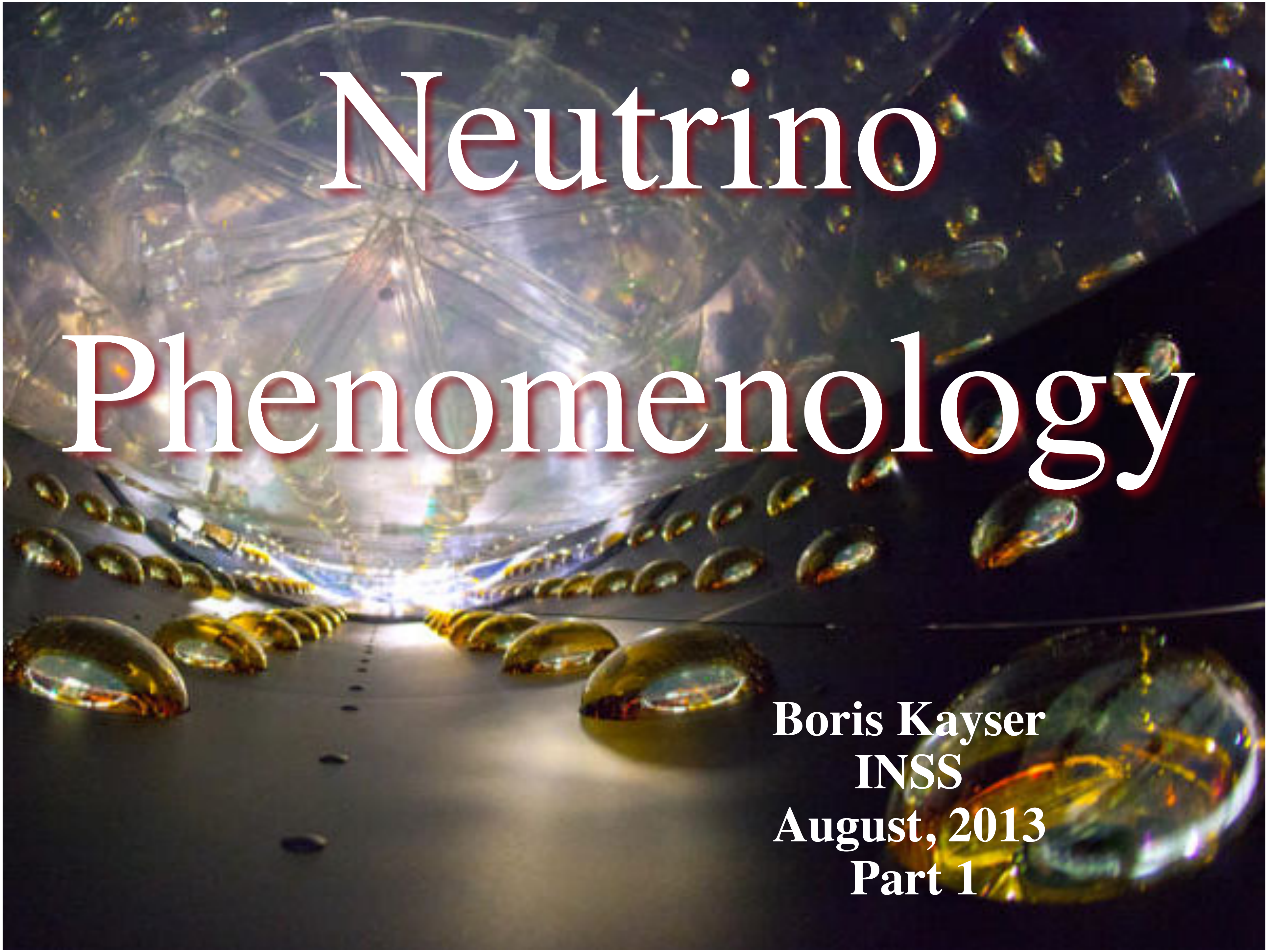

Neutrino Phenomenology

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INSS
August, 2013
Part 1

What Are Neutrinos Good For?

Energy generation in the sun starts with the reaction —

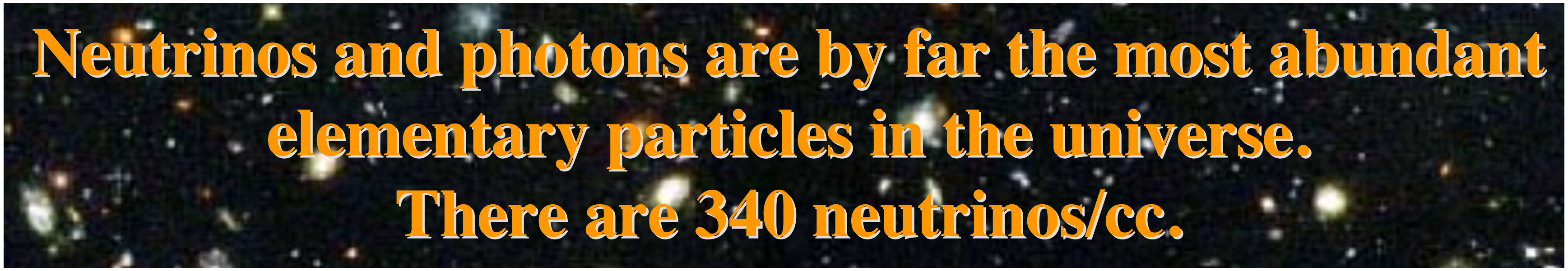
$$p + p \rightarrow d + e^{+} + \nu$$

Spin: $\frac{1}{2}$ $\frac{1}{2}$ 1 $\frac{1}{2}$ $\frac{1}{2}$

Without the neutrino, angular momentum would not be conserved.

Uh, oh ……

The Neutrinos



Neutrinos and photons are by far the most abundant elementary particles in the universe.
There are 340 neutrinos/cc.

The neutrinos are spin – 1/2, electrically neutral, leptons.

The only known forces they experience are the weak force and gravity.

This means that their interactions with other matter have very low strength.
Thus, neutrinos are difficult to detect and study.

Their weak interactions are successfully described by the Standard Model.

The Neutrino Revolution
(1998 – ...)

Neutrinos have nonzero masses!

Leptons mix!

These discoveries come from the observation of ***neutrino flavor change (neutrino oscillation)***.

The Physics of Neutrino Oscillation

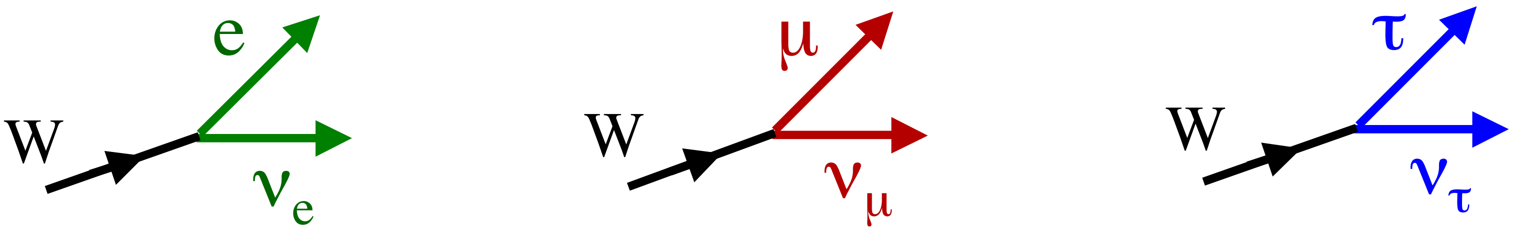
— Preliminaries

The Neutrino Flavors

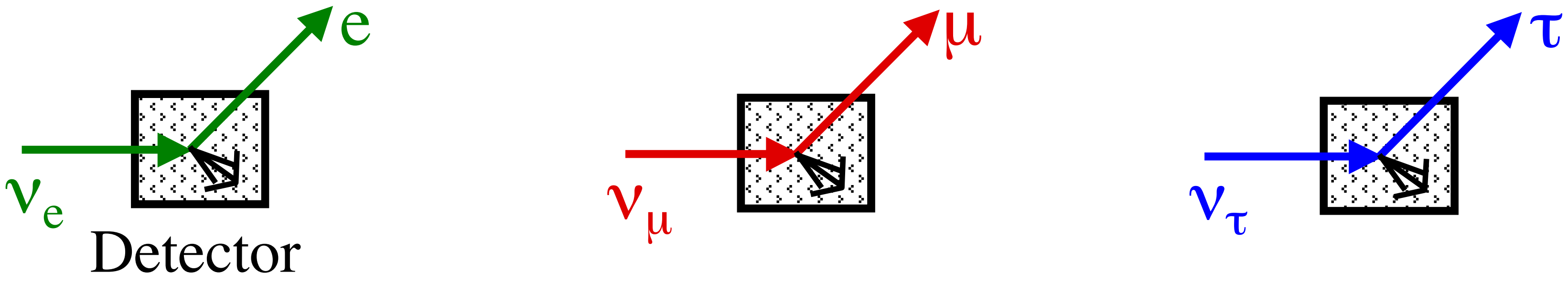
There are three flavors of charged leptons: e , μ , τ

There are three known flavors of neutrinos: ν_e, ν_μ, ν_τ

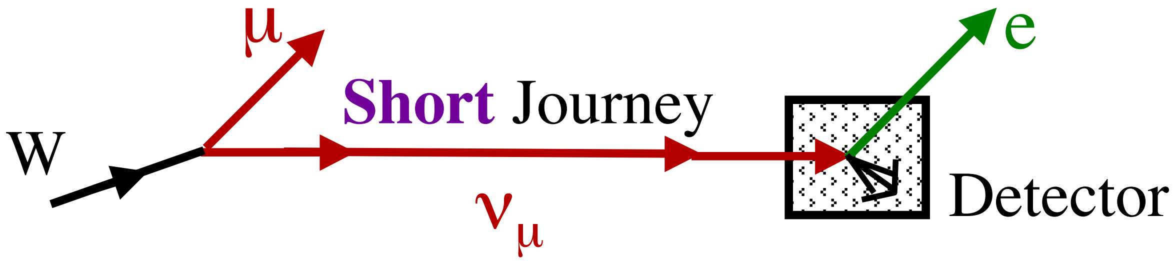
We ***define*** the neutrinos of specific flavor, ν_e, ν_μ, ν_τ, by W boson decays:



As far as we know, when interacting,
a neutrino of given flavor creates
only the charged lepton of the same flavor.



As far as we know, neither



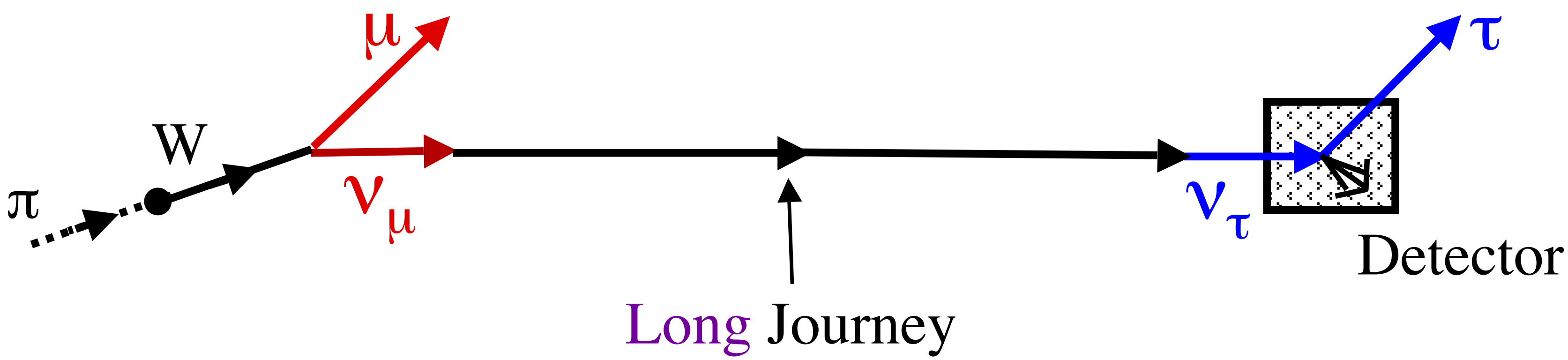
Lederman
Schartz
Steinberger

nor any other change of flavor in the $\nu \to \ell$ ***interaction***
ever occurs.

Notation: ℓ denotes a charged lepton. $\ell_e \equiv e$, $\ell_\mu \equiv \mu$, $\ell_\tau \equiv \tau$.

Neutrino Flavor Change (“Oscillation”)

If neutrinos have masses, and leptons mix, we can have —



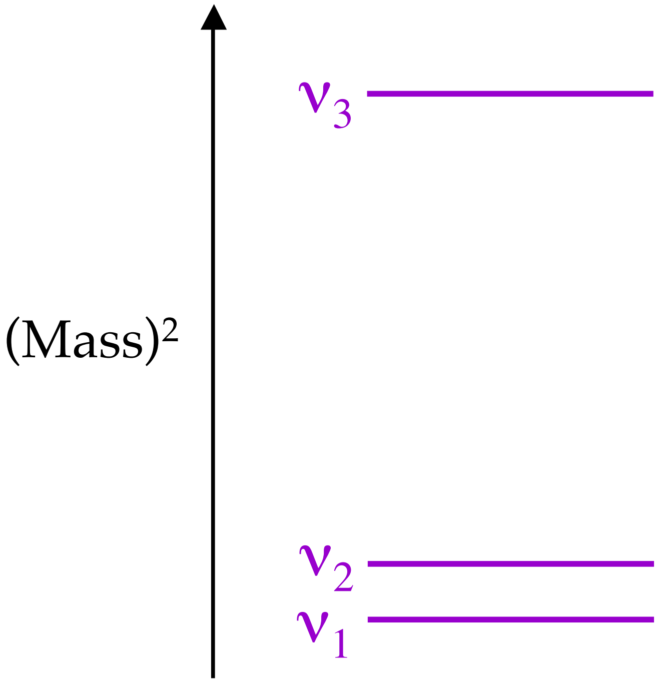
Give a ν time to change character, and you can have

for example: $\nu_\mu \longrightarrow \nu_\tau$

The last 15 years have brought us compelling evidence that such flavor changes actually occur.

Flavor Change Requires ***Neutrino Masses***

There must be some spectrum
of neutrino mass eigenstates ν_i:



Mass $(\nu_i) \equiv m_i$

Flavor Change Requires ***Leptonic Mixing***

The neutrinos $\nu_{e,\mu,\tau}$ of definite flavor

$(W \rightarrow e\nu_e$ or $\mu\nu_\mu$ or $\tau\nu_\tau)$

must be superpositions of the mass eigenstates:

$$|\nu_\alpha> = \sum_i U^*_{\alpha i} |\nu_i> \quad .$$

Neutrino of flavor
$\alpha = e, \mu,$ or τ

Neutrino of definite mass m_i

"PMNS" Leptonic Mixing Matrix

There must be ***at least 3*** mass eigenstates ν_i, because there are 3 orthogonal neutrinos of definite flavor ν_α.

This *mixing* is easily incorporated into the Standard Model (SM) description of the $\ell\nu W$ interaction.

For this interaction, we then have —

Semi-weak coupling

Left-handed

$$\mathcal{L}_{SM} = -\frac{g}{\sqrt{2}} \sum_{\alpha=e,\mu,\tau} \left(\bar{\ell}_{L\alpha}\gamma^{\lambda}\nu_{L\alpha}W_{\lambda}^{-} + \bar{\nu}_{L\alpha}\gamma^{\lambda}\ell_{L\alpha}W_{\lambda}^{+} \right)$$

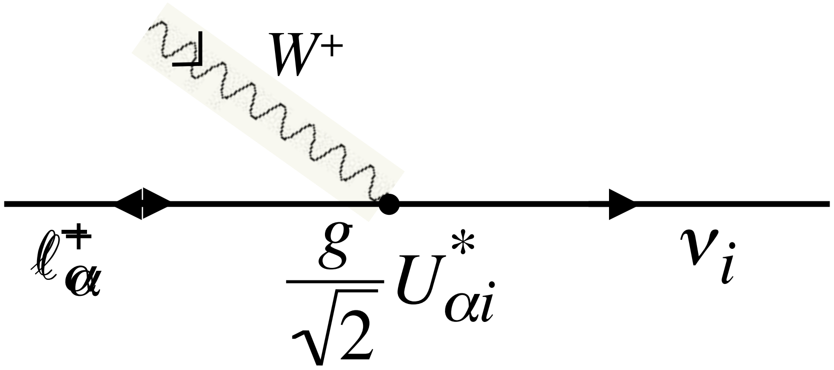
$$= -\frac{g}{\sqrt{2}} \sum_{\substack{\alpha=e,\mu,\tau \\ i=1,2,3}} \left(\bar{\ell}_{L\alpha}\gamma^{\lambda}U_{\alpha i}\nu_{Li}W_{\lambda}^{-} + \bar{\nu}_{Li}\gamma^{\lambda}U_{\alpha i}^{*}\ell_{L\alpha}W_{\lambda}^{+} \right)$$

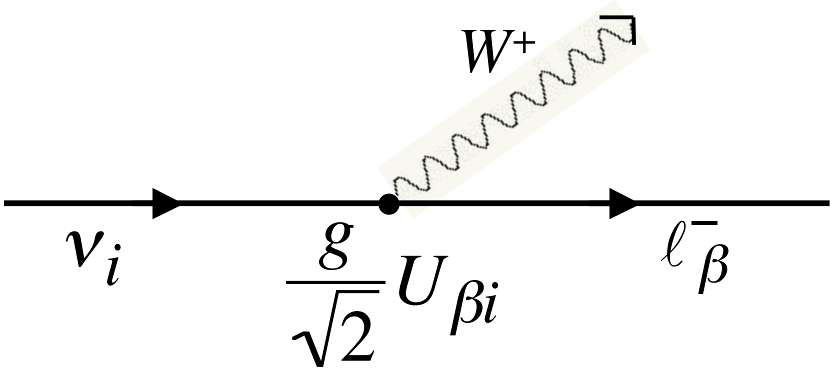
Taking mixing into account

$$\text{Amp}\left(W^{+} \rightarrow \ell_{\alpha}^{+} + \nu_{i}\right) = \frac{g}{\sqrt{2}}U_{\alpha i}^{*} \qquad \text{Amp}\left(\nu_{i} \rightarrow \ell_{\beta}^{-} + W^{+}\right) = \frac{g}{\sqrt{2}}U_{\beta i}$$

The SM interaction conserves the Lepton Number L, defined by $L(\nu) = L\left(\ell^{-}\right) = -L(\bar{\nu}) = -L\left(\ell^{+}\right) = 1$.

The Meaning of *U*





$$U = \begin{array}{c} \\ e \\ \mu \\ \tau \end{array} \begin{array}{c} \begin{array}{ccc} \nu_1 & \nu_2 & \nu_3 \end{array} \\ \begin{bmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{bmatrix} \end{array}$$

The e row of U: The linear combination of neutrino mass eigenstates that couples to e.

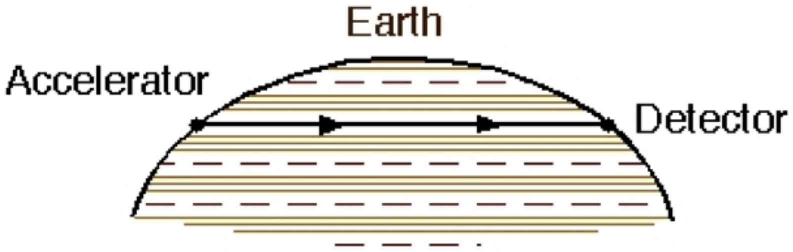
The ν_1 column of U: The linear combination of charged-lepton mass eigenstates that couples to ν_1 .

Slides on

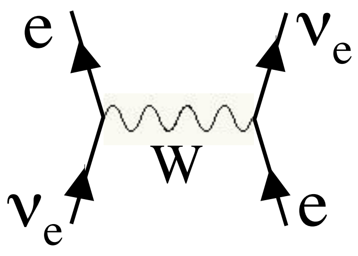
The Physics of Neutrino Oscillation

go here.

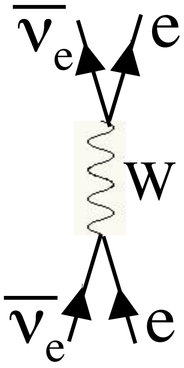
Neutrino Flavor Change In Matter



involves



or



Coherent forward scattering via this W-exchange interaction leads to an extra interaction potential energy —

$$V_W = \begin{cases} +\sqrt{2}G_F N_e, & \nu_e \\ -\sqrt{2}G_F N_e, & \overline{\nu_e} \end{cases}$$

Fermi constant (arrow to G_F) — Electron density (arrow to N_e)

This raises the effective mass of ν_e, and lowers that of $\overline{\nu_e}$.

The fractional importance of matter effects on an oscillation involving a vacuum splitting Δm^2 is —

$$\overbrace{[\sqrt{2}G_F N_e]}^{\text{Interaction energy}} \,/\, \overbrace{[\Delta m^2/2E]}^{\text{Vacuum energy}} \equiv x\ .$$

The matter effect —

— Grows with neutrino energy E

— Is sensitive to Sign(Δm^2)

— Reverses when ν is replaced by $\bar{\nu}$

This last is a “fake CP violation”, but
the matter effect is negligible when $x << 1$.

Evidence For Flavor Change

Neutrinos	Evidence of Flavor Change
Solar	Compelling
Reactor (Long-Baseline)	Compelling
Atmospheric	Compelling
Accelerator (Long-Baseline)	Compelling
Accelerator & Reactor (Short-Baseline)	“Interesting”

KamLAND Evidence for Oscillatory Behavior

The KamLAND detector studies $\overline{\nu}_e$ produced by Japanese nuclear power reactors ~ 180 km away.

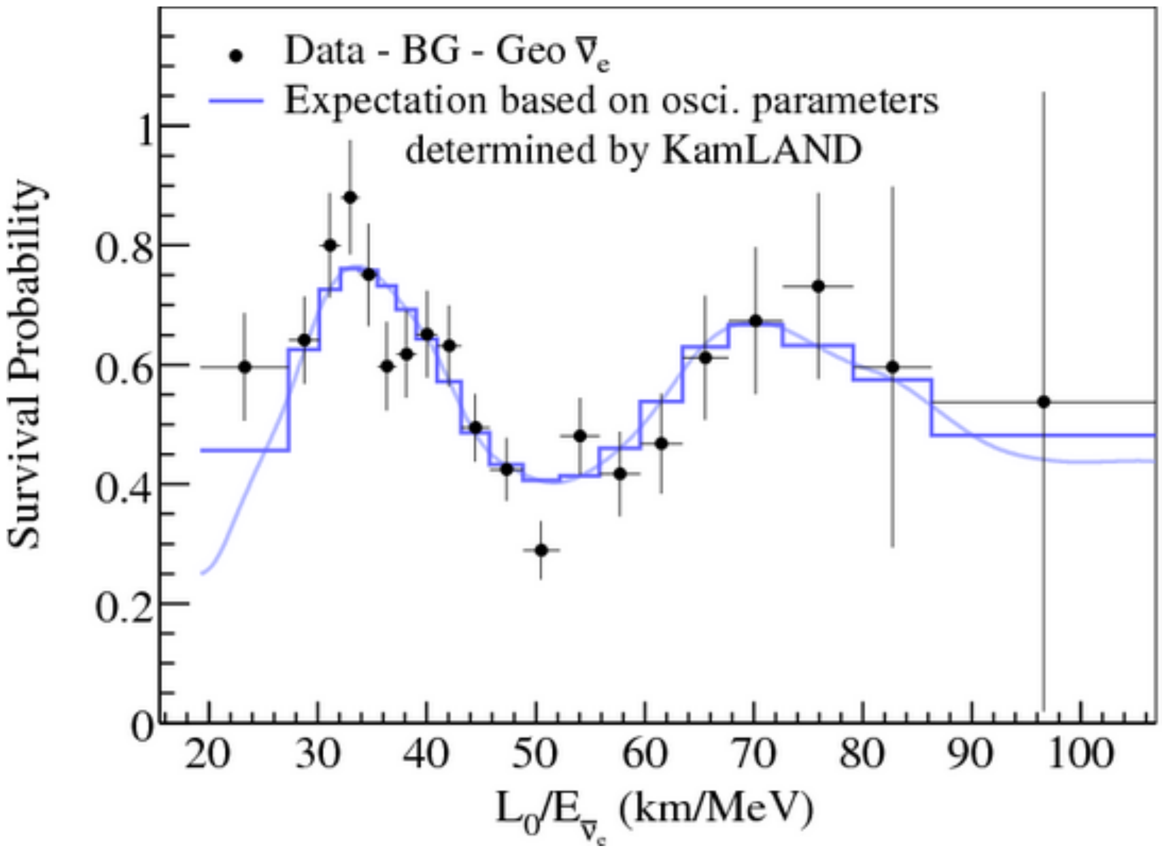
For KamLAND, $x_{\mathrm{Matter}} < 10^{-2}$. Matter effects are negligible.

The $\bar{\nu}_e$ survival probability, $P(\bar{\nu}_e \to \bar{\nu}_e)$, should oscillate as a function of L/E following the vacuum oscillation formula.

In the two-neutrino approximation, we expect —

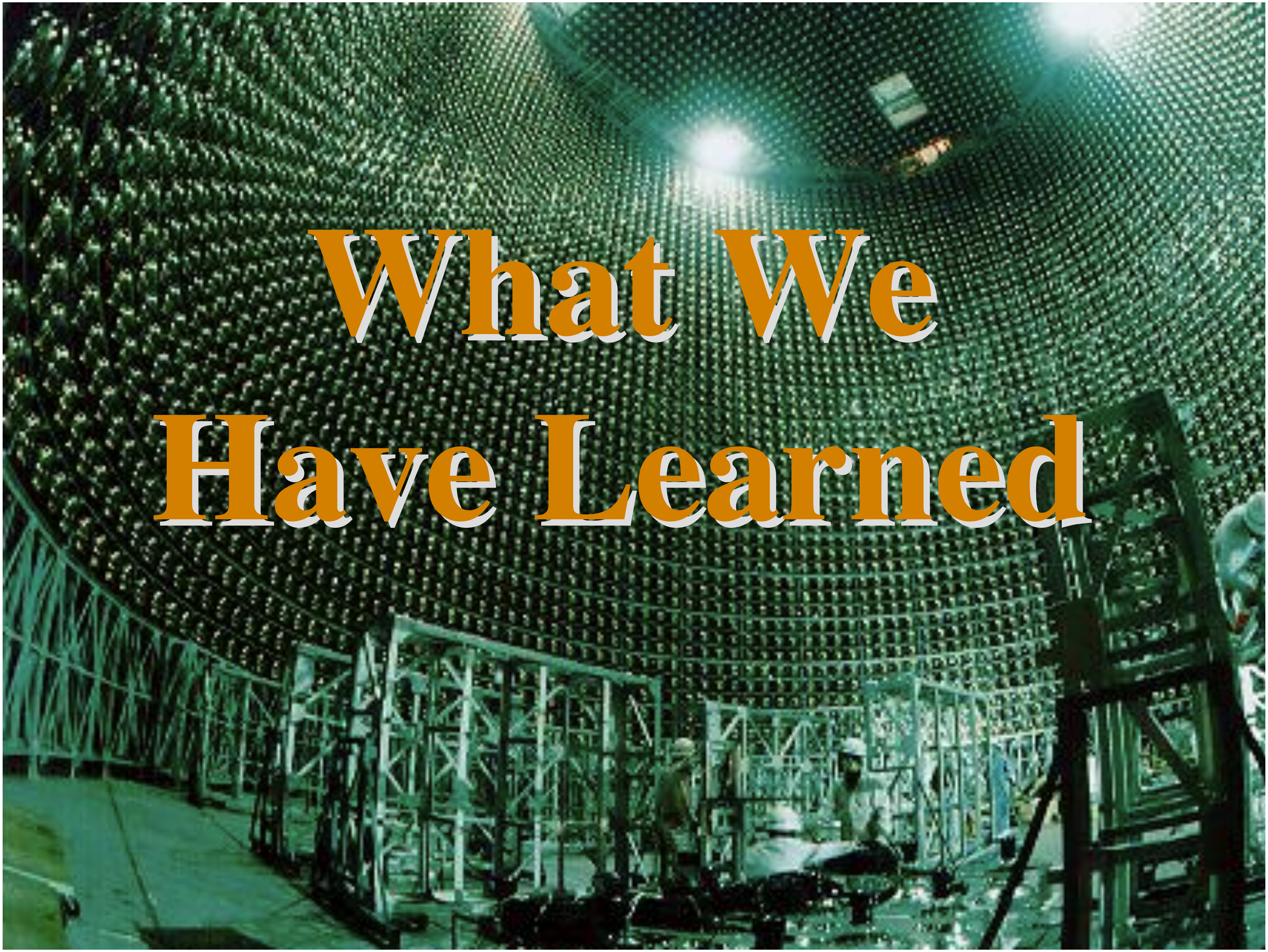
$$P(\bar{\nu}_e \to \bar{\nu}_e) = 1 - \sin^2 2\theta \sin^2\left[1.27\Delta m^2\left(eV^2\right)\frac{L(km)}{E(GeV)}\right].$$

Survival probability $P(\bar{\nu}_e \rightarrow \bar{\nu}_e)$ of reactor $\bar{\nu}_e$



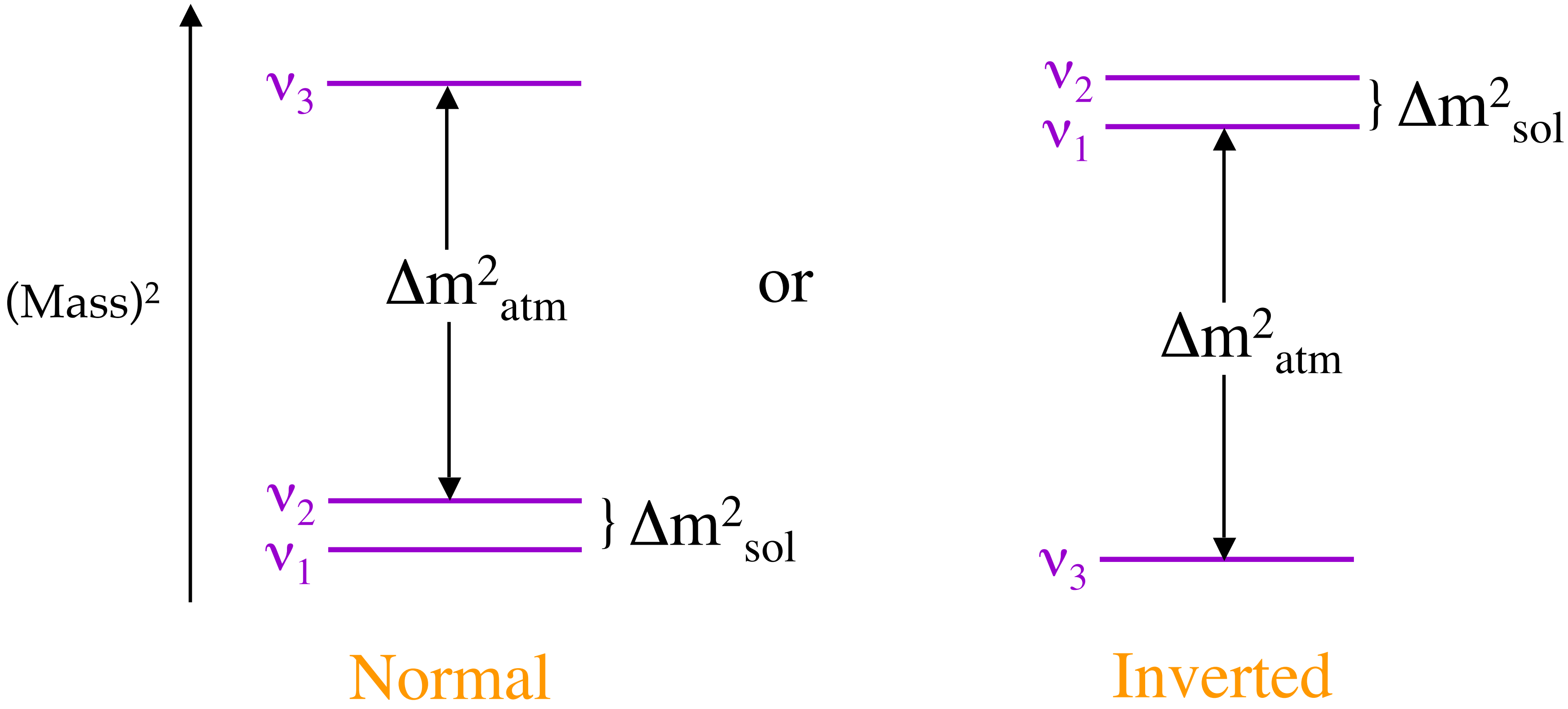
L_0 = 180 km is a flux-weighted average travel distance.

$P(\bar{\nu}_e \rightarrow \bar{\nu}_e)$ actually oscillates!



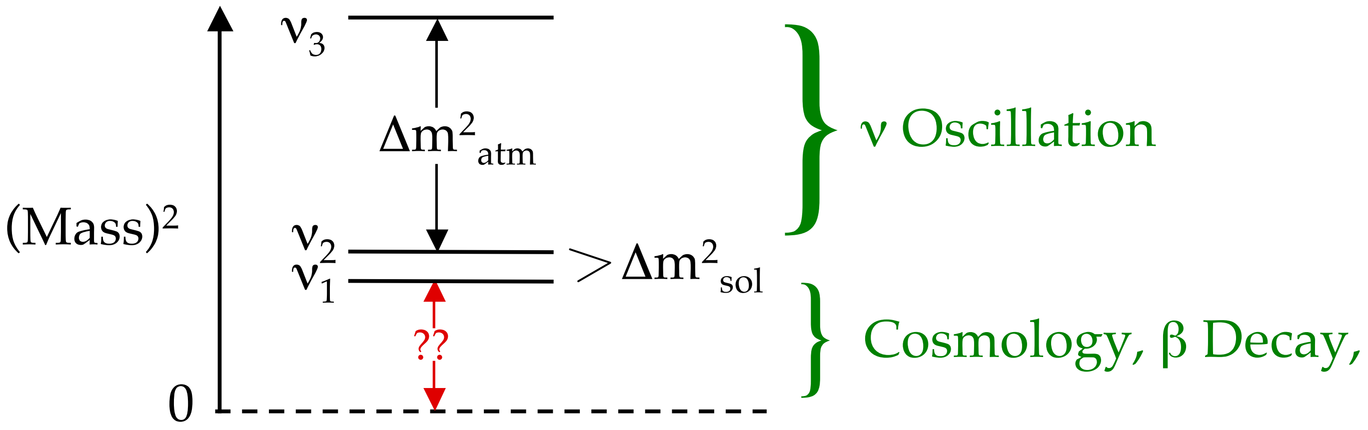
What We Have Learned

The $(\text{Mass})^2$ Spectrum



$\Delta m^2_{sol} \cong 7.5 \times 10^{-5}\ \text{eV}^2$, $\Delta m^2_{atm} \cong 2.4 \times 10^{-3}\ \text{eV}^2$

The Absolute Scale of Neutrino Mass



How far above zero is the whole pattern?

Oscillation Data $\Rightarrow \sqrt{\Delta m^2_{atm}} <$ Mass[Heaviest ν_i]

Mass[Heaviest ν_i] $\gtrsim 0.04$ eV

The Upper Bound From Cosmology

(Jenni Adams)

$\sum_i m(\nu_i)$ In the Early Universe

Large Scale Structure in the universe and the CMB probe this sum of the neutrino masses, ***assuming*** that all ν_i have thermalized in the early universe.

$$\sum_i m(\nu_i) < 0.23 \text{ eV} \qquad \begin{pmatrix} \text{Planck + WP +} \\ \text{high L + BAO} \end{pmatrix}$$

Possible tension with terrestrial experiments
if one or more $\Delta m^2 > 1 \text{ eV}^2$.

However, in cosmology, there are parameter degeneracies.

The Upper Bound From Tritium

(Liang Yang)

Cosmology is wonderful, but there are known loopholes in its argument concerning neutrino mass.

The absolute neutrino mass can in principle also be measured by the kinematics of β decay.

Tritium decay: $^3H \to {}^3He + e^- + \overline{\nu_i}$; $i = 1, 2,$ or 3

$$BR\left({}^3H \to {}^3He + e^- + \overline{\nu_i}\right) \propto |U_{ei}|^2$$

In $^3H \to {}^3He + e^- + \overline{\nu_i}$, the bigger m_i is, the smaller the maximum electron energy is.

There are 3 separate thresholds in the β energy spectrum.

The β energy spectrum is modified according to —

$$(E_0 - E)^2 \Theta[E_0 - E] \Rightarrow \sum_i |U_{ei}|^2 (E_0 - E)\sqrt{(E_0 - E)^2 - m_i^2}\,\Theta[(E_0 - m_i) - E]$$

Maximum β energy when
there is no neutrino mass

β energy

Present experimental energy resolution
is insufficient to separate the thresholds.

Measurements of the spectrum bound the average neutrino mass —

$$\langle m_\beta \rangle = \sqrt{\sum_i |U_{ei}|^2 m_i^2}$$

$$\boxed{\text{Presently: } \langle m_\beta \rangle < 2\ \text{eV}}$$

Mainz &
Troitzk

Leptonic Mixing

This has the consequence that —

Mass eigenstate

Flavor eigenstate

$$|\nu_i > = \sum_{\alpha} U_{\alpha i} \, |\nu_\alpha > \; .$$

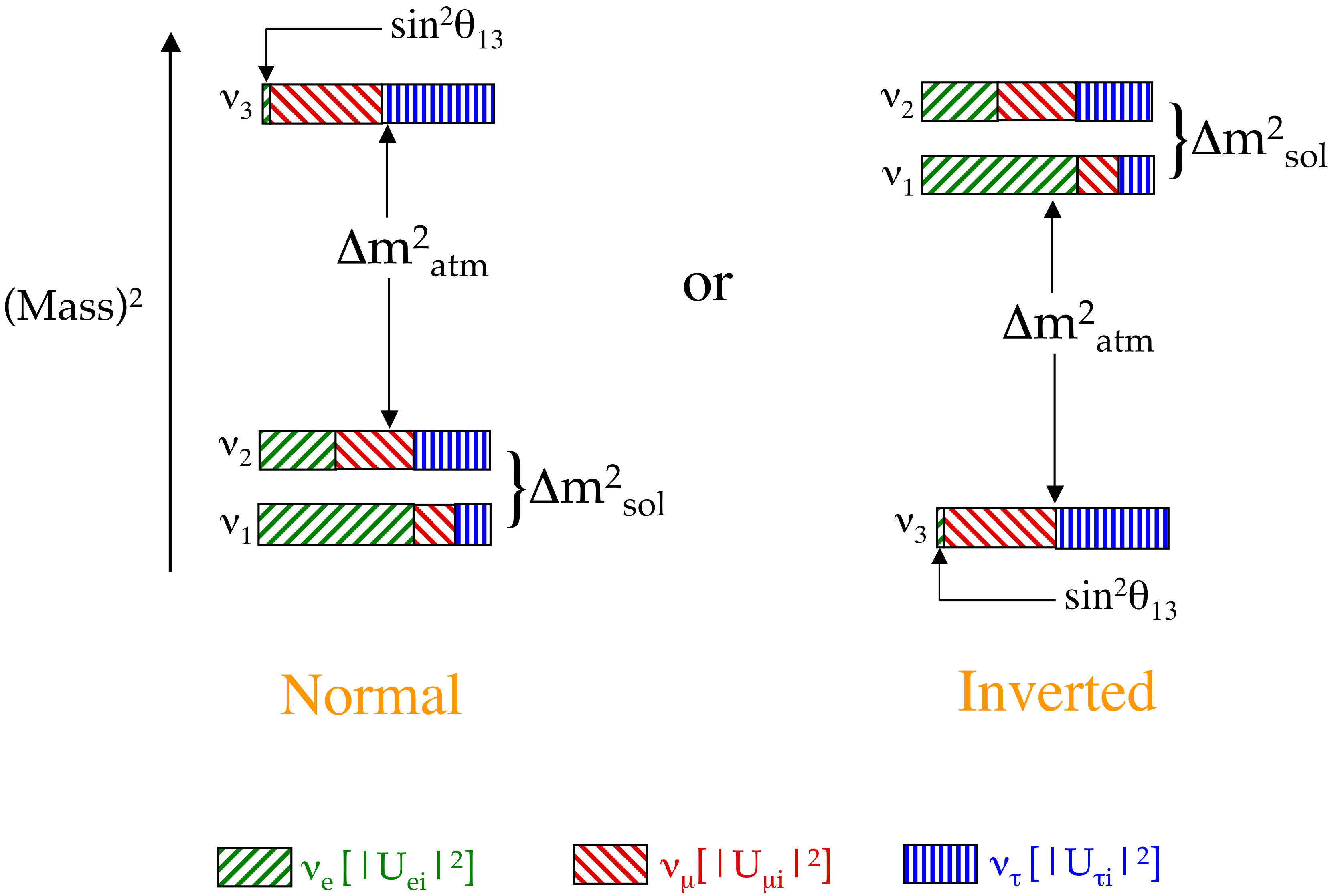
e, μ, or τ

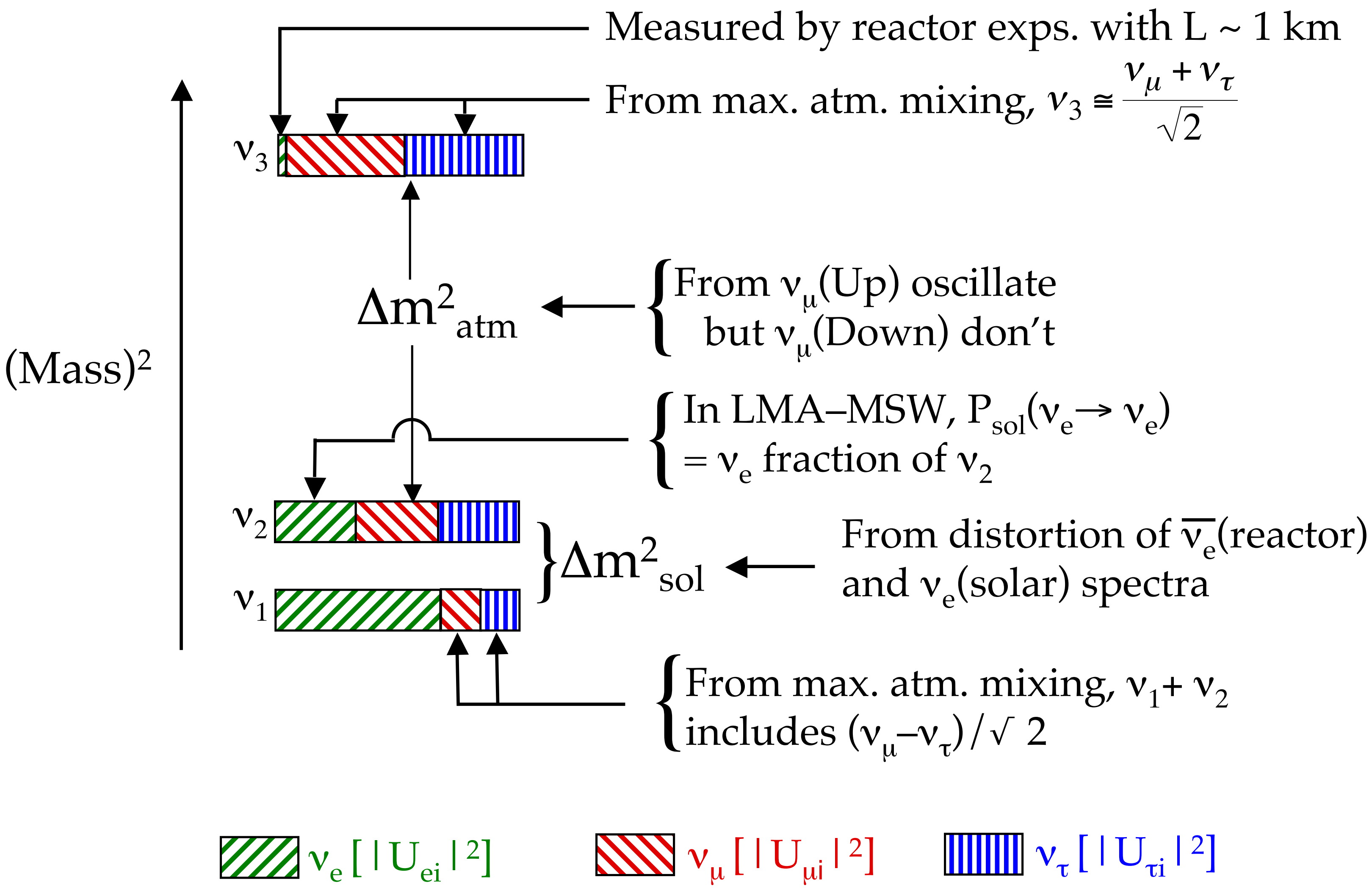
Leptonic Mixing Matrix

Flavor-α fraction of $\nu_i = |U_{\alpha i}|^2$.

When a ν_i interacts and produces a charged lepton, the probability that this charged lepton will be of flavor α is $|U_{\alpha i}|^2$.

The spectrum, showing its approximate flavor content, is





Measured by reactor exps. with L ~ 1 km

From max. atm. mixing, $\nu_3 \cong \frac{\nu_\mu + \nu_\tau}{\sqrt{2}}$

From ν_μ(Up) oscillate but ν_μ(Down) don't

In LMA–MSW, $P_{sol}(\nu_e \rightarrow \nu_e)$ = ν_e fraction of ν_2

From distortion of $\overline{\nu}_e$(reactor) and ν_e(solar) spectra

From max. atm. mixing, $\nu_1 + \nu_2$ includes $(\nu_\mu - \nu_\tau)/\sqrt{2}$

The 3 X 3 Unitary Mixing Matrix

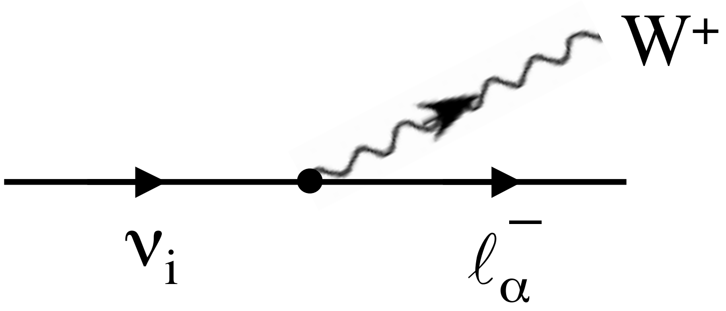
Caution: We are *assuming* the mixing matrix U to be 3 x 3 and unitary.

$$\mathcal{L}_{SM} = -\frac{g}{\sqrt{2}} \sum_{\substack{\alpha=e,\mu,\tau \\ i=1,2,3}} \left(\overline{\ell}_{L\alpha} \gamma^{\lambda} U_{\alpha i} \nu_{Li} W_{\lambda}{}^{-} + \overline{\nu}_{Li} \gamma^{\lambda} U_{\alpha i}{}^{*} \ell_{L\alpha} W_{\lambda}{}^{+} \right)$$

$$(CP)\left(\overline{\ell}_{L\alpha} \gamma^{\lambda} U_{\alpha i} \nu_{Li} W_{\lambda}{}^{-} \right)(CP)^{-1} = \overline{\nu}_{Li} \gamma^{\lambda} U_{\alpha i} \ell_{L\alpha} W_{\lambda}{}^{+}$$

Phases in U will lead to CP violation, unless they are removable by redefining the leptons.

$U_{\alpha i}$ describes —



$$U_{\alpha i} \sim \langle \ell_\alpha^- W^+ | H | \nu_i \rangle$$

$$\text{When } |\nu_i\rangle \rightarrow |e^{i\varphi} \nu_i\rangle , \; U_{\alpha i} \rightarrow e^{i\varphi} U_{\alpha i}, \text{ all } \alpha$$

$$\text{When } |\ell_\alpha^-\rangle \rightarrow |e^{i\varphi} \ell_\alpha^-\rangle, \; U_{\alpha i} \rightarrow e^{-i\varphi} U_{\alpha i}, \text{ all } i$$

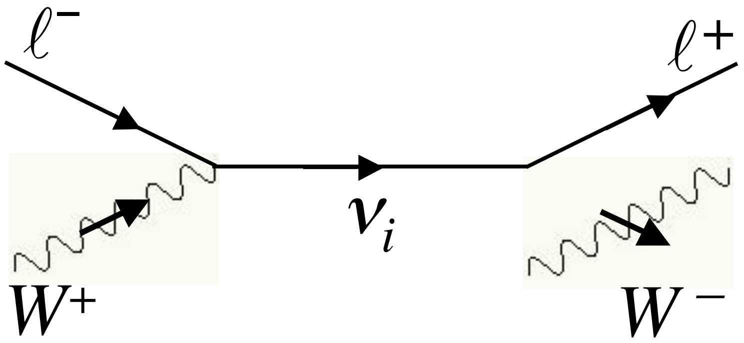
Thus, one may multiply any column, or any row, of U by a complex phase factor without changing the physics.

Some phases may be removed from U in this way.

When the Neutrino Mass Eigenstates Are Their Own Antiparticles

When this is the case, processes that
do not conserve the lepton number
$L \equiv$ #(Leptons) – #(Antileptons) can occur.

Example:



The amplitude for any such L-violating process contains an extra factor.

When we phase-redefine ν_i to remove a phase from U, that phase just moves to the extra factor.

It does not disappear from the physics.

Hence, when $\overline{\nu}_i = \nu_i$, U can contain extra physically-significant phases.

These are called Majorana phases.

How Many Mixing Angles and ~~CP~~ Phases Does U Contain?

Real parameters before constraints:	18
Unitarity constraints — $\sum_i U_{\alpha i}{}^* U_{\beta i} = \delta_{\alpha\beta}$	
Each row is a vector of length unity:	– 3
Each two rows are orthogonal vectors:	– 6
Rephase the three ℓ_α :	– 3
Rephase two ν_i , if $\overline{\nu}_i \neq \nu_i$:	– 2
Total physically-significant parameters:	4
Additional (Majorana) ~~CP~~ phases if $\overline{\nu}_i = \nu_i$:	2

How Many Of The Parameters Are Mixing Angles?

The *mixing angles* are the parameters in U when it is *real*.

U is then a three-dimensional rotation matrix.

Everyone knows such a matrix is described in terms of 3 angles.

Thus, U contains 3 mixing angles.

Summary

Mixing angles	$\cancel{CP}$ phases if $\bar{\nu}_i \neq \nu_i$	$\cancel{CP}$ phases if $\bar{\nu}_i = \nu_i$
3	1	3

The Mixing Matrix *U*

$$U = \overset{\text{Atmospheric}}{\begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{bmatrix}} \times \overset{\text{Reactor (L ~ 1 km)}}{\begin{bmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{bmatrix}} \times \overset{\text{Solar}}{\begin{bmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix}}$$

$c_{ij} \equiv \cos \theta_{ij}$

$s_{ij} \equiv \sin \theta_{ij}$

$$\times \underbrace{\begin{bmatrix} e^{i\alpha_1/2} & 0 & 0 \\ 0 & e^{i\alpha_2/2} & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{\text{Majorana phases}}$$

$$\overbrace{\theta_{12} \approx 33°, \theta_{23} \approx 36\text{-}42° \ \underline{\text{or}}\ 48\text{-}54°,}^{\textit{Note big mixing!}} \ \theta_{13} \approx 8\text{-}9°$$

No more worry!

δ would lead to $P(\bar{\nu}_\alpha \rightarrow \bar{\nu}_\beta) \neq P(\nu_\alpha \rightarrow \nu_\beta)$. *CP violation*

But note the crucial role of $s_{13} \equiv \sin \theta_{13}$.

The Majorana ~~CP~~ Phases

The phase α_i is associated with
neutrino mass eigenstate ν_i:

$U_{\alpha i} = U^0_{\alpha i}\exp(i\alpha_i/2)$ for all flavors α.

$$\mathrm{Amp}(\nu_\alpha \to \nu_\beta) = \sum_i U_{\alpha i}^* \exp(-\,i m_i^2 L/2E)\, U_{\beta i}$$

is insensitive to the Majorana phases α_i.

Only the phase δ can cause CP violation in
neutrino oscillation.

There Is Nothing Special About θ_{13}

All mixing angles must be nonzero for ~~CP~~ in oscillation.

For example —

$$P\left(\bar{\nu}_\mu \to \bar{\nu}_e\right) - P\left(\nu_\mu \to \nu_e\right) = 2\cos\theta_{13}\sin 2\theta_{13}\sin 2\theta_{12}\sin 2\theta_{23}\sin\delta$$
$$\times \sin\left(\Delta m^2{}_{31}\frac{L}{4E}\right)\sin\left(\Delta m^2{}_{32}\frac{L}{4E}\right)\sin\left(\Delta m^2{}_{21}\frac{L}{4E}\right)$$

In the factored form of U, one can put
δ next to θ_{12} instead of θ_{13}.