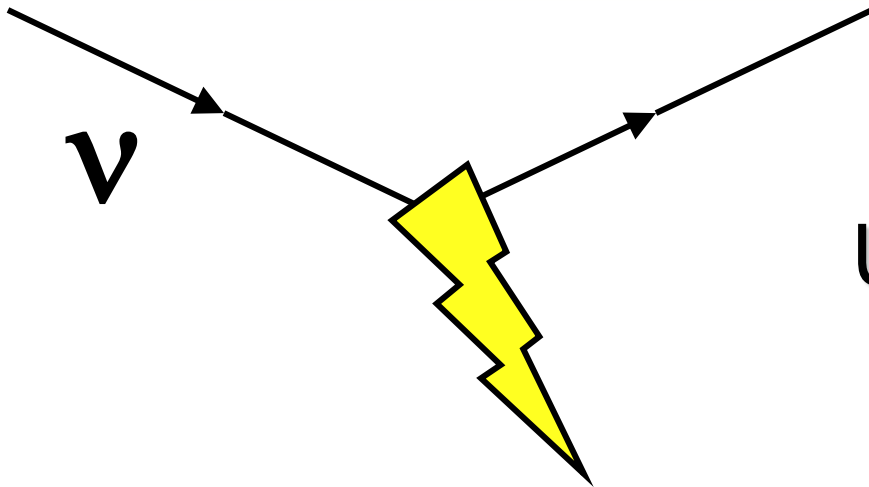
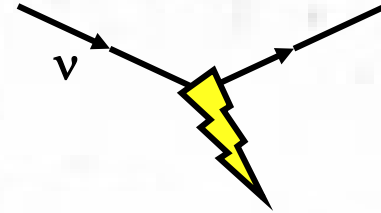


# ***Interactions of Neutrinos***



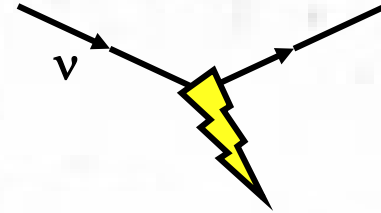
Kevin McFarland  
University of Rochester  
INSS 2013, Beijing  
6-8 August 2013

# Outline



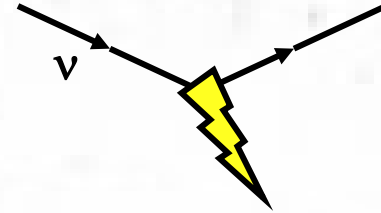
- Brief Motivation for and History of Measuring Interactions
  - Key reactions and thresholds
- Weak interactions and neutrinos
  - Elastic and quasi-elastic processes, e.g.,  $\nu e$  scattering
  - Complication of Targets with Structure
  - Deep inelastic scattering ( $\nu q$ ) and UHE neutrinos
  - Quasielastic and nearly elastic scattering
- Special problems at accelerator energies
  - Nuclear Effects
  - Generators, theory and experimental data
- Conclusions

# ***Focus of These Lectures***



- This is not a comprehensive review of all the interesting physics associated with neutrino interactions
- Choice of topics will focus on:
  - Cross-sections useful for studying neutrino properties
  - Estimating cross-sections
  - Understanding the most important effects qualitatively or semi-quantitatively
  - Understanding how we use our knowledge of cross-sections in experiments

# Weak Interactions



- Current-current interaction  $\mathcal{H}_w = \frac{G_F}{\sqrt{2}} \mathcal{J}^\mu \mathcal{J}_\mu$   
Fermi, Z. Physik, 88, 161 (1934)

- Paper famously rejected by *Nature*:  
*“it contains speculations too remote from reality to be of interest to the reader”*

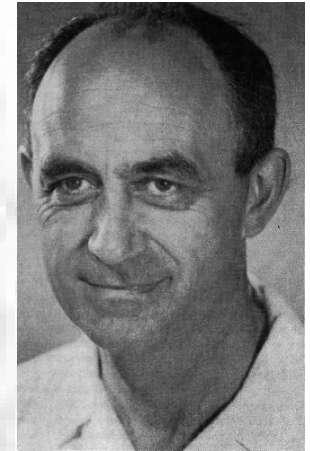
- Prediction for neutrino interactions

- If  $n \rightarrow pe^- \bar{\nu}$ , then  $\bar{\nu} p \rightarrow e^+ n$
- Better yet, it is robustly predicted by Fermi theory
  - Bethe and Peirels, Nature 133, 532 (1934)

- For neutrinos of a few MeV from a reactor, a typical cross-section was found to be

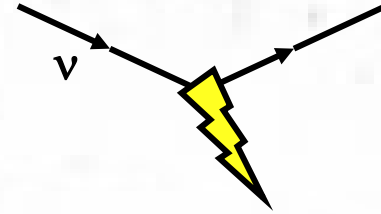
$$\sigma_{\bar{\nu} p} \sim 5 \times 10^{-44} \text{ cm}^2$$

This is wrong by a factor of two (parity violation)





# How Weak is This?



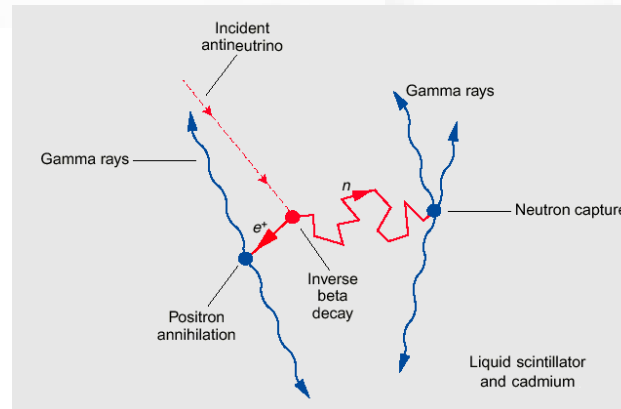
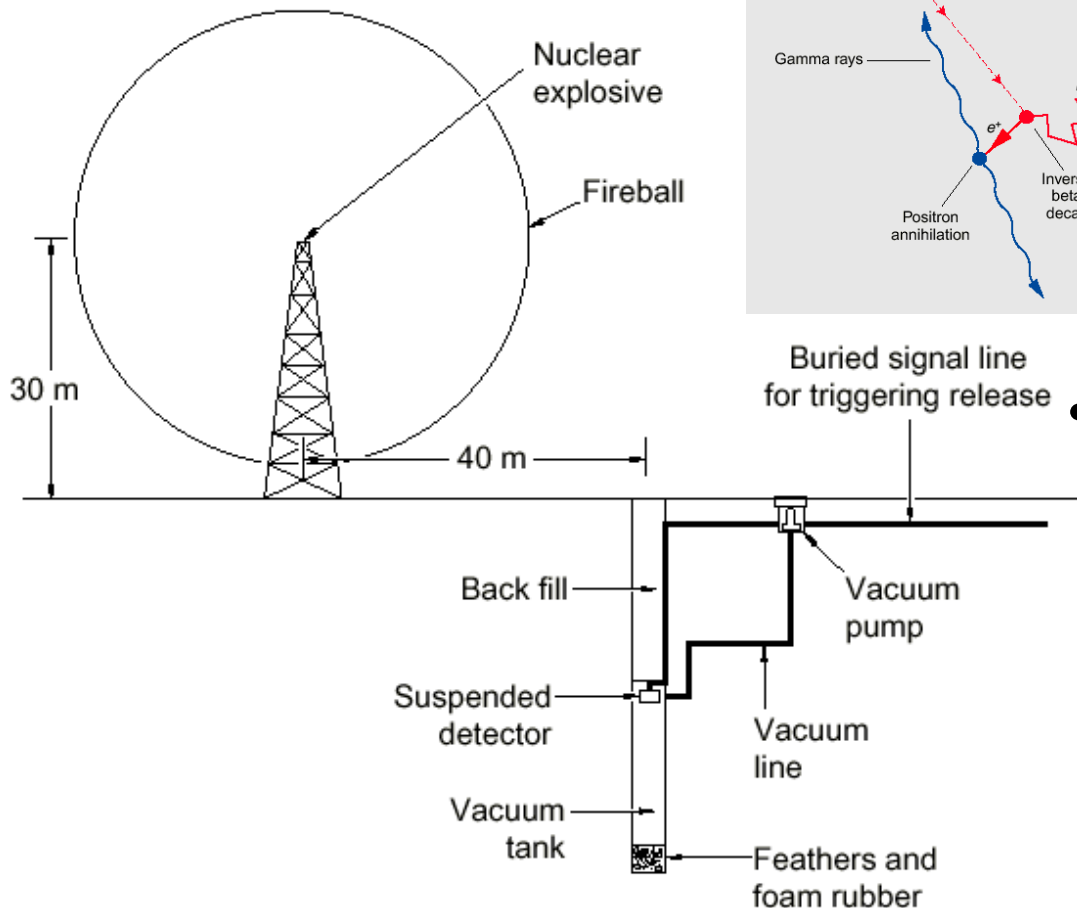
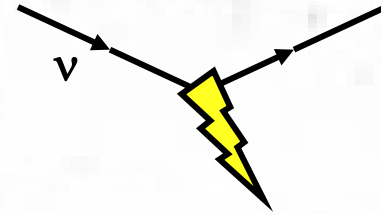
- $\sigma \sim 5 \times 10^{-44} \text{ cm}^2$  compared with
  - $\sigma_{yp} \sim 10^{-25} \text{ cm}^2$  at similar energies, for example
- The cross-section of these few MeV neutrinos is such that the mean free path in steel would be 10 light-years

*"I have done something very bad today by proposing a particle that cannot be detected; it is something no theorist should ever do."*

Wolfgang Pauli



# Extreme Measures to Overcome Weakness (Reines and Cowan, 1946)



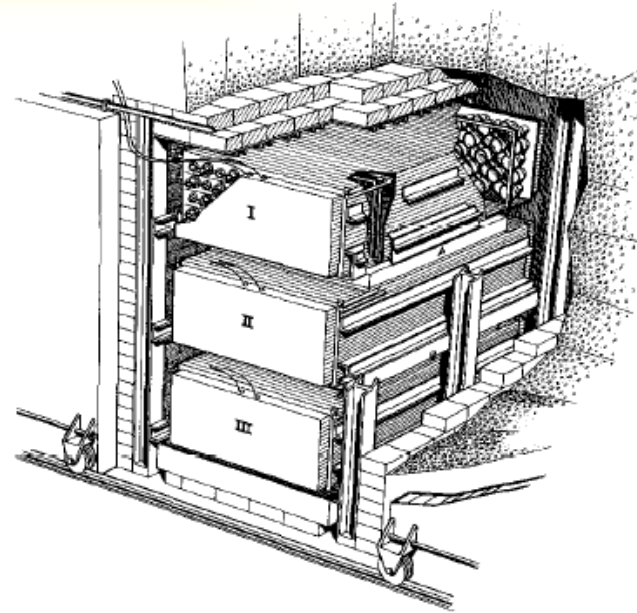
Why inverse neutron beta decay?

- clean prediction of Fermi weak theory
- clean signature of prompt gammas from  $e^+$  plus delayed neutron signal.
  - o Latter not as useful with bomb source.

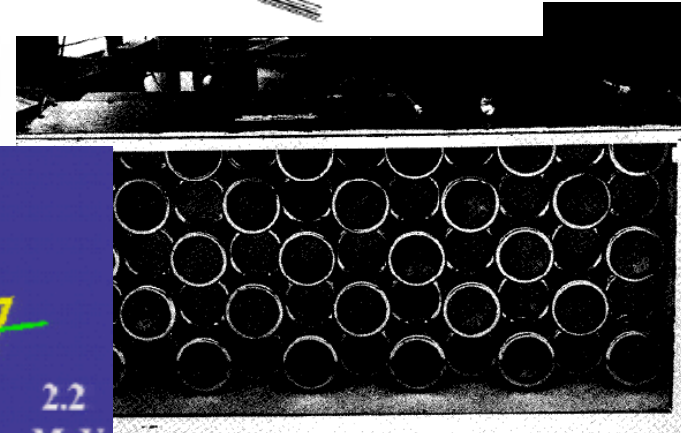
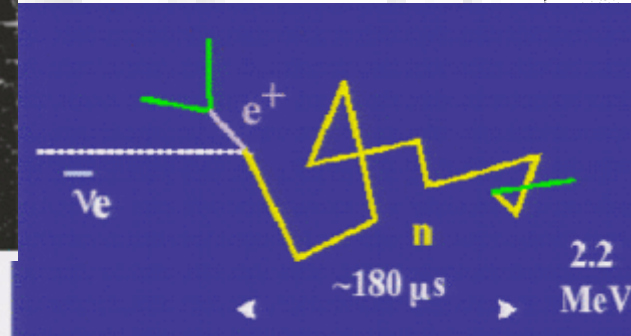
# Discovery of the Neutrino

- Reines and Cowan (1955)

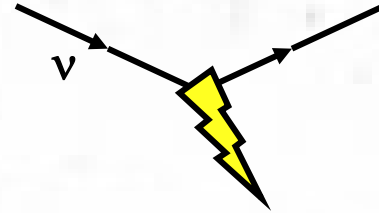
- Chose a constant source, nuclear reactor (Savannah River)
- 1956 message to Pauli: "We are happy to inform you [Pauli] that we have definitely detected neutrinos..."
- 1995 Nobel Prize for Reines



$$\bar{\nu} p \rightarrow e^+ n$$



# Better than the Nobel Prize?



Frederick REINES and Clyde COWAN  
Box 1663, LOS ALAMOS, New Mexico  
Thanks for message. Everything comes to  
him who knows how to wait.

Pauli

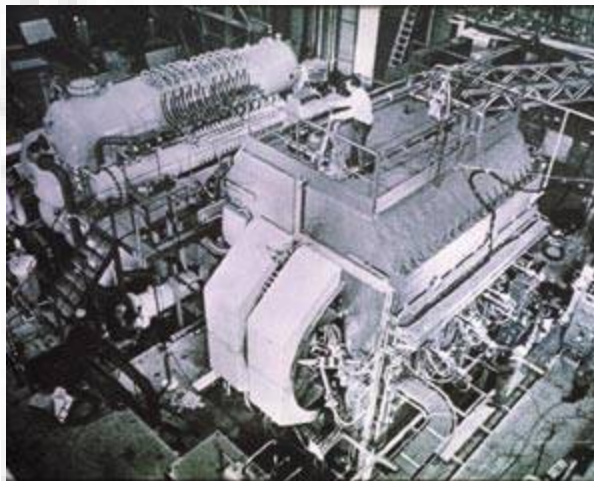
Thanks for the message. Everything  
comes to him who knows how to wait.

Aug. 15.6.13 / 15.31R  
also might better

# Another Neutrino Interaction Discovery

- Neutrinos only feel the weak force
  - a great way to study the weak force!
- Search for neutral current
  - arguably the most famous neutrino interaction ever observed is shown at right

$$\bar{\nu}_{\mu} e^{-} \rightarrow \bar{\nu}_{\mu} e^{-}$$

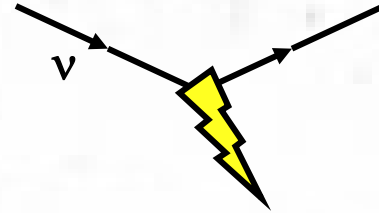


AEROMETRIC photo



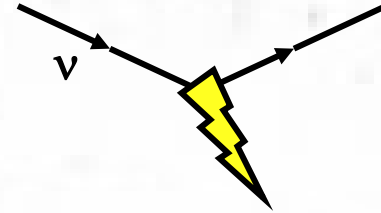
$\nu$

Gargamelle, event from  
neutral weak force



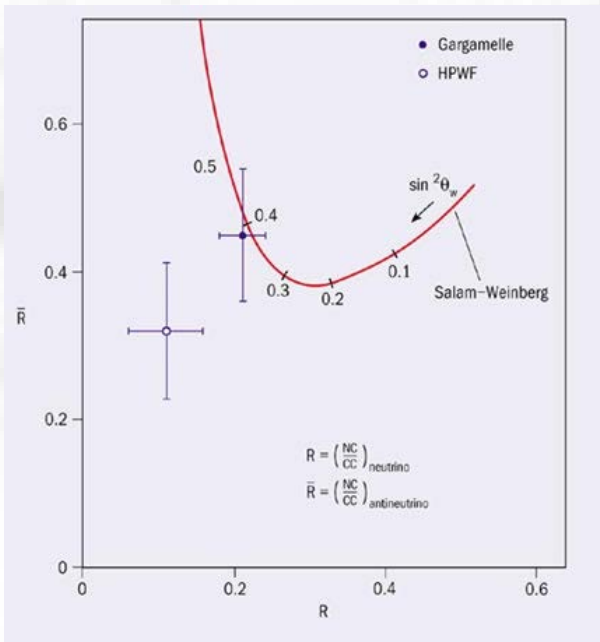


# An Illuminating Aside



- The “discovery signal” for the neutral current was really neutrino scattering from nuclei
  - usually quoted as a ratio of muon-less interactions to events containing muons

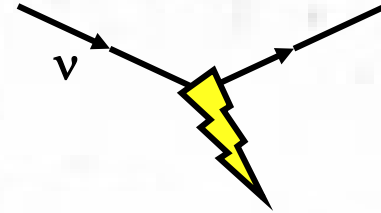
$$R^{\nu} = \frac{\sigma(\nu_{\mu} N \rightarrow \nu_{\mu} X)}{\sigma(\nu_{\mu} N \rightarrow \mu^{-} X)}$$



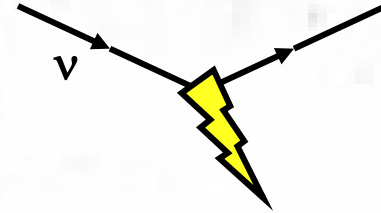
- But this discovery was complicated for 12-18 months by a lack of understanding of neutrino interactions
  - backgrounds from neutrons induced by neutrino interactions outside the detector
  - not understanding fragmentation to high energy hadrons which then “punched through” to fake muons

*Great article: P. Gallison, Rev Mod Phys 55, 477 (1983)*

# ***The Future: Interactions and Oscillation Experiments***



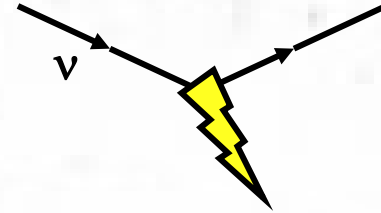
- Oscillation experiments point us to a rich physics potential at  $L/E \sim 400 \text{ km/GeV}$  (*and  $L/E \sim N \cdot (400 \text{ km/GeV})$  as well*)
  - mass hierarchy, CP violation
- But there are difficulties
  - transition probabilities as a function of energy must be precisely measured for mass hierarchy and CP violation
  - the neutrinos must be at difficult energies of 1-few GeV for electron appearance experiments, few-many GeV for atmospheric neutrino and  $\tau$  appearance experiments.
  - or use neutrinos from a reactor ☺
- *Our generation doesn't have neutrino flavor measurements in which distinguishing 1 from 0 or 1/3 buys a ticket to Stockholm*
  - Difficulties are akin to neutral current experiments
  - Is there a message for us here?



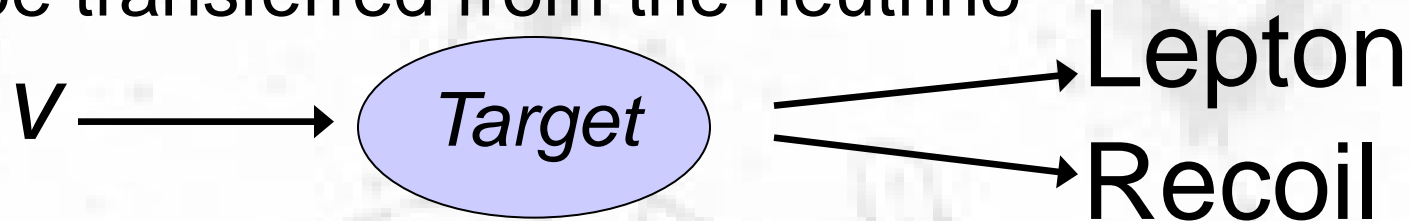
# ***Kinematics of Neutrino Reactions***



# Thresholds and Processes

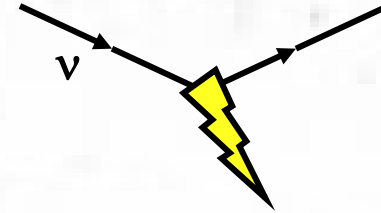


- We detect neutrino interactions only in the final state, and often with poor knowledge of the incoming neutrinos
- Creation of that final state may require energy to be transferred from the neutrino



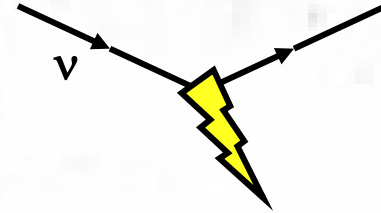
- In charged-current reactions, where the final state lepton is charged, this lepton has mass
- The recoil may be a higher mass object than the initial state, or it may be in an excited state

# Thresholds and Processes



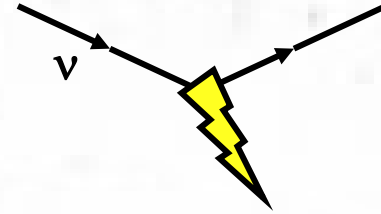
Process	Considerations	Threshold (typical)
$\nu N \rightarrow \nu N$ (elastic)	Target nucleus is often free (recoil is very small)	none
$\nu_e n \rightarrow e^- p$	In some nuclei (mostly metastable ones), this reaction is exothermic if proton not ejected	None for free neutron some others.
$\nu e \rightarrow \nu e$ (elastic)	Most targets have atomic electrons	$\sim 10\text{eV} - 100\text{keV}$
$\text{anti-}\nu_e p \rightarrow e^- n$	$m_n > m_p$ & $m_e$ . Typically more to make recoil from stable nucleus.	1.8 MeV (free p). More for nuclei.
$\nu_\ell n \rightarrow \ell^- p$ (quasielastic)	Final state nucleon is ejected from nucleus. Massive lepton	$\sim 10\text{s MeV}$ for $\nu_e$ + $\sim 100\text{ MeV}$ for $\nu_\mu$
$\nu_\ell N \rightarrow \ell^- X$ (inelastic)	Must create additional hadrons. Massive lepton.	$\sim 200\text{ MeV}$ for $\nu_e$ + $\sim 100\text{ MeV}$ for $\nu_\mu$

- Energy of neutrinos determines available reactions, and therefore experimental technique



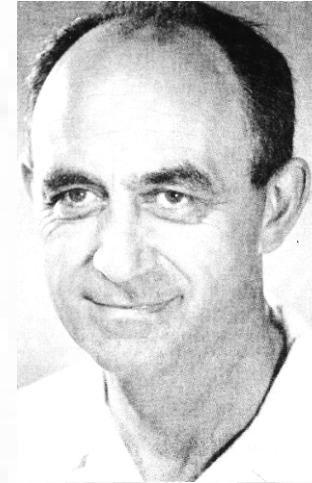
# ***Calculating Neutrino Interactions from Electroweak Theory***

# Weak Interactions Revisited



- Current-current interaction (Fermi 1934)

$$\mathcal{H}_w = \frac{G_F}{\sqrt{2}} \mathcal{J}^\mu \mathcal{J}_\mu$$

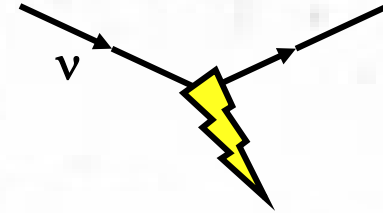


- Modern version:

$$\mathcal{H}_{weak} = \frac{G_F}{\sqrt{2}} \left[ \bar{l} \gamma_\mu (1 - \gamma_5) \nu \right] \left[ \bar{f} \gamma^\mu (V - A \gamma_5) f \right] + h.c.$$

- $P_L = 1/2(1 - \gamma_5)$  is a projection operator onto left-handed states for fermions and right-handed states for anti-fermions

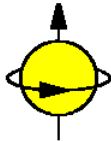
# Helicity and Chirality



- **Helicity** is projection of spin along the particles direction
  - Frame dependent (if massive)

The operator:  $\sigma \cdot \mathbf{p}$

right-helicity



left-helicity



- However, **chirality** (“handedness”) is Lorentz-invariant

– Only same as helicity for massless particles.

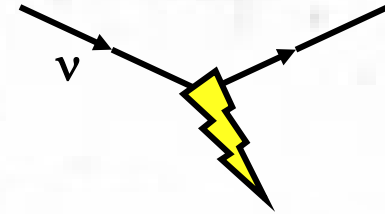
- Neutrinos only interact weakly with a (V-A) interaction
  - All neutrinos are left-handed
  - All antineutrinos are right-handed
    - because of production!
  - Weak interaction maximally violates parity

- If neutrinos have mass then left-handed neutrino is:
  - Mainly left-helicity
  - But also small right-helicity component  $\propto m/E$
- Only left-handed charged-leptons ( $e^-, \mu^-, \tau^-$ ) interact weakly but mass brings in right-helicity:

$$\pi^+ (J=0) \rightarrow \mu^+ (J=\frac{1}{2}) \nu_\mu (J=\frac{1}{2})$$

$$R_{theory} = \frac{\Gamma(\pi^\pm \rightarrow e^\pm \nu_e)}{\Gamma(\pi^\pm \rightarrow \mu^\pm \nu_\mu)} = \left(\frac{m_e}{m_\mu}\right)^2 \left(\frac{m_\pi^2 - m_e^2}{m_\pi^2 - m_\mu^2}\right)^2 = 1.23 \times 10^{-4}$$

# Two Weak Interactions



- W exchange gives Charged-Current (CC) events and Z exchange gives Neutral-Current (NC) events

In charged-current events,

Flavor of outgoing lepton tags flavor of neutrino

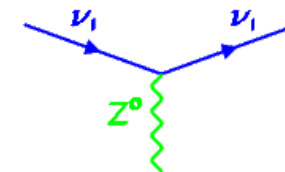
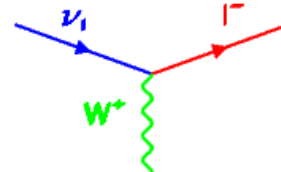
Charge of outgoing lepton determines if neutrino or antineutrino

$$l^- \Rightarrow \nu_l$$

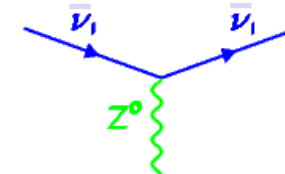
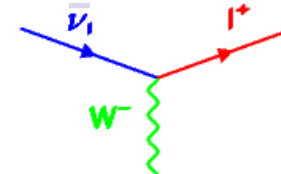
$$l^+ \Rightarrow \bar{\nu}_l$$

Charged-Current (CC) Interactions      Neutral-Current (NC) Interactions

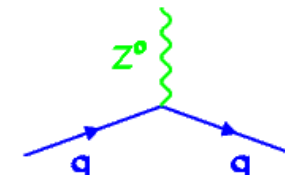
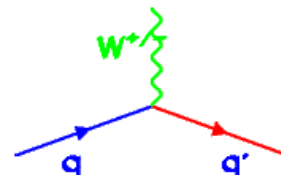
Neutrinos



Anti-Neutrinos



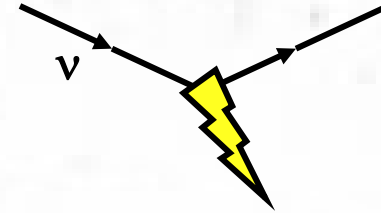
Quarks



Flavor Changing

Flavor Conserving

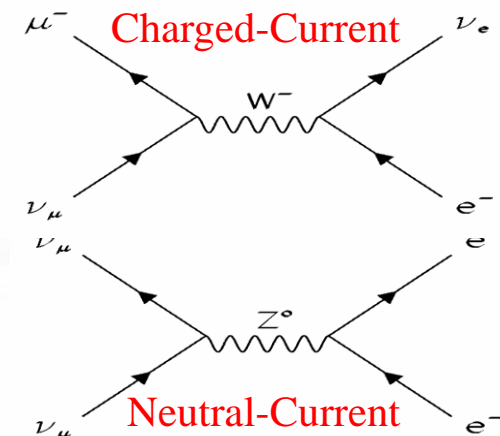
# Electroweak Theory



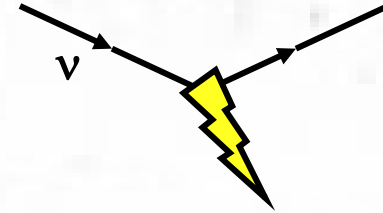
- Standard Model
  - $SU(2) \otimes U(1)$  gauge theory unifying weak/EM  
 $\Rightarrow$  **weak NC follows from EM, Weak CC**
  - Physical couplings related to mixing parameter for the interactions in the high energy theory

$$\mathcal{L}_{EW}^{\text{int}} = -Q_e A_\mu \bar{e} \gamma^\mu e + \frac{g}{\sqrt{2}} W_\mu^+ \bar{\nu}_L \gamma^\mu e_L + \frac{g}{\sqrt{2}} W_\mu^- \bar{e}_L \gamma^\mu \nu_L$$

$$+ \frac{g}{\cos \theta_W} Z_\mu^0 \left\{ \begin{aligned} &\frac{1}{2} \bar{\nu}_L \gamma^\mu \nu_L \\ &+ \left( \sin^2 \theta_W - \frac{1}{2} \right) \bar{e}_L \gamma^\mu e_L \\ &+ \sin^2 \theta_W \bar{e}_R \gamma^\mu e_R \end{aligned} \right\}$$



# Electroweak Theory

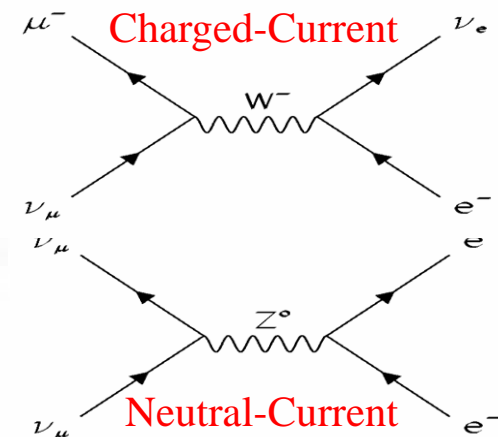


- Standard Model
  - $SU(2) \otimes U(1)$  gauge theory unifying weak/EM  
 $\Rightarrow$  weak NC follows from EM, Weak CC
  - Measured physical parameters related to mixing parameter for the couplings.

Z Couplings	$g_L$	$g_R$
$\nu_e, \nu_\mu, \nu_\tau$	1/2	0
$e, \mu, \tau$	$-1/2 + \sin^2\theta_W$	$\sin^2\theta_W$
$u, c, t$	$1/2 - 2/3 \sin^2\theta_W$	$-2/3 \sin^2\theta_W$
$d, s, b$	$-1/2 + 1/3 \sin^2\theta_W$	$1/3 \sin^2\theta_W$

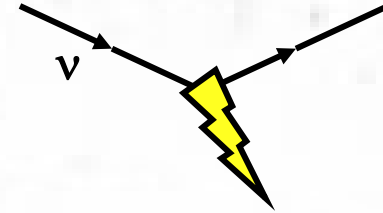
$$e = g \sin \theta_W, G_F = \frac{g^2 \sqrt{2}}{8M_W^2}, \frac{M_W}{M_Z} = \cos \theta_W$$

- Neutrinos are special in SM
  - Right-handed neutrino has **NO** interactions!





# Why “Weak”?



- Weak interactions are weak because of the massive W and Z bosons exchange

$$\frac{d\sigma}{dq^2} \propto \frac{1}{(q^2 - M^2)^2}$$

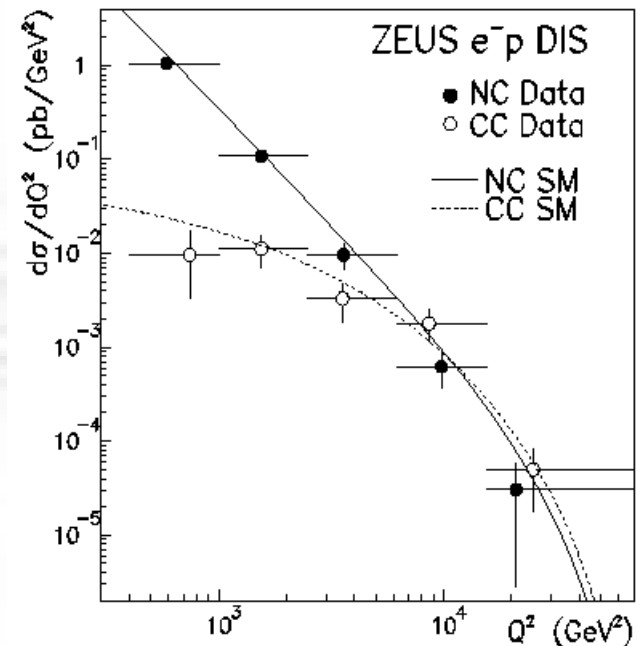
$q$  is 4-momentum carried by exchange particle  
 $M$  is mass of exchange particle

At HERA see W and Z propagator effects  
 - Also weak ~ EM strength

- Explains dimensions of Fermi “constant”

$$G_F = \frac{\sqrt{2}}{8} \left( \frac{g_W}{M_W} \right)^2$$

$$= 1.166 \times 10^{-5} / \text{GeV}^2 \quad (g_W \approx 0.7)$$



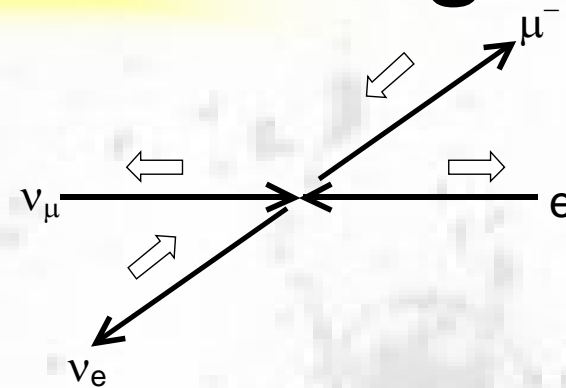
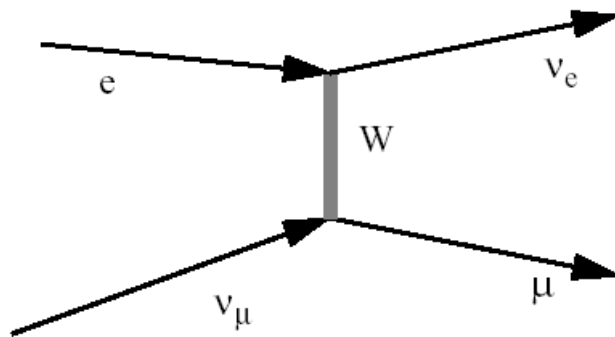
# Neutrino-Electron Scattering

- Inverse  $\mu$ -decay:**

$$\nu_{\mu} + e^{-} \rightarrow \mu^{-} + \nu_e$$

- Total spin  $J=0$

(Assuming massless muon, helicity=chirality)



$$Q^2 \equiv -(\underline{e} - \underline{\nu}_e)^2$$

$$\sigma_{TOT} \propto \int_0^{Q_{\max}^2} dQ^2 \frac{1}{(Q^2 + M_W^2)^2}$$

$$\approx \frac{Q_{\max}^2}{M_W^4}$$

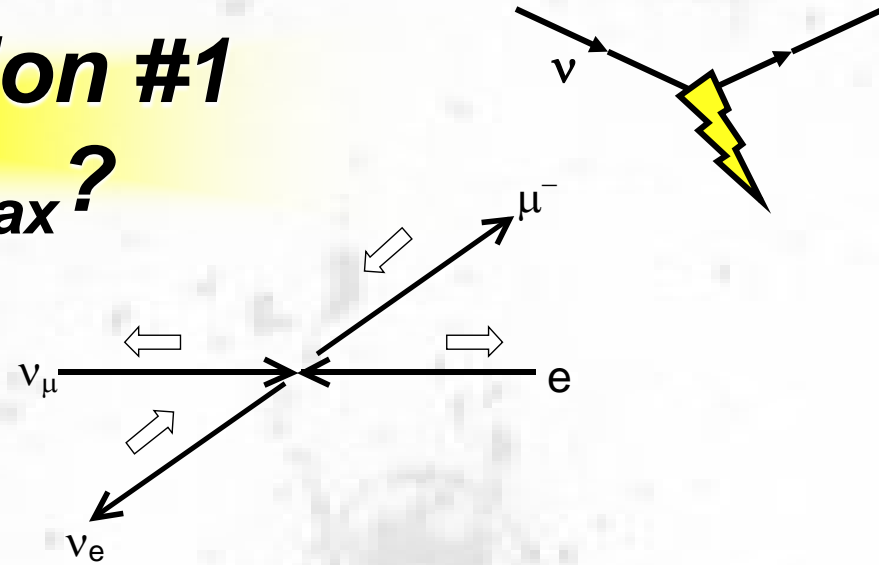
# Lecture Question #1

## What is $Q^2_{max}$ ?

$$\nu_\mu + e^- \rightarrow \mu^- + \nu_e$$

$$Q^2 \equiv -(\underline{e} - \underline{\nu}_e)^2$$

Work in the center-of-mass frame and assume, **for now**, that we can neglect the masses.



# Lecture Question #1

## What is $Q^2_{max}$ ?

$$\nu_\mu + e^- \rightarrow \mu^- + \nu_e$$

$$Q^2 \equiv -(\underline{e} - \underline{\nu}_e)^2$$

Work in the center-of-mass frame and assume, **for now**,

that we can neglect the masses.

$$\underline{e} \approx (E_v^*, 0, 0, -E_v^*)$$

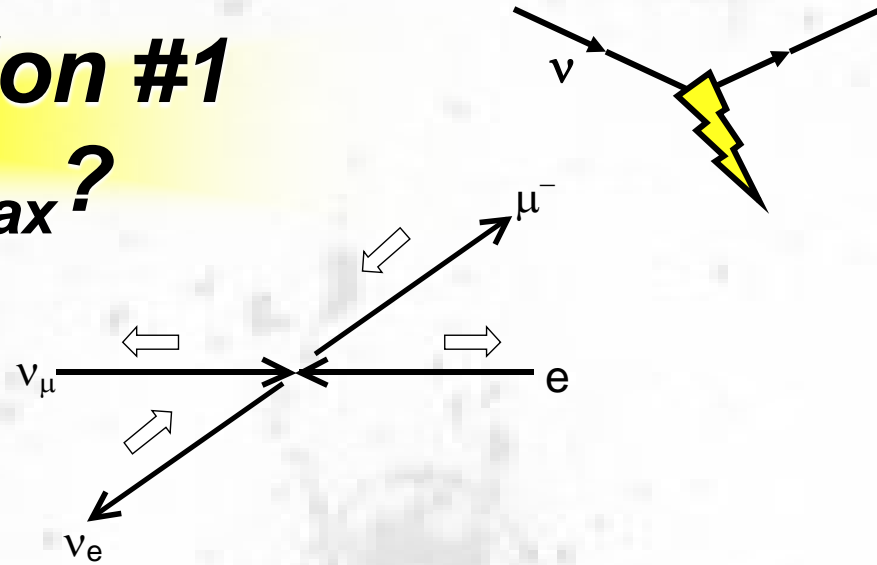
$$\underline{\nu}_e \approx (E_v^*, -E_v^* \sin \theta^*, 0, -E_v^* \cos \theta^*)$$

$$Q^2 = -(\underline{e}^2 + \underline{\nu}_e^2 - 2\underline{e} \cdot \underline{\nu}_e)^2$$

$$\approx -\left[-2E_v^{*2}(1 - \cos \theta^*)\right]$$

$$0 < Q^2 < (2E_v^*)^2 \approx (\underline{e} + \underline{\nu}_\mu)^2$$

$$0 < Q^2 < s$$



# Neutrino-Electron (cont'd)

$$\sigma_{TOT} \propto Q_{\max}^2 = s$$

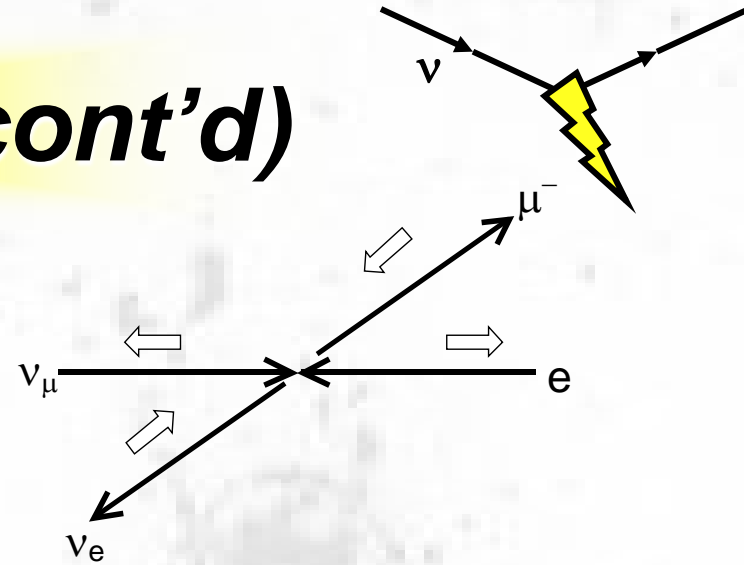
$$\sigma_{TOT} = \frac{G_F^2 s}{\pi}$$

$$= 17.2 \times 10^{-42} \text{ cm}^2 / \text{GeV} \cdot E_\nu (\text{GeV})$$

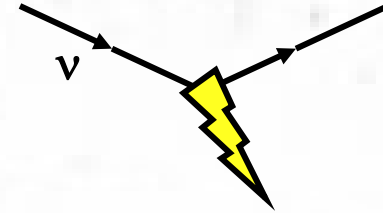
- Why is it proportional to beam energy?

$$s = (\underline{p}_{\nu_\mu} + \underline{p}_e)^2 = m_e^2 + 2m_e E_\nu \text{ (e}^-\text{ rest frame)}$$

- Proportionality to energy is a generic feature of point-like scattering!
  - because  $d\sigma/dQ^2$  is constant (at these energies)



# Neutrino-Electron (cont'd)



- **Elastic scattering:**

$$\nu_{\mu} + e^{-} \rightarrow \nu_{\mu} + e^{-}$$

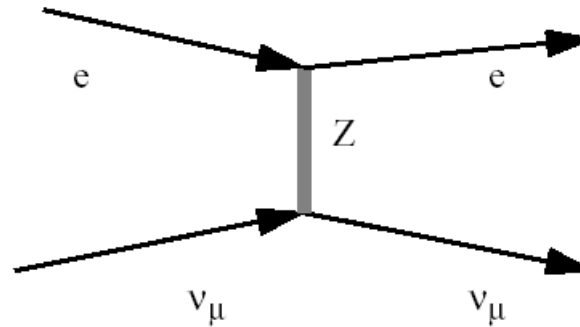
- Recall, EW theory has coupling to left or right-handed electron

- Total spin,  $J=0,1$

- **Electron- $Z^0$  coupling**

- Left-handed:  $-1/2 + \sin^2\theta_W$

- Right-handed:  $\sin^2\theta_W$

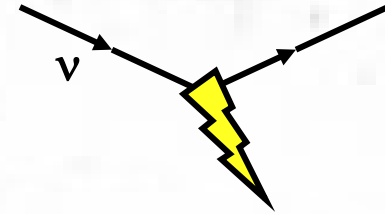


Z Couplings	$g_L$	$g_R$
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$e, \mu, \tau$	$-1/2 + \sin^2\theta_W$	$\sin^2\theta_W$
$u, c, t$	$1/2 - 2/3 \sin^2\theta_W$	$-2/3 \sin^2\theta_W$
$d, s, b$	$-1/2 + 1/3 \sin^2\theta_W$	$1/3 \sin^2\theta_W$

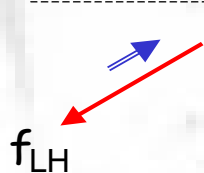
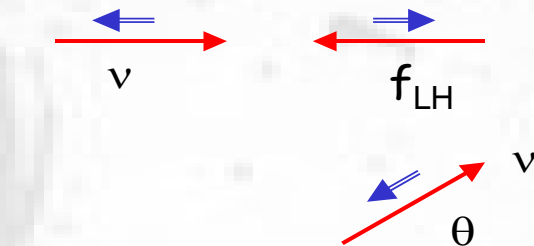
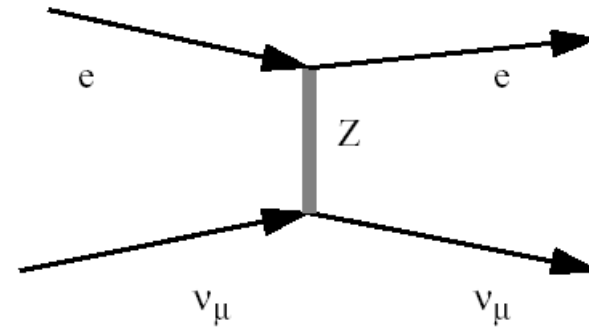
$$\sigma \propto \frac{G_F^2 s}{\pi} \left( \frac{1}{4} - \sin^2 \theta_W + \sin^4 \theta_W \right)$$

$$\sigma \propto \frac{G_F^2 s}{\pi} (\sin^4 \theta_W)$$

# Neutrino-Electron (cont'd)

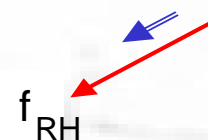
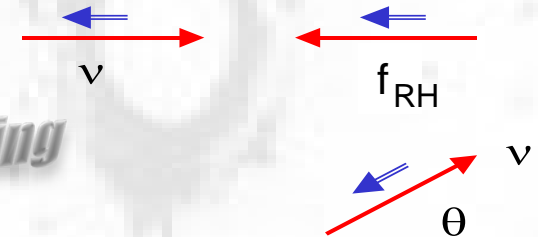


- What are relative contributions of scattering from left *and* right-handed electrons?



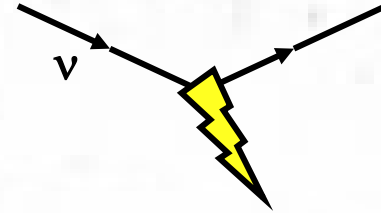
$$\frac{d\sigma}{d\cos\theta} = \text{const}$$

*Backwards scattering is disfavored*



$$\frac{d\sigma}{d\cos\theta} = \text{const} \times \left( \frac{1 + \cos\theta}{2} \right)^2$$

# Neutrino-Electron (cont'd)



- **Electron- $Z^0$  coupling**  $\sigma \propto \frac{G_F^2 s}{\pi} \left( \frac{1}{4} - \sin^2 \theta_W + \sin^4 \theta_W \right)$ 
  - (LH, V-A):  $-1/2 + \sin^2 \theta_W$
  - (RH, V+A):  $\sin^2 \theta_W$

$$\sigma \propto \frac{G_F^2 s}{\pi} (\sin^4 \theta_W)$$

Let  $y$  denote inelasticity.  
Recoil energy is related to  
CM scattering angle by

$$y = \frac{E_e}{E_\nu} \approx 1 - \frac{1}{2} (1 - \cos \theta)$$

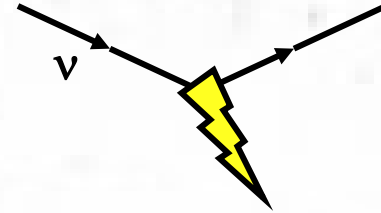
$$\int dy \frac{d\sigma}{dy} = \begin{cases} \text{LH:} & \int dy = 1 \\ \text{RH:} & \int (1-y)^2 dy = 1/3 \end{cases}$$

$$\sigma_{TOT} = \frac{G_F^2 s}{\pi} \left( \frac{1}{4} - \sin^2 \theta_W + \frac{4}{3} \sin^4 \theta_W \right) = 1.4 \times 10^{-42} \text{ cm}^2 / \text{GeV} \cdot E_\nu (\text{GeV})$$



# **Lecture Question #2:**

## **Flavors and $\nu_e$ Scattering**



**The reaction**

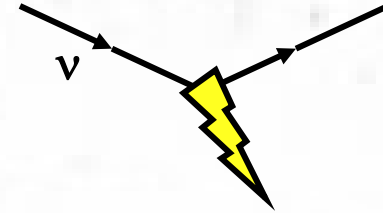
$$\nu_{\mu} + e^{-} \rightarrow \nu_{\mu} + e^{-}$$

**has a much smaller cross-section than**

$$\nu_e + e^{-} \rightarrow \nu_e + e^{-}$$

**Why?**

# Lecture Question #2: Flavors and $\nu_e$ Scattering



The reaction

$$\nu_{\mu} + e^{-} \rightarrow \nu_{\mu} + e^{-}$$

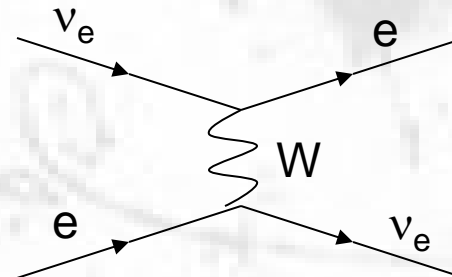
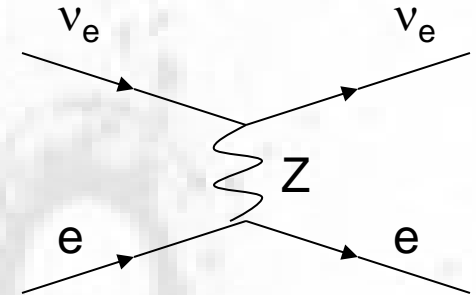
has a much smaller cross-section than

$$\nu_e + e^{-} \rightarrow \nu_e + e^{-}$$

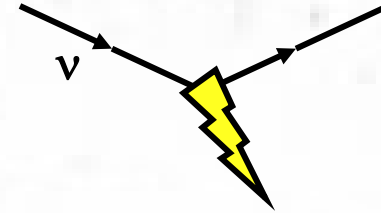
Why?

$$\nu_e + e^{-} \rightarrow \nu_e + e^{-}$$

has a second contributing  
reaction, charged current



# Lecture Question #2: Flavors and $\nu_e$ Scattering



**Let's show that this increases the rate**

(Recall from the previous pages...

$$\sigma_{TOT} = \int dy \frac{d\sigma}{dy}$$

$$= \int dy \left[ \frac{d\sigma^{LH}}{dy} + \frac{d\sigma^{RH}}{dy} \right]$$

$$= \sigma_{TOT}^{LH} + \frac{1}{3} \sigma_{TOT}^{RH}$$

$$\sigma_{TOT}^{LH} \propto \left| \text{total coupling}_{e^-}^{LH} \right|^2$$

For electron...	LH coupling	RH coupling
Weak NC	$-1/2 + \sin^2\theta_W$	$\sin^2\theta_W$
Weak CC	$-1/2$	0

)

We have to show the interference between CC and NC is constructive.

The total RH coupling is unchanged by addition of CC because there is no RH weak CC coupling

There are two LH couplings: NC coupling is  $-1/2 + \sin^2\theta_W \approx -1/4$  and the CC coupling is  $-1/2$ . We add the associated amplitudes... and get  $-1 + \sin^2\theta_W \approx -3/4$

# Lepton Mass Effects

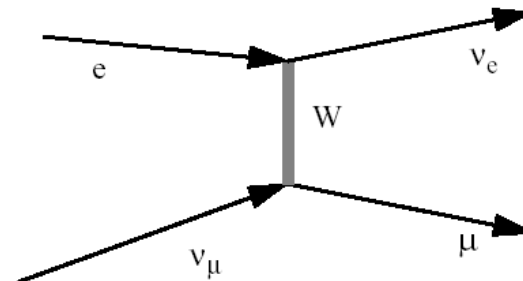
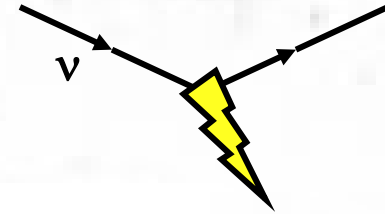
- Let's return to Inverse  $\mu$ -decay:

$$\nu_\mu + e^- \rightarrow \mu^- + \nu_e$$

- What changes in the presence of final state mass?
  - pure CC so always left-handed
  - BUT there must be finite  $Q^2$  to create muon in final state!

$$Q_{\min}^2 = m_\mu^2$$

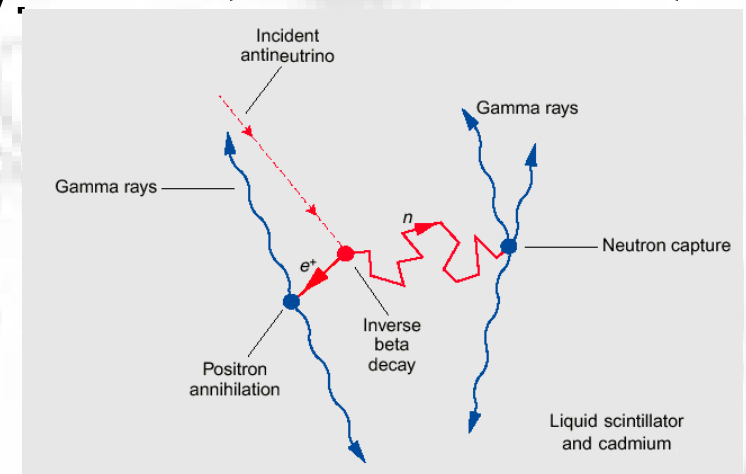
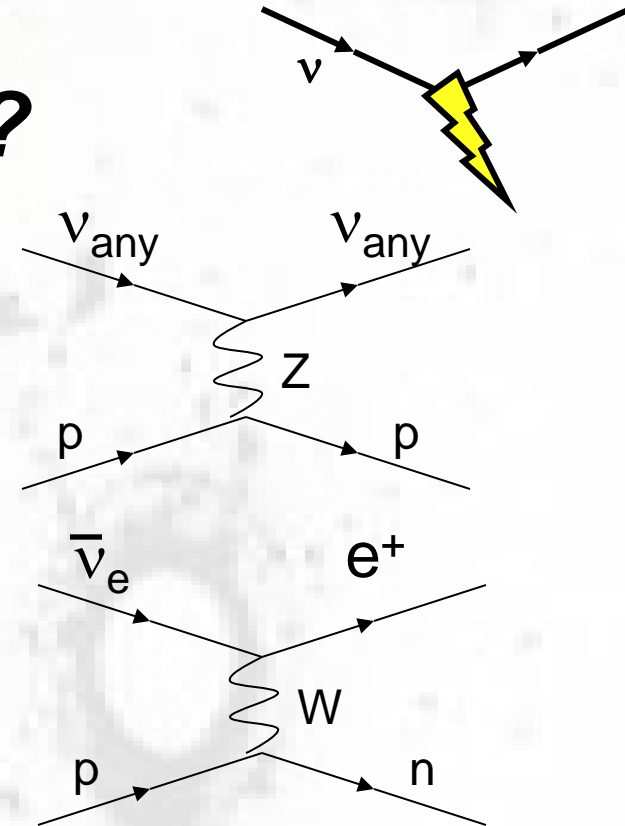
- see a suppression scaling with **(mass/CM energy)<sup>2</sup>**
  - This can be generalized...



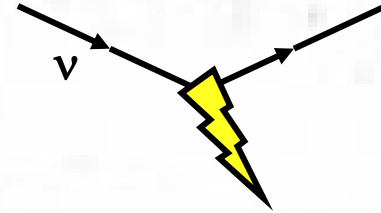
$$\begin{aligned} \sigma_{TOT} &\propto \int_{Q_{\min}^2}^{Q_{\max}^2} dQ^2 \frac{1}{(Q^2 + M_W^2)^2} \\ &\approx \frac{Q_{\max}^2 - Q_{\min}^2}{M_W^4} \\ \sigma_{TOT} &= \frac{G_F^2 (s - m_\mu^2)}{\pi} \\ &= \left[ \sigma_{TOT}^{(\text{massless})} \right] \left( 1 - \frac{m_\mu^2}{s} \right) \end{aligned}$$

# What about other targets?

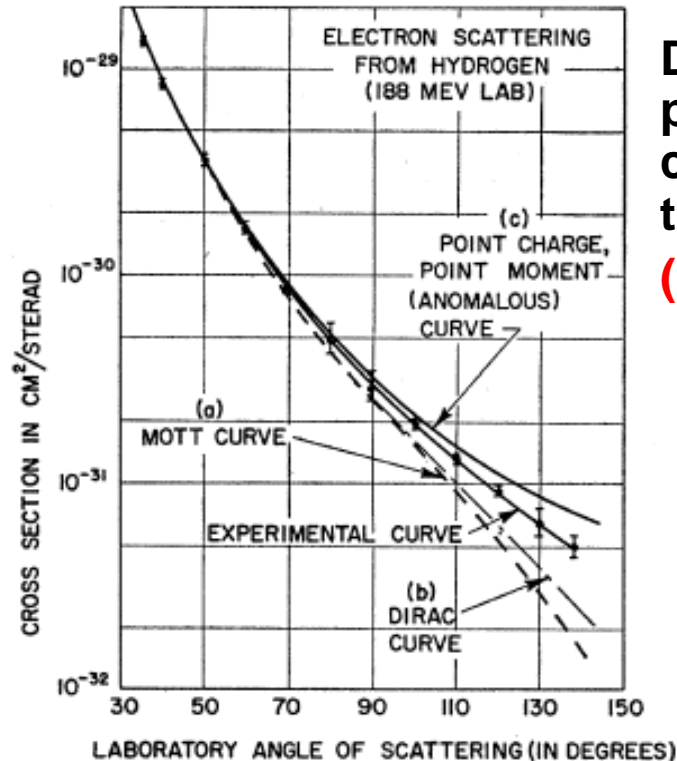
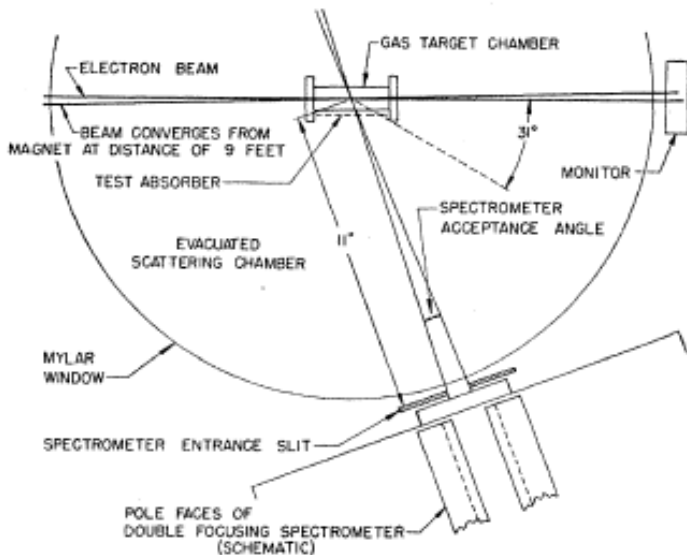
- Imagine now a proton target
  - Neutrino-proton elastic scattering:
$$\nu_e + p \rightarrow \nu_e + p$$
  - “Inverse beta-decay” (IBD):
$$\bar{\nu}_e + p \rightarrow e^+ + n$$
  - and “stimulated” beta decay:
$$\nu_e + n \rightarrow e^- + p$$
  - Recall that IBD was the Reines and Cowan discovery signal



# Proton Structure



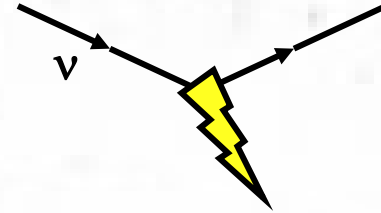
- How is a proton different from an electron?
  - anomalous magnetic moment,  $\kappa \equiv \frac{g-2}{2} \neq 1$
  - “form factors” related to finite size



Determined  
proton RMS  
charge radius  
to be  
 **$(0.7 \pm 0.2)$   
 $\times 10^{-13}$  cm**

**McAllister and Hofstadter 1956**  
188 MeV and 236 MeV electron beam  
from linear accelerator at Stanford

# Final State Mass Effects

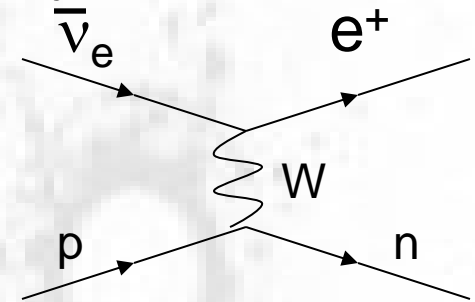


- In IBD,  $\bar{\nu}_e + p \rightarrow e^+ + n$ , have to pay a mass penalty *twice*

- $M_n - M_p \approx 1.3 \text{ MeV}$ ,  $M_e \approx 0.5 \text{ MeV}$

- What is the threshold?

- kinematics are simple, at least to zeroth order in  $M_e/M_n$   
 $\rightarrow$  heavy nucleon kinetic energy is zero

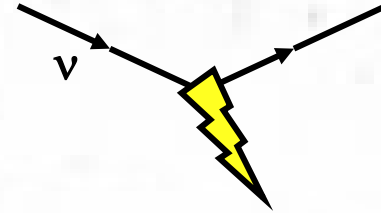


$$s_{\text{initial}} = (\underline{p}_\nu + \underline{p}_p)^2 = M_p^2 + 2M_p E_\nu \quad (\text{proton rest frame})$$

$$s_{\text{final}} = (\underline{p}_e + \underline{p}_n)^2 \approx M_n^2 + m_e^2 + 2M_n \left( E_\nu - (M_n - M_p) \right)$$

- Solving...  $E_\nu^{\text{min}} \approx \frac{(M_n + m_e)^2 - M_p^2}{2M_p} \approx 1.806 \text{ MeV}$

# Final State Mass Effects (cont'd)



- Define  $\delta E$  as  $E_\nu - E_\nu^{\min}$ , then

$$\begin{aligned} s_{\text{initial}} &= M_p^2 + 2M_p \left( \delta E + E_\nu^{\min} \right) \\ &= M_p^2 + 2\delta E \times M_p + (M_n + m_e)^2 - M_p^2 \\ &= 2\delta E \times M_p + (M_n + m_e)^2 \end{aligned}$$

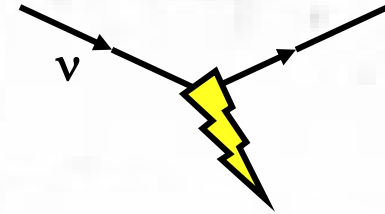
- Remember the suppression generally goes as

$$\xi_{\text{mass}} = 1 - \frac{m_{\text{final}}^2}{s} = 1 - \frac{(M_n + m_e)^2}{(M_n + m_e)^2 + 2M_p \times \delta E}$$

$$= \frac{2M_p \times \delta E}{(M_n + m_e)^2 + 2M_p \times \delta E} \approx \begin{cases} \delta E \times \frac{2M_p}{(M_n + m_e)^2} & \text{low energy} \\ 1 - \frac{(M_n + m_e)^2}{2M_p^2} \frac{M_p}{\delta E} & \text{high energy} \end{cases}$$



# Putting it all together...



$$\sigma_{TOT} = \frac{G_F^2 S}{\pi} \times \cos^2 \theta_{Cabibbo} \times (\xi_{mass}) \times (g_V^2 + 3g_A^2)$$

*quark mixing!*

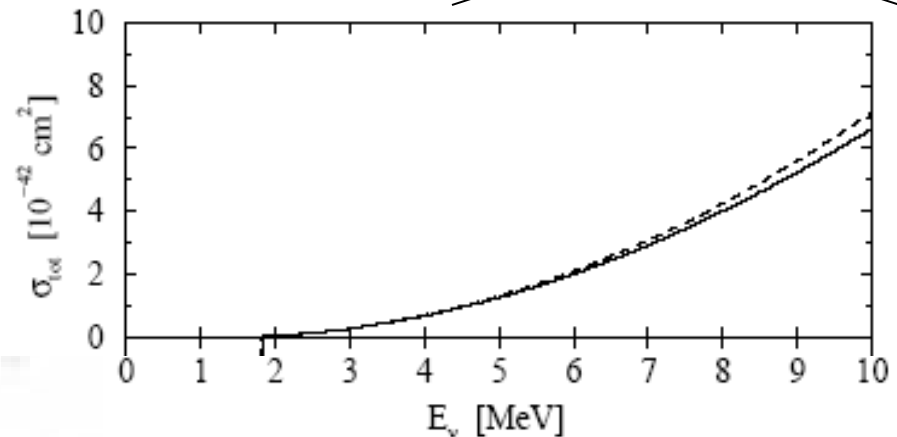
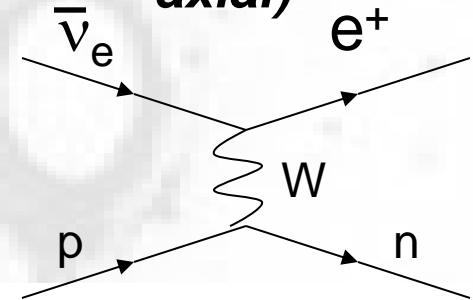
*final state mass suppression*

*proton form factors (vector, axial)*

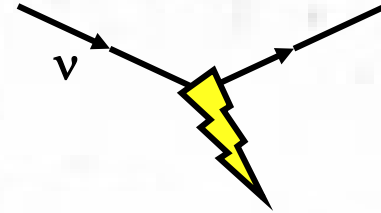
- mass suppression is proportional to  $\delta E$  at low  $E_\nu$ , so quadratic near threshold
- vector and axial-vector form factors (for IBD usually referred to as  $f$  and  $g$ , respectively)

$$g_V, g_A \approx 1, 1.26.$$

- FFs,  $\theta_{Cabibbo}$ , best known from  $\tau_n$



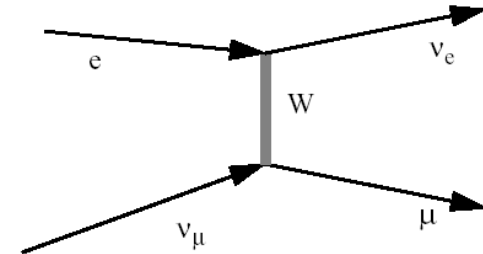
# Lecture Question #3: Quantitative Lepton Mass Effect



- Which is closest to the minimum beam energy in which the reaction

$$\nu_{\mu} + e^{-} \rightarrow \mu^{-} + \nu_e$$

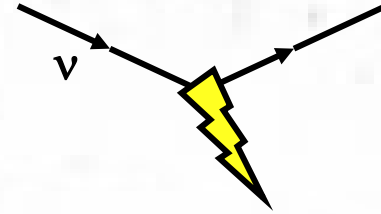
can be observed?



(a) 100 MeV      (b) 1 GeV      (c) 10 GeV

(It might help you to remember that  $Q_{\min}^2 = m_{\mu}^2$   
or you might just want to think about the total CM energy required  
to produce the particles in the final state.)

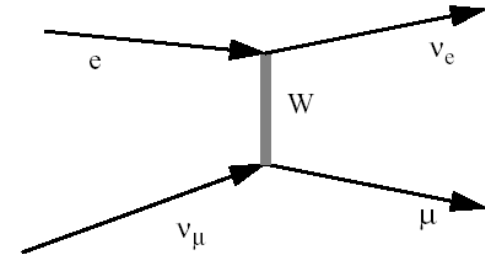
# Lecture Question #3: Quantitative Lepton Mass Effect



- Which is closest to the minimum beam energy in which the reaction

$$\nu_{\mu} + e^{-} \rightarrow \mu^{-} + \nu_e$$

can be observed?



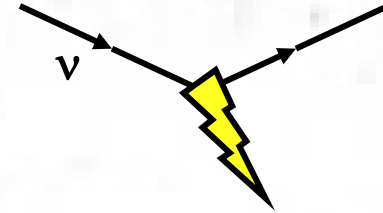
$$Q^2_{\min} = m_{\mu}^2 \text{ (a) 100 MeV (b) 1 GeV}$$

(c) 10 GeV

$$Q^2 < s = (\underline{p}_e + \underline{p}_{\nu})^2$$

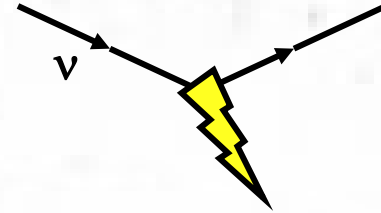
$$= (m_e + E_{\nu}, 0, 0, \sqrt{E_{\nu}^2 - m_{\nu}^2})^2 \approx m_e^2 + 2m_e E_{\nu}$$

$$\therefore E_{\nu} > \frac{m_{\mu}^2}{2m_e} \approx 10.9 \text{ GeV}$$



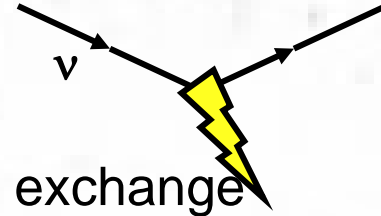
# Summary... and Next Topic

- We know  $\nu e^-$  scattering and IBD cross-sections!
- In point-like weak interactions, key features are:
  - $d\sigma/dQ^2$  is  $\approx$  constant.
    - Integrating gives  $\sigma \propto E_\nu$
  - LH coupling enters w/  $d\sigma/dy \propto 1$ , RH w/  $d\sigma/dy \propto (1-y)^2$ 
    - Integrating these gives 1 and 1/3, respectively
  - Lepton mass effect gives minimum  $Q^2$ 
    - Integrating gives correction factor in  $\sigma$  of  $(1-Q_{\min}^2/s)$
  - Structure of target can add form factors
- Deep Inelastic Scattering is also a point-like limit where interaction is  $\nu$ -quark scattering

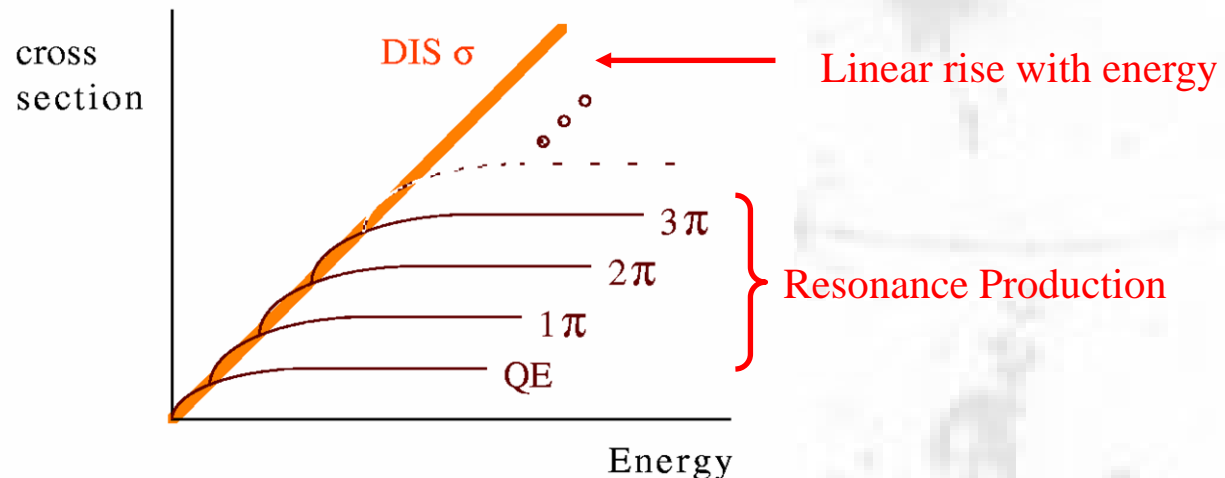


# ***Neutrino-Nucleon Deep Inelastic Scattering***

# Neutrino-Nucleon Scattering

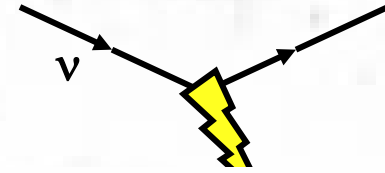


- Charged - Current:  $W^\pm$  exchange
  - Quasi-elastic Scattering:  
(Target changes but no break up)  
 $\nu_\mu + n \rightarrow \mu^- + p$
  - Nuclear Resonance Production:  
(Target goes to excited state)  
 $\nu_\mu + n \rightarrow \mu^- + p + \pi^0$  ( $N^*$  or  $\Delta$ )  
 $n + \pi^+$
  - Deep-Inelastic Scattering:  
(Nucleon broken up)  
 $\nu_\mu + \text{quark} \rightarrow \mu^- + \text{quark}'$
- Neutral - Current:  $Z^0$  exchange
  - Elastic Scattering:  
(Target unchanged)  
 $\nu_\mu + N \rightarrow \nu_\mu + N$
  - Nuclear Resonance Production:  
(Target goes to excited state)  
 $\nu_\mu + N \rightarrow \nu_\mu + N + \pi$  ( $N^*$  or  $\Delta$ )
  - Deep-Inelastic Scattering  
(Nucleon broken up)  
 $\nu_\mu + \text{quark} \rightarrow \nu_\mu + \text{quark}$



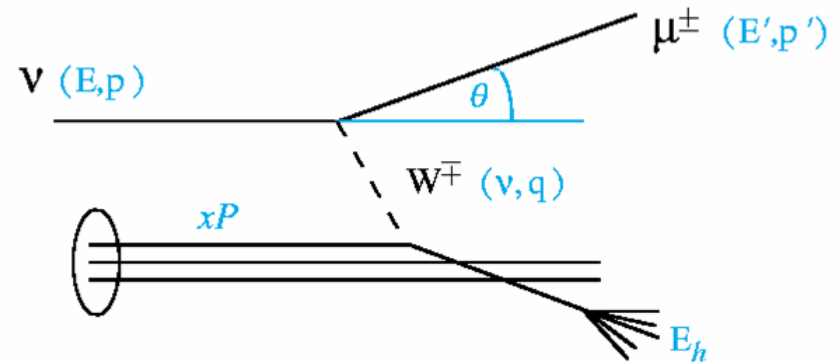
# Scattering Variables

## DEEP INELASTIC NEUTRINO SCATTERING



Scattering variables given in terms of invariants

- More general than just deep inelastic (neutrino-quark) scattering, although interpretation may change.



Measured quantities:  $E_h$ ,  $E'$ ,  $\theta$

$$\text{4-momentum Transfer}^2: Q^2 = -q^2 = -(p' - p)^2 \approx \left( 4EE' \sin^2(\theta/2) \right)_{\text{Lab}}$$

$$\text{Energy Transfer: } \nu = (q \cdot P) / M_T = (E - E')_{\text{Lab}} = (E_h - M_T)_{\text{Lab}}$$

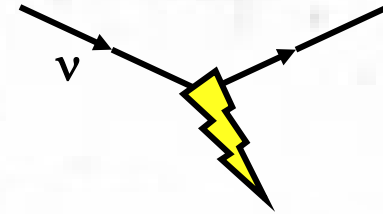
$$\text{Inelasticity: } y = (q \cdot P) / (p \cdot P) = (E_h - M_T) / (E_h + E')_{\text{Lab}}$$

$$\text{Fractional Momentum of Struck Quark: } x = -q^2 / 2(p \cdot q) = Q^2 / 2M_T \nu$$

$$\text{Recoil Mass}^2: W^2 = (q + P)^2 = M_T^2 + 2M_T \nu - Q^2$$

$$\text{CM Energy}^2: s = (p + P)^2 = M_T^2 + \frac{Q^2}{xy}$$

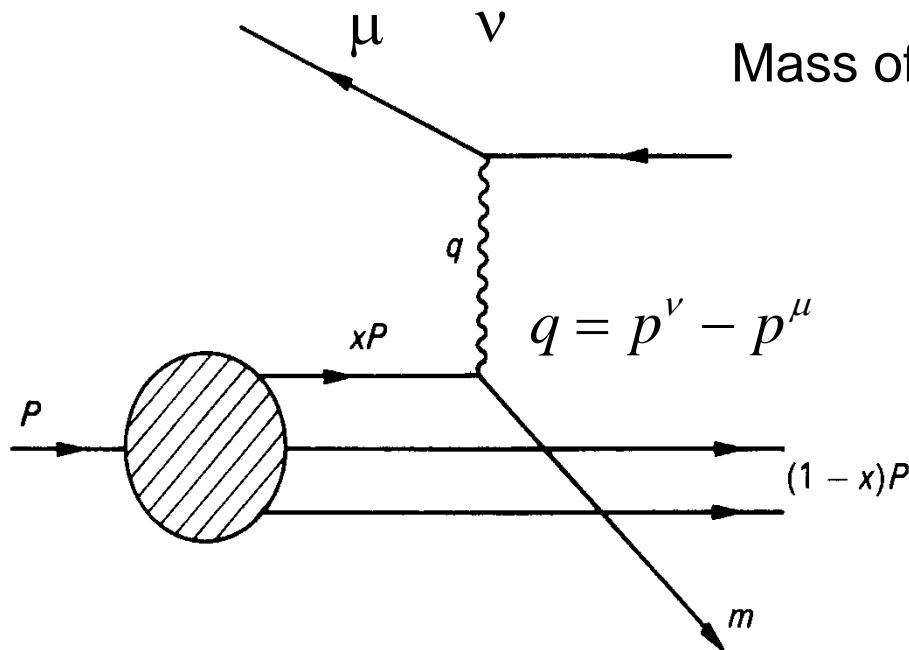
# Parton Interpretation of High Energy Limit



Mass of target quark  $m_q^2 = x^2 P^2 = x^2 M_T^2$

Mass of final state quark

$$m_{q'}^2 = (xP + q)^2$$



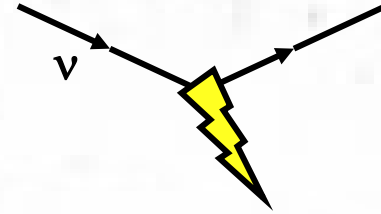
Neutrino scatters off a parton inside the nucleon

In “infinite momentum frame”,  $xP$  is momentum of partons inside the nucleon

$$x = \frac{Q^2}{2P \cdot q} = \frac{Q^2}{2M_T \nu}$$



# So why is cross-section so large?

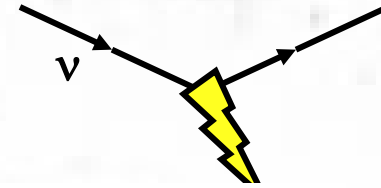


- (at least compared to  $\nu e^-$  scattering!)
- Recall that for neutrino beam and target at rest

$$\sigma_{TOT} \approx \frac{G_F^2}{\pi} \int_0^{Q_{\max}^2 \equiv s} dQ^2 = \frac{G_F^2 s}{\pi}$$
$$s = m_e^2 + 2m_e E_\nu$$

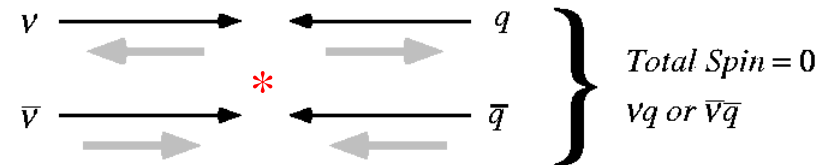
- But we just learned for DIS that effective mass of each target quark is  $m_q = x m_{\text{nucleon}}$
- So much larger target mass means larger  $\sigma_{TOT}$

# Chirality, Charge in CC $\nu$ -q Scattering

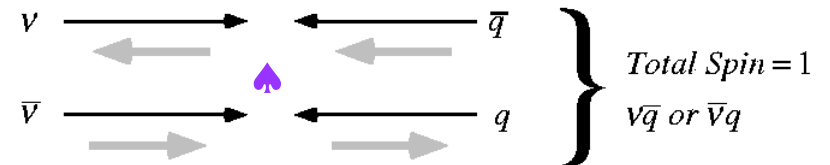


- Total spin determines inelasticity distribution
  - Familiar from neutrino-electron scattering

*point-like scattering  
implies linear with energy*



Flat in  $y$



$$1/4(1+\cos\theta^*)^2 = (1-y)^2$$

$$\int (1-y)^2 dy = 1/3$$

$$\frac{d\sigma^{\nu p}}{dxdy} = \frac{G_F^2 S}{\pi} \left( x \overset{*}{d}(x) + x \overset{\spadesuit}{\bar{u}}(x)(1-y)^2 \right)$$

$$\frac{d\sigma^{\bar{\nu} p}}{dxdy} = \frac{G_F^2 S}{\pi} \left( x \overset{*}{\bar{d}}(x) + x \overset{\spadesuit}{u}(x)(1-y)^2 \right)$$

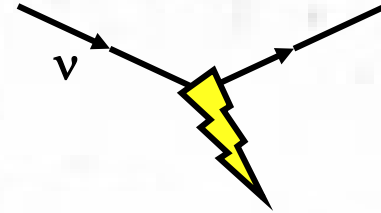
- Neutrino/Anti-neutrino CC each produce particular  $\Delta q$  in scattering

$$\nu d \rightarrow \mu^- u$$

$$\bar{\nu} u \rightarrow \mu^+ d$$

but what is this " $q(x)$ "?

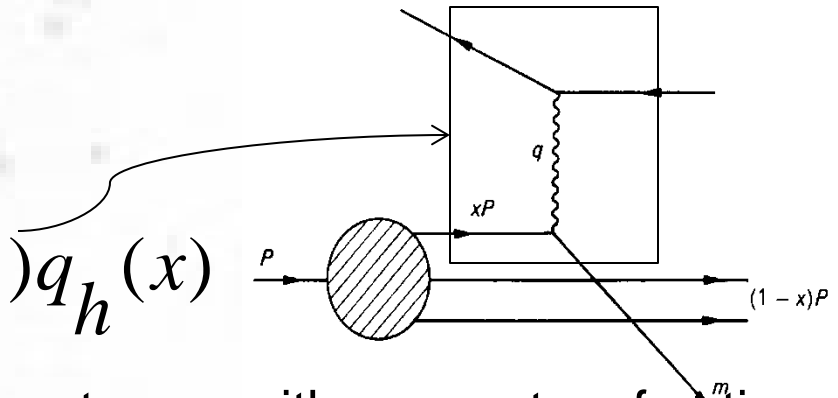
# Factorization and Partons



- Factorization Theorem of QCD allows cross-sections for hadronic processes to be written as:

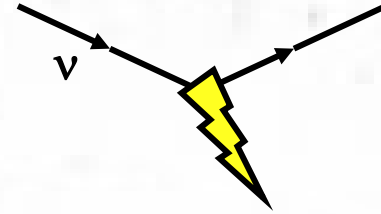
$$\sigma(l + h \rightarrow l + X)$$

$$= \sum_q \int dx \sigma(l + q(x) \rightarrow l + X) q_h(x)$$



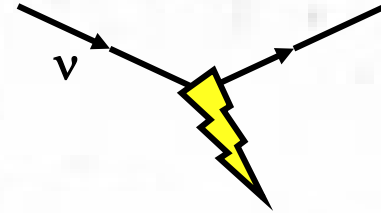
- $q_h(x)$  is the probability of finding a parton,  $q$ , with momentum fraction  $x$  inside the hadron,  $h$ . It is called a parton distribution function (PDF).
  - PDFs are universal
  - PDFs are not (yet) calculable from first principles in QCD
- “Scaling”: parton distributions are largely independent of  $Q^2$  scale, and depend on fractional momentum,  $x$ .

# Brief Summary of Neutrino-Quark Scattering so Far

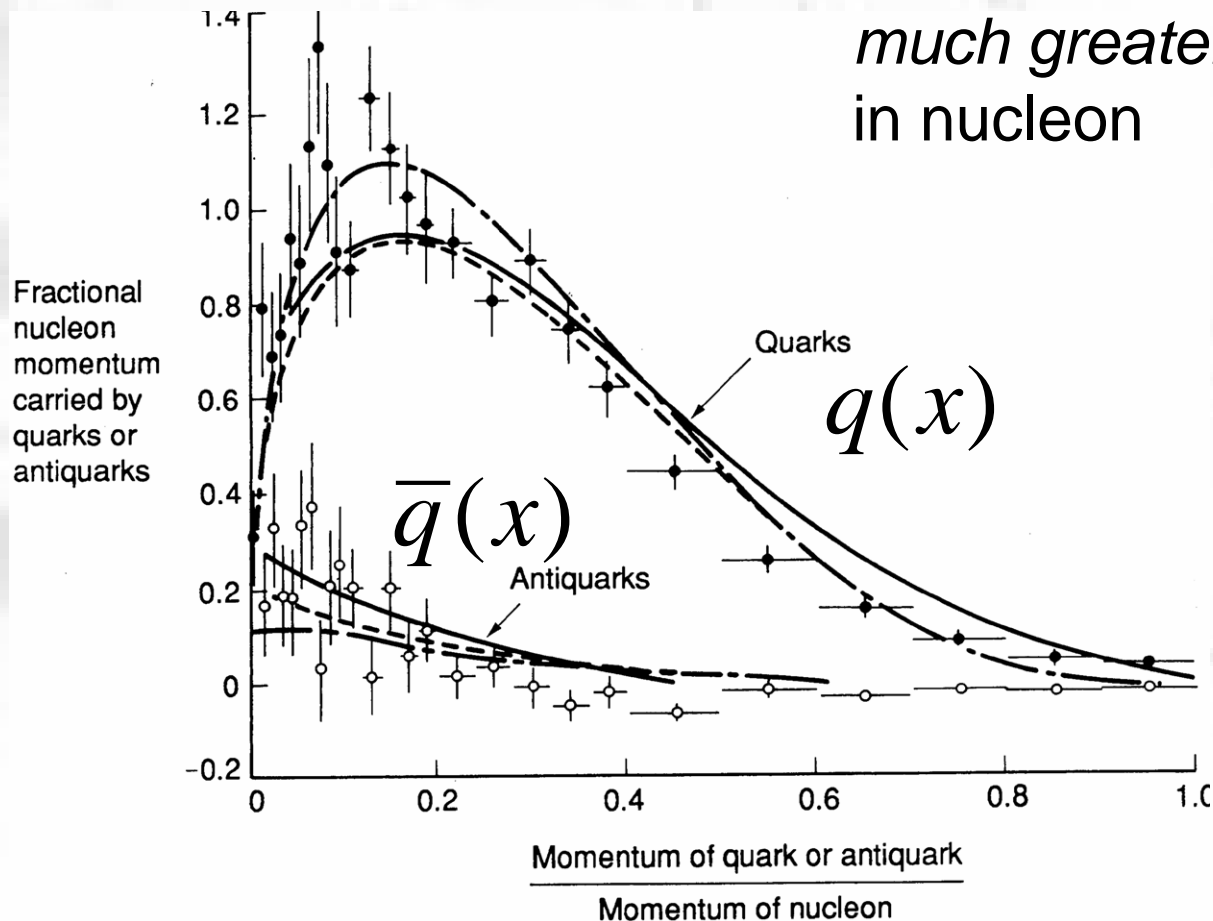


- $x \equiv Q^2/2M_T v$  is the fraction of the nucleon 4-momentum carried by a quark in the infinite momentum frame
  - Effective mass for struck quark,  $M_q = \sqrt{(xP)^2} = xM_T$
  - Parton distribution functions,  $q(x)$ , incorporate information about the “flux” of quarks inside the hadron
- Quark and anti-quark scattering from neutrinos or anti-neutrinos defines total spin
  - $\nu q$  and  $\bar{\nu} \bar{q}$  are spin 0, isotropic
  - $\nu \bar{q}$  and  $\bar{\nu} q$  are spin 1, backscattering is suppressed
- Neutrinos and anti-neutrinos pick out definite quark and anti-quark flavors (charge conservation)

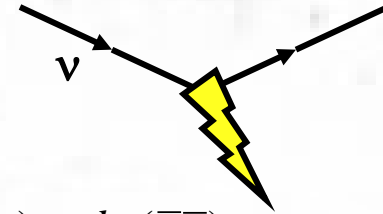
# Momentum of Quarks & Antiquarks



- Momentum carried by quarks *much greater* than anti-quarks in nucleon



# *y* distribution in Neutrino CC DIS



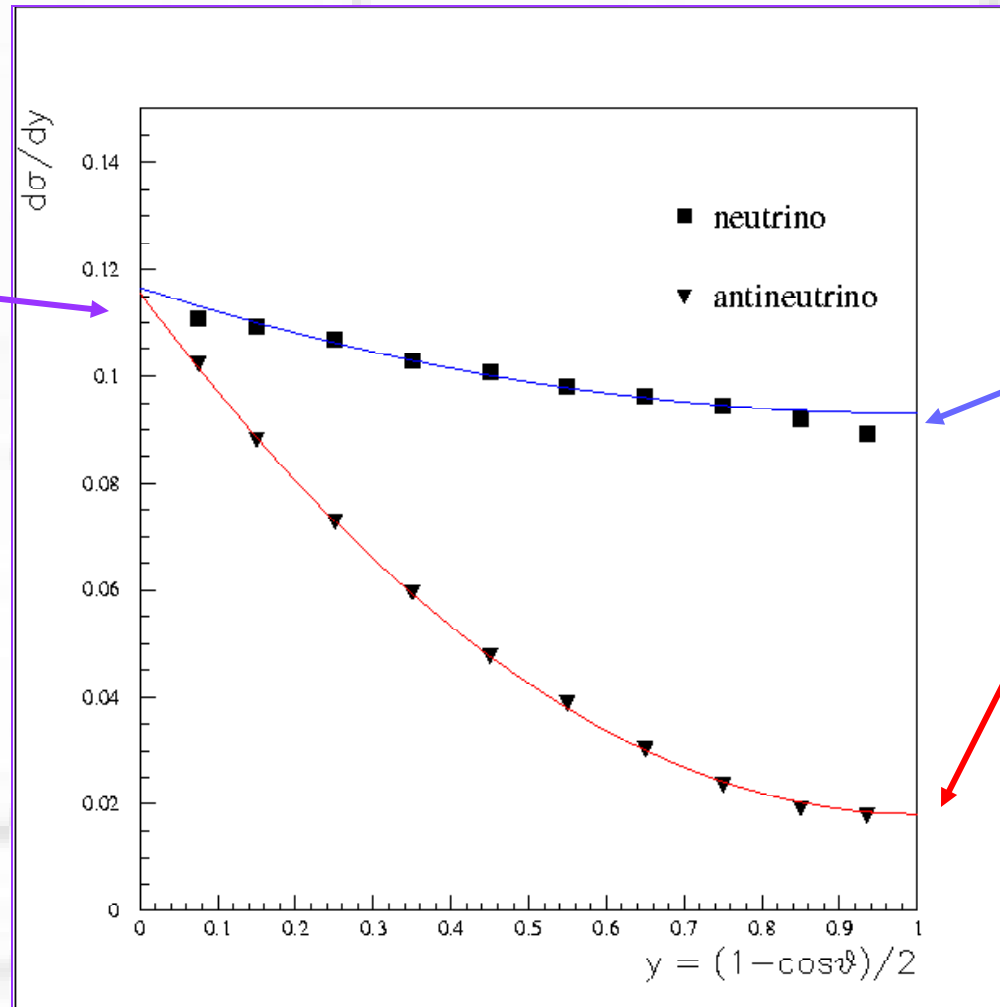
$$\frac{d\sigma(\nu q)}{dx dy} = \frac{d\sigma(\bar{\nu} \bar{q})}{dx dy} \propto 1$$

$$\frac{d\sigma(\nu \bar{q})}{dx dy} = \frac{d\sigma(\bar{\nu} q)}{dx dy} \propto (1-y)^2$$

At  $y=0$ :

Quarks & anti-quarks

Neutrino and anti-neutrino identical



At  $y=1$ :

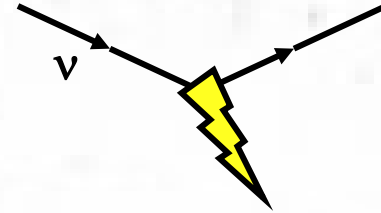
Neutrinos see only quarks.

Anti-neutrinos see only anti-quarks

Averaged over protons and neutrons,

$$\sigma^{\bar{\nu}} \approx \frac{1}{2} \sigma^{\nu}$$

# Structure Functions (SFs)



- A model-independent picture of these interactions can also be formed in terms of nucleon “structure functions”
  - All Lorentz-invariant terms included
  - Approximate zero lepton mass (small correction)

$$\frac{d\sigma^{\nu,\bar{\nu}}}{dxdy} \propto \left[ y^2 2xF_1(x, Q^2) + \left( 2 - 2y - \frac{M_T xy}{E} \right) F_2(x, Q^2) \pm y(2-y)xF_3(x, Q^2) \right]$$

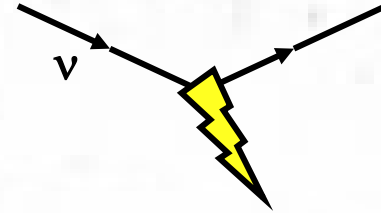
- For massless free spin-1/2 partons, one simplification...
  - Callan-Gross relationship,  $2xF_1 = F_2$
  - Implies intermediate bosons are completely transverse

Can parameterize longitudinal cross-section by  $R_L$ .

Callan-Gross violations result from  $M_T$ , NLO pQCD,  $g \rightarrow qq$

$$R_L = \frac{\sigma_L}{\sigma_T} = \frac{F_2}{2xF_1} \left( 1 + \frac{4M_T^2 x^2}{Q^2} \right)$$

# SFs to PDFs



- Can relate SFs to PDFs in naïve quark-parton model by matching  $y$  dependence

- Assuming Callan-Gross, massless targets and partons...

- $F_3$ :  $2y - y^2 = (1-y)^2 - 1$  ,  $2xF_1 = F_2$ :  $2 - 2y + y^2 = (1-y)^2 + 1$

$$2xF_1^{\nu p, CC} = x \left[ d_p(x) + \bar{u}_p(x) + s_p(x) + \bar{c}_p(x) \right]$$

$$xF_3^{\nu p, CC} = x \left[ d_p(x) - \bar{u}_p(x) + s_p(x) - \bar{c}_p(x) \right]$$

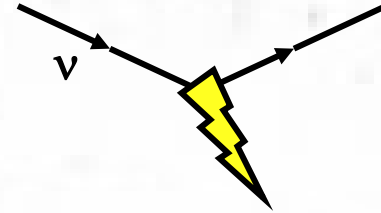
- In analogy with neutrino-electron scattering, **CC** only involves **left-handed quarks**
- However, **NC** involves both chiralities (**V-A** and **V+A**)
  - Also **couplings** from EW Unification
  - And **no selection by quark charge**

$$2xF_1^{\nu p, NC} = x \left[ (u_L^2 + u_R^2) \left( u_p(x) + \bar{u}_p(x) + c_p(x) + \bar{c}_p(x) \right) + (d_L^2 + d_R^2) \left( d_p(x) + \bar{d}_p(x) + s_p(x) + \bar{s}_p(x) \right) \right]$$

$$xF_3^{\nu p, NC} = x \left[ (u_L^2 - u_R^2) \left( u_p(x) - \bar{u}_p(x) + c_p(x) - \bar{c}_p(x) \right) + (d_L^2 - d_R^2) \left( d_p(x) - \bar{d}_p(x) + s_p(x) - \bar{s}_p(x) \right) \right]$$



# Isoscalar Targets



- Heavy nuclei are roughly neutron-proton isoscalar
- Isospin symmetry implies  $u_p = d_n, d_p = u_n$
- Structure Functions have a particularly simple interpretation in quark-parton model for this case...

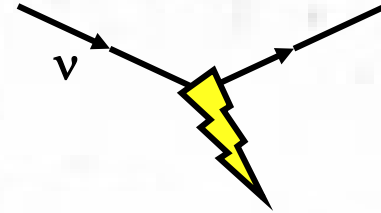
$$\frac{d^2 \sigma^{\nu(\bar{\nu})N}}{dx dy} = \frac{G_F^2 s}{2\pi} \left\{ \left( 1 + (1-y)^2 \right) F_2(x) \pm \left( 1 - (1-y)^2 \right) x F_3^{\nu(\bar{\nu})}(x) \right\}$$

$$2xF_1^{\nu(\bar{\nu})N,CC}(x) = x(u(x) + d(x) + \bar{u}(x) + \bar{d}(x) + s(x) + \bar{s}(x) + c(x) + \bar{c}(x)) = xq(x) + x\bar{q}(x)$$

$$xF_3^{\nu(\bar{\nu})N,CC}(x) = xu_{Val}(x) + xd_{Val}(x) \pm 2x(s(x) - \bar{s}(x))$$

where  $u_{Val}(x) = u(x) - \bar{u}(x)$

# Lecture Question #4: Neutrino and Anti-Neutrino $\sigma^{\nu N}$



- Given that  $\sigma_{CC}^{\bar{\nu}N} \approx \frac{1}{2} \sigma_{CC}^{\nu N}$  in the DIS regime (CC)

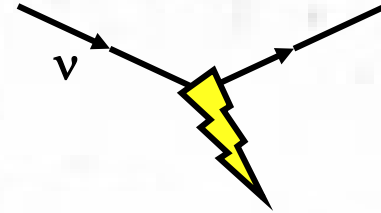
and that 
$$\frac{d\sigma(\nu q)}{dx} = \frac{d\sigma(\bar{\nu} \bar{q})}{dx} = 3 \frac{d\sigma(\nu \bar{q})}{dx} = 3 \frac{d\sigma(\bar{\nu} q)}{dx}$$

for CC scattering from quarks or anti-quarks of a given momentum,

and that cross-section is proportional to parton momentum, what is the approximate ratio of anti-quark to quark momentum in the nucleon?

- (a)  $\bar{q} / q \sim 1/3$       (b)  $\bar{q} / q \sim 1/5$       (c)  $\bar{q} / q \sim 1/8$

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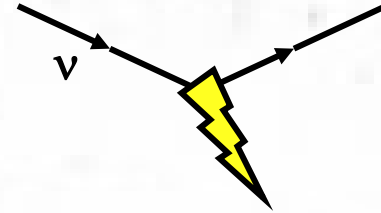
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- **Given:**  $\sigma_{CC}^{\bar{\nu}N} \approx \frac{1}{2} \sigma_{CC}^{\nu N}$  **in the DIS regime (CC)**

**and** 
$$\frac{d\sigma(\nu q)}{dx} = \frac{d\sigma(\bar{\nu} \bar{q})}{dx} = 3 \frac{d\sigma(\nu \bar{q})}{dx} = 3 \frac{d\sigma(\bar{\nu} q)}{dx}$$

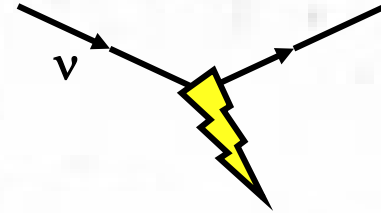
$$\sigma_{\nu} = \int_{q, \bar{q}} dx \left( \frac{d\sigma(\nu q)}{dx} + \frac{d\sigma(\nu \bar{q})}{dx} \right)$$

$$\sigma_{\bar{\nu}} = \int_{q, \bar{q}} dx \left( \frac{d\sigma(\bar{\nu} q)}{dx} + \frac{d\sigma(\bar{\nu} \bar{q})}{dx} \right) = \int_{q, \bar{q}} dx \left( \frac{d\sigma(\nu q)}{3dx} + \frac{3d\sigma(\nu \bar{q})}{dx} \right)$$

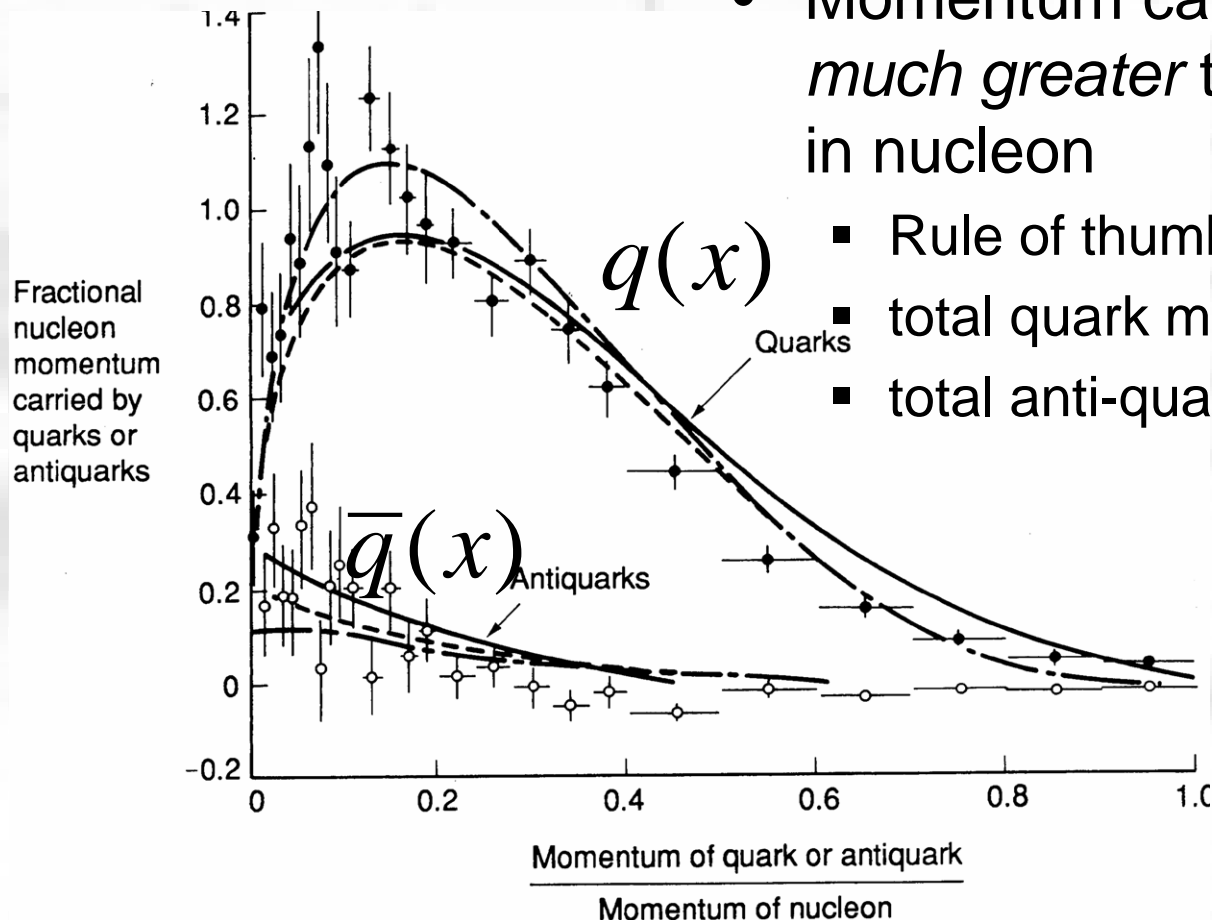
$$\therefore \int_{q, \bar{q}} dx \left( \frac{d\sigma(\nu q)}{dx} + \frac{d\sigma(\nu \bar{q})}{dx} \right) = 2 \int_{q, \bar{q}} dx \left( \frac{d\sigma(\nu q)}{3dx} + \frac{3d\sigma(\nu \bar{q})}{dx} \right)$$

$$\frac{1}{3} \int_q dx \frac{d\sigma(\nu q)}{dx} = 5 \int_{\bar{q}} dx \frac{d\sigma(\nu \bar{q})}{dx} = \frac{5}{3} \int_{\bar{q}} dx \frac{d\sigma(\bar{\nu} \bar{q})}{dx}$$

# Momentum of Quarks & Antiquarks

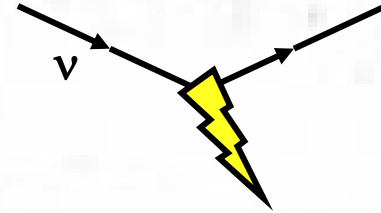


- Momentum carried by quarks *much greater* than anti-quarks in nucleon



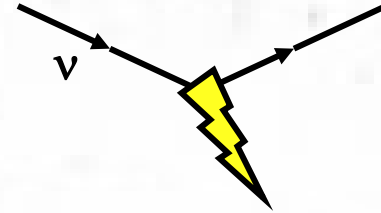
- Rule of thumb: at  $Q^2$  of  $10 \text{ GeV}^2$ :
- total quark momentum is  $1/3$ ,
- total anti-quark is  $1/15$ .

# ***From SFs to PDFs***



- As you all know, there is a large industry in determining Parton Distributions for hadron collider simulations.
  - to the point where some of my colleagues on collider experiments might think of parton distributions as an annoying piece of FORTRAN code in their software package
- The purpose, of course, is to use factorization to predict cross-sections for various processes
  - combining deep inelastic scattering data from various sources together allows us to “measure” parton distributions
  - which then are applied to predict hadron-hadron processes at colliders, and can also be used in predictions for neutrino scattering, as we shall see.

# From SFs to PDFs (cont'd)



- We just learned that...

$$2xF_1^{\nu(\bar{\nu})N,CC}(x) = xq(x) + x\bar{q}(x)$$

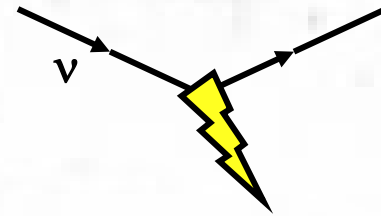
$$xF_3^{\nu(\bar{\nu})N,CC}(x) = xu_{val}(x) + xd_{val}(x) \pm 2x(s(x) - \bar{c}(x))$$

$$\text{where } u_{val}(x) = u(x) - \bar{u}(x)$$

- In charged-lepton DIS

$$2xF_1^{\gamma p}(x) = \left(\frac{2}{3}\right)^2 \sum_{\text{up type quarks}} q(x) + \bar{q}(x) \\ + \left(\frac{1}{3}\right)^2 \sum_{\text{down type quarks}} q(x) + \bar{q}(x)$$

- So you begin to see how one can combine neutrino and charged lepton DIS and separate
  - the quark sea from valence quarks
  - up quarks from down quarks



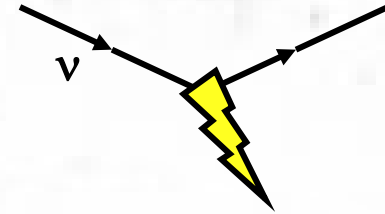
# ***DIS: Massive Quarks and Leptons***



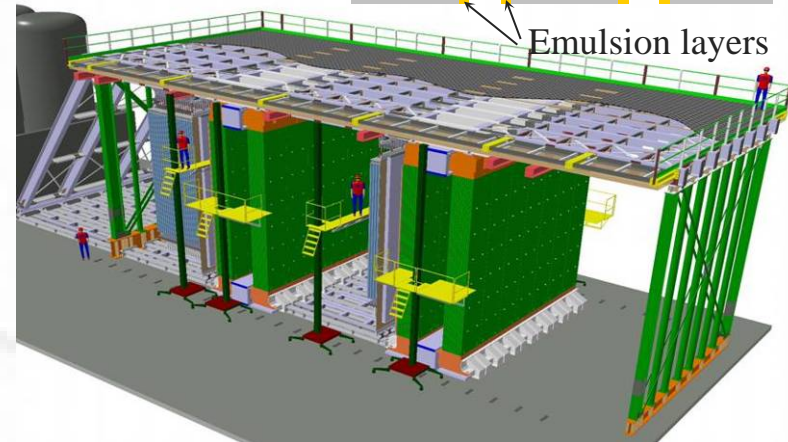
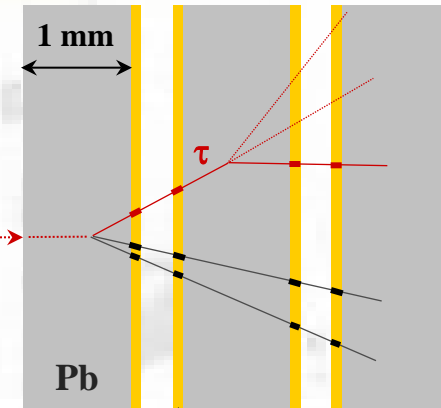
# Opera at CNGS

Goal:  $\nu_\tau$  appearance

- 0.15 MWatt source
- high energy  $\nu_\mu$  beam
- 732 km baseline
- handfuls of events/yr

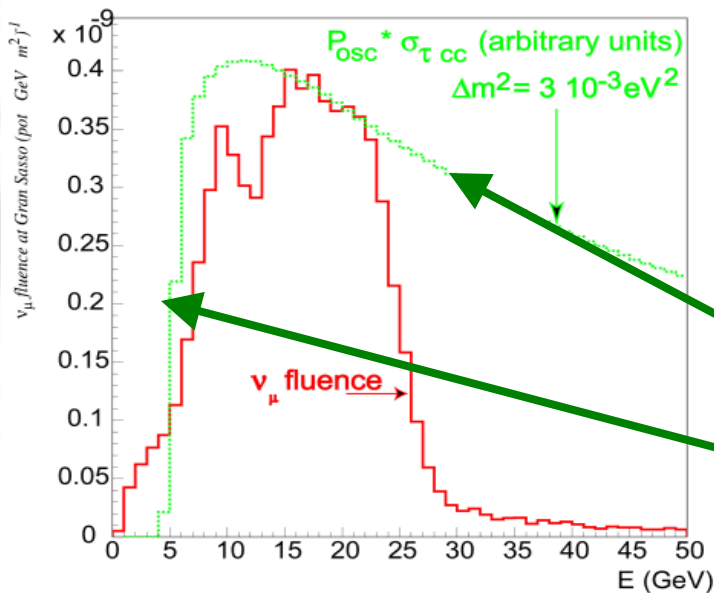


1.8kTon



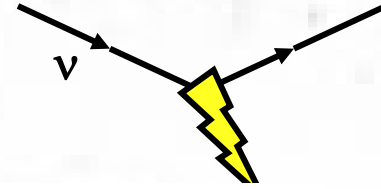
Emulsion layers

figures courtesy D. Autiero



*oscillation probability  
but what is this effect?*

# Lepton Mass Effects in DIS



- Recall that final state mass effects enter as corrections:

$$1 - \frac{m_{\text{lepton}}^2}{s_{\text{point-like}}} \rightarrow 1 - \frac{m_{\text{lepton}}^2}{x s_{\text{nucleon}}}$$

- relevant center-of-mass energy is that of the “point-like” neutrino-parton system
  - this is high energy approximation
- For  $\nu_\tau$  charged-current, there is a threshold of

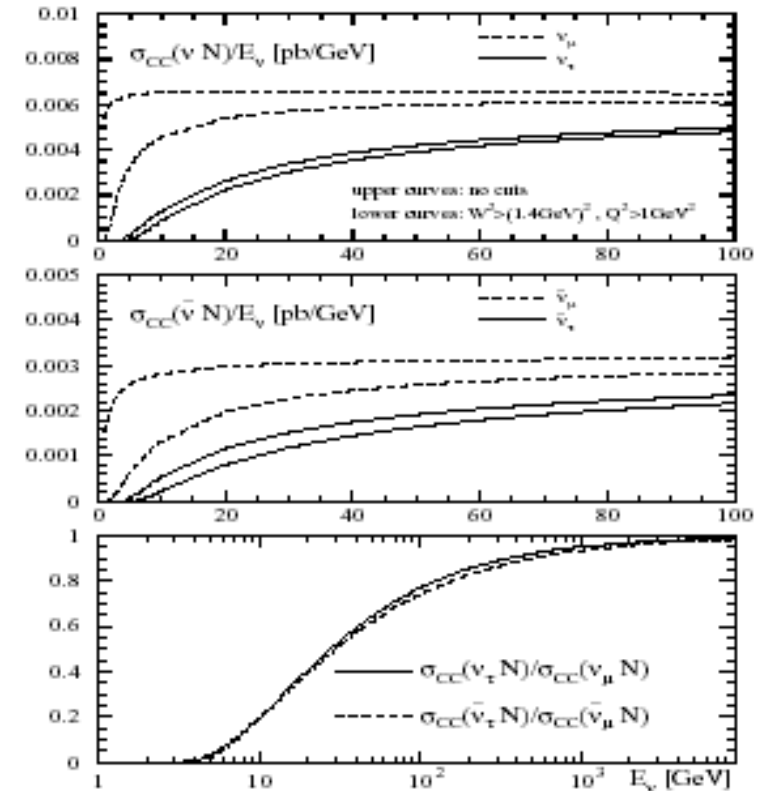
$$s_{\text{min}} = (m_{\text{nucleon}} + m_\tau)^2$$

where

$$s_{\text{initial}} = m_{\text{nucleon}}^2 + 2E_\nu m_{\text{nucleon}}$$

$$\therefore E_\nu > \frac{m_\tau^2 + 2m_\tau m_{\text{nucleon}}}{2m_{\text{nucleon}}} \approx 3.5 \text{ GeV}$$

" $m_{\text{nucleon}}$ " is  $M_T$  elsewhere,  
but don't want to confuse with  $m_\tau$ ...

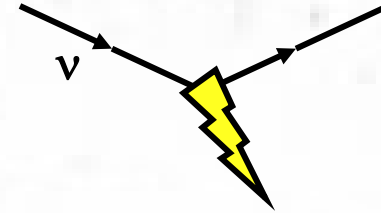


(Kretzer and Reno)

- This is threshold for partons with *entire* nucleon momentum
  - effects big at higher  $E_\nu$  also

# Lecture Question #5:

## What if Taus were Lighter?

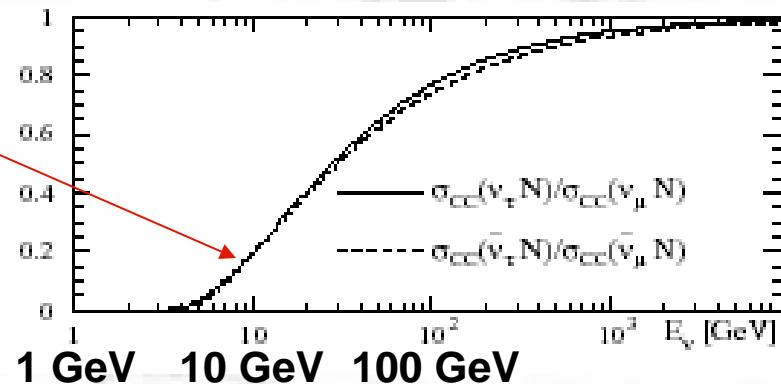


- Imagine we lived in a universe where the tau mass was not 1.777 GeV, but was 0.888 GeV
- By how much would the tau appearance cross-section for an 8 GeV tau neutrino increase at OPERA?

mass suppression:

$$1 - \frac{m_{\text{lepton}}^2}{\chi S_{\text{nucleon}}}$$

$$S_{\text{nucleon}} = m_{\text{nucleon}}^2 + 2E_{\nu}m_{\text{nucleon}}$$



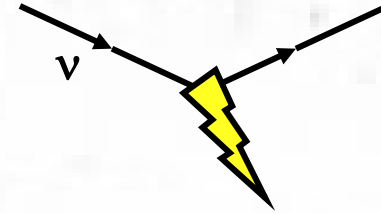
(a)  $\frac{\sigma_{\text{Light Tau}}}{\sigma_{\text{Reality}}} \sim 1.4$

(b)  $\frac{\sigma_{\text{Light Tau}}}{\sigma_{\text{Reality}}} \sim 2$

(c)  $\frac{\sigma_{\text{Light Tau}}}{\sigma_{\text{Reality}}} \sim 3$

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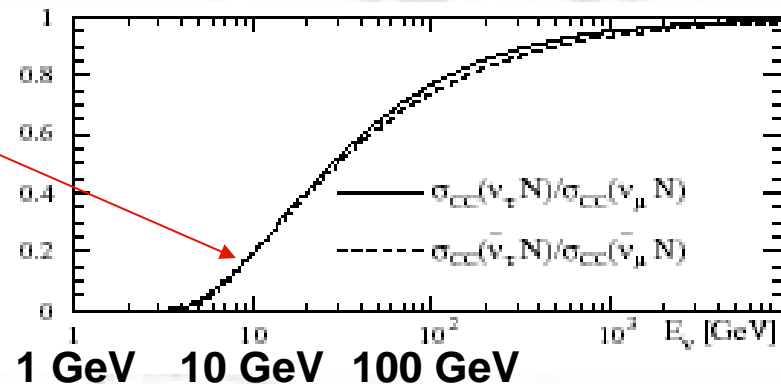


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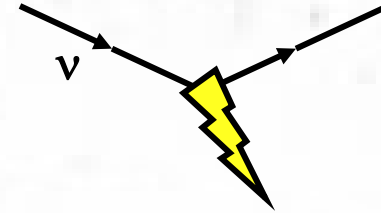


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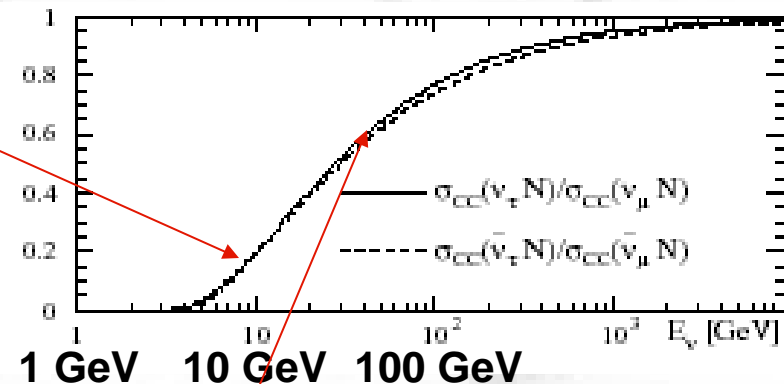
mass suppression:

$$1 - \frac{m_{\text{lepton}}^2}{\chi S_{\text{nucleon}}}$$

Numerator goes down by factor of four. Equivalent to denominator increasing by factor of four and tau mass unchanged...

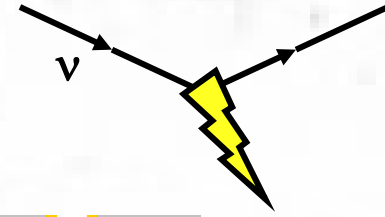
$$S_{\text{nucleon}} = m_{\text{nucleon}}^2 + 2E_{\nu}m_{\text{nucleon}}$$

energy term dominates...  
so set energy a factor of four higher



$$\frac{\sigma_{\text{Light Tau}}}{\sigma_{\text{Reality}}} \sim 3$$

# Opera at CNGS

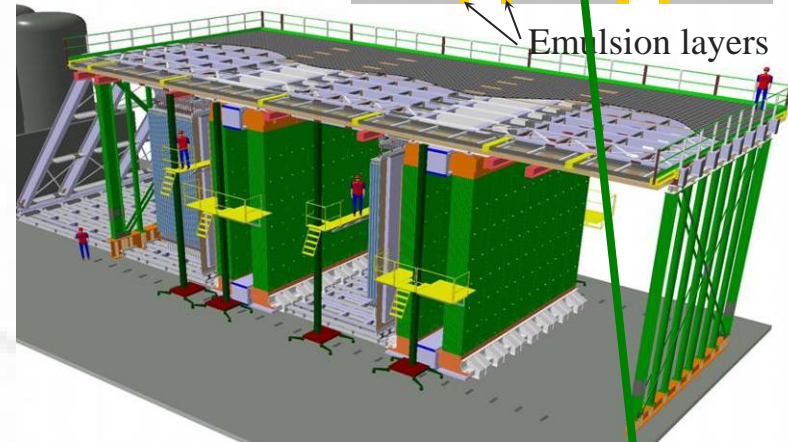
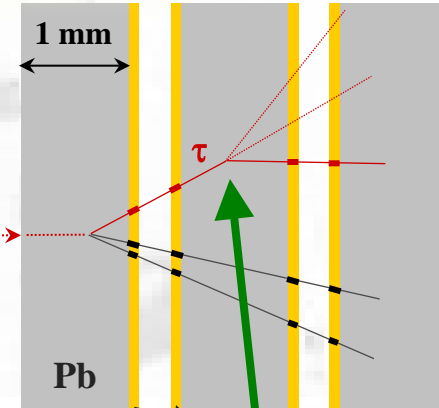


Goal:  $\nu_\tau$  appearance

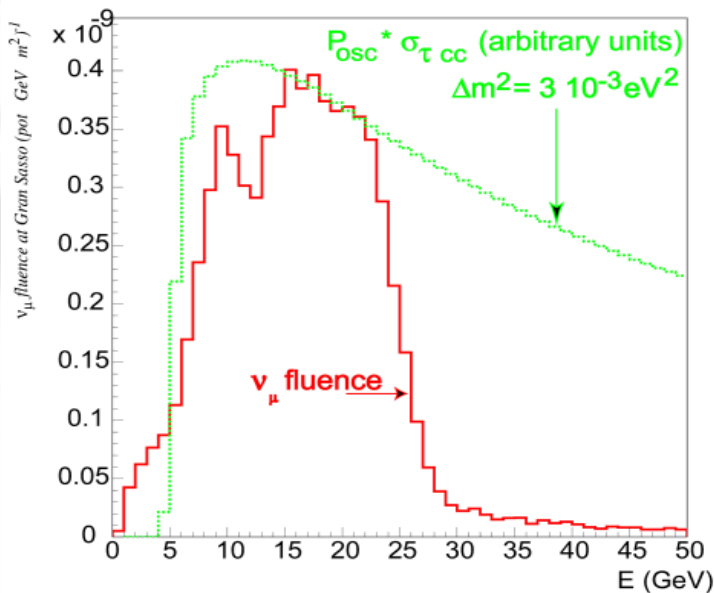
- 0.15 MWatt source
- high energy  $\nu_\mu$  beam
- 732 km baseline
- handfuls of events/yr



1.8kTon



figures courtesy D. Autiero



*what else is copiously produced in neutrino interactions with  $c\tau \sim 100\mu\text{m}$  and decays to hadrons?*



# Heavy Quark Production

- Production of heavy quarks modifies kinematics of our earlier definition of  $x$ .
  - Charm is heavier than proton; hints that its mass is not a negligible effect...

$$(q + \zeta p)^2 = p'^2 = m_c^2$$

$$q^2 + 2\zeta p \cdot q + \zeta^2 M^2 = m_c^2$$

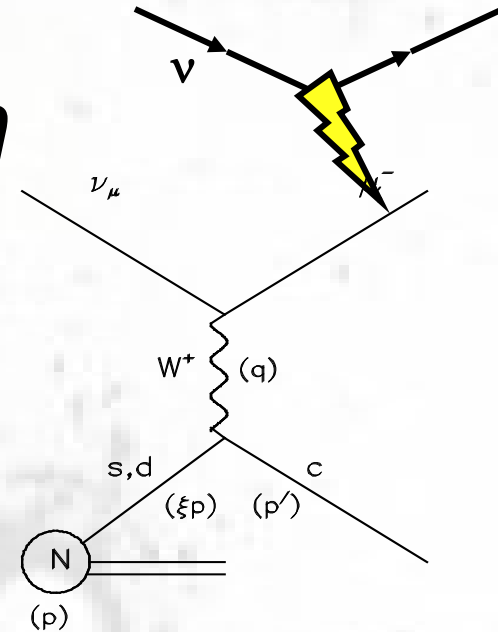
$$\text{Therefore } \zeta \cong \frac{-q^2 + m_c^2}{2p \cdot q}$$

$$\zeta \cong \frac{Q^2 + m_c^2}{2M\nu} = \frac{Q^2 + m_c^2}{Q^2 / x}$$

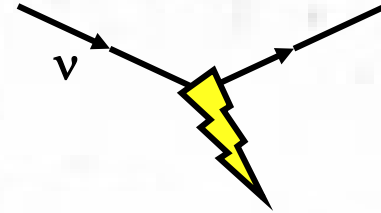
$$\zeta \cong x \left( 1 + \frac{m_c^2}{Q^2} \right)$$

*Note different definition  
of fractional momentum*

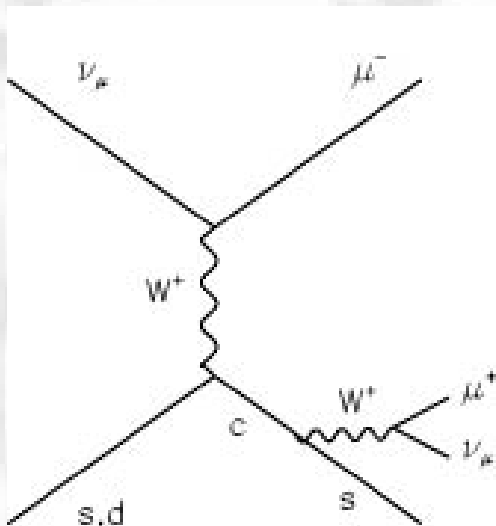
“slow rescaling” leads to  
kinematic suppression of  
charm production



# Neutrino Dilepton Events

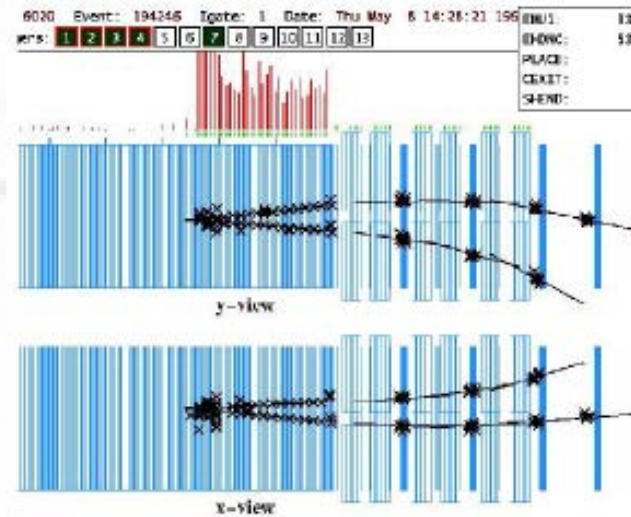


- Neutrino induced charm production has been extensively studied
  - Emulsion/Bubble Chambers (low statistics, 10s of events).  
Reconstruct the charm final state, but limited by target mass.
  - “Dimuon events” (high statistics, 1000s of events)



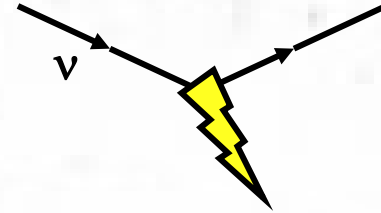
$$\nu_{\mu} + \begin{pmatrix} d \\ s \end{pmatrix} \rightarrow \mu^{-} + c + X, \quad c \rightarrow \mu^{+} + \nu_{\mu} + X'$$

$$\bar{\nu}_{\mu} + \begin{pmatrix} \bar{d} \\ -s \end{pmatrix} \rightarrow \mu^{+} + \bar{c} + X, \quad \bar{c} \rightarrow \mu^{-} + \bar{\nu}_{\mu} + X'$$

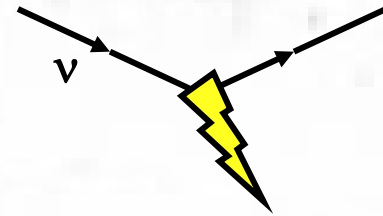




# ***Deep Inelastic Scattering: Conclusions and Summary***

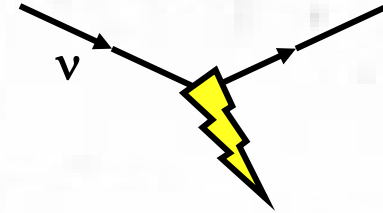


- Neutrino-quark scattering is elastic scattering!
  - complicated by fact that quarks live in nucleons
- Important lepton and quark mass effects for tau neutrino appearance experiments
- Neutrino DIS important for determining parton distributions
  - particularly valence and strange quarks

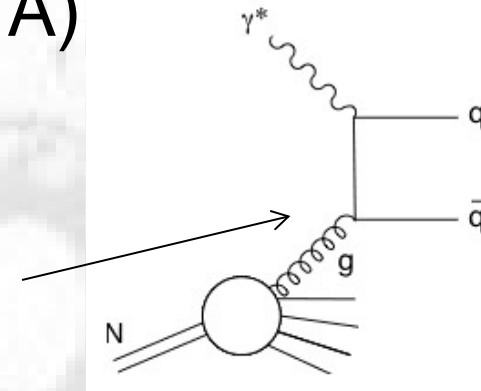


# ***Ultra-High Energy Cross-Sections***

# Ultra-High Energies

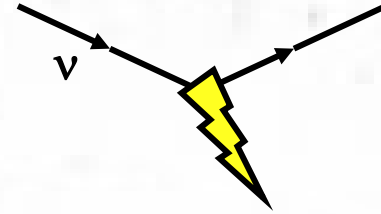


- At energies relevant for UHE Cosmic Ray studies (e.g., IceCube, Antares, ANITA)
  - $\nu$ -parton cross-section is dominated by high  $Q^2$ , since  $d\sigma/dQ^2$  is constant
    - o at high  $Q^2$ , gluon radiation and splitting lead to more sea quarks at fewer high  $x$  partons (see supplemental material: scaling violations)
    - o see a rise in  $\sigma/E_\nu$  from growth of sea at low  $x$
    - o neutrino & anti-neutrino cross-sections nearly equal
  - *Until*  $Q^2 \gg M_W^2$ , then propagator term starts decreasing and cross-section stops growing linearly with energy



$$\frac{d\sigma}{dq^2} \propto \frac{1}{(q^2 - M^2)^2}$$

# Lecture Question #6: Where does $\sigma$ Level Off?

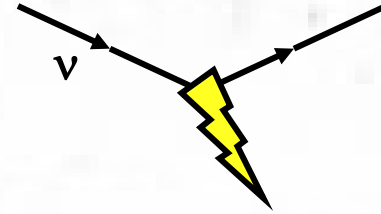


- *Until  $Q^2 \gg M_W^2$ , then propagator term starts decreasing and cross-section becomes constant*
- *To within a few orders of magnitude, at what beam energy for a target at rest will this happen?*

$$\frac{d\sigma}{dq^2} \propto \frac{1}{(q^2 - M^2)^2}$$

(a)  $E_\nu \sim 10\text{TeV}$     (b)  $E_\nu \sim 10,000\text{TeV}$     (c)  $E_\nu \sim 10,000,000\text{TeV}$

# Lecture Question #6: Where does $\sigma$ Level Off?

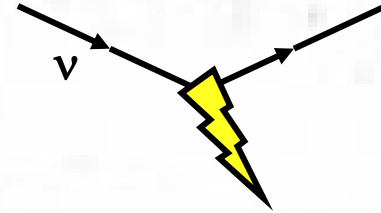


- *Until  $Q^2 \gg M_W^2$ , then propagator term starts decreasing and cross-section becomes constant*
- *To within a few orders of magnitude, at what beam energy for a target at rest will this happen?*

$$\frac{d\sigma}{dq^2} \propto \frac{1}{(q^2 - M^2)^2}$$

**(a)**  $E_\nu \sim 10\text{TeV}$     **(b)**  $E_\nu \sim 10,000\text{TeV}$     **(c)**  $E_\nu \sim 10,000,000\text{TeV}$

# Lecture Question #6: Where does $\sigma$ Level Off?



- *Until  $Q^2 \gg M_W^2$ , then propagator term starts decreasing and cross-section becomes constant*
- *At what beam energy for a target at rest will this happen?*

$$\frac{d\sigma}{dq^2} \propto \frac{1}{(q^2 - M^2)^2}$$

$$Q^2 < s_{\text{nucleon}} = m_{\text{nucleon}}^2 + 2E_\nu m_{\text{nucleon}}$$

$$Q^2 < s_{\text{nucleon}} \approx 2E_\nu m_{\text{nucleon}}$$

$$\frac{M_W^2}{2m_{\text{nucleon}}} < E_\nu$$

$$\therefore E_\nu \gtrsim \frac{(80.4)^2 \text{ GeV}^2}{2(938) \text{ GeV}} \sim 3000 \text{ GeV}$$

*$Q^2$  limit is  $s$ .  
So won't start to  
plateau until  $s > M_W^2$*

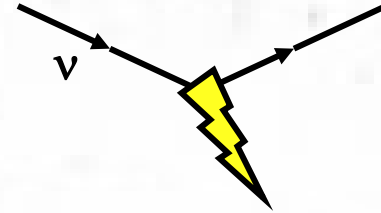
*Bonus point realization...*

*In reality, that is only correct for a parton at  $x=1$ . Typical quark  $x$  is much less, say  $\sim 0.03$*

$$\frac{M_W^2}{2m_{\text{nucleon}} x} < E_\nu$$

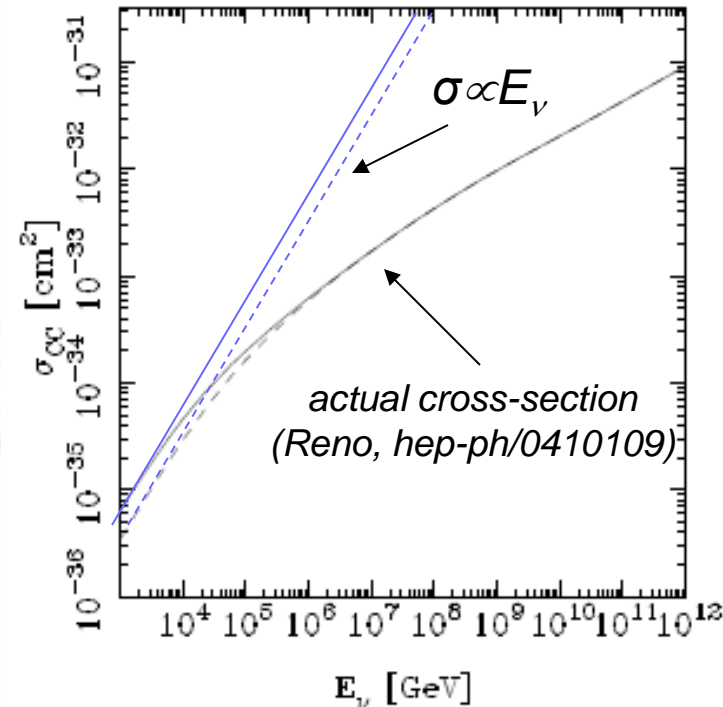
$$\therefore E_\nu \gtrsim \frac{3000 \text{ GeV}}{0.03} \sim 100 \text{ TeV}$$

# Ultra-High Energies

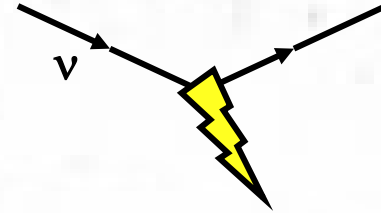


- $\nu$ -parton cross-section is dominated by high  $Q^2$ , since  $d\sigma/dQ^2$  is constant
  - at high  $Q^2$ , scaling violations have made most of nucleon momentum carried by sea quarks
  - see a rise in  $\sigma/E_\nu$  from growth of sea at low  $x$
  - neutrino & anti-neutrino cross-sections nearly equal
- *Until*  $Q^2 \gg M_W^2$ , then propagator term starts decreasing and cross-section becomes constant

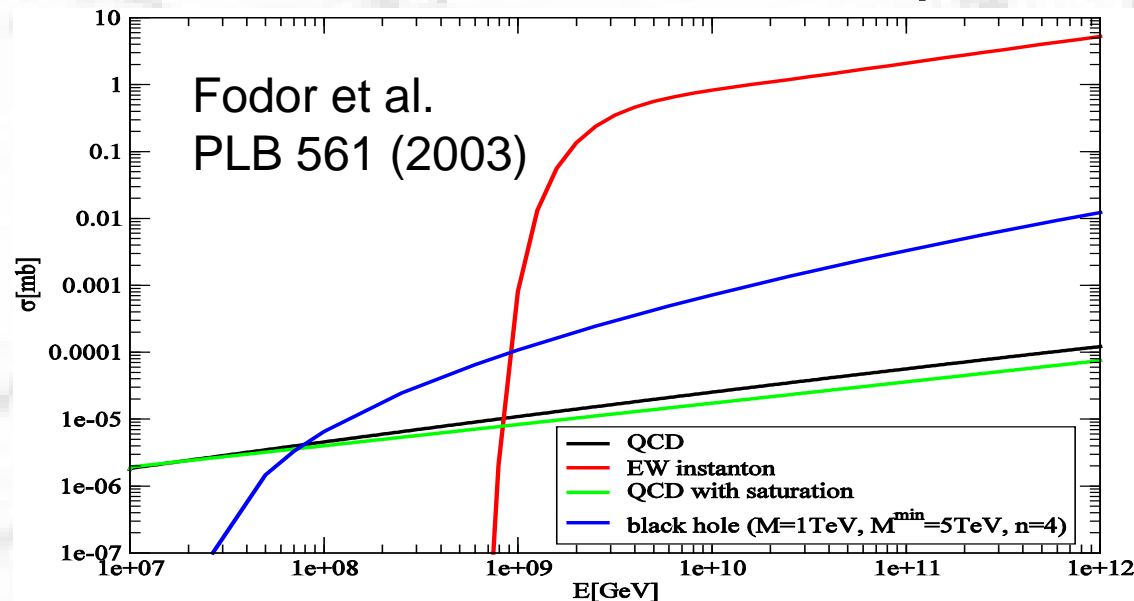
$$\frac{d\sigma}{dq^2} \propto \frac{1}{(q^2 - M^2)^2}$$



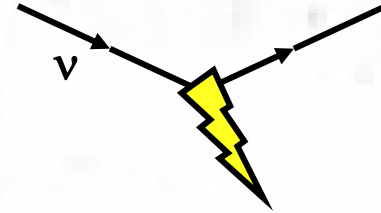
# Example: Ultra-High Energies



- At UHE, can we reach thresholds of non-SM processes?
  - E.g., structure of quark or leptons, black holes from extra dimensions, etc.
  - Then no one knows what to expect...







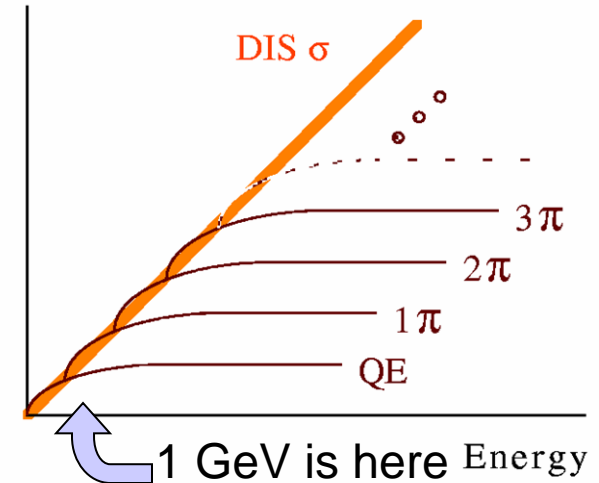
# ***Motivation for Understanding GeV Cross-Sections***

# What's special about it?

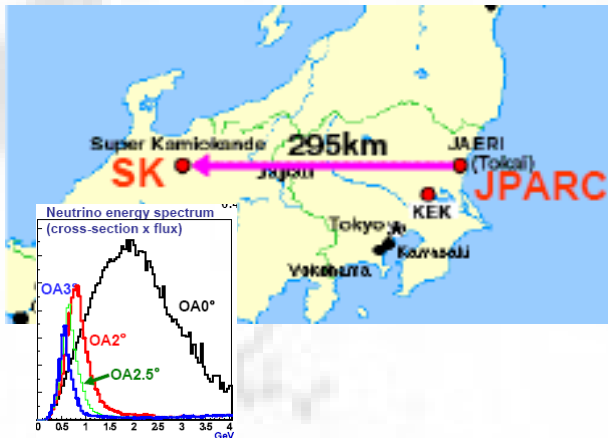
## Why do we care?



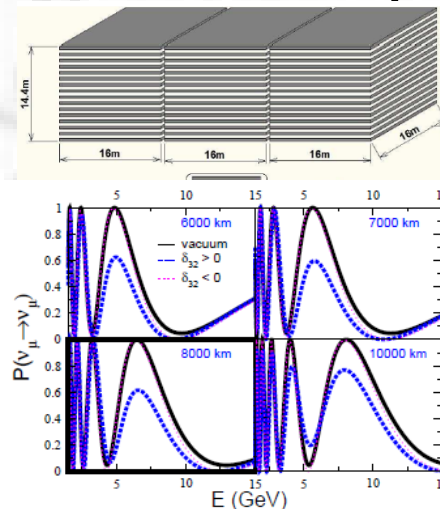
cross  
section



- Remember this picture?
  - 1-few GeV is exactly where these additional processes are turning on
  - It's not DIS yet! Final states & threshold effects matter
- Why is it important? Examples from T2K, ICAL



6-8 August 2013



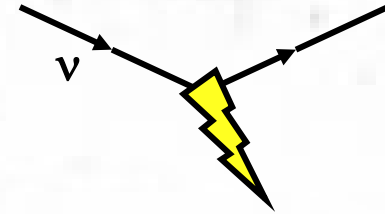
Kevin McFarland, Interactions of Neutrinos

Goals:

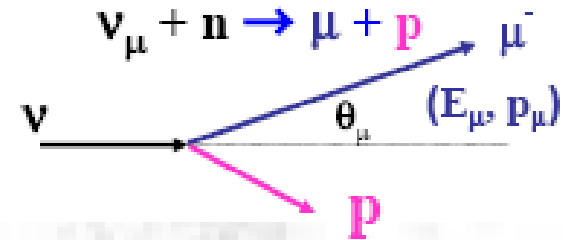
- $\nu_\mu \rightarrow \nu_e$
- $\nu_\mu$  disappearance

$E_\nu$  is 0.4-2.0 GeV  
(T2K) or 3-10 GeV  
(INO ICAL)

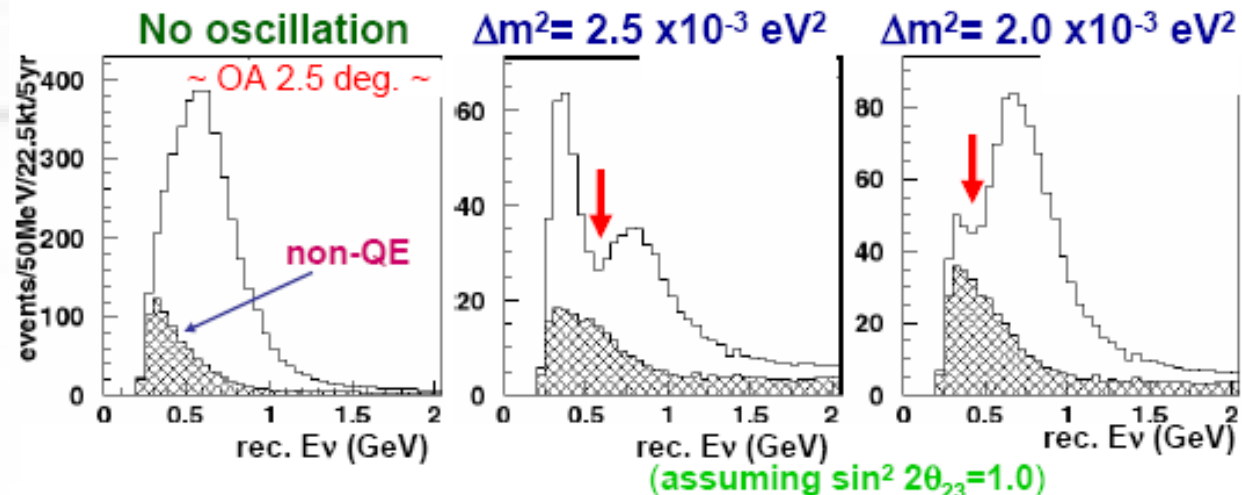
# How do cross-sections effect oscillation analysis?



- $\nu_\mu$  disappearance (low energy)
  - at Super-K reconstruct these events by muon angle and momentum (proton below Cerenkov threshold in  $\text{H}_2\text{O}$ )
  - other final states with more particles below threshold (“non-QE”) will disrupt this reconstruction
- T2K must know these events at few % level to do disappearance analysis to measure  $\Delta m^2_{23}, \theta_{23}$

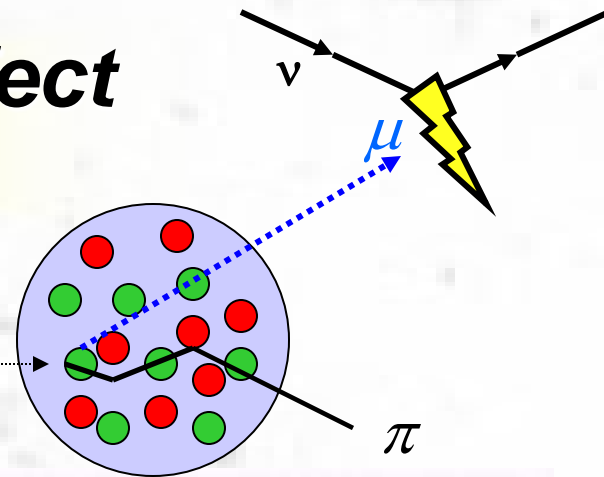


(fig. courtesy  
Y. Hayato)

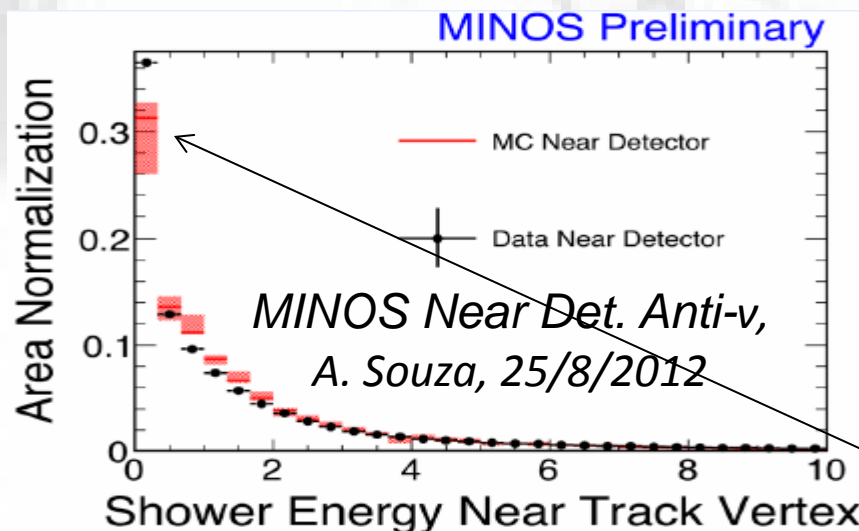
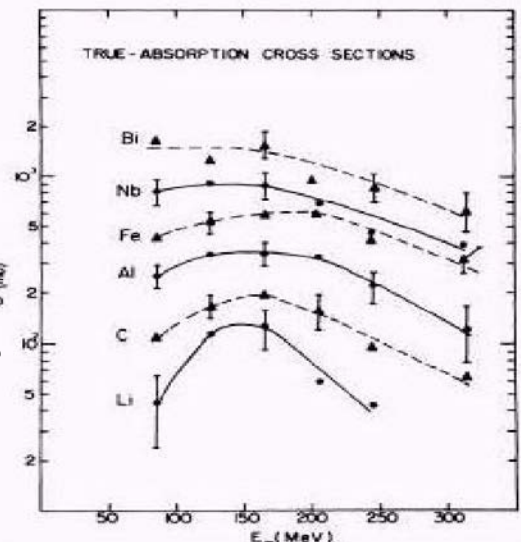


# How do cross-sections effect oscillation analysis?

- $\nu_\mu$  disappearance (high energy)
- Visible Energy in a calorimeter is NOT the  $\nu$  energy transferred to the hadronic system
  - $\pi$  absorption,  $\pi$  re-scattering, final state rest mass effect the calorimetric response
  - Can use external data to constrain

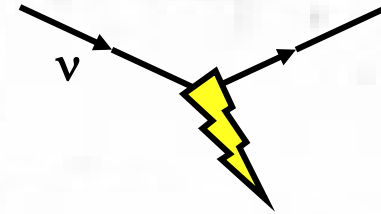


D. Ashery et al, PRC 23, 1993

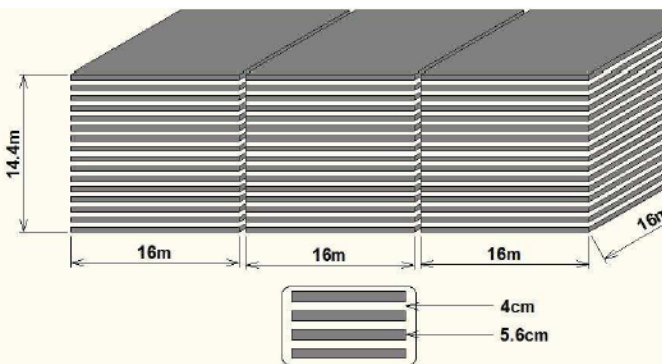


- At very high energies, particle multiplicities are high and these effects will average out
- Low energy is more difficult

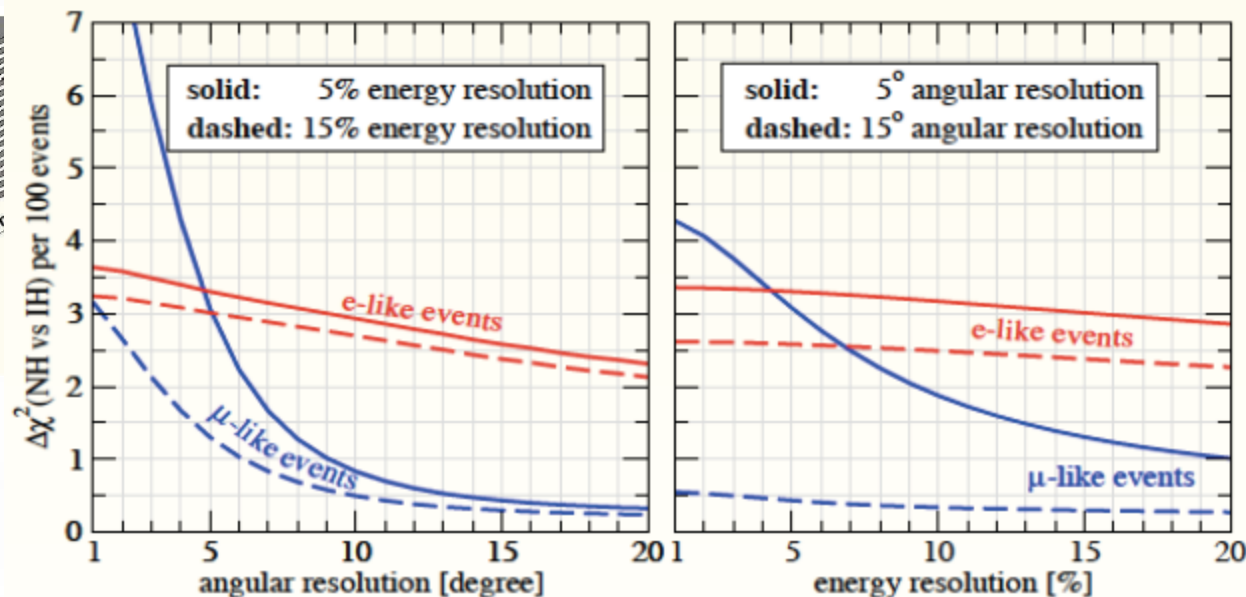
# How do cross-sections effect oscillation analysis?



- In the case of INO ICAL, need good energy and angle resolution to separate normal and inverted hierarchy
  - Best sensitivity requires survival probability in both  $E_\nu$  and  $L$

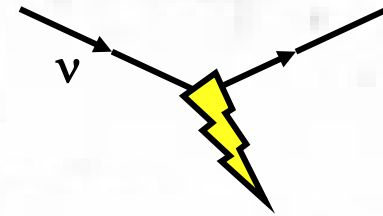


- Interaction models are understanding of detector response both needed to optimize resolution

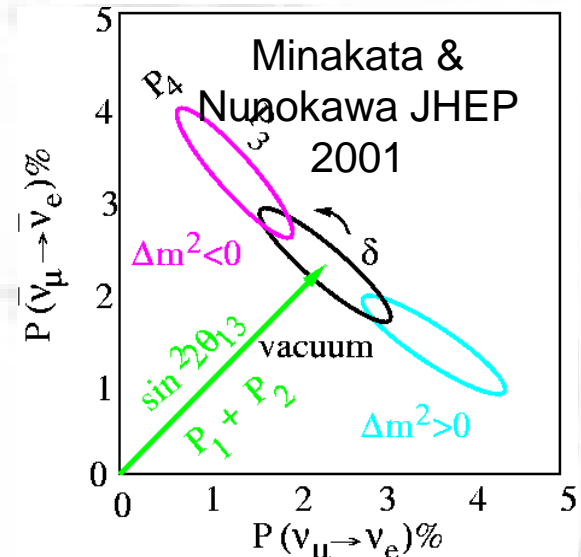
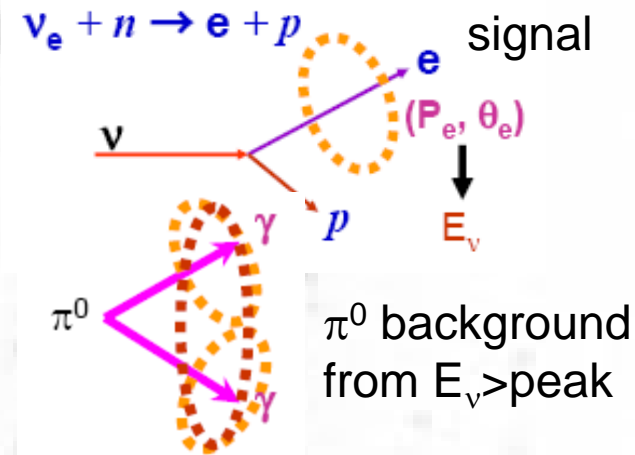


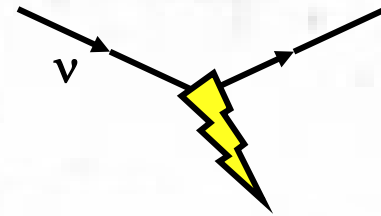
Petcov, Schwetz, hep-ph/0511277

# How do cross-sections effect oscillation analysis?



- $\nu_e$  appearance
  - different problem: signal rate is very low so even rare backgrounds contribute!
- Remember the end goal of electron neutrino appearance experiments
- Want to compare two signals with two different sets of backgrounds and signal reactions
  - with sub-percent precision
  - Requires precise knowledge of background and signal reactions

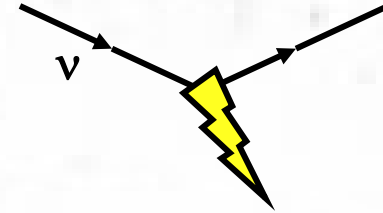




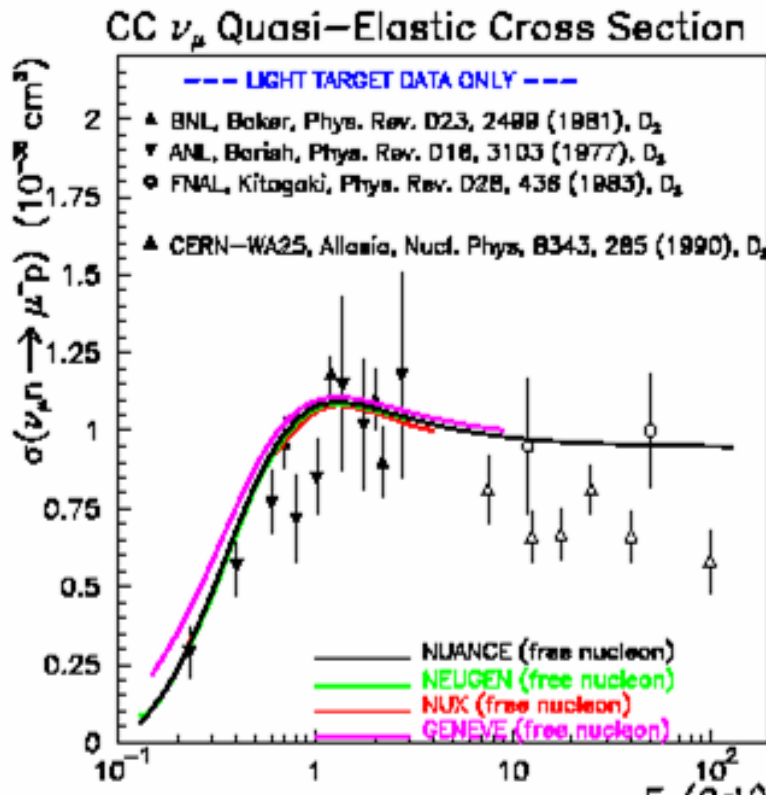
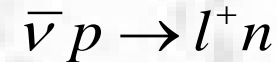
# ***Models for GeV Cross-Sections***



# (Quasi-)Elastic Scattering



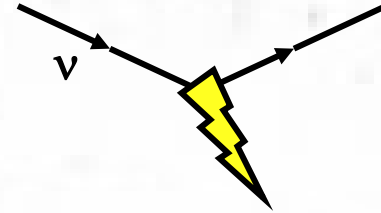
- Elastic scattering leaves a single nucleon in the final state
  - CC “quasi-elastic” easier to observe



- State of data is marginal
  - No free neutrons implies nuclear corrections
  - Low energy statistics poor
- Cross-section is calculable
  - But depends on incalculable form-factors of the nucleon
- Theoretically and experimentally constant at high energy
  - $1 \text{ GeV}^2$  is  $\sim$  a limit in  $Q^2$

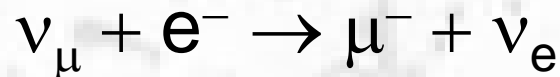


# What was that last cryptic remark?



- Theoretically and experimentally constant at high energy
  - $1 \text{ GeV}^2$  is  $\sim$  a limit in  $Q^2$

- **Inverse  $\mu$ -decay:**

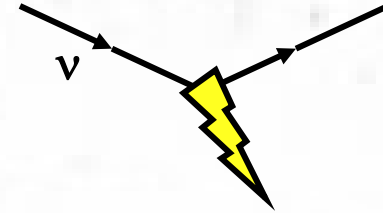


a maximum  $Q^2$  independent of beam energy  $\Rightarrow$  constant  $\sigma_{TOT}$

$$\sigma_{TOT} \propto \int_0^{Q_{\max}^2} dQ^2 \frac{1}{(Q^2 + M_W^2)^2} \approx \frac{Q_{\max}^2}{M_W^4}$$

- OK, but why does cross-section have a  $Q_{\max}^2$  limit?
  - If  $Q^2$  is too large, then the probability for the final state nucleon to stay intact (elastic scattering) becomes low
  - This information is encoded in “form factors” of the nucleons

# Elastic Scattering (cont'd)



- As with IBD, nucleon structure alters cross-section

- Can write down in terms of all possible “form factors” of the nucleon allowed by Lorentz invariance

C.H. Llewellyn Smith, *Phys. Rep.* 3C, 261 (1972)

$$\frac{d\sigma}{dQ^2}(\nu n \rightarrow l^- p) = \left[ A(Q^2) \mp B(Q^2) \frac{s-u}{M^2} + C(Q^2) \frac{(s-u)^2}{M^4} \right] \times \frac{M^2 G_F^2 \cos^2 \theta_c}{8\pi E_\nu^2}$$

$$A(Q^2) = \frac{m^2 + Q^2}{4M^2} \left[ \left(4 + \frac{Q^2}{M^2}\right) |F_A|^2 - \left(4 - \frac{Q^2}{M^2}\right) |F_V^1|^2 + \frac{Q^2}{M^2} \xi |F_V^2|^2 \left(1 - \frac{Q^2}{4M^2}\right) + \frac{4Q^2 \text{Re} F_V^{1*} \xi F_V^2}{M^2} - \frac{Q^2}{M^2} \left(4 + \frac{Q^2}{M^2}\right) |F_A^3|^2 - \frac{m^2}{M^2} \left( |F_V^1 + \xi F_V^2|^2 + |F_A + 2F_P|^2 - \left(4 + \frac{Q^2}{M^2}\right) (|F_V^3|^2 + |F_P|^2) \right) \right],$$

$$B(Q^2) = \frac{Q^2}{M^2} \text{Re} F_A^* (F_V^1 + \xi F_V^2) - \frac{m^2}{M^2} \text{Re} \left[ \left( F_V^1 - \frac{Q^2}{4M^2} \xi F_V^2 \right)^* F_V^3 - \left( F_A - \frac{Q^2 F_P}{2M^2} \right)^* F_A^3 \right] \text{ and}$$

$$C(Q^2) = \frac{1}{4} \left( |F_A|^2 + |F_V^1|^2 + \frac{Q^2}{M^2} \left| \frac{\xi F_V^2}{2} \right|^2 + \frac{Q^2}{M^2} |F_A^3|^2 \right).$$

$$\nu n \rightarrow l^- p$$

$$\bar{\nu} p \rightarrow l^+ n$$

$$\begin{matrix} (-) & & (-) \\ \nu & N \rightarrow & \nu N \end{matrix}$$

Occupants of the form factor zoo:

$F_V^1, F_V^2$  are vector form factors;

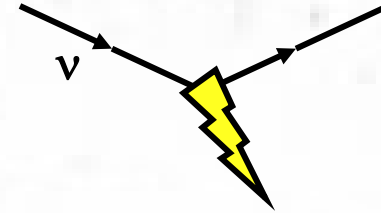
$F_A$  is the axial vector form factor;

$F_P$  is the pseudo-scalar form factor;

$F_V^3$  and  $F_A^3$  are form factors

related to currents requiring G-parity violation, small?

# Elastic Scattering (cont'd)



- Form factors representing second class currents,  $F_V^3$  and  $F_A^3$ , are usually assumed to be zero
- Pseudoscalar form factor,  $F_P$ , can be calculated from  $F_A$  with reasonable assumptions (Adler's theorem and the Goldberger-Treiman relation)
- The leading form factors,  $F_V^1$ ,  $F_V^2$  and  $F_A$ , are approximately dipole in form

$$F_V(q^2) \sim \frac{1}{(1 - q^2/M_V^2)^2} \quad F_A(q^2) = \frac{F_A(0)}{(1 - q^2/M_A^2)^2} \quad \leftarrow \text{"dipole approximation"}$$

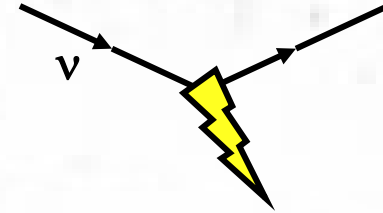
$$\left. \begin{array}{l} M_V \approx 0.71 \text{ GeV} \\ M_A \approx 1.01 \text{ GeV} \\ F_A(0) \approx -1.267 \\ F_V(0) \text{ is charge of proton} \end{array} \right\}$$

parameters  
determined from data

*n.b.: we've seen  $F_V(0)$  and  $F_A(0)$  before in IBD discussion ( $g_V$  and  $g_A$ )*

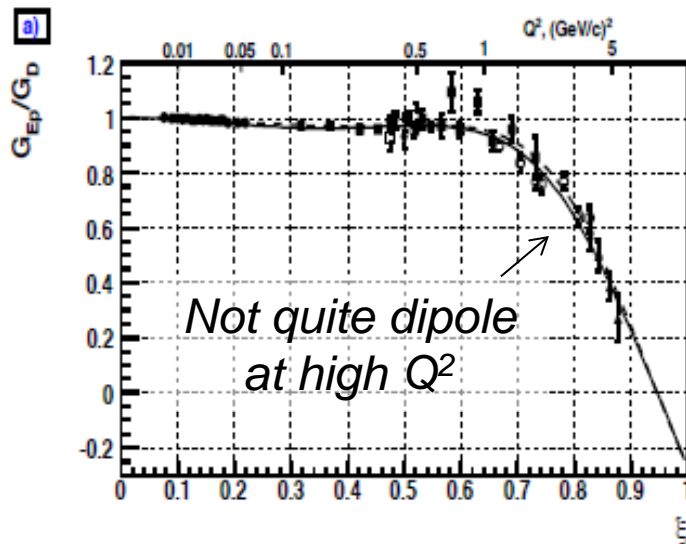
- Note that those masses which "cut off" the form factor are of order 1 GeV, so form factors are low beyond 1 GeV<sup>2</sup>

# Elastic Scattering (cont'd)



## Vector form factors

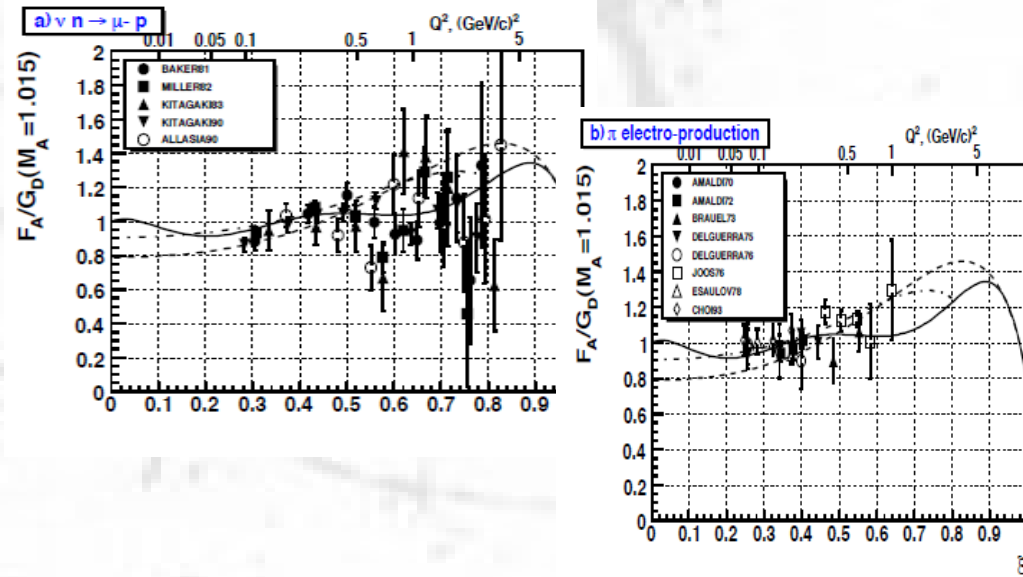
- Measured in charged lepton scattering



e.g., Bradford-Bodek-Budd-Arrington ("BBBA"),  
Nucl.Phys.Proc.Suppl. 159:127-132,2006

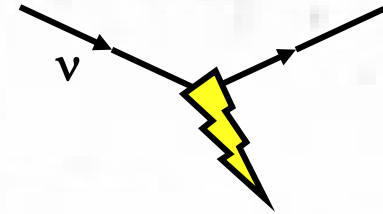
## Axial vector form factors

- Measured in pion electro-production & neutrino scattering



Bodek, Avvakumov, Bradford and Budd,  
J. Phys. Conf. Ser. 110, 082004 (2008).

# Low $W$ , the Baryon Resonance Region



- Intermediate to elastic and DIS regions is a region of resonance production

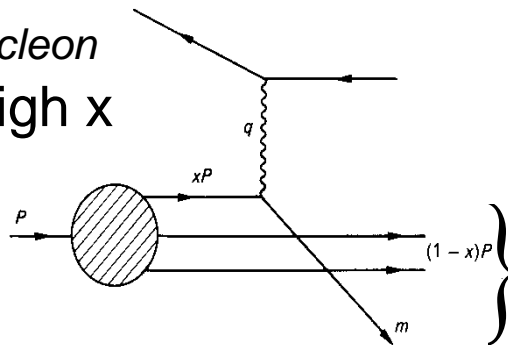
- Recall mass<sup>2</sup> of hadronic final state is given by

$$W^2 = M_T^2 + 2M_T\nu - Q^2 = M_T^2 + 2M_T\nu(1-x)$$

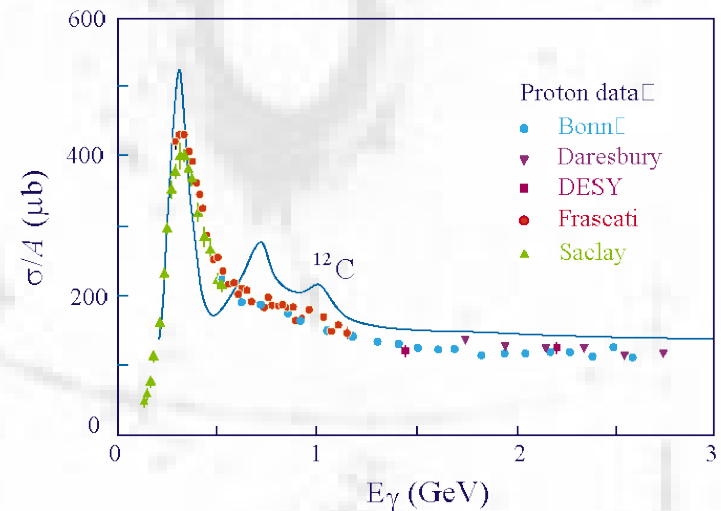
- At low energy, nucleon-pion states dominated by  $N^*$  and  $\Delta$  resonances

- Leads to cross-section with significant structure in  $W$  just above  $M_{\text{nucleon}}$

- Low  $\nu$ , high  $x$

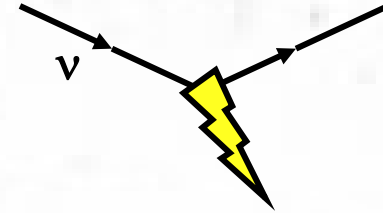


$W^2$



photoabsorption vs  $E_\gamma$ .  
Line shows protons.  
More later...

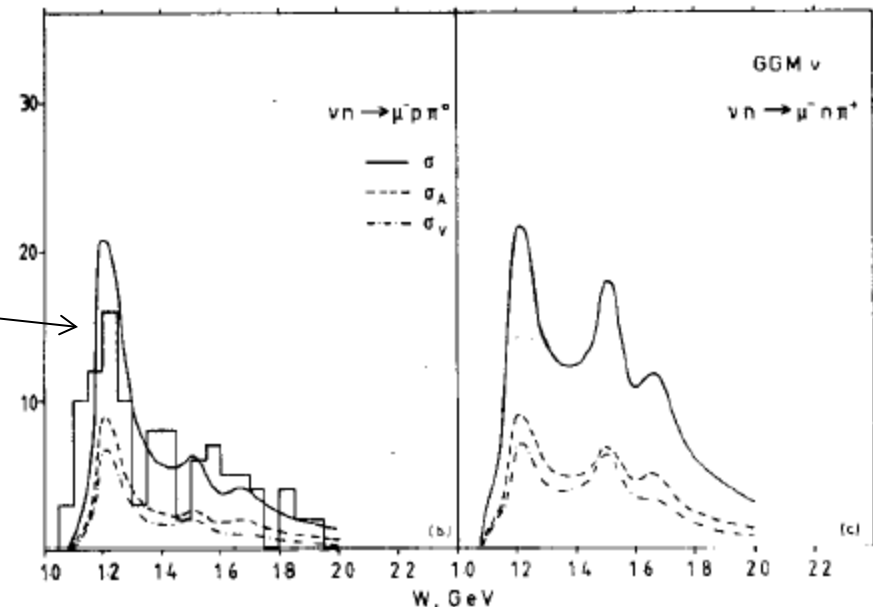
# The Resonance Region



- Models of the resonance region are complicated
  - In principle, many baryon resonances can be excited in the scattering and they all can contribute
  - They de-excite mostly by radiating pions

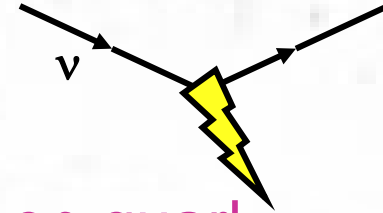
Nucleon Resonances below 2 GeV/c<sup>2</sup> according to Ref. [4]

Resonance Symbol <sup>a</sup>	Central mass value $M$ [MeV/c <sup>2</sup> ]	Total width $\Gamma_0$ [MeV]	Elasticity $x_E = \pi N^c$ branching ratio	Quark-Model/ $SU_6$ -assignment
$P_{33}(1234)$	1234	124	1	$^4(10)_{3/2} [56, 0^+]_0$
$P_{11}(1450)$	1450	370	0.65	$^2(8)_{1/2} [56, 0^+]_2$
$D_{13}(1525)$	1525	125	0.56	$^2(8)_{3/2} [70, 1^-]_1$
$S_{11}(1540)$	1540	270	0.45	$^2(8)_{1/2} [70, 1^-]_1$
$S_{31}(1620)$	1620	140	0.25	$^2(10)_{1/2} [70, 1^-]_1$
$S_{11}(1640)$	1640	140	0.60	$^4(8)_{1/2} [70, 1^-]_1$
$P_{33}(1640)$	1640	370	0.20	$^4(10)_{3/2} [56, 0^+]_2$
$D_{13}(1670)$	1670	80	0.10	$^4(8)_{3/2} [70, 1^-]_1$
$D_{13}(1680)$	1680	180	0.35	$^4(8)_{3/2} [70, 1^-]_1$
$F_{13}(1680)$	1680	120	0.62	$^2(8)_{3/2} [56, 2^+]_2$
$P_{11}(1710)$	1710	100	0.19	$^2(8)_{1/2} [70, 0^+]_3$
$D_{33}(1730)$	1730	300	0.12	$^2(10)_{3/2} [70, 1^-]_1$
$P_{13}(1740)$	1740	210	0.19	$^2(8)_{3/2} [56, 2^+]_2$
$P_{31}(1920)$	1920	300	0.19	$^4(10)_{1/2} [56, 2^+]_9$
$F_{35}(1920)$	1920	340	0.15	$^4(10)_{3/2} [56, 2^+]_2$
$F_{37}(1950)$	1950	340	0.40	$^4(10)_{7/2} [56, 2^+]_3$
$P_{33}(1960)$	1960	300	0.17	$^4(10)_{3/2} [56, 2^+]_2$
$F_{17}(1970)$	1970	325	0.06	$^4(8)_{7/2} [70, 2^+]_3$



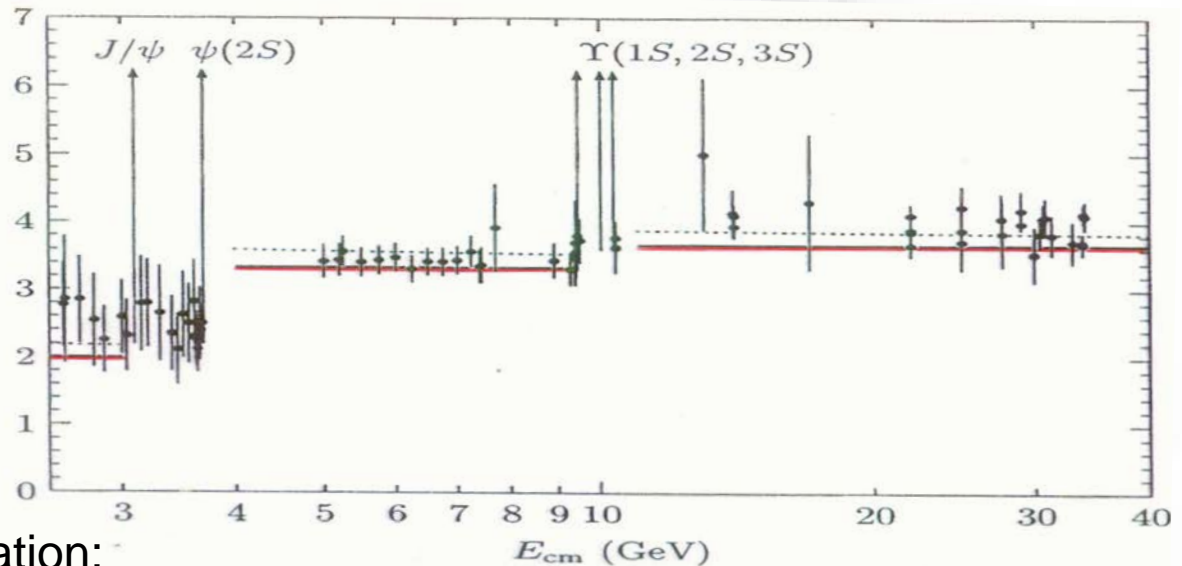
*D. Rein and L. Sehgal, Ann. Phys. 133, 79 (1981)*

# Quark-Hadron Duality



- Bloom-Gilman Duality is the relationship between quark and hadron descriptions of reactions. It reflects:
  - link between *confinement* and *asymptotic freedom*
  - transition from *non-perturbative* to *perturbative* QCD

$$R \equiv \frac{\sigma(e^+e^- \rightarrow \text{hadrons})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)}$$



quark-parton model calculation:

$$R = N_C \sum_{q \not\equiv s > m_q^2} \left( Q_q^{EM} \right)^2 + O(\alpha_{EM} + \alpha_S)$$

*but of course, final state is really sums over discrete hadronic systems*



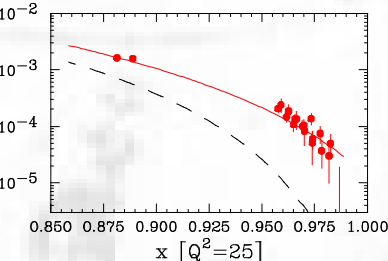
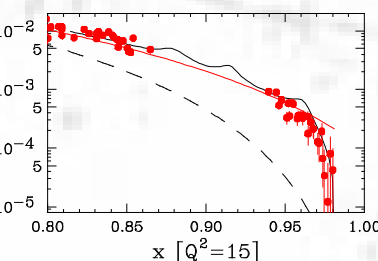
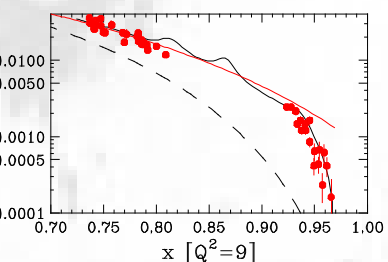
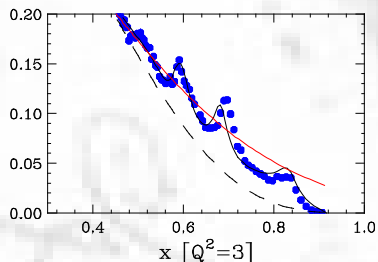
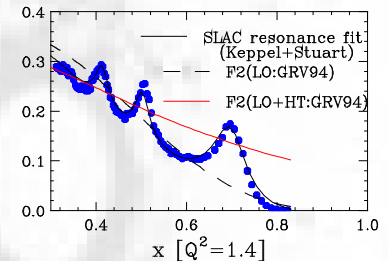
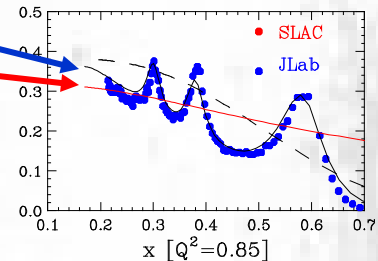
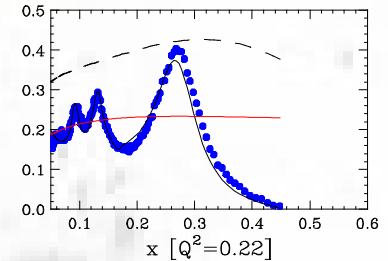
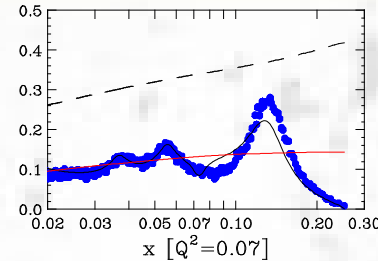
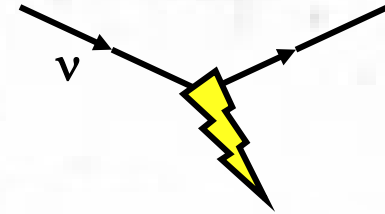
# Duality and $\nu$

$$W^2 = M_T^2 + Q^2 \left( \frac{1}{x} - 1 \right)$$

Low  $Q^2$  data

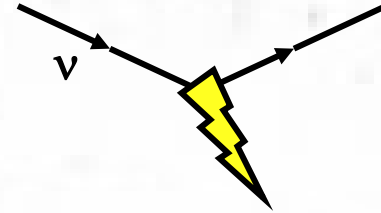
DIS-Style PDF prediction

- Governs transition between resonance and DIS region
- Sums of discrete resonances approaches DIS cross-section
- Bodek-Yang: *Observe in electron scattering data; apply to  $\nu$  cross-sections*





# *Duality's Promise*



- In principle, a duality based approach can be applied over the entire kinematic region
- The problem is that duality gives “averaged” differential cross-sections, and not details of a final state

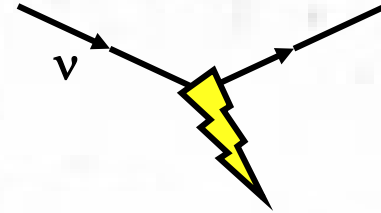
*Microphysical models of  
exclusive processes*



*Duality models based on  
data of inclusive rates*

- Microphysical models may lack important physics, but duality models may not predict all we need to know
  - How to scale the mountain between the two?

# Lecture Question #7: Duality meets Reality



A difficulty in relating cross-sections of electron scattering (photon exchange) to charged-current neutrino scattering ( $W^\pm$  exchange) is that some e-scattering reactions have imperfect  $\nu$ -scattering analogues.

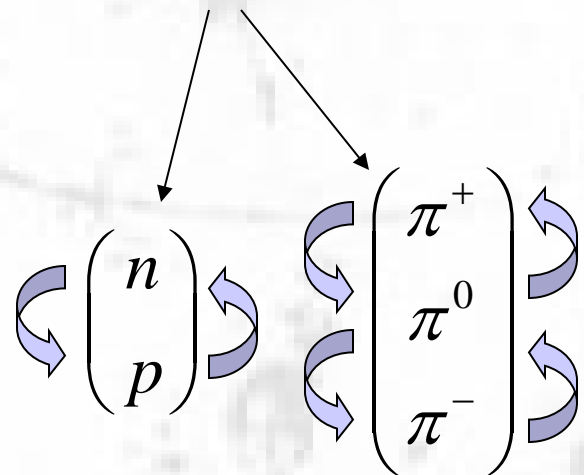
Write all possible  $\nu_\mu$  CC reactions involving the same target particle and isospin rotations of the final state for each of the following...

(a)  $e^- n \rightarrow e^- n$

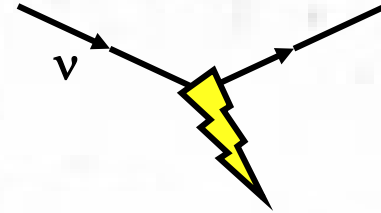
(b)  $e^- p \rightarrow e^- p$

(c)  $e^- p \rightarrow e^- n \pi^+$

(d)  $e^- n \rightarrow e^- p \pi^-$



# Lecture Question #7: Duality meets Reality



Write all possible  $\nu$  reactions involving the same target particle and isospin rotations of the final state for each of the following...

(a)  $e^- n \rightarrow e^- n$

$$\nu_\mu n \rightarrow \mu^- p$$

(c)  $e^- p \rightarrow e^- n \pi^+$

$$\nu_\mu p \rightarrow \mu^- p \pi^+$$

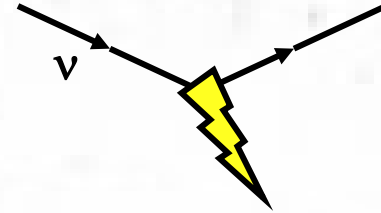
(b)  $e^- p \rightarrow e^- p$

there are none!

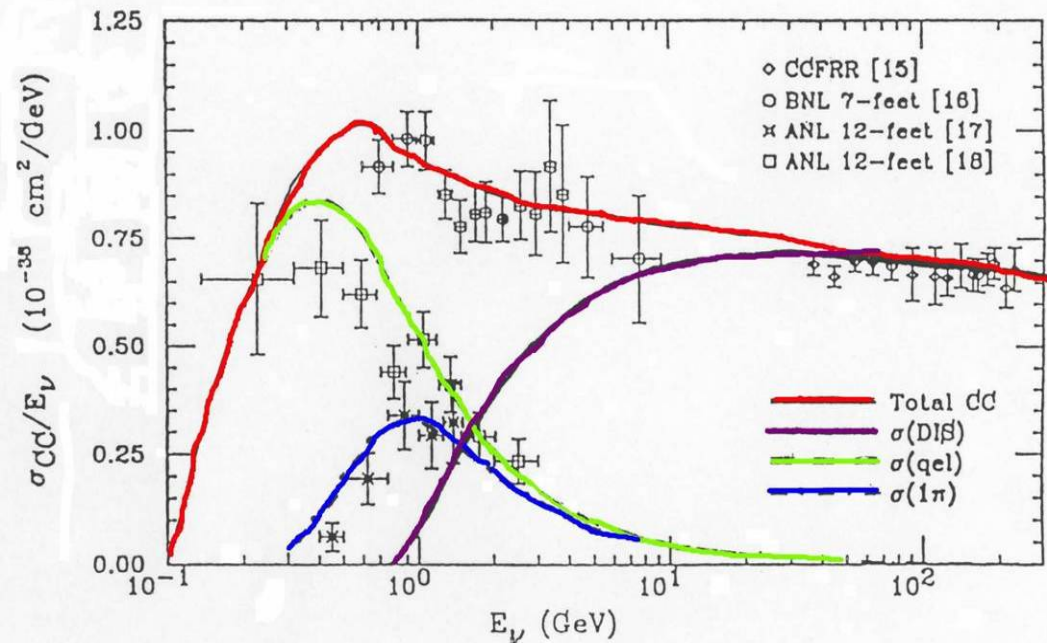
(d)  $e^- n \rightarrow e^- p \pi^-$

$$\begin{aligned}\nu_\mu n &\rightarrow \mu^- n \pi^+ \\ \nu_\mu n &\rightarrow \mu^- p \pi^0\end{aligned}$$

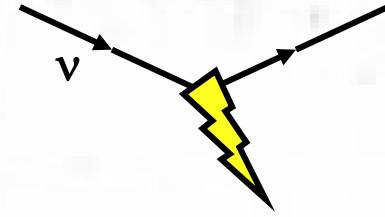
# Building a Unified Model



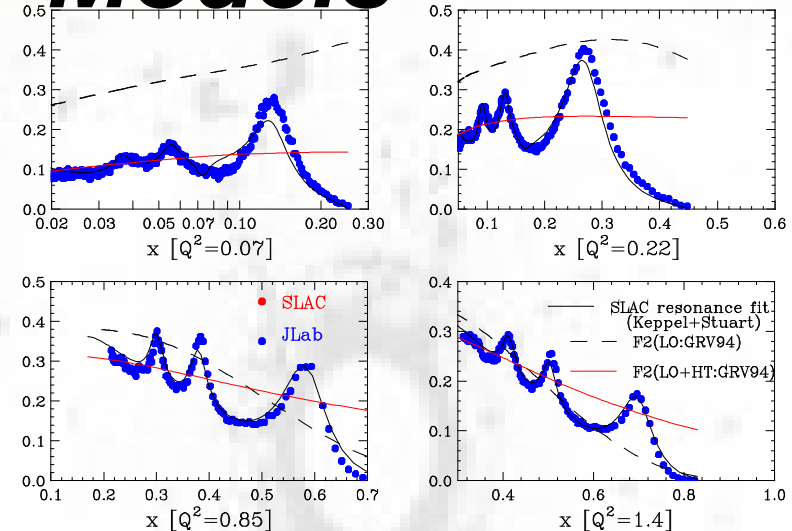
- In the relevant energy regime around 1 GeV, need a model that smoothly manages exclusive (elastic, resonance) to inclusive (DIS) transition
- Duality argues that the transition from the high  $W$  part of the resonance region (many resonances) to deep inelastic scattering should be smooth.

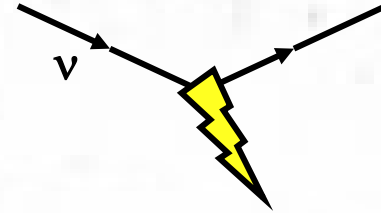


# Exclusive Resonance Models and Duality Models



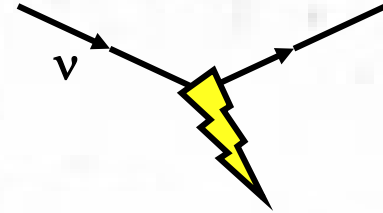
- Duality models agree with inclusive data by construction
  - However, in a generator context, have to add details of final state
- Typical approach (GENIE, NEUT and NUANCE) is to use a resonance model (Rein & Sehgal) below  $W < 2$  GeV, and duality + string fragmentation model for  $W > 2$  GeV
  - This is far from an idea solution
  - Discrete resonance model (probably) disagrees with total cross-section data below  $W < 2$  GeV and is difficult to tune
  - Average cross-section at high  $W$  does agree with data, but final state simulation is of unknown quality and difficult to tune also.



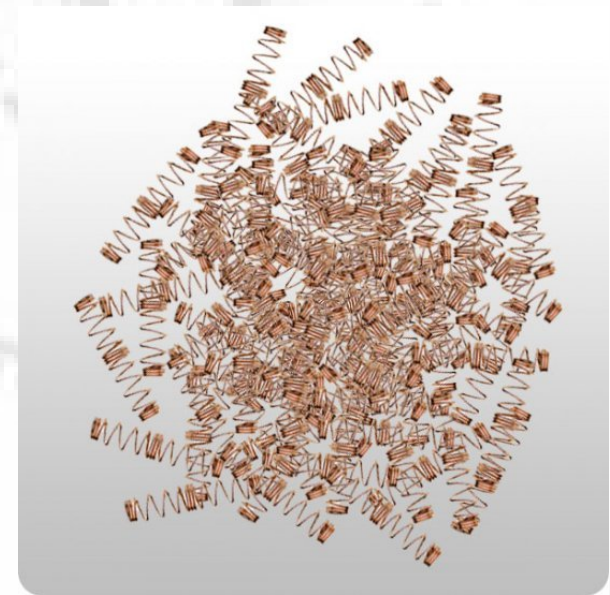
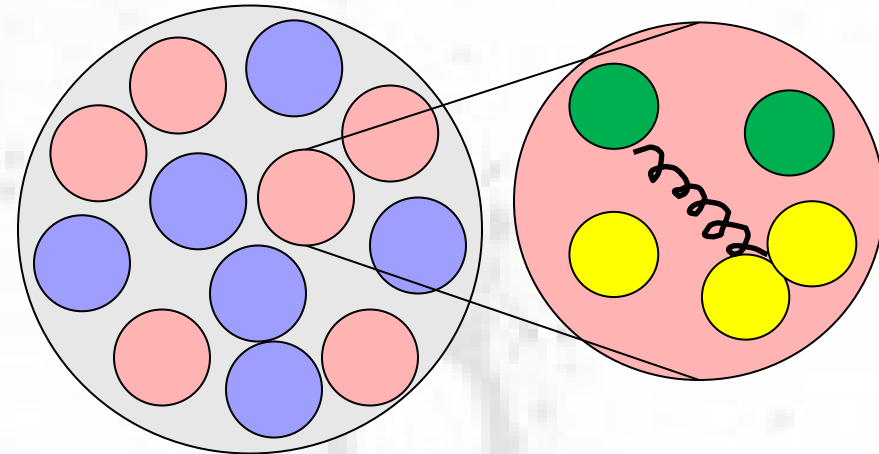


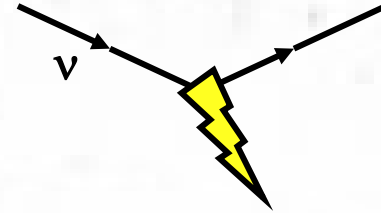
# ***From Nucleons to Nuclei***

# Why are *Nuclei* So Difficult?



- The fundamental theory allows a complete calculation of neutrino scattering from quarks
- But those quarks are in nucleons (PDFs), and those nucleons are in a strongly interacting tangle
- Imagine calculating the excitations of a pile of coupled springs. Very hard in general.

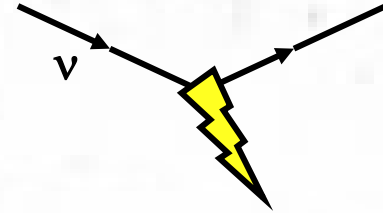




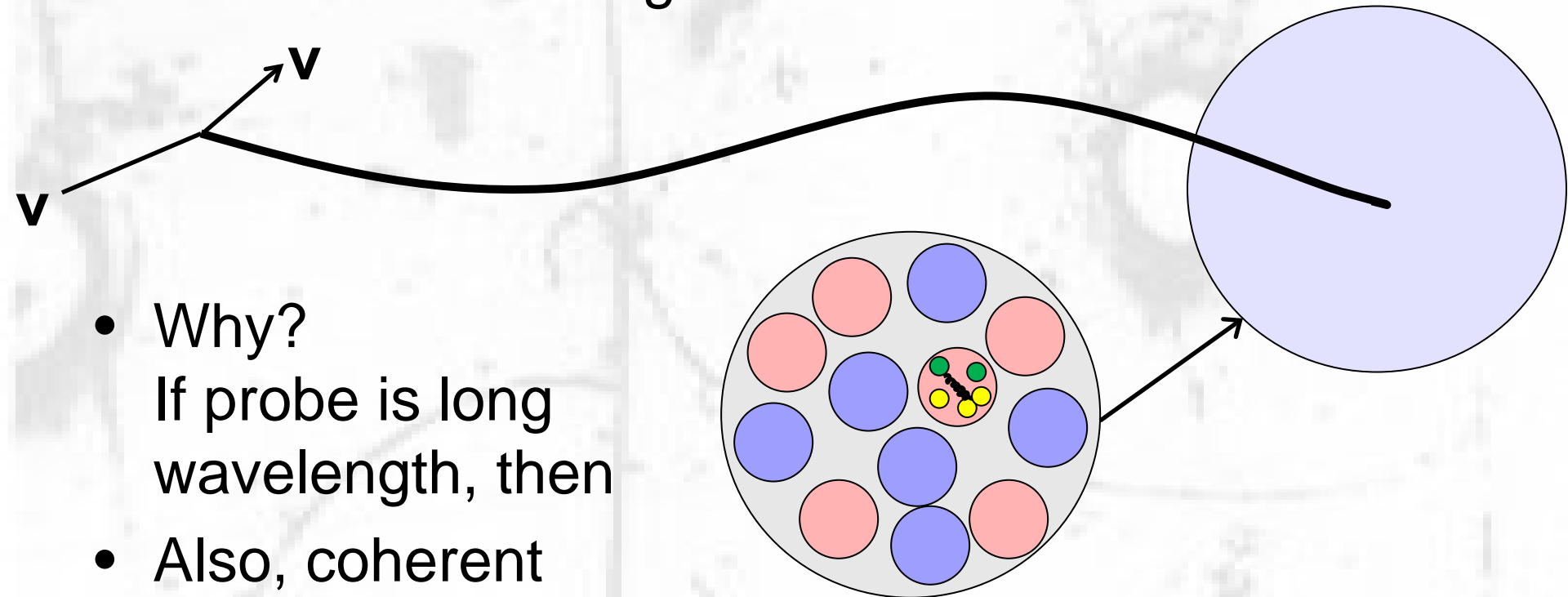
# ***Coherent Neutrino-Nucleus Scattering***



# Coherent and Elastic

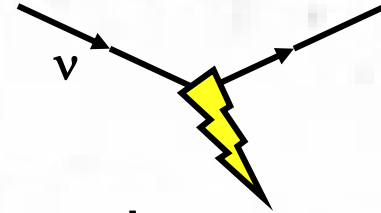


- Here is a limit in which, in principle, we can calculate scattering from the nucleus



- Why?  
If probe is long wavelength, then
- Also, coherent implies significant enhancement of rate

# Coherence Condition



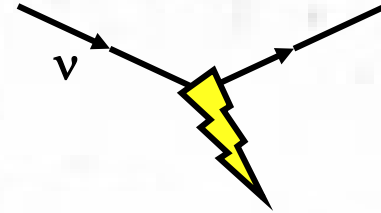
- Wavelength of probe, must be much larger than target, so momentum transfer:  $Q \ll 1/R$
- If coherent, *amplitudes* from nucleons add
  - Therefore rate goes as  $(\text{\#nucleons})^2$
- Limited momentum transfer, means limited kinetic energy of recoil:  $T_{\text{max}} \ll 1/M_A R^2$ 
  - Typical nuclear size in “natural” units  $\sim 100$  MeV, so maximum recoil energy is  $\sim 100$  keV or less for  $^{40}\text{Ar}$

$$T \approx \frac{Q^2}{2M_A} \text{ for } Q \ll M_A$$

$$\frac{d\sigma}{dT} = \frac{G_F^2}{4\pi} \underbrace{\left[ N - Z \left( 1 - 4 \sin^2 \theta_w \right) \right]^2}_{\text{Weak NC coupling : nearly zero for proton}} \left( 1 - \frac{M_A T}{2E_\nu^2} \right) \left( F(Q^2) \right)^2$$

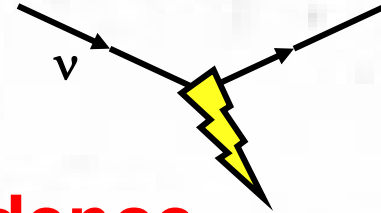
*Form factor with coherence condition... goes to 0 except for very low  $Q^2$*

# ***Comments on Coherent Nuclear Scattering***



- No one has ever observed this because of the difficulties of finding such low recoils in nuclear matter
  - Most promising approaches have much in common with dark matter detectors
- Very useful practically if this can be overcome since it is a reaction perfect for “counting” neutrinos from a beam, a reactor, etc.

## Lecture Question #8



I would be willing to assert at high confidence that the discovery of neutrinos from the big bang would earn you a Nobel prize.

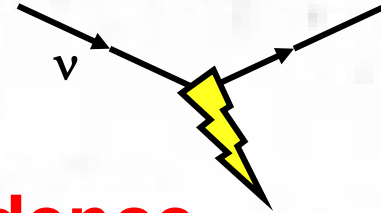
Coherent scattering has no threshold, so can use it to detect neutrinos with energies  $\sim 1$  meV

What makes this difficult?

$$Q \ll \frac{1}{R} \Rightarrow T_{\max} \ll \frac{1}{M_A R^2} \quad T \approx \frac{Q^2}{2M_A} \text{ for } Q \ll M_A$$

$$\frac{d\sigma}{dT} = \frac{G_F^2}{4\pi} \left[ N - Z \left( 1 - 4 \sin^2 \theta_w \right) \right]^2 \left( 1 - \frac{M_A T}{2E_\nu^2} \right) \left( F(Q^2) \right)^2$$

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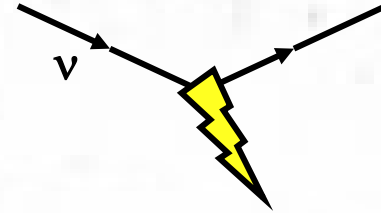
What makes this difficult to detect?

*The maximum momentum that can be transferred to a heavy stationary target is no more than twice the lab frame momentum.*

$$T \approx \frac{Q^2}{2M_A} \text{ for } Q \ll M_A$$

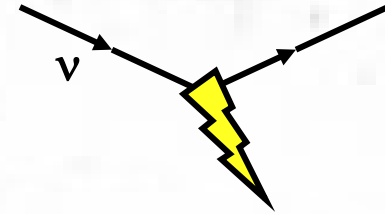
$$\text{So } T \approx \frac{Q^2}{2M_A} < \frac{2p_\nu^2}{M_A} < 10^{-15} \text{ eV}$$

*Bummer! I was looking forward to that sauna.*



# ***Inverse Beta Decay and Related Reactions in Nuclei***

# Recall: Inverse Beta Decay



$$\frac{d\sigma}{d\cos\theta} = \frac{G_F^2 s}{\pi} \times \cos^2 \theta_{\text{Cabibbo}} \times (\xi_{\text{mass}}) \times \left( g_V^2 (1 + \beta_e \cos\theta) + 3g_A^2 \left( 1 - \frac{\beta_e}{3} \cos\theta \right) \right)$$

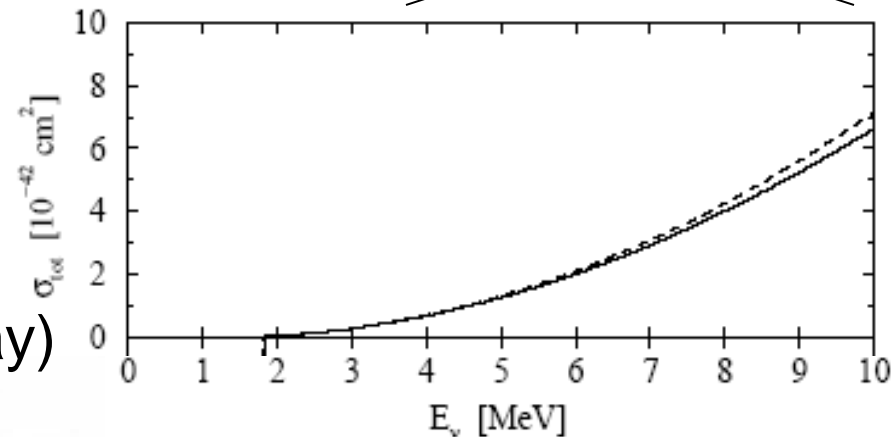
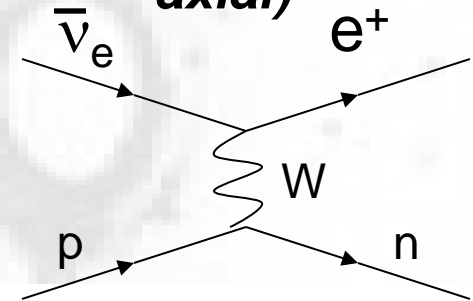
**quark mixing!**
**final state mass suppression**
**proton form factors (vector, axial)**

- mass suppression is proportional to  $\delta E$  at low  $E_\nu$ , so quadratic near threshold

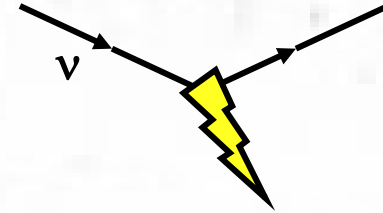
- vector and axial-vector form factors (for IBD usually referred to as  $f$  and  $g$ , respectively)

$$g_V, g_A \approx 1, 1.26.$$

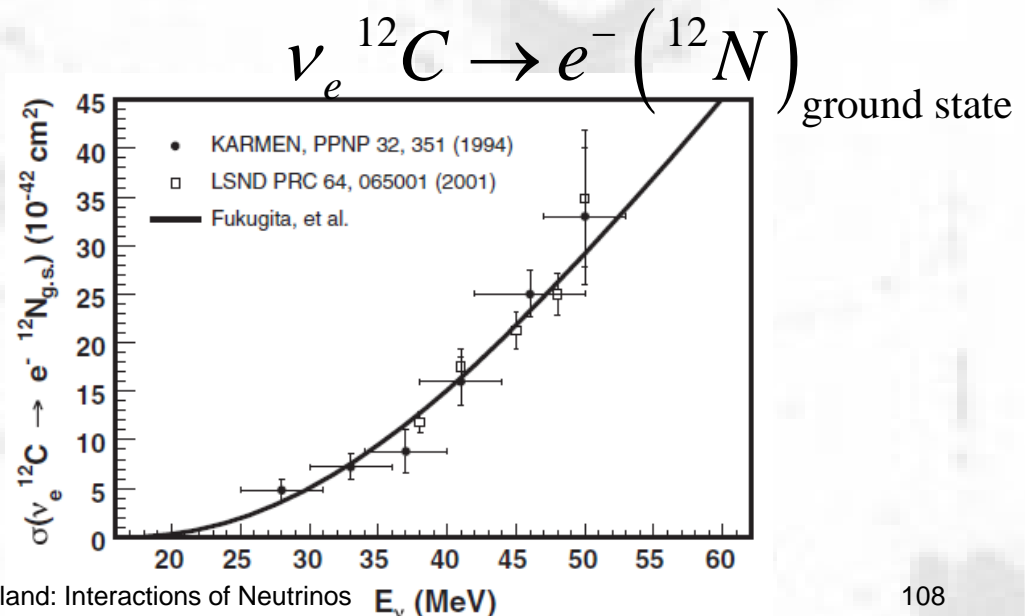
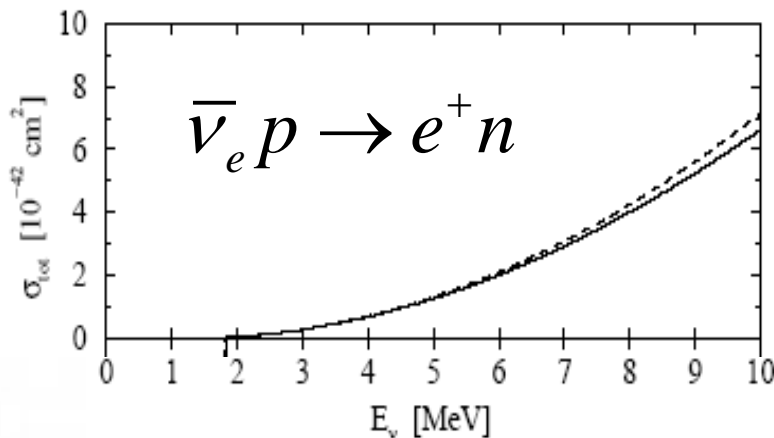
- FFs,  $\theta_{\text{Cabibbo}}$ , best known from  $\tau_n$  (neutron beta decay)



# Inside a Nucleus

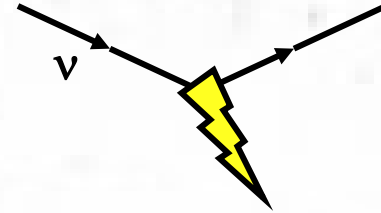


- Near threshold, have to account for discrete excitations of final state nucleus
  - If reaction is inclusive, then this is a sum over states
  - That can be difficult if many states are involved
- Exclusive reactions behave like free nucleon beta decay, but with a different threshold





# Nuclei for Solar Neutrinos



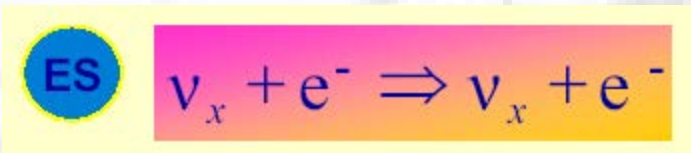
- Here are some nuclei historically important for Solar neutrino experiments. Low thresholds.

<i>Experiment</i>	<i>Nuclear Target</i>	<i>Reaction</i>	$\sigma_0$ [ $10^{-46}\text{cm}^2$ ]	$\Delta E_{\text{nucl}}$ [MeV] (no det. Thres.)
GALLEX/GNO SAGE	$^{71}\text{Ga}_{33}$	$\nu_e + ^{71}\text{Ga} \rightarrow e^- + ^{71}\text{Ge}$	$8.611 \pm 0.4\%$ (GT)	0.2327
HOMESTAKE	$^{37}\text{Cl}_{17}$	$\nu_e + ^{37}\text{Cl} \rightarrow e^- + ^{37}\text{Ar}$	1.725 (F)	0.814
SNO	$^2\text{H}_1$	$\nu_e + ^2\text{H} \rightarrow e^- + p + p$	(GT)	1.442
ICARUS	$^{40}\text{Ar}_{18}$	$\nu_e + ^{40}\text{Ar} \rightarrow e^- + ^{40}\text{K}^*$	148.58 (F)	1.505 +
		...	44.367 (GT <sub>2</sub> )	
		...	41.567 (GT <sub>6</sub> )	
		...	...	

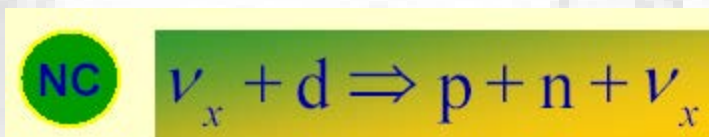
*table courtesy F. Cavanaugh*

# SNO

- Three reactions for observing  $\nu$  from sun ( $E_\nu \sim \text{few MeV}$ )

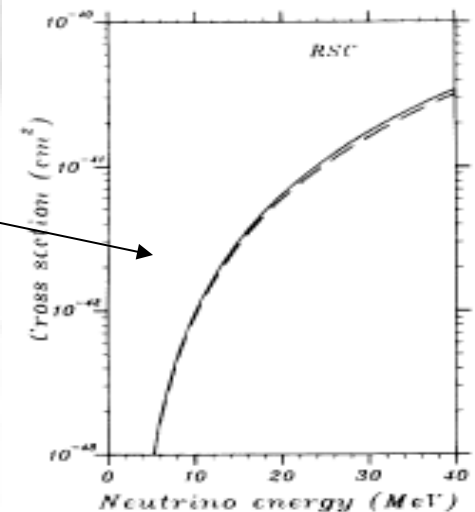
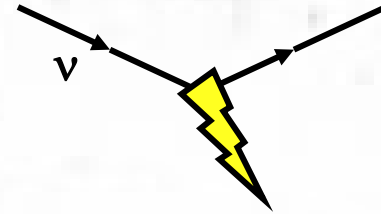
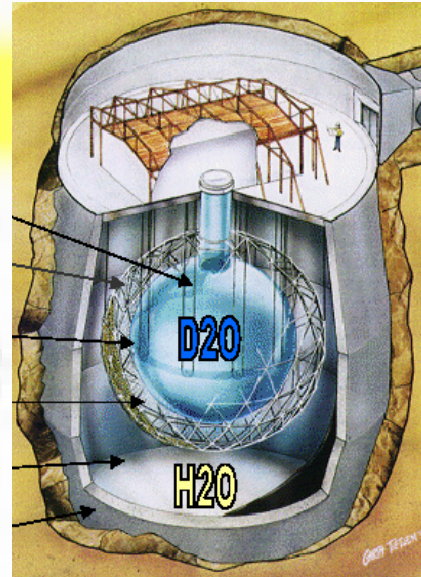


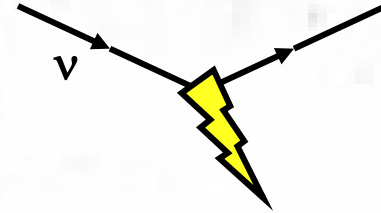
- $^2\text{H}$ ,  $^{16}\text{O}$  binding energies are 13.6eV,  $\sim 1$  keV.
- Therefore,  $e^-$  are “free”.  $\sigma \propto E_\nu$



Deuteron binding energy is 2.2 MeV

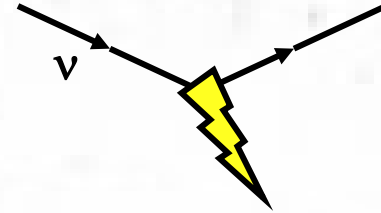
- Energy threshold of a few MeV for neutral current. Less for the charged current because  $m_n > m_p + m_e$



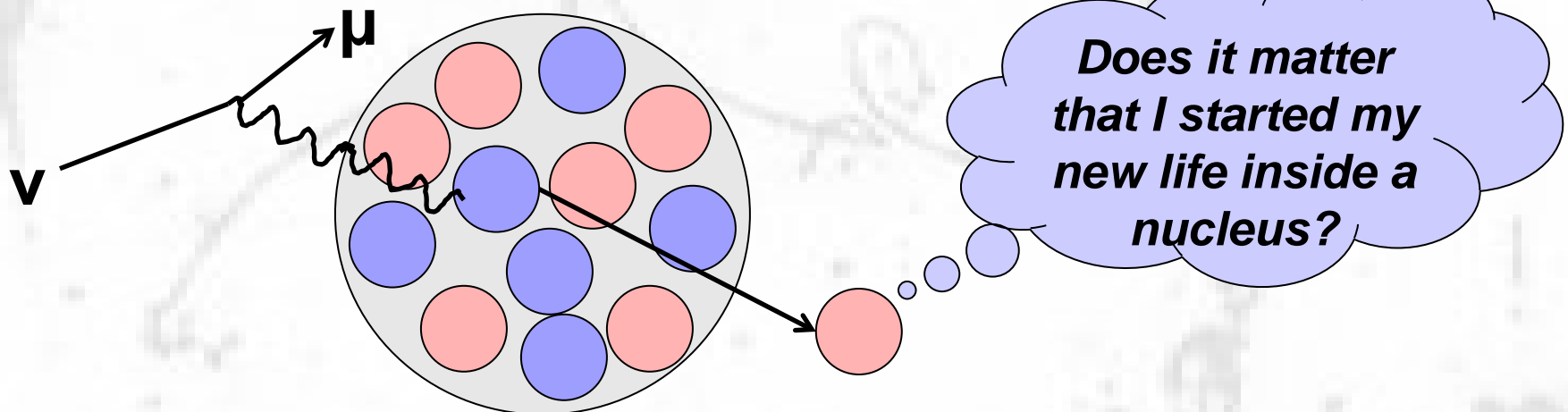


# ***GeV Cross-Sections on Nucleons in a Nucleus***

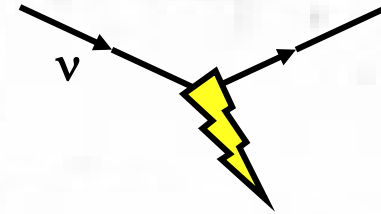
# ***Elastic? Fantastic!***



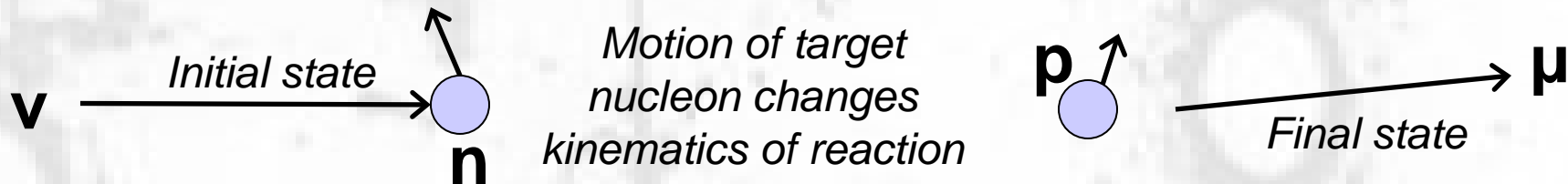
- Last time, we showed that the elastic scattering of neutrinos from nucleons is (nearly) predicted
  - Charged-current reaction allows tagging of neutrino flavor and reconstruction of energy
- Unfortunately, practical neutrino experiments have these nucleons inside nuclei



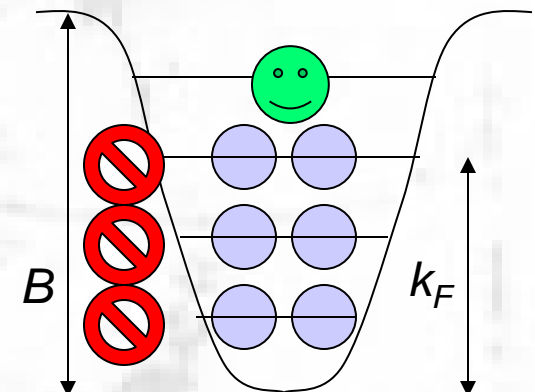
# Fermi Motion, Binding and Pauli “Blocking”



- In a nucleus, target nucleon has some initial momentum which modifies the observed scattering
  - Simple model is a “Fermi Gas” model of nucleons filling available states up to some initial state Fermi momentum,  $k_F$

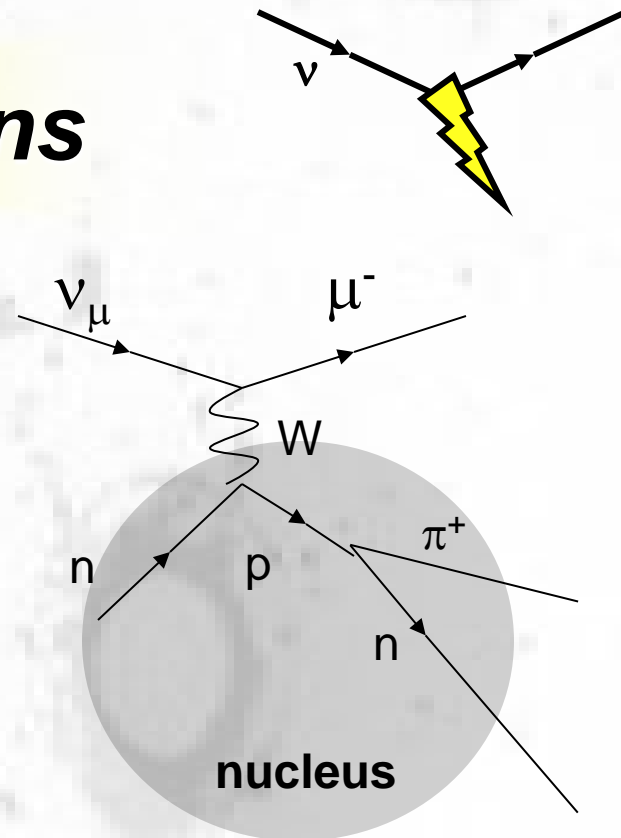


- The nucleon is bound in the nucleus, so it takes energy to remove it
- Pauli blocking for nucleons not escaping nucleus... states are already filled with identical nucleon



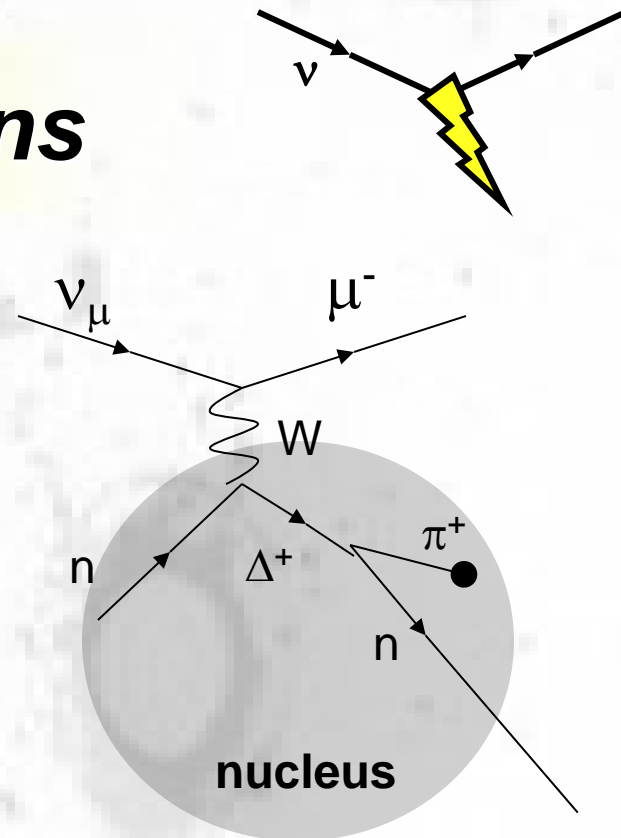
# ***“Final State” Interactions***

- The outgoing nucleon could create another particle as it travels in nucleus
  - If it is a pion, event would appear inelastic
- Also other final states can contribute to apparent “quasi-elastic” scattering through absorption in the nucleus...
  - kinematics may or may not distinguish the reaction from elastic
- Theoretical uncertainties in these reactions are **large**
  - At least at the 10% level. More on this later.
  - If precise knowledge is needed for target (e.g., water, liquid argon, hydrocarbons), dedicated measurements will be needed
    - Most relevant for low energy experiments, i.e., T2K



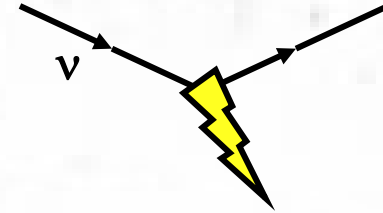
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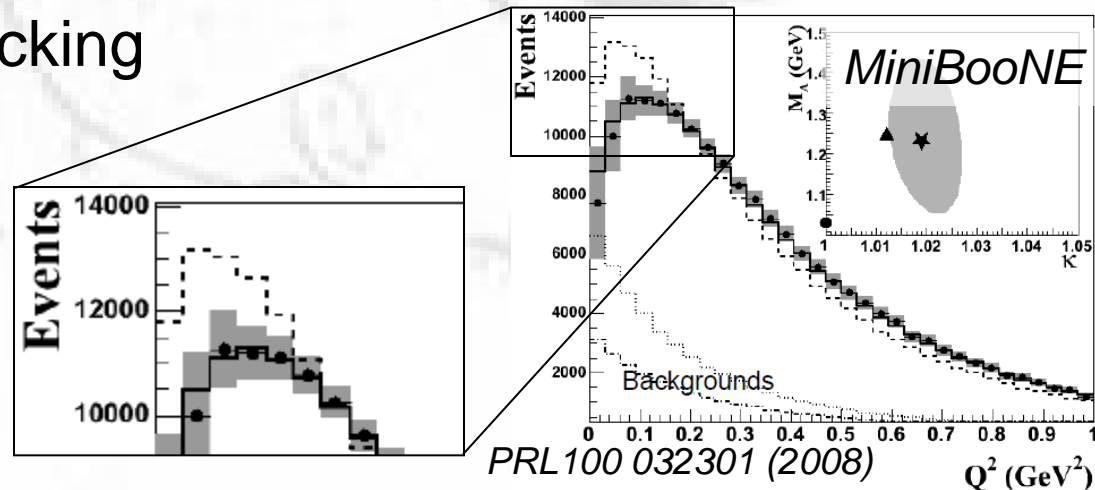
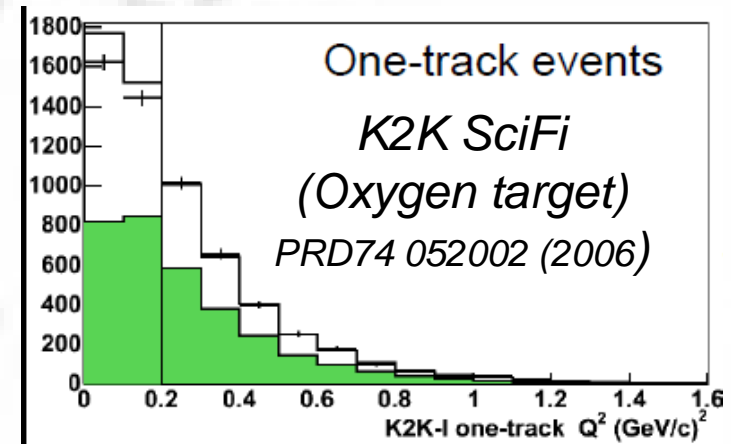




# Measurements of CCQE on Nuclei: Backgrounds

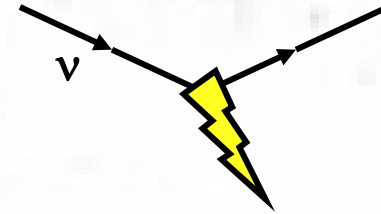


- K2K famously observed a “low  $Q^2$  deficit” in its analysis
- MiniBooNE originally had a significant discrepancy at low  $Q^2$  as well
  - Original approach was to enhance Pauli blocking to “fix” low  $Q^2$
  - Was resolved by tuning single pion background to data w/ pions

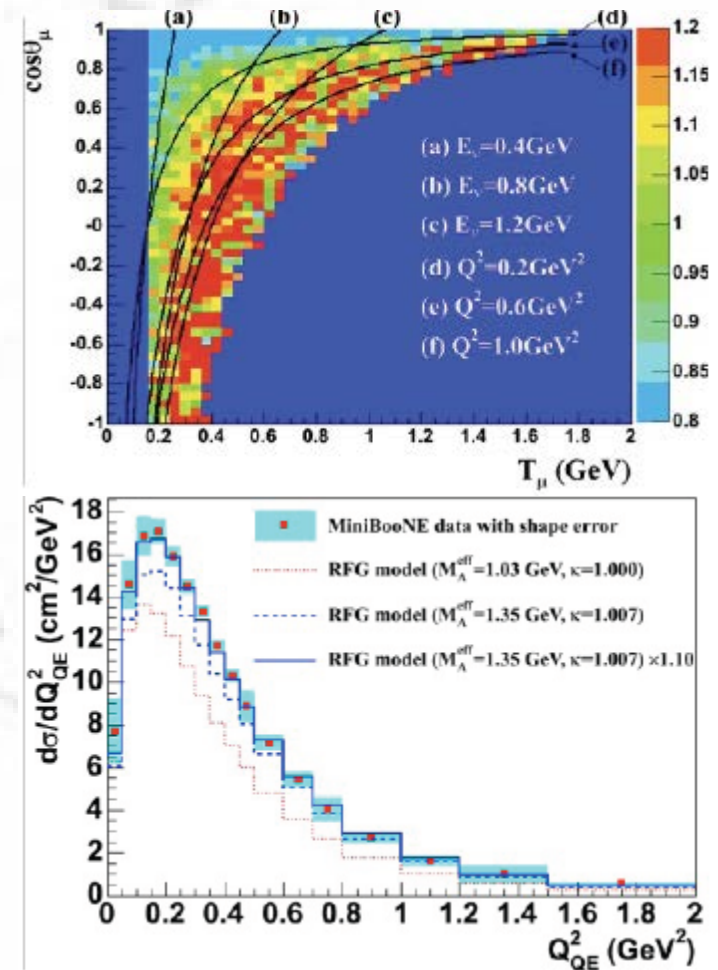




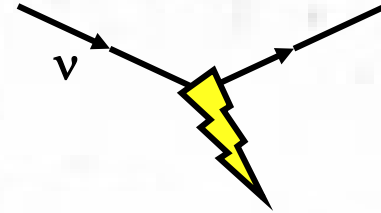
# MiniBooNE (Phys. Rev. D81 092005, 2010)



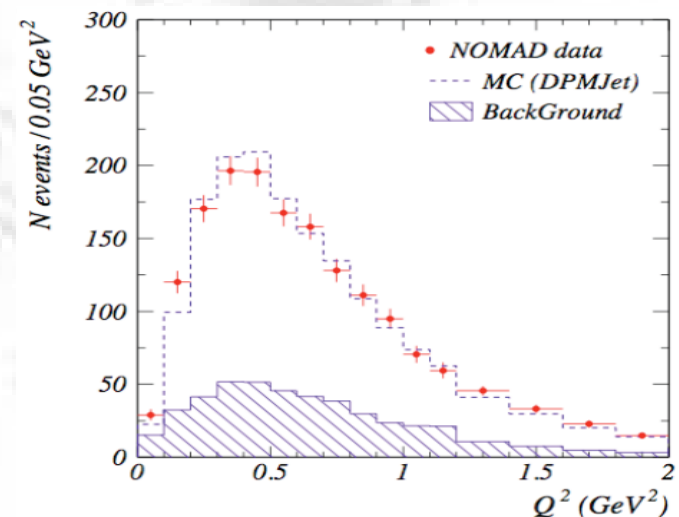
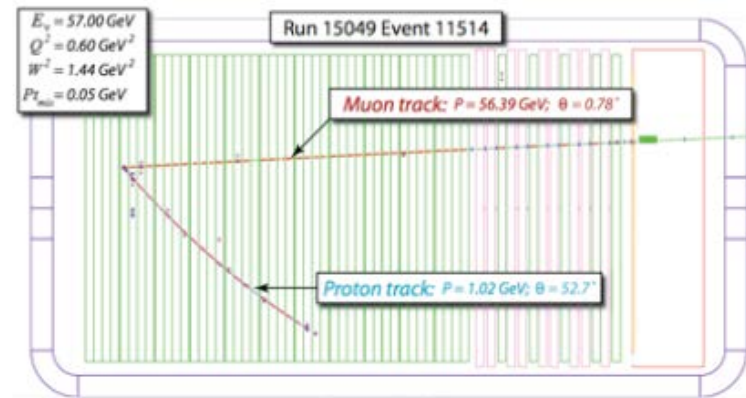
- Oil Cerenkov detector (carbon), views only muon
- Fit to observables, muon energy & angle find a discrepancy with expectation from free nucleons
- It looks like a distortion of the  $Q^2$  distribution
- MiniBooNE fits for an “effective” axial mass,  $M_A$ , higher than expected
  - Good consistency between total cross-section and this  $Q^2$  shape in this high  $M_A$  explanation



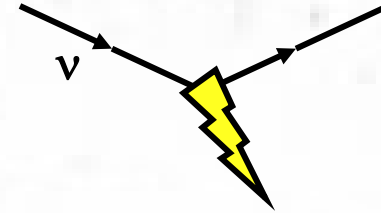
# NOMAD (Eur.Phys.J.C63:355-381,2009)



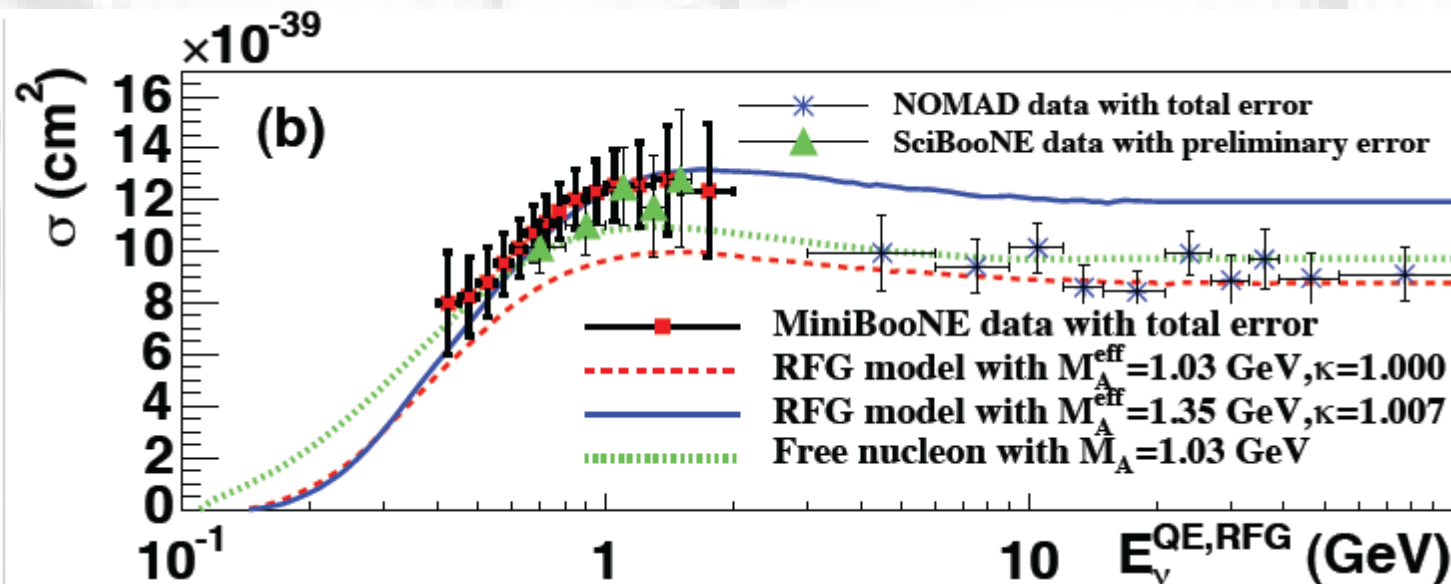
- Like MiniBooNE, target is mostly carbon (drift chamber walls)
- Reconstruct both recoiling proton and muon
- Total cross-section and  $Q^2$  distribution are both consistent with expectation from free nucleon
- *Two experiments, same target, but different energies and reconstruction...*  
*... incompatible results?*



# MiniBooNE and NOMAD

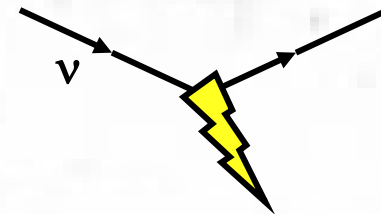


- Current data cannot be fit by a single prediction for low energy data (BooNEs) and high energy data (NOMAD)
  - In effective dipole form-factor picture, different “ $M_A$ ”
  - Free nucleon  $M_A$  is  $\sim 1$  GeV from both pion electroproduction and neutrino scattering on

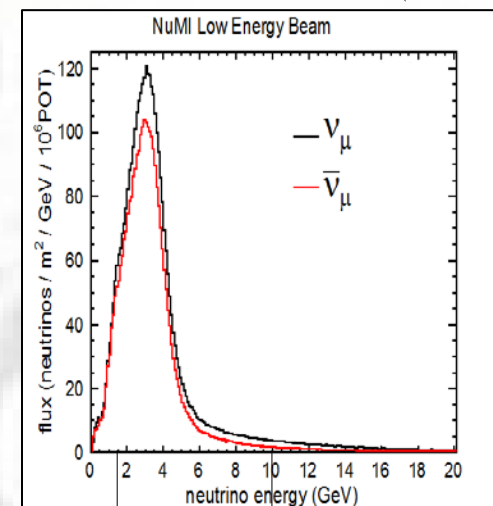


Plot courtesy  
of T. Katori

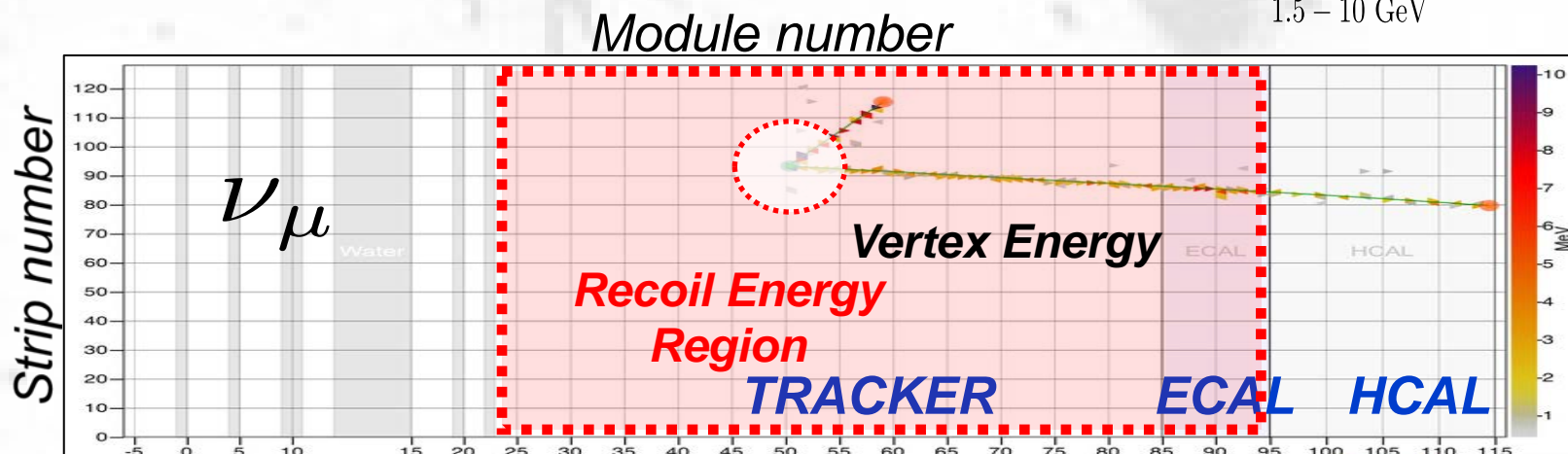
# MINERvA CCQE on Carbon



- MINERvA has measured CCQE in neutrino and anti-neutrino beams
  - Flux integrated from 1.5 to 10 GeV.  
It's a measurement “near” 3.5 GeV
- Sample is selected by muon and “low” calorimetric recoil away from vertex



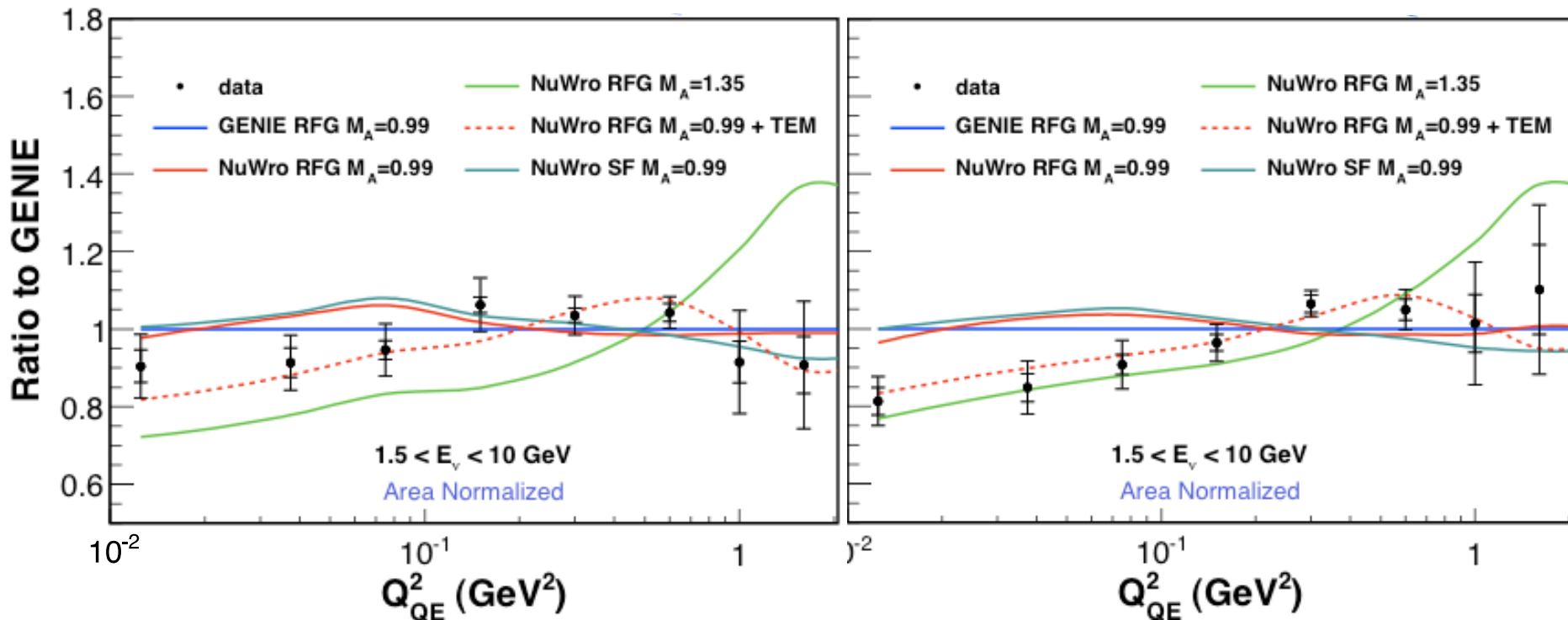
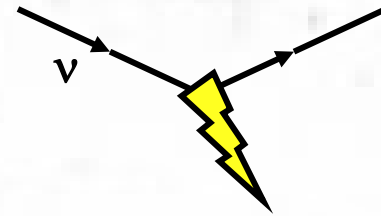
1.5 – 10 GeV



# $d\sigma/dQ^2$ Shape

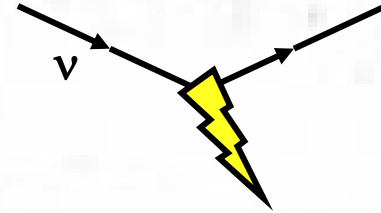
$\nu_\mu$  CCQE

$\bar{\nu}_\mu$  CCQE

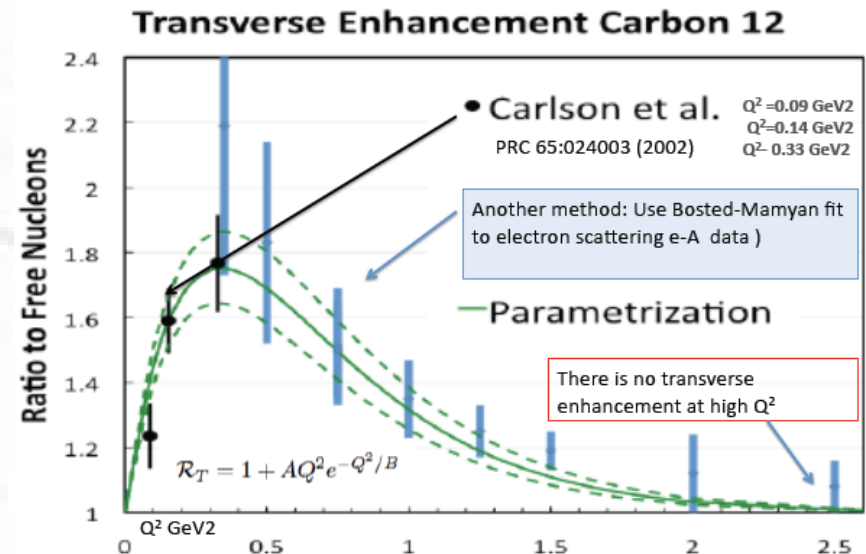


- $Q^2$  distribution doesn't agree well with “high effective  $M_A$ ”, but there is a clear disagreement with free nucleon result
- Best fit is to “transverse enhancement model”

# Multi-Nucleon Correlations

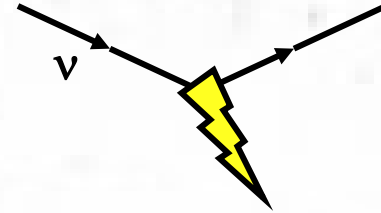


- Inclusion correlations among nucleons in nucleus would add another quasielastic like process knocking two nucleons from nucleus
  - Could alter kinematics and rate in a way that would make a better fit to the data
- How to implement?
  - Microphysical models don't yet give complete final state description
  - “Ad hoc” enhancement scaled from electron scattering data?  
(Carlson & Bodek, Budd, Christy)

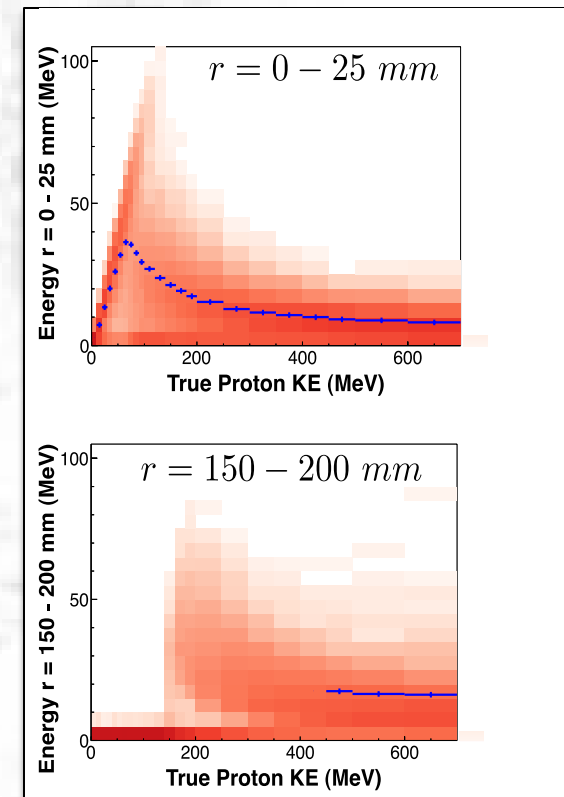
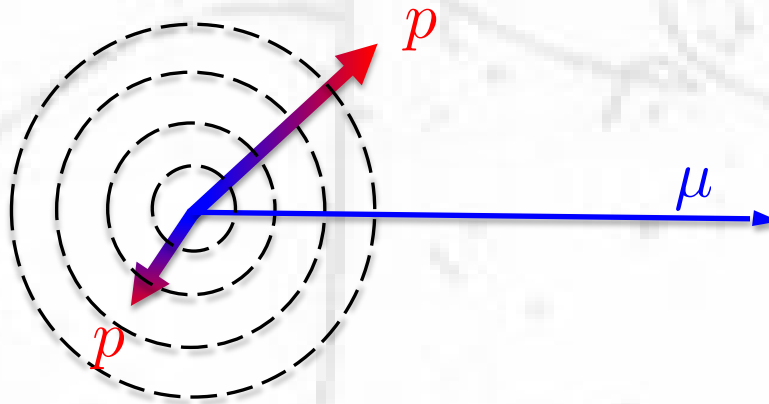




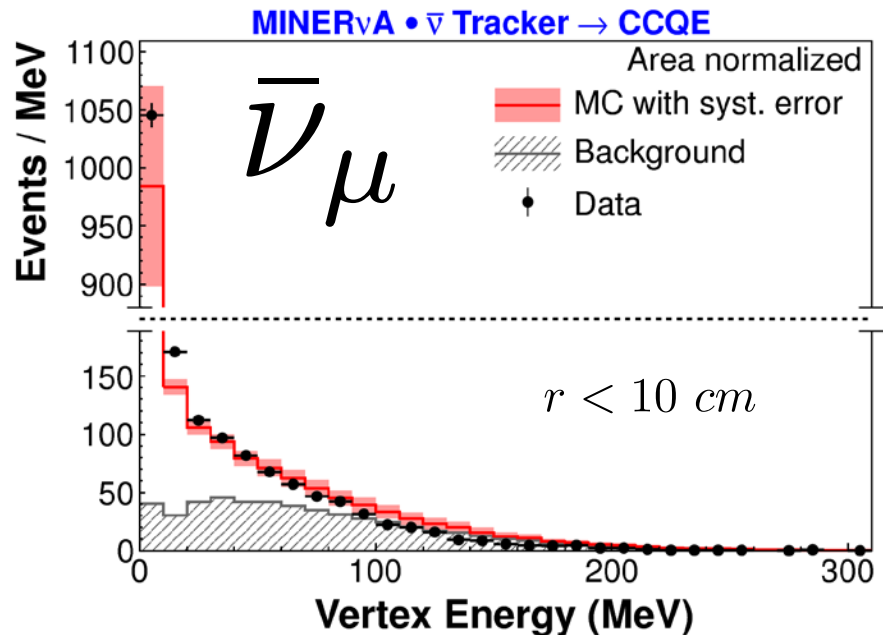
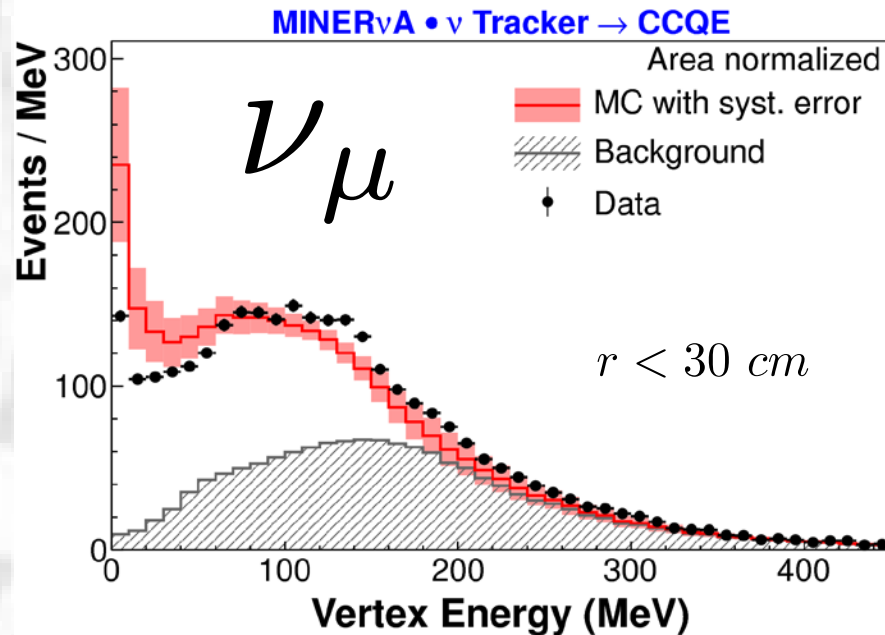
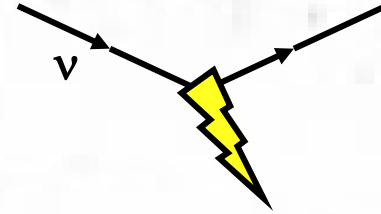
# Vertex Region Energy



- Vertex region ignored in MINERvA recoil cut
  - Therefore selection is mostly insensitive to low energy nucleons in the final state
- Study energy near vertex
  - Vertex is precisely located, so distance of energy from vertex is sensitive to range of extra protons



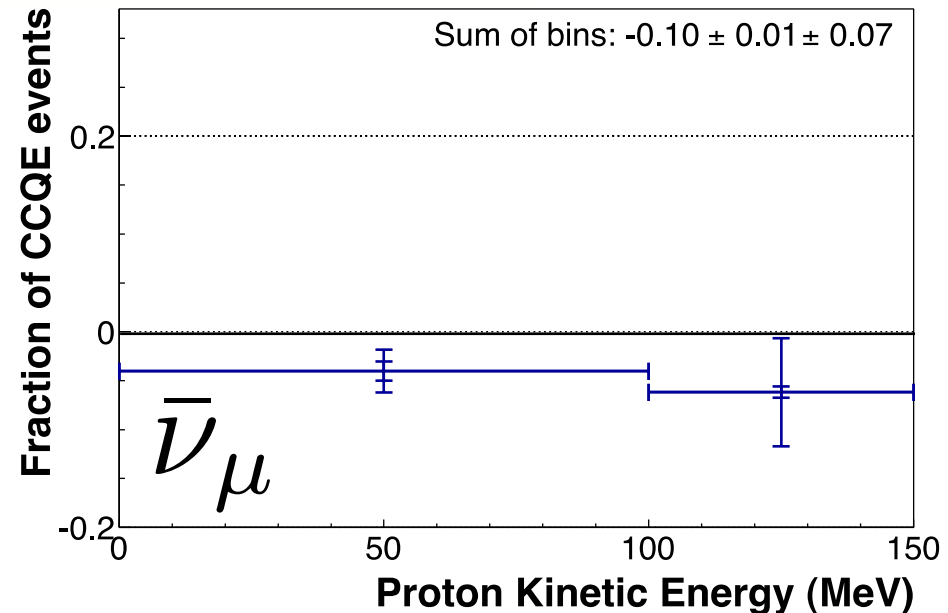
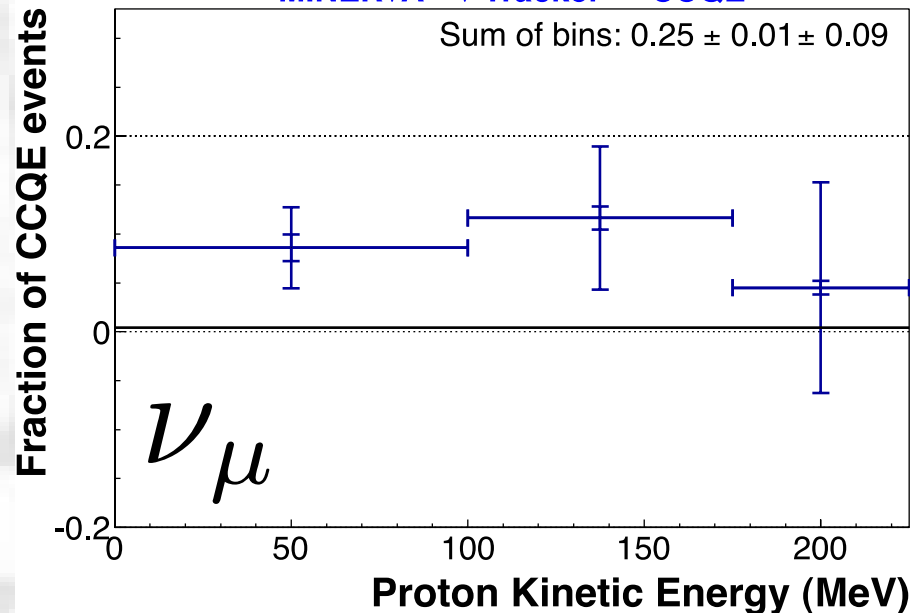
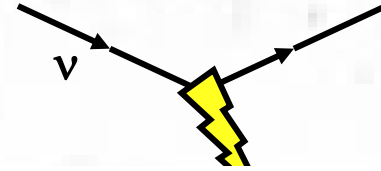
# MINERvA: Vertex Energy



- A trend toward higher vertex energy is observed in the neutrino data, but not in anti-neutrino data
- Red band represents uncertainties on energy reconstruction and final state interactions
- Assume extra energy is due to additional protons

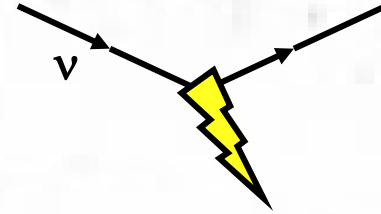


# Extra Protons in MINERvA?



- Data wants to add low energy protons in  $25 \pm 9\%$  of neutrino events, but prefers  $10 \pm 7\%$  fewer protons in anti-neutrino
- Suggests correlated pairs are dominantly n+p in initial state, and therefore p+p or n+n in CCQE

# Summary of CCQE in Nuclear Targets

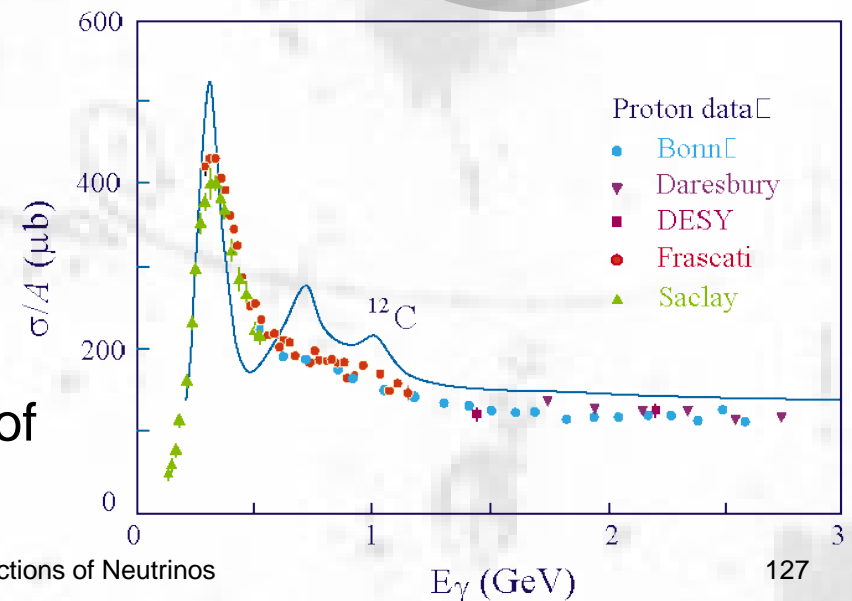
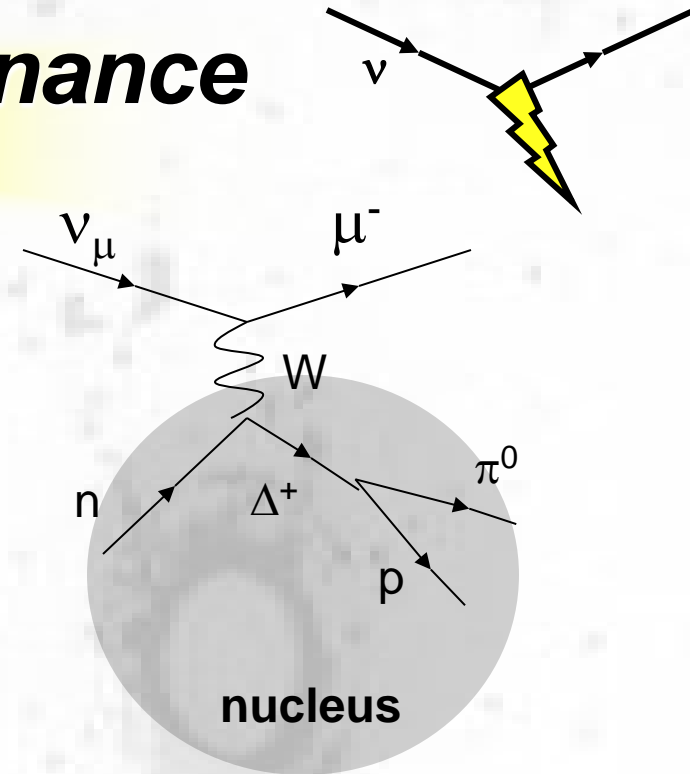


- There is evidence for nuclear modification of quasielastic neutrino-nucleon reactions
  - Kinematics of nucleons: Fermi motion, Pauli blocking
  - Multi-nucleon processes seem to also be present
- There are other possible effects
  - More complete nucleon kinematics (spectral function)
  - A suppression is expected at low  $Q^2$  (long probe wavelength) from interactions of probe with multiple nuclei in “random phase approximation” calculations
- Some of these effects contain overlapping physics! A challenge for the prediction.

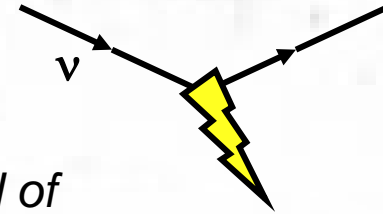
# Nuclear Effects in Resonance Region

- An important reaction like  

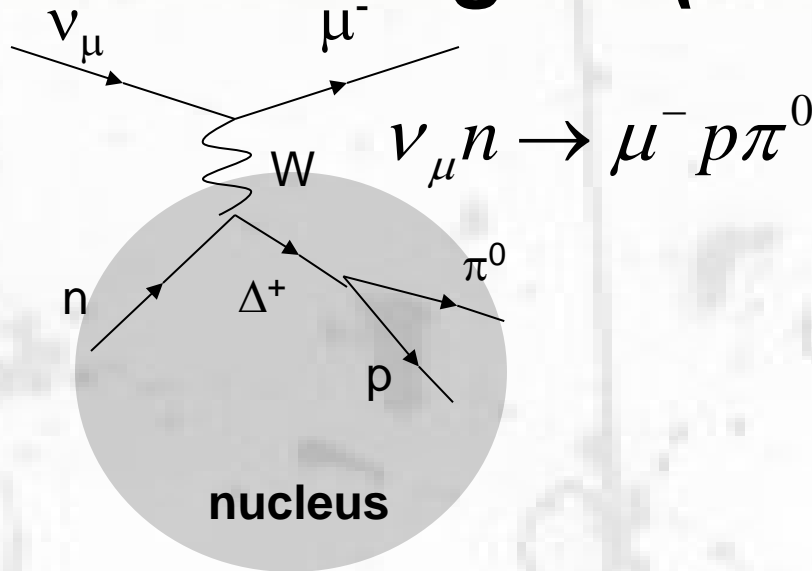
$$\nu_{\mu} n \rightarrow \mu^{-} p \pi^0$$
 ( $\nu_e$  background) can be modified in a nucleus
- Production kinematics are modified by nuclear medium
  - at right have photoabsorption showing resonance structure
  - line is proton; data is  $^{12}\text{C}$
  - except for first  $\Delta$  peak, the structure is washed out
  - Fermi motion and interactions of resonance inside nucleus



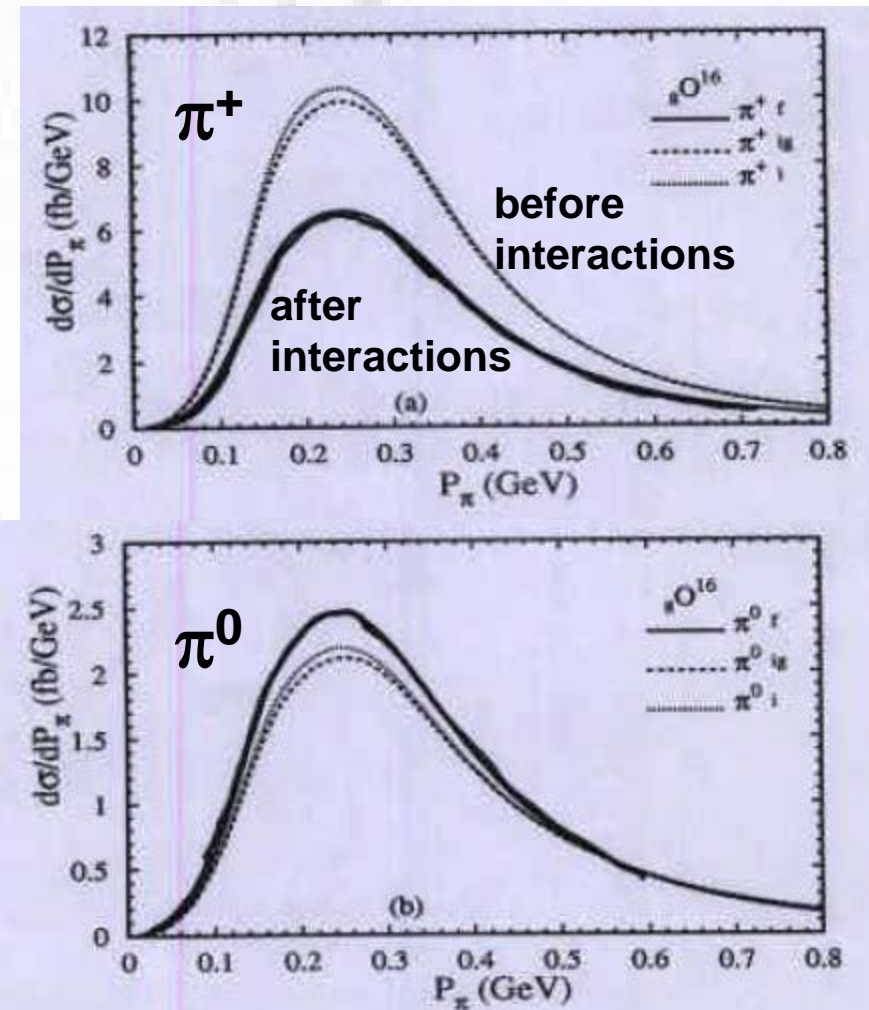
# Nuclear Effects in Resonance Region (cont'd)



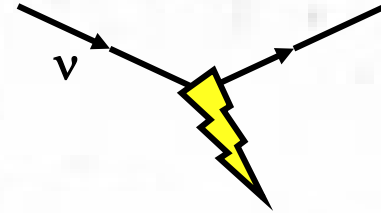
model of  
E. Paschos, NUINT04



- How does nucleus affect  $\pi^0$  after production?
- “Final State Interactions”: migration of one state to another and pion absorption

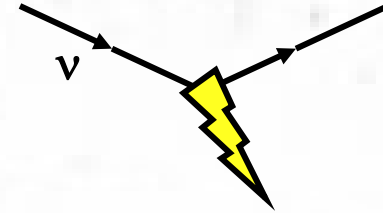


# ***Approaches to Final State Interactions***



- Propagate final state particles through the nuclear medium with varying degrees of sophistication where they interact according the measured cross-sections or models
- Issues:
  - Are the hadrons modified by the nuclear medium?
  - Are hadrons treated as only on-shell or is off-shell transport allowed?
  - How to cleanly separate the initial state particles from their final state interactions?
  - How to relate scattering of external pions or nucleons from nuclei to scattering of particle created in nucleus?

# Lecture Question #9

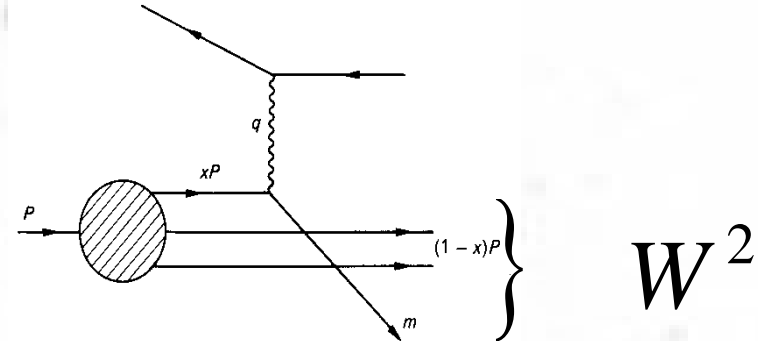


- Two questions with (*hint*) related answers...

## 1. Remember that $W^2$ is...

$$W^2 = M_P^2 + 2M_P\nu - Q^2$$

$$= M_P^2 + 2M_P\nu(1-x)$$



the square of the invariant mass of the hadronic system. ( $\nu=E_\nu-E_\mu$ ;  $x$  is the parton fractional momentum)  
It can be measured, as you see above with only leptonic quantities (neutrino and muon 4-momentum).

In neutrino scattering on a scintillator target, you observe an event with a recoiling proton and with  $W$  reconstructed (perfectly) from leptonic variables  $\langle M_p$ . Explain this event.

## 2. In the same scintillator target, you observe the

reaction...  $\nu_\mu {}^{12}\text{C} \rightarrow \mu^- p \pi^- + \text{remnant nucleus}$

Why might this be puzzling? Explain the process.

# Lecture Question #9

- Both phenomena occur because of nuclear effects!

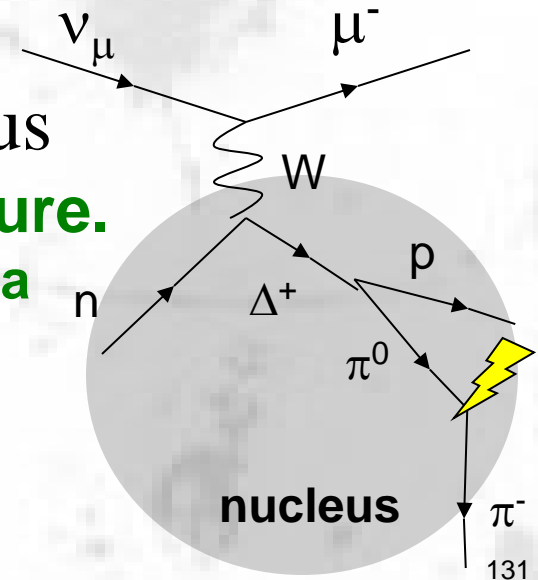
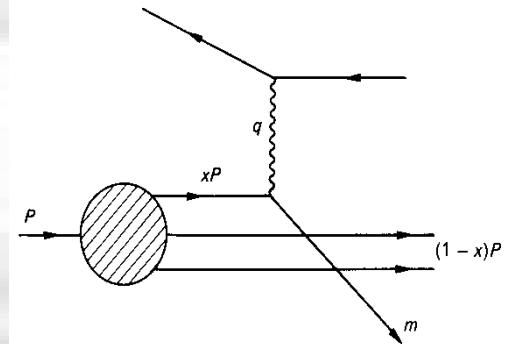
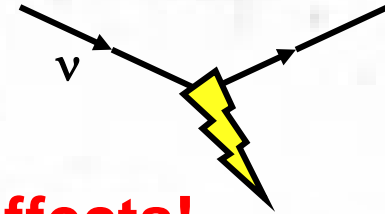
1.  $M_P > W^2 = M_P^2 + 2M_P \nu (1-x)$   
can only be true if  $x > 1$ .

That means the fractional momentum by the struck target parton is  $>1$ ! This can only happen for in a nucleon boosted towards the collision in the CM frame by interactions within the nucleus ("Fermi momentum")

3.  $\nu_\mu {}^{12}\text{C} \rightarrow \mu^- p \pi^- + \text{remnant nucleus}$   
is nonsense in a free nucleon picture.  
It is forbidden to occur off of a proton or a neutron target by charge conservation!

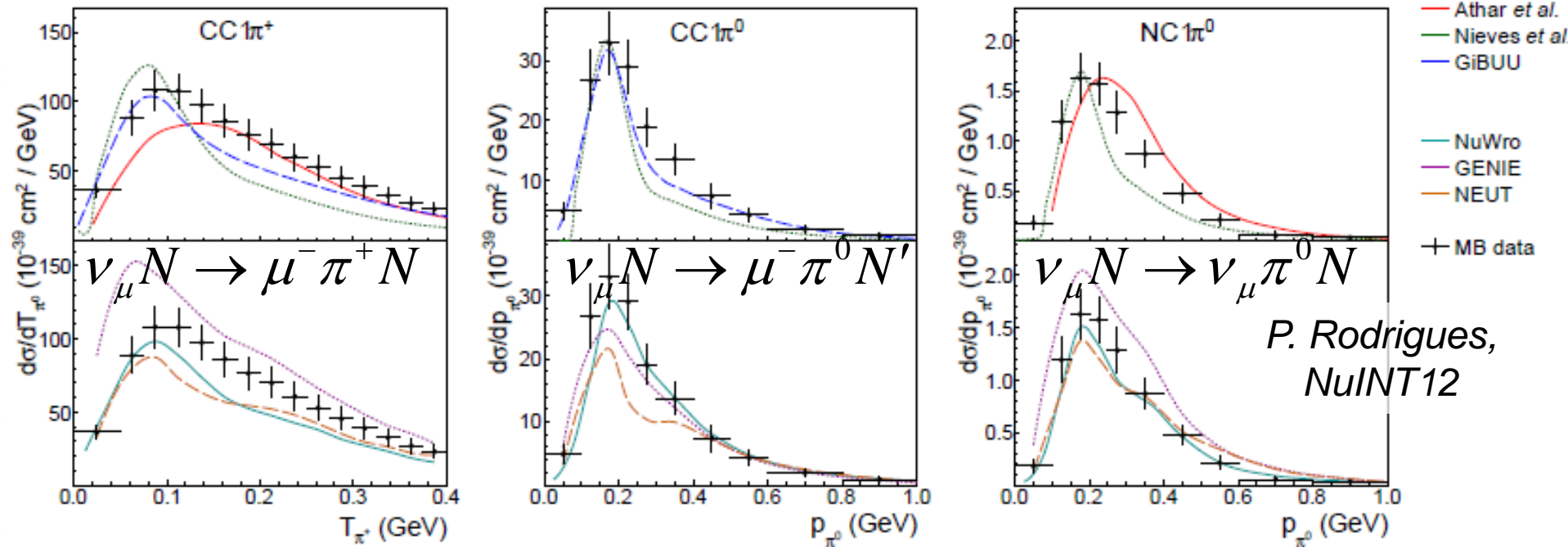
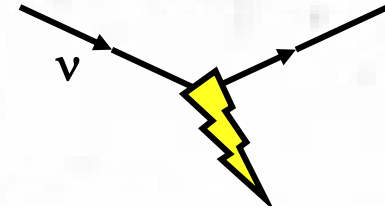
But remember...

reinteraction of pions!





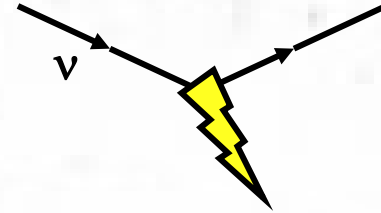
# Single Pion Production Data



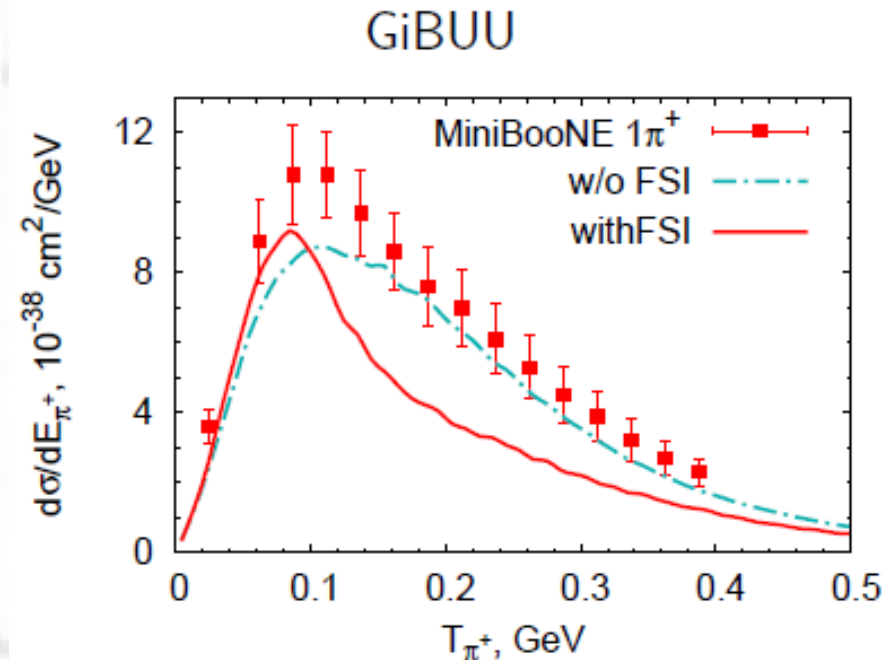
- Comparison of models to MiniBooNE single pion production on  $\text{CH}_2$
- Some models do better on one process than another, but no model reproduces features of all processes
- That's crazy! These are processes related by isospin!



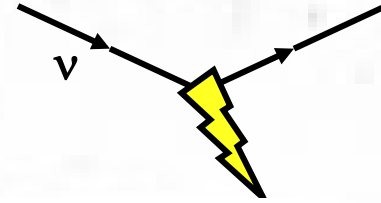
# What is Failing?



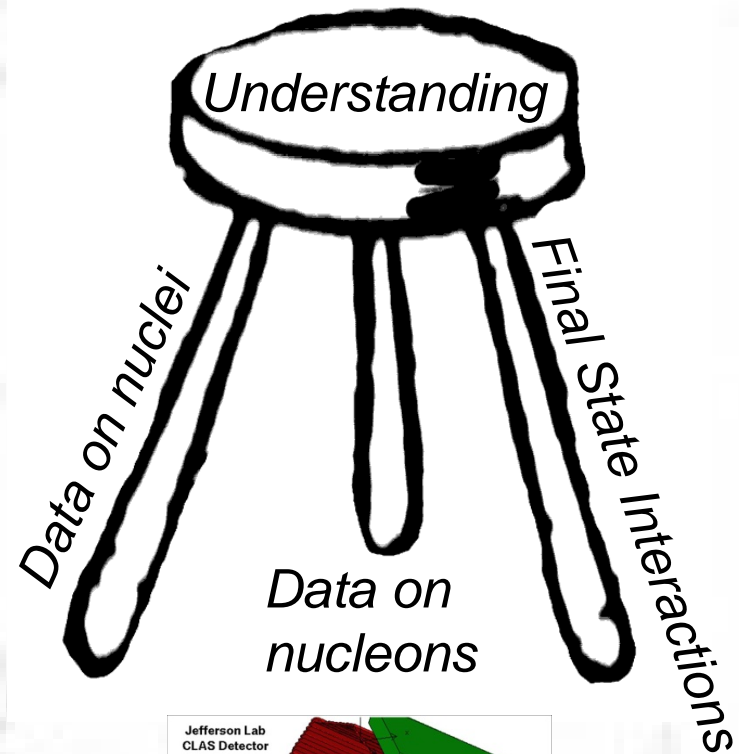
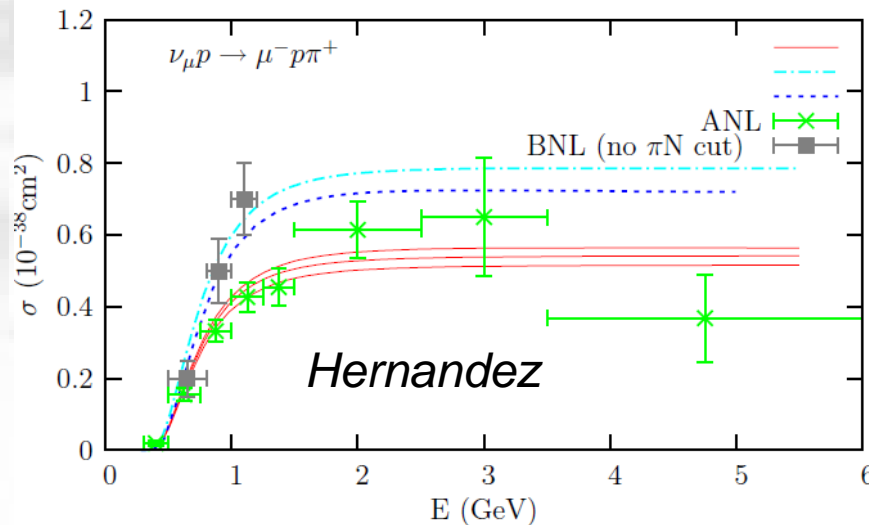
- The honest answer: we don't know
  - Comparison at right is:  
(1) the best model for pion production tuned to electron scattering  
+ (2) a sophisticated final state model tuned to photoproduction
- This disagreement is large compared to precision needed for current oscillation experiments



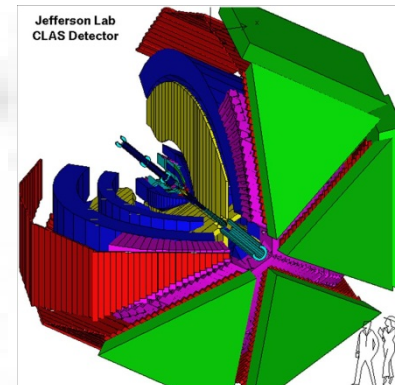
# ***D<sub>2</sub> : Disappointing Data?***

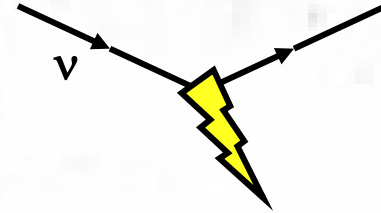


- Ideally to resolve our pion conundrum, we would go to *reliable* nucleon level data
  - Unfortunately, we don't have it.



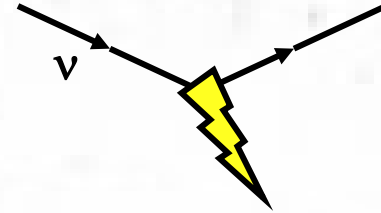
- eN vs. eA data: our only hope for exclusive states? (MINERvA is proposing a D<sub>2</sub> target, but for DIS.)



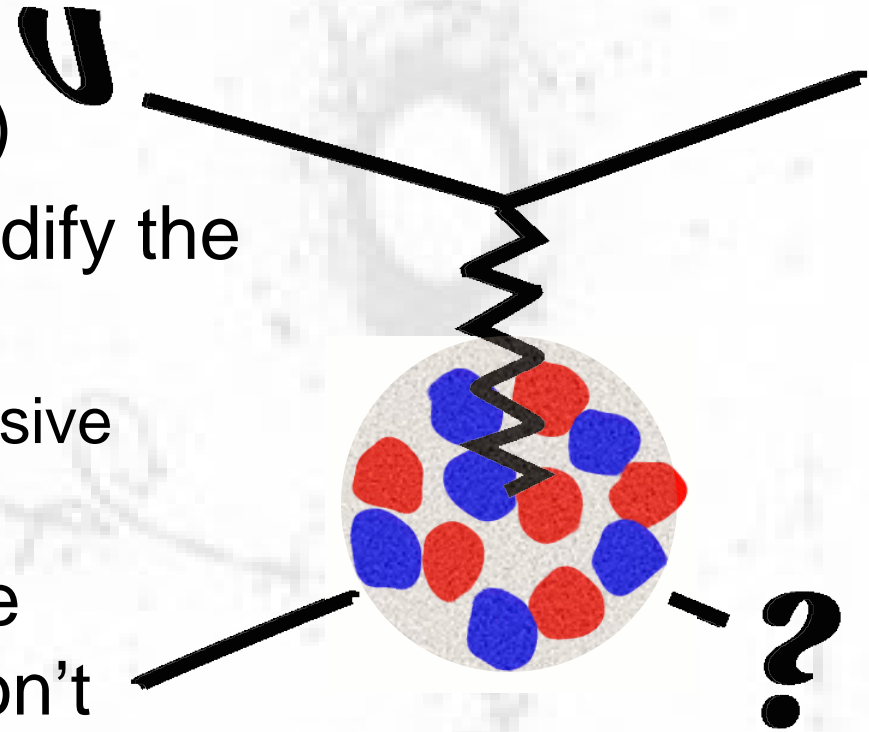


# ***Nuclear Effects in Deep Inelastic Scattering***

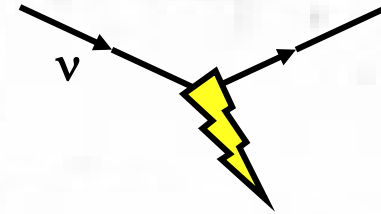
# ***For Inclusive Scattering, Does Nucleus Matter?***



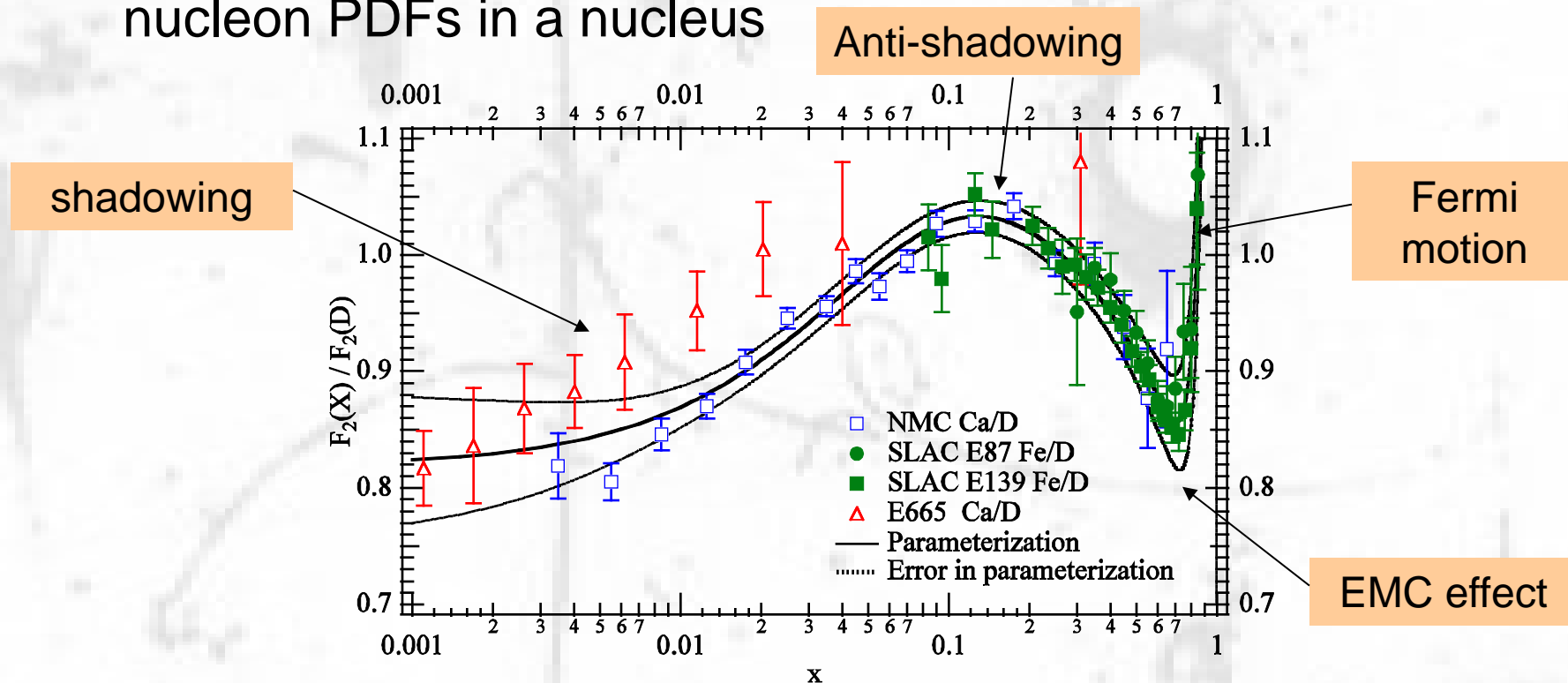
- In high energy limit, calculate of strongly coupled system should be “easy”. However...
- Nucleon are not at rest in nucleus (Fermi motion)
- Nuclear medium may modify the structure of free nucleon
  - Evidence of this from inclusive charged lepton scattering
- Less important: final state interactions, since you don't care about exclusive final states



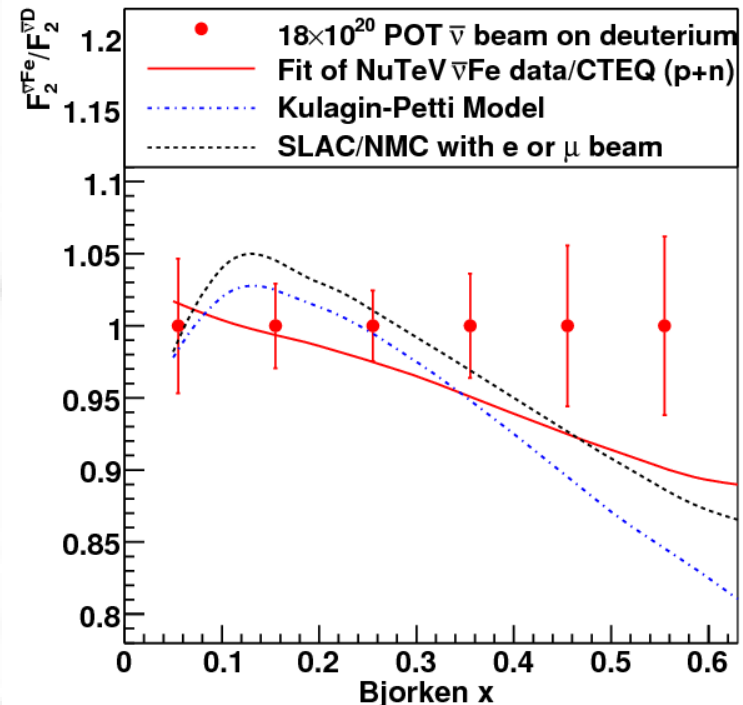
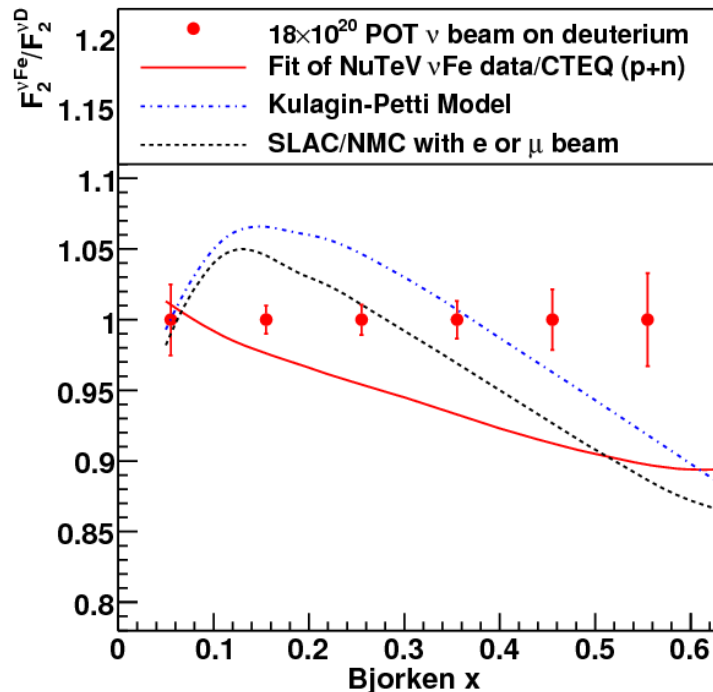
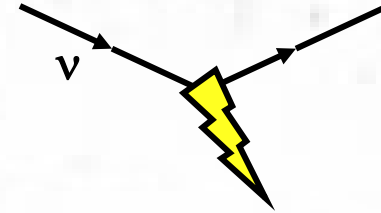
# Is the *DIS Limit Simple*?



- Well measured effects in charged-lepton DIS
  - Maybe the same for neutrino DIS; maybe not... all precise neutrino data is on Ca or Fe targets!
  - Conjecture: these can be absorbed into effective nucleon PDFs in a nucleus



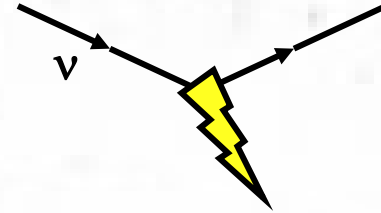
# But that conjecture may be wrong...



Curves from: Ingo Schienbein et al., *Phys.Rev.D*80(2009)094004; *PRD*77(2008)054013

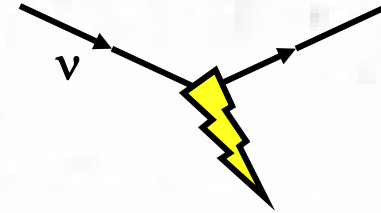
- Only answer is to measure... red points would be precision of MINERvA experiment if it could add a deuterium target in the NOvA running of NuMI beamline.

# Measuring Inclusive Interactions



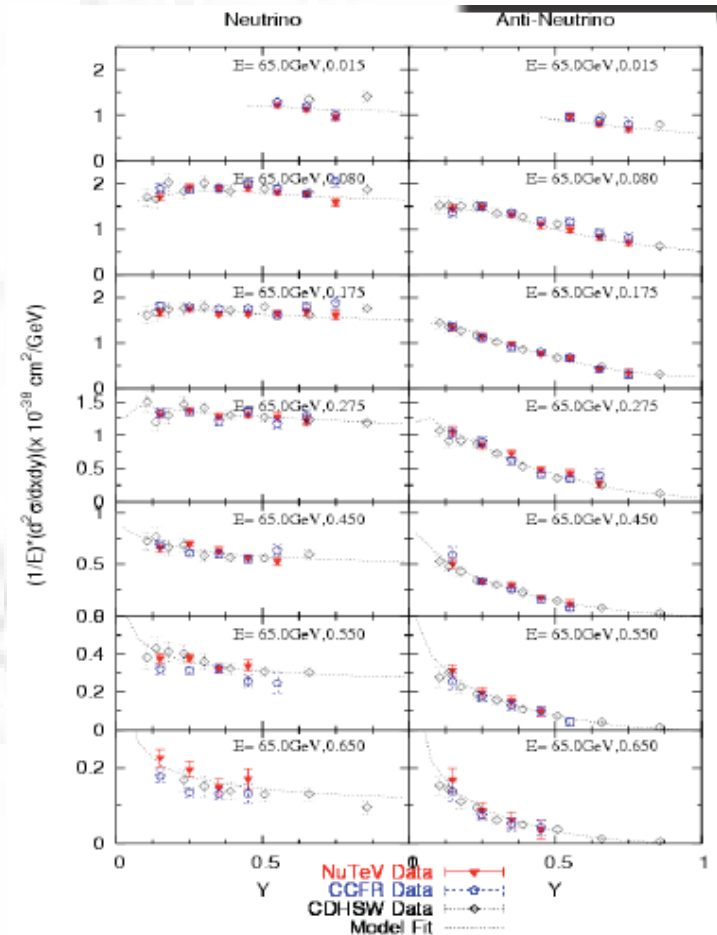
- Much of the data we have is at high energies
  - Neutrino flux is usually poorly known. Common wideband technique is “low recoil” method which uses the observation that  $\lim_{\nu \rightarrow 0} \frac{d\sigma}{d\nu}$  is independent of  $E_\nu$
  - Cross-section normalized from narrow band expt's which counted secondary particles to measure flux
- Typical goal is to extract structure functions  $2xF_1(x, Q^2)$ ,  $F_2(x, Q^2)$ ,  $xF_3(x, Q^2)$  from dependence in  $y$  and  $E_\nu$ .
- Most recently, NuTeV, CHORUS, NOMAD, MINOS

# NuTeV CC Differential Cross-Sections



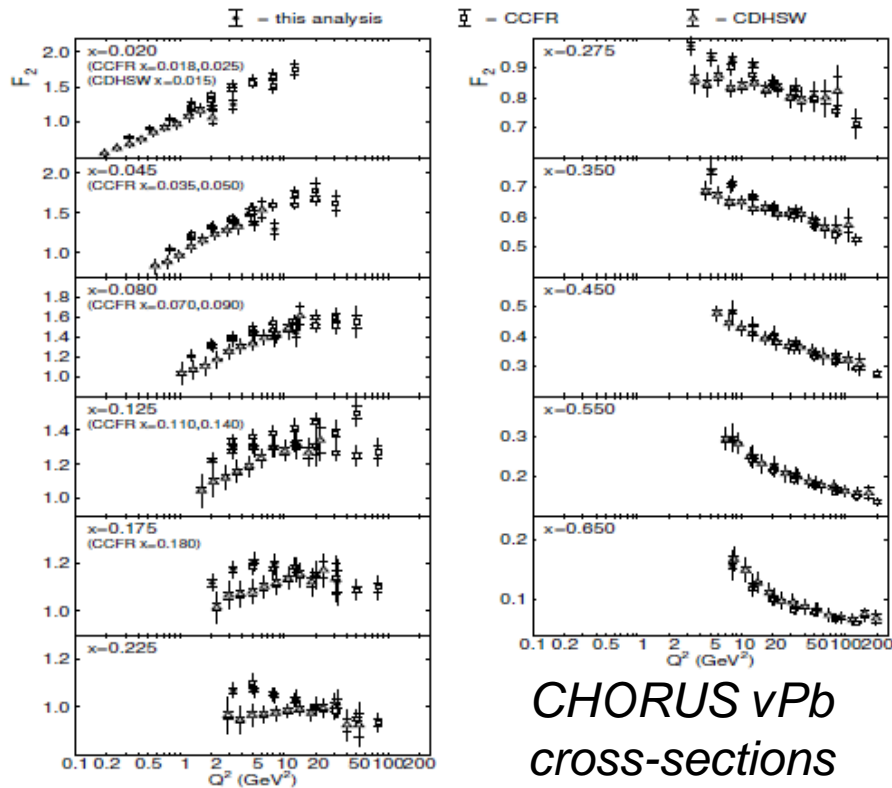
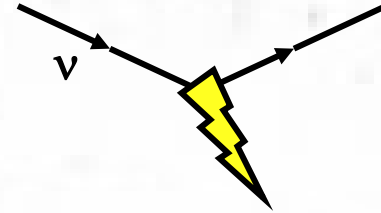
*Phys.Rev.D74:012008,2006*

- NuTeV has a very large data sample on iron
  - High energies, precision calibration from testbeam
- Uses:
  - pQCD fits for  $\Lambda_{\text{QCD}}$
  - Extract structure functions for comparisons with other experiments

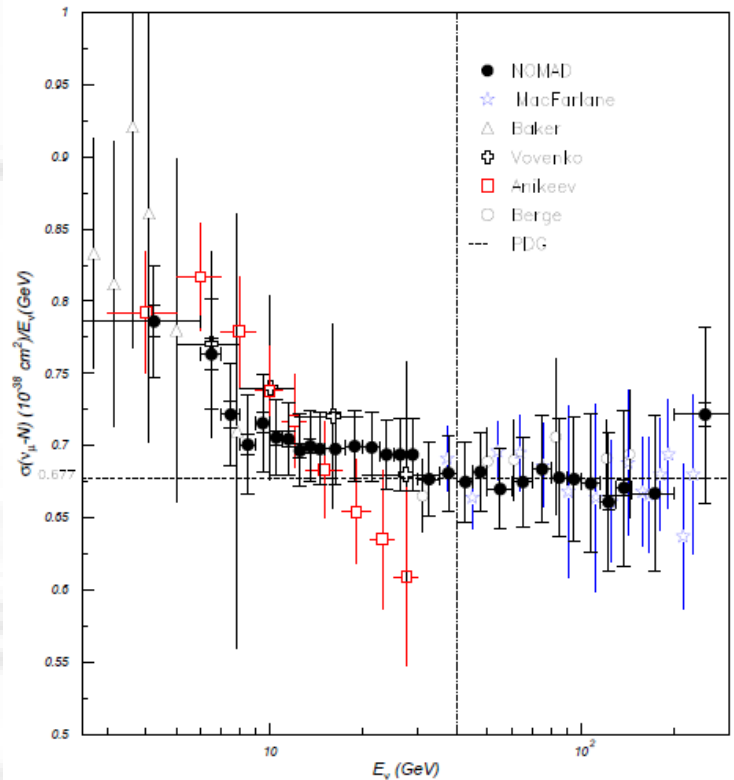




# CHORUS and NOMAD

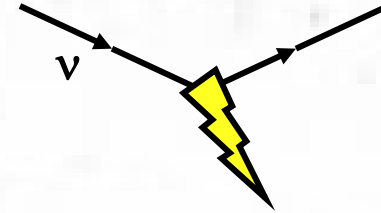


*Phys.Lett..632(2006) 65*

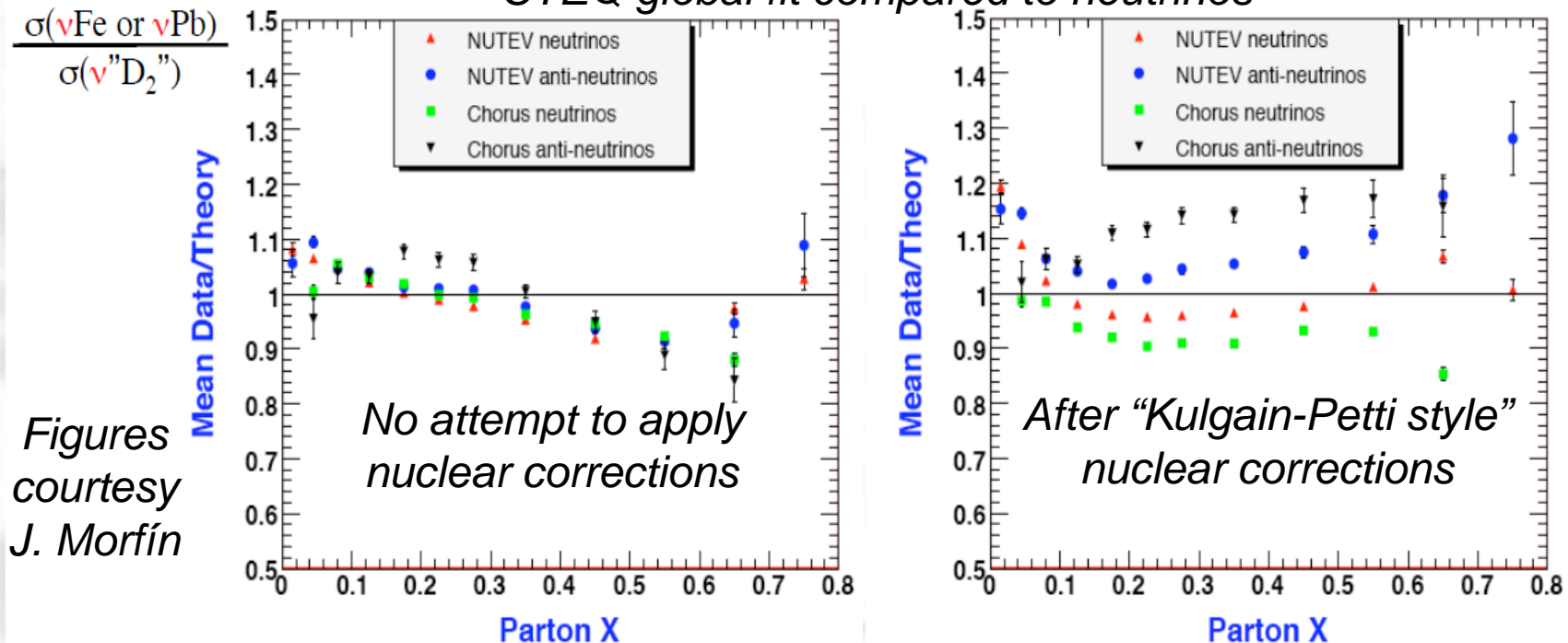


*Phys.Lett.B660:19-25,2008*

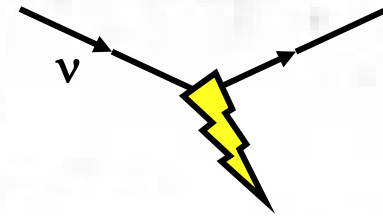
# Nuclear Corrections and High- $x$ PDFs



CTEQ global fit compared to neutrinos

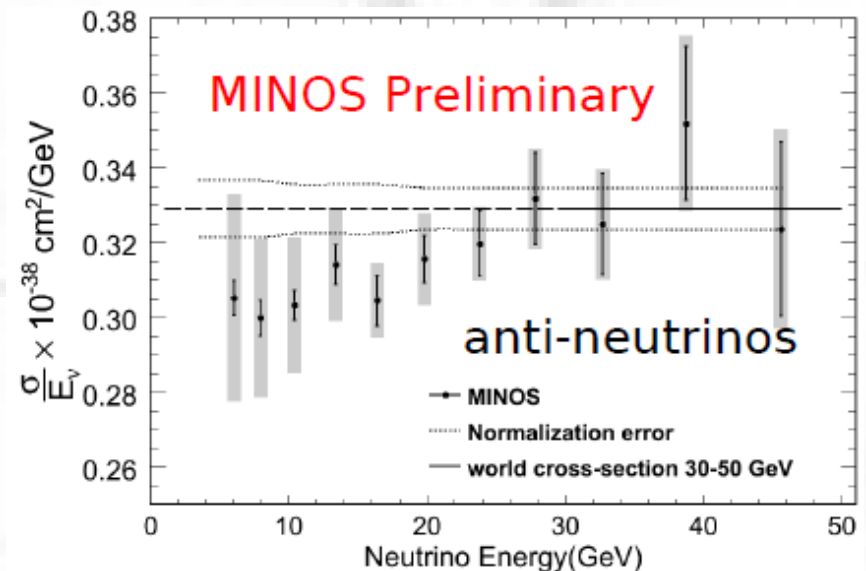
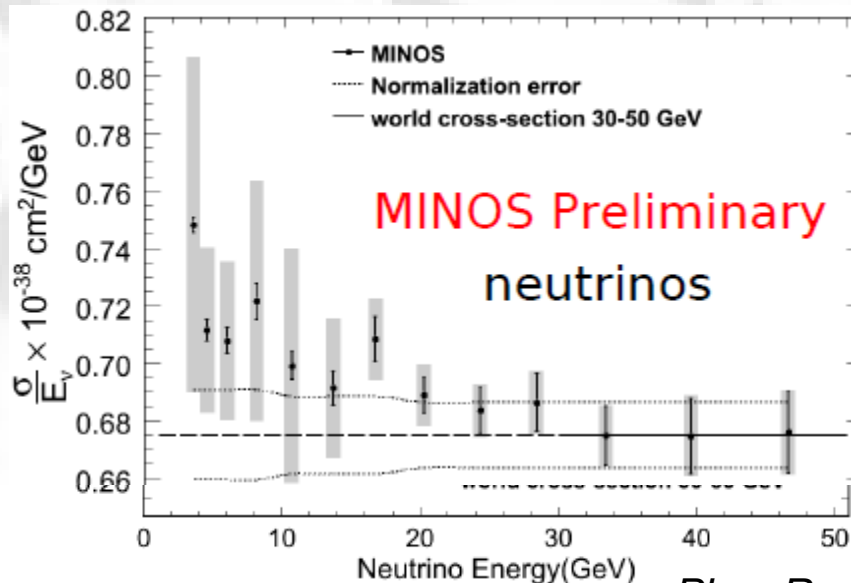


- There are two confusing aspect of these comparisons
  - We observed problems before in nuclear corrections from models
  - Also, some strange behavior at high  $x$ ... difficult to incorporate both data sets in one model

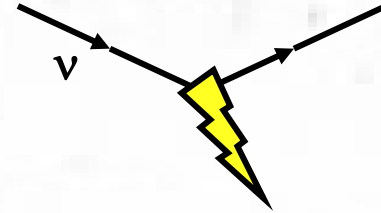


# MINOS Total Cross-Section

- Attempt to bravely extend low recoil technique to very low energies
  - “Low recoil” sample is visible hadronic energy below 1 GeV, so a fair fraction of the cross-section at the lowest energy (3 GeV)

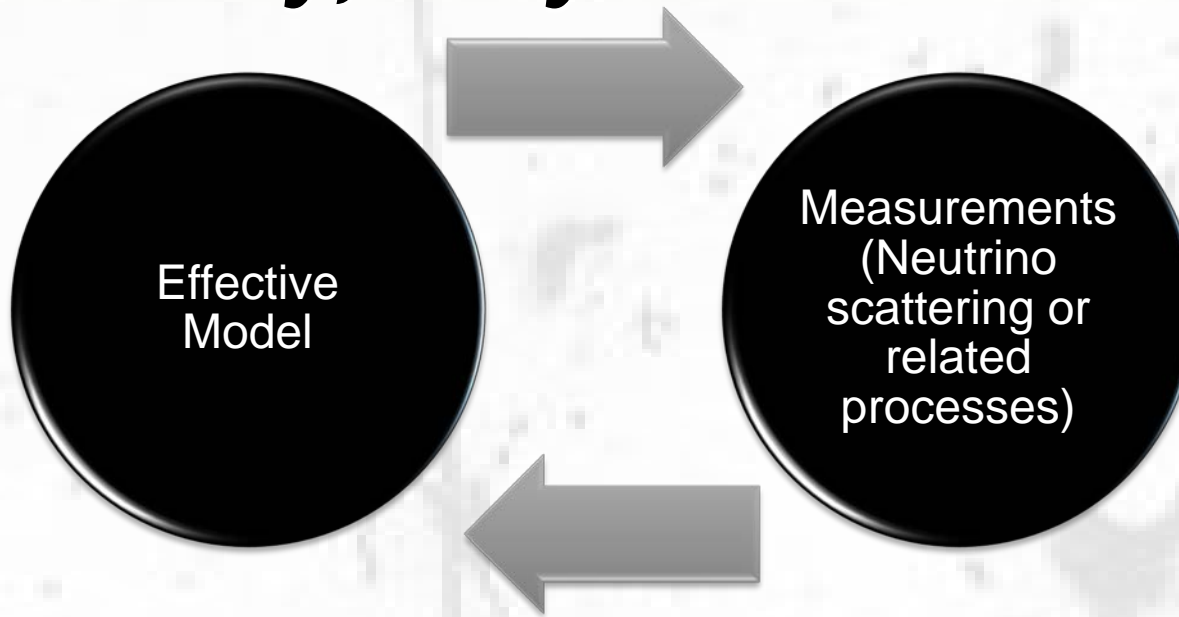
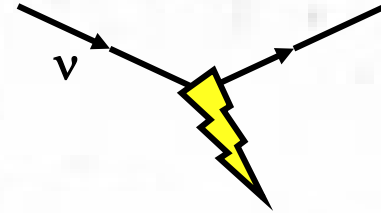


*Phys.Rev.D81:072002,2010*



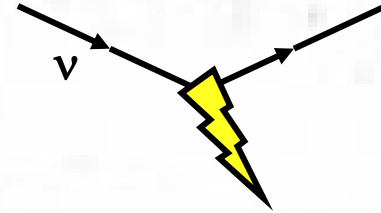
# ***Thoughts on Effective Models and Neutrino Interaction Generators***

# *The Problem of the Nucleus is Very, Very Hard*



- Our iterative process uses data to improve models
- Our models are effective theories, ranging from pure parameterizations of data to microphysical models with simplifying assumptions.
  - “Effective” has both positive and negative meanings, but in particular here I mean that these are not first-principles calculations from QCD.

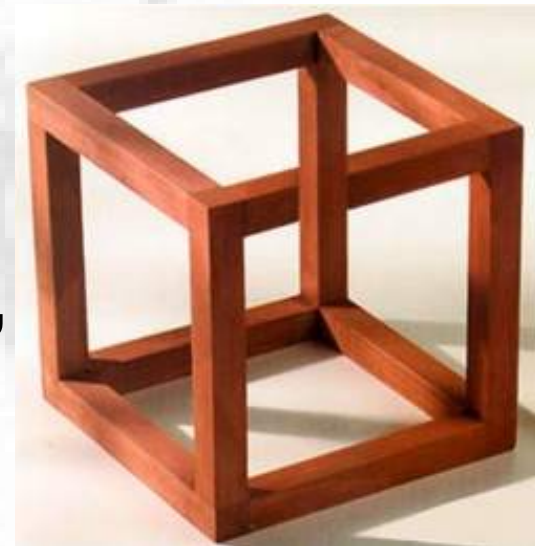
# ***The Mosel Paradox***



We don't have models which fit (all) the available data, although many models provide valuable insight into features of this data

Theorist's paradigm: "A good generator does not have to fit the data, provided [its model] is right"

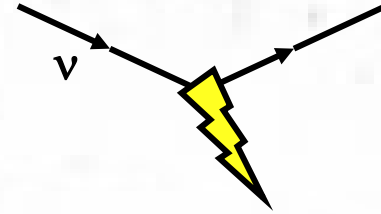
Experimentalist's paradigm: "A good generator does not have to be right, provided it fits the data"



*Ulrich Mosel, first articulated at NuINT11 conference*



# ***Feynman Weighs In...***



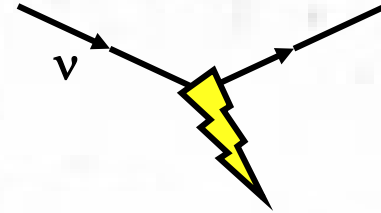
*“It doesn't matter how beautiful your theory is; it doesn't matter how smart you are.*

*If it doesn't agree with experiment, it's wrong.”*

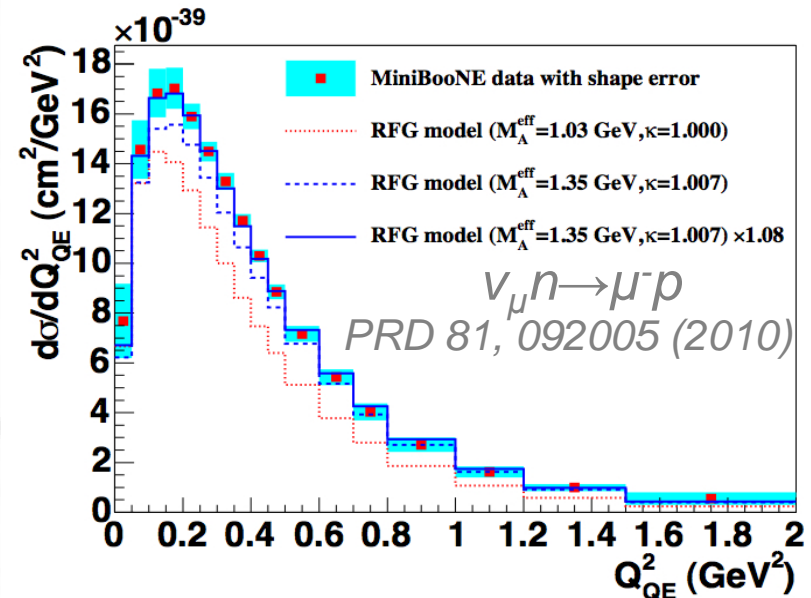
*— Richard Feynman*

This is surely true, but  
invalidating one side of an  
argument doesn't make  
the other side correct!

# Counter Argument

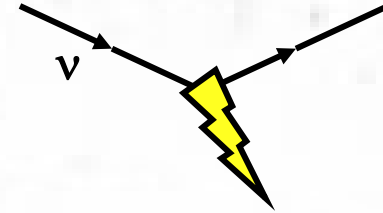


- *Experimentalists can do (and have done, and will do) shameful things when confronted with data and model disagreements!*
- MiniBooNE oscillation analysis approach:
  - Modify the dipole axial mass and Pauli blocking until model fits data.
  - But there is nothing fundamental backing this approach. It's a mechanical convenience to parameterize the data for the oscillation analysis.





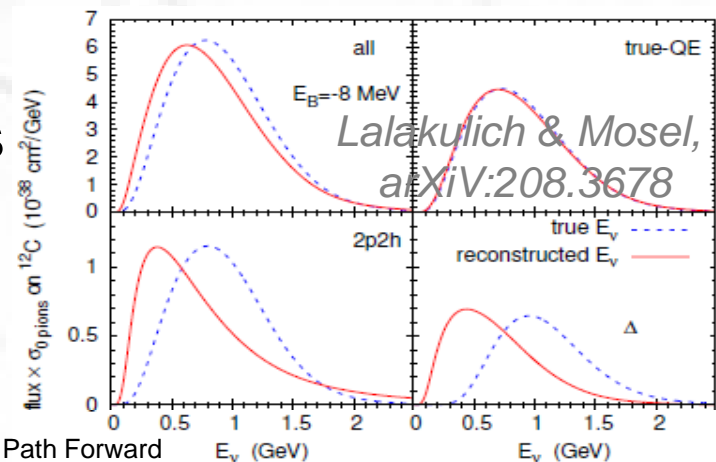
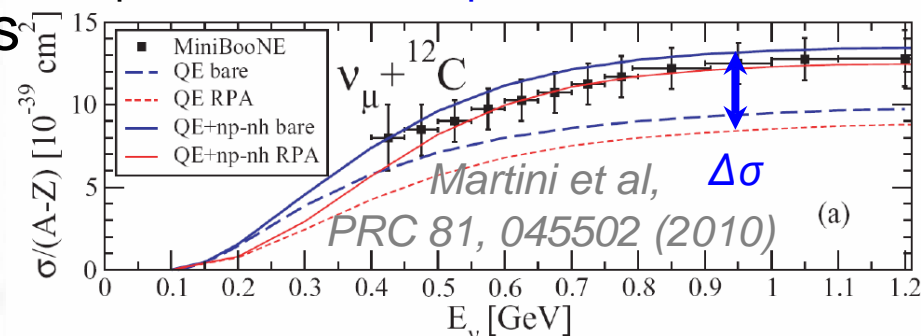
# Counter Argument (cont'd)



- What we now believe about the MiniBooNE oscillation analysis approach:

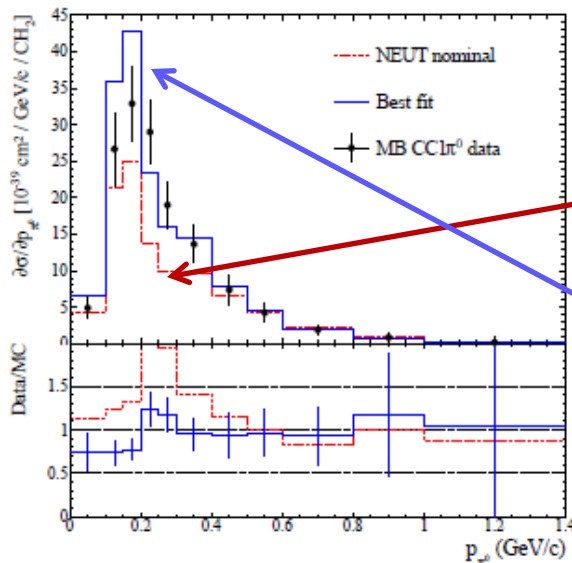
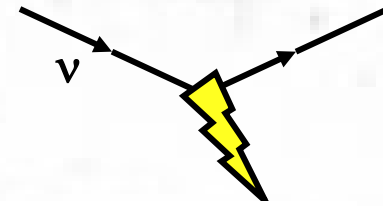
- In a simplistic view, there are neglected contributions from multi-nucleon pairs.
- Those pairs alter the kinematics.
- MiniBooNE got its energy reconstruction wrong by picking the wrong physics to modify.
- OK within uncertainties? If so, only by luck.

$$\nu_{\mu} n \rightarrow \mu^{-} p + \nu_{\mu} (np)_{\text{corr.}} \rightarrow \mu^{-} pp$$



Also demonstrated by  
Nieves, Ankowski here  
at NuINT12

# Counter Argument (cont'd)

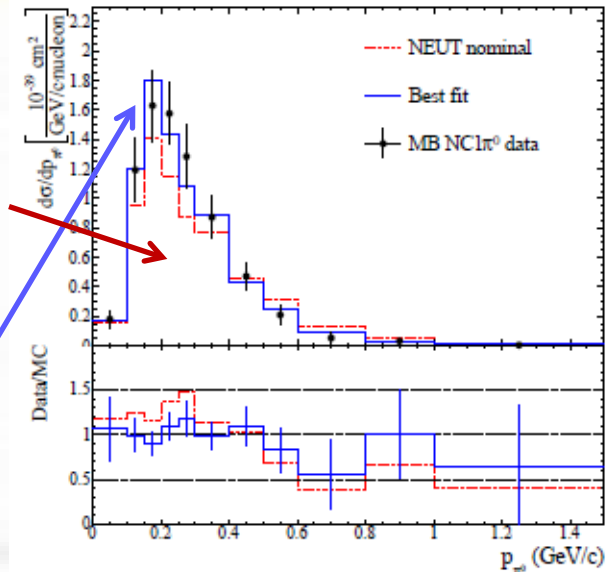


(a) CC1 $\pi^0$  | $p_{\pi^0}$ |

P. Rodrigues, NuFact 2012  
and NuINT12

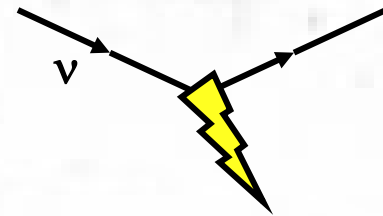
**Rein-Sehgal**  
[Ann. Phys. 133, 79-153 (1981)]  
implementation in NEUT

“Tuned” Rein-Sehgal  
to modify  $Q^2$  distribution,  
pion spectrum, rate



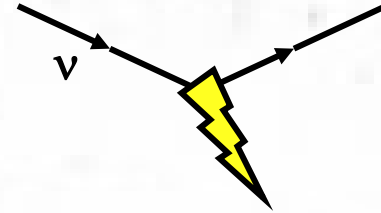
(c) NC1 $\pi^0$  | $p_{\pi^0}$ |

- *But what else can experimentalists do? **Mea culpa.***
- T2K finds poor agreement between Rein-Sehgal and MiniBooNE  $\nu_\mu N \rightarrow \mu^- \pi^{(+)\pi^0} N^{(\prime)}$  and  $\nu_\mu N \rightarrow \nu_\mu \pi^0 N$  data.
- *Ad hoc* tuning “breaks” assumptions of underlying model, e.g. CC-NC universality of process and relation among resonances, to force good agreement.

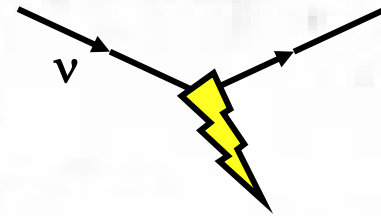


# ***Conclusions***

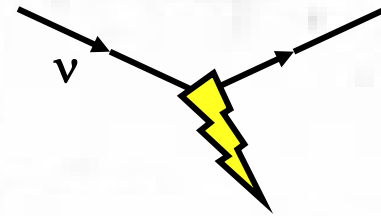
# ***What Should I Remember from These Lectures?***



- Understanding neutrino interactions is necessary for precision measurements of neutrino oscillations
- Point like scattering: weak interactions couple differently to each chirality of fermions, neutrino scattering rate proportional to energy (until real boson exchange)
- Target (proton, nucleus) structure is a significant complication to theoretical prediction of cross-section
  - Particularly problematic near inelastic thresholds
- Our best models are incomplete, and even those best models often aren't the ones in generators
- Resolving differences between data and models is a major conceptual challenge

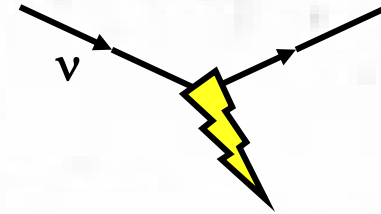


# ***Supplemental Slides***

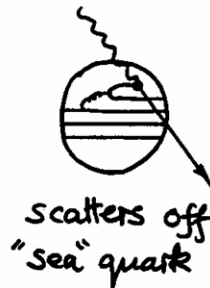
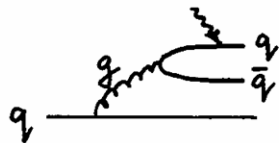


# ***SUPPLEMENT: Scaling Violations***

# Strong Interactions among Partons

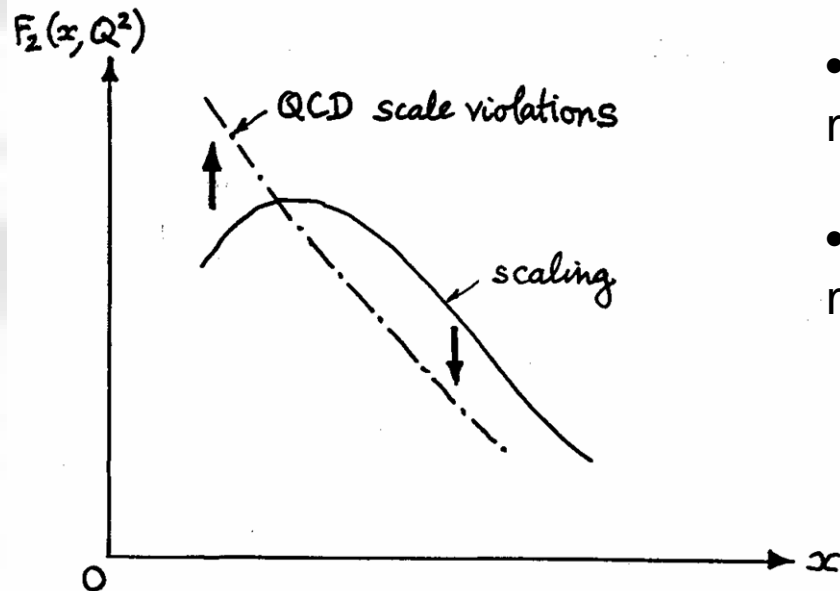


$Q^2$  Scaling fails due to these interactions



$$\frac{\partial q(x, Q^2)}{\partial \log Q^2} = \frac{\alpha_s(Q^2)}{2\pi} \int_x^1 \frac{dy}{y}$$

$$\left[ P_{qq} \left( \frac{x}{y} \right) q(y, Q^2) + P_{qg} \left( \frac{x}{y} \right) g(y, Q^2) \right]$$



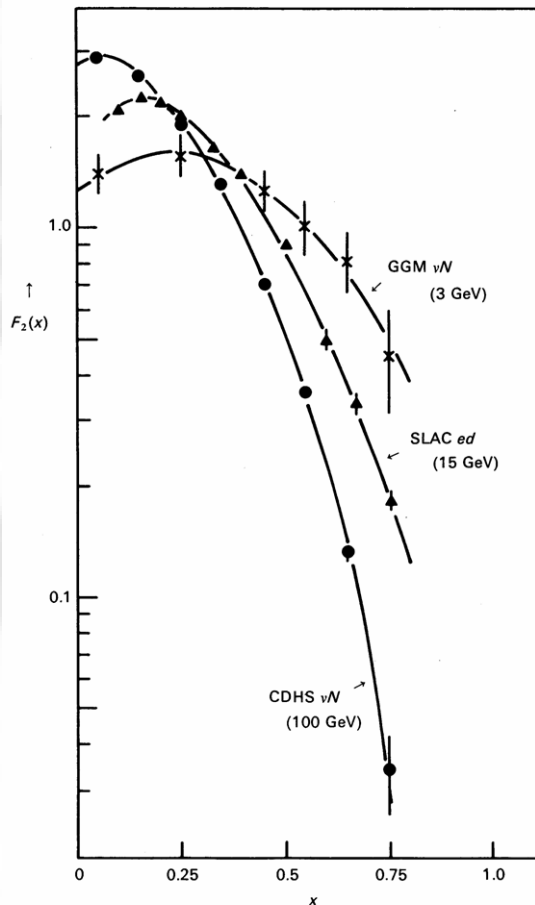
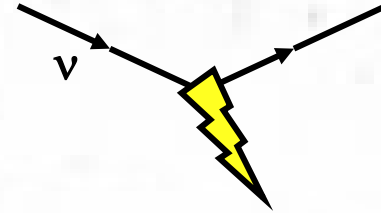
•  $P_{qq}(x/y)$  = probability of finding a quark with momentum  $x$  within a quark with momentum  $y$

•  $P_{qg}(x/y)$  = probability of finding a  $q$  with momentum  $x$  within a gluon with momentum  $y$

$$P_{qq}(z) = \frac{4}{3} \frac{1+z^2}{1-z} + 2\delta(1-z)$$

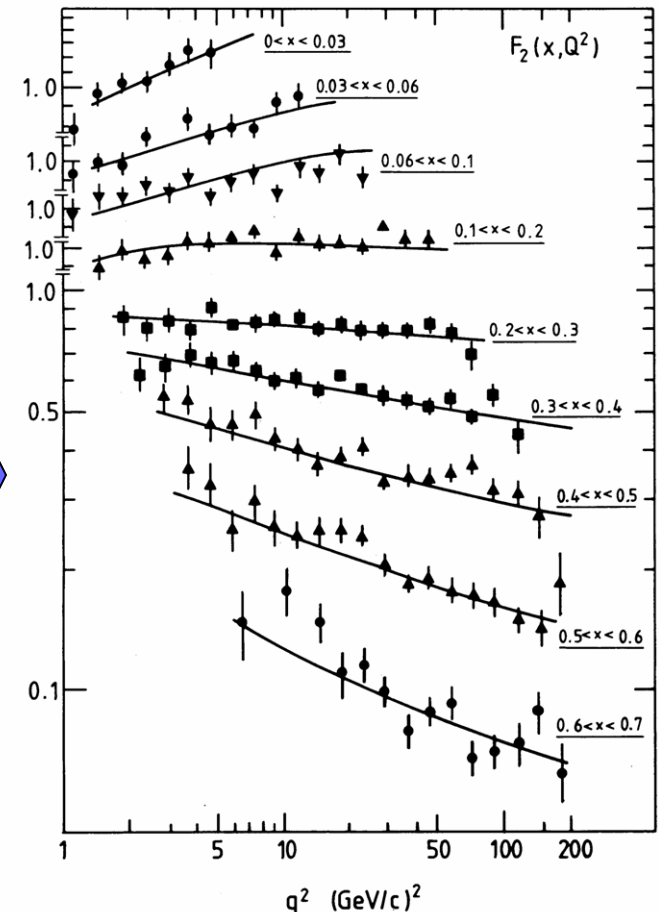
$$P_{gq}(z) = \frac{1}{2} \left[ z^2 + (1-z)^2 \right]$$

# Scaling from QCD

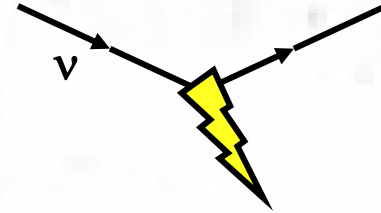


Observed quark distributions vary with  $Q^2$

Scaling well modeled by perturbative QCD with a single free parameter ( $\alpha_s$ )

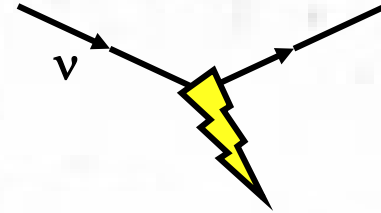




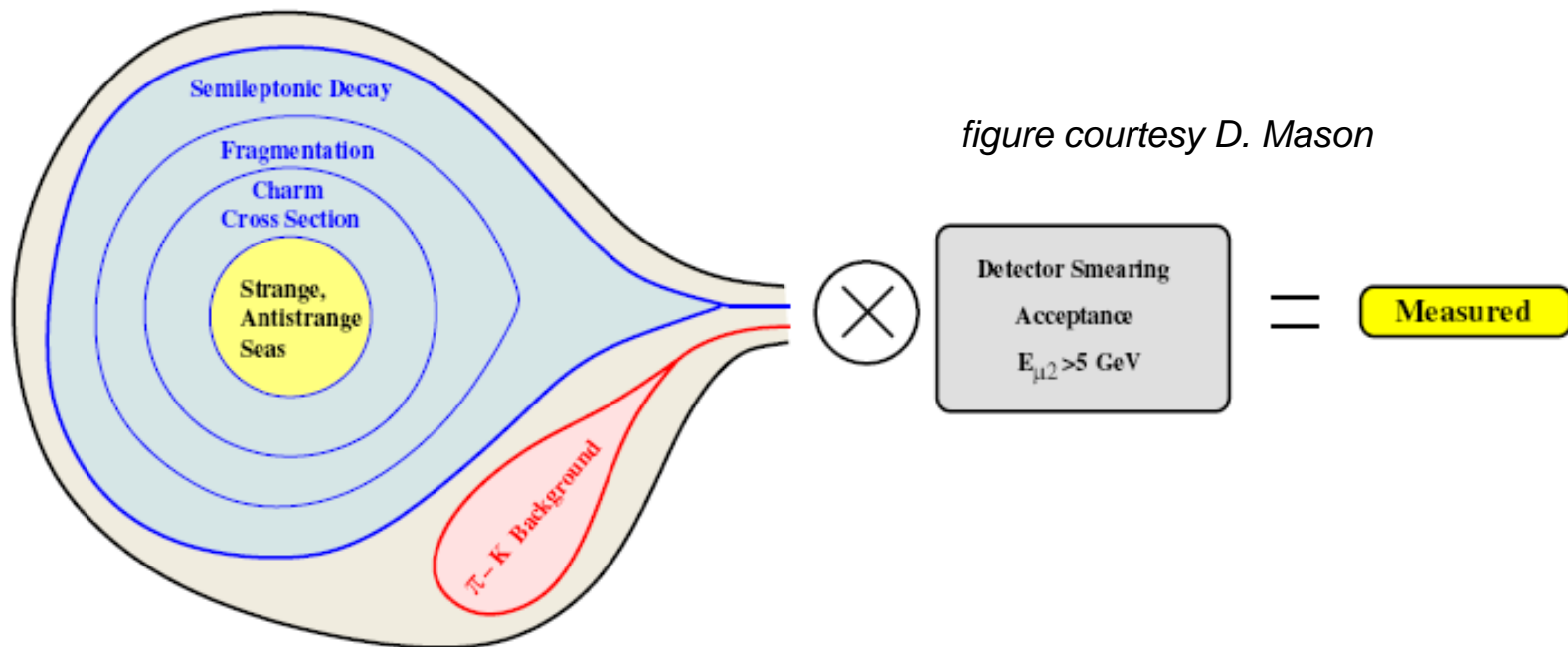


# ***SUPPLEMENT: NuTeV Measurement of Strange Sea***

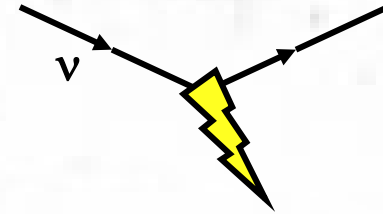
# Neutrino Dilepton Events



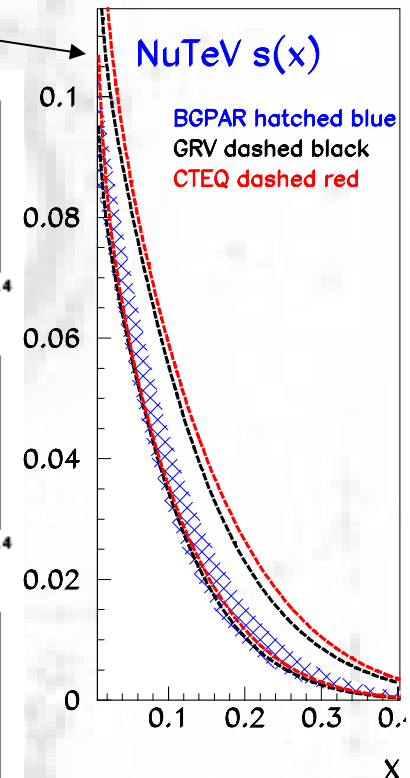
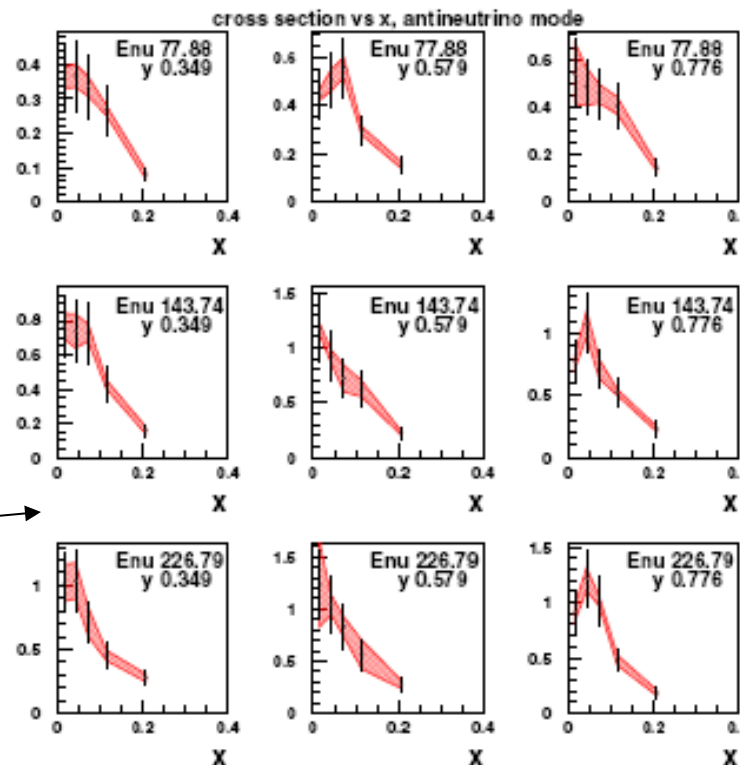
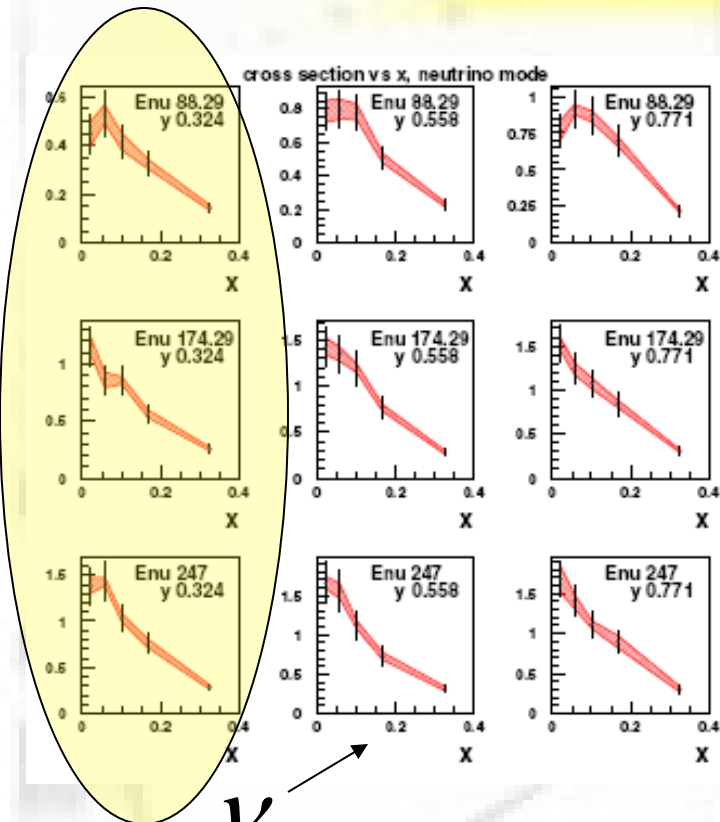
- Rate depends on:
  - d, s quark distributions,  $|V_{cd}|$
  - Semi-leptonic branching ratios of charm
  - Kinematic suppression and fragmentation



# NuTeV Dimuon Sample

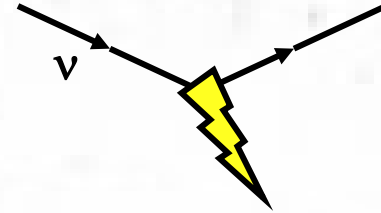


- Lots of data!
- Separate data in energy, x and y (inelasticity)
  - Energy important for charm threshold,  $m_c$
  - x important for  $s(x)$

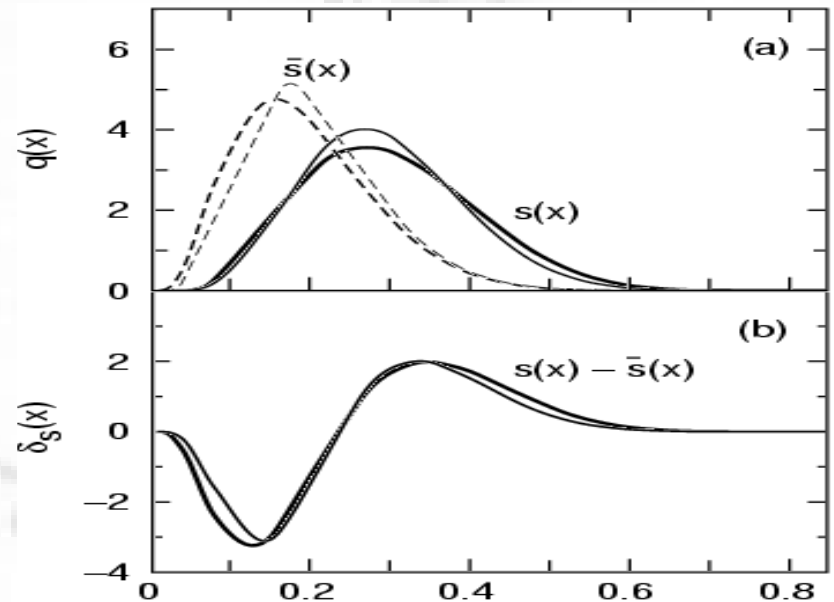
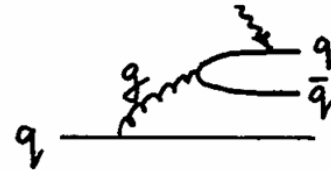


$$\frac{\pi \times \frac{d^2 \sigma(\nu N \rightarrow \mu \mu X)}{dx dy}}{G_F^2 M_N E_\nu}$$

# QCD at Work: Strange Asymmetry?

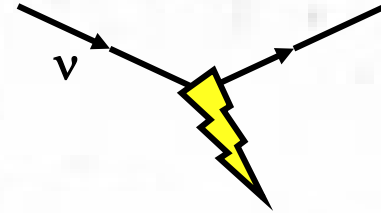


- An interesting aside...
  - The strange sea can be generated perturbatively from  $g \rightarrow s + \bar{s}$ .
  - BUT, in perturbative generation the momenta of strange and anti-strange quarks is equal
    - o well, in the leading order splitting at least. At higher order get a vanishingly small difference.
  - SO  $s$  &  $\bar{s}$  difference probe non-perturbative (“intrinsic”) strangeness
    - o Models: Signal&Thomas, Brodsky&Ma, etc.



(Brodsky & Ma,  $s$ - $\bar{s}$ )

# NuTeV's Strange Sea



- NuTeV has tested this
  - NB: very dependent on what is assumed about non-strange sea
  - Why? Recall CKM mixing...

$$V_{cd} d(x) + V_{cs} s(x) \rightarrow s'(x)$$

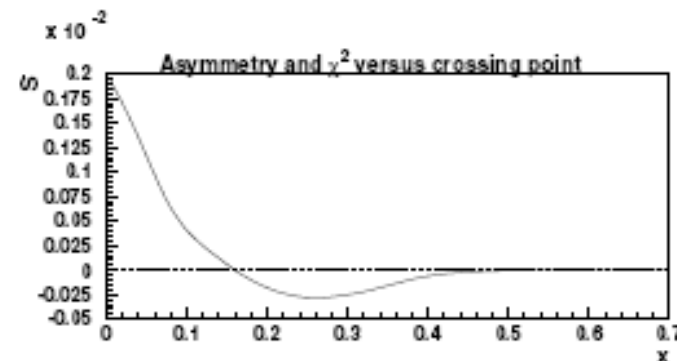
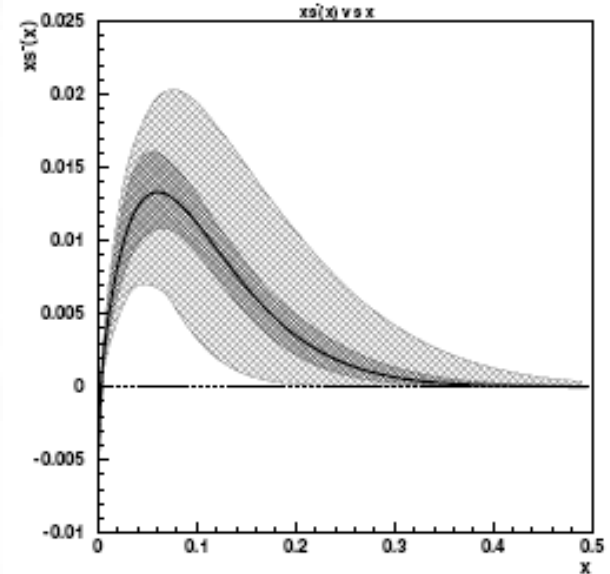
$$V_{cd} \bar{d}(x) + V_{cs} \bar{s}(x) \rightarrow \bar{s}'(x)$$

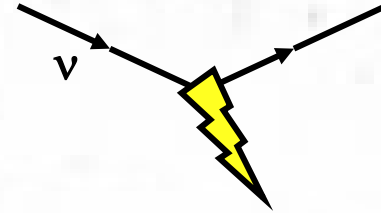
*small*      *big*

- Using CTEQ6 PDFs...

$$\int dx \left[ x(s - \bar{s}) \right] = 0.0019 \pm 0.0005 \pm 0.0014$$

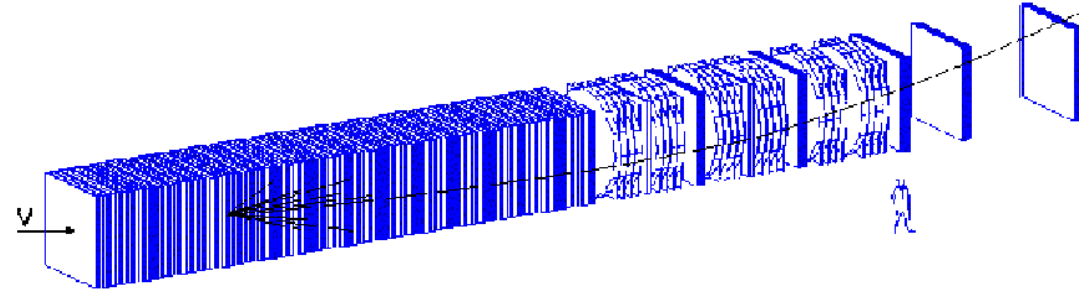
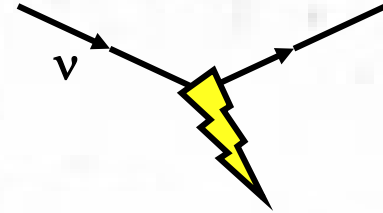
$$\text{c.f., } \int dx \left[ x(s + \bar{s}) \right] \approx 0.02$$



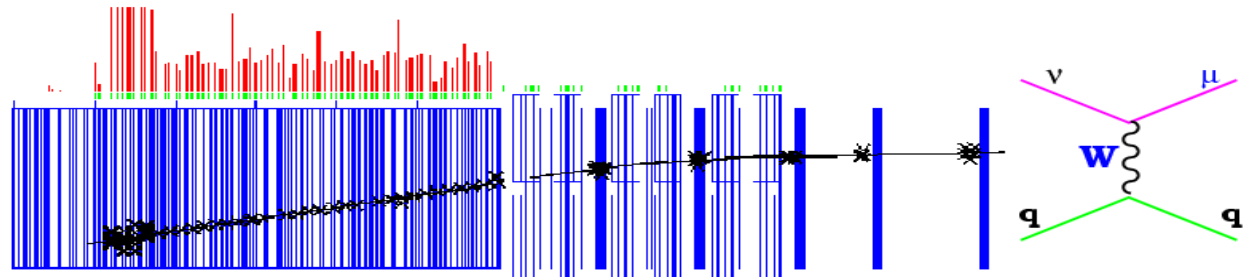


# ***SUPPLEMENT: NuTeV $\sin^2\theta_w$***

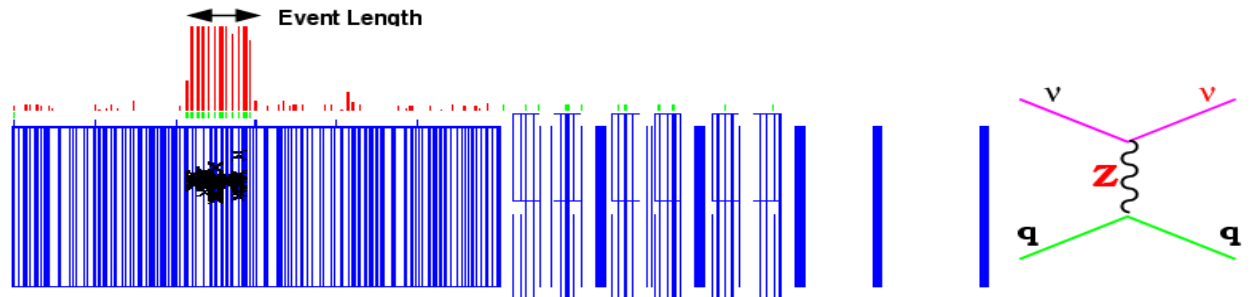
# NuTeV at Work...



← Event Length →

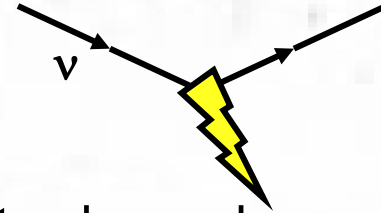


← Event Length →

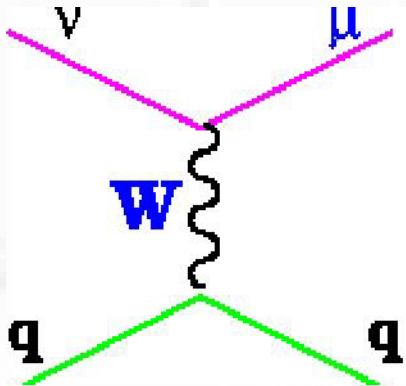




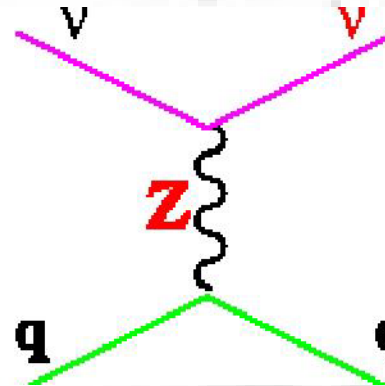
# DIS NC/CC Ratio



- Experimentally, it's "simple" to measure ratios of neutral to charged current cross-sections on an isoscalar target to extract NC couplings



W-q coupling is  $I_3$



Z-q coupling is  $I_3 - Q \sin^2 \theta_W$

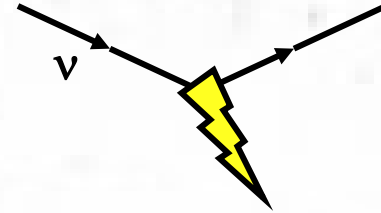
## Llewellyn Smith Formulae

$$R^{\nu(\bar{\nu})} = \frac{\sigma_{NC}^{\nu(\bar{\nu})}}{\sigma_{CC}^{\nu(\bar{\nu})}} = \left( (u_L^2 + d_L^2) + \frac{\sigma_{CC}^{\bar{\nu}(\nu)}}{\sigma_{CC}^{\nu(\bar{\nu})}} (u_R^2 + d_R^2) \right)$$

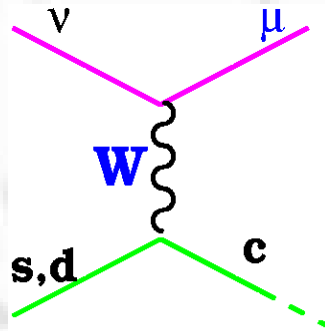
- Holds for isoscalar targets of u and d quarks only
  - Heavy quarks, differences between u and d distributions are corrections
- Isospin symmetry causes PDFs to drop out, even outside of naïve quark-parton model



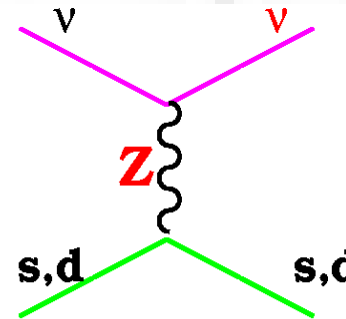
# Lecture Question #6: Paschos-Wolfenstein Relation



Charged-Current

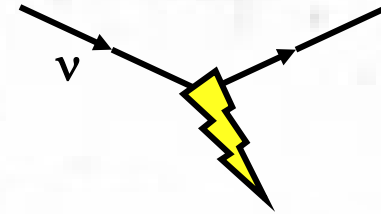


Neutral-Current

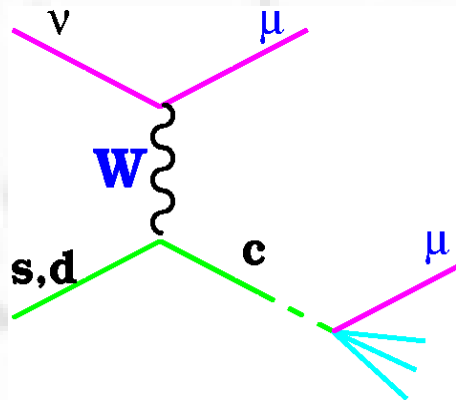


- If we want to measure electroweak parameters from the ratio of charged to neutral current cross-sections, what problem will we encounter from these processes?

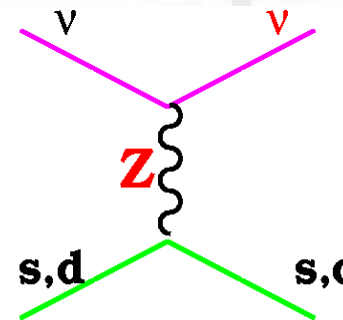
# Lecture Question #6: Paschos-Wolfenstein Relation



Charged-Current

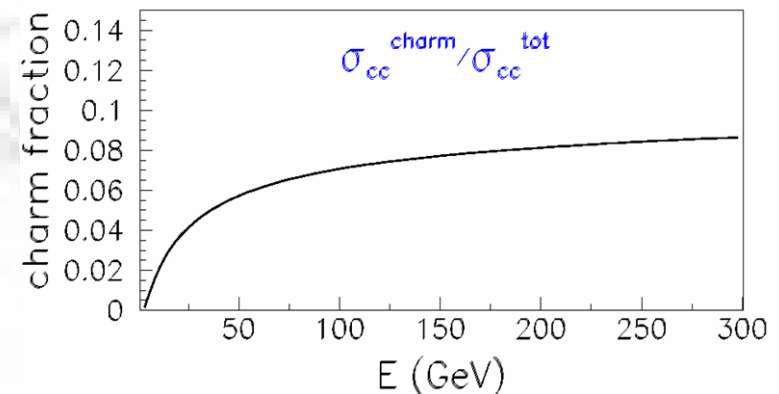


Neutral-Current



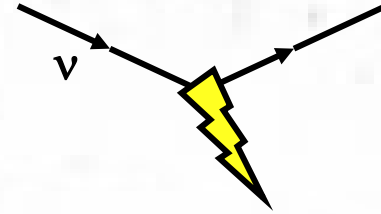
- CC is suppressed due to final state charm quark  
 $\Rightarrow$  Need strange sea and  $m_c$ 
  - Remember heavy quark mass effect:

$$x \rightarrow \xi = x \left( 1 + \frac{m_c^2}{Q^2} \right)$$



# Lecture Question #6:

## Paschos-Wolfenstein Relation



- The NuTeV experiment employed a complicated design to measure

Paschos - Wolfenstein Relation

$$R^- = \frac{\sigma_{NC}^{\nu} - \sigma_{NC}^{\bar{\nu}}}{\sigma_{CC}^{\nu} - \sigma_{CC}^{\bar{\nu}}} = \rho^2 \left( \frac{1}{2} - \sin^2 \theta_W \right)$$

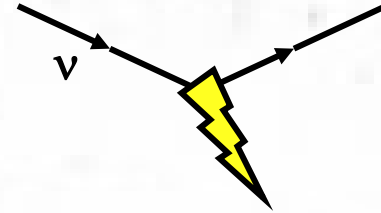
- How did this help with the heavy quark problem of the previous question?

*Hint: what to you know about the relationship of:*

$$\sigma(\nu q) \text{ and } \sigma(\bar{\nu} \bar{q})$$

# Lecture Question #6:

## Paschos-Wolfenstein Relation



- The NuTeV experiment employed a complicated design to measure

Paschos - Wolfenstein Relation

$$R^- = \frac{\sigma_{NC}^{\nu} - \sigma_{NC}^{\bar{\nu}}}{\sigma_{CC}^{\nu} - \sigma_{CC}^{\bar{\nu}}} = \rho^2 \left( \frac{1}{2} - \sin^2 \theta_W \right)$$

- How did this help with the heavy quark problem of the previous question?

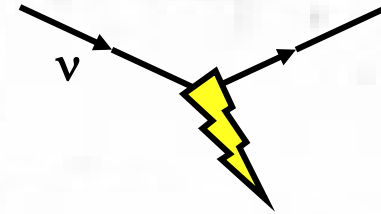
$$\sigma(\nu q) = \sigma(\bar{\nu} \bar{q})$$

$$\sigma(\nu \bar{q}) = \sigma(\bar{\nu} q)$$

$$\therefore \sigma(\nu q) - \sigma(\bar{\nu} \bar{q}) = 0$$

*So any quark-antiquark symmetric part is not in difference, e.g., strange sea.*

# NuTeV Fit to $R^\nu$ and $R^{\nu\text{bar}}$



- NuTeV result:

$$\sin^2 \theta_W^{(on-shell)} = 0.2277 \pm \pm 0.0013(stat.) \pm 0.0009(syst.)$$

$$= 0.2277 \pm 0.0016$$

(Previous neutrino measurements gave  $0.2277 \pm 0.0036$ )

- Standard model fit (LEPEWWG):  $0.2227 \pm 0.00037$

**A  $3\sigma$  discrepancy...**

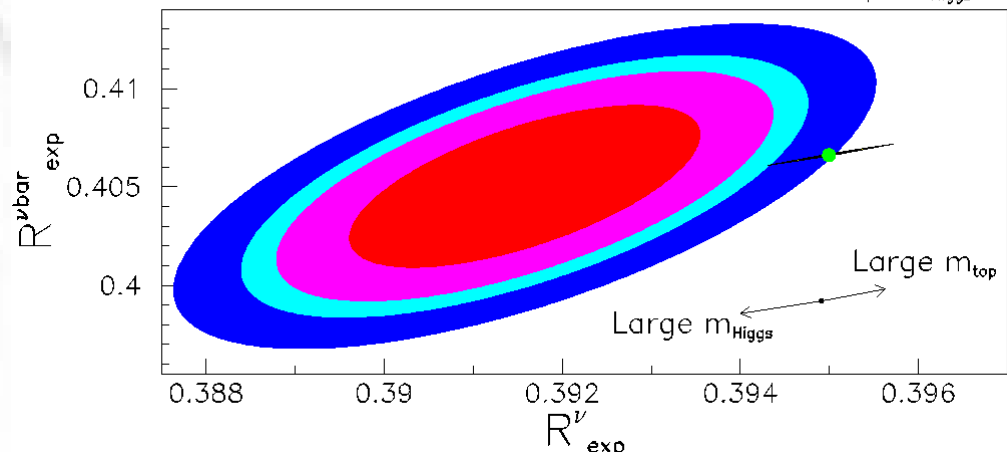
$$R_{\text{exp}}^\nu = 0.3916 \pm 0.0013$$

(SM : 0.3950)  $\Leftarrow 3\sigma$  difference

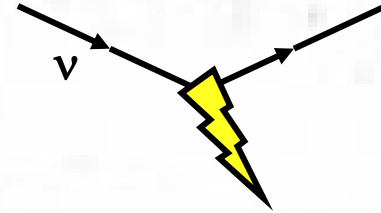
$$R_{\text{exp}}^{\nu\text{bar}} = 0.4050 \pm 0.0027$$

(SM : 0.4066)  $\Leftarrow$  Good agreement

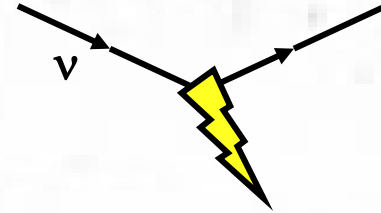
68%,90%,95%,99% C.L. Contours, Grid of SM  $\pm 1\sigma$   $m_{\text{top}}$ ,  $m_{\text{Higgs}}$



# ***NuTeV Electroweak: What does it Mean?***

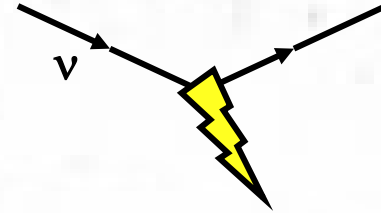


- If I knew, I'd tell you.
- It could be BSM physics. Certainly there is no exclusive of a  $Z'$  that could cause this. But why?
- It could be the asymmetry of the strange sea...
  - it would contribute because the strange sea would not cancel in
  - but it's been measured; not anywhere near big enough
- It could be very large isospin violation
  - if  $d_p(x) \neq u_n(x)$  at the 5% level... it would shift charge current (normalizing) cross-sections enough.
  - no data to forbid it. any reason to expect it?

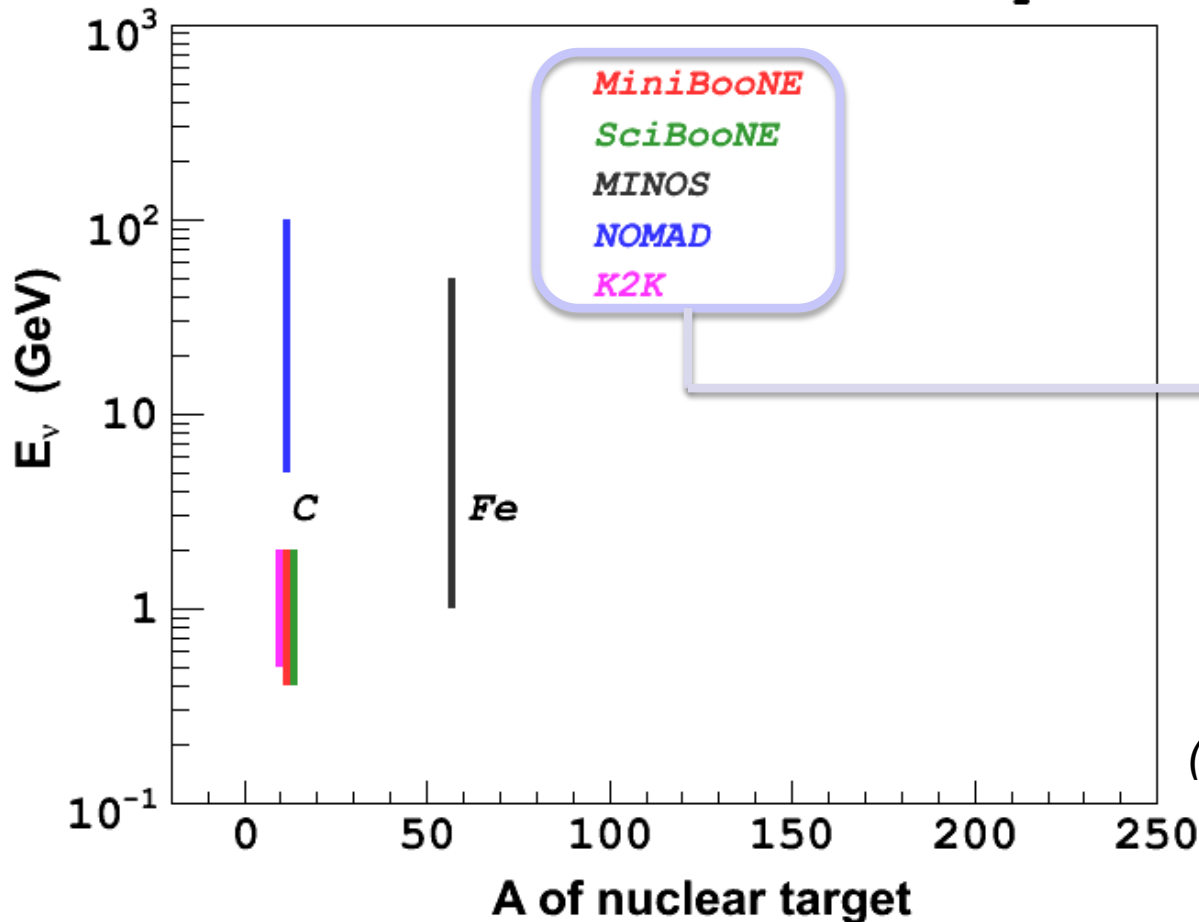


# ***SUPPLEMENT: Experiments to Measure GeV Cross-Sections***

# Energies and Targets of Cross-Section Measurements



Modern Neutrino Cross-Section Experiments

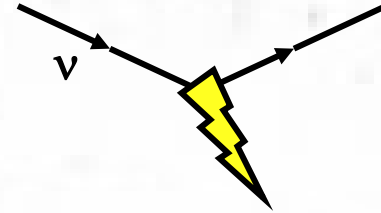


recent results and/or  
currently analyzing and  
publishing new cross-  
section data

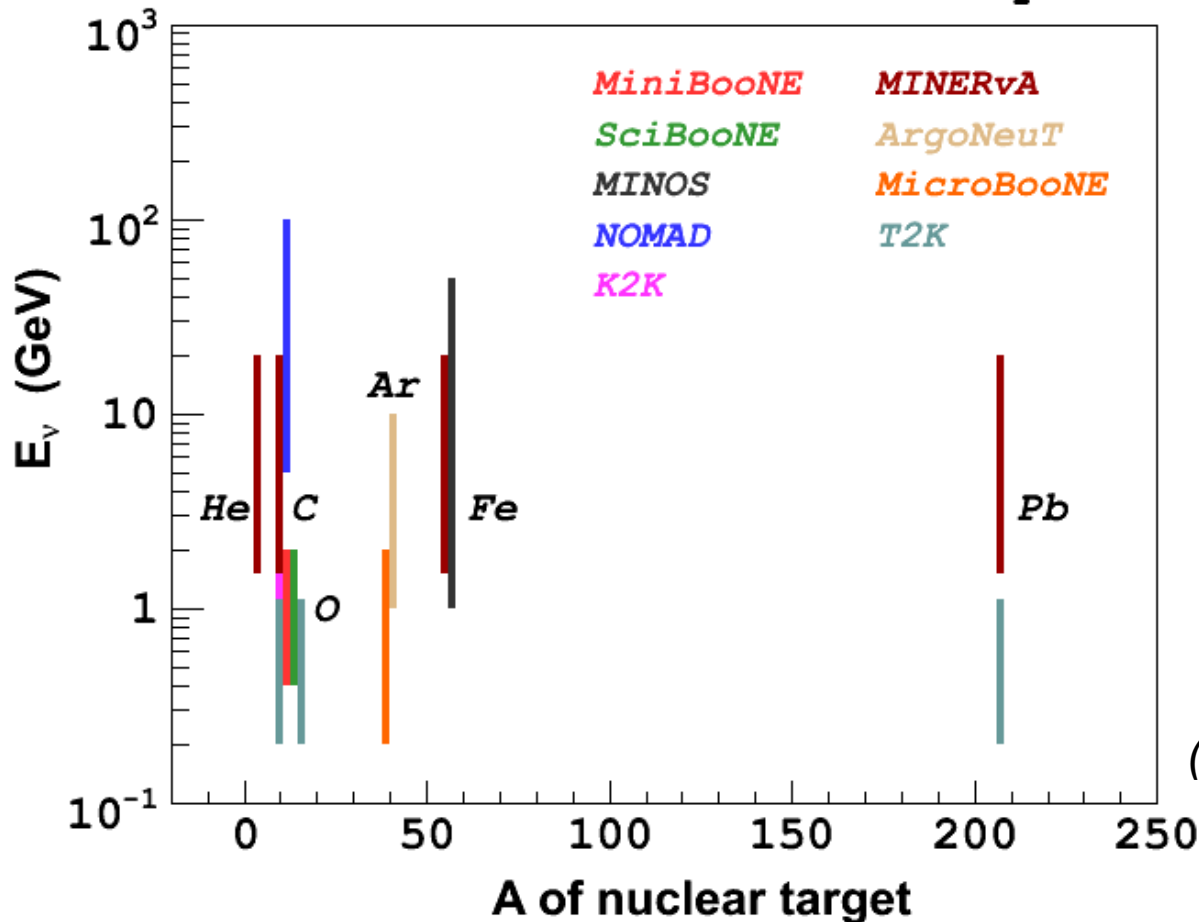
(Compilation from D. Schmitz)



# Energies and Targets of Cross-Section Measurements

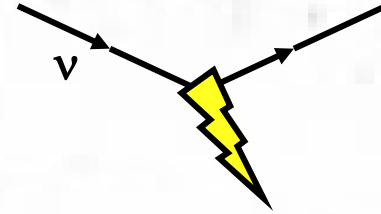


Modern Neutrino Cross-Section Experiments



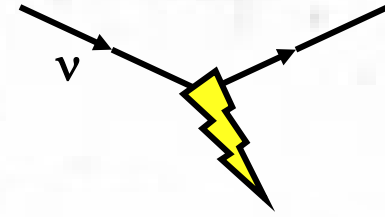
(Compilation from D. Schmitz)

# ***Technologies of “Old” Experiments***



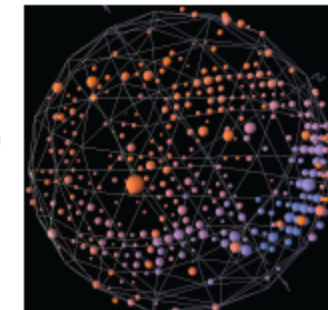
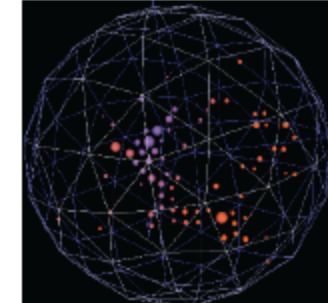
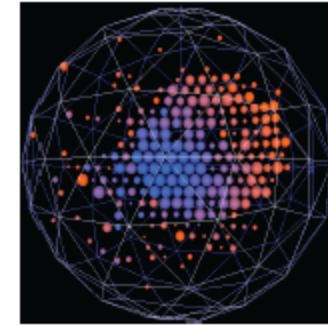
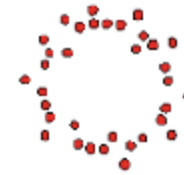
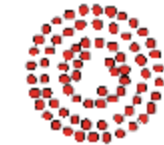
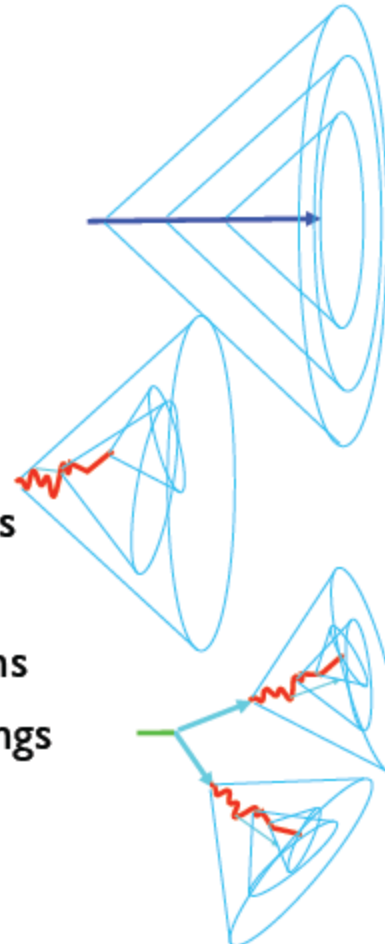
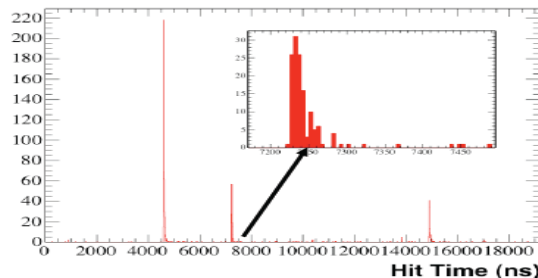
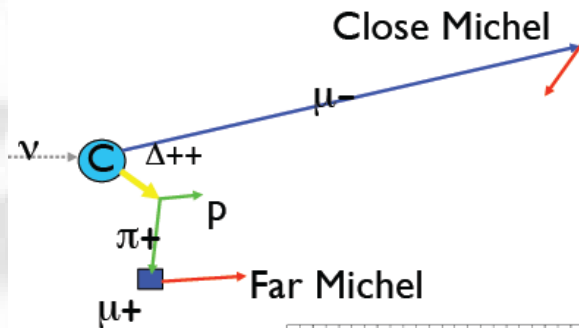
- BooNE and K2K: both have Cerenkov and Scintillator Bar detectors for measuring neutrino interactions
  - Cerenkov detectors have uniform acceptance, but high thresholds for massive particles
  - Scintillator bar detectors usually have a directional bias, typically smaller and may not contain interaction, but thresholds are lower than Cerenkov and particles can be identified by  $dE/dx$
- NOMAD: drift chambers in an analyzing magnet
  - Good momentum measurement and possibly better particle identification by  $dE/dx$ , but diffuse material makes photon reconstruction difficult
- MINOS: coarse sampling iron detector
  - Difficult to distinguish particles other than muons, but very high rate

# Technologies: Cerenkov Detectors



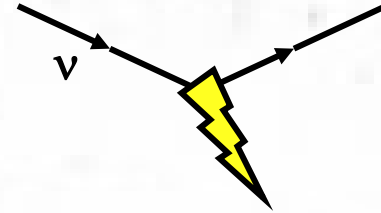
- Cerenkov gives efficient muon or e/γ identification
- Also, tag soft pions by decay

- Muons
  - full rings
- Electrons
  - fuzzy rings
- Neutral pions
  - double rings

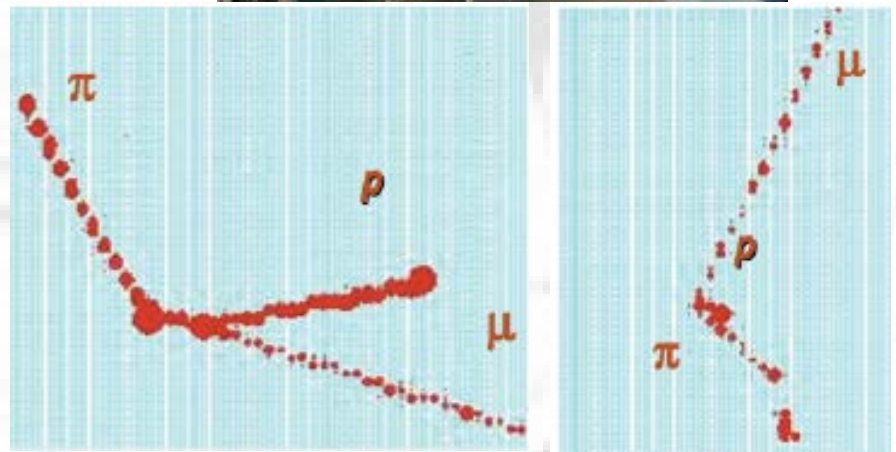
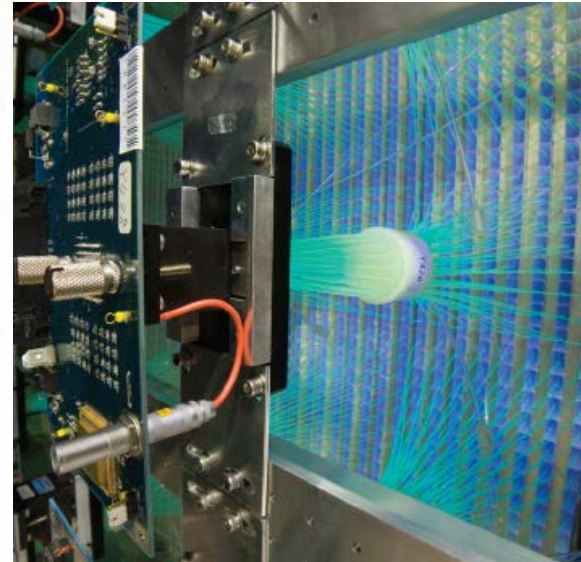


Figures from M. Wascko

# Technologies: Segmented Scintillator



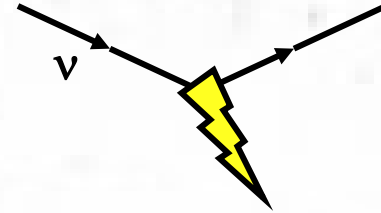
- Lower thresholds, particle ID by  $dE/dx$ , calorimetric energy reconstruction
  - i.e., vertex activity
- But detectors must be smaller (cost), so escaping particles
- Reconstruction not uniform in angle



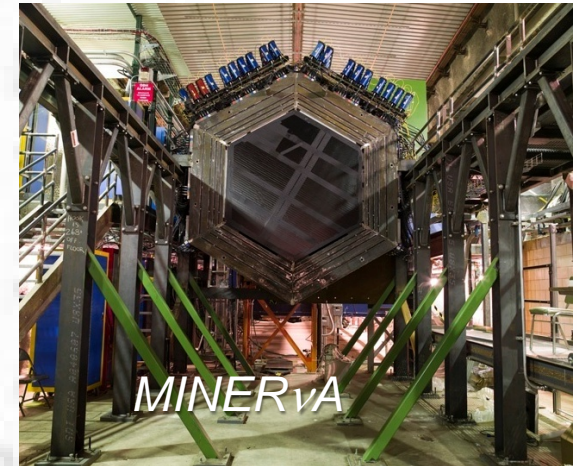
Figures from M. Wascko



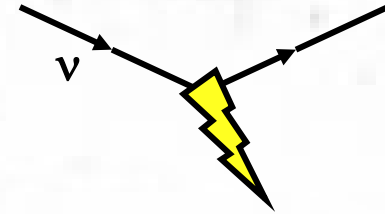
# Current and Near Future Experiments



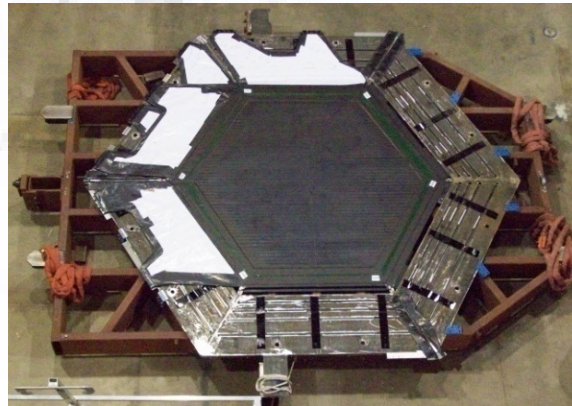
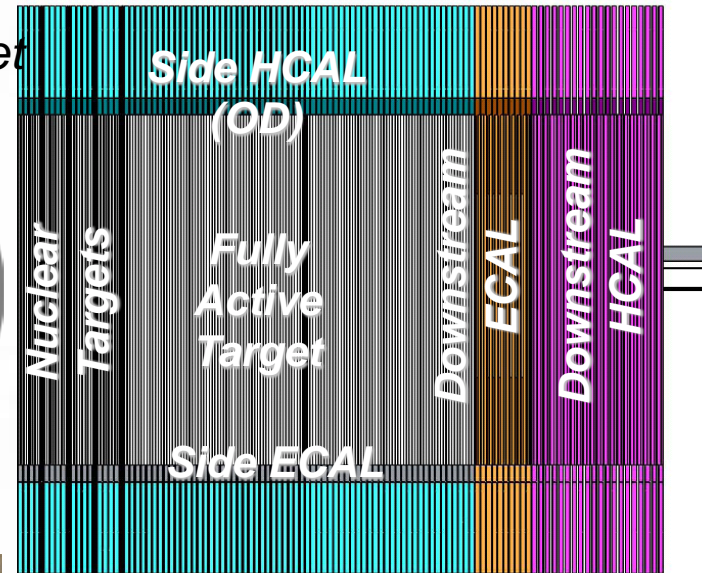
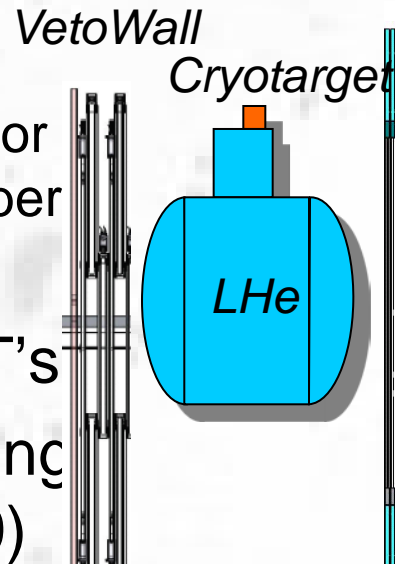
- MINER $\nu$ A: in NuMI at Fermilab
  - Fine-grained scintillator detector
  - Nuclear targets of He, C, H<sub>2</sub>O, Fe, Pb
- T2K 280m Near Detector at J-PARC
  - Fine-grained scintillator, water, and TPC's in a magnetic field
- NO $\nu$ A near detector: to run in 2014
  - Segmented Liquid scintillator in off-axis beam
- MicroBooNE: to run in 2014
  - Liquid Argon TPC in FNAL Booster Beam
  - Some data from ArgoNeuT, a test in NuMI



# MINERvA Detector



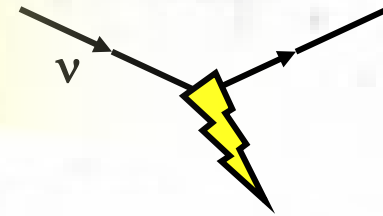
- 120 modules
  - Finely segmented scintillator planes read out by WLS fiber
  - Side calorimetry
- Signals to 64-anode PMT's
- Front End Electronics using Trip-t chips (thanks to D0)
- Side and downstream EM and hadron calorimetry
- MINOS Detector gives muon momentum and charge



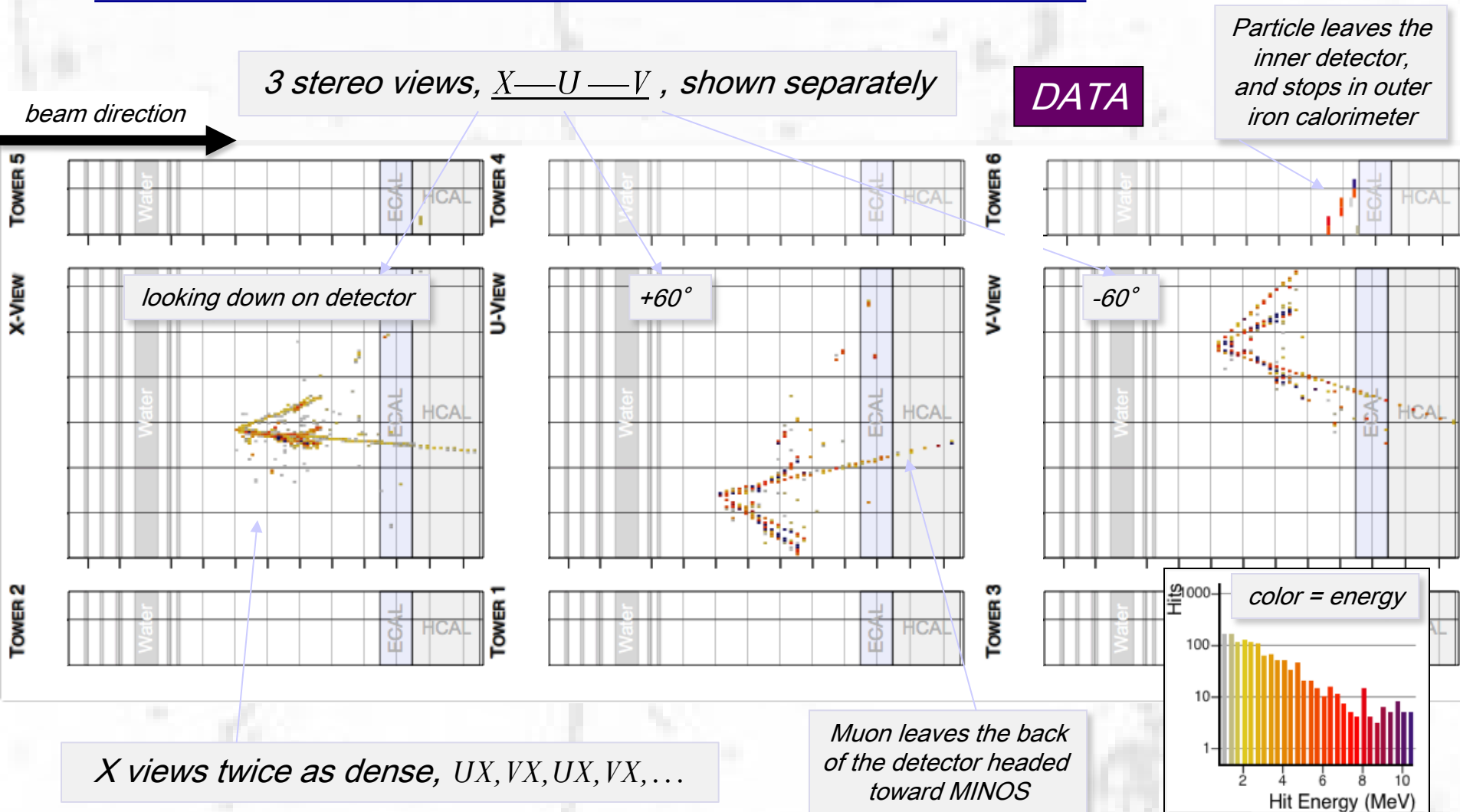
Kevin McFarland: Interactions of Neutrinos

6-8 August 2013

# $\nu$ Events in MINER $\nu$ A

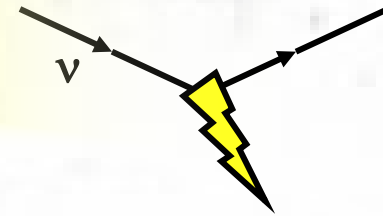


- So what does an event look like in MINER $\nu$ A...

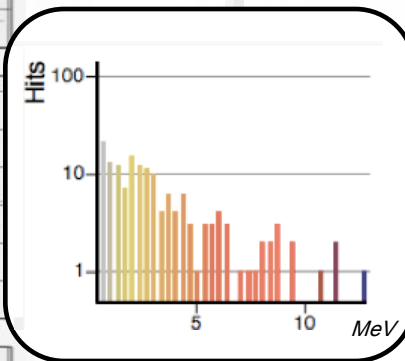
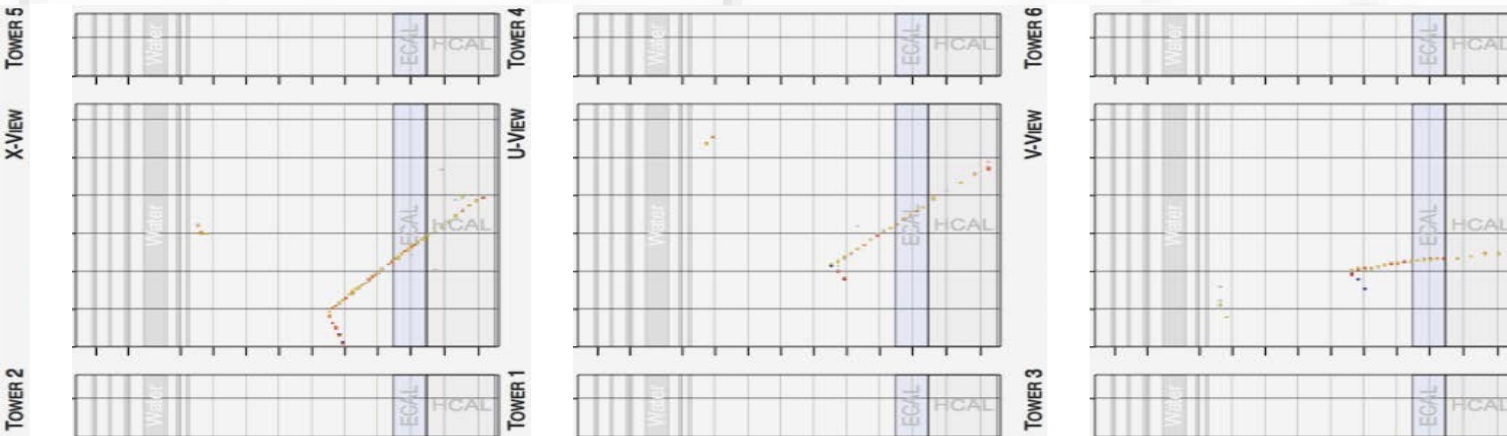




# $\nu$ Events in MINER $\nu$ A

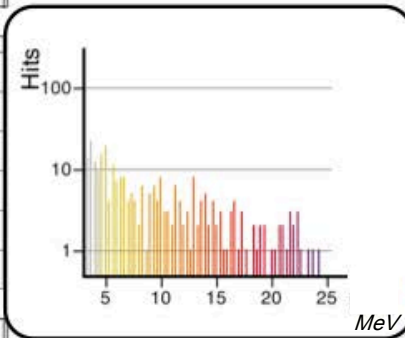
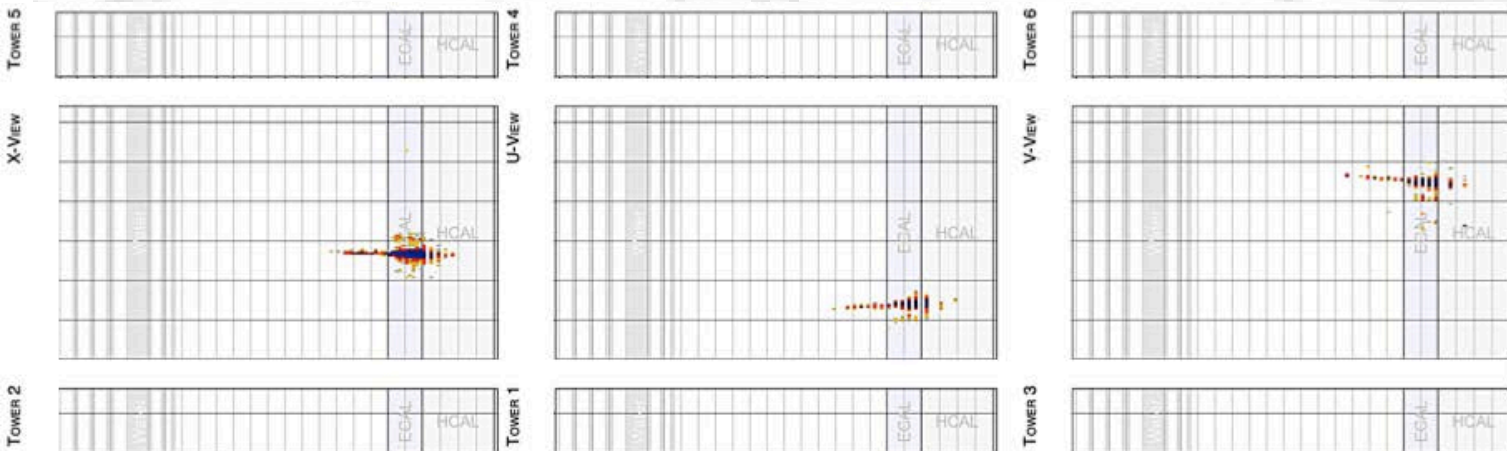


- Charged-current Quasi-elastic candidate



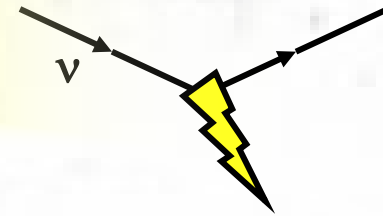
**DATA**

- Single Electron Candidate

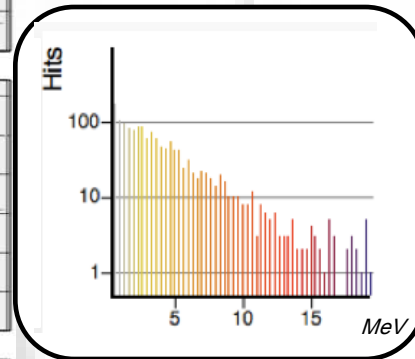
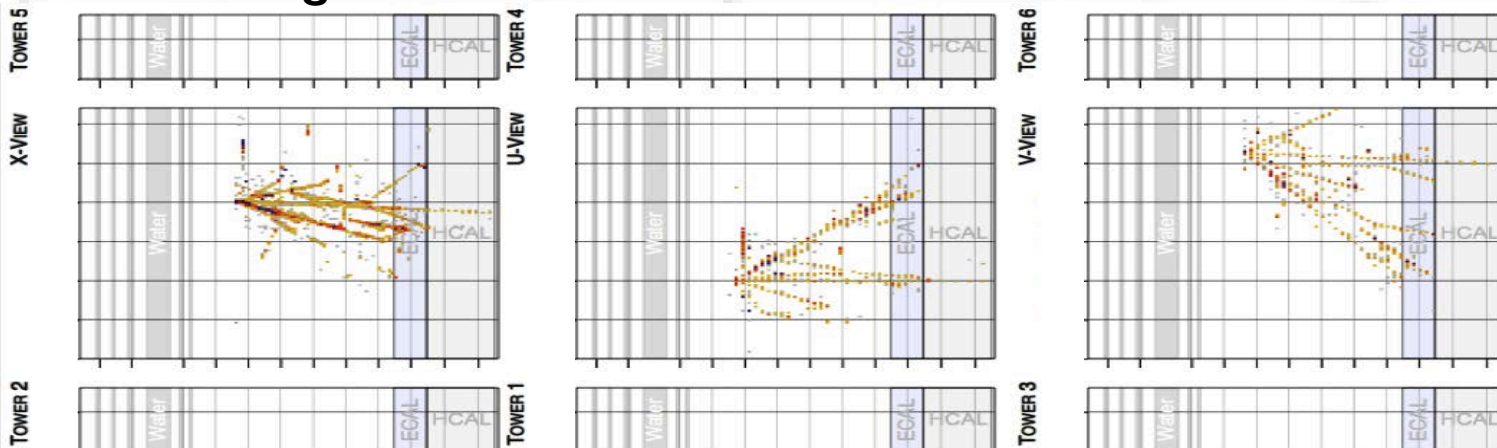




# $\nu$ Events in MINER $\nu$ A

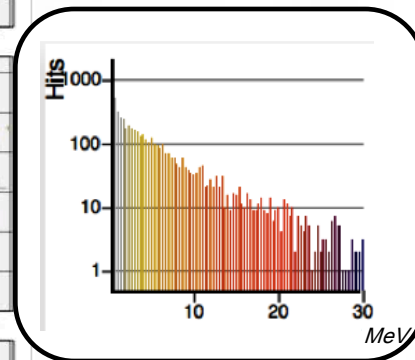
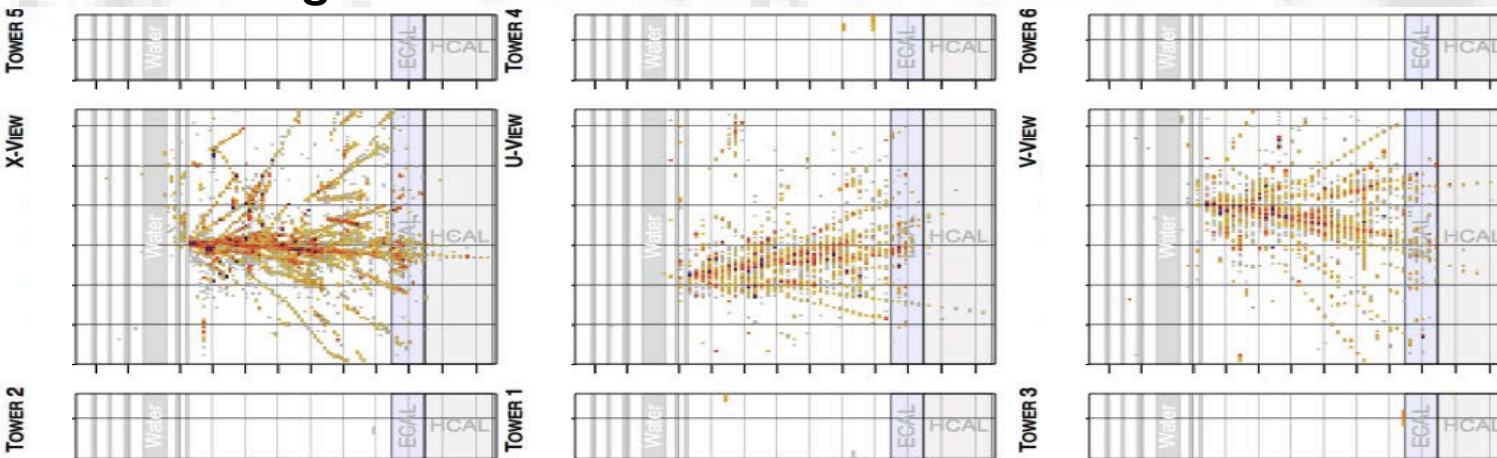


- Charged-current DIS candidate

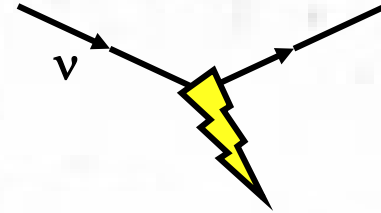


**DATA**

- Charged-current DIS candidate

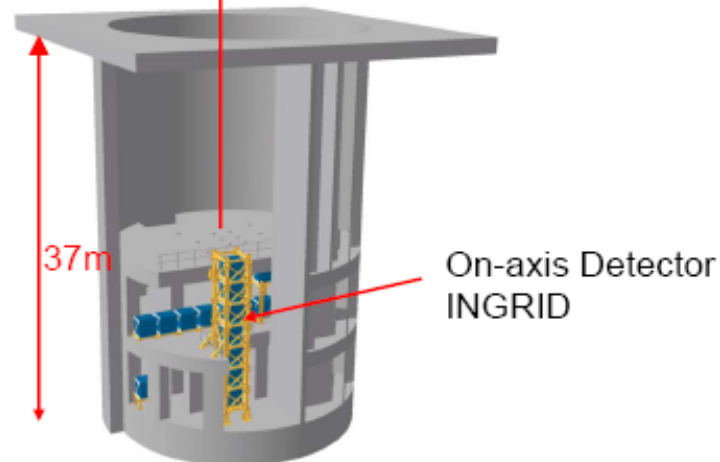
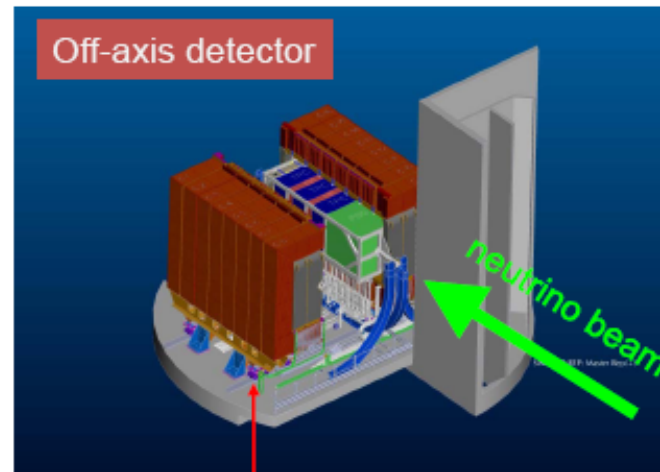


# T2K Near Detectors



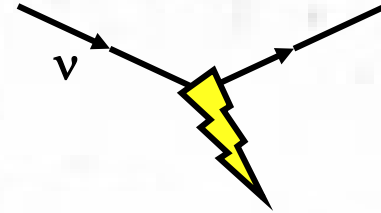
## T2K Near Detector Suite

- Understand the neutrino beam before oscillations occur
- On – Axis Detector
  - Monitor beam direction
  - Monitor beam intensity
- Off – Axis Detector
  - Beam flux
  - Beam  $\nu_e$  contamination
  - Cross sections

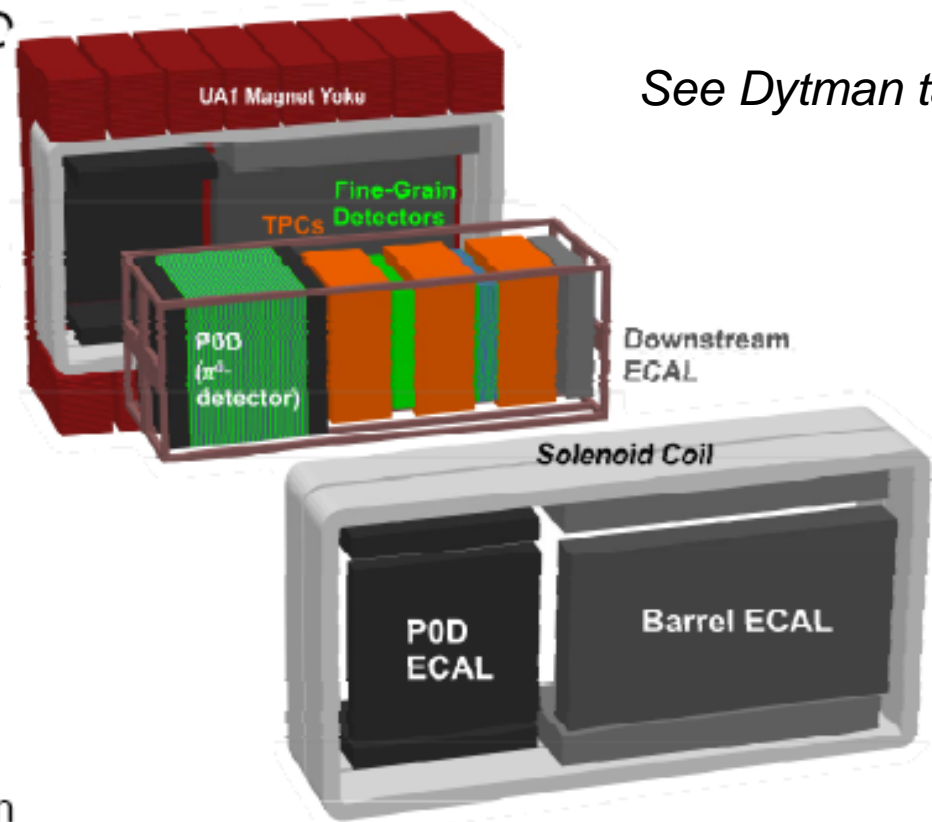


*slide courtesy of R. Terri*

# Off-Axis Detector



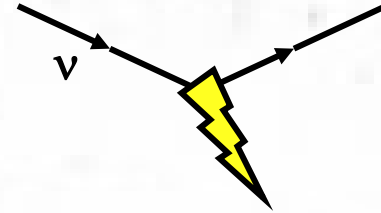
- UA1 Magnet 0.2T field
- Includes a water target in POD and Tracker
  - Understand interactions at SK
- Tracker Region
  - Fine Grained Detectors (FGDs) & TPCs
  - Particle Tracking
- POD
  - Measure NC  $\pi^0$  rate
- ECAL
  - Surrounds tracker and POD
  - Capture EM energy
- SMRD
  - Muon ranging instrumentation in the magnet yoke



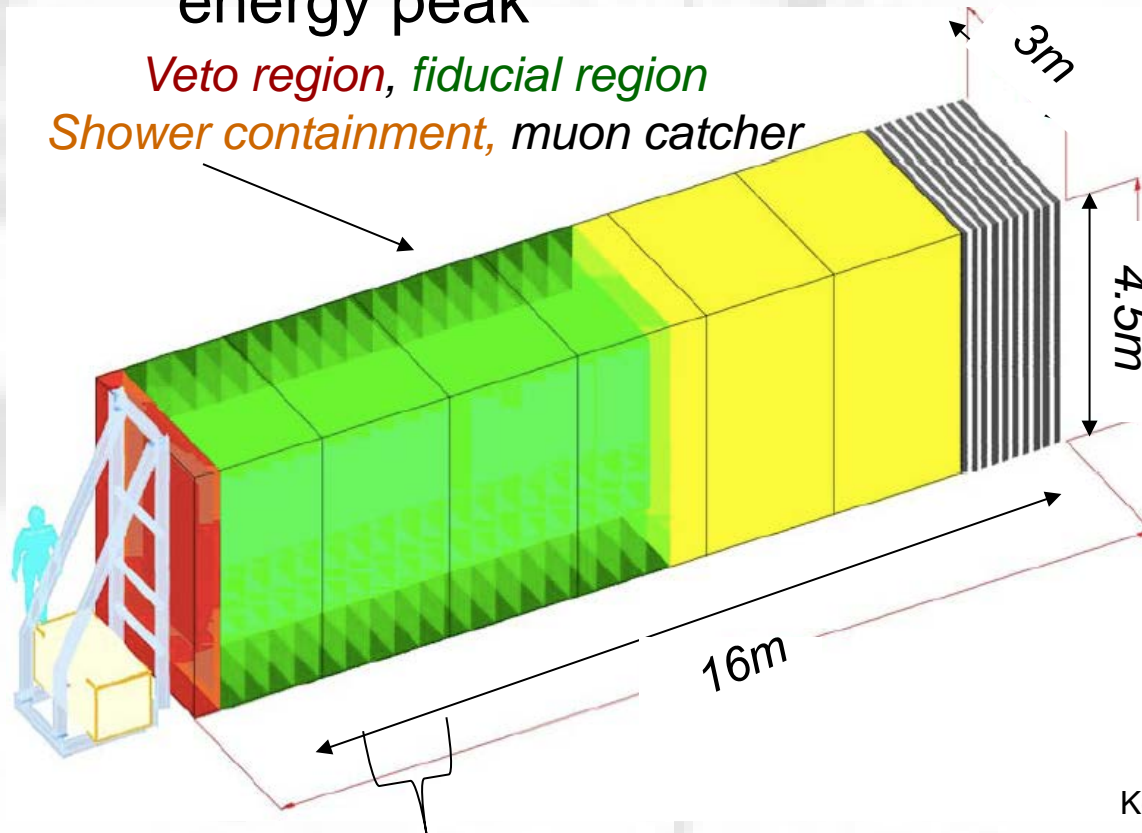
*See Dytman talk*

*slide courtesy of R. Terri*

# NOvA Near Detector

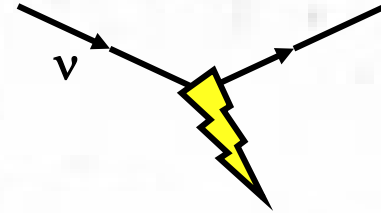


- Scintillator extrusion cross section of 3.87cm x 6cm , but with added muon range stack to see 2 GeV energy peak



- Range stack: 1.7 meters long, steel interspersed with 10 active planes of liquid scintillator
- First located on the surface, then moved to final underground location

# MicroBooNE

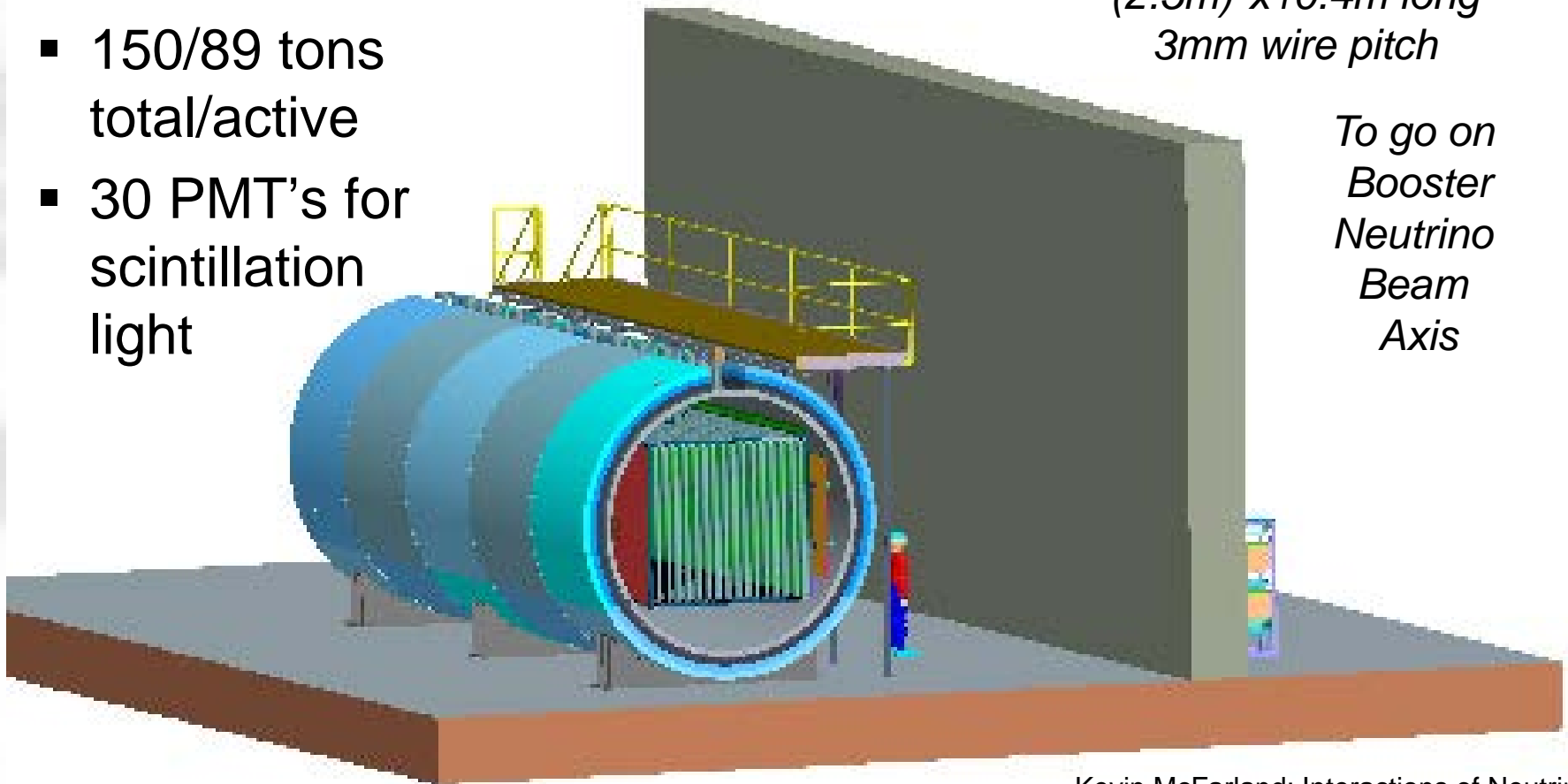


- Liquid Argon TPC

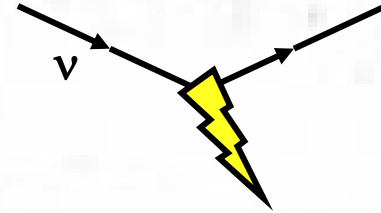
- 150/89 tons total/active
- 30 PMT's for scintillation light

TPC:  
 $(2.5\text{m})^2 \times 10.4\text{m}$  long  
3mm wire pitch

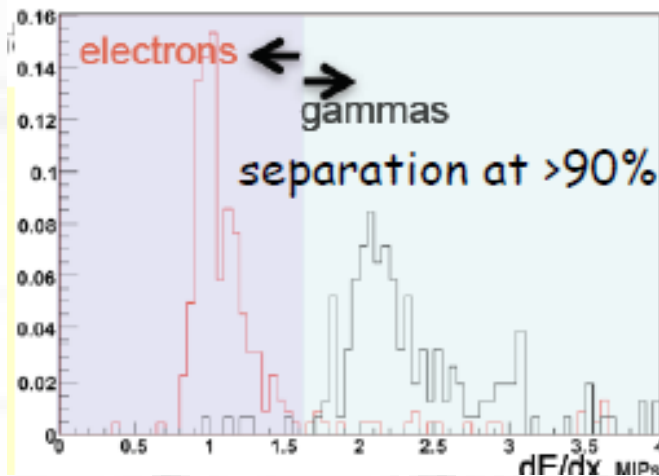
To go on  
Booster  
Neutrino  
Beam  
Axis



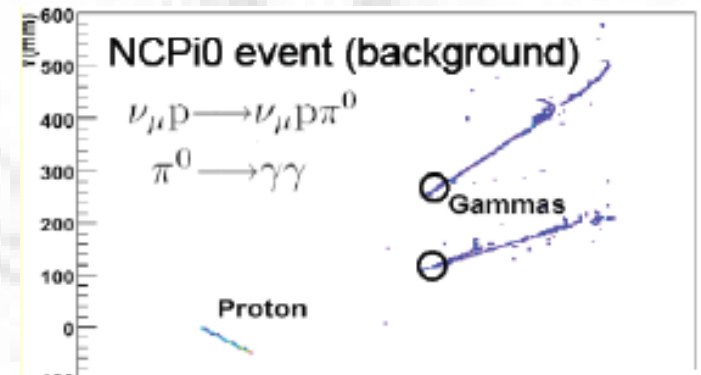
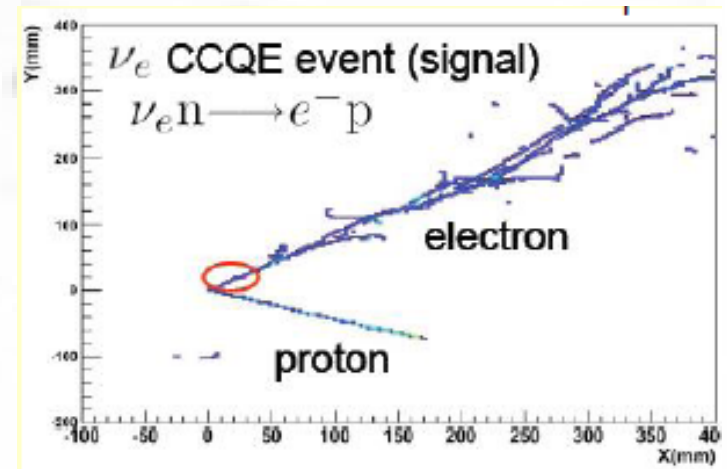
# Technologies: Liquid Argon



- Very low threshold, excellent particle ID
  - Even electron/photon separation!



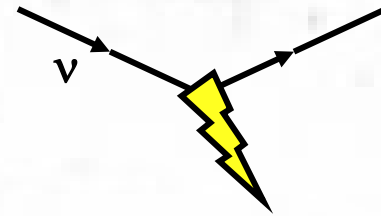
- Reconstruction is not always so straightforward with this level of detail available



Figures from G. Barker



# ***Future Experiments at a Neutrino Factory***



- Early on in the consideration of neutrino factories, this possibility was pointed out by a number of groups
  - Concepts for experiments tried to leverage flux in high energy beams
  - Precision weak interaction physics through  $\nu e \rightarrow \nu e$
  - Separated flavor structure functions through neutrino and anti-neutrino scattering on  $H_2$  and  $D_2$  targets
- Expect proposals for these experiments, or sensible versions thereof, to match parameters of whatever we eventually build

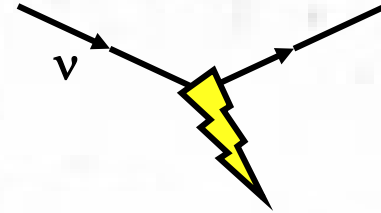
*D. Harris, KSM, AIP Conf.Proc.435:376-383,1998;*

*AIP Conf.Proc.435:505-510,1998,*

*R. Ball, D. Harris, KSM, hep-ph/0009223*

*M. Mangano et al. CERN-TH-2001-131, 2001*

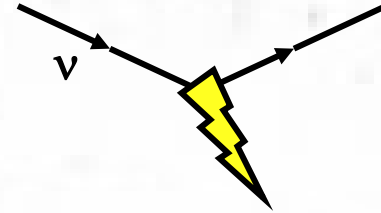
*I.I. Bigi et al, Phys.Rept.371:151-230,2002.*



# ***Slides with Animations (not good for PDF)***



# Nuclear Effects in Elastic Scattering



- Several effects:
  - In a nucleus, target nucleon has some initial momentum which modifies the observed scattering
    - Simple model is a “Fermi Gas” model of nucleons filling available states up to some initial state Fermi momentum,  $k_F$

$v$



- The nucleon is bound in the nucleus, so it takes energy to remove it
- Pauli blocking for nucleons not escaping nucleus... states are already filled with identical nucleon
- Outgoing nucleon can interact with the target

