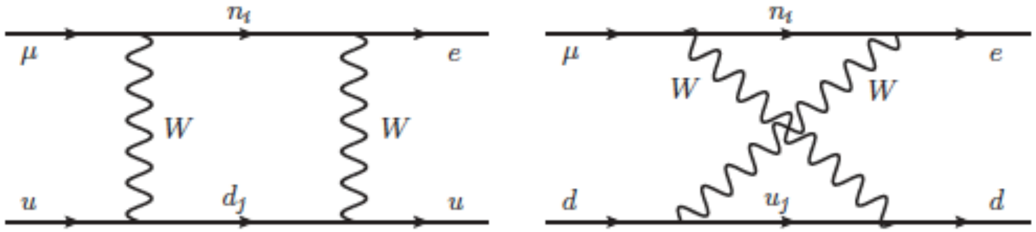
(Alonso, Dhen, Gavela, Hambye)

μ-->e conversion

mass eigenstates $n_i=\nu_1$, ν_2, ν_3, N_1....N_k



(a) Photon Penguin Diagram (b) Z Penguin Diagram (c) Z Penguin Diagram



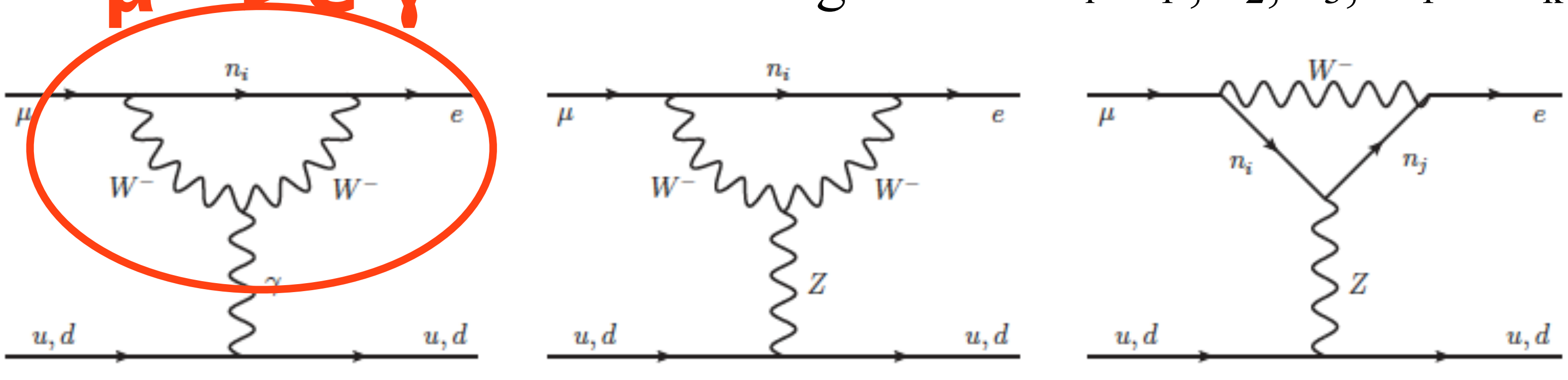
(d) Box Diagram (e) Box Diagram

Figure 1: The five classes of diagrams contributing to μ to e conversion in the type-I seesaw model.

μ-->e conversion

μ-->e γ

mass eigenstates $n_i=\nu_1$, ν_2, ν_3, $N_1....N_k$



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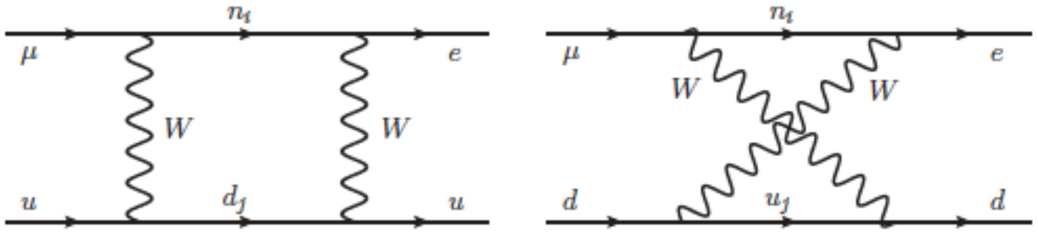


(d) Box Diagram (e) Box Diagram

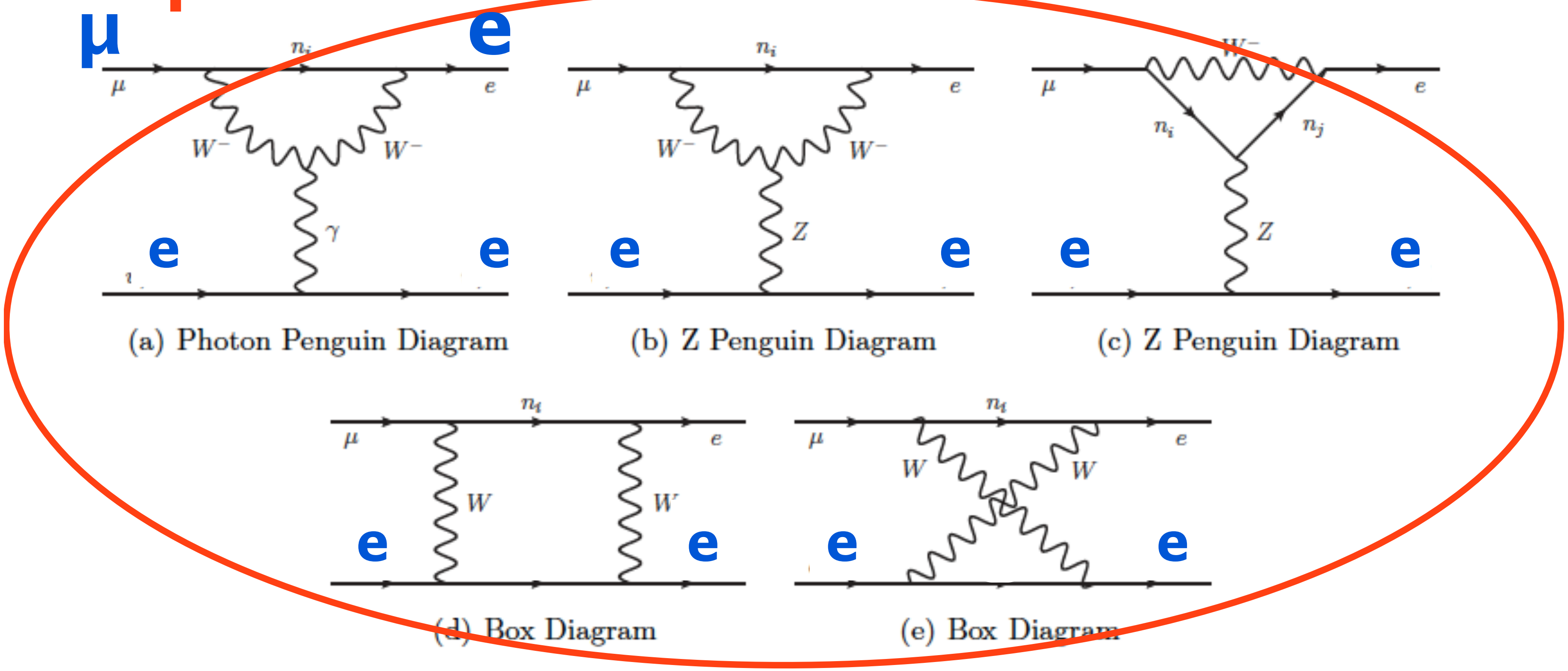
Figure 1: The five classes of diagrams contributing to μ to e conversion in the type-I seesaw model.

They share just one form factor ("dipole")

μ-->e conversion

μ-->e ee

mass eigenstates $n_i = \nu_1 , \nu_2, \nu_3, N_1....N_k$



(a) Photon Penguin Diagram (b) Z Penguin Diagram (c) Z Penguin Diagram

(d) Box Diagram (e) Box Diagram

Figure 1: The five classes of diagrams contributing to μ to e conversion in the type-I seesaw model.

Share all form factors,
in different combinations

Type I seesaw

μ-->e conversion

Many people before us computed it for singlet fermions:

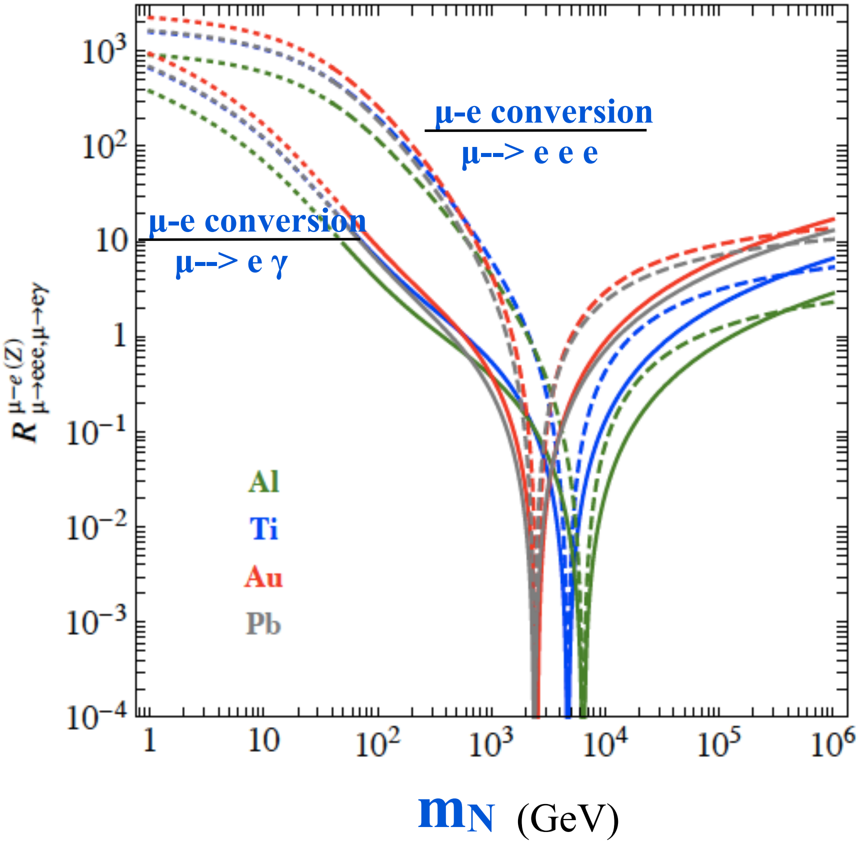
De Gouvea
Mohapatra
Riazzudin+Marshak+Mohapatra 91,
Chang+Ng 94,
Ioannisian+Pilaftsis00,
Grimus + Lavoura
Pilaftsis and Underwood05,
Deppish+Kosmas+Valle06,
Ilakovac+Pilaftsis09,
Deppish+Pilaftsis11,
Dinh+Ibarra+Molinaro+Petcov12,
Aristizabal Sierra+Degee+Kamenik12

We agree for logarithmic dependence

Not two among those papers completely agree with each other, or they are not complete

typical applications assumed masses over 100 GeV or TeV

- **we computed all contributions (logarithmic and constant)**
- **μ--> e conversion vanishes for masses in the 2-7 TeV mass regime; (degenerate or hierarchical heavy neutrinos)**

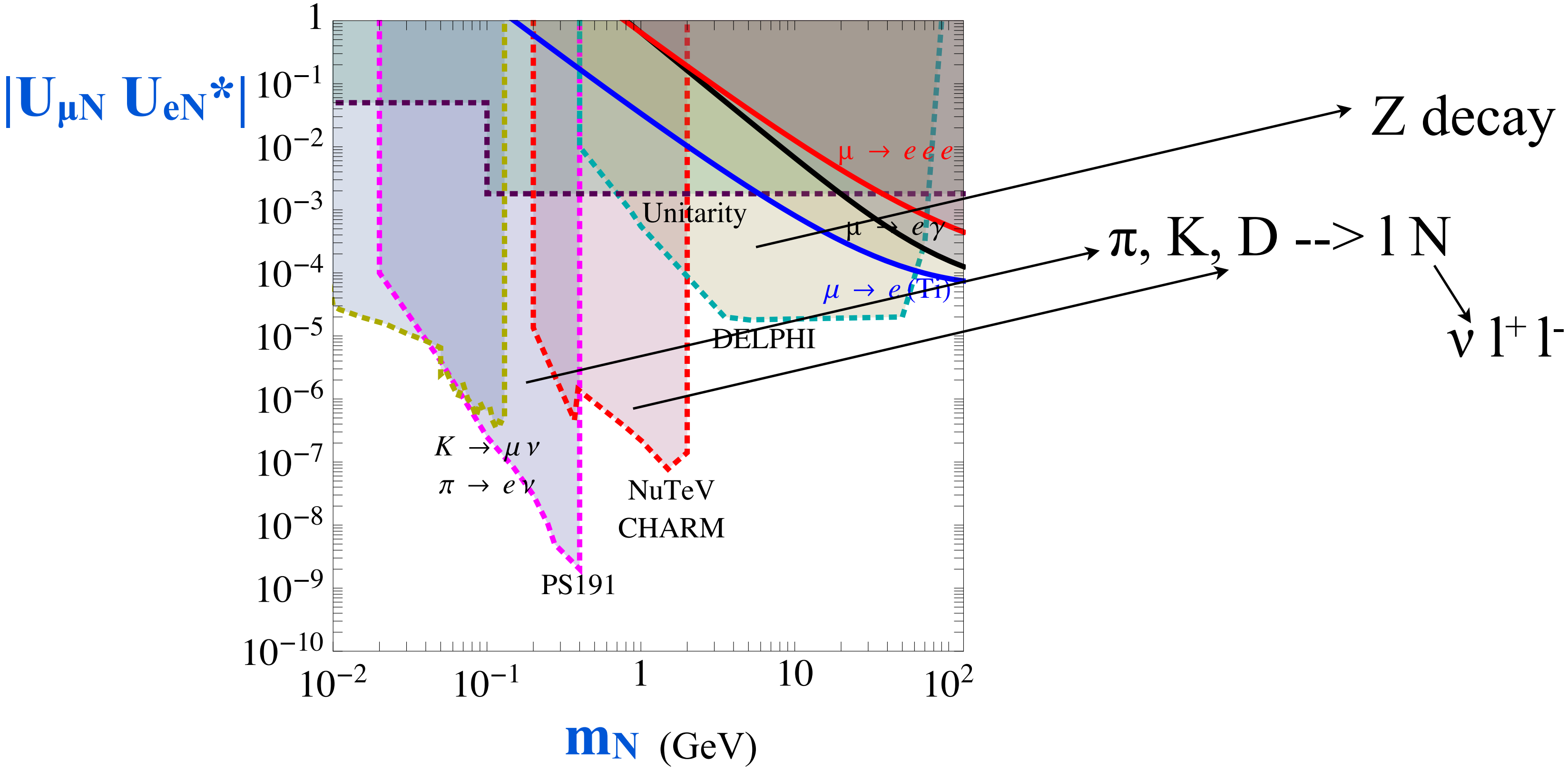


- **we also considered the low mass region, sweeping over eV< m_N < thousands GeV**

(Alonso, Dhen, Gavela, Hambye)

* Low mass regime eV << m_N << m_W

(realistic neutrino masses <--> degenerate heavy neutrinos)



Z decay

π, K, D --> l N

$\nu\, l^+ l^-$

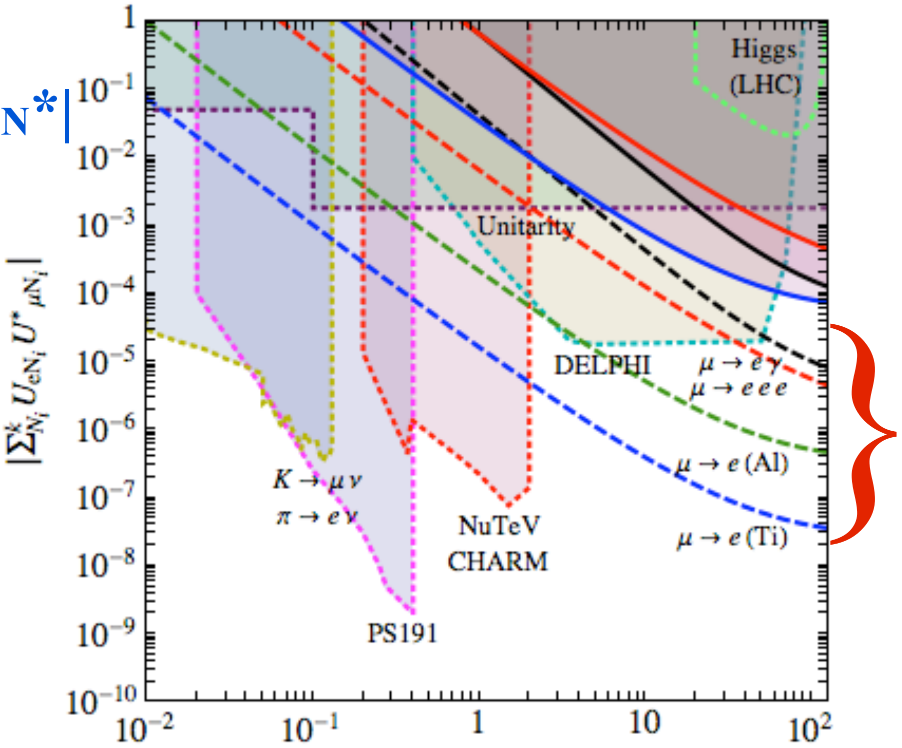
Peak decays+PS191+NuTev/CHARM+Delphi: Atre+Han+Pascoli+Zhang 09...... Richayskiy+Ivashko 12

Unitarity: Antusch+Biggio+Fdez-Martinez+Gavela+Lopez-Pavon 06; Antusch+Bauman+ Fedez-Martinez 09

* Low mass regime eV << m_N << m_W

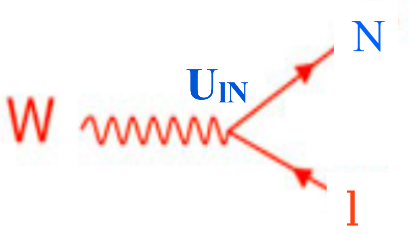
(realistic neutrino masses <--> degenerate heavy neutrinos)

$|U_{\mu N}\, U_{eN}{}^*|$



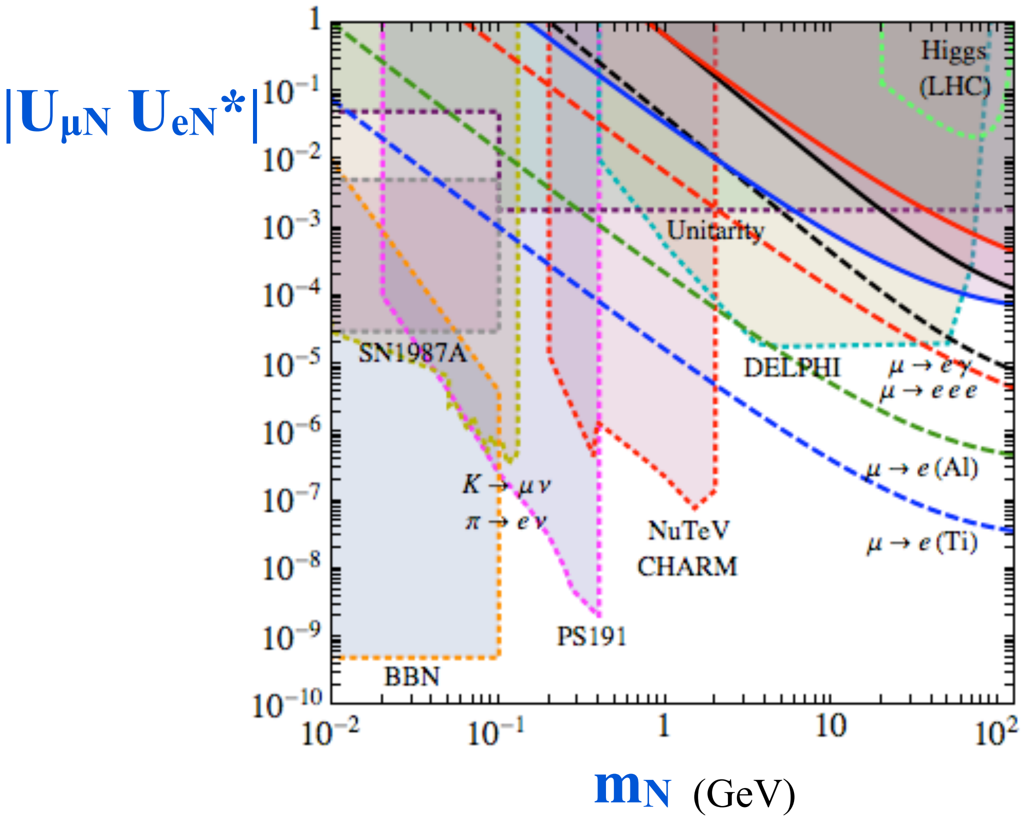
Future

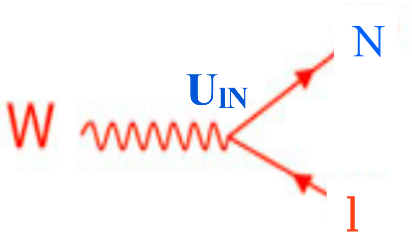
m_N (GeV)



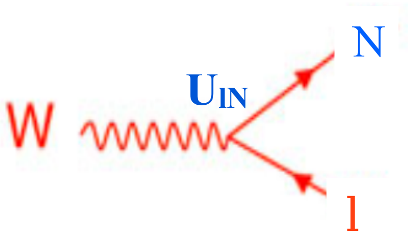
* Low mass regime eV << m_N << m_W

(realistic neutrino masses <--> degenerate heavy neutrinos)

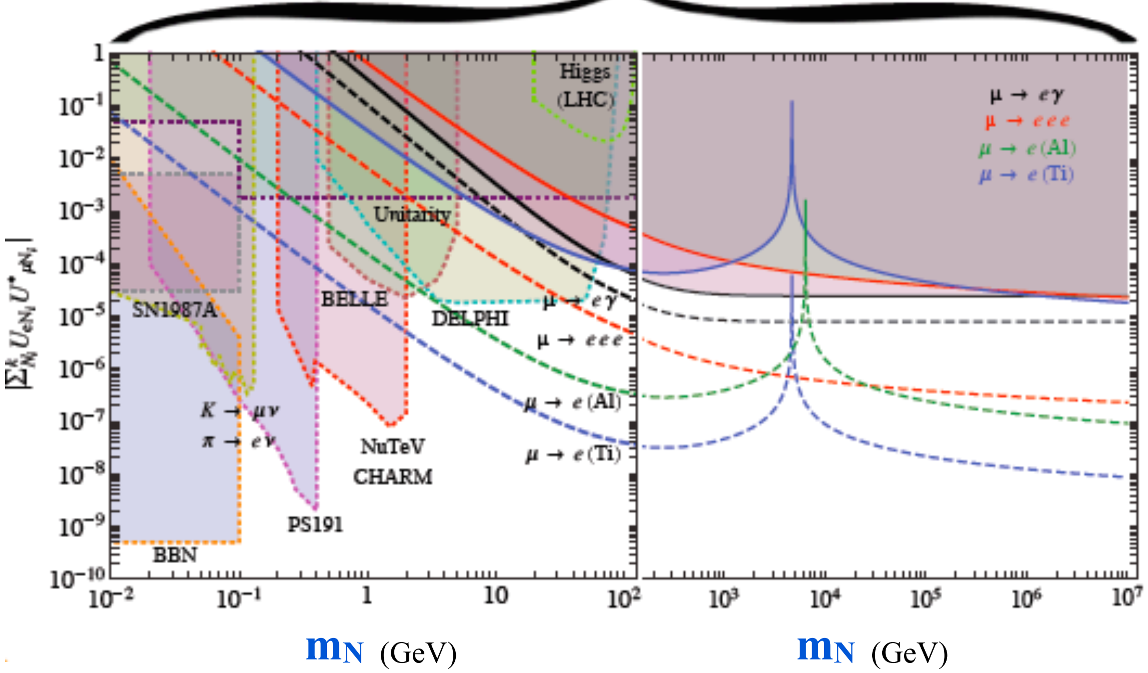


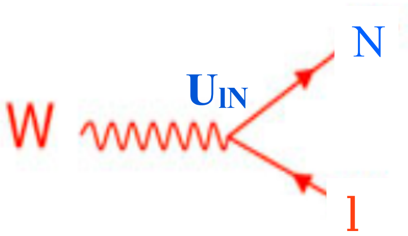


BBN and SN: Kainulainen+Maalampi+Peltoniemi91, Kusenko+Pascoli+Semikoz 05, Mangano+Serpico 11, Ruchaysiliy +Ivashko 12,
Kufflick+McDermott+Zurek 12



$$\sum_{Ni} U_{eNi} U^{*}_{\mu Ni}$$

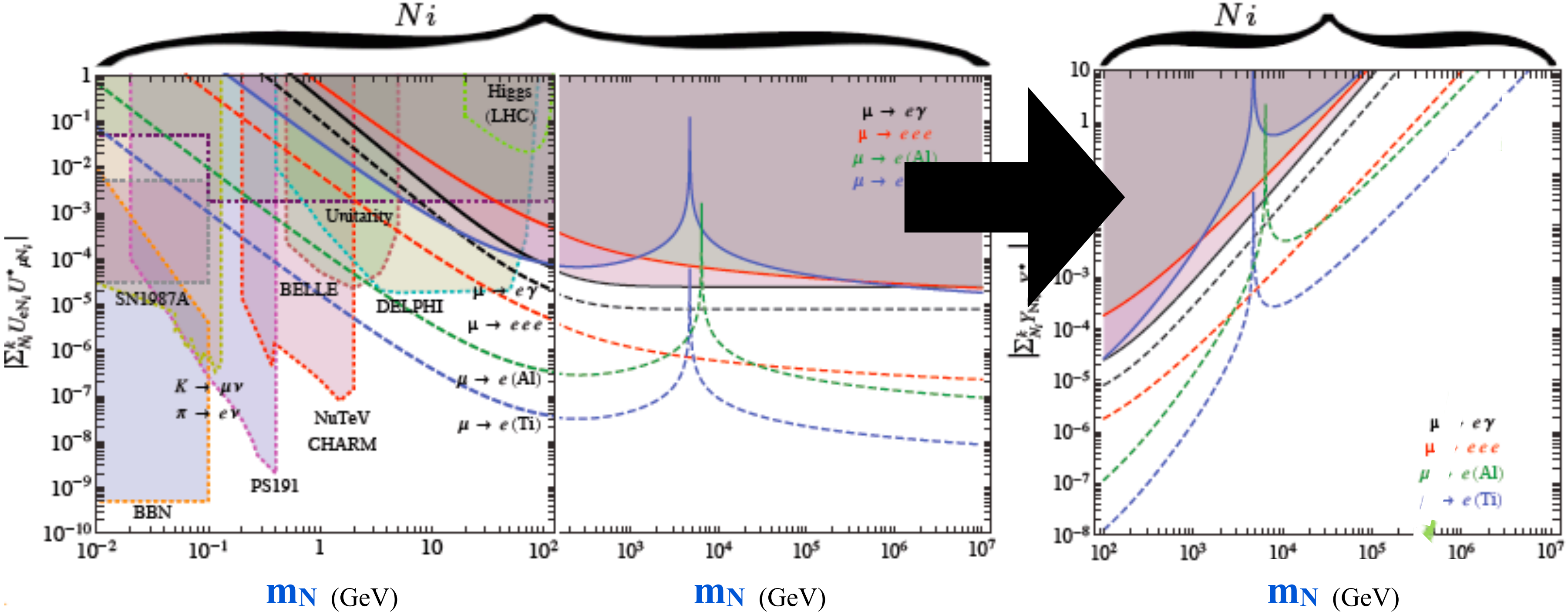




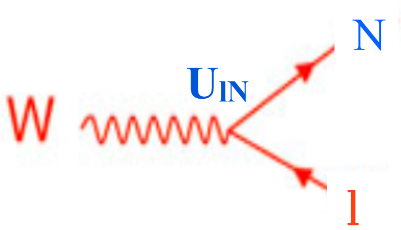
$U_{lN} \sim Y\, v / M$

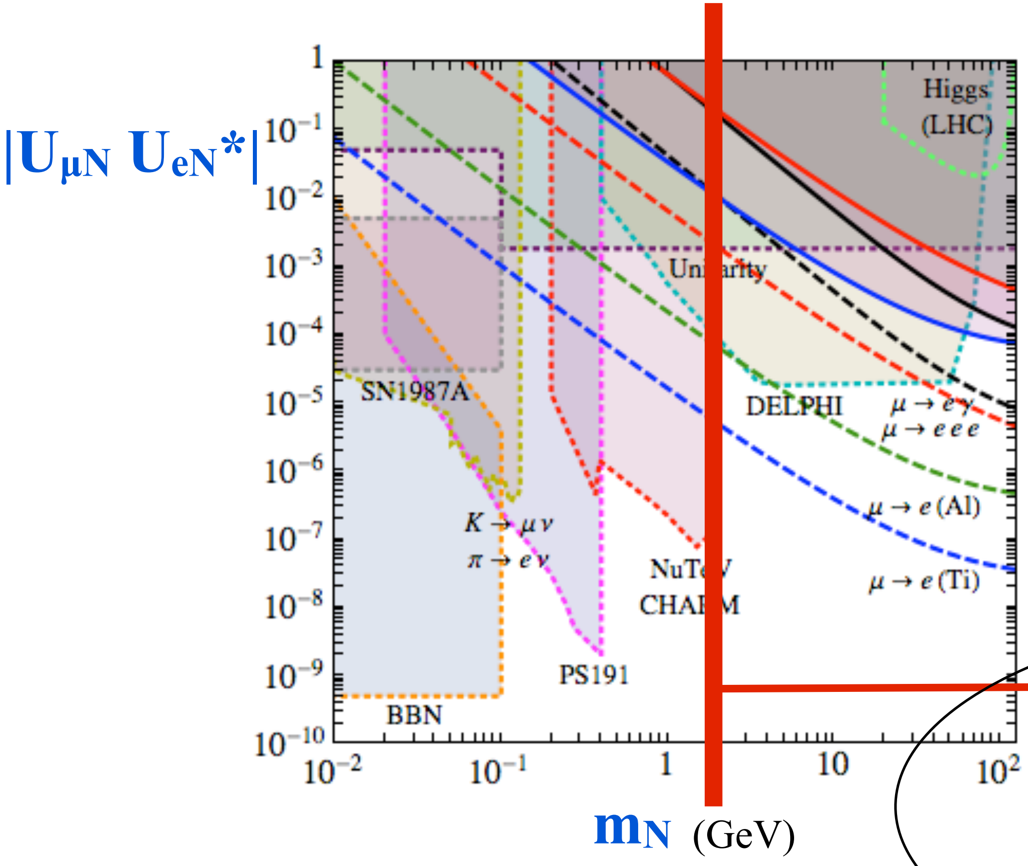
$$\sum_{Ni} U_{eNi} U^*_{\mu Ni}$$

$$\sum_{Ni} Y^*_{eNi} Y_{\mu Ni}$$



In summary



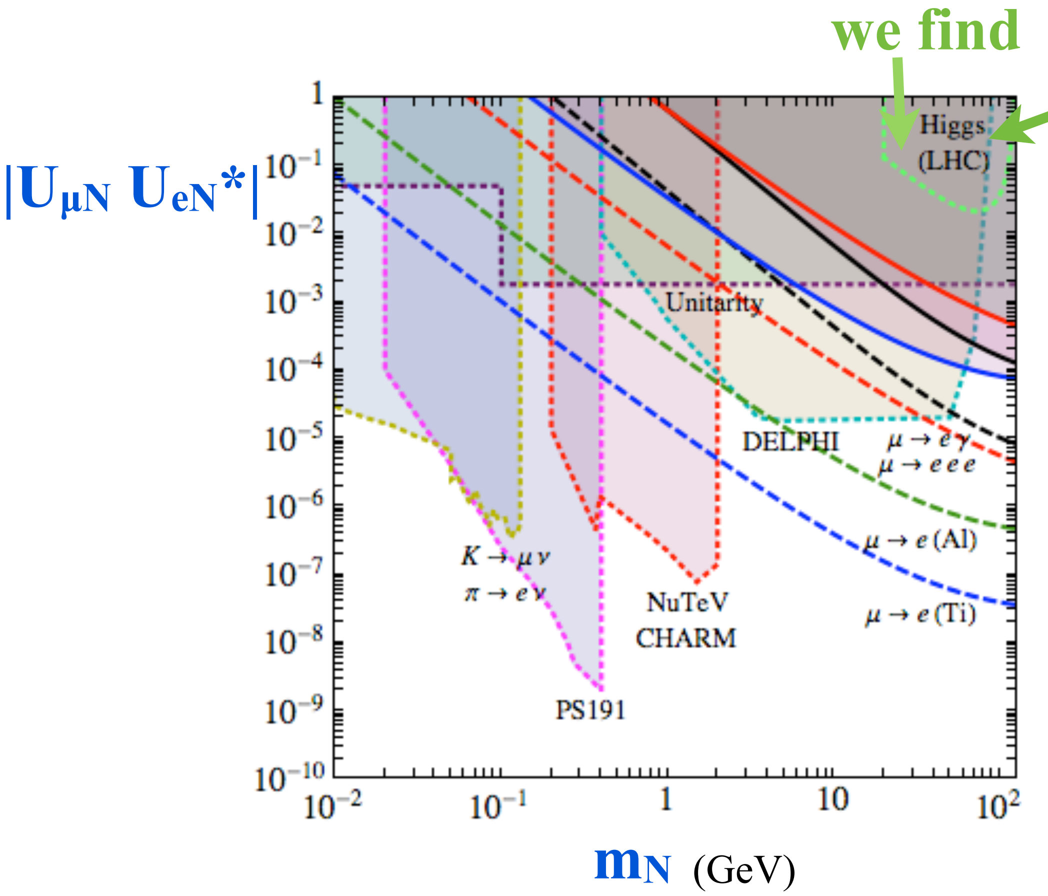


model-independent

future sensitivity reach of
μ --> e conversion:

2 Gev --- 6000 TeV !

(Alonso, Dhen, Gavela, Hambye)



Absolute bound from the decay of the SM scalar doublet, from absence of

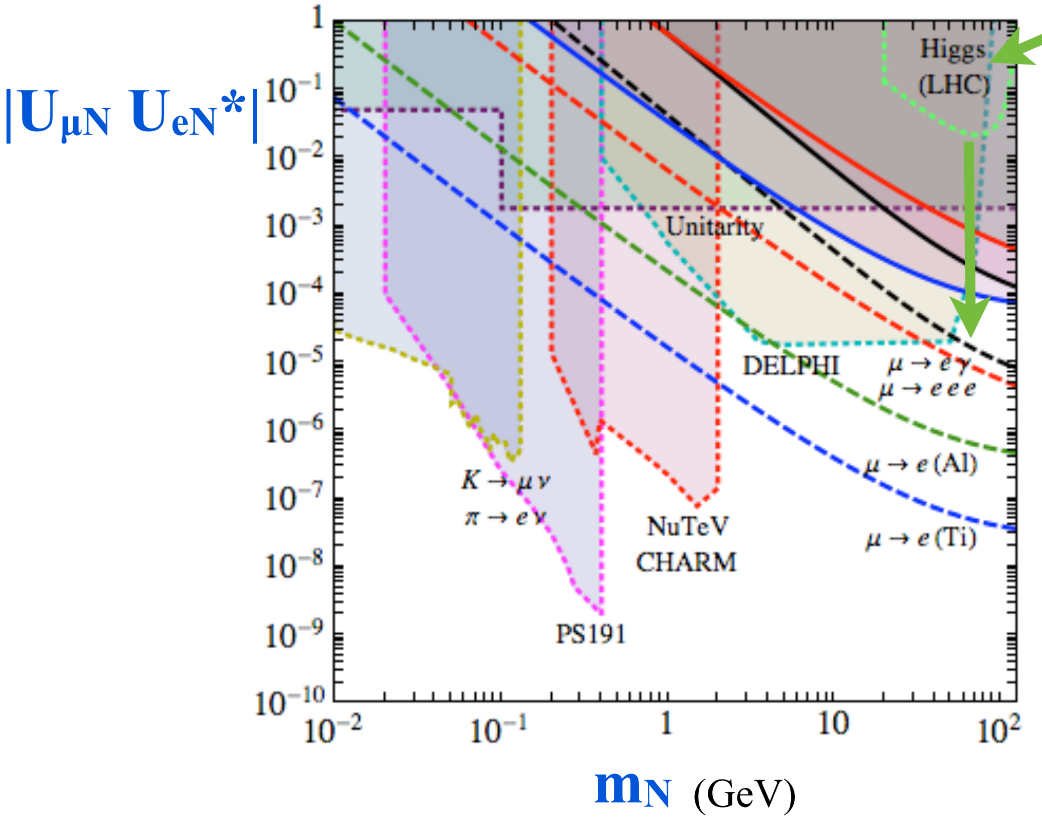
H --> ν N

↘ $\nu\, l^+ l^-$

at LHC:

Br (H -> ν N) < 0.4

(Espinosa, Grojean, Muhlleitner, Trott, 12
Dev+Franceschini+Mohapatra12, Cely+
Ibarra+Molinaro+Petcov 12)



LHC is more competitive for concrete seesaw models.

For instance those with approximate $U(1)_{LN}$ and low scale ~TeV (inverse seesaws etc.)

see backup slides

(Cely at al.)
(Alonso, Dhen, Gavela, Hambye)

Comparing the seesaw scales reached by

Neutrino Oscillations **vs** **μ-e experiments** **vs** **LHC**

e.g. for Seesaw type I (heavy singlet fermions):

* **ν-oscillations:** **10^{-3}eV - $M_{GUT} \sim 10^{15}$ GeV**, because interferometry

* **μ-e conversion:** **2MeV - 6000 GeV** (type I inverse seesaw class)

* **LHC:** **~ # TeV**

Warning: all LFV searches complementary in impact, e.g.:

μ-->e conversion

type III contributes
tree level

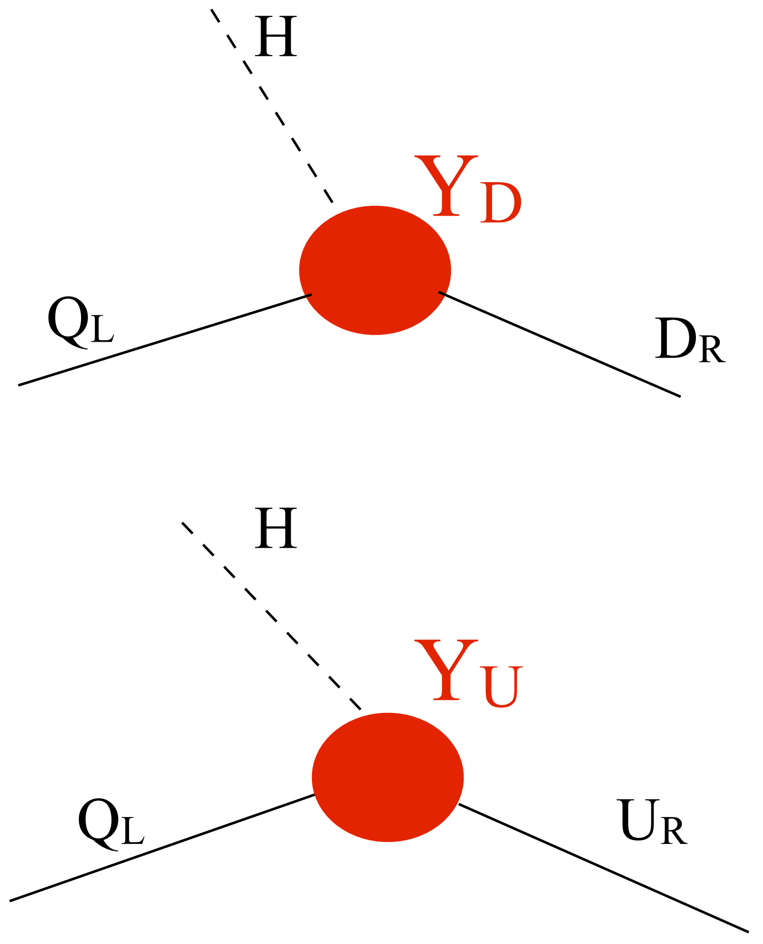
μ-->e γ

μ-->e e e

type II contributes tree level

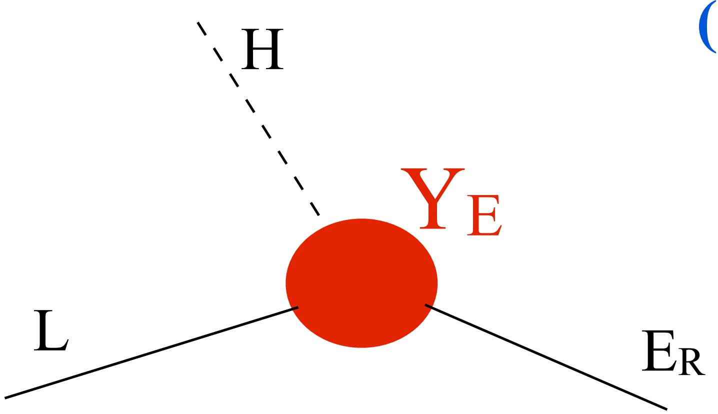
- Dynamical Yukawas

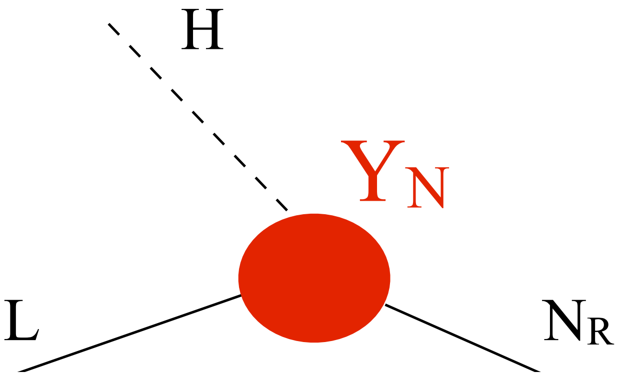
Yukawa couplings are the source of flavour in the SM



Yukawa couplings are a source of flavour in the ν-SM

(i.e. Seesaw type-I)





$$\mathcal{L} = \mathcal{L}_{SM} + i\overline{N_R}\not{\partial} N_R - \left[\overline{N_R} Y_N \tilde{\phi}^\dagger \ell_L + \frac{1}{2}\overline{N_R} M N_R{}^c + h.c.\right]$$

May they correspond to dynamical fields (e.g. vev of fields that carry flavor) ?

In many BSM the Yukawas do not come from dynamical fields:

Some good ideas:

D.B. Kaplan-Georgi in the 80's proposed a light SM scalar because being a (quasi) goldstone boson: ***composite Higgs***

(D.B. Kaplan, Georgi, Dimopoulos, Banks, Dugan, Galison.......Contino, Nomura, Pomarol; Agashe, Contino, Pomarol; Giudice, Pomarol, Ratazzi, Grojean; Contino, Grojean, Moretti; Azatov, Galloway, Contino... Frigerio, Pomarol, Riva, Urbano...)

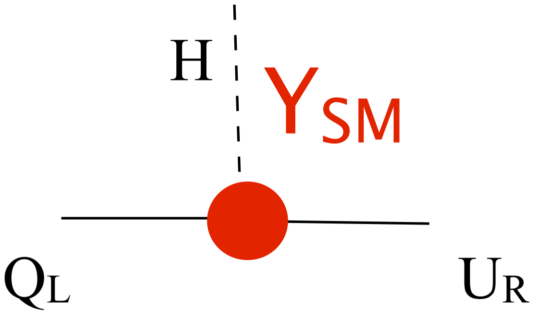
Some good ideas:

D.B. Kaplan-Georgi in the 80's proposed a light SM scalar because being a (quasi) goldstone boson: ***composite Higgs***

Flavour "Partial compositeness" D.B Kaplan 91:

A sort of **"seesaw for quarks"**

(nowadays sometimes justified from extra-dim physics)



$m_q = v\, Y_{SM}$

(D.B Kaplan 91; Redi, Weiler; Contino, Kramer, Son, Sundrum; da Rold, Delauney, Grojean, G. Perez; Contino, Nomura, Pomarol, Agashe, Giudice, Perez, Panico, Redi, Wulzer...)

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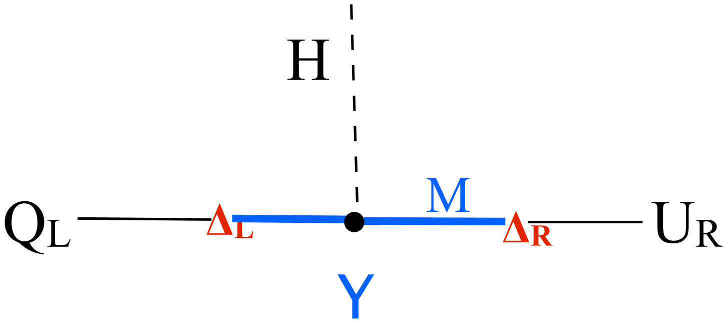

$$Y_{SM} = Y \Delta_L \Delta_R / M^2$$

$$m_q = v\, Y_{SM}$$

(D.B Kaplan 91; Redi, Weiler; Contino, Kramer, Son, Sundrum; da Rold, Delauney, Grojean, G. Perez; Contino, Nomura, Pomarol, Agashe, Giudice, Perez, Panico, Redi, Wulzer...)

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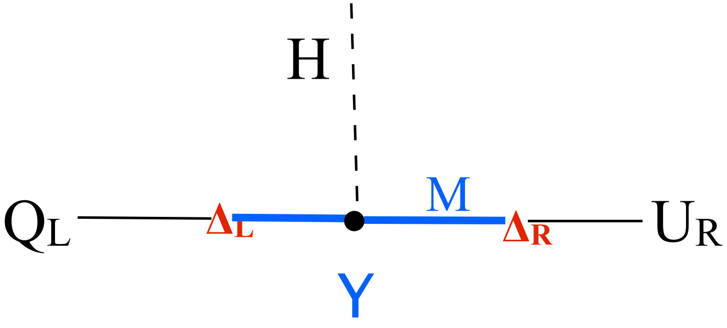

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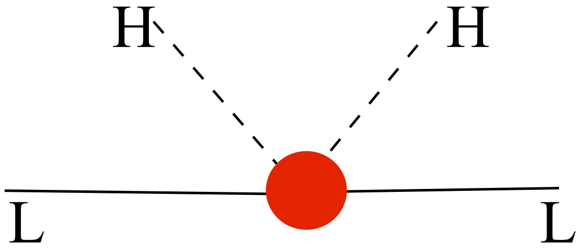
$$m_q = v \, Y_{SM}$$

Neutrino masses:



d=5 Weinberg operator

(D.B Kaplan 91; Redi, Weiler; Contino, Kramer, Son, Sundrum; da Rold, Delauney, Grojean, G. Perez; Contino, Nomura, Pomarol, Agashe, Giudice, Perez, Panico, Redi, Wulzer...)

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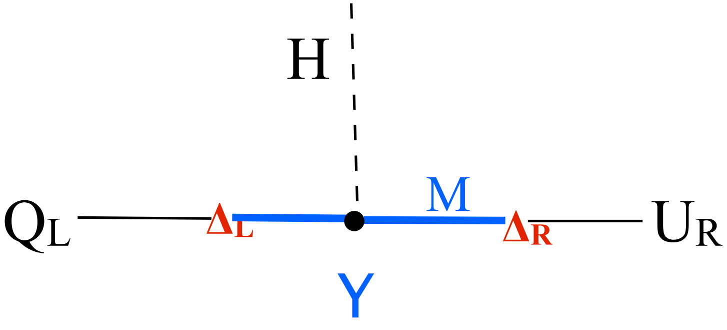

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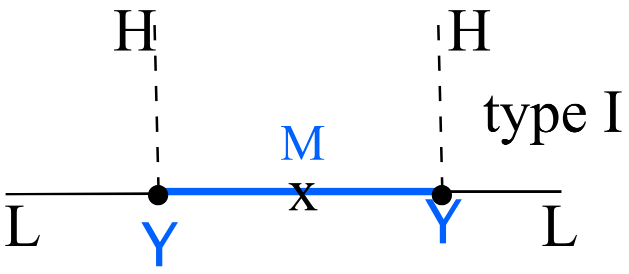
Neutrino masses:



$$m_\nu = Y \, v^2 / M \, Y^T$$

(D.B Kaplan 91; Redi, Weiler; Contino, Kramer, Son, Sundrum; da Rold, Delauney, Grojean, G. Perez; Contino, Nomura, Pomarol, Agashe, Giudice, Perez, Panico, Redi, Wulzer...)

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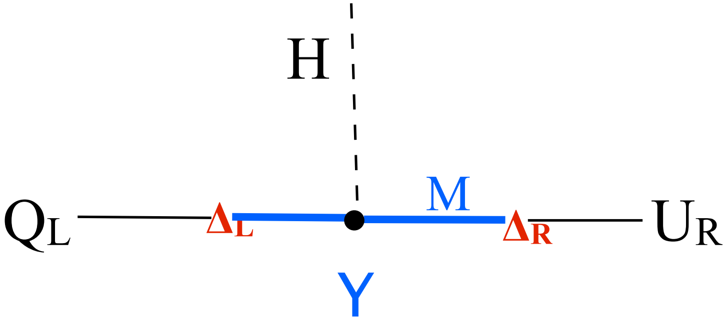

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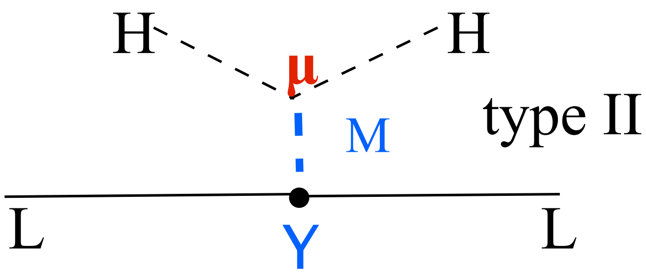
Neutrino masses:



$$m_\nu = Y\, \mu\, v^2/M^2$$

(D.B Kaplan 91; Redi, Weiler; Contino, Kramer, Son, Sundrum; da Rold, Delauney, Grojean, G. Perez; Contino, Nomura, Pomarol, Agashe, Giudice, Perez, Panico, Redi, Wulzer...)

In other BSM Yukawas do correspond to dynamical fields:

Discrete symmetry ideas:

The Yukawas are indeed explained in terms of dynamical fields.

In spite of θ_{13} not very small, some activity.
e.g. combine generalized CP (Bernabeu, Branco, Gronau 80s) with Z_2 :
maximal θ_{23}, strong constraints on values of CP phases
(Feruglio, Hagedorn and Ziegler 2013; Holthausen, Lindner and Schmidt 2013; Girardi, Meroni, Petcov 2013)

- Discrete approaches do not relate mixing to spectrum
- Difficulties to consider both quarks and leptons

Instead of inventing an ad-hoc symmetry group,

why not use the continuous flavour group

suggested by the SM itself?

We have realized that the different pattern for

quarks versus leptons

may be a simple consequence of the

continuous flavour group of the SM (+ seesaw)

(Alonso, Gavela, D.Hernandez, Merlo, Rigolin)

(Alonso, Gavela, Isidori, Maiani)

We have realized that the different pattern for

quarks versus leptons

may be a simple consequence of the

continuous flavour group of the SM (+ seesaw)

Our guideline is to use:

- maximal symmetry
- minimal field content

(Alonso, Gavela, D.Hernandez, Merlo, Rigolin)

(Alonso, Gavela, Isidori, Maiani)

Global flavour symmetry of the SM

* QCD has a global -chiral- symmetry in the limit of massless quarks. For n generations:

$$\mathcal{L}_{QCD}^{\text{fermions}} = \bar{\Psi}(i\not{D} - m)\Psi \rightarrow \bar{\Psi} i\not{D}\Psi = \overline{\Psi_L} i\not{D}\Psi_L + \overline{\Psi_R} i\not{D}\Psi_R$$

$$SU(n)_L \times SU(n)_R \times U(1)'s$$

* In the SM, fermion masses and mixings result from Yukawa couplings. For massless quarks, the SM has a global flavour symmetry:

Quarks

$$\mathscr{L}_{\text{SM}}^{\text{fermions}} = i \sum_{\psi=Q_L}^{D_R} \overline{\psi} \not{D} \psi$$

$$G_{\text{flavour}} = U(n)_{Q_L} \times U(n)_{U_R} \times U(n)_{D_R}$$

[Georgi, Chivukula, 1987]

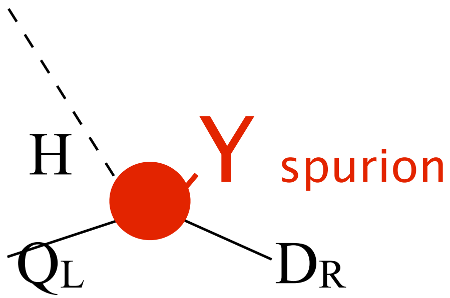
This continuous symmetry of the SM

$$G_{\text{flavour}} = U(n)_{Q_L} \times U(n)_{U_R} \times U(n)_{D_R}$$

is phenomenologically very successful and

at the basis of Minimal Flavour Violation

in which the Yukawa couplings are only spurions



D'Ambrosio+Giudice+Isidori+Strumia;
Cirigliano+Isidori+Grinstein+Wise

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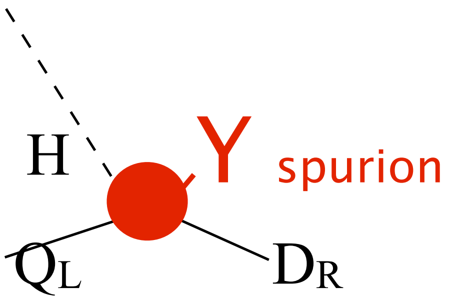


$$\frac{Y_{\alpha\beta}^{+}\,Y_{\delta\gamma}}{\Lambda_f^2}\;\bar{Q}_\alpha\,\gamma_\mu Q_\beta\,\bar{Q}_\gamma\,\gamma^\mu\,Q_\delta$$

D’Ambrosio+Giudice+Isidori+Strumia;
Cirigliano+Isidori+Grinstein+Wise

One step further

(Alonso, Gavela, D.Hernandez, Merlo, Rigolin, 2012 -2013)

(Alonso, Gavela, Isidori, Maiani, 2013)

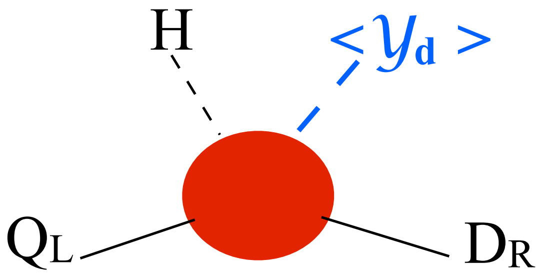
Quarks

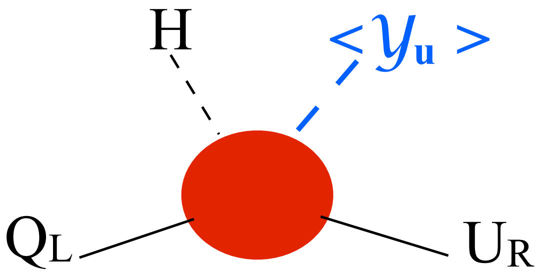
For this talk:

each Y_{SM} -- >one single field $\mathcal{Y}$

$$Y_{SM} \sim \frac{< \mathcal{Y} >}{\Lambda_f}$$

quarks:





Anselm+Berezhiani 96; Berezhiani+Rossi 01... Alonso+Gavela+Merlo+Rigolin 11...

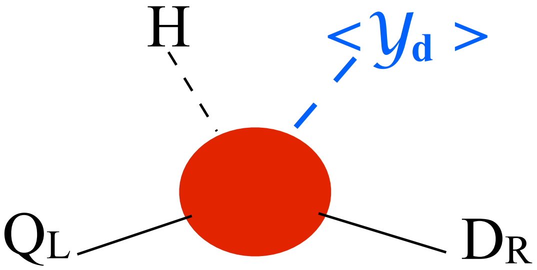
$$G_{flavour} = SU(3)_{Q_L} \times SU(3)_{U_R} \times SU(3)_{D_R} \; ....$$

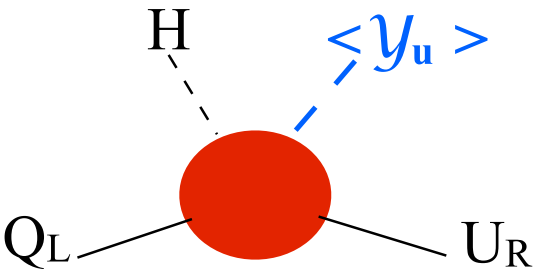
For this talk:

each Y_{SM} -- >one single field $\mathcal{Y}$

$$Y_{SM} \sim \frac{<\mathcal{Y}>}{\Lambda_f}$$

quarks:



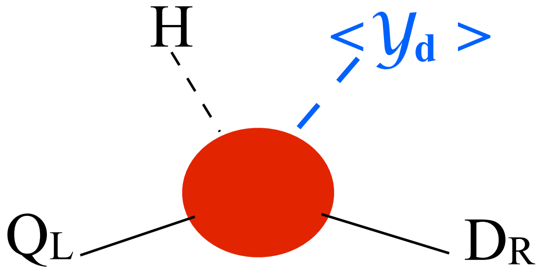


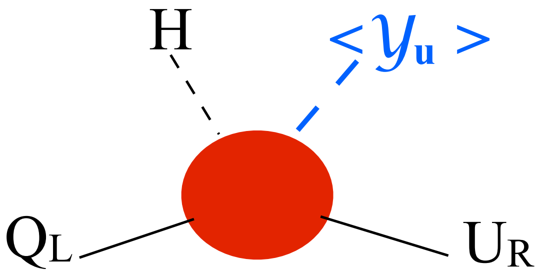
$\mathcal{Y}_d \sim (3,1,\bar{3})$ “bifundamentals” $\mathcal{Y}_u \sim (3,\bar{3},1)$

$$G_{flavour} = SU(3)_{Q_L} \times SU(3)_{U_R} \times SU(3)_{D_R} \;\; ....$$

$$G_{\text{flavour}} = SU(3)_{Q_L} \times SU(3)_{U_R} \times SU(3)_{D_R} \quad \ldots.$$

$\mathcal{Y}_d \sim (3,1,\bar{3})$ $\mathcal{Y}_u \sim (3,\bar{3},1)$

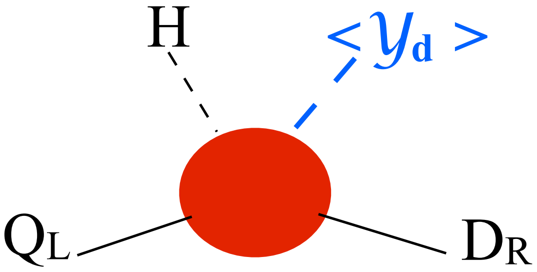


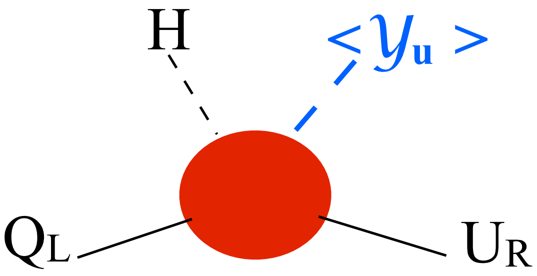


¿$V(\mathcal{Y}_d, \mathcal{Y}_u)$?

$G_{\text{flavour}} = SU(3)_{Q_L} \times SU(3)_{U_R} \times SU(3)_{D_R}$ ....

$\mathcal{Y}_d \sim (3,1,\bar{3})$ $\mathcal{Y}_u \sim (3,\bar{3},1)$





*** Does the minimum of the scalar potential justify the observed masses and mixings?**

$$\mathcal{Y}_d \sim (3, \bar{3}, 1) \qquad \mathcal{Y}_u \sim (3, 1, \bar{3})$$

$$\boxed{\frac{<\mathcal{Y}_d>}{\Lambda_f} = Y_D = V_{CKM} \begin{pmatrix} y_d & 0 & 0 \\ 0 & y_s & 0 \\ 0 & 0 & y_b \end{pmatrix}} \, , \quad \boxed{\frac{<\mathcal{Y}_u>}{\Lambda_f} = Y_U = \begin{pmatrix} y_u & 0 & 0 \\ 0 & y_c & 0 \\ 0 & 0 & y_t \end{pmatrix}} \, .$$

$$\mathbf{V}(\mathcal{Y}_{\mathbf{d}},\ \mathcal{Y}_{\mathbf{u}})$$

* Invariant under the SM gauge symmetry

* Invariant under its global flavour symmetry G_{flavour}

$$G_{\text{flavour}} = U(3)_{Q_L} \times U(3)_{U_R} \times U(3)_{D_R}$$

$$\mathbf{V}(\mathcal{Y}_d, \mathcal{Y}_u)$$

* Invariant under the SM gauge symmetry

* Invariant under its global flavour symmetry **G_{flavour}**

$$G_{\text{flavour}} = U(3)_{Q_L} \times U(3)_{U_R} \times U(3)_{D_R}$$

There are as many independent invariants I as physical variables

$$\mathbf{V}(\mathcal{Y}_d, \mathcal{Y}_u) = \mathbf{V}(\mathbf{I}(\mathcal{Y}_d, \mathcal{Y}_u))$$

Minimization

a variational principle fixes the vevs of the Fields

$$\delta V = 0$$

$$\sum_j \frac{\partial I_j}{\partial y_i} \frac{\partial V}{\partial I_j} \equiv J_{ij} \frac{\partial V}{\partial I_j} = 0\,,$$

masses, mixing angles etc.

This is an homogenous linear equation;
if the rank of the Jacobian $J_{ij} = \partial I_j / \partial y_i$, is:

Maximum:
then the only solution is: $\frac{\partial V}{\partial I_j} = 0\,,$

Less than Maximum:
then the number of equations reduces to a number equal to the rank

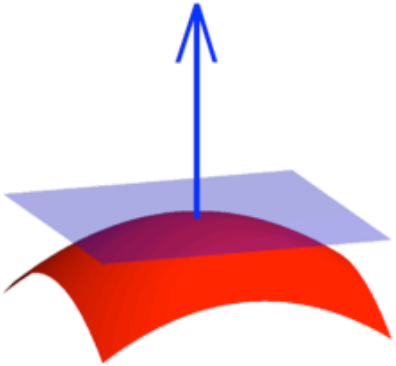
Boundaries

for a reduced rank of the Jacobian,

$\det(J) = 0$

there exists (at least) a direction δy_i for which a variation of the field variables does not vary the invariants

$$\delta I_j = \sum_i \frac{\partial I_j}{\partial y_i} \delta y_i = 0$$



that is a Boundary of the *I-manifold*

[Cabibbo, Maiani,1969]

Boundaries Exhibit Unbroken Symmetry [Michel, Radicati, 1969]

(maximal subgroups)

quark case

Bi-fundamental Flavour Fields

For quarks: 10 independent invariants (because 6 masses+ 3 angles + 1 phase) that we may choose as

$$I_U = \mathrm{Tr}\left[\mathcal{Y}_U \mathcal{Y}_U^\dagger\right] , \qquad I_D = \mathrm{Tr}\left[\mathcal{Y}_D \mathcal{Y}_D^\dagger\right] ,$$

$$I_{U^2} = \mathrm{Tr}\left[\left(\mathcal{Y}_U \mathcal{Y}_U^\dagger\right)^2\right] , \qquad I_{D^2} = \mathrm{Tr}\left[\left(\mathcal{Y}_D \mathcal{Y}_D^\dagger\right)^2\right] ,$$

$$I_{U^3} = \mathrm{Tr}\left[\left(\mathcal{Y}_U \mathcal{Y}_U^\dagger\right)^3\right] , \qquad I_{D^3} = \mathrm{Tr}\left[\left(\mathcal{Y}_D \mathcal{Y}_D^\dagger\right)^3\right] ,$$

$$I_{U,D} = \mathrm{Tr}\left[\mathcal{Y}_U \mathcal{Y}_U^\dagger \mathcal{Y}_D \mathcal{Y}_D^\dagger\right] , \qquad I_{U,D^2} = \mathrm{Tr}\left[\mathcal{Y}_U \mathcal{Y}_U^\dagger \left(\mathcal{Y}_D \mathcal{Y}_D^\dagger\right)^2\right] ,$$

$$I_{U^2,D} = \mathrm{Tr}\left[\mathcal{Y}_U \mathcal{Y}_U^\dagger \left(\mathcal{Y}_D \mathcal{Y}_D^\dagger\right)^2\right] , \qquad I_{(U,D)^2} = \mathrm{Tr}\left[\left(\mathcal{Y}_U \mathcal{Y}_U^\dagger \mathcal{Y}_D \mathcal{Y}_D^\dagger\right)^2\right] .$$

[Feldmann, Jung, Mannel;
Jenkins, Manohar]

quark case

Bi-fundamental Flavour Fields

$$\mathrm{Tr}[\mathbf{y}_U \mathbf{y}_U] = \sum y_\alpha^2$$

$$I_U = \mathrm{Tr}\left[\mathcal{Y}_U \mathcal{Y}_U^\dagger\right], \qquad I_D = \mathrm{Tr}\left[\mathcal{Y}_D \mathcal{Y}_D^\dagger\right],$$

$$I_{U^2} = \mathrm{Tr}\left[\left(\mathcal{Y}_U \mathcal{Y}_U^\dagger\right)^2\right], \qquad I_{D^2} = \mathrm{Tr}\left[\left(\mathcal{Y}_D \mathcal{Y}_D^\dagger\right)^2\right],$$

$$I_{U^3} = \mathrm{Tr}\left[\left(\mathcal{Y}_U \mathcal{Y}_U^\dagger\right)^3\right], \qquad I_{D^3} = \mathrm{Tr}\left[\left(\mathcal{Y}_D \mathcal{Y}_D^\dagger\right)^3\right],$$

only masses

$$I_{U,D} = \mathrm{Tr}\left[\mathcal{Y}_U \mathcal{Y}_U^\dagger \mathcal{Y}_D \mathcal{Y}_D^\dagger\right], \qquad I_{U,D^2} = \mathrm{Tr}\left[\mathcal{Y}_U \mathcal{Y}_U^\dagger \left(\mathcal{Y}_D \mathcal{Y}_D^\dagger\right)^2\right],$$

$$I_{U^2,D} = \mathrm{Tr}\left[\mathcal{Y}_U \mathcal{Y}_U^\dagger \left(\mathcal{Y}_D \mathcal{Y}_D^\dagger\right)^2\right], \qquad I_{(U,D)^2} = \mathrm{Tr}\left[\left(\mathcal{Y}_U \mathcal{Y}_U^\dagger \mathcal{Y}_D \mathcal{Y}_D^\dagger\right)^2\right].$$

masses and mixing

[Feldmann, Jung, Mannel; Jenkins, Manohar
Alonso. Gavela. Isidori. Maiani 20131

Jacobian Analysis: Mixing

$$\begin{aligned}\det\left(J_{UD}\right) = &\left(y_u^2 - y_t^2\right)\left(y_t^2 - y_c^2\right)\left(y_c^2 - y_u^2\right)\\ &\left(y_d^2 - y_b^2\right)\left(y_b^2 - y_s^2\right)\left(y_s^2 - y_d^2\right)\\ &\times |V_{ud}||V_{us}||V_{cd}||V_{cs}|\end{aligned}$$

the rank is reduced the most for:

V_{CKM}= PERMUTATION

no mixing: reordering of states

(Alonso, Gavela, Isidori, Maiani 2013)

Quark Natural Flavour Pattern

Summarizing, a possible and natural breaking pattern arises:

$G_{\text{flavour (quarks)}}$: $U(3)^3 \rightarrow U(2)^3 \times U(1)$

giving a hierarchical mass spectrum **without mixing**

$$< \mathcal{Y}_{\mathrm{D}} > = \Lambda_f \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & y_b \end{pmatrix} , \quad < \mathcal{Y}_{\mathrm{U}} > = \Lambda_f \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & y_t \end{pmatrix} ,$$

a good approximation to the observed Yukawas to order $(\lambda_C)^2$

And what happens for leptons ?

Any difference with Majorana neutrinos?

Global flavour symmetry of the SM + seesaw

* In the SM, for quarks the maximal global symmetry in the limit of massless quarks was:

$$\mathscr{L}^{\text{quarks}}_{\text{SM}} = i \sum_{\psi=Q_L}^{D_R} \overline{\psi} \not{D} \psi \qquad G_{\text{flavour}} = U(n)_{Q_L} \times U(n)_{U_R} \times U(n)_{D_R}$$

* In SM +type I seesaw, for leptons

$$\mathcal{L} = \mathcal{L}_{SM} + i\overline{N_R}\not{\partial}N_R - \left[\overline{N_R}Y_N\tilde{\phi}^\dagger \ell_L + \frac{1}{2}\overline{N_R}M{N_R}^c + h.c.\right]$$

the maximal leptonic global symmetry in the limit of massless light leptons is $U(n)_L \times U(n)_{E_R} \times O(n)_{N_R}$

-> degenerate heavy neutrinos

Bi-fundamental Flavour Fields

Physical parameters
=Independent Invariants

Very direct results using the bi-unitary parametrization:

$$Y_\nu = \langle \frac{\mathcal{Y}_\nu}{\Lambda_f} \rangle = \mathcal{U}_L \mathbf{y}_\nu \mathcal{U}_R \,, \qquad Y_E = \langle \frac{\mathcal{Y}_E}{\Lambda_f} \rangle = \mathbf{y}_E$$

$$\mathcal{U}_L \mathcal{U}_L^\dagger = 1 \,, \quad \mathcal{U}_R \mathcal{U}_R^\dagger = 1 \,,$$

➡ * $m_{e,\mu,\tau} = v\,\mathbf{y}_E$

*But the relation of $\mathcal{Y}_\nu$ with light neutrino masses is through:

$$m_\nu = Y \frac{v^2}{M} Y^T$$

Bi-fundamental Flavour Fields

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=Independent Invariants

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$$\mathbf{Y}_\nu = \frac{<\mathcal{Y}_\nu>}{\Lambda_f} = \mathcal{U}_L \mathbf{y}_\nu \mathcal{U}_R\,, \qquad \mathbf{Y}_E = \frac{<\mathcal{Y}_E>}{\Lambda_f} = \mathbf{y}_E$$

$$\mathcal{U}_L \mathcal{U}_L^\dagger = 1\,, \quad \mathcal{U}_R \mathcal{U}_R^\dagger = 1\,,$$

→ * $m_{e,\mu,\tau} = v\, \mathbf{y}_E$

*But the relation of $\mathcal{Y}_\nu$ with light neutrino masses is through:

→ $$U_{PMNS}\, \mathbf{m}_\nu\, U_{PMNS}^T = \frac{v^2}{2M} \mathcal{U}_L\, \mathbf{y}_\nu \mathcal{U}_R \mathcal{U}_R^T\, \mathbf{y}_\nu \mathcal{U}_L^T\,,$$

Bi-fundamental Flavour Fields

Physical parameters
=Independent Invariants

Very direct results using the bi-unitary parametrization:

$$\mathbf{Y}_\nu = < \frac{\mathcal{Y}_\nu}{\Lambda_f} > = \mathcal{U}_L \mathbf{y}_\nu \mathcal{U}_R \,, \qquad \mathbf{Y}_E = < \frac{\mathcal{Y}_E}{\Lambda_f} > = \mathbf{y}_E$$

$$\mathcal{U}_L \mathcal{U}_L^\dagger = 1 \,, \quad \mathcal{U}_R \mathcal{U}_R^\dagger = 1 \,,$$

→ * $m_{e,\mu,\tau} = v\, \mathbf{y}_E$

*But the relation of $\mathcal{Y}_\nu$ with light neutrino masses is through:

$\mathcal{U}_R$ is relevant for leptons

→ $$U_{PMNS}\, \mathbf{m}_\nu \, U_{PMNS}^T = \frac{v^2}{2M} \mathcal{U}_L \, \mathbf{y}_\nu \, \mathcal{U}_R \mathcal{U}_R^T \, \mathbf{y}_\nu \, \mathcal{U}_L^T \,,$$

* For instance for two generations: $O(2)_{NR}$

e.g. two families

$$m_\nu \sim Y_\nu \frac{v^2}{M} Y_\nu{}^T = y_1\, y_2 \frac{v^2}{M} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\longrightarrow \left\{ \begin{array}{l} U_{PMNS} = \boxed{\frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} e^{i\pi/4} & 0 \\ 0 & e^{-i\pi/4} \end{pmatrix}} \\ \\ \textbf{Degenerate neutrino masses} \end{array} \right.$$

Generically, O(2) allows :

- **one mixing angle maximal**
- **one relative Majorana phase of $\pi/2$**
- **two degenerate light neutrinos**

Now for three generations and considering all possible independent invariants

easier using the bi-unitary parametrization as we did for quarks

Number of Physical parameters = number of Independent Invariants

15 invariants for $G_{\text{flavour (leptons)}} = U(3)_L \times U(3)_{E_R} \times O(3)_{N_R}$

Leptons

$$I_E = \mathrm{Tr}\left[\mathcal{Y}_E \mathcal{Y}_E^\dagger\right] , \qquad I_\nu = \mathrm{Tr}\left[\mathcal{Y}_\nu \mathcal{Y}_\nu^\dagger\right] ,$$

$$I_{E^2} = \mathrm{Tr}\left[\left(\mathcal{Y}_E \mathcal{Y}_E^\dagger\right)^2\right] , \qquad I_{\nu^2} = \mathrm{Tr}\left[\left(\mathcal{Y}_\nu \mathcal{Y}_\nu^\dagger\right)^2\right] ,$$

$$I_{E^3} = \mathrm{Tr}\left[\left(\mathcal{Y}_E \mathcal{Y}_E^\dagger\right)^3\right] , \qquad I_{\nu^3} = \mathrm{Tr}\left[\left(\mathcal{Y}_\nu \mathcal{Y}_\nu^\dagger\right)^3\right] ,$$

$$I_L = \mathrm{Tr}\left[\mathcal{Y}_\nu \mathcal{Y}_\nu^\dagger \mathcal{Y}_E \mathcal{Y}_E^\dagger\right] ,$$

$$I_{L^2} = \mathrm{Tr}\left[\mathcal{Y}_\nu \mathcal{Y}_\nu^\dagger \left(\mathcal{Y}_E \mathcal{Y}_E^\dagger\right)^2\right] ,$$

$$I_{L^3} = \mathrm{Tr}\left[\mathcal{Y}_E \mathcal{Y}_E^\dagger \left(\mathcal{Y}_\nu \mathcal{Y}_\nu^\dagger\right)^2\right] ,$$

$$I_{L^4} = \mathrm{Tr}\left[\left(\mathcal{Y}_\nu \mathcal{Y}_\nu^\dagger \mathcal{Y}_E \mathcal{Y}_E^\dagger\right)^2\right] ,$$

U_L and eigenvalues

$$I_R = \mathrm{Tr}\left[\mathcal{Y}_\nu^\dagger \mathcal{Y}_\nu \mathcal{Y}_\nu^T \mathcal{Y}_\nu^*\right] ,$$

$$I_{R^2} = \mathrm{Tr}\left[\left(\mathcal{Y}_\nu^\dagger \mathcal{Y}_\nu\right)^2 \mathcal{Y}_\nu^T \mathcal{Y}_\nu^*\right] ,$$

$$I_{R^3} = \mathrm{Tr}\left[\left(\mathcal{Y}_\nu^\dagger \mathcal{Y}_\nu \mathcal{Y}_\nu^T \mathcal{Y}_\nu^*\right)^2\right] ,$$

U_R and eigenvalues

$$I_{LR} = \mathrm{Tr}\left[\mathcal{Y}_\nu \mathcal{Y}_\nu^T \mathcal{Y}_\nu^* \mathcal{Y}_\nu^\dagger \mathcal{Y}_E \mathcal{Y}_E^\dagger\right] , \qquad I_{RL} = \mathrm{Tr}\left[\mathcal{Y}_\nu \mathcal{Y}_\nu^T \mathcal{Y}_E^* \mathcal{Y}_E^T \mathcal{Y}_\nu^* \mathcal{Y}_\nu^\dagger \mathcal{Y}_E \mathcal{Y}_E^\dagger\right] ,$$

New Invariants wrt Quarks

Number of Physical parameters = number of Independent Invariants

15 invariants for $G_{\text{flavour (leptons)}} = U(3)_L \times U(3)_{E_R} \times O(3)_{N_R}$

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$$I_{L^4} = \mathrm{Tr}\left[\left(\mathcal{Y}_\nu\mathcal{Y}_\nu^\dagger\mathcal{Y}_E\mathcal{Y}_E^\dagger\right)^2\right] ,$$

U_L and eigenvalues

$$I_R = \mathrm{Tr}\left[\mathcal{Y}_\nu^\dagger\mathcal{Y}_\nu\left(\mathcal{Y}_\nu^\dagger\mathcal{Y}_\nu\right)^{\mathrm{T}}\right]$$

$$I_{R^2} = \mathrm{Tr}\left[\left(\mathcal{Y}_\nu^\dagger\mathcal{Y}_\nu\right)^2\mathcal{Y}_\nu^T\mathcal{Y}_\nu^*\right] ,$$

$$I_{R^3} = \mathrm{Tr}\left[\left(\mathcal{Y}_\nu^\dagger\mathcal{Y}_\nu\mathcal{Y}_\nu^T\mathcal{Y}_\nu^*\right)^2\right] ,$$

U_R and eigenvalues

New Invariants wrt Quarks

$$I_{LR} = \mathrm{Tr}\left[\mathcal{Y}_\nu\mathcal{Y}_\nu^T\mathcal{Y}_\nu^*\mathcal{Y}_\nu^\dagger\mathcal{Y}_E\mathcal{Y}_E^\dagger\right] , \qquad I_{RL} = \mathrm{Tr}\left[\mathcal{Y}_\nu\mathcal{Y}_\nu^T\mathcal{Y}_E^*\mathcal{Y}_E^T\mathcal{Y}_\nu^*\mathcal{Y}_\nu^\dagger\mathcal{Y}_E\mathcal{Y}_E^\dagger\right] ,$$

Number of Physical parameters = number of Independent Invariants

15 invariants for $G_{\text{flavour (leptons)}} = U(3)_L \times U(3)_{E_R} \times O(3)_{N_R}$

Leptons

$$I_E = \mathrm{Tr}\left[\mathcal{Y}_E \mathcal{Y}_E^\dagger\right] , \qquad I_\nu = \mathrm{Tr}\left[\mathcal{Y}_\nu \mathcal{Y}_\nu^\dagger\right] ,$$

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$$I_{L^3} = \mathrm{Tr}\left[\mathcal{Y}_E \mathcal{Y}_E^\dagger \left(\mathcal{Y}_\nu \mathcal{Y}_\nu^\dagger\right)^2\right] ,$$

$$I_{L^4} = \mathrm{Tr}\left[\left(\mathcal{Y}_\nu \mathcal{Y}_\nu^\dagger \mathcal{Y}_E \mathcal{Y}_E^\dagger\right)^2\right] ,$$

U_L and eigenvalues

$$\mathrm{Tr}(\mathbf{y}_\nu^2 \mathcal{U}_R \mathcal{U}_R^T \mathbf{y}_\nu^2 \mathcal{U}_R^* \mathcal{U}_R^\dagger)$$

$$I_{R^2} = \mathrm{Tr}\left[\left(\mathcal{Y}_\nu^\dagger \mathcal{Y}_\nu\right)^2 \mathcal{Y}_\nu^T \mathcal{Y}_\nu^*\right] ,$$

$$I_{R^3} = \mathrm{Tr}\left[\left(\mathcal{Y}_\nu^\dagger \mathcal{Y}_\nu \mathcal{Y}_\nu^T \mathcal{Y}_\nu^*\right)^2\right] ,$$

U_R and eigenvalues

New Invariants wrt Quarks

$$I_{LR} = \mathrm{Tr}\left[\mathcal{Y}_\nu \mathcal{Y}_\nu^T \mathcal{Y}_\nu^* \mathcal{Y}_\nu^\dagger \mathcal{Y}_E \mathcal{Y}_E^\dagger\right] , \qquad I_{RL} = \mathrm{Tr}\left[\mathcal{Y}_\nu \mathcal{Y}_\nu^T \mathcal{Y}_E^* \mathcal{Y}_E^T \mathcal{Y}_\nu^* \mathcal{Y}_\nu^\dagger \mathcal{Y}_E \mathcal{Y}_E^\dagger\right] ,$$

Jacobian Analysis: Mixing

$$\det\left(J_{\mathcal{U}_L}\right) = \left(y_{\nu_1}^2 - y_{\nu_2}^2\right)\left(y_{\nu_2}^2 - y_{\nu_3}^2\right)\left(y_{\nu_3}^2 - y_{\nu_1}^2\right)$$
$$\left(y_e^2 - y_\mu^2\right)\left(y_\mu^2 - y_\tau^2\right)\left(y_\tau^2 - y_e^2\right)|\mathcal{U}_L^{e1}||\mathcal{U}_L^{e2}||\mathcal{U}_L^{\mu 1}||\mathcal{U}_L^{\mu 2}|\,.$$

same as for V_{CKM}

O(3) vs U(3)

$$\det J_{\mathcal{U}_R} = \left(y_{\nu_1}^2 - y_{\nu_2}^2\right)^3\left(y_{\nu_2}^2 - y_{\nu_3}^2\right)^3\left(y_{\nu_3}^2 - y_{\nu_1}^2\right)^3$$
$$\times\,|\left(\mathcal{U}_R\mathcal{U}_R^T\right)_{11}||\left(\mathcal{U}_R\mathcal{U}_R^T\right)_{22}||\left(\mathcal{U}_R\mathcal{U}_R^T\right)_{12}|$$

the rank is reduced the most for $\boxed{\mathcal{U}_R\mathcal{U}_R^T \text{ being a permutation}}$

Jacobian Analysis: Mixing

...which is now **not** trivial mixing...

$$\frac{v^2}{M}\begin{pmatrix} y_{\nu_1}^2 & 0 & 0 \\ 0 & 0 & y_{\nu_2}y_{\nu_3} \\ 0 & y_{\nu_2}y_{\nu_3} & 0 \end{pmatrix} = U_{PMNS}\begin{pmatrix} m_{\nu_1} & 0 & 0 \\ 0 & m_{\nu_2} & 0 \\ 0 & 0 & m_{\nu_2} \end{pmatrix} U_{PMNS}^T ,$$

...in fact it allows maximal mixing:

Jacobian Analysis: Mixing

...which is now **not** trivial mixing...

$$\frac{v^2}{M}\begin{pmatrix} y_{\nu_1}^2 & 0 & 0 \\ 0 & 0 & y_{\nu_2}y_{\nu_3} \\ 0 & y_{\nu_2}y_{\nu_3} & 0 \end{pmatrix} = U_{PMNS}\begin{pmatrix} m_{\nu_1} & 0 & 0 \\ 0 & m_{\nu_2} & 0 \\ 0 & 0 & m_{\nu_2} \end{pmatrix} U^T_{PMNS}\,,$$

...in fact it leads to one maximal mixing angle:

$$U_{PMNS} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{i}{\sqrt{2}} \\ 0 & -\frac{1}{\sqrt{2}} & \frac{i}{\sqrt{2}} \end{pmatrix}, \quad m_{\nu2} = m_{\nu3} = \frac{v^2}{M} y_{\nu_2}y_{\nu_3}\,, \quad m_{\nu_1} = \frac{v^2}{M} y_{\nu_1}^2\,.$$

and maximal Majorana phase

Jacobian Analysis: Mixing

...which is now **not** trivial mixing...

$$\frac{v^2}{M}\begin{pmatrix} y_{\nu_1}^2 & 0 & 0 \\ 0 & 0 & y_{\nu_2}y_{\nu_3} \\ 0 & y_{\nu_2}y_{\nu_3} & 0 \end{pmatrix} = U_{PMNS}\begin{pmatrix} m_{\nu_1} & 0 & 0 \\ 0 & m_{\nu_2} & 0 \\ 0 & 0 & m_{\nu_2} \end{pmatrix} U_{PMNS}^T\,,$$

...in fact it leads to one maximal mixing angle:

θ_{23} =45°;

Majorana Phase Pattern (1,1,i)

& at this level mass degeneracy: $m_{\nu 2} = m_{\nu 3}$

related to the O(2) substructure [Alonso, Gavela, D. Hernández, L. Merlo;

[Alonso, Gavela, D. Hernández, L. Merlo, S. Rigolin] if the three neutrinos are quasidegenerate,

$$U_{PMNS}\begin{pmatrix} m_0 & 0 & 0 \\ 0 & m_0 & 0 \\ 0 & 0 & m_0 \end{pmatrix}U^T_{PMNS} = \frac{y_\nu v^2}{M}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

This very simple structure is signaled by the extrema of the potential and

has eigenvalues (1,1,-1) →

3 degenerate light neutrinos
+ a maximal Majorana phase

and is diagonalized by a maximal θ = 45°

Generalization to any seesaw model

the effective Weinberg Operator

$$\bar{\ell}_L \tilde{H} \frac{C^{d=5}}{M} \tilde{H}^T \ell_L^c$$

shall have a flavour structure that breaks $U(3)_L$ to $O(3)$

$$\frac{v^2 \; C^{d=5}}{M} = m_\nu \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

then the results apply to any seesaw model

First conclusion:

* at the same order in which the minimum of the potential

does NOT allow quark mixing,

it allows:

- hierarchical charged leptons
- quasi-degenerate neutrino masses
- one angle of ~45 degrees
- one maximal Majorana phase and the other one trivial

Perturbations can produce a second large angle

if the three neutrinos are quasidegenerate, perturbations:

$$: U_{PMNS} \begin{pmatrix} m_0 & 0 & 0 \\ 0 & m_0 & 0 \\ 0 & 0 & m_0 \end{pmatrix} U_{PMNS}^T = \frac{y_\nu v^2}{M} \begin{pmatrix} 1+\delta+\sigma & \epsilon+\eta & \epsilon-\eta \\ \epsilon+\eta & \delta+\kappa & 1 \\ \epsilon-\eta & 1 & \delta-\kappa \end{pmatrix}$$

produce a second large angle and a perturbative one together with mass splittings

$$\theta_{23} \simeq \pi/4 \;,\; \theta_{12} \text{ large} \;\;,\;\; \theta_{13} \simeq \epsilon$$

Fixed Majorana phases: $(1, 1, i)$

degenerate spectrum

Perturbations can produce a second large angle

if the three neutrinos are quasidegenerate, perturbations:

$$U_{PMNS}\begin{pmatrix} m_0 & 0 & 0 \\ 0 & m_0 & 0 \\ 0 & 0 & m_0 \end{pmatrix} U_{PMNS}^T = \frac{y_\nu v^2}{M}\begin{pmatrix} 1+\delta+\sigma & \epsilon+\eta & \epsilon-\eta \\ \epsilon+\eta & \delta+\kappa & 1 \\ \epsilon-\eta & 1 & \delta-\kappa \end{pmatrix}$$

produce a second large angle and a perturbative one together with mass splittings

$$\theta_{23} \simeq \pi/4 \ , \ \theta_{12} \text{ large} \ , \ \theta_{13} \simeq \epsilon$$

only this vanishes with the perturbations

Fixed Majorana phases: $(1, 1, i)$

this angle does not vanish with vanishing perturbations

degenerate spectrum

Perturbations can produce a second large angle

if the three neutrinos are quasidegenerate, perturbations:

$$U_{PMNS}\begin{pmatrix} m_0 & 0 & 0 \\ 0 & m_0 & 0 \\ 0 & 0 & m_0 \end{pmatrix}U_{PMNS}^T = \frac{y_\nu v^2}{M}\begin{pmatrix} 1+\delta+\sigma & \epsilon+\eta & \epsilon-\eta \\ \epsilon+\eta & \delta+\kappa & 1 \\ \epsilon-\eta & 1 & \delta-\kappa \end{pmatrix}$$

produce a second large angle and a perturbative one together with mass splittings

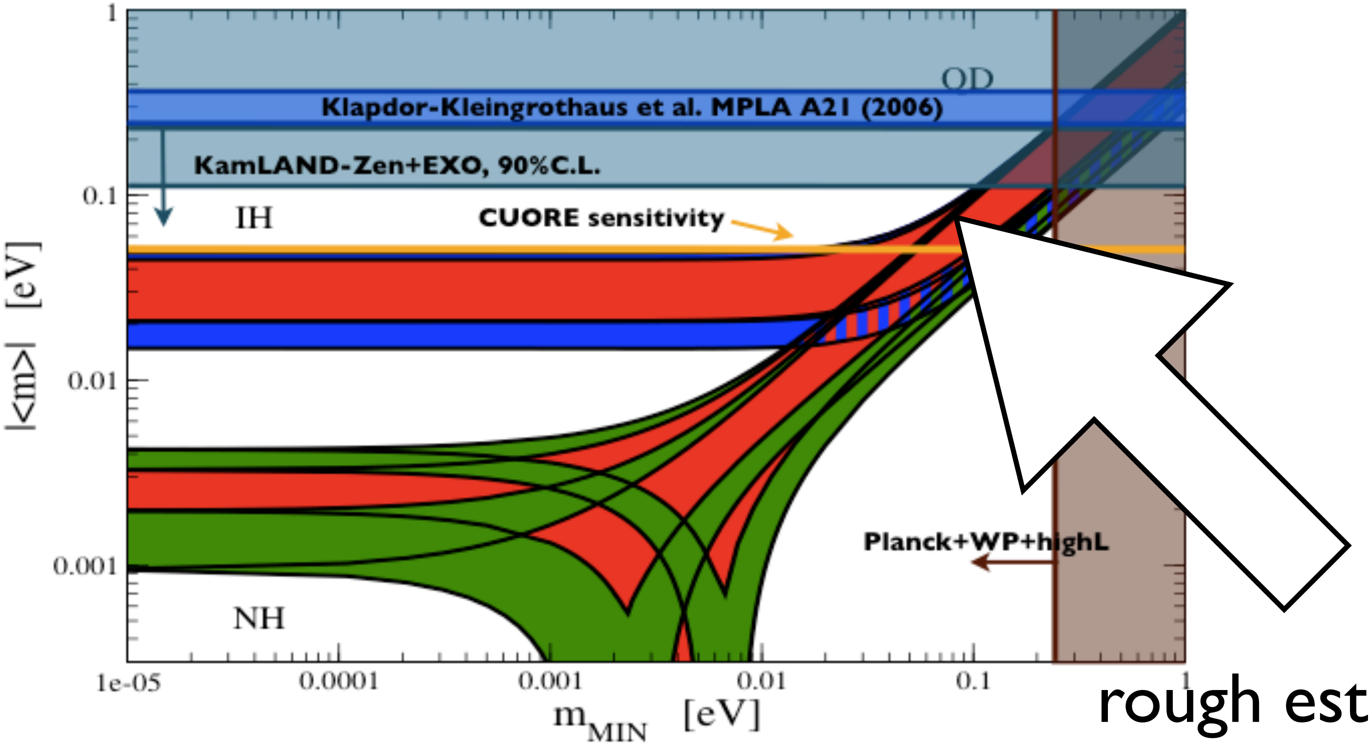
$$\theta_{23} \simeq \pi/4 \ , \ \theta_{12} \text{ large} \ , \ \theta_{13} \simeq \epsilon$$

Fixed Majorana phases: $(1, 1, i)$

~ degenerate spectrum

accommodation of angles requires degenerate spectrum

at reach in future neutrinoless double β exps.!

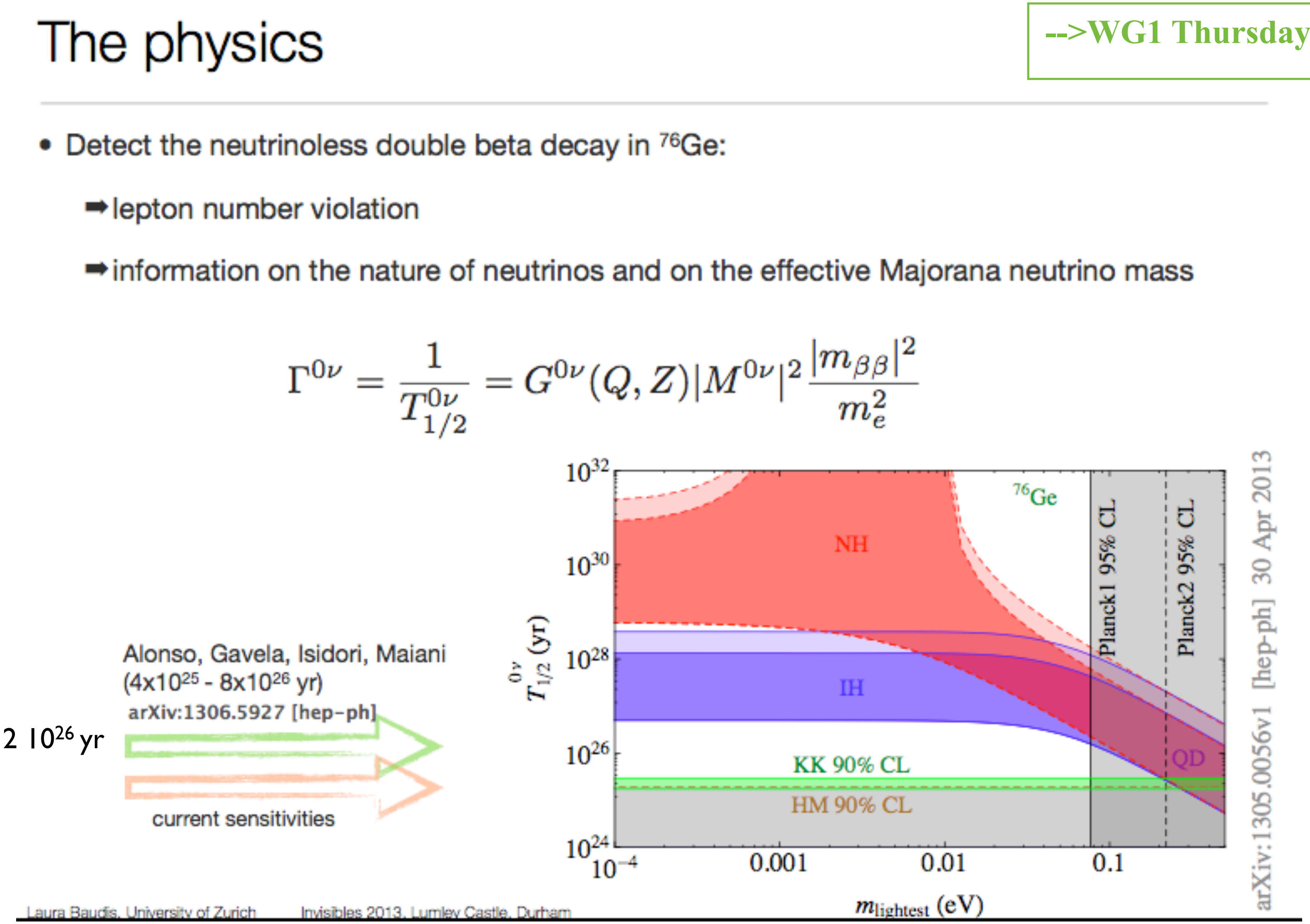


rough estimate

m~0.1eV

Slide from Laura Baudis talk presenting the new Gerda data at Invisibles13 workshop 3 weeks ago

-->WG1 Thursday



The physics

- Detect the neutrinoless double beta decay in ^{76}Ge:
  - ➡lepton number violation
  - ➡information on the nature of neutrinos and on the effective Majorana neutrino mass

$$\Gamma^{0\nu} = \frac{1}{T_{1/2}^{0\nu}} = G^{0\nu}(Q,Z)|M^{0\nu}|^2 \frac{|m_{\beta\beta}|^2}{m_e^2}$$

Alonso, Gavela, Isidori, Maiani
($4x10^{25}$ - $8x10^{26}$ yr)
arXiv:1306.5927 [hep-ph]

~ 2 10^{26} yr

current sensitivities

Laura Baudis, University of Zurich Invisibles 2013, Lumley Castle, Durham

latest from Planck....

$$\sum m_\nu = 0.22 \pm 0.09 \text{ eV}$$

Planck Collaboration: Cosı

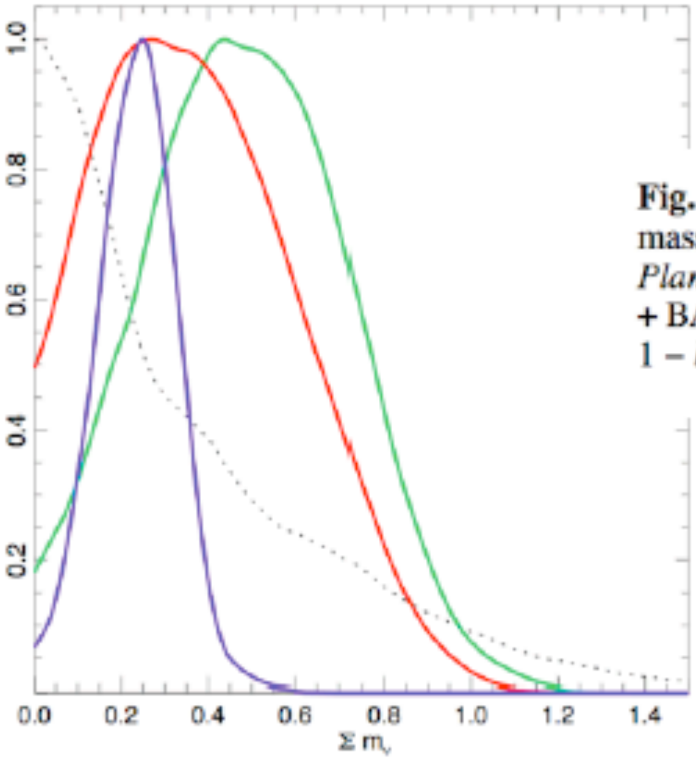


Fig. 12. Cosmological constraints when including neutrino masses $\sum m_\nu$ from: *Planck* CMB data alone (black dotted line); *Planck* CMB + SZ with $1-b$ in [0.7, 1] (red); *Planck* CMB + SZ + BAO with $1-b$ in [0.7, 1] (blue); and *Planck* CMB + SZ with $1-b=0.8$ (green).

Where do the differences in Mixing originated?

in the symmetries of the
Quark and Lepton sectors

$\mathcal{G}^q_\mathcal{F} \sim U(3)^3$ $\mathcal{G}^l_\mathcal{F} \sim U(3)^2 \times O(3)$

for the type I seesaw employed here;

in general $U(n_g)$ vs $O(n_g)$

Where do the differences in Mixing originate?

From the
MAJORANA vs DIRAC nature of fermions

We set the perturbations by hand. Can we predict them also dynamically?

Fundamental Fields

May provide dynamically the perturbations

In the case of quarks they can give
the right corrections:

$$\frac{\mathcal{Y}_U}{\Lambda_f} + \frac{\chi_U^L \chi_U^{R\dagger}}{\Lambda_f^2} \sim \begin{pmatrix} 0 & \sin\theta_c \, y_c & 0 \\ 0 & \cos\theta_c \, y_c & 0 \\ 0 & 0 & y_t \end{pmatrix}$$

[Alonso, Gavela, Merlo, Rigolin]

under study in the lepton sector

Conclusions

* Exciting experimental windows ahead into neutrino(and/or DM) physics: **μ-e conversion** will test SM-singlet fermions in the **2 GeV- 6000 TeV** mass range !

* **Spontaneous flavour symmetry breaking** is very predictive. The **SM+seesaw maximal global flavor symm.**(**U(3)'s and O(3)**) points dynamically to patterns of masses and mixings close to nature, both for quarks and leptons. The differences stem from the Majorana character:

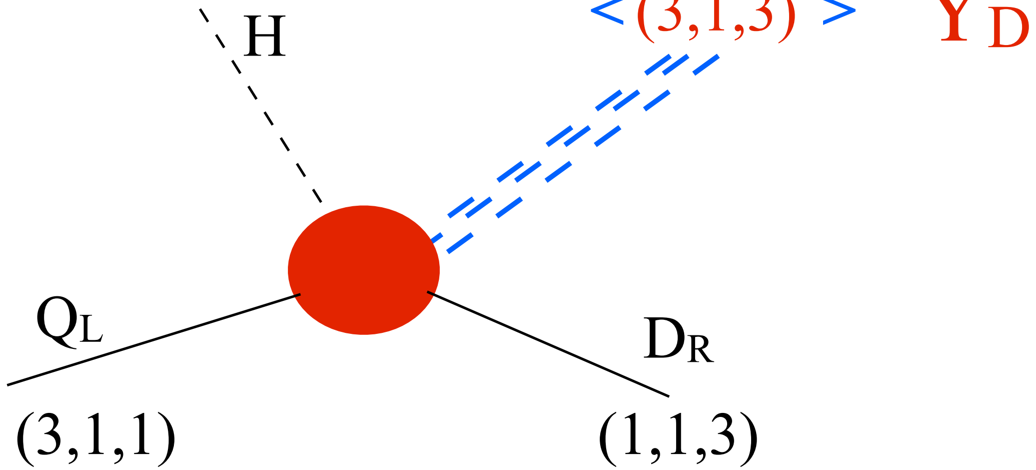
- A correlation between large angles and degenerate ν spectrum emerges, with: i) fixed Majorana phases (1,1,i), ii) θ_{23} =45°, iii) θ_{12} large, θ_{13} small.
- This scenario will be tested in the near future by 0ν2β experiments (m~.1eV).... or cosmology!!!

Back-up slides

Use the flavour symmetry of the SM with masless fermions:

$$G_f = U(3)_{Q_L} \text{ x } U(3)_{U_R} \text{ x } U(3)_{D_R}$$

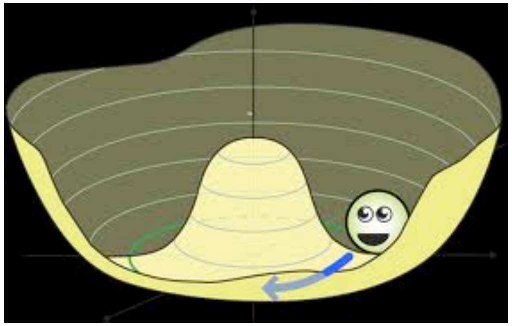
replace Yukawas by fields:



Spontaneous breaking of flavour symmetry dangerous

Flavour Symmetry Breaking

To prevent Goldstone Bosons the symmetry can be Gauged

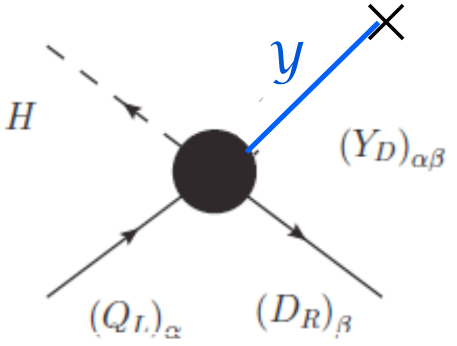


[Grinstein, Redi, Villadoro
Guadagnoli, Mohapatra, Sung
Feldman]

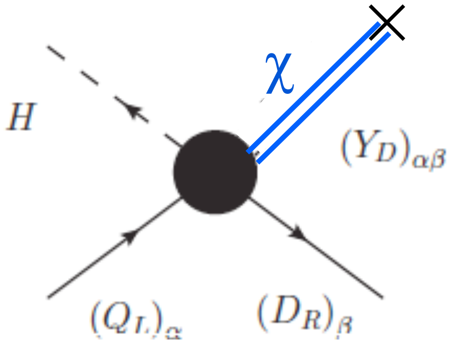
*a good possibility for the other angles :

Yukawas --> add fields in the fundamental of the flavour group

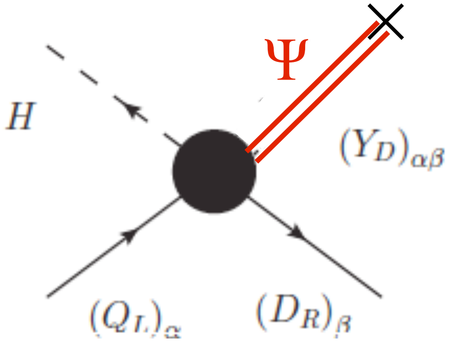
1) Y -- > one single scalar $\mathcal{Y} \sim (3, 1, \bar{3})$



2) Y -- > two scalars $\chi\,\chi^+ \sim (3, 1, \bar{3})$



3) Y -- > two fermions $\bar{\Psi}\Psi \sim (3, 1, 3)$



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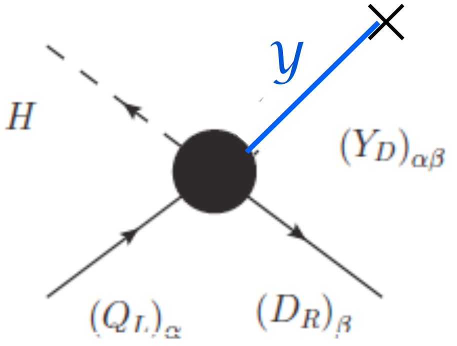
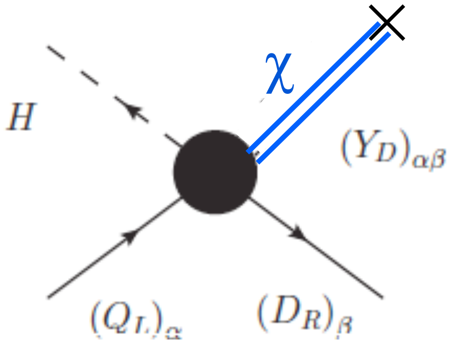


2) Y --> two scalars $\chi\,\chi^{+} \sim (3, 1, \bar{3})$

$\chi \sim (3, 1, 1)$



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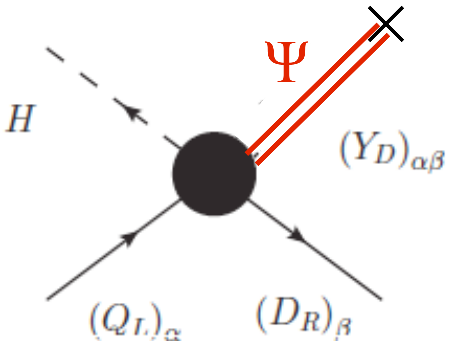
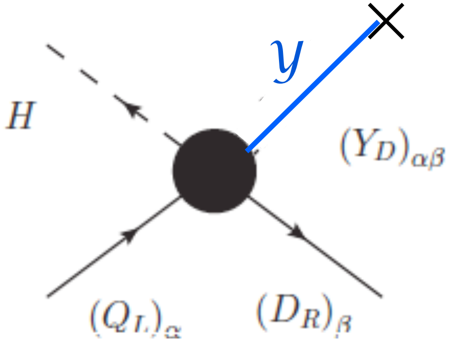


1) Y -- > one single scalar $\mathcal{Y} \sim (3, 1, \bar{3})$

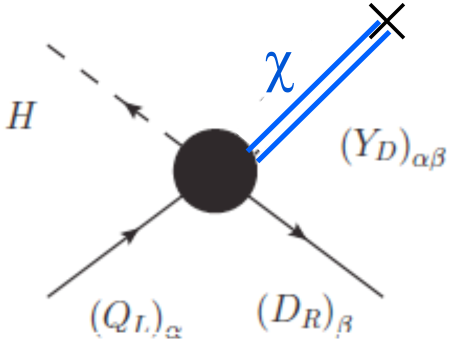
d=5 operator



2) Y -- > two scalars $\chi\,\chi^{+} \sim (3, 1, \bar{3})$

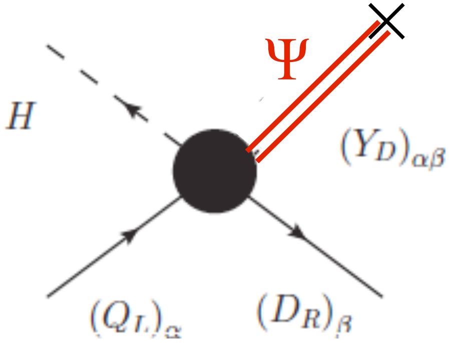
d=6 operator

$\chi \sim (3, 1, 1)$



3) Y -- > two fermions $\bar{\Psi}\Psi \sim (3, 1, 3)$

d=7 operator



i.e. for quarks, a possible path:

*** At leading (renormalizable) order:**

$$Y_u \equiv \frac{<\mathcal{Y}_u>}{\Lambda_f} + \frac{\langle\chi_u^L\rangle\,\langle\chi_u^{R\dagger}\rangle}{\Lambda_f^2} = \begin{pmatrix} 0 & \sin\theta_c\, y_c & 0 \\ 0 & \cos\theta_c\, y_c & 0 \\ 0 & 0 & y_t \end{pmatrix},$$

$$Y_d \equiv \frac{<\mathcal{Y}_d>}{\Lambda_f} + \frac{\langle\chi_d^L\rangle\,\langle\chi_d^{R\dagger}\rangle}{\Lambda_f^2} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & y_s & 0 \\ 0 & 0 & y_b \end{pmatrix}.$$

without unnatural fine-tunings

*** The masses of the first family and the other angles from non-renormalizable terms or other corrections or replicas ?**

....and analogously for leptonic mixing ?

We set the perturbations by hand.
Can we predict them also dynamically?

Fundamental Fields

May provide dynamically the perturbations

In the case of quarks they can give
the right corrections:

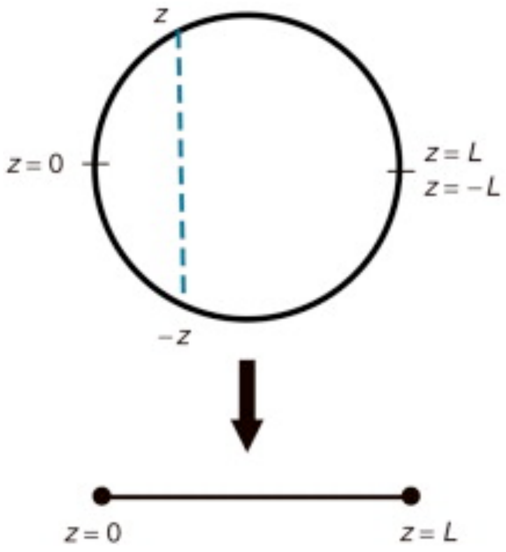
$$\frac{\mathcal{Y}_U}{\Lambda_f} + \frac{\chi_U^L \chi_U^{R\dagger}}{\Lambda_f^2} \sim \begin{pmatrix} 0 & \sin\theta_c\, y_c & 0 \\ 0 & \cos\theta_c\, y_c & 0 \\ 0 & 0 & y_t \end{pmatrix}$$

[Alonso, Gavela, Merlo, Rigolin]

under study in the lepton sector

Boundaries Exhibit Unbroken Symmetry

Extra-Dimensions Example



<u>The smallest boundaries are extremal points of any function</u>

[Michel, Radicati, 1969]

Jacobian Analysis: [40 years ago...]

Breaking of $SU(3) \times SU(3)$ [Cabibbo, Maiani]

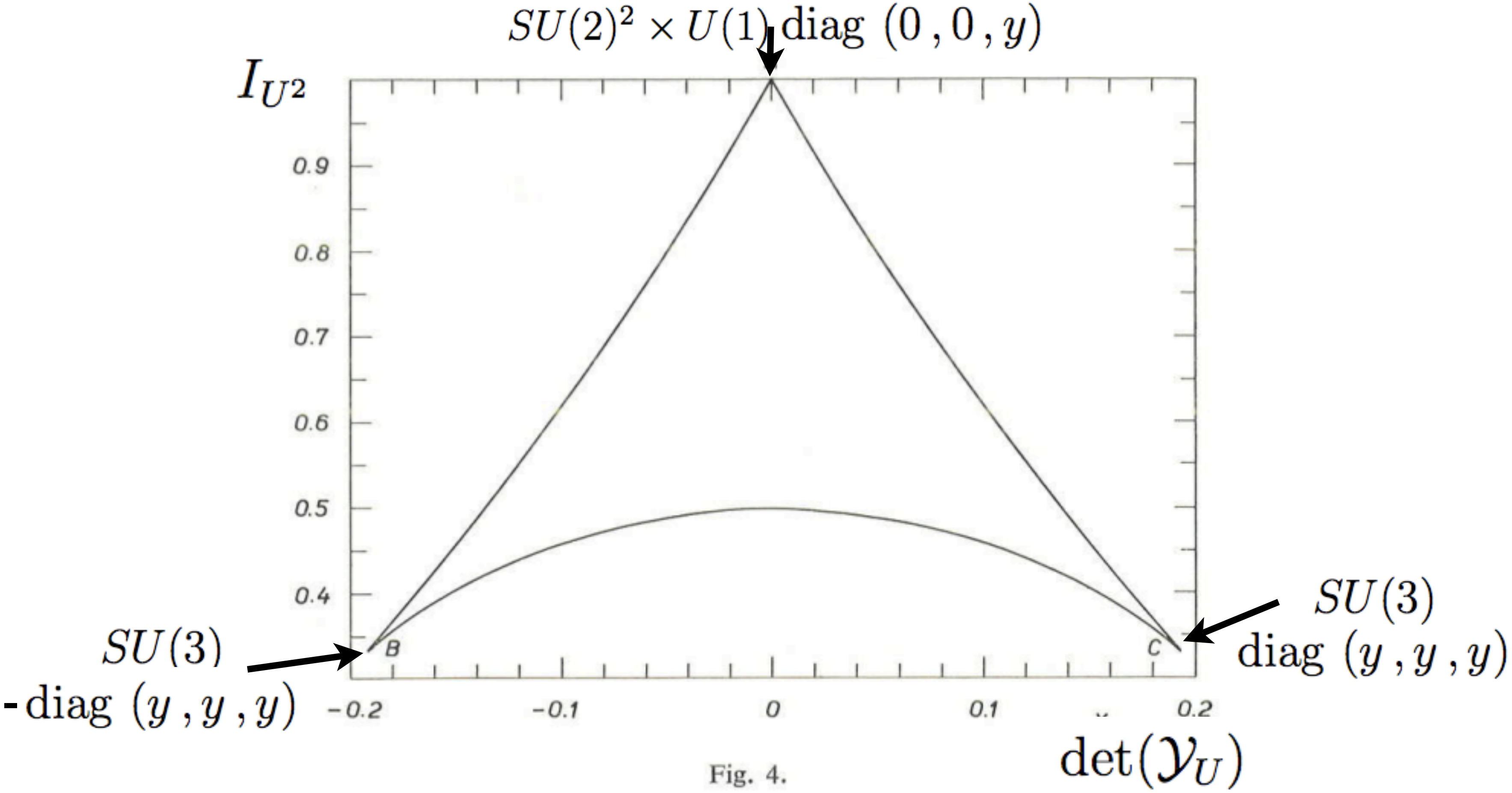
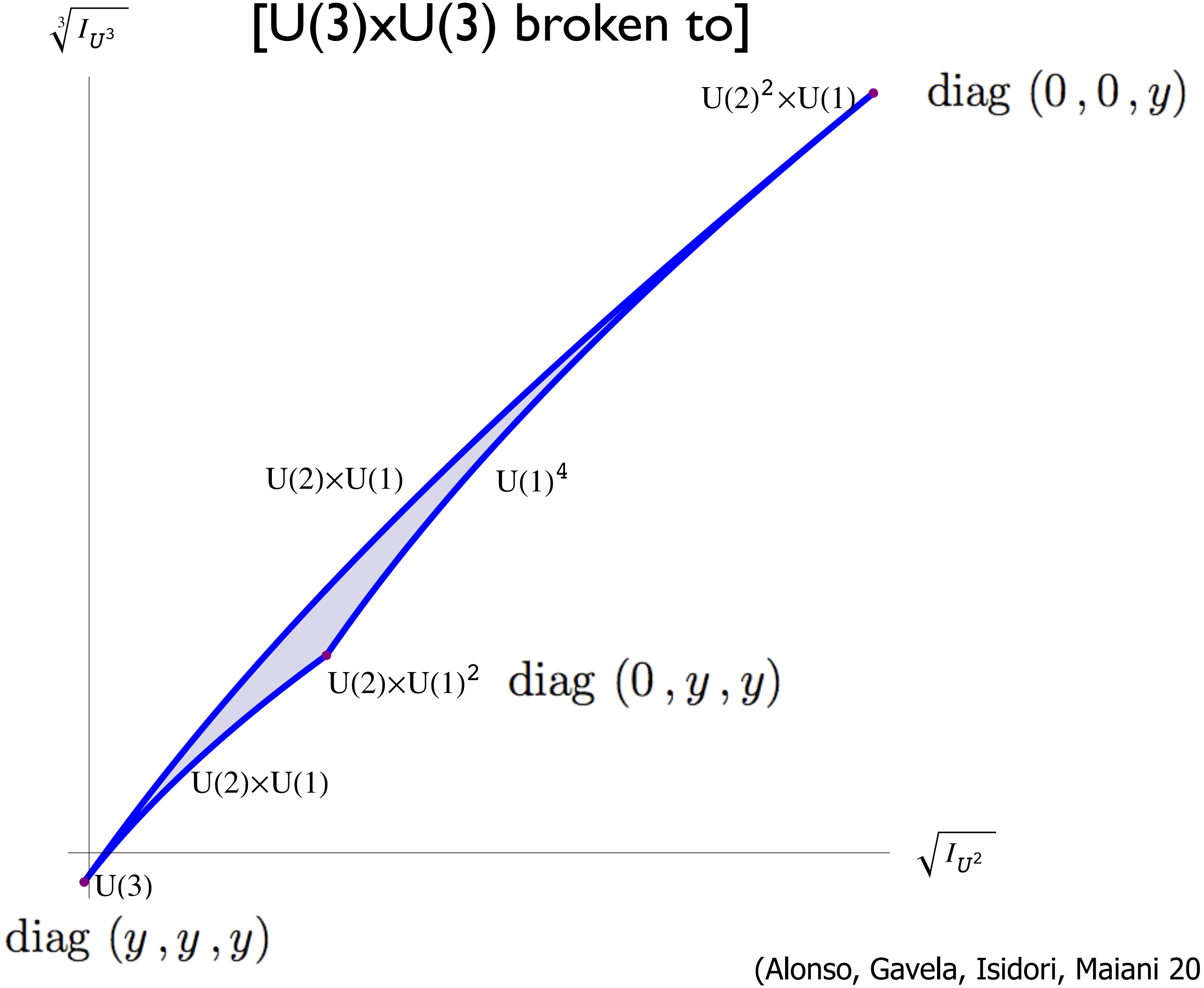


Fig. 4.

Jacobian Analysis: Masses

[U(3)xU(3) broken to]



(Alonso, Gavela, Isidori, Maiani 2013)

Jacobian Analysis: Mixing

$$\begin{aligned}\det\left(J_{UD}\right) = &\left(y_u^2 - y_t^2\right)\left(y_t^2 - y_c^2\right)\left(y_c^2 - y_u^2\right)\\ &\left(y_d^2 - y_b^2\right)\left(y_b^2 - y_s^2\right)\left(y_s^2 - y_d^2\right)\\ &\times |V_{ud}||V_{us}||V_{cd}||V_{cs}|\end{aligned}$$

the rank is reduced the most for:

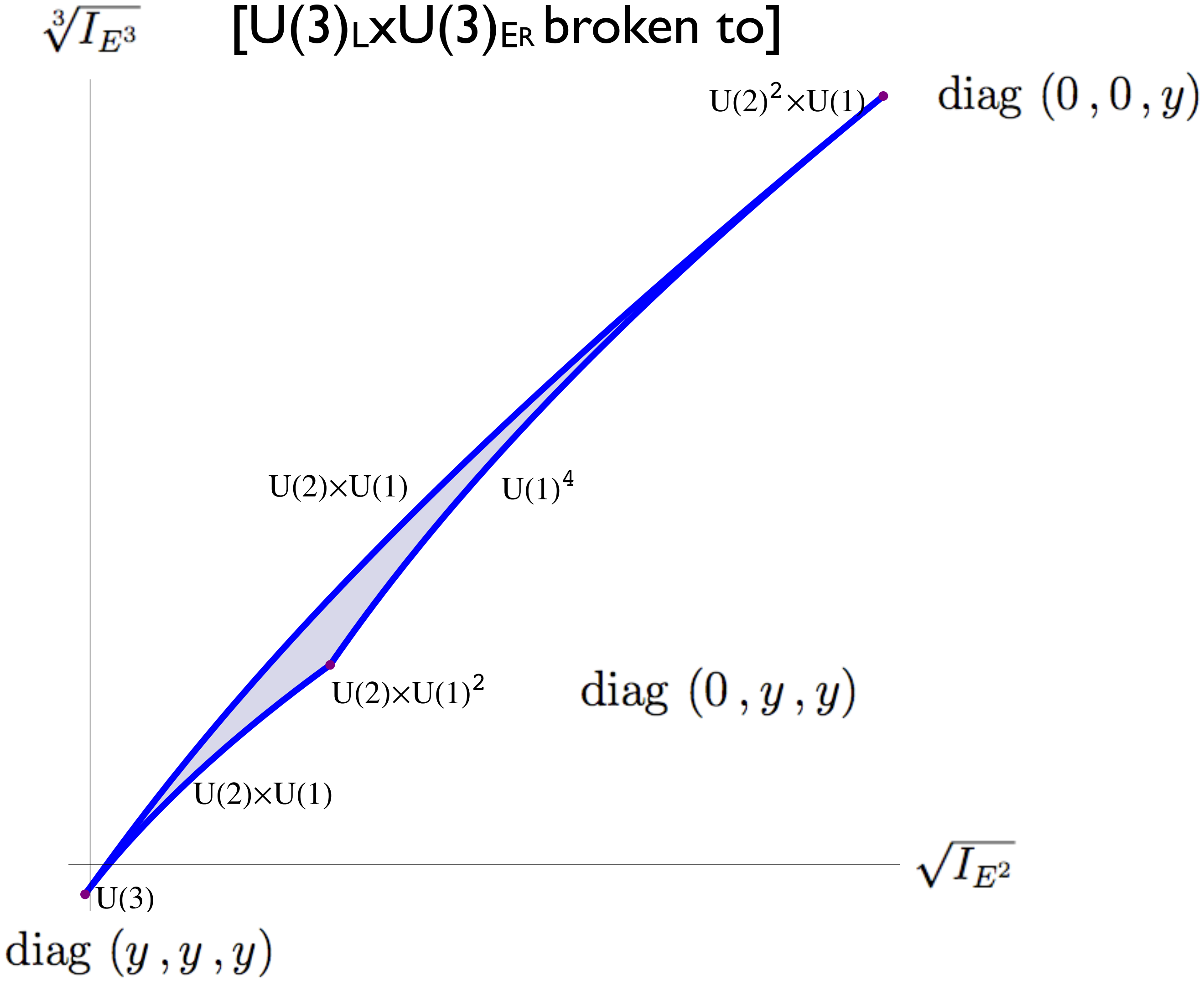
V_{CKM}= PERMUTATION

no mixing: reordering of states

(Alonso, Gavela, Isidori, Maiani 2013)

Jacobian Analysis: Eigenvalues

[U(3)$_L$xU(3)$_{ER}$ broken to]



Renormalizable Potential

Invariants at the Renormalizable Level

$$\boxed{\begin{aligned} I_U &= \mathrm{Tr}\left[\mathcal{Y}_U \mathcal{Y}_U^\dagger\right] , & I_D &= \mathrm{Tr}\left[\mathcal{Y}_D \mathcal{Y}_D^\dagger\right] , \\ I_{U^2} &= \mathrm{Tr}\left[\left(\mathcal{Y}_U \mathcal{Y}_U^\dagger\right)^2\right] , & I_{D^2} &= \mathrm{Tr}\left[\left(\mathcal{Y}_D \mathcal{Y}_D^\dagger\right)^2\right] ,\end{aligned}}$$

$$I_{U^3} = \mathrm{Tr}\left[\left(\mathcal{Y}_U \mathcal{Y}_U^\dagger\right)^3\right] , \qquad I_{D^3} = \mathrm{Tr}\left[\left(\mathcal{Y}_D \mathcal{Y}_D^\dagger\right)^3\right] ,$$

$$\boxed{I_{U,D} = \mathrm{Tr}\left[\mathcal{Y}_U \mathcal{Y}_U^\dagger \mathcal{Y}_D \mathcal{Y}_D^\dagger\right] ,} \qquad I_{U,D^2} = \mathrm{Tr}\left[\mathcal{Y}_U \mathcal{Y}_U^\dagger \left(\mathcal{Y}_D \mathcal{Y}_D^\dagger\right)^2\right] ,$$

$$I_{U^2,D} = \mathrm{Tr}\left[\mathcal{Y}_U \mathcal{Y}_U^\dagger \left(\mathcal{Y}_D \mathcal{Y}_D^\dagger\right)^2\right] , \qquad I_{(U,D)^2} = \mathrm{Tr}\left[\left(\mathcal{Y}_U \mathcal{Y}_U^\dagger \mathcal{Y}_D \mathcal{Y}_D^\dagger\right)^2\right] .$$

Renormalizable Potential

with the definition

$$X \equiv (I_U, I_D)^T = \left(\mathrm{Tr}\left(\mathcal{Y}_U \mathcal{Y}_U^\dagger\right), \mathrm{Tr}\left(\mathcal{Y}_D \mathcal{Y}_D^\dagger\right)\right)^T ,$$

the potential

$$\begin{aligned} V^{(4)} = & - \mu^2 \cdot X + X^T \cdot \lambda \cdot X + g\, \mathrm{Tr}\left(\mathcal{Y}_U \mathcal{Y}_U^\dagger \mathcal{Y}_D \mathcal{Y}_D^\dagger\right) \\ & + \boxed{h_U \mathrm{Tr}\left(\mathcal{Y}_U \mathcal{Y}_U^\dagger \mathcal{Y}_U \mathcal{Y}_U^\dagger\right)} + \boxed{h_D \mathrm{Tr}\left(\mathcal{Y}_D \mathcal{Y}_D^\dagger \mathcal{Y}_D \mathcal{Y}_D^\dagger\right)} , \end{aligned}$$

mass spectrum

which contains 8 parameters

Renormalizable Potential

with the definition

$$X \equiv (I_U, I_D)^T = \left(\mathrm{Tr}\left(\mathcal{Y}_U \mathcal{Y}_U^\dagger\right), \mathrm{Tr}\left(\mathcal{Y}_D \mathcal{Y}_D^\dagger\right)\right)^T ,$$

the potential

mixing

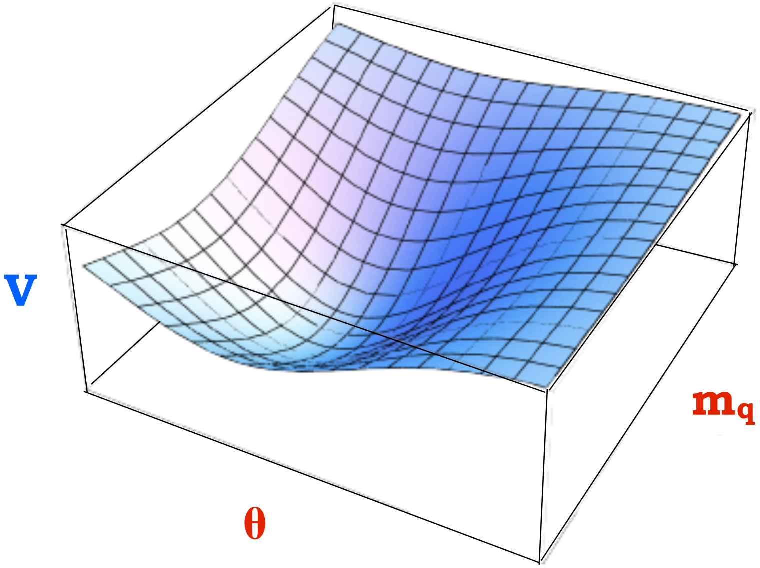
$$V^{(4)} = -\,\mu^2 \cdot X + X^T \cdot \lambda \cdot X + \boxed{g\,\mathrm{Tr}\left(\mathcal{Y}_U \mathcal{Y}_U^\dagger \mathcal{Y}_D \mathcal{Y}_D^\dagger\right)}$$
$$+\, h_U \mathrm{Tr}\left(\mathcal{Y}_U \mathcal{Y}_U^\dagger \mathcal{Y}_U \mathcal{Y}_U^\dagger\right) + h_D \mathrm{Tr}\left(\mathcal{Y}_D \mathcal{Y}_D^\dagger \mathcal{Y}_D \mathcal{Y}_D^\dagger\right) ,$$

which contains 8 parameters

e.g. for the case of two families:

$$\mathrm{Tr}(\mathcal{Y}_u\, \mathcal{Y}_u^+\, \mathcal{Y}_d\, \mathcal{Y}_d^+) \propto (m_c^2 - m_u^2)(m_s^2 - m_d^2) \cos 2\theta$$

at the minimum: $\boxed{(m_c^2 - m_u^2)(m_s^2 - m_d^2) \sin 2\theta = 0}$



-> NO MIXING

same conclusion for 3 families

Berezhiani-Rossi; Anselm, Berezhiani; Alonso, Gavela, Merlo, Rigolin

Renormalizable Potential, mixing **three families**

Von Neumann Trace Inequality

$$y_u^2 y_b^2 + y_s^2 y_c^2 + y_d^2 y_t^2 \leq \mathrm{Tr}\left(\mathcal{Y}_U \mathcal{Y}_U^\dagger \mathcal{Y}_D \mathcal{Y}_D^\dagger\right) \leq y_u^2 y_d^2 + y_s^2 y_c^2 + y_b^2 y_t^2 \,.$$

So the Potential selects:

coefficient in the potential

"normal"
Hierarchy

$$g < 0\,, \quad V_{CKM} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} ;$$

"inverted"
Hierarchy

$$g > 0\,, \quad V_{CKM} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} .$$

No mixing, independently of the mass spectrum

Example: 2 families; consider the renormalizable set of invariants:

The flavour symmetry is $G_f = U(2)_L \times U(2)_{E_R} \times O(2)_{N_R}$

which adds a new invariant for the lepton sector. In total:

$$\mathrm{Tr}\,(\,\mathcal{Y}_E\ \mathcal{Y}_E^+\,) \qquad \mathrm{Tr}\,(\,\mathcal{Y}_E\ \mathcal{Y}_E^+\,)^2$$

$$\mathrm{Tr}\,(\,\mathcal{Y}_\nu\ \mathcal{Y}_\nu^+\,) \qquad \mathrm{Tr}\,(\,\mathcal{Y}_\nu\ \mathcal{Y}_\nu^+\,)^2$$

$$\mathrm{Tr}\,(\,\mathcal{Y}_E\ \mathcal{Y}_E^+\ \mathcal{Y}_\nu\ \mathcal{Y}_\nu^+\,) \longleftarrow$$ **mixing**

$$\mathrm{Tr}\,(\,\mathcal{Y}_\nu^+\mathcal{Y}_\nu\ \mathcal{Y}_\nu^T\ \mathcal{Y}_\nu^*\,)$$ <-- **$O(2)_N$**

Example: 2 families; consider the renormalizable set of invariants:

The flavour symmetry is $G_f = U(2)_L \times U(2)_{E_R} \times O(2)_{N_R}$

which adds a new invariant for the lepton sector. In total:

$$\mathrm{Tr}\,(\mathcal{Y}_E\,\mathcal{Y}_E^{+}) \qquad \mathrm{Tr}\,(\mathcal{Y}_E\,\mathcal{Y}_E^{+})^2$$

$$\mathrm{Tr}\,(\mathcal{Y}_\nu\,\mathcal{Y}_\nu^{+}) \qquad \mathrm{Tr}\,(\mathcal{Y}_\nu\,\mathcal{Y}_\nu^{+})^2$$

$$\mathrm{Tr}\,(\mathcal{Y}_E\,\mathcal{Y}_E^{+}\,\mathcal{Y}_\nu\,\mathcal{Y}_\nu^{+}) \longleftarrow \textbf{mixing}$$

$$\mathrm{Tr}\,(\mathcal{Y}_\nu^{+}\mathcal{Y}_\nu\,(\mathcal{Y}_\nu^{+}\,\mathcal{Y}_\nu)^T) \text{<--}\ \mathbf{O(2)_N}$$

2 families, leptons; let us analyze the mixing invariant

diagonal eigenvalues

Using Casas-Ibarra parametrization $\mathbf{Y_\nu = U_{PMNS}\, m_\nu^{1/2}\, R\, M_N^{1/2}}$

complex orthogonal; it encodes our ignorance of the high energy theory

it follows that:

$$\mathrm{Tr}(\mathcal{Y}_E\, \mathcal{Y}_E^+\, \mathcal{Y}_\nu\, \mathcal{Y}_\nu^+) = \mathrm{Tr}(m_i^{1/2}\, U^+\, m_l^2\, U\, m_i^{1/2}\, R^+\, M_N\, R)$$

* In degenerate limit of heavy neutrinos $M_{N1}=M_{N2}=M$

$$\mathbf{R} = \begin{pmatrix} \mathrm{ch}\ \omega & -i\ \mathrm{sh}\ \omega \\ i\ \mathrm{sh}\ \omega & \mathrm{ch}\ \omega \end{pmatrix}$$ with ω real,

for 2 generations, the mixing terms in $\mathbf{V}(\mathcal{Y}_E, \mathcal{Y}_\nu)$ is :

Leptons

$$\mathrm{Tr}(\mathcal{Y}_E\, \mathcal{Y}_E^{+}\, \mathcal{Y}_\nu\, \mathcal{Y}_\nu^{+}) \propto$$

$$(m_\mu^2 - m_e^2)\left[\cos 2\omega (m_{\nu_2} - m_{\nu_1}) \cos 2\theta + 2 \sin 2\omega \sqrt{m_{\nu_2} m_{\nu_1}}\, sin 2\alpha \sin 2\theta\right]$$

where $U_{PMNS} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} e^{-i\alpha} & 0 \\ 0 & e^{i\alpha} \end{pmatrix}$

Quarks

$$\mathrm{Tr}(\mathcal{Y}_u\, \mathcal{Y}_u^{+}\, \mathcal{Y}_d\, \mathcal{Y}_d^{+}) \propto (m_c^2 - m_u^2)(m_s^2 - m_d^2) \cos 2\theta$$

e.g., **for 2 generations**, the mixing terms in $\mathbf{V}(\mathcal{Y}_{\mathbf{E}}, \mathcal{Y}_{\mathbf{\nu}})$ is :

Leptons

$$\mathrm{Tr}(\mathcal{Y}_E \mathcal{Y}_E^+ \mathcal{Y}_\nu \mathcal{Y}_\nu^+) \propto$$

$$(m_\mu^2 - m_e^2)\left[\cos 2\omega(m_{\nu_2} - m_{\nu_1}) \cos 2\theta + 2 \sin 2\omega \sqrt{m_{\nu_2} m_{\nu_1}}\, sin2\alpha \sin 2\theta\right]$$

This mixing term unphysical if either "up" or "down" fermions degenerate

Mixing physical even with degenerate neutrino masses, if Majorana phase non-trivial

Quarks

$$\mathrm{Tr}(\mathcal{Y}_u \mathcal{Y}_u^+ \mathcal{Y}_d \mathcal{Y}_d^+) \propto (m_c^2 - m_u^2)(m_s^2 - m_d^2) \cos 2\theta$$

e.g., **for 2 generations**, the mixing terms in $\mathbf{V}(\mathcal{Y}_\mathbf{E}, \mathcal{Y}_\mathbf{\nu})$ is :

Minimisation (for non trivial sin2ω)

$\mathrm{Tr}(\mathcal{Y}_E\, \mathcal{Y}_E^+\, \mathcal{Y}_\nu\, \mathcal{Y}_\nu^+)$

* $\sin 2\omega \sqrt{m_{\nu_2} m_{\nu_1}} \sin 2\theta \cos 2\alpha = 0 \longrightarrow \boxed{\alpha = \pi/4 \text{ or } 3\pi/4}$

Maximal Majorana phase

* $\mathrm{tg} 2\theta = \sin 2\alpha \dfrac{2\sqrt{m_{\nu_2} m_{\nu_1}}}{m_{\nu_2} - m_{\nu_1}}$ **tgh 2ω**

Large angles correlated with degenerate masses

* What is the role of the neutrino flavour group?

$O(2)_{N_R}$

e.g. two families

$$m_\nu \sim \mathbf{Y_\nu} \frac{v^2}{M} \mathbf{Y_\nu}^{\mathbf{T}} = y_1\, y_2\, \frac{v^2}{M} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\longrightarrow \left\{ \begin{array}{l} U_{PMNS} = \boxed{\frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} e^{i\pi/4} & 0 \\ 0 & e^{-i\pi/4} \end{pmatrix}} \\ \\ \textbf{Degenerate neutrino masses} \end{array} \right.$$

Generically, O(2) allows :

- **one mixing angle maximal**
- **one relative Majorana phase of $\pi/2$**
- **two degenerate light neutrinos**

Anarchy: alive with not so small θ_{13} and not θ_{23} not maximal

no symmetry in the lepton sector, just random numbers

$$m_\nu \sim \begin{pmatrix} \sim 1 & \sim 1 & \sim 1 \\ \sim 1 & \sim 1 & \sim 1 \\ \sim 1 & \sim 1 & \sim 1 \end{pmatrix}$$

still looks "good" in GU context, coupled to U(1)s

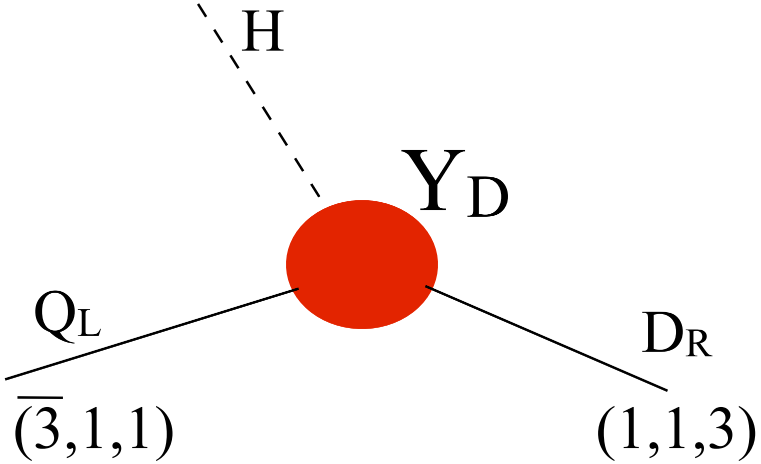
- Does not relate mixing to spectrum
- Does not address both quarks and leptons

(Hall, Murayama, Weiner; Haba, Murayama; De Gouvea, Murayama...
Going towards hierarchy: Altarelli, Feruglio, Masina, Merlo..)

The non-abelian part of the flavour symmetry of the SM:

$$G_f = SU(3)_{Q_L} \times SU(3)_{U_R} \times SU(3)_{D_R}$$

broken by Yukawas:



H

Y_D

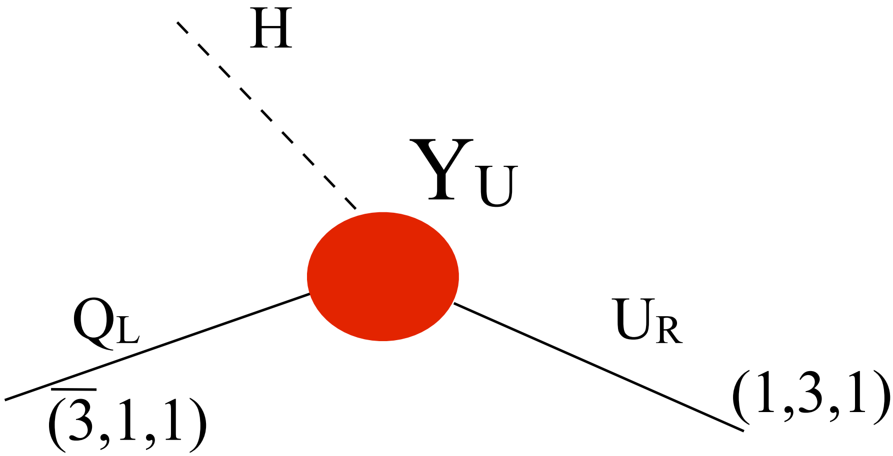
Q_L

$(\bar{3},1,1)$

D_R

$(1,1,3)$

$\sim (\bar{3},1,3)$



H

Y_U

Q_L

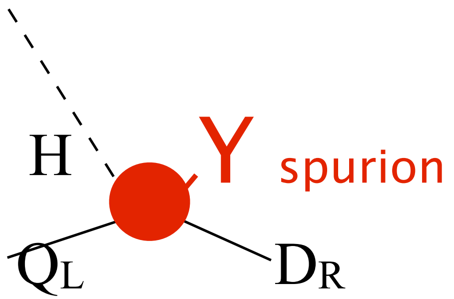
$(\bar{3},1,1)$

U_R

$(1,3,1)$

$\sim (\bar{3},3,1)$

Some good ideas:



Minimal Flavour Violation:

- Use the flavour symmetry of the SM in the limit of massless fermions (**Chivukula+ Georgi**)

quarks: $\mathbf{G_{flavour} = U(3)_{QL} \times U(3)_{UR} \times U(3)_{DR}}$

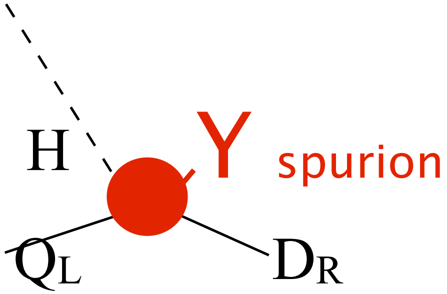
- Assume that Yukawas are the only source of flavour in the SM and beyond

$$\frac{\mathbf{Y_{\alpha\beta}^{+} Y_{\delta\gamma}}}{\mathbf{\Lambda_{flavour}^{2}}} \; \overline{\mathbf{Q}}_\alpha \gamma_\mu \mathbf{Q}_\beta \overline{\mathbf{Q}}_\gamma \gamma^\mu \mathbf{Q}_\delta$$

... agrees with flavour data being aligned with SM
... allows to bring down $\mathbf{\Lambda_{flavour}}$ --> TeV

D’Ambrosio+Giudice+Isidori+Strumia;
Cirigliano+Isidori+Grinstein+Wise

Some good ideas:



Minimal Flavour Violation:

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- Assume that Yukawas are the only source of flavour in the SM and beyond

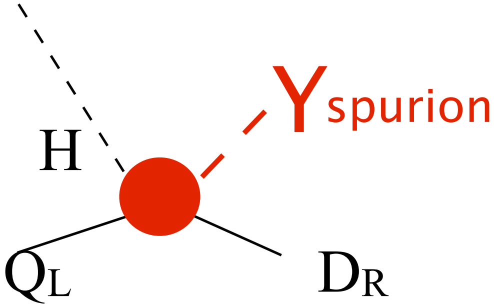
$$\frac{Y_{\alpha\beta}^{+} Y_{\delta\gamma}}{\Lambda_{flavour}^{2}} \overline{Q}_{\alpha} \gamma_{\mu} Q_{\beta} \overline{Q}_{\gamma} \gamma^{\mu} Q_{\delta}$$

... agrees with flavour data being aligned with SM
... allows to bring down $\Lambda_{flavour}$ --> TeV

(Chivukula+Georgi 87; Hall+Randall; D'Ambrosio+Giudice+Isidori+Strumia; Cirigliano+Isidori+Grisnstein +Wise; Davidson+Pallorini; Kagan+G. Perez + Volanski+Zupan,...)

Lalak, Pokorski, Ross; Fitzpatrick, Perez, Randall; Grinstein, Redi, Villadoro

Some good ideas:



Related to MFV:

- Use the flavour symmetry of the SM in the limit of massless fermions

quarks: $G_{\rm flavour}= U(3)_{QL}$ x $U(3)_{UR}$ x $U(3)_{DR}$

Hybrid dynamical-non-dynamical Yukawas:

U(2) (Pomarol, Tomasini; Barbieri, Dvali, Hall, Romanino...)....

$U(2)^3$ (Craig, Green, Katz; Barbieri, Isidori, Jones-Peres, Lodone, Straub.. ..Sala)

Sequential ideas (Feldman, Jung, Mannel; Berezhiani+Nesti; Ferretti et al., Calibbi et al. ...)

The basis of the game is to find the minima of the invariants that you can construct out of Yukawa couplings

L. Michel+Radicati 70, Cabibbo+Maiani71 for the spectrum of masses

List of possible invariants: Hanani, Jenkins, Manohar 2010

$V(\mathcal{Y}_d,\ \mathcal{Y}_u)$

Construction of the Potential

* 5 invariants at d=4 level:
(Feldman, Jung, Mannel)

$$\mathrm{Tr}\,(\,\mathcal{Y}_u\,\mathcal{Y}_u^{+}\,) \qquad \mathrm{Tr}\,(\,\mathcal{Y}_u\,\mathcal{Y}_u^{+}\,)^2$$

$$\mathrm{Tr}\,(\,\mathcal{Y}_d\,\mathcal{Y}_d^{+}\,) \qquad \mathrm{Tr}\,(\,\mathcal{Y}_d\,\mathcal{Y}_d^{+}\,)^2$$

$$\mathrm{Tr}\,(\,\mathcal{Y}_u\,\mathcal{Y}_u^{+}\,\mathcal{Y}_d\,\mathcal{Y}_d^{+}\,)$$

* results following general; for this talk we will illustrate in 2-generation

(Alonso, Gavela, Merlo, Rigolin, arXiv 11; Nardi 11, Espinosa, Fong, Nardi 13)

$V(\mathcal{Y}_d, \mathcal{Y}_u)$

Construction of the Potential

* 5 invariants at d=4 level:
(Feldman, Jung, Mannel)

$$\mathrm{Tr}\,(\mathcal{Y}_u \mathcal{Y}_u^{+}) \qquad \mathrm{Tr}\,(\mathcal{Y}_u \mathcal{Y}_u^{+})^2$$

$$\mathrm{Tr}\,(\mathcal{Y}_d \mathcal{Y}_d^{+}) \qquad \mathrm{Tr}\,(\mathcal{Y}_d \mathcal{Y}_d^{+})^2$$

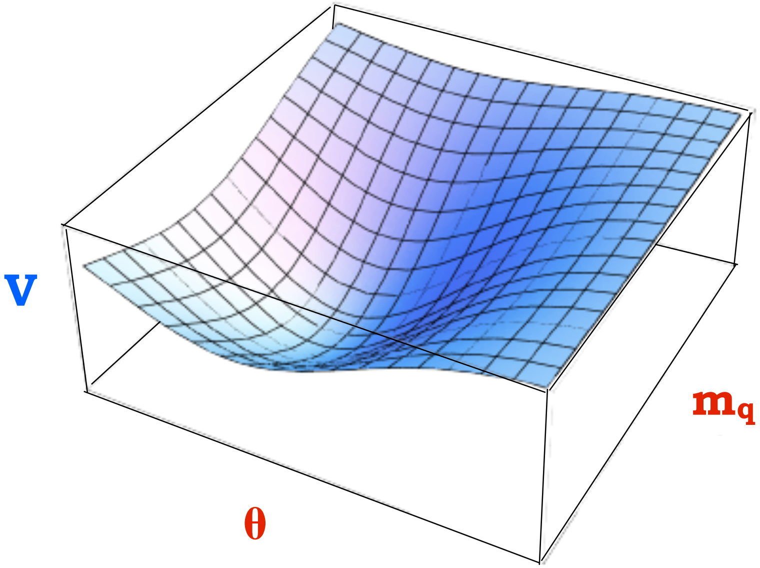
$$\mathrm{Tr}\,(\mathcal{Y}_u \mathcal{Y}_u^{+} \mathcal{Y}_d \mathcal{Y}_d^{+})$$ <-- **mixing**

(Alonso, Gavela, Merlo, Rigolin, arXiv 11; Nardi 11, Espinosa, Fong, Nardi 13)

e.g. for the case of two families:

$$\mathrm{Tr}(\mathcal{Y}_u\,\mathcal{Y}_u^+\,\mathcal{Y}_d\,\mathcal{Y}_d^+) \propto (m_c^2 - m_u^2)(m_s^2 - m_d^2)\cos 2\theta$$

at the minimum: $\boxed{(m_c^2 - m_u^2)(m_s^2 - m_d^2)\sin 2\theta = 0}$



-> NO MIXING

same conclusion for 3 families

Berezhiani-Rossi; Anselm, Berezhiani; Alonso, Gavela, Merlo, Rigolin

And what happens for leptons ?

Any difference with Majorana neutrinos?

Alonso, B.G., D. Hernandez, Merlo, Rigolin.

Just TWO heavy neutrinos

$$\mathcal{L}_{\mathcal{M}_\nu} = (\bar{\ell}_L\,,\,\bar{N}^c\,,\,\bar{N}'^c)\begin{pmatrix} 0 & vY & vY' \\ vY^T & 0 & \mathbf{M} \\ vY'^T & \mathbf{M} & 0 \end{pmatrix}\begin{pmatrix} \ell_L^c \\ N \\ N' \end{pmatrix}$$

the Yukawas are determined up to their overal magnitude

N.H. $$Y = \frac{y}{\sqrt{m_{\nu_2} + m_{\nu_3}}}\, U_{PMNS} \begin{pmatrix} 0 \\ -i\sqrt{m_{\nu_2}}e^{-i\alpha} \\ \sqrt{m_{\nu_3}}e^{i\alpha} \end{pmatrix}$$

The flavour symmetry is G$_f$= $U(3)_{\ell_L} \times U(3)_{E_R} \times O(2)_N$

(Alonso, Gavela, D. Hernandez, Merlo, Rigolin)

The flavour symmetry is $G_f = U(3)_{\ell_L} \times U(3)_{E_R} \times O(2)_N$

adds a new invariant for the lepton sector, in total:

$$\mathrm{Tr}\,(\mathcal{Y}_E\,\mathcal{Y}_E^+) \qquad \mathrm{Tr}\,(\mathcal{Y}_E\,\mathcal{Y}_E^+)^2$$

$$\mathrm{Tr}\,(\mathcal{Y}_\nu\,\mathcal{Y}_\nu^+) \qquad \mathrm{Tr}\,(\mathcal{Y}_\nu\,\mathcal{Y}_\nu^+)^2$$

$$\mathrm{Tr}\,(\mathcal{Y}_E\,\mathcal{Y}_E^+\,\mathcal{Y}_\nu\,\mathcal{Y}_\nu^+) \longleftarrow \textbf{mixing}$$

$$\mathrm{Tr}\,(\mathcal{Y}_\nu\,\sigma_2\,\mathcal{Y}_\nu^+)^2 \quad \texttt{<--}\ \mathbf{O(2)_N}$$

$O(2)_N$ is simply associated to Lepton Number

e.g., **for 2 generations**, the mixing terms in $\mathbf{V}(\mathcal{Y}_{\mathbf{E}}, \mathcal{Y}_{\mathbf{\nu}})$ is :

Leptons

$$\mathrm{Tr}(\mathcal{Y}_{\mathrm{E}}\,\mathcal{Y}_{\mathrm{E}}^{+}\,\mathcal{Y}_{\nu}\,\mathcal{Y}_{\nu}^{+}) \propto$$

$$(m_\mu^2 - m_e^2)\left[(y^2 + y'^2)(m_{\nu_2} - m_{\nu_1})\cos 2\theta + (y^2 - y'^2)2\sqrt{m_{\nu_2} m_{\nu_1}}\sin 2\alpha \sin 2\theta\right]$$

$$\text{where } U_{PMNS} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} e^{-i\alpha} & 0 \\ 0 & e^{i\alpha} \end{pmatrix}$$

Quarks

$$\mathrm{Tr}(\mathcal{Y}_{\mathrm{u}}\,\mathcal{Y}_{\mathrm{u}}^{+}\,\mathcal{Y}_{\mathrm{d}}\,\mathcal{Y}_{\mathrm{d}}^{+}) \propto (m_c^2 - m_u^2)(m_s^2 - m_d^2)\cos 2\theta$$

e.g., **for 2 generations**, the mixing terms in $\mathbf{V}(\mathcal{Y}_\mathbf{E}, \mathcal{Y}_\mathbf{\nu})$ is :

Leptons

$$\mathrm{Tr}(\mathcal{Y}_\mathrm{E}\, \mathcal{Y}_\mathrm{E}^+\, \mathcal{Y}_\nu\, \mathcal{Y}_\nu^+) \propto$$

$$(m_\mu^2 - m_e^2)\left[(y^2 + y'^2)(m_{\nu_2} - m_{\nu_1}) \cos 2\theta + (y^2 - y'^2) 2\sqrt{m_{\nu_2} m_{\nu_1}} \sin 2\alpha \sin 2\theta\right]$$

Mixing term unphysical if either “up” or “down” fermions degenerate

Mixing physical even with degenerate neutrino masses, if Majorana phase non-trivial

Quarks

$$\mathrm{Tr}(\mathcal{Y}_\mathrm{u}\, \mathcal{Y}_\mathrm{u}^+\, \mathcal{Y}_\mathrm{d}\, \mathcal{Y}_\mathrm{d}^+) \propto (m_c^2 - m_u^2)(m_s^2 - m_d^2) \cos 2\theta$$

e.g., for 2 generations, the mixing terms in **V**($\mathcal{Y}_E$, $\mathcal{Y}_\nu$) is :

Minimisation

Tr($\mathcal{Y}_E \, \mathcal{Y}_E^+ \, \mathcal{Y}_\nu \, \mathcal{Y}_\nu^+$)

* $(y^2 - y'^2)\sqrt{m_{\nu_2} m_{\nu_1}} \sin 2\theta \cos 2\alpha = 0$ ⟶ $\boxed{\alpha = \pi/4 \text{ or } 3\pi/4}$

* $$\boxed{\mathrm{tg} 2\theta = 2\frac{y^2 - y'^2}{y^2 + y'^2} \sin 2\alpha \frac{\sqrt{m_{\nu_2} m_{\nu_1}}}{m_{\nu_2} - m_{\nu_1}}}$$

Large angles correlated with degenerate masses

Maximal Majorana phase

What makes the difference?

- The Majorana character?
- The flavour group?
- The particular model?

Let us try to generalize to any model

-- for 2 families
-- for 3 families

* Generalize to arbitrary seesaw model

in progress: Alonso, D. Hernandez, Merlo, Rigolin, B.G.

diagonal eigenvalues

Use Casas-Ibarra parametrization $\mathbf{Y_\nu = U_{\text{PMNS}}\, m_\nu^{1/2}\, R\, M_N^{1/2}}$

complex orthogonal matrix

* Generalize to arbitrary seesaw model

in progress: Alonso, D. Hernandez, Merlo, Rigolin, B.G.

diagonal eigenvalues

Use Casas-Ibarra parametrization $\mathbf{Y}_\nu = \mathbf{U}_{PMNS}\, \mathbf{m}_\nu^{1/2}\, \mathbf{R}\, \mathbf{M}_N^{1/2}$

complex orthogonal matrix

The mixing invariant shown before:

$$Tr(\mathcal{Y}_E\, \mathcal{Y}_E^{+}\, \mathcal{Y}_\nu\, \mathcal{Y}_\nu^{+}) = Tr(m_i^{1/2}\; U^{+}\, m_l^{2}\, U\, m_i^{1/2}\, R^{+}\, M_N\, R)$$

define $P=(R^{+} M_N R) \xrightarrow[\text{2 fam.}]{}$

$$* \quad \sqrt{m_1 m_2}\,|P_{12}|\,\sin\left[2\alpha - arg(P_{12})\right] = 0$$

$$* \quad tg 2\theta = 2|P_{12}|\sin 2\alpha \frac{\sqrt{m_{\nu_2} m_{\nu_1}}}{m_{\nu_1} P_{11} - m_{\nu_2} P_{22}}$$

* Generalize to arbitrary seesaw model

in progress: Alonso, D. Hernandez, Merlo, Rigolin, B.G.

diagonal eigenvalues

Use Casas-Ibarra parametrization $Y_\nu = U_{\text{PMNS}}\, \mathbf{m}_\nu^{1/2}\, \mathbf{R}\, M_N^{1/2}$

complex orthogonal matrix

The mixing invariant shown before:

$$\mathrm{Tr}(\,\mathcal{Y}_E\ \mathcal{Y}_E^{+}\ \mathcal{Y}_\nu\ \mathcal{Y}_\nu^{+}) = \mathrm{Tr}(\,m_i^{1/2}\ U^{+}\, m_l^{2}\ U\, m_i^{1/2}\ R^{+}\ M_N\ R)$$

define $P = (\,R^{+}\ M_N\ R)$ $\xrightarrow[\text{2 fam.}]{}$

$$\boxed{\begin{aligned} &*\ \ \sqrt{m_1 m_2}\,|P_{12}|\,\sin\left[2\alpha - arg(P_{12})\right] = 0 \\ &*\ \ \mathrm{tg}2\theta = 2|P_{12}|\,\sin 2\alpha\,\frac{\sqrt{m_{\nu_2} m_{\nu_1}}}{m_{\nu_1} P_{11} - m_{\nu_2} P_{22}} \end{aligned}}$$

* In degenerate limit of heavy neutrinos $M_{N_1} = M_{N_2} = M$

$$\mathbf{R} = \begin{pmatrix} \mathrm{ch}\ \omega & -i\ \mathrm{sh}\ \omega \\ i\ \mathrm{sh}\ \omega & \mathrm{ch}\ \omega \end{pmatrix}$$ with ω real,

$$\mathrm{tg}2\theta = \sin 2\alpha\,\frac{2\sqrt{m_{\nu_2} m_{\nu_1}}}{m_{\nu_2} - m_{\nu_1}}\ \mathbf{tgh}\ 2\omega$$

$$\boxed{\alpha = \pi/4 \text{ or } 3\pi/4}$$

* Generalize to arbitrary seesaw model

in progress: Alonso, D. Hernandez, Merlo, Rigolin, B.G.

diagonal eigenvalues

Use Casas-Ibarra parametrization $Y_\nu = U_{PMNS}\, m_\nu^{1/2}\, R\, M_N^{1/2}$

complex orthogonal matrix

The mixing invariant shown before:

$$\mathrm{Tr}(\mathcal{Y}_E\, \mathcal{Y}_E^+\, \mathcal{Y}_\nu\, \mathcal{Y}_\nu^+) = \mathrm{Tr}(m_i^{1/2}\, U^+ m_l^2\, U\, m_i^{1/2}\, R^+ M_N\, R)$$

define $P = (R^+ M_N R)$ $\xrightarrow[\text{2 fam.}]{}$

$$\boxed{\begin{aligned} &* \quad \sqrt{m_1 m_2}\,|P_{12}|\,\sin\left[2\alpha - arg(P_{12})\right] = 0 \\ &* \quad \mathrm{tg}2\theta = 2|P_{12}|\sin 2\alpha \frac{\sqrt{m_{\nu_2} m_{\nu_1}}}{m_{\nu_1} P_{11} - m_{\nu_2} P_{22}} \end{aligned}}$$

*** In degenerate limit of heavy neutrinos $M_{N_1} = M_{N_2} = M$**

$$\mathbf{R} = \begin{pmatrix} \mathrm{ch}\,\omega & -i\,\mathrm{sh}\,\omega \\ i\,\mathrm{sh}\,\omega & \mathrm{ch}\,\omega \end{pmatrix}$$ with ω real,

e.g. in Previous model

$$\mathrm{tg}2\theta = \sin 2\alpha \frac{2\sqrt{m_{\nu_2} m_{\nu_1}}}{m_{\nu_2} - m_{\nu_1}} \quad \frac{y^2 - y'^2}{y^2 - y'^2}$$

$$\boxed{\alpha = \pi/4 \text{ or } 3\pi/4}$$

* What is the role of the neutrino flavour group?

Leptons: $G_{\text{flavour}} = U(2)_L \times U(2)_{ER} \times$?

O(2), SU(n), O(n) .... ?

Inmediate results using for both quark and leptons

$$Y = U_L \, y^{\text{diag}} \, U_R$$

* **What is the role of the neutrino flavour group?**

$$\mathrm{U}(n)$$

* What is the role of the neutrino flavour group?

$$U(n)$$

i.e.: $U(3)_L \text{ x } U(3)_{E_R} \text{ x } U(2)_{N_R}$

or: $U(3)_L \text{ x } U(3)_{E_R} \text{ x } U(3)_{N_R}$

* What is the role of the neutrino flavour group?

To analyze this in general, use common parametrization for quarks and leptons:

$$\mathbf{Y} = U_L \; y^{\text{diag.}} \, U_R$$

* **Quarks**, for instance: U_R unphysical, U_L --> U_{CKM}

$$\mathbf{Y_D} = \mathbf{U_{CKM}} \, \text{diag}(y_d, y_s, y_b) \quad ; \quad \mathbf{Y_U} = \text{diag}(y_u, y_c, y_t)$$

* **Leptons**:

$$\mathbf{Y_E} = \text{diag}(y_e, y_\mu, y_\tau) \quad ; \quad \mathbf{Y_\nu} = U_L \; y^{\text{diag.}} \, U_R$$

U_{PMNS} diagonalize

$$\boxed{m_\nu \sim \mathbf{Y_\nu} \frac{v^2}{M} \mathbf{Y_\nu}^{\mathbf{T}} = U_L \, y_\nu^{\text{diag.}} \, U_R \frac{v^2}{M} U_R^T \, y_\nu^{\text{diag.}} \, U_L^T}$$

SU(n)

* What is the role of the neutrino flavour group?

e.g. $SU(n)_{N_R}$ ... leptons

e.g. generic seesaw

$$\mathcal{L} = \mathcal{L}_{SM} + i\overline{N_R}\partial\!\!\!/ N_R - \left[\overline{N_R} Y_N \tilde{\phi}^\dagger \ell_L + \frac{1}{2}\overline{N_R}\,\mathbf{M}\,N_R{}^c + h.c.\right]$$

with **M** carrying flavour → **M** spurion

More invariants in this case:

$\mathrm{Tr}\,(\,\mathcal{Y}_E\,\mathcal{Y}_E^+\,)$ $\mathrm{Tr}\,(\,\mathcal{Y}_E\,\mathcal{Y}_E^+\,)^2$ $\mathrm{Tr}\,(\,\mathcal{Y}_E\,\mathcal{Y}_E^+\,\mathcal{Y}_\nu\,\mathcal{Y}_\nu^+\,)$

$\mathrm{Tr}\,(\,\mathcal{Y}_\nu\,\mathcal{Y}_\nu^+\,)$ $\mathrm{Tr}\,(\,\mathcal{Y}_\nu\,\mathcal{Y}_\nu^+\,)^2$

$\mathrm{Tr}\,(\,\mathcal{M}_N\,\mathcal{M}_N^+\,)$ $\mathrm{Tr}\,(\,\mathcal{M}_N\,\mathcal{M}_N^+\,)^2$ $\mathrm{Tr}\,(\,\mathcal{M}_N\,\mathcal{M}_N^+\,\mathcal{Y}_\nu^+\mathcal{Y}_\nu)$

At the minimum:

$*\ \mathrm{Tr}\,(\,\mathcal{Y}_\nu\,\mathcal{Y}_\nu^+\,\mathcal{Y}_E\,\mathcal{Y}_E^+\,) = \mathrm{Tr}\,(\,U_L\,y_\nu^{\mathrm{diag.}\,2}\,U_L{}^+\,y_l^{\mathrm{diag.}\,2}) \longrightarrow U_L=1$

$*\ \mathrm{Tr}\,(\,\mathcal{M}_N\,\mathcal{M}_N^+\,\mathcal{Y}_\nu\,\mathcal{Y}_\nu^+\,) = \mathrm{Tr}\,(\,U_R\,y_\nu^{\mathrm{diag.}\,2}\,U_R{}^+\,M_i^{\mathrm{diag.}\,2}) \longrightarrow U_R=1$

same conclusion for 3 families of quarks:

$$Y = U_L \;\; y^{\mathrm{diag.}} \, U_R$$

* **Quarks**, for instance: U_R unphysical, U_L --> U_{CKM}

$$Y_D = U_{CKM} \, \mathrm{diag}(y_d, y_s, y_b) \quad ; \quad Y_U = \mathrm{diag}(y_u, y_c, y_t)$$

$$\mathrm{Tr} \left(\mathcal{Y}_u \, \mathcal{Y}_u^+ \, \mathcal{Y}_d \, \mathcal{Y}_d^+ \right) = \mathrm{Tr} \left(U_L \, y_u^{\mathrm{diag.}\,2} \, U_L^{\,+} \, y_d^{\mathrm{diag.}\,2} \right)$$

$\longrightarrow$ $U_L = U_{\,CKM} \sim 1$ at the minimum

NO MIXING

* What is the role of the neutrino flavour group?

e.g. $U(n)_{NR}$ ... leptons

e.g. generic seesaw

$$\mathcal{L} = \mathcal{L}_{SM} + i\overline{N_R}\not{\partial} N_R - \left[\overline{N_R} Y_N \tilde{\phi}^\dagger \ell_L + \frac{1}{2}\overline{N_R}\,\mathbf{M} N_R{}^c + h.c.\right]$$

with **M** carrying flavour → **M** spurion

More invariants in this case:

$\mathrm{Tr}\,(\mathcal{Y}_E\,\mathcal{Y}_E^+)$ $\mathrm{Tr}\,(\mathcal{Y}_E\,\mathcal{Y}_E^+)^2$

$\mathrm{Tr}\,(\mathcal{Y}_E\,\mathcal{Y}_E^+\,\mathcal{Y}_\nu\,\mathcal{Y}_\nu^+)$

$\mathrm{Tr}\,(\mathcal{Y}_\nu\,\mathcal{Y}_\nu^+)$ $\mathrm{Tr}\,(\mathcal{Y}_\nu\,\mathcal{Y}_\nu^+)^2$

$\mathrm{Tr}\,(\mathcal{M}_N\,\mathcal{M}_N^+)$ $\mathrm{Tr}\,(\mathcal{M}_N\,\mathcal{M}_N^+)^2$

$\mathrm{Tr}\,(\mathcal{M}_N\,\mathcal{M}_N^+\,\mathcal{Y}_\nu^+\mathcal{Y}_\nu)$

Result: no mixing for flavour groups U(n)

$O(n)$

*3 families with $O(2)_{N_R}$:

- 3 light + 2 heavy N degenerate: bad θ_{12} quadrant. It cannot accomodate data!

- 3 light + 3 heavy N : **OK for θ_{23} maximal and spectrum**

experimentally $\sin^2\theta_{23}$= 0.41 +-0.03 or 0.59+-0.02

Gonzalez-Garcia, Maltoni, Salvado, Schwetz Sept. 2012

T2K -> 45º in 2-fam.

*What about the other angles?

* What is the role of the neutrino flavour group?

e.g. SU(n)$_{NR}$ ... leptons

e.g. generic seesaw

$$\mathcal{L} = \mathcal{L}_{SM} + i\overline{N_R}\not{\partial}N_R - \left[\overline{N_R}Y_N\tilde{\phi}^\dagger \ell_L + \frac{1}{2}\overline{N_R}\mathbf{M}N_R{}^c + h.c.\right]$$

with **M** carrying flavour $\longrightarrow$ **M** spurion

More invariants in this case:

$\mathrm{Tr}\,(\,\mathcal{Y}_E\,\mathcal{Y}_E^+\,)$ $\quad$ $\mathrm{Tr}\,(\,\mathcal{Y}_E\,\mathcal{Y}_E^+\,)^2$ $\quad$ $\mathrm{Tr}\,(\,\mathcal{Y}_E\,\mathcal{Y}_E^+\,\mathcal{Y}_\nu\,\mathcal{Y}_\nu^+\,)$

$\mathrm{Tr}\,(\,\mathcal{Y}_\nu\,\mathcal{Y}_\nu^+\,)$ $\quad$ $\mathrm{Tr}\,(\,\mathcal{Y}_\nu\,\mathcal{Y}_\nu^+\,)^2$

$\mathrm{Tr}\,(\,\mathcal{M}_N\,\mathcal{M}_N^+\,)$ $\quad$ $\mathrm{Tr}\,(\,\mathcal{M}_N\,\mathcal{M}_N^+\,)^2$ $\quad$ $\mathrm{Tr}\,(\,\mathcal{M}_N\,\mathcal{M}_N^+\,\mathcal{Y}_\nu^+\mathcal{Y}_\nu)$

At the minimum:

$* \;\mathrm{Tr}\,(\,\mathcal{Y}_\nu\,\mathcal{Y}_\nu^+\,\mathcal{Y}_E\,\mathcal{Y}_E^+\,) = \mathrm{Tr}\,(\;U_L\,y_\nu^{diag.\,2}\,U_L^{\,+}\,y_l^{diag.\,2}) \longrightarrow U_L=1$

$* \;\mathrm{Tr}\,(\,\mathcal{M}_N\,\mathcal{M}_N^+\,\mathcal{Y}_\nu\,\mathcal{Y}_\nu^+\,) = \mathrm{Tr}\,(\;U_R\,y_\nu^{diag.\,2}\,U_R^{\,+}\,M_i^{diag.\,2}) \longrightarrow U_R=1$

$$G_f = U(3)_Q \;\; x \;\; U(3)_U \;\; x \;\; U(3)_D$$

$$V\,(\mathcal{Y}_{\mathbf{u}},\ \mathcal{Y}_{\mathbf{u}}) = \Sigma_i \,[\, -\,\mu_i^2\, \mathrm{Tr}\,(\,\mathcal{Y}_{\mathbf{i}}\ \mathcal{Y}_{\mathbf{i}}^+\,) - \lambda_i\, \mathrm{Tr}\,(\,\mathcal{Y}_{\mathbf{i}}\ \mathcal{Y}_{\mathbf{i}}^+\,)^2\,]$$

$$+\,\Sigma_{i\neq j}\,[\,\lambda_{ij}\ \mathrm{Tr}\,(\,\mathcal{Y}_{\mathbf{i}}\ \mathcal{Y}_{\mathbf{i}}^+\ \mathcal{Y}_{\mathbf{j}}\ \mathcal{Y}_{\mathbf{j}}^+\,)\,] + \ \ldots$$

it only relies on G_f symmetry and SM gauge symmetry

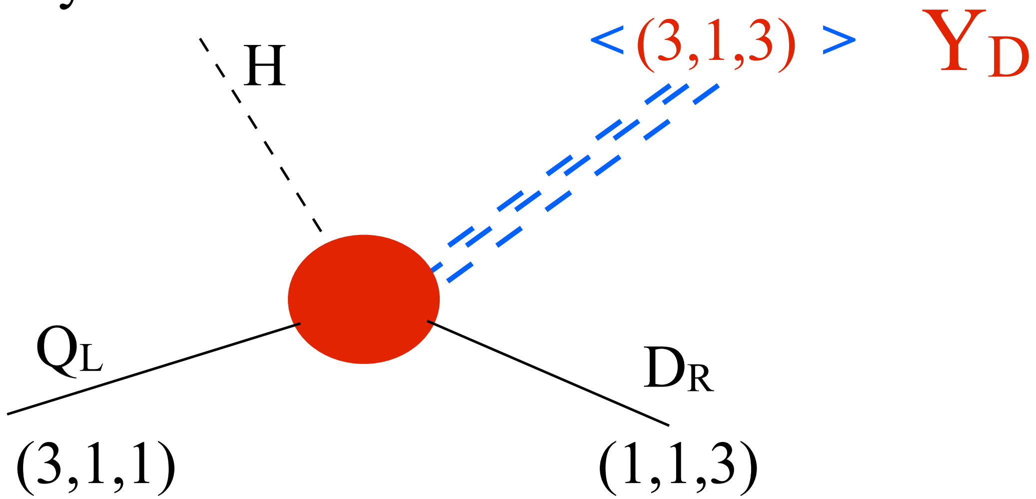
It allows for either (too) hierarchical or degenerate spectrum

(Alonso, Gavela, Merlo, Rigolin 11; Nardi 11; Espinosa, Fong, Nardi 13)

Use the flavour symmetry of the SM with masless fermions:

$$G_f = U(3)_{Q_L} \times U(3)_{U_R} \times U(3)_{D_R}$$

replace Yukawas by fields:



Spontaneous breaking of flavour symmetry dangerous

--> i.e. gauge it (Grinstein, Redi, Villadoro, 2010)
(Feldman, 2010)
(Guadagnoli, Mohapatra, Sung, 2010)

$$G_f = U(3)_Q \ \ x \ \ U(3)_U \ \ x \ \ U(3)_D$$

$$V\,(\mathcal{Y}_{\mathbf{u}},\ \mathcal{Y}_{\mathbf{u}}) = \Sigma_i\, [\, -\, \mu_i^2\, \mathrm{Tr}\,(\, \mathcal{Y}_{\mathbf{i}}\ \mathcal{Y}_{\mathbf{i}}^+) - \lambda_i\, \mathrm{Tr}\,(\, \mathcal{Y}_{\mathbf{i}}\ \mathcal{Y}_{\mathbf{i}}^+)^2\,]$$

$$+\ \Sigma_{i\neq j}\, [\, \lambda_{ij}\ \mathrm{Tr}\,(\, \mathcal{Y}_{\mathbf{i}}\ \mathcal{Y}_{\mathbf{i}}^+\ \mathcal{Y}_{\mathbf{j}}\ \mathcal{Y}_{\mathbf{j}}^+)\,] + \ \ldots$$

it only relies on G_f symmetry and SM gauge symmetry

and analyzed its minima

(Alonso, Gavela, Merlo, Rigolin, arXiv 11; Nardi 11, Espinosa, Fong, Nardi 13)

Can its minimum correspond **naturally** to the observed masses and mixings?

i.e. with all dimensionless λ's ~ 1

and dimensionful μ's $\leqq \Lambda_f$

Y --> one single field Σ

Spectrum for flavons Σ in the bifundamental:

* **3 generations: for the largest fraction of the parameter space, the stable solution is a degenerate spectrum**

$$\begin{pmatrix} y_u & & \\ & y_c & \\ & & y_t \end{pmatrix} \sim \begin{pmatrix} y & & \\ & y & \\ & & y \end{pmatrix}$$

instead of the observed hierarchical spectrum, i.e.

$$\begin{pmatrix} y_u & & \\ & y_c & \\ & & y_t \end{pmatrix} \sim \begin{pmatrix} 0 & & \\ & 0 & \\ & & y \end{pmatrix}$$

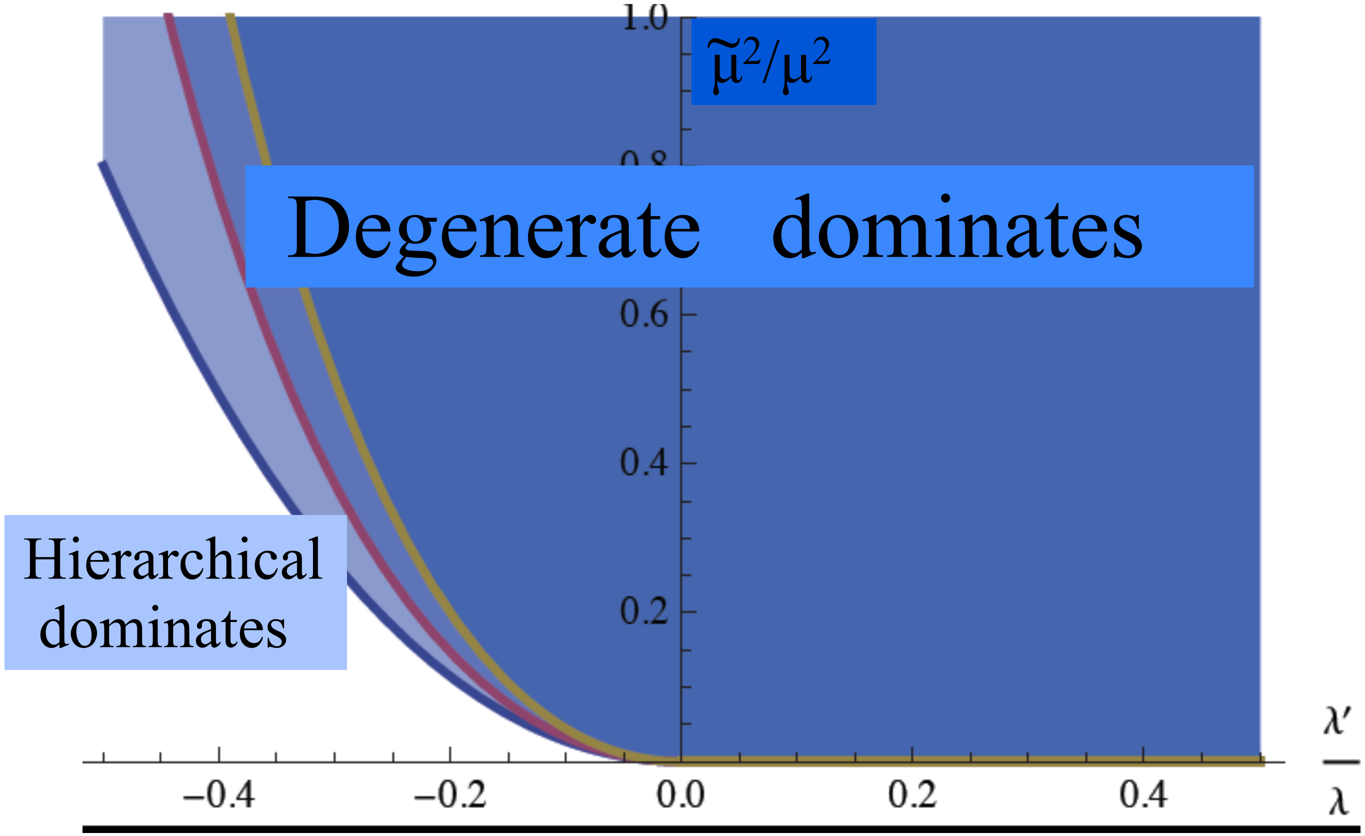
(at leading order)

Spectrum: the hierarchical solution is unstable in most of the parameter space.

Stability: $\boxed{\frac{\tilde{\mu}^2}{\mu^2} < \frac{2\lambda'^2}{\lambda}}$

$$V^{(4)} = \sum_{i=u,d} \left(-\mu_i^2 A_i + \tilde{\mu}_i B_i + \lambda_i A_i^2 + \lambda_i' A_{ii}\right) + g_{ud} A_u A_d + \lambda_{ud} A_{ud} \,.$$

ie, the u-part: $V^{(4)} = -\mu_u^2 A_u + \tilde{\mu}_u B_u + \lambda_u A_u^2 + \lambda_u' A_{uu}$



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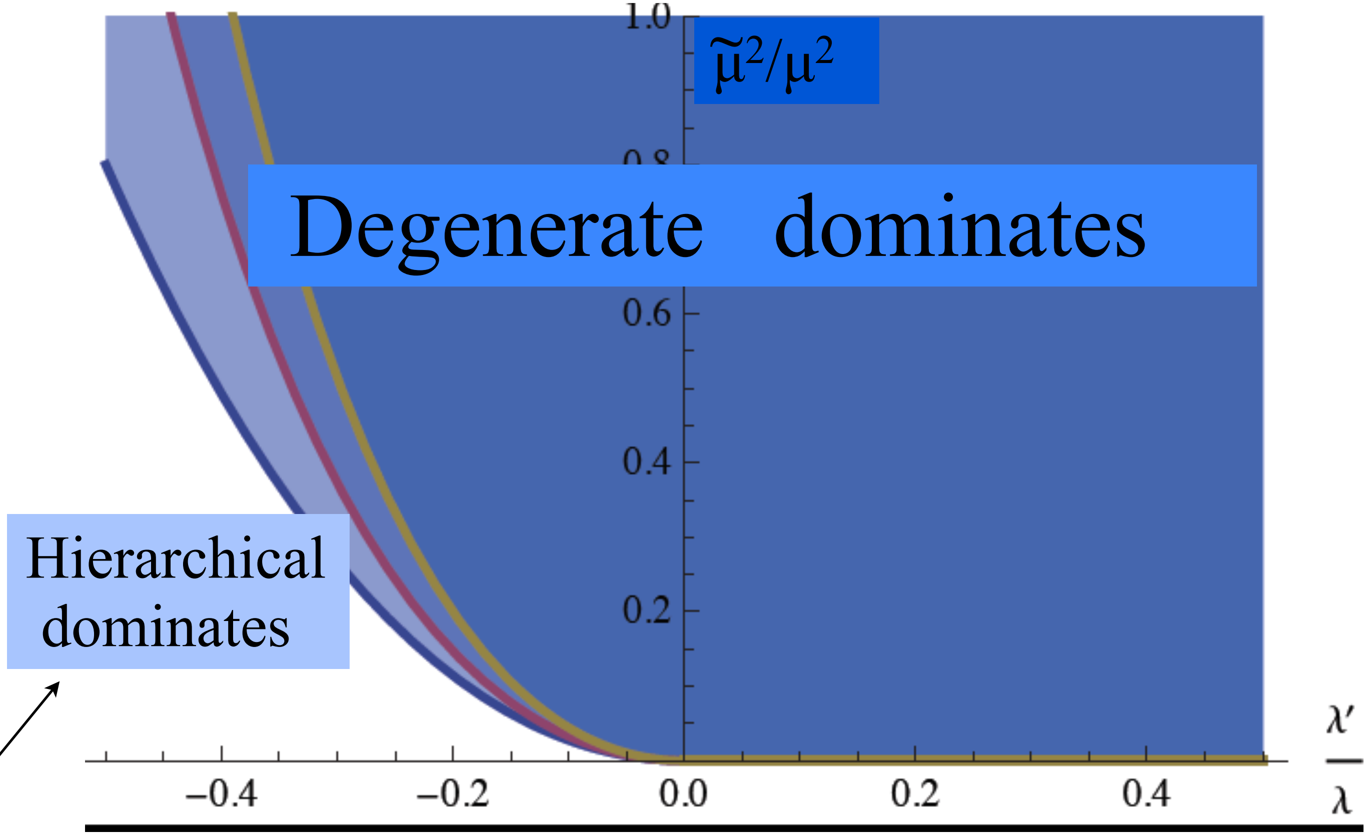


Degenerate dominates

Hierarchical dominates

Nardi emphasized this solution (and extended the analysis to include also U(1) factors)

Normal hierarchy:

Up to terms of $\mathcal{O}(\sqrt{r}, s_{13})$, we find

$$r = \frac{|\Delta m^2_{12}|}{|\Delta m^2_{13}|}$$

$$Y_N^T \simeq y \begin{pmatrix} e^{i\delta} s_{13} + e^{-i\alpha} s_{12} r^{1/4} \\ s_{23}\left(1 - \frac{\sqrt{r}}{2}\right) + e^{-i\alpha} r^{1/4} c_{12} c_{23} \\ c_{23}\left(1 - \frac{\sqrt{r}}{2}\right) - e^{-i\alpha} r^{1/4} c_{12} s_{23} \end{pmatrix} .$$

Inverted hierarchy:

$$Y_N^T \simeq \frac{y}{\sqrt{2}} \begin{pmatrix} c_{12} e^{i\alpha} + s_{12} e^{-i\alpha} \\ c_{12}\left(c_{23} e^{-i\alpha} - s_{23} s_{13} e^{i(\alpha-\delta)}\right) - s_{12}\left(c_{23} e^{i\alpha} + s_{23} s_{13} e^{-i(\alpha+\delta)}\right) \\ -c_{12}\left(s_{23} e^{-i\alpha} + c_{23} s_{13} e^{i(\alpha-\delta)}\right) + s_{12}\left(s_{23} e^{i\alpha} - c_{23} s_{13} e^{-i(\alpha+\delta)}\right) \end{pmatrix}$$

Y --> one single field Σ

The invariants can be written in terms of masses and mixing

* two families:

$$< \Sigma_d > = \Lambda_f \,.\, \mathrm{diag}\,(y_d)\,; \qquad < \Sigma_u > = \Lambda_f \,. V_{\mathrm{Cabibbo}}\, \mathrm{diag}(y_u)$$

$$Y_D = \begin{pmatrix} y_d & 0 \\ 0 & y_s \end{pmatrix}, \qquad Y_U = \mathcal{V}_C^\dagger \begin{pmatrix} y_u & 0 \\ 0 & y_c \end{pmatrix} \qquad V_{\mathrm{Cabibbo}} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

$$<\mathrm{Tr}\,(\,\Sigma_u\, \Sigma_u^+\,)> = \Lambda_f^{\,2}\;\, (y_u^2 + y_c^2)\,; \quad <\det\,(\,\Sigma_u\,)> = \Lambda_f^{\,2}\, y_u y_c$$

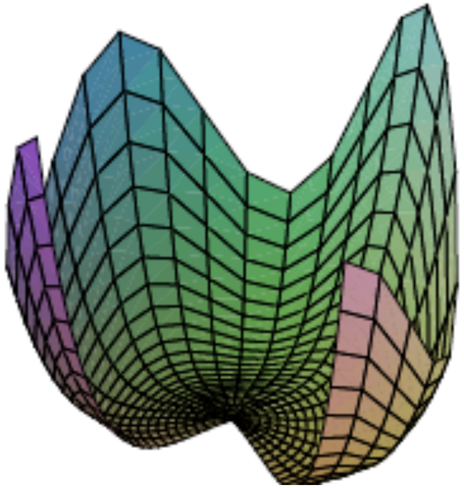
$$<\mathrm{Tr}\,(\,\Sigma_u\, \Sigma_u^+ \Sigma_d\, \Sigma_d^+\,)>=\Lambda_f^4\, [(\,y_c^2 - y_u^2)\,(\,y_s^2 - y_d^2)\, \cos 2\theta\, + \ldots]/2$$

(Alonso, Gavela, Merlo, Rigolin, arXiv 1103.2915)

Y --> one single field Σ

Minimum of the Potential

Dimension 5 Yukawa Operator



The minimum of the Potential is given by:

$$\frac{\partial V}{\partial y_i} = 0 \qquad \frac{\partial V}{\partial \theta_i} = 0$$

Take the angle for example:

$$\frac{\partial V}{\partial \theta_c} \propto (y_c^2 - y_u^2)\,(y_s^2 - y_d^2)\sin 2\theta_c = 0$$

Non-degenerate masses ⟶ $\sin 2\theta_c = 0$ No mixing !

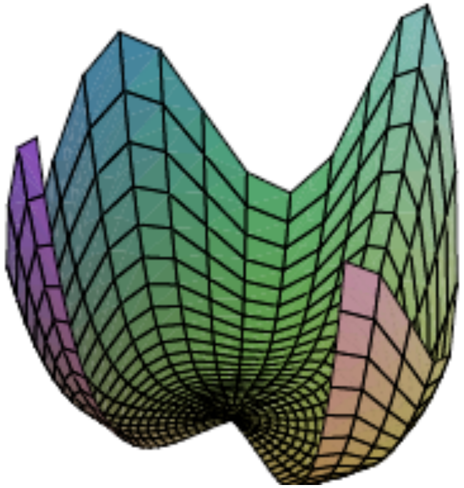
Notice also that $\frac{\partial V^{(4)}}{\partial \theta} \sim \sqrt{J}$ (Jarlskog determinant)

(Alonso, Gavela, Merlo, Rigolin, arXiv 1103.2915)

Y --> one single field Σ

Minimum of the Potential

Dimension 5 Yukawa Operator



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Non-degenerate masses $\longrightarrow \sin 2\theta_c = 0$ No mixing !

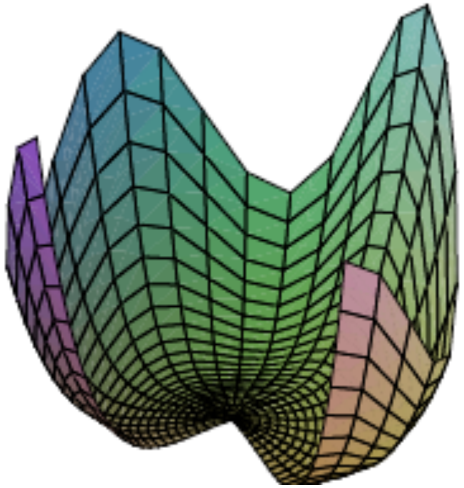
Can the actual masses and mixings fit naturally in the minimum of the Potential? e.g. adding non-renormalizable terms... **NO**

(Alonso, Gavela, Merlo, Rigolin, arXiv 1103.2915)

Y --> one single field Σ

Minimum of the Potential

Dimension 5 Yukawa Operator



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Non-degenerate masses $\sin 2\theta_c = 0$ No mixing !

Can the actual masses and mixings fit naturally in the minimum of the Potential? e.g. adding non-renormalizable terms...

NO

* Without fine-tuning, for two families the spectrum is degenerate

* To accomodate realistic mixing one must introduce wild fine tunnings of O(10^{-10}) and nonrenormalizable terms of dimension 8

(Alonso, Gavela, Merlo, Rigolin)

Y --> one single field Σ

three families

* at renormalizable level: 7 invariants instead of the 5 for two families

$$\mathrm{Tr}\left(\Sigma_u\Sigma_u^\dagger\right) \overset{vev}{=} \Lambda_f^2\left(y_t^2+y_c^2+y_u^2\right)\,,$$

$$\mathrm{Tr}\left(\Sigma_d\Sigma_d^\dagger\right) \overset{vev}{=} \Lambda_f^2\left(y_b^2+y_s^2+y_d^2\right)\,,$$

$$= \mathrm{Tr}\left(\Sigma_u\Sigma_u^\dagger\Sigma_u\Sigma_u^\dagger\right) \overset{vev}{=} \Lambda_f^4\left(y_t^4+y_c^4+y_u^4\right)\,,$$

$$= \mathrm{Tr}\left(\Sigma_d\Sigma_d^\dagger\Sigma_d\Sigma_d^\dagger\right) \overset{vev}{=} \Lambda_f^4\left(y_b^4+y_s^4+y_d^4\right)\,,$$

$$= \mathrm{Tr}\left(\Sigma_u\Sigma_u^\dagger\Sigma_d\Sigma_d^\dagger\right) \overset{vev}{=} \Lambda_f^4\left(P_0+P_{int}\right)\,,$$

$$Det\left(\Sigma_u\right) \overset{vev}{=} \Lambda_f^3 y_u y_c y_t\,,$$

$$Det\left(\Sigma_d\right)\Big| \overset{vev}{=} \Lambda_f^3 y_d y_s y_b\,,$$

Interesting angular dependence:

$$P_0 \equiv -\sum_{i<j}\left(y_{u_i}^2-y_{u_j}^2\right)\left(y_{d_i}^2-y_{d_j}^2\right)\sin^2\theta_{ij}\,,$$

$$\begin{aligned}P_{int} \equiv &\sum_{i<j,k}\left(y_{d_i}^2-y_{d_k}^2\right)\left(y_{u_j}^2-y_{u_k}^2\right)\sin^2\theta_{ik}\sin^2\theta_{jk}+\\ &-\left(y_d^2-y_s^2\right)\left(y_c^2-y_t^2\right)\sin^2\theta_{12}\sin^2\theta_{13}\sin^2\theta_{23}+\\ &+\frac{1}{2}\left(y_d^2-y_s^2\right)\left(y_c^2-y_t^2\right)\cos\delta\,\sin 2\theta_{12}\sin 2\theta_{23}\sin\theta_{13}\,,\end{aligned}$$

The real, unavoidable, problem is again mixing:

* Just one source:

$$\mathrm{Tr}\,(\,\Sigma_u\,\Sigma_u^+\,\Sigma_d\,\Sigma_d^+\,) = \Lambda_f^4\,(P_0 + P_{int})$$

P_0 and P_{int} encode the angular dependence,

$$P_0 \equiv -\sum_{i<j}\left(y_{u_i}^2 - y_{u_j}^2\right)\left(y_{d_i}^2 - y_{d_j}^2\right)\sin^2\theta_{ij}\,,$$

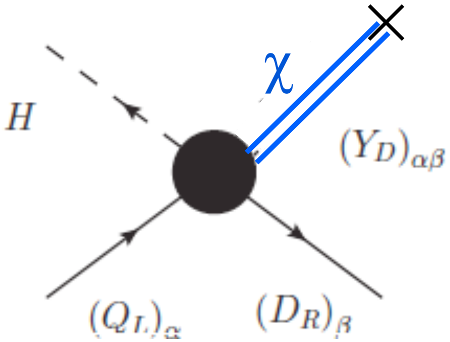
$$\begin{aligned} P_{int} \equiv & \sum_{i<j,k}\left(y_{d_i}^2 - y_{d_k}^2\right)\left(y_{u_j}^2 - y_{u_k}^2\right)\sin^2\theta_{ik}\sin^2\theta_{jk} + \\ & -\left(y_d^2 - y_s^2\right)\left(y_c^2 - y_t^2\right)\sin^2\theta_{12}\sin^2\theta_{13}\sin^2\theta_{23} + \\ & +\frac{1}{2}\left(y_d^2 - y_s^2\right)\left(y_c^2 - y_t^2\right)\cos\delta\,\sin 2\theta_{12}\sin 2\theta_{23}\sin\theta_{13} \end{aligned}$$

Sad conclusions as for 2 families:

needs non-renormalizable + super fine-tuning

Y --> quadratic in fields χ

$$Y \sim \frac{<\chi\chi^{\dagger}>}{\Lambda_f^2}$$

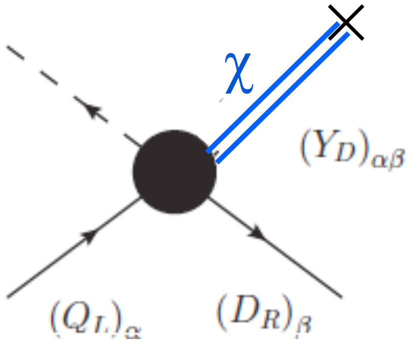


→ Automatic strong mass hierarchy and one mixing angle already at the renormalizable level

Holds for 2 and 3 families !

2) Y --> quadratic in fields χ

$$Y \sim \frac{<\chi\chi^{\dagger}>}{\Lambda_f^2}$$



→ i.e. $Y_D \sim \frac{\chi^L{}_d\ (\chi^R{}_d)^+}{\Lambda_f^2} \sim (3, 1, 1)\ (1, 1, \overline{3}) \sim (3, 1, \overline{3})$

Y --> quadratic in fields χ

It is very simple:

- a square matrix built out of 2 vectors

$$\begin{pmatrix} d \\ e \\ f \\ \vdots \end{pmatrix} (a , b , c \ldots\ldots.)$$

has only one non-vanishing eigenvalue

→ strong mass hierarchy at leading order:

-- only 1 heavy "up" quark

-- only 1 heavy "down" quark

only $|\chi|$´s relevant for scale

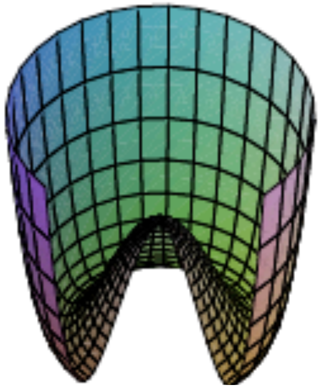
Y --> quadratic in fields χ

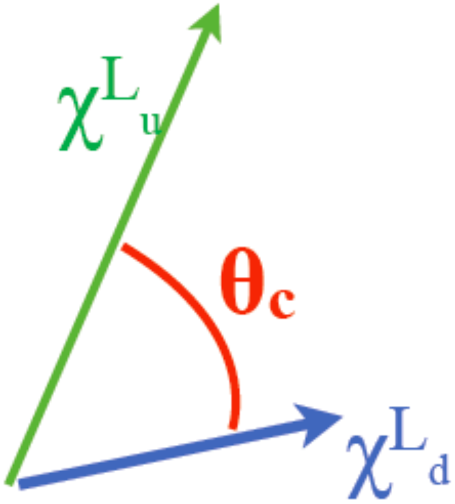
Minimum of the Potential

Dimension 6 Yukawa Operator

The invariants are:

$$\chi_u^{L\dagger}\chi_u^L\,,\qquad \chi_u^{R\dagger}\chi_u^R\,,\qquad \chi_d^{L\dagger}\chi_d^L\,,$$
$$\chi_d^{R\dagger}\chi_d^R\,,\qquad \chi_u^{L\dagger}\chi_d^L = \left|\chi_u^L\right|\left|\chi_d^L\right|\cos\theta_c\,.$$





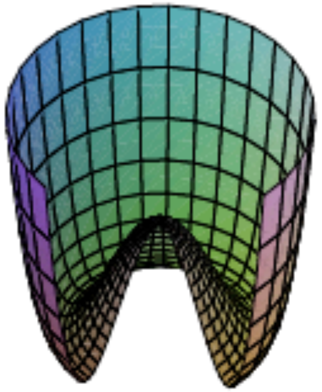
θ_c is the angle between up and down L vectors

Y --> quadratic in fields χ

Minimum of the Potential

Dimension 6 Yukawa Operator

The invariants are:



$$\chi_u^{L\dagger}\chi_u^L\,, \qquad \chi_u^{R\dagger}\chi_u^R\,, \qquad \chi_d^{L\dagger}\chi_d^L\,,$$

$$\chi_d^{R\dagger}\chi_d^R\,, \qquad \chi_u^{L\dagger}\chi_d^L = \left|\chi_u^L\right| \left|\chi_d^L\right| \cos\theta_c\,.$$

We can fit the angle and the masses in the Potential; as an example:

$$V' = \lambda_u \left(\chi_u^{L\dagger}\chi_u^L - \frac{\mu_u^2}{2\lambda_u}\right)^2 + \lambda_d \left(\chi_d^{L\dagger}\chi_d^L - \frac{\mu_d^2}{2\lambda_d}\right)^2$$

$$+\lambda_{ud} \left(\chi_u^{L\dagger}\chi_d^L - \frac{\mu_{ud}^2}{2\lambda_{ud}}\right)^2 + \cdots$$

Whose minimum sets (2 generations):

$$y_c^2 = \frac{\mu_u^2}{2\lambda_u\Lambda^2_f} \qquad y_s^2 = \frac{\mu_d^2}{2\lambda_d\Lambda^2_f} \qquad \cos\theta = \frac{\mu_{ud}^2\sqrt{\lambda_u\lambda_d}}{\mu_u\mu_d\lambda_{ud}}$$

Y --> quadratic in fields χ

Towards a realistic 3 family spectrum

e.g. replicas of χ^L , χ^R_u , χ^R_d

???

Suggests sequential breaking:

$$\mathbf{SU(3)^3} \xrightarrow[\mathbf{mt,\ mb}]{} \mathbf{SU(2)^3} \xrightarrow[\mathbf{mc,\ ms,\ \theta_C}]{} \ldots\ldots\ldots\ldots$$

$$Y_u \equiv \frac{\langle \chi^L \rangle \, \langle \chi_u^{R\dagger} \rangle}{\Lambda_f^2} + \frac{\langle \chi_u'^L \rangle \, \langle \chi_u'^{R\dagger} \rangle}{\Lambda_f^2} = \begin{pmatrix} 0 & \sin\theta \, y_c & 0 \\ 0 & \cos\theta \, y_c & 0 \\ 0 & 0 & y_t \end{pmatrix}$$

$$Y_d \equiv \frac{\langle \chi^L \rangle \, \langle \chi_d^{R\dagger} \rangle}{\Lambda_f^2} + \frac{\langle \chi_d'^L \rangle \, \langle \chi_d'^{R\dagger} \rangle}{\Lambda_f^2} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & y_s & 0 \\ 0 & 0 & y_b \end{pmatrix} .$$

Y --> quadratic in fields χ

Towards a realistic 3 family spectrum

e.g. replicas of χ^L , χ^R_u , χ^R_d

???

Suggests sequential breaking:

$$\mathbf{SU(3)^3} \xrightarrow[\text{mt, mb}]{} \mathbf{SU(2)^3} \xrightarrow[\text{mc, ms, } \theta_C]{} \text{............}$$

$$\frac{\quad}{\Lambda_f^2} \qquad \frac{\quad}{\Lambda_f^2} \qquad \begin{pmatrix} 0 & \sin\theta\, y_c & 0 \\ 0 & 0 & y_t \end{pmatrix}$$

Maybe some connection to: Berezhiani+Nesti; Ferretti et al., Calibbi et al. ??

$$Y_d \equiv \frac{\langle\chi^L\rangle\,\langle\chi_d^{R\dagger}\rangle}{\Lambda_f^2} + \frac{\langle\chi_d'^L\rangle\,\langle\chi_d'^{R\dagger}\rangle}{\Lambda_f^2} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & y_s & 0 \\ 0 & 0 & y_b \end{pmatrix} .$$

* From bifundamentals: $<\mathcal{Y}_u> = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & y_t \end{pmatrix}$

$$<\mathcal{Y}_d> = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & y_b \end{pmatrix}$$

* From fundamentals χ : y_c, y_s and θ_C

Y --> linear + quadratic in fields

Towards a realistic 3 family spectrum

Combining fundamentals and bi-fundamentals

i.e. combining d=5 and d =6 Yukawa operators

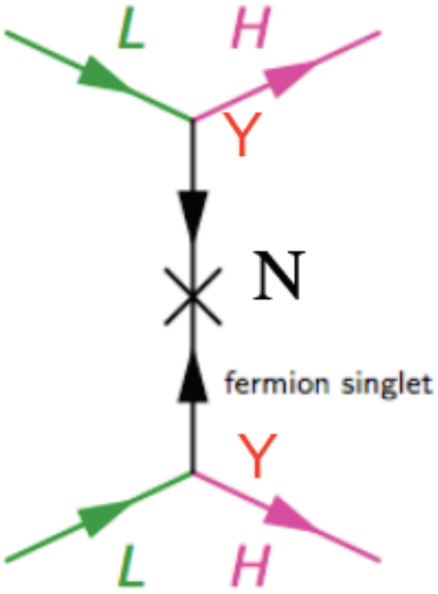
$$\Sigma_u \sim (3,\overline{3},1)\ , \qquad \Sigma_d \sim (3,1,\overline{3})\ , \qquad \Sigma_R \sim (1,3,\overline{3})\ ,$$

$$\chi_u^L \in (3,1,1)\ , \qquad \chi_u^R \in (1,3,1)\ , \qquad \chi_d^L \in (3,1,1)\ , \qquad \chi_d^R \in (1,1,3)\ .$$

The Yukawa Lagrangian up to the second order in $1/\Lambda_f$ is given by:

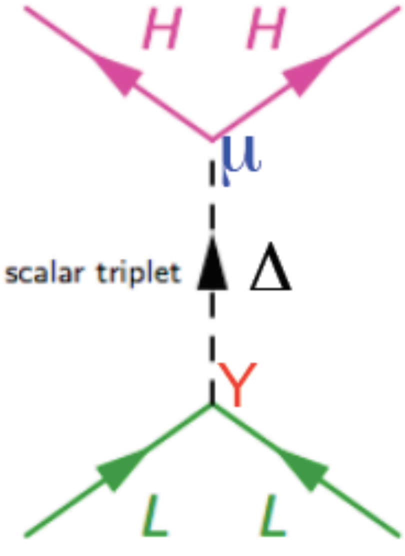
$$\mathscr{L}_Y = \overline{Q}_L \left[\frac{\Sigma_d}{\Lambda_f} + \frac{\chi_d^L \chi_d^{R\dagger}}{\Lambda_f^2} \right] D_R H + \overline{Q}_L \left[\frac{\Sigma_u}{\Lambda_f} + \frac{\chi_u^L \chi_u^{R\dagger}}{\Lambda_f^2} \right] U_R \tilde{H} + \text{h.c.}\ ,$$

- Three types of models yield the Weinberg operator at tree level



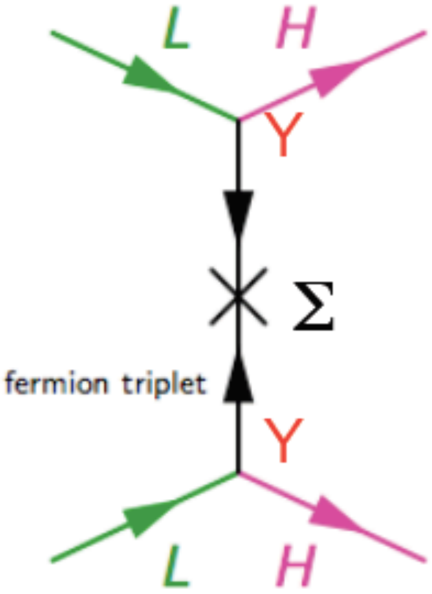
Type I

$$m_\nu \sim v^2 \; Y_N{}^T \frac{1}{M_N} Y_N$$



Type II

$$m_\nu \sim v^2 \; Y_\Delta \; \frac{\mu}{M_\Delta{}^2}$$



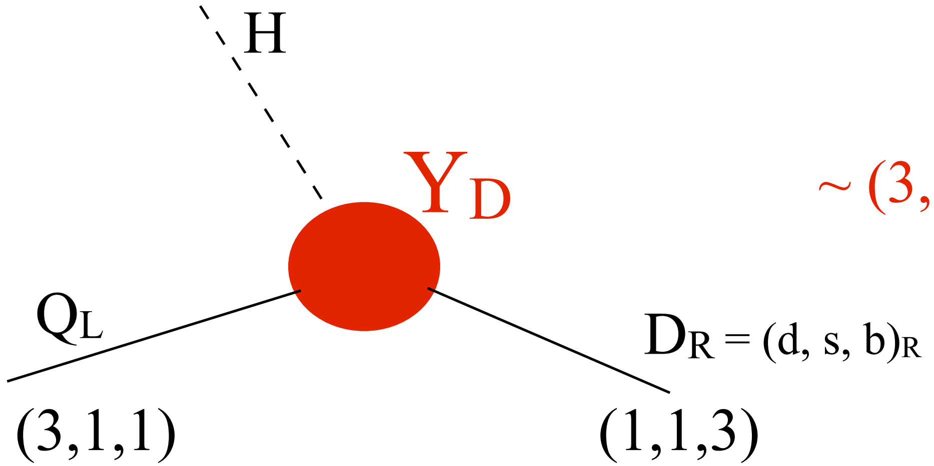
Type III

$$m_\nu \sim v^2 \; Y_\Sigma{}^T \frac{1}{M_\Sigma} Y_\Sigma$$

Use the flavour symmetry of the SM with masless fermions:

$$G_f = SU(3)_{Q_L} \times SU(3)_{U_R} \times SU(3)_{D_R} \times U(1)_S$$

which is broken by Yukawas:



H

Y_D

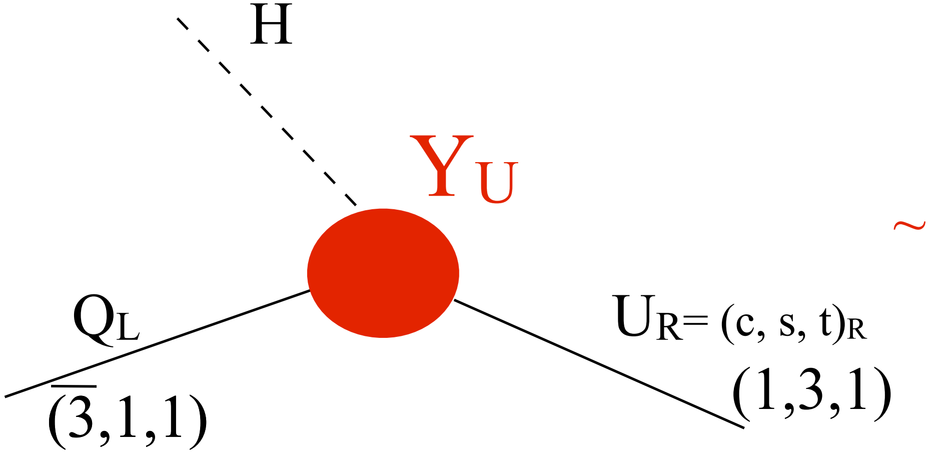
Q_L

$(3,1,1)$

$D_R = (d, s, b)_R$

$(1,1,3)$

$\sim (3,1,3)$



H

Y_U

Q_L

$(\bar{3},1,1)$

$U_R = (c, s, t)_R$

$(1,3,1)$

$\sim (3,3,1)$

*In the O(2)model used before: $\tanh 2\omega = \dfrac{y^2 - y'^2}{y^2 - y'^2}$ and

$$\mathrm{tg}2\theta = \sin 2\alpha \frac{2\sqrt{m_{\nu_2} m_{\nu_1}}}{m_{\nu_2} - m_{\nu_1}} \frac{y^2 - y'^2}{y^2 - y'^2}$$

$\boxed{\alpha = \pi/4 \text{ or } 3\pi/4}$

*If we had used instead a flavor SU(2)model $\sinh 2\omega = 0$ -->NO MIXING

Some good ideas:

“Partial compositeness”:

D.B. Kaplan-Georgi in the 80s proposed a composite Higgs:

* **Higgs light because the whole Higgs doublet is multiplet of goldstone bosons**

They explored **SU(5)--> SO(5)**.

Explicit breaking of SU(2)xU(1) symmetry via external gauged U(1)
(Kaplan, Georgi, Dimopoulos, Banks, Dugan, Galison)

Nowadays **SO(5)--> SO(4)** and explicit breaking via SM weak interaction
(Contino, Nomura, Pomarol; Agashe, Contino, Pomarol; Giudice, Pomarol, Ratazzi, Grojean; Contino, Grojean, Moretti; Azatov, Galloway, Contino...)

SO(6) --> SO(5) to get also DM (Frigerio, Pomarol, Riva, Urbano)

Anarchy: alive with not so small θ_{13} and not θ_{23} not maximal

no symmetry in the lepton sector, just random numbers

$$m_\nu \sim \begin{pmatrix} \sim 1 & \sim 1 & \sim 1 \\ \sim 1 & \sim 1 & \sim 1 \\ \sim 1 & \sim 1 & \sim 1 \end{pmatrix}$$

- Does not relate mixing to spectrum
- Does not address both quarks and leptons

(Hall, Murayama, Weiner; Haba, Murayama; De Gouvea, Murayama...
Going towards hierarchy: Altarelli, Feruglio, Masina, Merlo)

μ-->e conversion

We performed an exact one-loop calculation, but for obvious approximations:

-- m_e=0 compared to m_μ

-- $m_{\nu 1}= m_{\nu 2} = m_{\nu 3} = 0$ compared to heavy neutrino masses (that is, assume $m_N > eV$)

-- higher orders in the external momentum neglected versus M_W, as usual

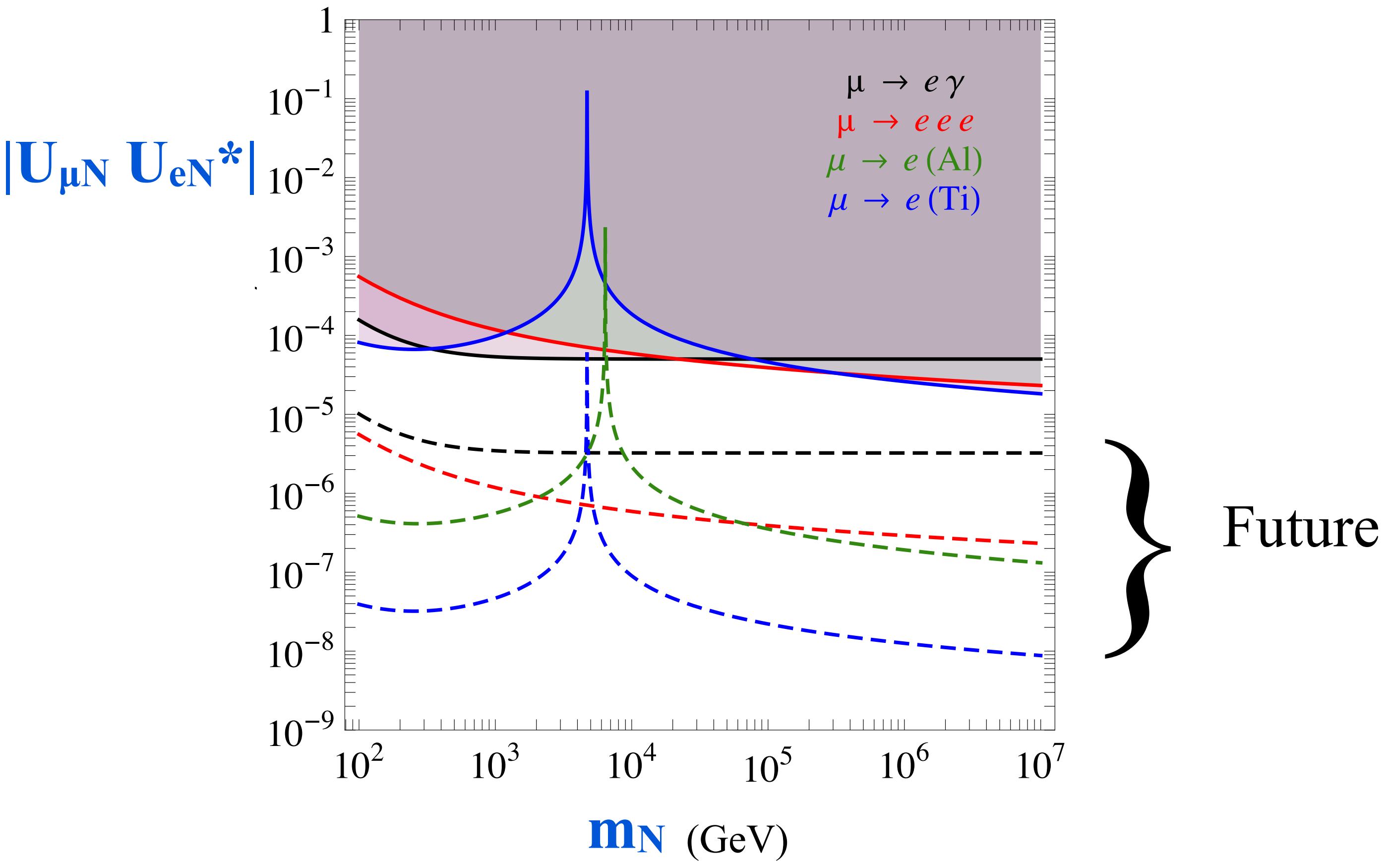
We did many checks to our results, e.g.:

* For “dipole” for factors .... check with b --> s l^+ l^-

* For the other form factors .... agreement with μ --> eee form factors

....

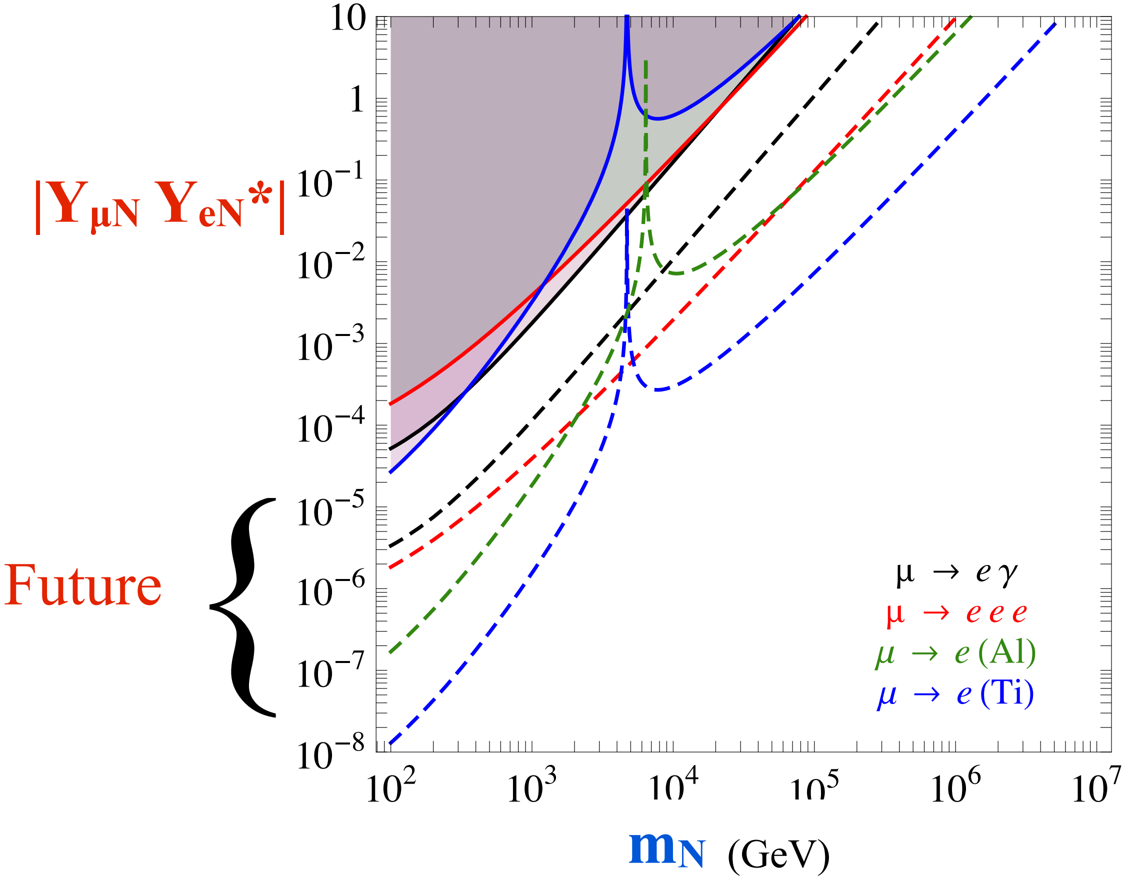
$|U_{\mu N}\, U_{eN}^*|$ versus m_N



Sensitivity up to $m_N \sim 6000$ TeV for Ti

For the particular case of seesaw I : $U_{lN} \sim Y\, v / M$

$|Y_{\mu N}\, Y_{eN}^{*}|$ versus m_N



Sensitivity up to $m_N \sim 6000$ TeV for Ti

* Large mass $m_N >> m_W$

When one **m_N** scale dominates (e.g. degenerate heavy neutrinos or hierarchical) the ratio of any two μ-e transitions only depends on **m_N** (Chu, Dhen, Hambye 11)

Besides, μ-e conversion vanishes at some large **m_N**

(Dinh, Ibarra, Molinaro, Petcov 12)

For instance, we find that for light nuclei (αZ<<1), it vanishes as

$$m_N^2\Big|_0 = M_W^2 \exp\left(\frac{\frac{9}{8}(A-Z)+\left(\frac{9}{8}+\frac{31s_W^2}{12}\right)Z}{\frac{3}{8}(A-Z)+\left(\frac{4s_W^2}{3}-\frac{3}{8}\right)Z}\right)$$

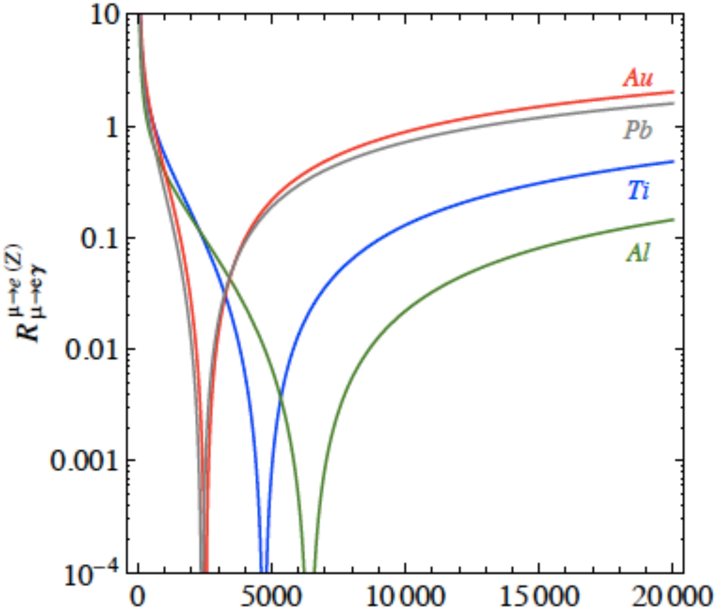
(Alonso, Dhen, Hambye, B.G.)

exponential sensitivity

The ratios of two e-μ transitions....

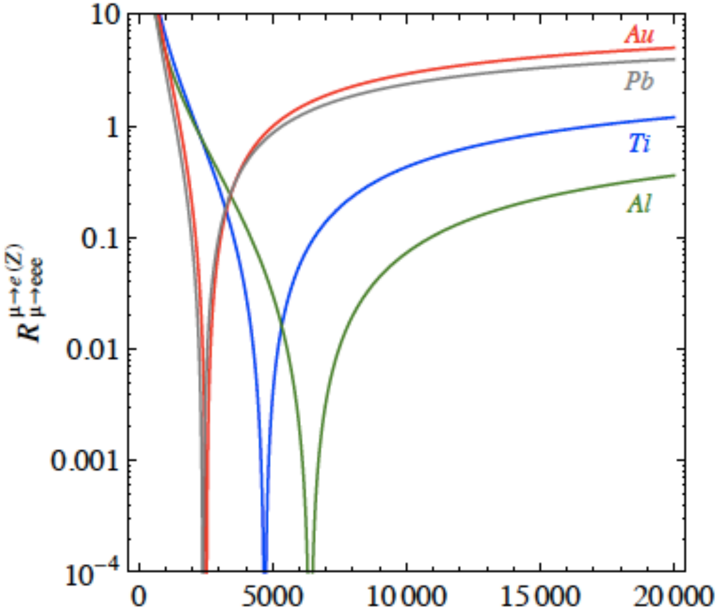
we obtain:

$$\frac{\mu\text{-e conversion}}{\mu\text{--> e }\gamma}$$



m_N (GeV)

$$\frac{\mu\text{-e conversion}}{\mu\text{--> e e e}}$$



m_N (GeV)

...typically vanishes for m_N in 2- 7 TeV range

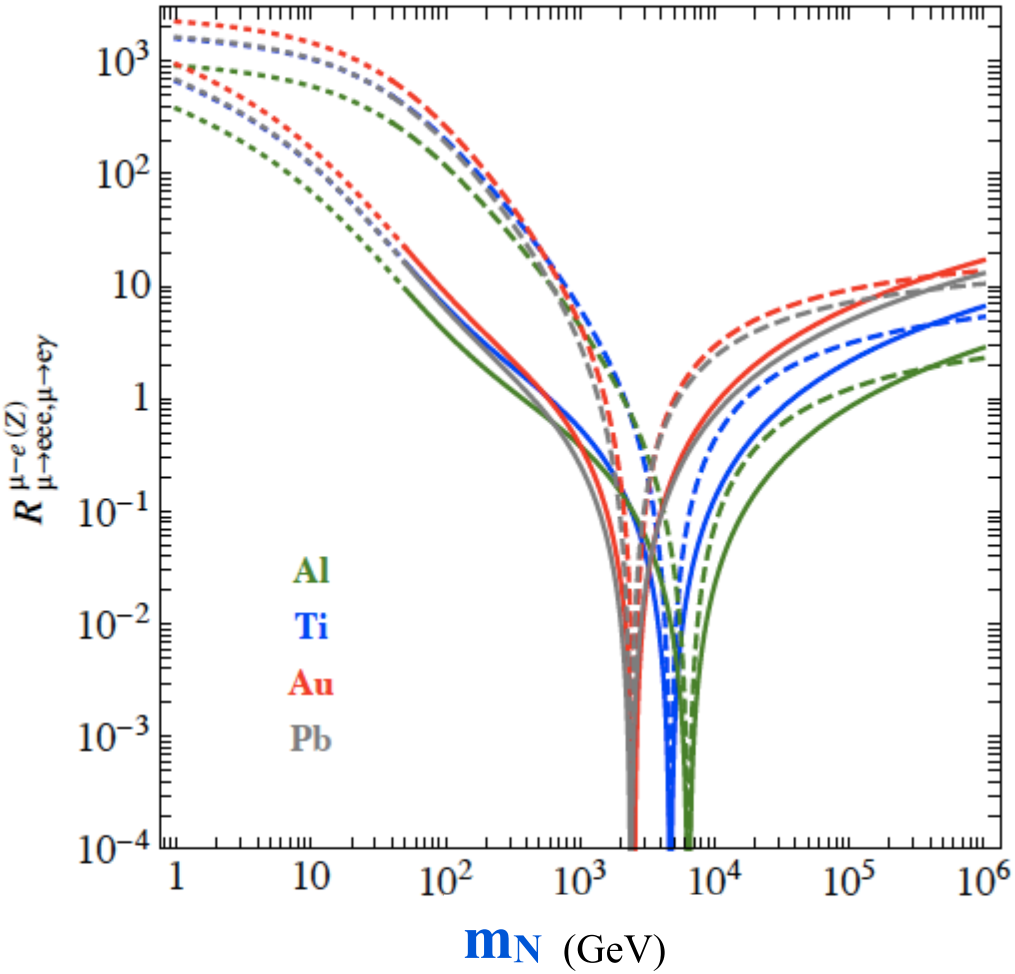
(Alonso, Dhen, Hambye, B.G.)

* Low mass regime eV $<<$ m_N $<<$ m_W

.... de Gouvea 05...

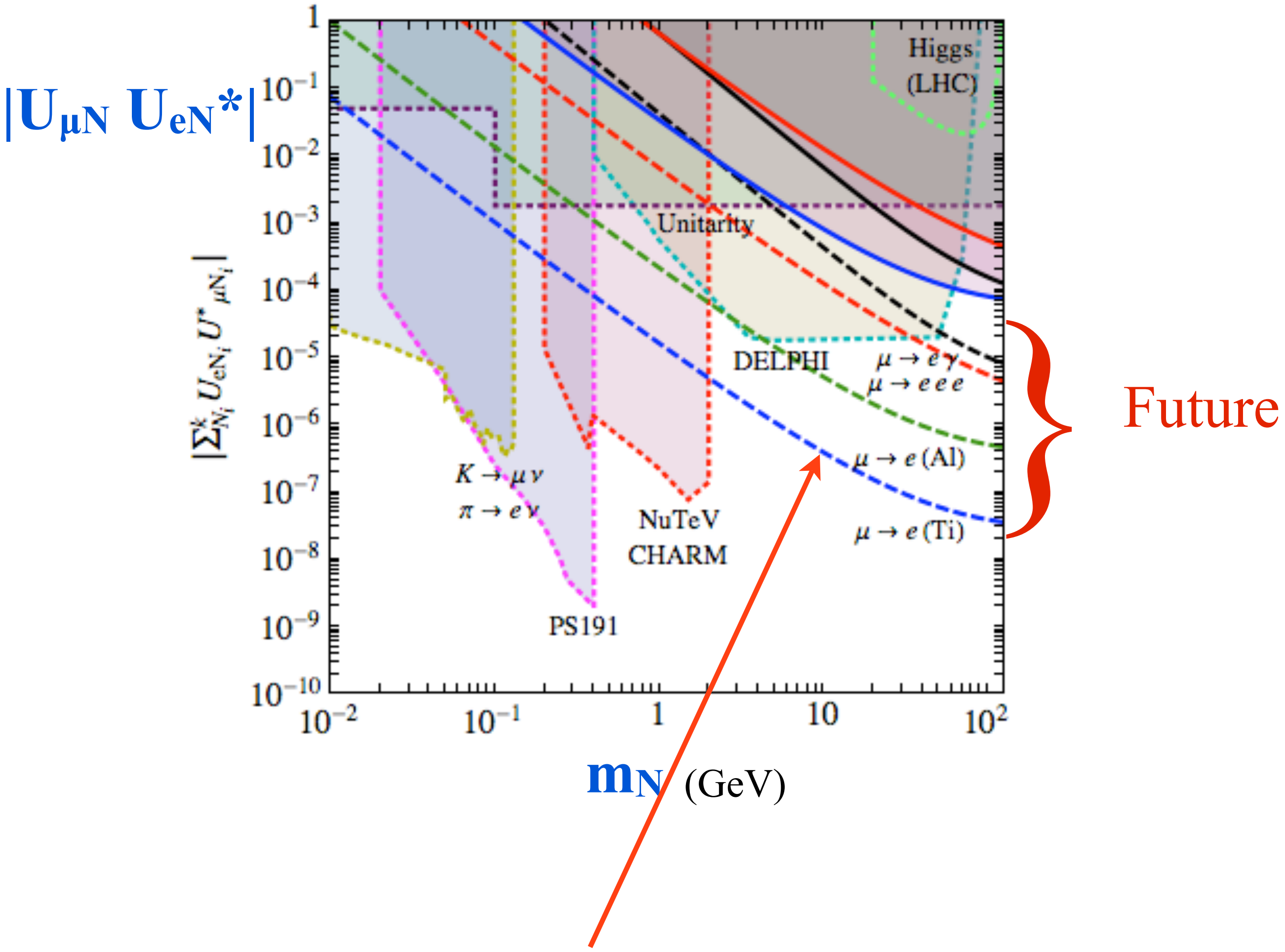
* Low mass regime eV << m_N << m_W

μ--> e conversion does not vanish for low mass



(Alonso, Dhen, Hambye, B.G.)

* Low mass regime eV << m_N << m_W



This experiment (considered alone) will probe masses down to m_N=2MeV

For instance, in the minimal seesaw I, Lepton number scale and flavour scale linked:

$$\mathscr{L}_{M_\nu} = \begin{pmatrix} 0 & Y^T v \\ Y v & M_N \end{pmatrix}$$

$$-\mathcal{L}_{\text{seesaw I}} = \overline{L} H Y_E E_R + \overline{L} \tilde{H} Y N + M \overline{N} N^c + h.c.$$

$$m_\nu = Y \frac{v^2}{M} Y^T \qquad U_{lN} \sim \frac{Y}{M}$$

LHC is more competitive for concrete seesaw models:

Low M, large Y is typical of seesaws with approximate Lepton Number conservation

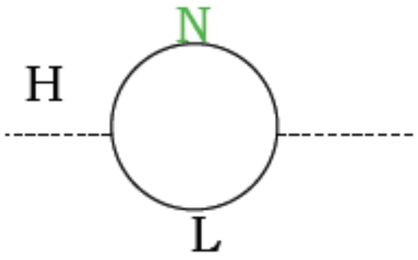
U(1)$_{LN}$

(-> ~ degenerate heavy neutrinos)

These models separate the flavor and the lepton number scale

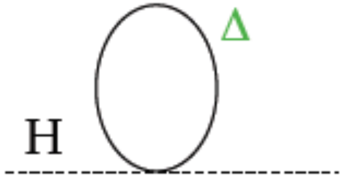
Wyler+Wolfenstein 83, Mohapatra+Valle 86, Branco+Grimus+Lavoura 89, Gonzalez-Garcia+Valle 89, Ilakovac+Pilaftsis 95, Barbieri+Hambye+Romanino 03, Raidal+Strumia+Turzynski 05, Kersten+Smirnov 07, Abada+Biggio+Bonnet+Gavela+Hambye 07, Shaposhnikov 07, Asaka+Blanchet 08, Gavela+Hambye+D. Hernandez+ P. Hernandez 09

M~1 TeV is suggested by electroweak hierarchy problem



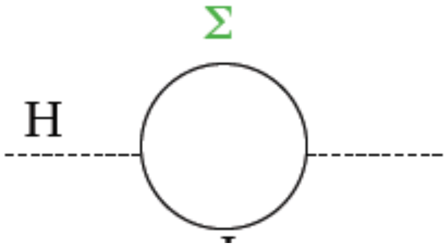
$$\delta m_H^2 = -\frac{Y_N^\dagger Y_N}{16\pi^2}\left[2\Lambda^2 + 2M_N^2 \log\frac{M_N^2}{\Lambda^2}\right]$$

(Vissani, Casas et al., Schmaltz)



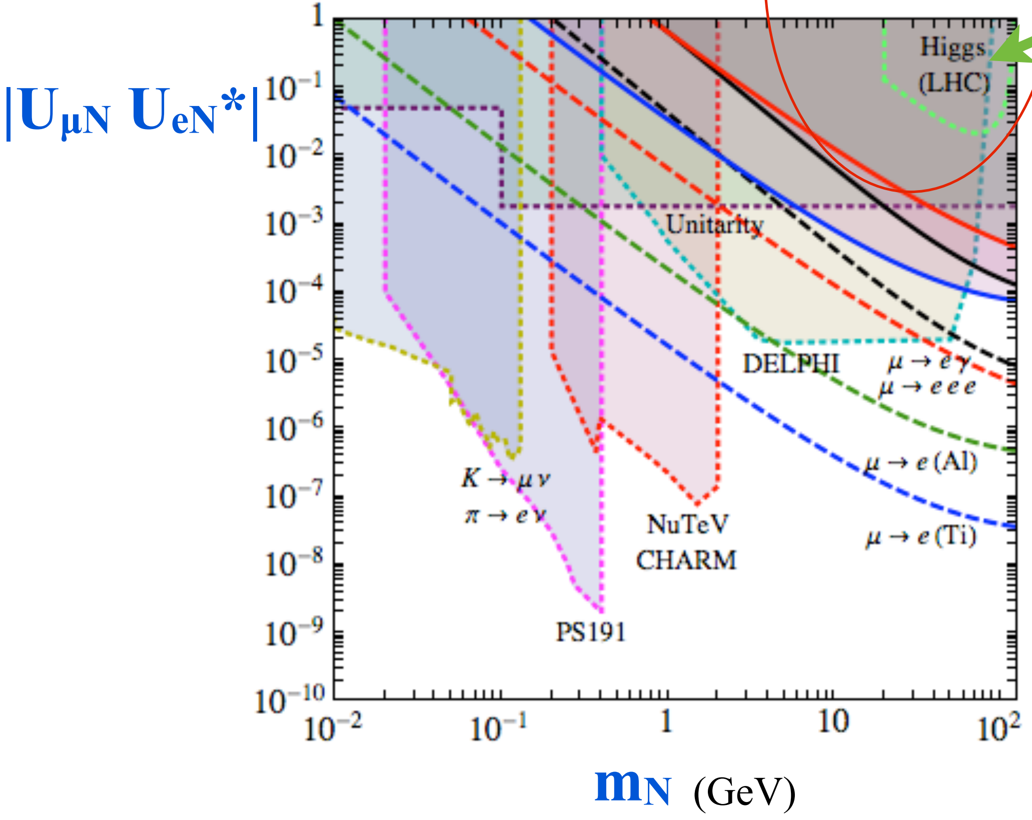
$$\delta m_H^2 = -3\frac{\lambda_3}{16\pi^2}\left[\Lambda^2 + M_\Delta^2\left(\log\frac{M_\Delta^2}{\Lambda^2} - 1\right)\right] - \frac{\mu_\Delta^2}{2\pi^2}\log\left(\left|\frac{M_\Delta^2 - \Lambda^2}{M_\Delta^2}\right|\right)$$

(Abada, Biggio, Bonnet, Hambye, M.B.G.)



$$\delta m_H{}^2 = -3\frac{Y_\Sigma^\dagger Y_\Sigma}{16\pi^2}\left[2\Lambda^2 + 2M_\Sigma^2 \log\frac{M_\Sigma^2}{\Lambda^2}\right]$$

let us zoom
on this region



LHC is more competitive for concrete seesaw models.

For instance those with approximate $U(1)_{LN}$ and low scale ~TeV (inverse seesaws etc.)

Higgs decay (LHC)

e.g. **H --> ν N**

Pilaftsis92....Chen et al.10, Dev+Franceschini+Mohapatra 12, Cely+Ibarra+Molinaro+Petcov

We get for the model-independent rate:

$$Br(h \to \nu N) = \frac{\alpha_W}{8 M_W^2 \Gamma_h^{tot}} \sum_i^k \left(|U_{eN_i}|^2 + |U_{\mu N_i}|^2 + |U_{\tau N_i}|^2 \right) m_h m_{N_i}^2 \left(1 - \frac{m_{N_i}^2}{m_h^2} \right)^2$$

and using $|\Sigma_i U_{eN_i} U^*_{\mu N_i}| < \Sigma_{i,\alpha} |U_{\alpha N_i}|^2$

Just TWO heavy neutrinos

$$\mathcal{L}_{\mathcal{M}_\nu} = (\bar{\ell}_L\,,\,\bar{N}^c\,,\,\bar{N}'^c)\begin{pmatrix} 0 & vY & vY' \\ vY^T & 0 & \mathbf{M} \\ vY'^T & \mathbf{M} & 0 \end{pmatrix}\begin{pmatrix} \ell_L^c \\ N \\ N' \end{pmatrix}$$

--> One massless neutrino and only one Majorana phase α

the Yukawas are determined up to their overal magnitude

$$\text{N.H.} \qquad Y = \frac{y}{\sqrt{m_{\nu_2} + m_{\nu_3}}}\, U_{PMNS} \begin{pmatrix} 0 \\ -i\sqrt{m_{\nu_2}}e^{-i\alpha} \\ \sqrt{m_{\nu_3}}e^{i\alpha} \end{pmatrix}$$

Gavela, Hambye, Hernandez[2]
Raidal, Strumia, Turszynski

Leptons

Just TWO heavy neutrinos

$$\mathcal{L}_{\mathcal{M}_\nu} = (\bar{\ell}_L\,,\,\bar{N}^c\,,\,\bar{N}'^c)\begin{pmatrix} 0 & vY & vY' \\ vY^T & 0 & \mathbf{M} \\ vY'^T & \mathbf{M} & 0 \end{pmatrix}\begin{pmatrix} \ell_L^c \\ N \\ N' \end{pmatrix}$$

the Yukawas are determined up to their overal magnitude

$$\text{N.H.} \qquad Y = \frac{y}{\sqrt{m_{\nu_2}+m_{\nu_3}}}\,U_{PMNS}\begin{pmatrix} 0 \\ -i\sqrt{m_{\nu_2}}e^{-i\alpha} \\ \sqrt{m_{\nu_3}}e^{i\alpha} \end{pmatrix}$$

The flavour symmetry is $G_f =$ $U(3)_{\ell_L} \times U(3)_{E_R} \times O(2)_N$

(Alonso, Gavela, D. Hernandez, Merlo, Rigolin) seesaw I with **Just TWO heavy neutrinos**

$$\mathcal{L}_{\mathcal{M}_\nu} = (\bar{\ell}_L\,,\,\bar{N}^c\,,\,\bar{N}'^c) \begin{pmatrix} 0 & vY & vY' \\ vY^T & 0 & \mathbf{M} \\ vY'^T & \mathbf{M} & 0 \end{pmatrix} \begin{pmatrix} \ell_L^c \\ N \\ N' \end{pmatrix}$$

Lepton number scale and flavour scale distinct

Raidal, Strumia, Turszynski
Gavela, Hambye, Hernandez^2

Just TWO heavy neutrinos

$$\mathcal{L}_{\mathcal{M}_\nu} = (\bar{\ell}_L\,,\,\bar{N}^c\,,\,\bar{N}'^c)\begin{pmatrix} 0 & vY & vY' \\ vY^T & 0 & \mathbf{M} \\ vY'^T & \mathbf{M} & 0 \end{pmatrix}\begin{pmatrix} \ell_L^c \\ N \\ N' \end{pmatrix}$$

$$m_\nu = \mathbf{Y}\,\frac{v^2}{M}\,\mathbf{Y}'^{\,T} \qquad U_{lN} \sim \frac{\mathbf{Y}}{M}$$

--> Lepton number conserved conserved if either Y or Y’ vanish:

Raidal, Strumia, Turszynski
Gavela, Hambye, Hernandez^2

* What is the role of the neutrino flavour group?

e.g. O(2)$_{NR}$ ... leptons

e.g. seesaw with approximately conserved lepton number

$$\mathcal{L}_{\mathcal{M}_\nu} = (\bar{\ell}_L \, , \, \bar{N}^c \, , \, \bar{N}'^c) \begin{pmatrix} 0 & vY & vY' \\ vY^T & 0 & \mathbf{M}^T \\ vY'^T & \mathbf{M} & 0 \end{pmatrix} \begin{pmatrix} \ell_L^c \\ N \\ N' \end{pmatrix}$$

* What is the role of the neutrino flavour group?

e.g. O(2)$_{NR}$ ... leptons

e.g. seesaw with approximately conserved lepton number

$$\mathcal{L}_{mass} = \bar{\ell}_L \phi Y_E E_R + \bar{\ell}_L \tilde{\phi} \tilde{Y}_\nu (N_1, N_2)^T + M(\overline{N}_1 N_1^c + \overline{N}_2 N_2^c) + h.c.$$

$$\tilde{Y}_\nu = \frac{1}{\sqrt{2}} U_{PMNS} \, f_{m_\nu} \begin{pmatrix} y+y' & -i(y-y') \\ i(y-y') & y+y' \end{pmatrix}$$

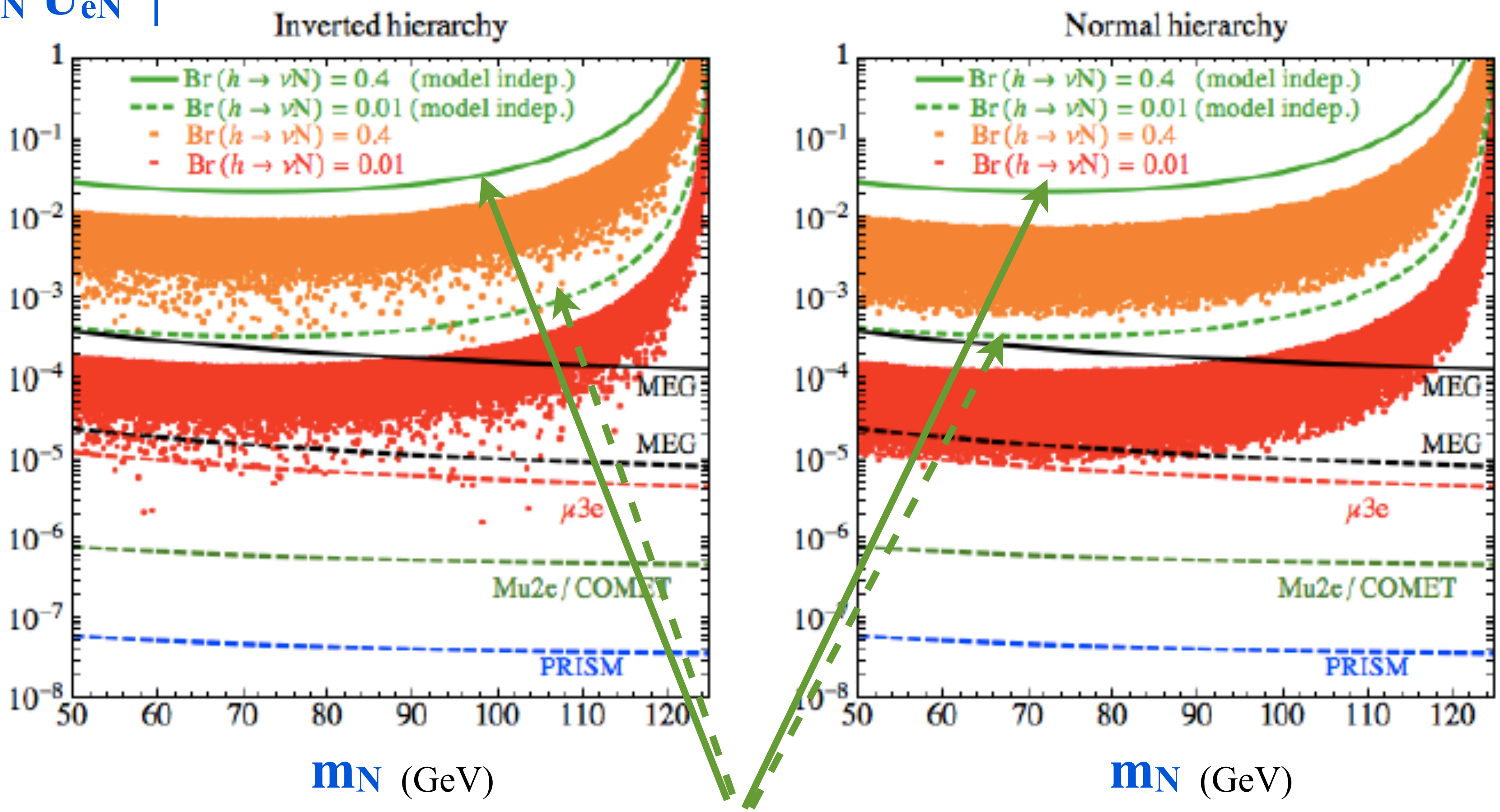
$$U(3)_{\ell_L} \times U(3)_{E_R} \times O(2)_N$$

$$Y_E = \frac{<\mathcal{Y}_E>}{\Lambda_f} \sim (3\,,\,\bar{3}\,,1)\,; \quad (Y\,,\,Y') = \frac{<\mathcal{Y}_\nu>}{\Lambda} \sim (3\,,\,1\,,2)$$

$$<\mathcal{Y}_E> \propto \begin{pmatrix} m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & m_\tau \end{pmatrix} \quad <\mathcal{Y}_\nu> \propto U_{PMNS} \begin{pmatrix} 0 & 0 \\ \sqrt{m_{\nu_2}} & 0 \\ 0 & \sqrt{m_{\nu_3}} \end{pmatrix} \begin{pmatrix} -iy & iy' \\ y & y' \end{pmatrix}$$

Varying the CP phases, we get:

$|U_{\mu N}\, U_{eN}^*|$



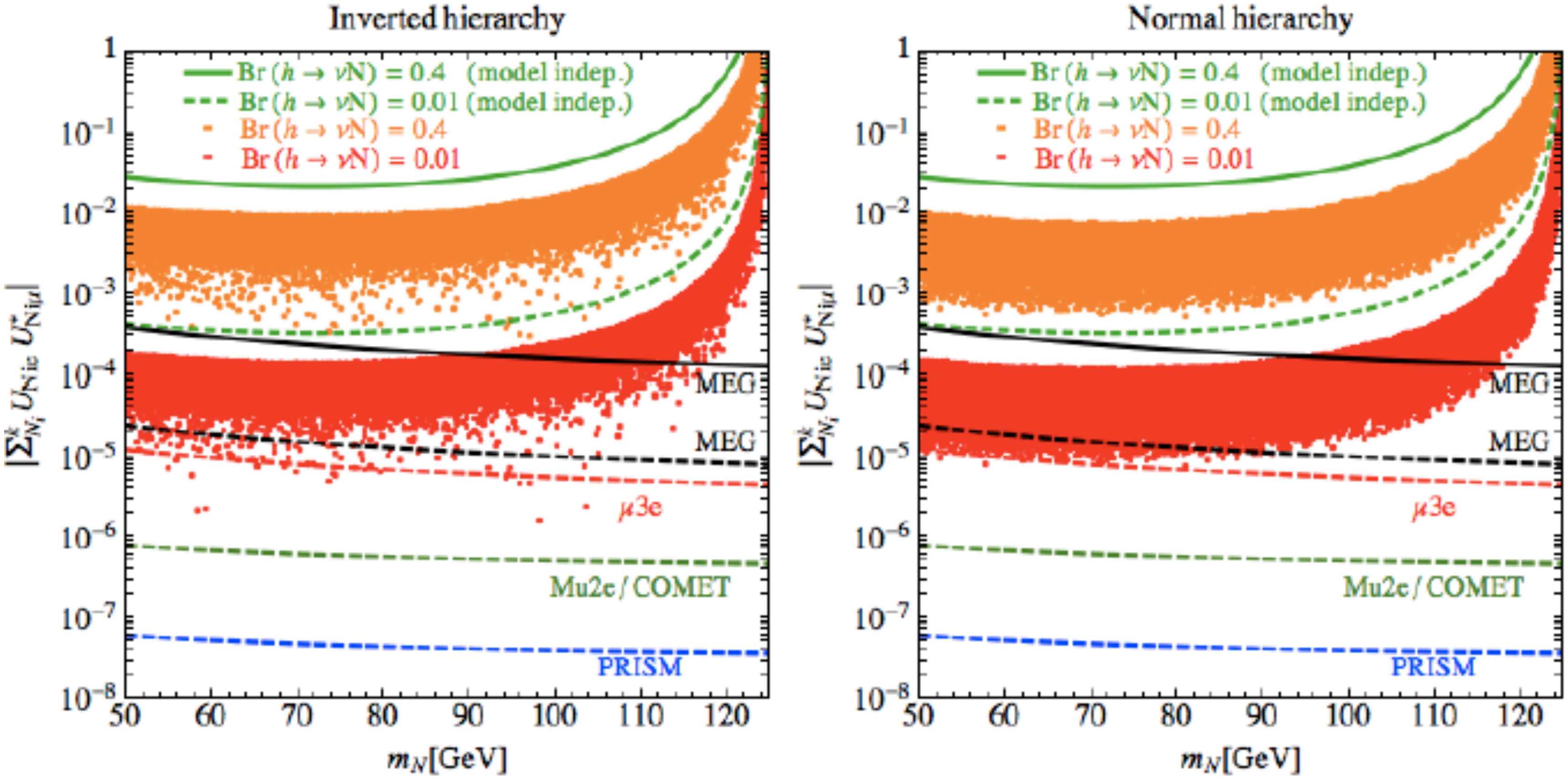
m_N (GeV) m_N (GeV)

Absolute bound in previous plot

$|U_{\mu N}\, U_{eN}^*|$ versus m_N

(Alonso, Dhen, Gavela, Hambye)

Varying the CP phases α and δ, we get:



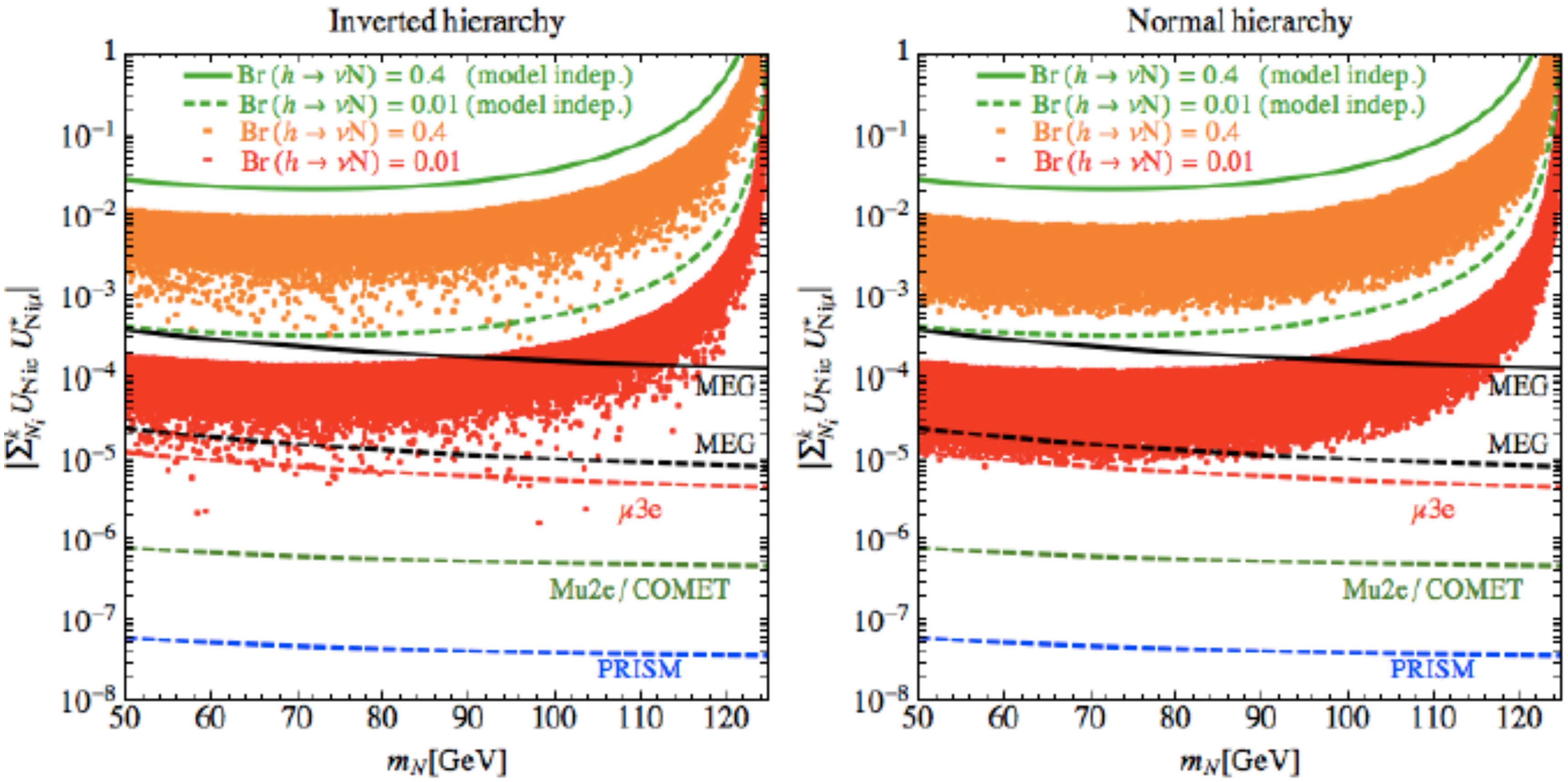
Orange and red model-dependent bounds limited by:

upper boundary: (α= π/2, δ= 0) (α=π, δ=3π/2)

lower boundary: ~(α=-π/2, δ= 0) (α=-π/4, δ=0)

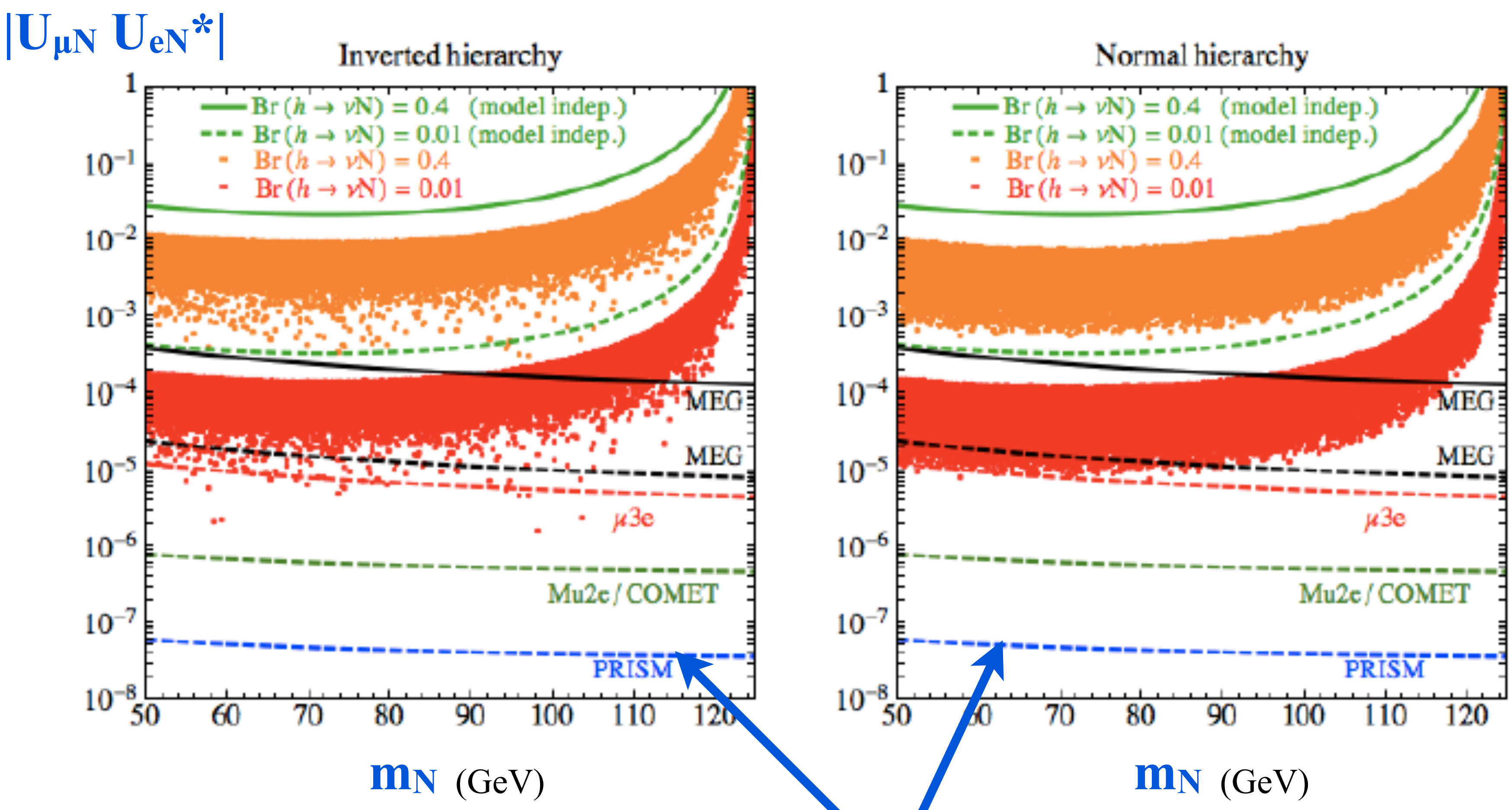
~ it could be consistent with Cely et al. 12, for α~0, δ~ 0

Varying the CP phases α and δ, we get:



For inverted hierarchy: some very low points for which μ-->e very small, because the Yukawas involved ---> 0 for particular values of α and δ (Alonso et al. 09, Alonso 08, Chu+Dhen+Hambye 11....)

Varying the CP phases, we get:



In any case, LHC expected sensitivity negligible compared with that of future μ--> e conversion expts.

* What is the role of the neutrino flavour group?

e.g. $O(2)_{NR}$ ... leptons

e.g. seesaw with approximately conserved lepton number

$$\mathcal{L}_{\mathcal{M}_\nu} = (\bar{\ell}_L\, , \bar{N}^c\, , \bar{N}'^c) \begin{pmatrix} 0 & vY & vY' \\ vY^T & 0 & \mathbf{M}^T \\ vY'^T & \mathbf{M} & 0 \end{pmatrix} \begin{pmatrix} \ell_L^c \\ N \\ N' \end{pmatrix}$$

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e.g. seesaw with approximately conserved lepton number

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$$< \mathcal{Y}_{\mathrm{E}} > \propto \begin{pmatrix} \mathrm{m_e} & 0 & 0 \\ 0 & \mathrm{m_\mu} & 0 \\ 0 & 0 & \mathrm{m_\tau} \end{pmatrix} \quad < \mathcal{Y}_{\nu} > \propto U_{PMNS} \begin{pmatrix} 0 & 0 \\ \sqrt{m_{\nu_2}} & 0 \\ 0 & \sqrt{m_{\nu_3}} \end{pmatrix} \begin{pmatrix} -iy & iy' \\ y & y' \end{pmatrix}$$

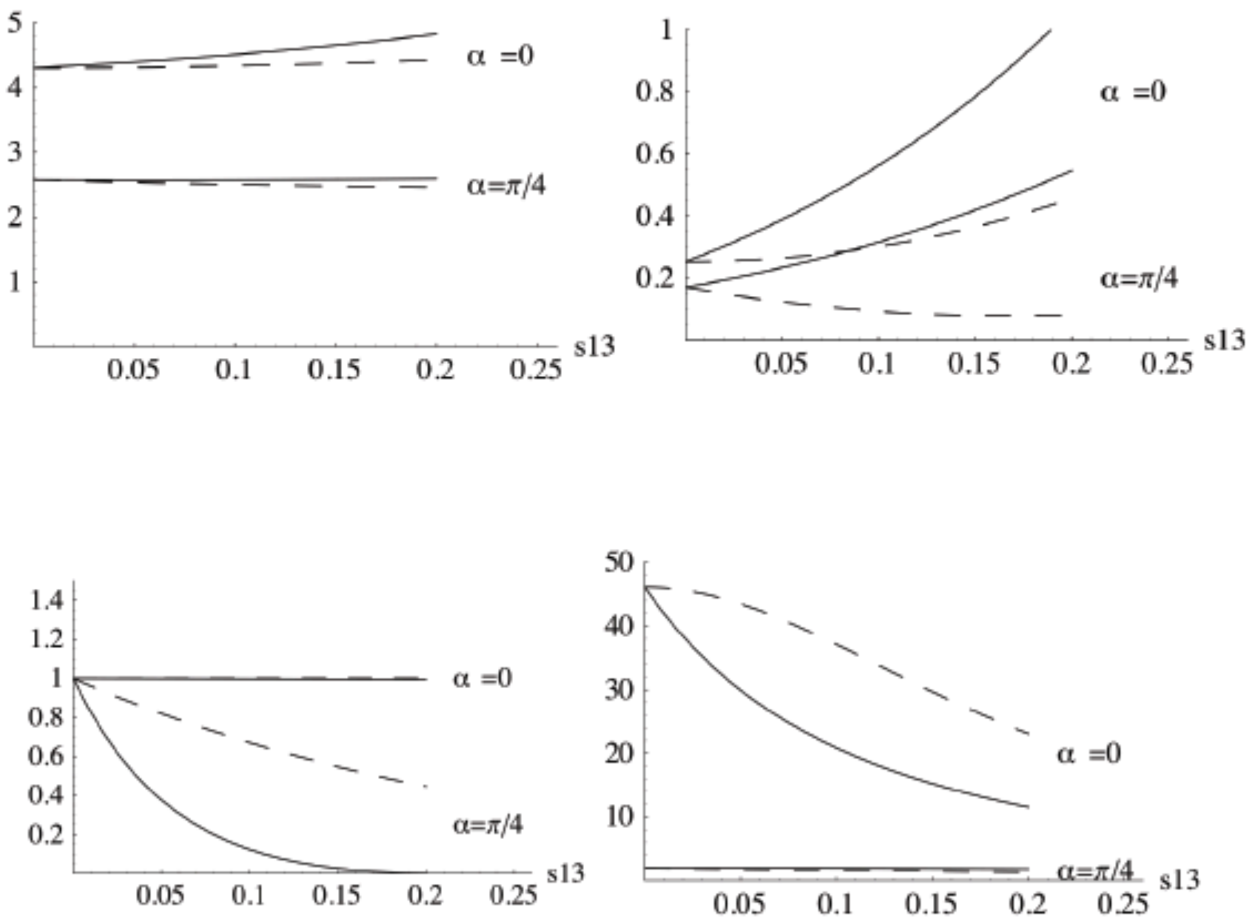
* e-μ, μ-τ etc. oscillations and rare decays studied:

Gavela, Hambye, Hernandez^2 ; .....

$Br(\mu \to e\gamma)/Br(\tau \to e\gamma)$ $Br(\mu \to e\gamma)/Br(\tau \to \mu\gamma)$

NH

IH



Gavela, Hambye, Hernandez^2 ;

Degeneracy in the Majorana phase α

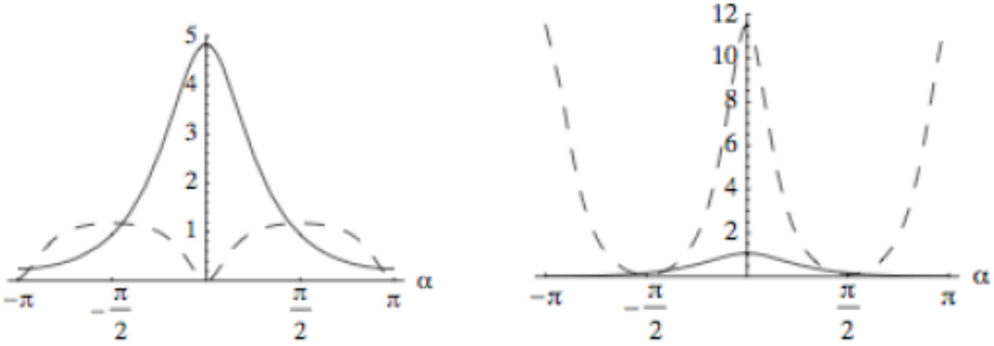
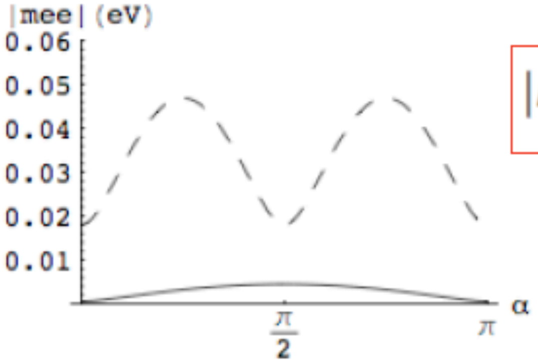


Figure 3: Left: Ratio $B_{e\mu}/B_{e\tau}$ for the normal hierarchy (solid) and the inverse hierarchy (dashed) as a function of α for $(\delta, s_{13}) = (0, 0.2)$. Right: the same for the ratio $B_{e\mu}/B_{\mu\tau}$.



$$|m_{ee}|_{IH} \simeq |s_{12}^2 e^{-2i\alpha} - c_{12}^2 e^{2i\alpha}|$$

Figure 5: m_{ee} as a function of α for the normal (solid) and inverted (dashed) hierarchies, for $(\delta, s_{13}) = (0, 0.2)$.

Gavela, Hambye, Hernandez^2 ;

$$\boxed{B_{\mu\to e\gamma} \propto |Y_{N_e} Y_{N_\mu}|^2}$$

cancellations
for large θ$_{13}$

i.e.

$$Y_N^T \simeq y \begin{pmatrix} e^{i\delta} s_{13} + e^{-i\alpha} s_{12} r^{1/4} \\ s_{23}\left(1 - \frac{\sqrt{r}}{2}\right) + e^{-i\alpha} r^{1/4} c_{12} c_{23} \\ c_{23}\left(1 - \frac{\sqrt{r}}{2}\right) - e^{-i\alpha} r^{1/4} c_{12} s_{23} \end{pmatrix} \qquad r = \frac{|\Delta m^2_{12}|}{|\Delta m^2_{13}|}$$

Normal hierarchy

* Alonso + Li, 2010, MINSIS report:
possible suppresion of μ-e transitions for large θ$_{13}$

- * e-μ, μ-τ etc. oscillations and rare decays studied:

  Gavela, Hambye, Hernandez^2 09 ; .....

- * Alonso + Li, 2010: possible suppresion of μ-e transitions

  ->important impact of ν_μ - ν_τ at a near detectors

$$\boxed{B_{\mu\to e\gamma} \propto |Y_{N_e} Y_{N_\mu}|^2}$$

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Normal hierarchy

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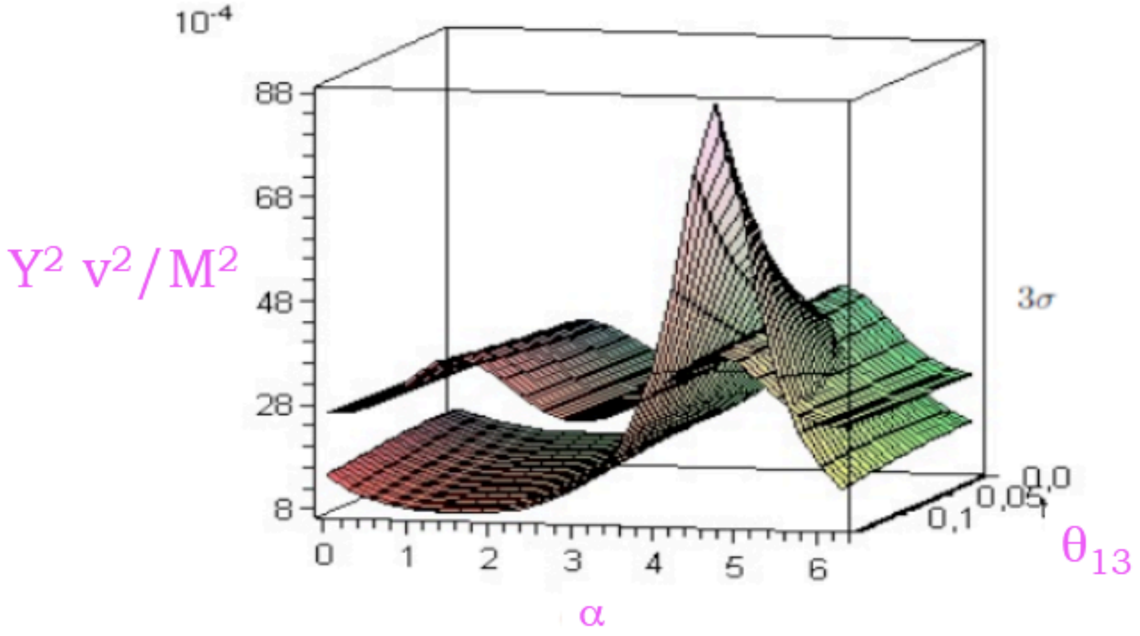


We find that there are regions where an experiment as MINSIS would improve the present bounds on our Model

For type III version of our 2 N model, signals observable at LHC up to Λ~ 500 GeV for 30 fb^{-1}

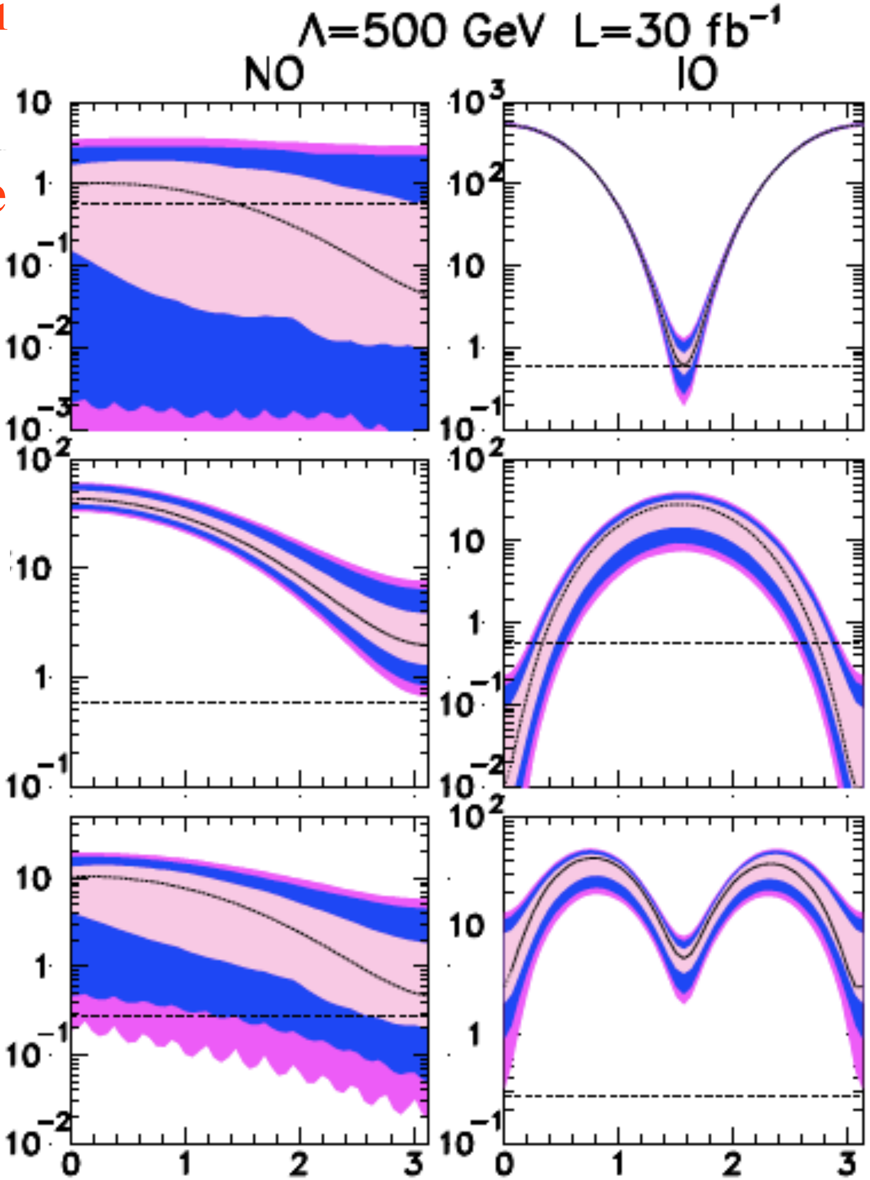
Eboli,
Gonzalez-Fraile,
Gonzalez-Garcia

$$pp \to \ell_a^\pm \ell_b^\mp jjjj$$

Nee

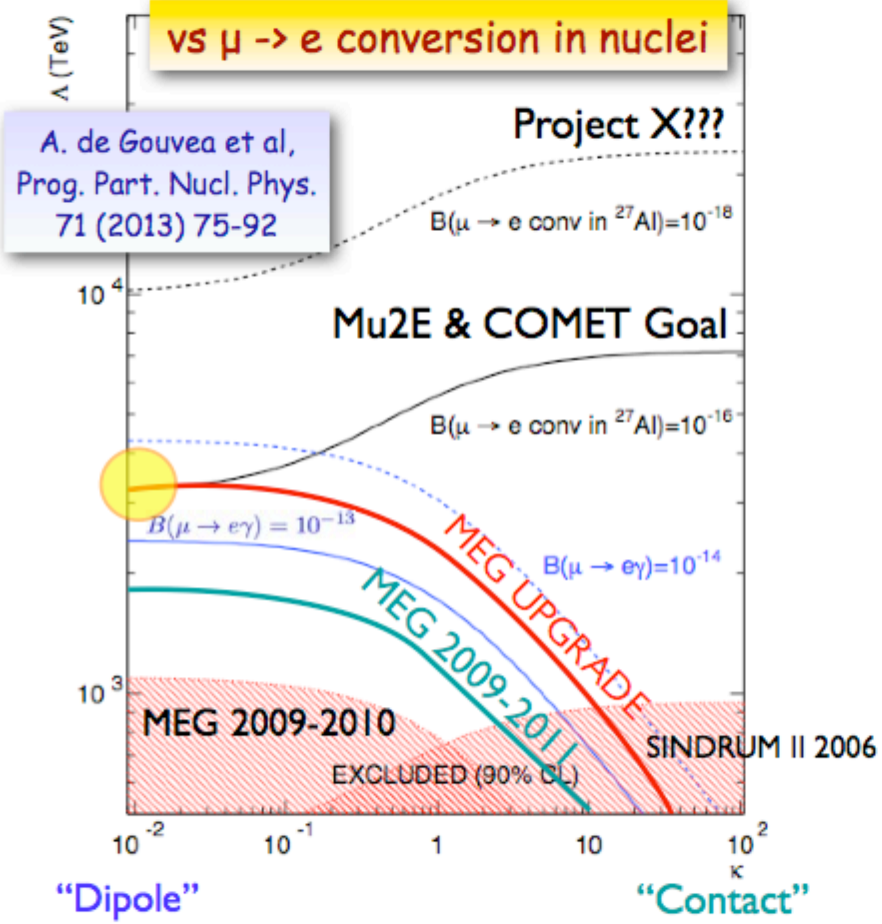
Neμ

Neτ



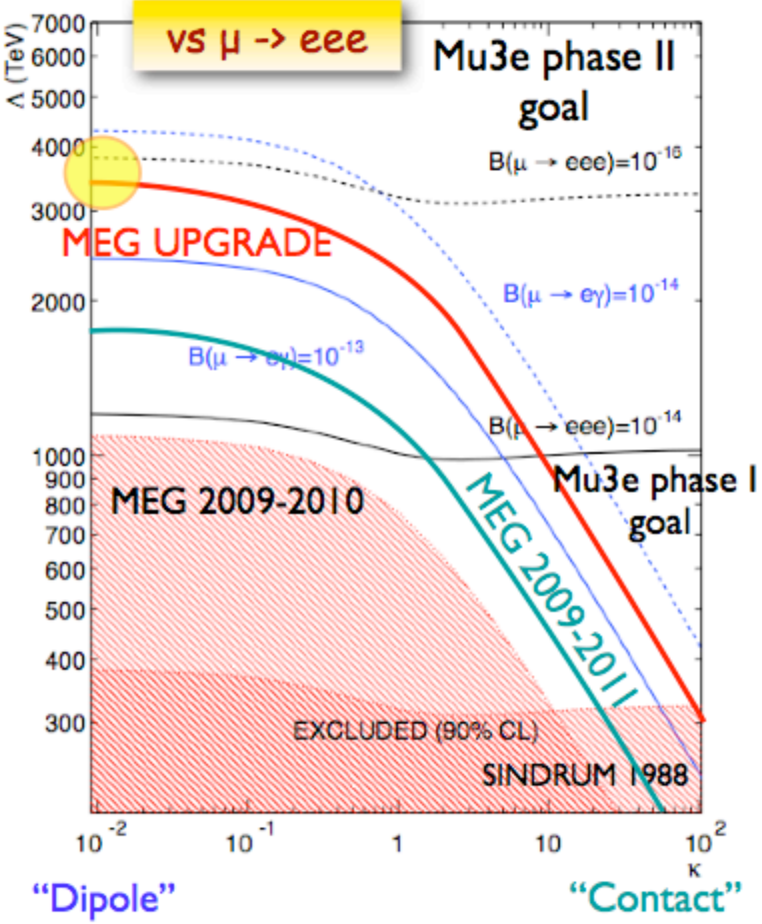
α **α** <-- Majorana phase

Sensitivity Comparison



vs μ -> e conversion in nuclei

A. de Gouvea et al,
Prog. Part. Nucl. Phys.
71 (2013) 75-92

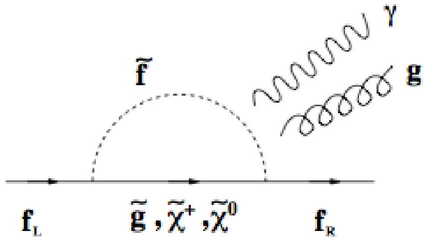


vs μ -> eee

MEG upgrade goal is competitive with next generation experiments for "dipole-type" coupling !!!!

The FLAVOUR WALL for BSM

i) Typically, BSMs have **electric dipole moments** at one loop
i.e susy MSSM:

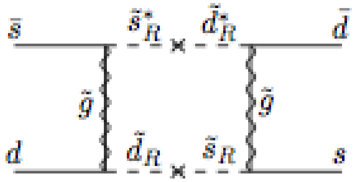


< 1 loop in SM ---> **Best (precision) window of new physics**

ii) **FCNC**

i.e susy MSSM:

$K^0 - \overline{K}^0$ mixing



$\mu \to e\gamma$

competing with SM at one-loop

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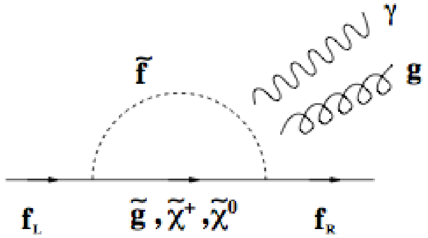

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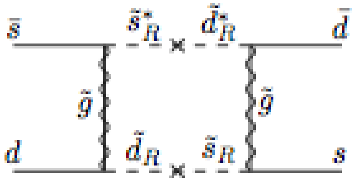

$\mu \to e$ conversion (MEG, μ2e...)

competing with SM at one-loop

Cabibbo’s dream

Beyond Standard Model because

1) **Experimental evidence** for new physics:

*** **"Dark energy"/cosmological cte.**

*** **Neutrino masses**

*** **Dark matter**

** **Matter-antimatter asymmetry**

2) **Uneasiness with SM fine-tunings**

Dark portals

Only three singlet combinations in SM with d < 4:

$H^+ H$	Scalar
$B_{\mu\nu}$	Vector
$\bar{L}\ H$	Fermionic

Any hidden sector, singlet under SM, can couple to the dark portals

Dark portals

Only three singlet combinations in SM with d < 4:

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Yukawa coupling

fermion singlets $\mathbf{\Psi}$ = “right-handed” neutrino

Analysis of SM-**DM** with higer-dimensional ops. (d>= 4) starting:

- with and withour flavour associated to DM:

$$\frac{1}{\Lambda_{DM}^2}\,\overline{\mathbf{Q}}_{\alpha}\,\gamma_{\mu}\mathbf{Q}_{\beta}\;\;\overline{\boldsymbol{\psi}}_{DM\gamma}\,\gamma^{\mu}\,\boldsymbol{\psi}_{DM\delta}$$

Flavour can stabilize DM (Batel et al.)

example of check: **Decoupling limits**

* Large mass $m_N >> m_W$

In the seesaw, for m_N-> ∞ the remaining theory is renormalizable (SM) --> rate must vanish then.
Our results do decouple for $x_N = m_N^2/M_W^2 \gg 1$

$$\begin{aligned} \Gamma &\sim (\log x_N)^2/x_N^2\,, && \text{for } \mu \to \text{eee} \quad \text{and} \quad \mu \to \text{e conversion}\,, \\ \Gamma &\sim 1/x_N^2\,, && \text{for } \mu \to \text{e}\gamma\,. \end{aligned}$$

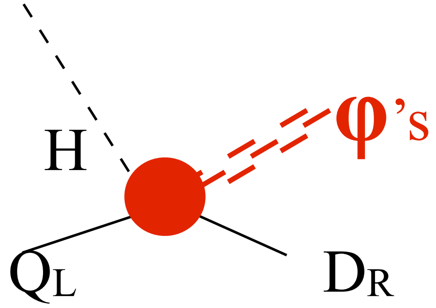
* Low mass $m_N << m_W$

they also vanish for m_N --> 0

$x_N = m_N^2/M_W^2 \ll 1$

$$\begin{aligned} \Gamma &\sim x_N^2(\log x_N)^2\,, && \text{for } \mu \to \text{eee} \quad \text{and} \quad \mu \to \text{e conversion}\,; \\ \Gamma &\sim x_N^2\,, && \text{for } \mu \to \text{e}\gamma\,. \end{aligned}$$

Some good ideas:



Frogatt-Nielsen '79: $\mathbf{U(1)_{flavour}}$ symmetry

$$Y \sim \left(\frac{<\varphi>}{\Lambda}\right)^n$$

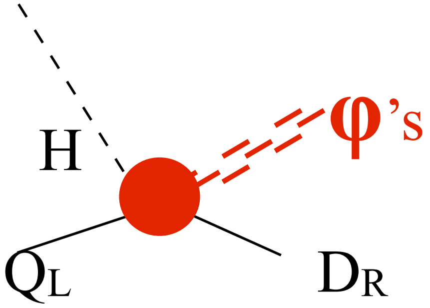
- Yukawa couplings are effective couplings,
- Fermions have $U(1)_{\text{flavour}}$ charges

$$\mathbf{Y}\ Q\ H\ q_R \;=\; \left(\frac{<\varphi>}{\Lambda}\right)^n Q\ H\ q_R$$

e.g. n=0 for the top, n large for light quarks, etc.

--> FCNC ?

A good idea with continuous groups:



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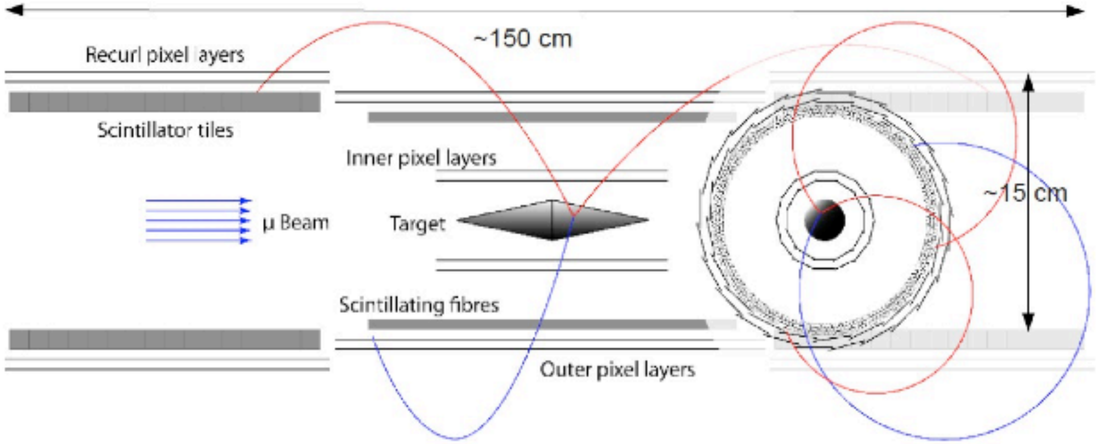
e.g. n=0 for the top, n large for light quarks, etc.

--> **FCNC ?**

PROPOSED SEARCH FOR $\mu^+ \rightarrow e^+ e^+ e^-$ @ PSI (Mu3e)

A search for
$\mu^+ \rightarrow e^+ e^+ e^-$
down to
BR ~ 10^{-16}



Anarchy

no symmetry in the lepton sector, just random numbers

$$m_\nu \sim \begin{pmatrix} \sim 1 & \sim 1 & \sim 1 \\ \sim 1 & \sim 1 & \sim 1 \\ \sim 1 & \sim 1 & \sim 1 \end{pmatrix}$$

- Does not relate mixing to spectrum
- Does not address both quarks and leptons

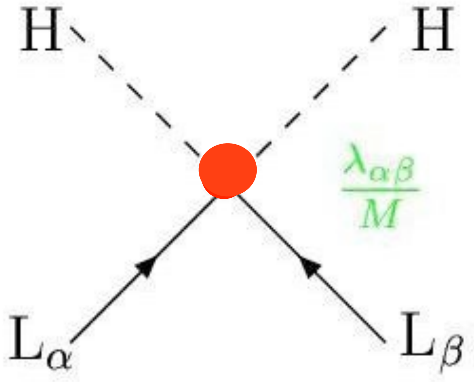
(Hall, Murayama, Weiner; Haba, Murayama; De Gouvea, Murayama...
Going towards hierarchy: Altarelli, Feruglio, Masina, Merlo)

ν masses beyond the SM

The Weinberg operator $O^{d=5}$

Dimension 5 operator:

$$\lambda/M\ \underbrace{(\overline{L^c}\ L\ \tilde{H}^+ H)}_{O^{d=5}} \rightarrow \lambda v^2/M\ (\overline{\nu_L{}^c}\nu)$$



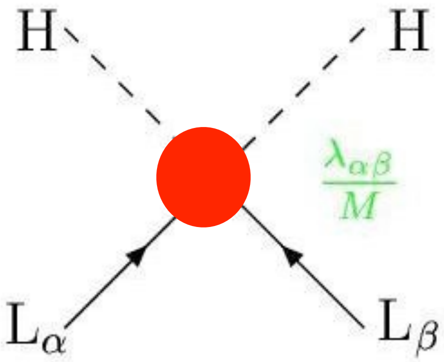
It's unique → very special role of ν masses:
lowest-order effect of higher energy physics

This mass term violates lepton number (B-L)
→ Majorana neutrinos

$O^{d=5}$ *is common to all models of Majorana* ν*s*

ν masses beyond the SM : tree level

$2 \times 2 = 1 + 3$



$SU(2) \, x U(1)_{em}$ inv.

$\lambda/M \, (L \, L \, H \, H)$

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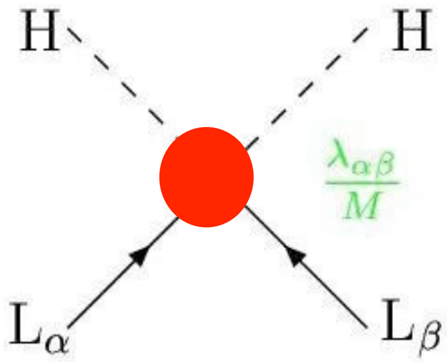

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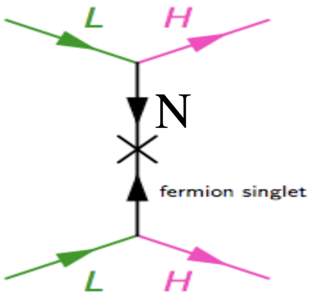
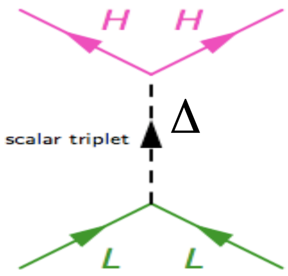
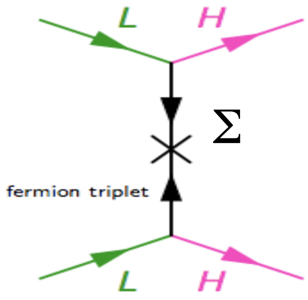


Type I



Type II



Type III

Dark portals

There are only three d≤4 combinations of SM and singlet fields:

$H^{+} H \mathbf{S}$ Scalar

$B_{\mu\nu} \mathbf{V}_{\mu\nu}$ Vector

$\overline{L}\ H\ \mathbf{\Psi}$ Fermionic

Any hidden sector, singlet under SM, can couple to the dark portals

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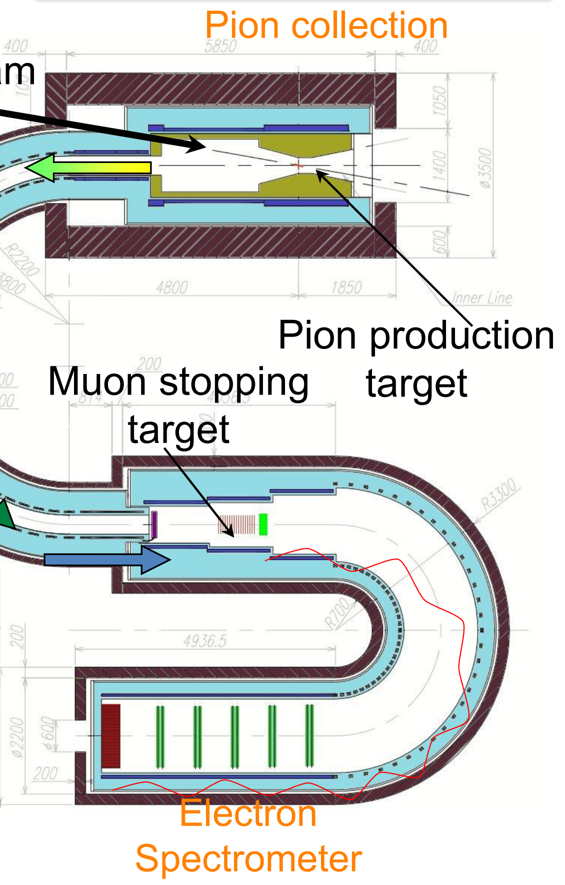
Yukawa coupling

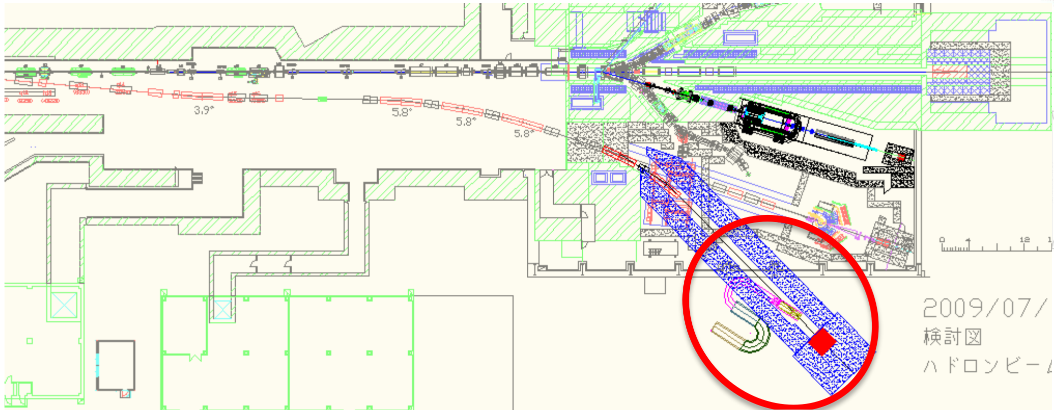
fermion singlets $\mathbf{\Psi}$ = “right-handed” neutrino

COMET μ-e conv. search

Phase-I phys run in 2017
Full COMET run in 2021-2022

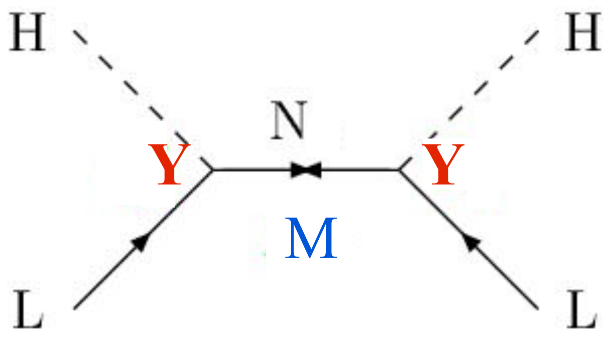
- Search for cLFV mu-e conv.
  - **10^{-16}** sensitivity (Target S.E.S. 2.6 × 10^{-17})
  - Improve **O(10^4)** than present upper bound such as SINDRUM-II BR[$\mu^- + Au \rightarrow e^- + Au$] < 7 × 10^{-13}
- Signature: 105MeV monochromatic electron
- Beam requirement
  - 8GeV bunched slow extraction
  - 1.6x10^{21} pot needed to reach goal
  - 7 uA (56kW) x 4 SN year (4x10^7sec)
  - Extinction <10^{-9}





courtesy of Yoshi Kuno

Type I seesaw

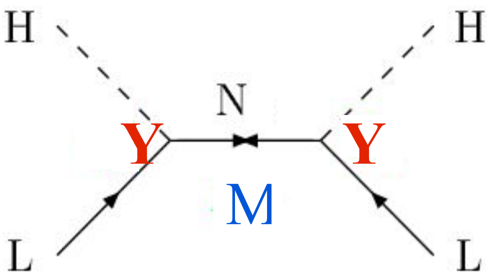


Fermionic Singlet
Seesaw (or type I)

2 x 2 = ① + 3

$$m_\nu \sim v^2 \, \mathbf{c}^{d=5} = Y \frac{v^2}{M} Y^{T}$$

Type I seesaw

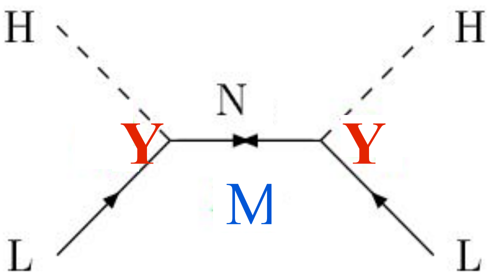


$$\mathcal{L}_{M_\nu} = \begin{pmatrix} 0 & \mathbf{Y}^{T} v \\ \mathbf{Y}\, v & \mathbf{M} \end{pmatrix}$$

$$-\mathcal{L}_{\text{seesaw I}} = \overline{L}\, H\, Y_E E_R + \overline{L}\, \tilde{H}\, Y N + M \overline{N} N^c + h.c.$$

$$m_\nu = \mathbf{Y} \frac{v^2}{M} \mathbf{Y}^{\mathbf{T}}$$

Type I seesaw



$$\mathscr{L}_{M_\nu} = \begin{pmatrix} 0 & Y^T v \\ Y v & M \end{pmatrix}$$

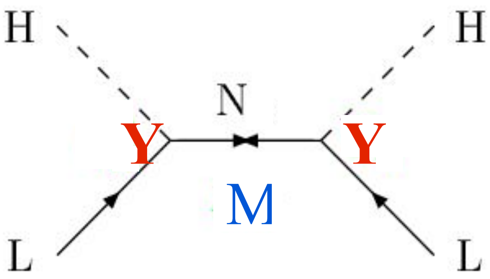
usual Dirac term

Majorana mass allowed by gauge invariance

$$-\mathcal{L}_{\text{seesaw I}} = \overline{L}\, H\, Y_E E_R + \overline{L}\, \tilde{H}\, Y N + M \overline{N} N^c + h.c.$$

$$m_\nu = Y \frac{v^2}{M} Y^T$$

Type I seesaw



$$\mathscr{L}_{M_\nu} = \begin{pmatrix} 0 & \mathbf{Y}^{T} v \\ \mathbf{Y}\, v & \mathbf{M} \end{pmatrix}$$

$$-\mathcal{L}_{\text{seesaw I}} = \overline{L}\, H\, Y_E E_R + \overline{L}\, \tilde{H}\, Y N + M \overline{N} N^c + h.c.$$

$$m_\nu = \mathbf{Y} \frac{v^2}{M} \mathbf{Y}^{\mathbf{T}}$$

$\mathbf{Y} \sim 1$ for $M \sim M_{GUT}$

$\mathbf{Y} \sim 10^{-6}$ for $M \sim \text{TeV}$