

Radiative Corrections in CCQE νN scattering

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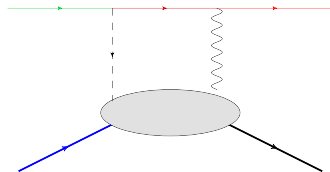
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NuFact13

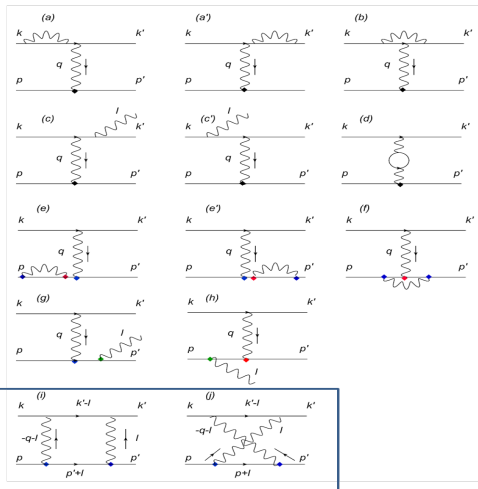


- $\nu_\mu \rightarrow \nu_e$: θ_{13} measurement and then CP violation parameter, in present and future neutrino experiments (e.g. T2K, NO ν A).
- - $E_\nu \sim 1$ GeV.
 - Charged Current Quasi-Elastic (CCQE) Scattering: a dominant process
 - estimate of the systematic differences between $\nu_e N$ and $\nu_\mu N$ CCQE cross sections important for data analysis: Day, McFarland, PRD86 (2012) 053003, also a talk by M. Day, NuFact12.
 - Are the Radiative Corrections (RC's) a potential source of difference between cross sections for ν_e and ν_μ scattering?
 - Bremsstrahlung radiation via leading log formula: $\sim 10\%$ between ν_e and ν_μ , Day, McFarland, PRD86 (2012) 053003.
- * $m_e \ll m_\mu$: **electron radiates more than muon!**

- Two Boson Exchange Effect (TBE) in CCQE
- TBE effect intensively studied in the ep scattering
 - Contribution from $\gamma\gamma$, $W^\pm\gamma$ and $Z_0\gamma$ box diagrams \rightarrow hadronic model dependent;
 - **The TBE effect in CCQE interaction: is it important to consider?**



α^3 corrections: Born $\times \mathcal{O}(\alpha^2)$



- 1 $e p$ data analysis: the radiative corrections from: Mo and Tsai, Rev. Mod. Phys. **41** (1969) 205;

$$\begin{aligned} \sigma_R &\rightarrow \sigma_{1\gamma,R}(1 + \delta_{rad.}) \\ \delta_{rad.} &= \delta_{ver.} + \delta_{prop.} \\ &+ \delta_{brem.}(soft.) \\ &+ \delta_{tpe}(soft.) \end{aligned}$$

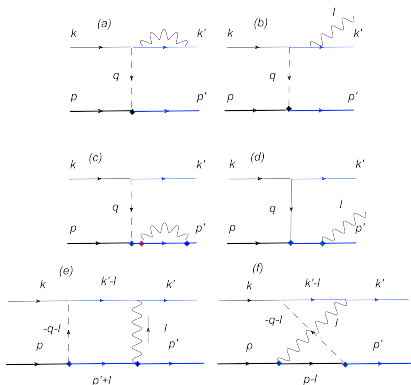
- 2 Re-Examined by Maximov and Tjon, Phys. Rev. C **62** (2000) 054320.

E_e (GeV)	Q^2 (GeV ²)	RC (total)	Electron Leg Leg
6.032	2.525	-25%	-24%
2.201	1.400	-25%	-22%
2.206	0.346	-22%	-21%

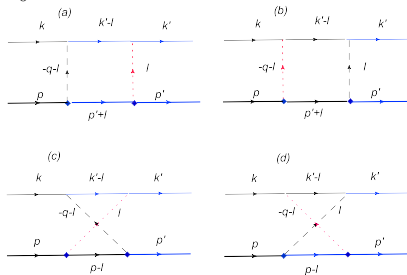
Table : Mo, Tsai, Rev. Mod. Phys. 41, 205 (1969)
 (elastic $e p$)



Radiative Corrections in νN Scattering



$Z_0 W$



additionally,

- Weak Vertex Corrections;
- Weak Propagator Self Energy Corrections;
- Vacuum Polarization;
- γ emission by W ;
- Separation of QED corrections in the case of νq does not have sense; QED and Weak corrections stand gauge invariant set!

Technical Remark:

- "Pure" lepton leg corrections: rather trivial to obtain;
- Model dependent part: hadronic part;
- Bremsstrahlung emission, one has to take into account the detector properties the way of performing the measurement.



Radiative Corrections in νN Interactions

β -decay, CCQE at the threshold scale (several MeV)

- T. Kinoshita and A. Sirlin, PR113 (1959) 1652;
- A. Sirlin, NPB **71** (1974) 29;
- M. Fukugita and T. Kubota, Acta Phys. Polon. B **35** (2004) 1687.
- * About 2% (outer correction, Brem.+Self.+Extracted from vertex.), + About (2.4% (inner correction: Box Contribution, quark+impact of FF))
- ** Inner-Outer: a technical problem how to cancel UV divergences, which appear in propagator corrections.
- U. Raha, F. Myhrer and K. Kubodera, PRC **85** (2012) 045502;

Bremsthalung Emission in CCQE in 1 GeV ν energy range.

- A. Bodek, arXiv:0709.4004 [hep-ex]
- M. Day and K. S. McFarland, PRD**86** (2012) 053003.
(relative effect between ν_e and ν_μ of the order of 10%)
- * De Rujula leading log formula applied for getting Brem. prediction 10% of relative difference between ν_e and ν_μ .

DIS

- Kiskis, PRD**8** 2129, $\nu_\mu \rightarrow \mu^- + X$, RC $\sim \pm 10\%$
- Barlow and Wolfram, (applied Kiskis) PRD**20**, 2198,
 $\nu_\mu \rightarrow \mu^- + X$ RC $\sim \pm 20\%$
- De Rujula et al., NPB**154**, 394, Leading Log Approx,
 $\nu_\mu + p \rightarrow \nu_e + X$, RC $\sim (-15\%, 10\%)$.
- A. Sirlin and W. J. Marciano, NPB**189** (1981) 442.
- H. Bohm, H. Spiesberger, NPB**304**, 749, DIS,
 $e^- p \rightarrow \nu_e + X$, RC of the order of $\sim -20\%$
increasing with growing x , leptonic part (Leading log
 -14%), $s = 10^5$ GeV, $\Delta E < 1$ GeV., Adding hard
photon Brem. contribution increases the total contribution.
- Arbutow et al., JHEP**06**, 078, $\nu_\mu + q \rightarrow \mu^- + q'$,
RS $\sim (-1\%, 5\%)$ but if Q^2 reconstructed only from
leptonic part, several times larger, applied for NOMAD.
- Notice the presence of the the hard and soft photon emission.

Remark

- In the case of the νq scattering the calculations are complex but straightforward (Standard Model);
- β decay, CCQE νN interaction \rightarrow
 - Lepton leg rather obvious, fermion self energy, vertex and Bremsstrahlung corrections rather simple to take into account.
 - **Hadron leg: model dependent, in particular Two-Boson Exchange Correction.**

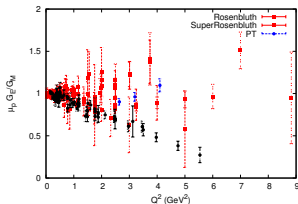


Two Boson Exchange Effect in ep and νN Scattering



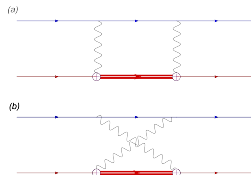
TPE Effect in ep Scattering

- Discrepancy between estimation of G_{Ep}/G_{Mp} based on Polarization Transfer (PT) and Rosenbluth technique estimations.



- CROSS SECTION data are consistent but one has to take into account normalization errors, Arrington, PRC68 (2003) 034325;

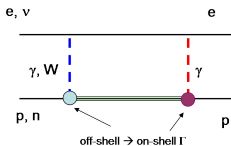
- Inclusion, into Rosenbluth analysis, of the TPE effect (hard photon contribution) solved partially the problem. Blunden *et al.*, PRL91 (2003) 142304, Chen *et al.* PRL 93 (2004) 122301.



- TPE effect in PV ep scattering: $\square \gamma Z^0$ diagrams. Zhou, Kao and Yang, PRL 99 (2007) 262001, Tjon, Blunden and Melnitchouk, PRC 79 (2009) 055201.
- What is the role of the TBE effect in CCQE νN scattering?**



TPE in $e p$ Scattering



- Quantum Field (QF) like approach:
 Blunden et al., PRL91 (2003) 142304, PRC72 (2005) 034612 ($\gamma\gamma$), Zhou et al., PRC 81 (2010) 035208 ($Z^0\gamma$).
 - take the Nucleon, $\Delta(1232)$, ..., to model hadronic intermediate state;
 - off-shell electro-weak \rightarrow on-shell nucleon vertices;
- Dominant contribution from Nucleon (elastic) state, Kondratyuk et al. PRL95 (2005) 172503;
- Agreement (in low and intermediate Q^2 range) with dispersion calculations by Borisjuk and Kobushkin, PRC 78 (2008) 025208;

$$\delta_{TPE} = \frac{2\text{Re} \left\{ \mathcal{M}_{Born}^* \left(i\Box_{\gamma\gamma} - i\Box_{\gamma\gamma}(M_0 - T_{sai}) \right) \right\}}{|\mathcal{M}_{Born}|^2}$$

- Let's apply above approach for predicting the TPE in CCQE.
- The same kinematical domain in (s, t) plane as in the case of 1 GeV neutrinos.

Remark

Reasonable predictions of QF-like approach at low and intermediate Q^2 range. A model dependence for $Q^2 > 4 \text{ GeV}^2$.

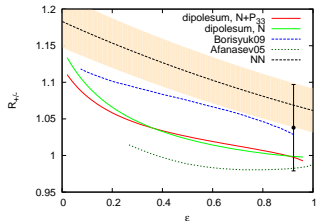


Figure : $R_{+/-} \approx 1 - 2\delta_{TPE}$, $Q^2 = 5 \text{ GeV}^2$, K.M.G., arXiv:1306.5991.



TPE in ep Scattering

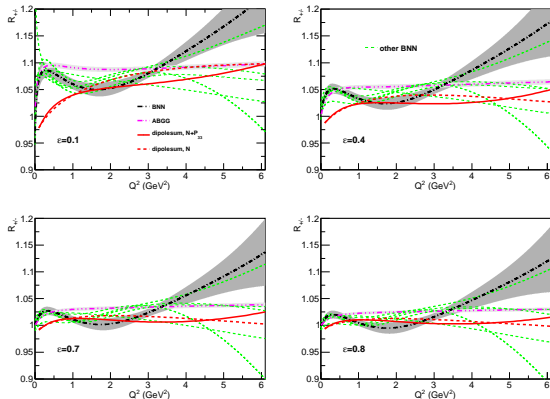
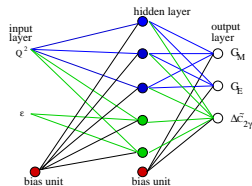


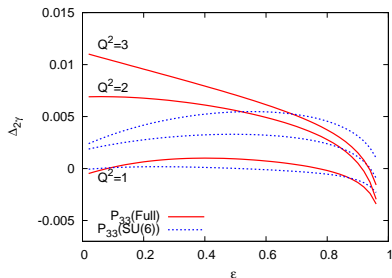
figure from K.M.G., arXiv:1306.5991

$$\epsilon = \left[1 + 2 \left(1 + \frac{Q^2}{4M_p^2} \right) \tan^2 \left(\frac{\theta}{2} \right) \right]^{-1},$$

- $R_{+/-} \approx 1 - 2\delta_{TPE}(2\gamma)$
- δ_{TPE} extracted from elastic ep data \rightarrow Bayesian Neural Network (BNN), K.M.G. PRC 84, 034314.
- * BNN breaks down below 1 GeV²!
- Good quantitative agreement between QF-like approach and phenomenological extraction (by Bayesian Neural Network) for $Q^2 \in (1, 3) \text{ GeV}^2$



$\Delta(1232)$ hadronic state

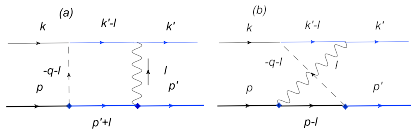


K.M.G., arXiv:1306.5991.

- Contribution from $\Delta(1232)$ negligible in the low Q^2 relevant for 1 GeV ν .
- It is also model dependent!

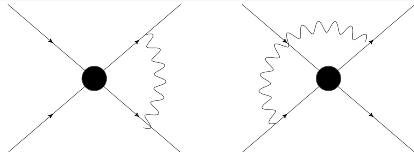


$W\gamma$ Contribution



If lack of form factors, it is UV finite!

$$\sim \int \frac{d^4 l}{(2\pi)^4} \frac{dl}{l^3}$$



If lack of form factors, it is UV (logarithmically) divergent!

$$\sim \int \frac{d^4 l}{(2\pi)^4} \frac{dl}{l}$$

- 1 TBE contribution corresponds to vertex correction in Fermi model.
- 2 TBE \times Born Amplitude $\sim \alpha^3$ correction.

$$\alpha = \frac{e^2}{4\pi} \approx \frac{1}{137}, \quad 4\pi\alpha \approx 0.09, \quad e^2 = g^2 \sin^2 \theta_W$$

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2}, \quad G_F = 1.16639 \times 10^{-5} \text{ GeV}^{-2}$$

$$\frac{g^2}{8} = M_W^2 \frac{G_F}{\sqrt{2}} \approx 0.05, \quad M_W = 80.385 \text{ GeV}/c^2$$

Born Amplitude

$$\nu(k) + p(p) \rightarrow l^-(k') + n(p')$$

$$i\mathcal{M}_{Born} = i \frac{g^2 \cos \theta_C}{8(t - M_W^2)} j_{\mu}^{cc} h_{cc}^{\mu}$$

$$\frac{g^2}{8} = M_W^2 \frac{G_F}{\sqrt{2}} \approx 0.05$$

- Leptonic current:

$$j_{cc}^{\mu} = \bar{u}(k') \gamma^{\mu} (1 - \gamma_5) u(k)$$

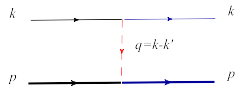
- CCQE one-body hadronic current:

$$h_{cc}^{\mu} = \bar{u}(p') \Gamma_{cc}^{\mu} (p' - p) u(p)$$

$$\Gamma_{cc}^{\mu}(q) = \Gamma_p^{\mu}(q) - \Gamma_n^{\mu}(q) - \gamma_{\mu} \gamma_5 F_A(q) - \frac{q^{\mu} \gamma_5}{2M} F_P(q)$$

$$F_A(q) = \frac{1.267}{(1 - q^2/M_A^2)^2}$$

$$F_P(q) = \frac{4M^2 F_A(q)}{m_{\pi}^2 - q^2}$$



$$\Gamma_{p,n}^{\mu}(q) = \gamma^{\mu} F_1^{p,n}(q) + \frac{i\sigma^{\mu\nu} q_{\nu}}{2M_{p,n}} F_2^{p,n}(q)$$

where

$$F_1^{p,n} = \left(G_E^{p,n} + \tau_{p,n} G_M^{p,n} \right) / (1 + \tau_{p,n})$$

$$F_2^{p,n} = \left(G_M^{p,n} - G_E^{p,n} \right) / (1 + \tau_{p,n})$$

$$\tau_{p,n} = -q^2 / 4M_{p,n}^2$$

$$G_E^p(q^2) = \frac{G_M^{p,n}(q^2)}{\mu_{p,n}} = \frac{\Lambda^4}{(q^2 - \Lambda^2)^2}$$

$$G_E^n(Q^2) = 0, \quad \Lambda = 0.84 \text{ GeV}$$



TBE in νN Scattering

$$i \square_{W+\gamma}^{\parallel} = -\cos \theta_C e^2 g^2 I_{W+\gamma}^{\parallel} / 8 \text{ and}$$

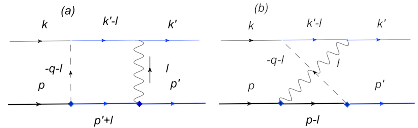
$$i \square_{W+\gamma}^{\times} = -\cos \theta_C e^2 g^2 I_{W+\gamma}^{\times} / 8, \text{ where}$$

$$I_{W+\gamma}^{\parallel} = \int \frac{d^4 l}{(2\pi)^4} \frac{l^{\mu\nu} h_{\mu\nu}^{\parallel}}{D(p', M_p)} \quad (1)$$

$$I_{W+\gamma}^{\times} = \int \frac{d^4 l}{(2\pi)^4} \frac{l^{\mu\nu} h_{\mu\nu}^{\times}}{D(-p, M_n)} \quad (2)$$

m denotes the lepton mass.

$$D_W(x, M_x) = [(q+l)^2 - M_W^2 + i\epsilon][l^2 + i\epsilon][(k' - l)^2 - m^2 + i\epsilon][(x+l)^2 - M_x^2 + i\epsilon].$$



$$l^{\mu\nu} = \bar{u}(k') \gamma^\mu (\hat{k}' - \hat{l} + m) \gamma^\nu (1 - \gamma_5) u(k)$$

$$h_{\mu\nu}^{\parallel} = \bar{u}(p') \Gamma_\mu^p(-l) (\hat{p}' + \hat{l} + M_p) \Gamma_\nu^{cc}(q+l) u(p)$$

$$h_{\mu\nu}^{\times} = \bar{u}(p') \Gamma_\nu^{cc}(q+l) (\hat{p} - \hat{l} + M_n) \Gamma_\mu^n(-l) u(p)$$

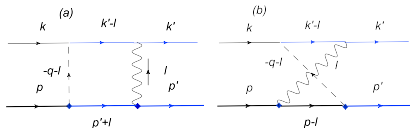
$$\Delta_{TBE} = \text{Re} \sum_{spin} \mathcal{M}_{Born}^* \left(\square_{W+\gamma}^{\parallel} + \square_{W+\gamma}^{\times} \right)$$

$$= \frac{g^4 e^2 \cos^2 \theta_C}{16(M_W^2 - t)} \text{Im} \underbrace{\sum_{spin} (j_\alpha h^\alpha)^* \left(I_{W+\gamma}^{\parallel} + I_{W+\gamma}^{\times} \right)}_{\diamond}$$

- How to compute \diamond ?
- Take the trace (Numerator of loop integral);
- Reduce the numerator with the denominator as much as possible;
- Express in terms of 1-, 2-, 3- and 4- point scalar loop integrals t'Hooft, Veltman;
- Compute numerical values (photon mass μ introduced to regularize)

* FeynCalc and LoopTool library used for algebraical and numerical calculations.





- Both box amplitudes are ultraviolet (UV) finite, because the presence of the form factors;
- $i\Box_{\gamma W+}^{\parallel}$ is infrared (IR) divergent;
- $i\Box_{\gamma W+}^{\times}$ is IR finite, in the limit $l \rightarrow 0$ $i\Box_{W+\gamma}^{\times}$ behaves as:

$$\mu_n \int \frac{d^4 l}{l^2 k' \cdot l p' \cdot l} \left(\frac{\gamma^\mu l^2}{l^2 - 4M_n^2} + \sigma^{\mu\nu} l_\nu \right).$$

•

$$\Delta_{TBE}^{\nu n}(\Box_{W+\gamma}^{\parallel}, soft) = -8k' \cdot p' e^2 |\mathcal{M}_{Born}|^2 \text{Im} C_0,$$

- $C_0 \equiv C_0(m^2, M_p^2, s, m^2, 0, M_p^2)$ a scalar one-loop integral, see 't Hooft79

$$C_0 = \int \frac{d^4 l}{(2\pi)^4} \frac{1}{l^2 [(k' - l)^2 - m^2][(p' + l)^2 - M_p^2]}$$

regularized by photon mass $1/l^2 \rightarrow 1/(l^2 - \mu^2)$

Fraction of TBA Correction

$$\delta_{TBE}^{\nu n} = \frac{\Delta_{TBE}^{\nu n} - \Delta_{TBE}^{\nu n}(\Box_{W+\gamma}^{\parallel}, soft)}{\frac{1}{2} \sum_{spin} |\mathcal{M}_{Born}|^2}$$

$$d\sigma = d\sigma_{Born} (1 + \delta_{TBE})$$

Soft Contribution

$$\int \frac{d^4 l}{(2\pi)^4} \frac{N(l \rightarrow 0)}{D(l)}$$



- The weak vertex $\Gamma_{nc,p(n)}^\mu$ for Z^0 proton (neutron) coupling Alberico, Bilenky and Maieron, PR358 (2002) 227,

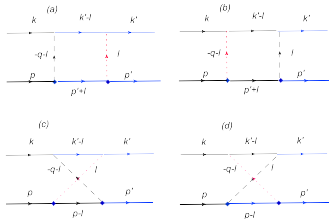
$$\Gamma_{nc,p(n)}^\mu = \gamma^\mu F_1^{nc,p(n)} + \frac{i\sigma^{\mu\nu} q_\nu}{2M} \gamma^\mu F_2^{nc,p(n)} - \gamma^\mu \gamma_5 F_A^{nc,p(n)}$$

where

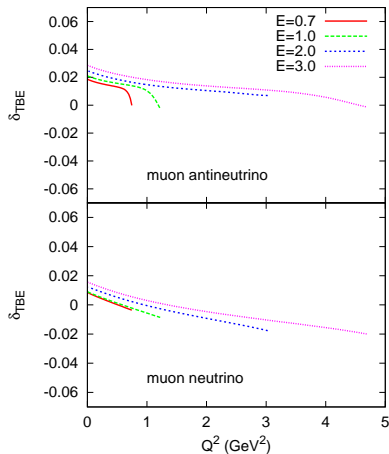
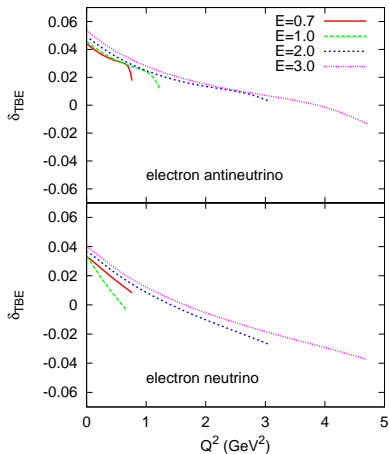
$$F_{1,2}^{nc,p(n)} = 1/2 \left[\left(1 - 4 \sin^2 \theta_W \right) F_{1,2}^{p(n)} - F_{1,2}^{n(n)} \right],$$

$$F_A^{nc,p(n)} = \pm F_A/2$$

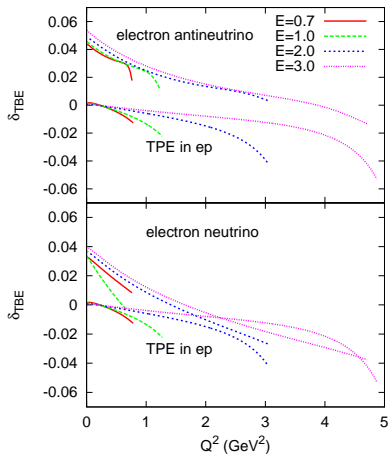
- It is negligible in comparison to $W\gamma$ contribution.



Q^2 dependence of TBE



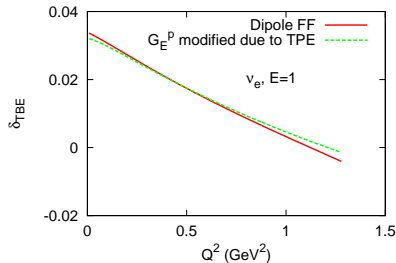
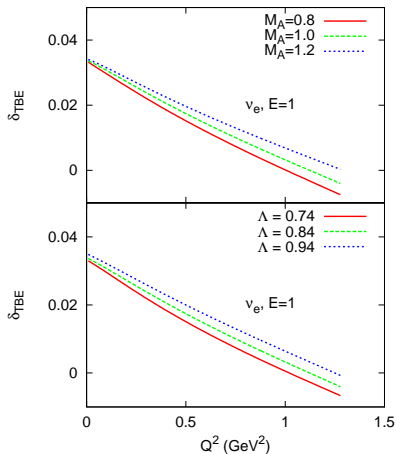
TPE vs. TBE



- TPE and TBE effects are of comparable size!



Parameter Dependence of TBE



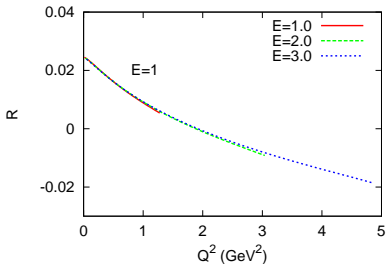
- Modification of the G_{Ep} due to (TPE effect, Alberico, Bilenky, Giunti, K.M.G, PRC **79** (2009) 065204).

$$G_E^P(Q^2) = (-0.130 Q^2 + 1.0022) G_M^P(Q^2) / \mu_P$$

- TBE weakly depends on the form factor parameters!



ν_e vs. ν_μ

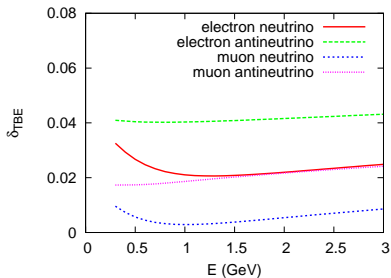


- The relative difference varies from (2%, -2%)

$$\begin{aligned}
 \mathcal{R}_\nu(Q^2) &= \frac{\sigma_{Born}(\nu_e) + \sigma_{TBE}(\nu_e)}{\sigma_{Born}(\nu_\mu) + \sigma_{TBE}(\nu_\mu)} \left(\frac{\sigma_{Born}(\nu_e)}{\sigma_{Born}(\nu_\mu)} \right)^{-1} - 1 \\
 &= \frac{1 + \delta_{TBE}(\nu_e)}{1 + \delta_{TBE}(\nu_\mu)} - 1 \approx \delta_{TBE}(\nu_e) - \delta_{TBE}(\nu_\mu),
 \end{aligned}$$



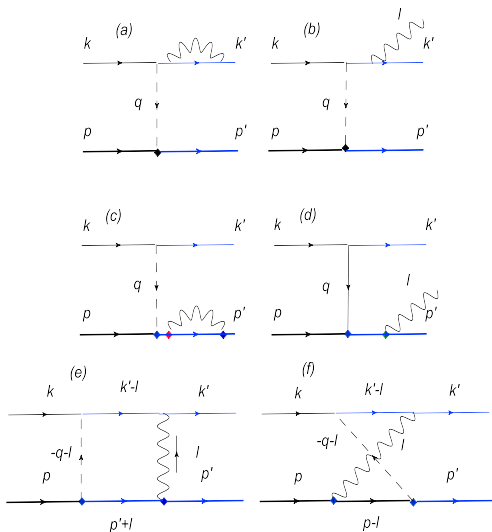
TBE in νN Scattering



- The relative difference between $\sigma(\nu_e)$ and $\sigma(\nu_\mu)$, due to TBE, is of the order of 2%;
- TBE effect is negligible in the case of ν_μ interactions.
- G_F established from μ decay, hence, one should subtract the analogical \square_μ contribution (Sirlin),
- at 20 MeV, $\delta_{TBE} - \square_\mu \sim 0.032$
- outer correction (dominated by TBE prediction given by quark box diagrams in the massless limit) ~ 0.024 by M. Fukugita and T. Kubota, Acta Phys. Polon. B 35 (2004) 1687.



QED-like only correction



Born $\times \mathcal{O}(\alpha^2) \sim \mathcal{O}(\alpha^3)$

- Minimal frame for validation of the calculations, only QED-like corrections;
- it allows to check the normalization, signs etc.
- lower bound for RC's corrections.

Fermion Propagator Correction

$$\delta_{Self.} = -\frac{\alpha}{\pi} \left\{ \frac{9}{4} + \ln \frac{\mu}{m} + \ln \frac{\mu}{M_p} + \frac{1}{2} \left(\ln \frac{\Lambda_{UV}}{m} + \ln \frac{\Lambda_{UV}}{M_p} \right) \right\}.$$



Cross Checks: Soft Bremsstrahlung

$$\nu(k) + n(p) \rightarrow l^-(k') + p(p') + \gamma(l)$$

$$i\mathcal{M}_{brem.} = i\mathcal{M}_{p',inel.} + i\mathcal{M}_{k',inel.}$$

$$i\mathcal{M}_{k',brem} = \bar{u}(k')(-ie)\gamma^\mu \epsilon_\mu^*(l) \frac{i(\hat{k}' + \hat{l} + m)}{(k' + l)^2 - m + i\epsilon} \times (\dots, born)$$

soft photon approximation, in the numerator $l \rightarrow 0, l^2 \rightarrow 0$ in the denominator,

$$|i\mathcal{M}_{brem}|^2 = -e^2 \left[\frac{m^2}{(k' \cdot l)^2} + \frac{M_p^2}{(p' \cdot l)^2} - \frac{2p' \cdot k'}{(p' \cdot l)(k' \cdot l)} \right] |\mathcal{M}_{born}|^2 \quad (3)$$

$$d\sigma_{brem} \sim e^2 d\sigma_{Born} \left[2p' \cdot k' L_{p'k'} - m^2 L_{k'k'} - M_p^2 L_{p'p'} \right], L_{xy} = \int_{|l| < \epsilon} \frac{d^3 l}{|l|} \frac{1}{(x \cdot l)(y \cdot l)}$$

analytic formulas: 't Hooft79, Maximon00 (by Lorentz invariant scalars)

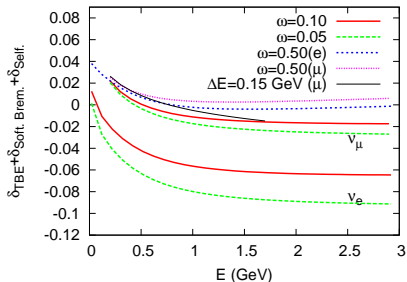
- IR divergent part of $L_{p'k'}$ cancels with IR singularity in TBE box diagrams.
- IR divergencies of $L_{p'p'}$ and $L_{k'k'}$ cancel with proton and electron (QED) self energy correction.

- ϵ is the maximal soft photon energy but in the frame $l + p' = 0!$

$$\Delta \epsilon = \underbrace{\eta}_{1 + (E/M)(1 - \cos \theta)} \underbrace{\Delta E}_{\omega E'}$$

- $\Delta E = \omega E'$ is the detector acceptance.
- * η - expression for electron

Soft Bremsstrahlung Emission



$$\sigma(E) = \sigma_{Born}(E)(1 + \delta_{TBE} + \delta_{Brem.Soft.} + \delta_{Self.})$$

- TBE + Brem. + Self. is IR finite as it should be, UV divergence remains.
- It is the lower bound for RCs!
- It depends on the detector acceptance. Notice that it is divergent when $\omega \rightarrow 0!$
- TBE effect reduces the Soft Bremsstrahlung contribution!
- Adding hard Bremsstrahlung reduces the RCs (setting $\omega = 0.5$ mimic hard photon limit and it shows only the tendency, does not correctly model the hard photon emission).
- At low E RC's (ν_e) \rightarrow 3.8%, which agrees with the threshold estimates by Fukugita and Kubota, Acta. Phys. Polon. B35 (2004) 1687.
- A proper estimate of RC's depends on the detector properties and the way of reconstructing muon energy and Q^2 .
- With hard photon emission the RC's tend to be on the percent level.

$$Bremsstrahlung\ Emission = \underbrace{\int_0^{\Delta E} \frac{d^3l}{|l|} \sigma_{Brem}(\nu N \rightarrow \nu P \gamma)}_{Soft(*)} + \underbrace{\sum_{\alpha=1}^2 \int_{\Delta E}^{E_{max}} \frac{d^3l}{|l|} \sigma_{\alpha}(\nu N \rightarrow \nu P \gamma)}_{Hard(**)}$$



- TBE exchange correction to CCQE νN scattering has been obtained.
- TBE is of comparable size as in the elastic ep scattering.
- The averaged difference between TBE for ν_e and ν_μ is of the order of 2%.
- QED set of corrections have been obtained. It gives the lower bound for RC's.
- TBE gives crucial contribution in total amount of RC's and it is necessary in order to reduce the Soft Bremsstrahlung correction.
- the proper inclusion of the RC's in the data analysis requires taking into account detector properties.
- Work in progress: calculations of the contribution from all diagrams required by gauge invariance;
- Confrontation with experiment: one should take into account the detector properties and the way of measurement of the particles, it seems to be not straightforward.
- **Let's include RS's corrections into NuWro Monte Carlo Generator.**

