Neutrino mass models

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Outline

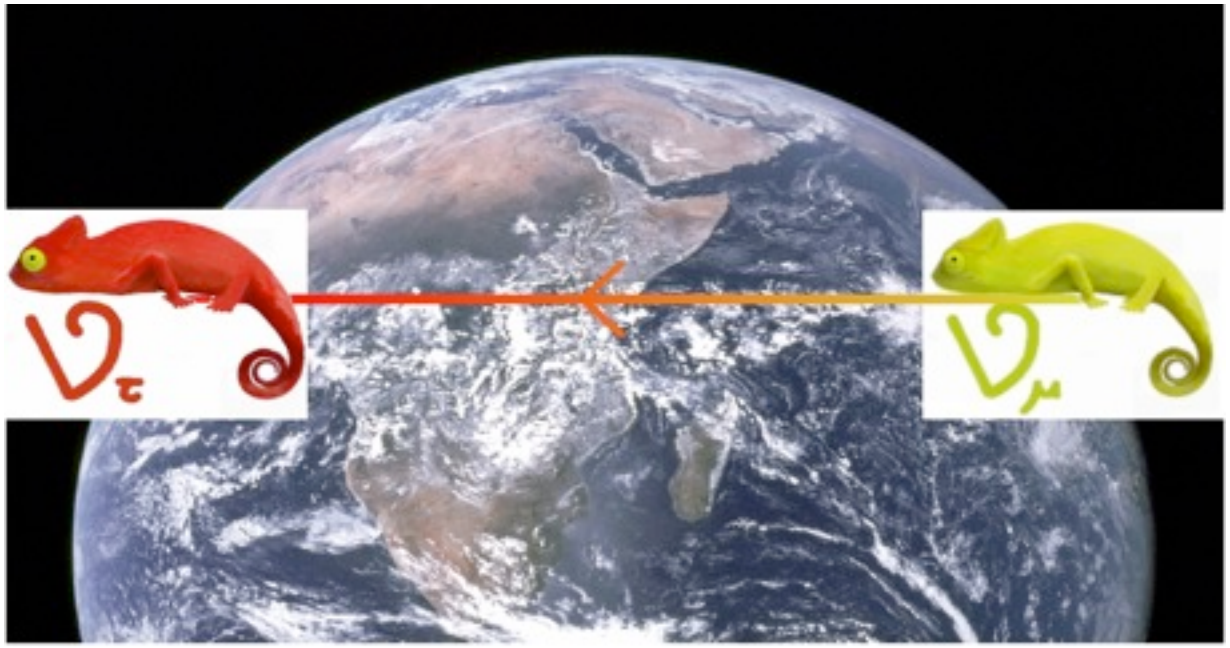
1. Present knowledge of neutrino masses

2. Questions for the future: How we will measure neutrino masses

3. Neutrino masses beyond the Standard Model: Dirac, Majorana and Dirac +Majorana masses

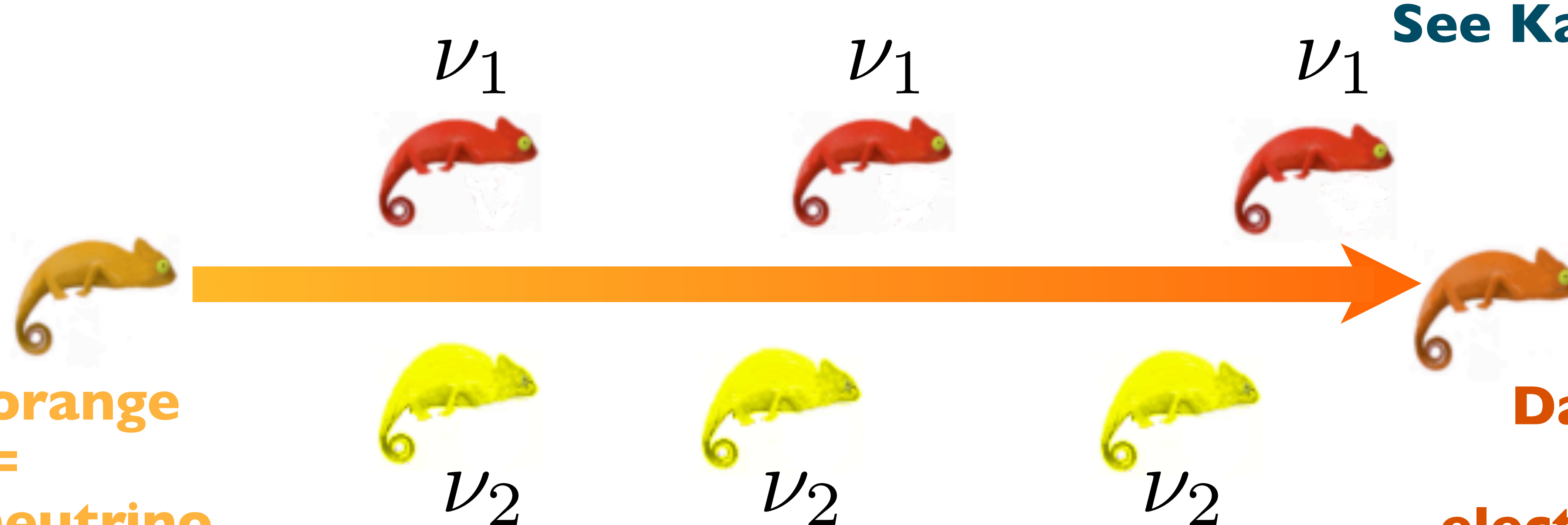
4. Models of masses BSM:
   - Dirac masses
   - see saw type I
   - see-saw type II
   - see-saw type III
   - extended-type see-saw models

The facts: Neutrinos oscillate



Contrary to what expected in the SM, neutrinos oscillate: after being produced, they can **change their "flavour"**.

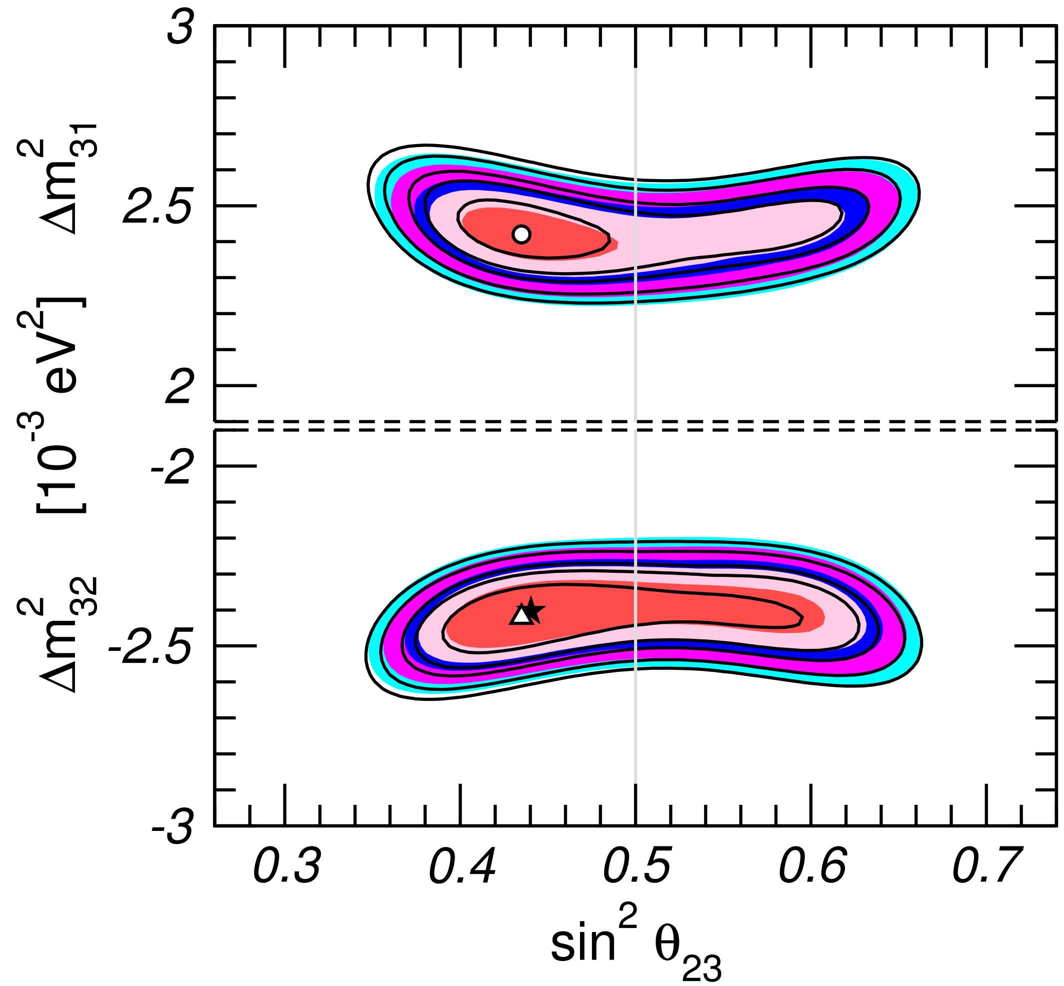
See Kayser's lectures

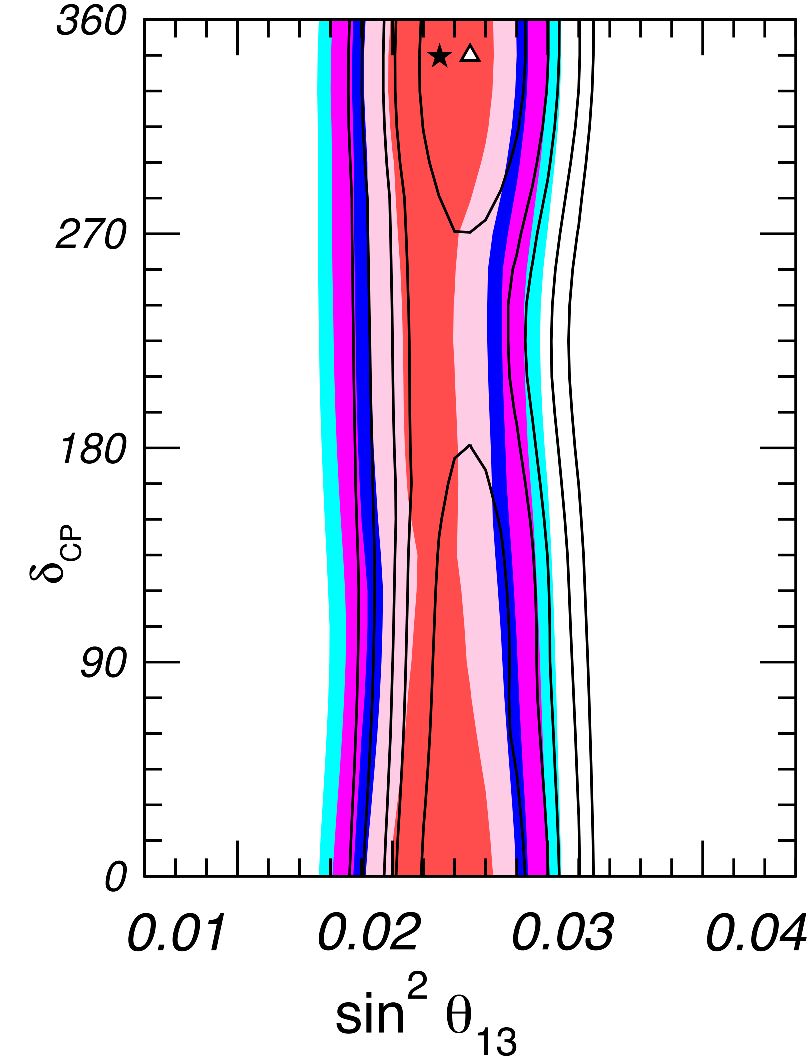


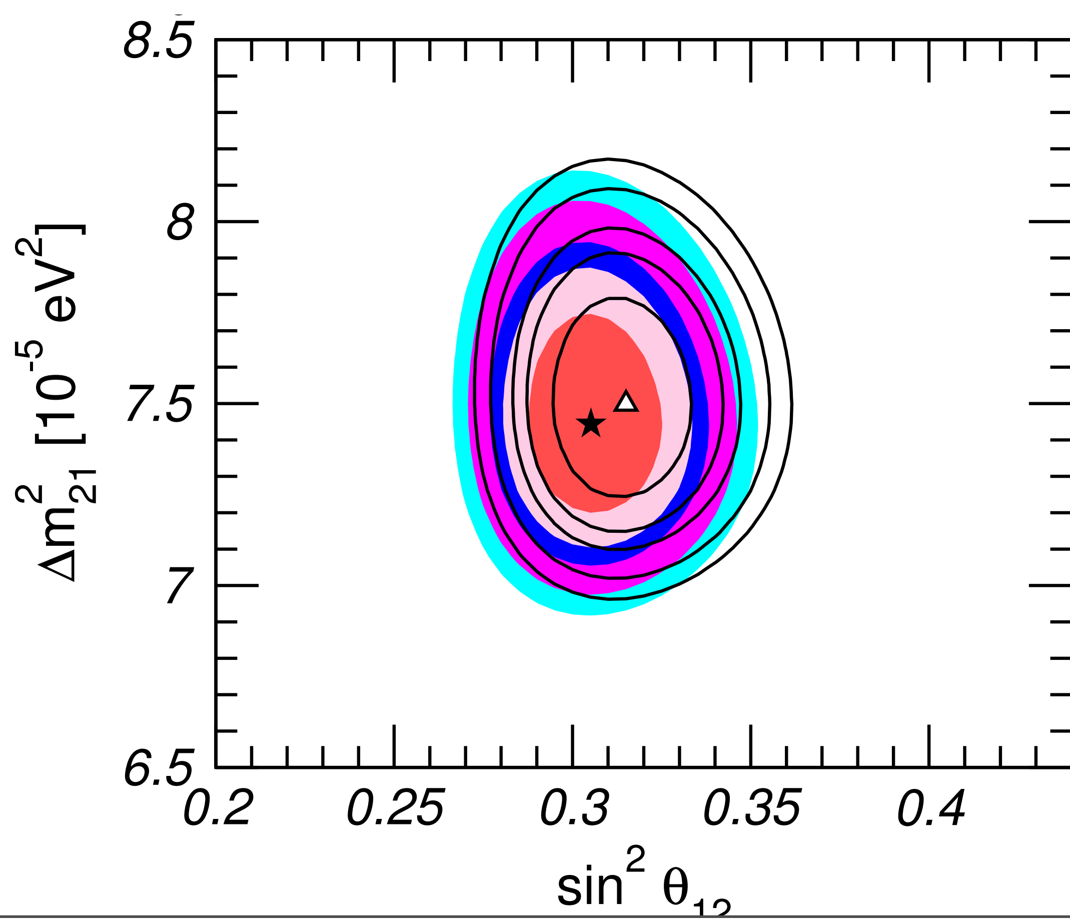
Light orange = muon neutrino

Dark orange = electron neutrino

Neutrino oscillations imply that neutrinos have mass and they mix.
First evidence of physics beyond the SM.







NuFit: M. C. Gonzalez-Garcia et al., 1209.3023

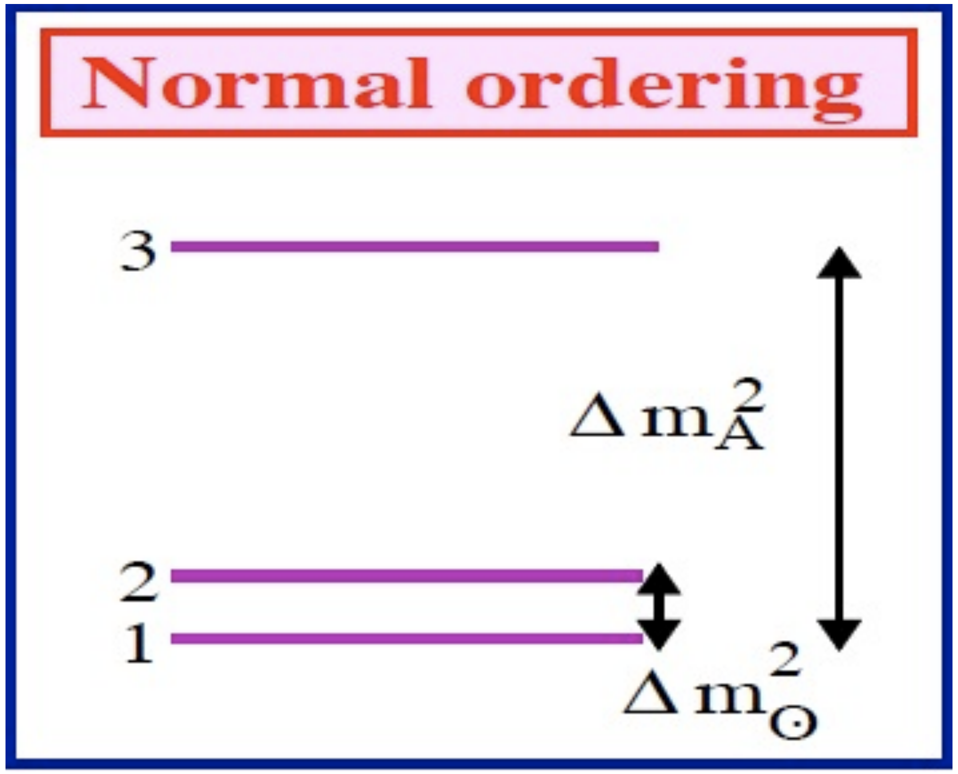
Global fit to neutrino oscillation data

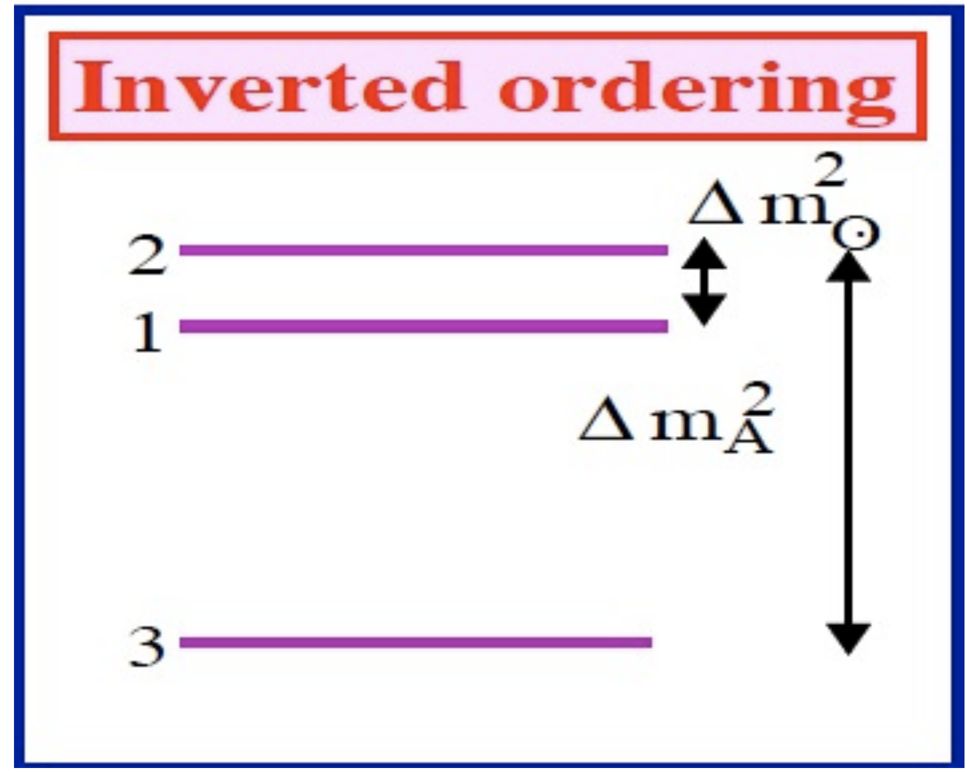
www.invisibles.eu

All oscillation parameters are measured with good precision, except for the mass hierarchy and the delta phase. One needs to check the 3-neutrino paradigm (sterile neutrino?).

Present status of neutrino masses

$\Delta m^2_{\rm s} \ll \Delta m^2_{\rm A}$ implies at least 3 massive neutrinos.





$$m_1 = m_{\rm min}$$
$$m_2 = \sqrt{m^2_{\rm min} + \Delta m^2_{\rm sol}}$$
$$m_3 = \sqrt{m^2_{\rm min} + \Delta m^2_{\rm A}}$$

$$m_3 = m_{\rm min}$$
$$m_1 = \sqrt{m^2_{\rm min} + \Delta m^2_{\rm A} - \Delta m^2_{\rm sol}}$$
$$m_2 = \sqrt{m^2_{\rm min} + \Delta m^2_{\rm A}}$$

Measuring the masses requires: $m_{\rm min}$ and the ordering.

Masses are much smaller than the other fermions.

Neutrino mixing

Mixing is described by the Pontecorvo-Maki-Nakagawa-Sakata matrix, which enters in the CC interactions

$$|\nu_\alpha\rangle = \sum_i U_{\alpha i} |\nu_i\rangle$$

Mass states

Flavour states

$$\mathcal{L}_{CC} = -\frac{g}{\sqrt{2}} \sum_{k\alpha} \left(U^*_{\alpha k} \bar{\nu}_{kL} \gamma^\rho l_{\alpha L} W_\rho + \text{h.c.} \right)$$

Large angles

$$U = \underbrace{\begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{\text{Solar, reactor } \theta_\odot \sim 30^o} \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}}_{\text{Atm, Acc. } \theta_A \sim 45^o}$$

CPV?

$$\underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{-i\delta} \end{pmatrix}}_{\text{CPV phase}} \underbrace{\begin{pmatrix} c_{13} & 0 & s_{13} \\ 0 & 1 & 0 \\ -s_{13} & 0 & c_{13} \end{pmatrix}}_{\text{Reactor, Acc. } \theta_{13} \sim 9^o} \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{-i\alpha_{21}/2} & 0 \\ 0 & 0 & e^{-i\alpha_{31}/2+i\delta} \end{pmatrix}}_{\text{CPV Majorana phases}}$$

Mixing angles are much larger than in the quark sector.

Phenomenology questions for the future

- **1. What is the nature of neutrinos?**
- **2. What are the values of the masses?** Absolute scale (KATRIN, ...?) and the ordering.
- **3. Is there CP-violation?** Its discovery in the next generation of LBL depends on the value of delta.
- **4. What are the precise values of mixing angles?** Do they suggest a underlying pattern?
- **5. Is the standard picture correct?** Are there NSI? Sterile neutrinos? Other effects?

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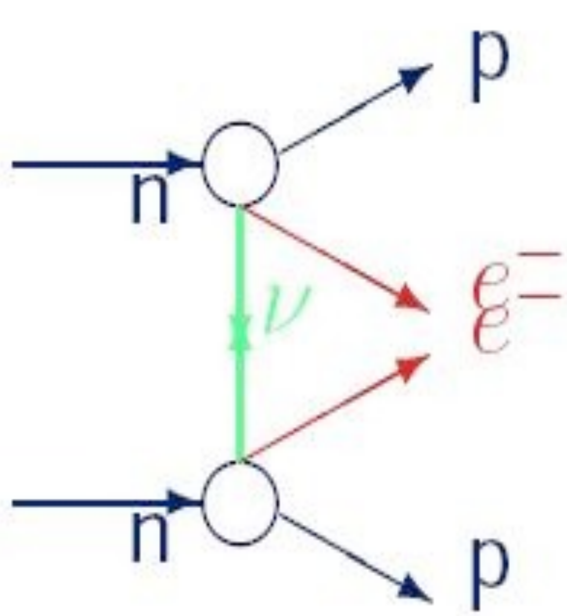
Nature of Neutrinos See Yang's lectures

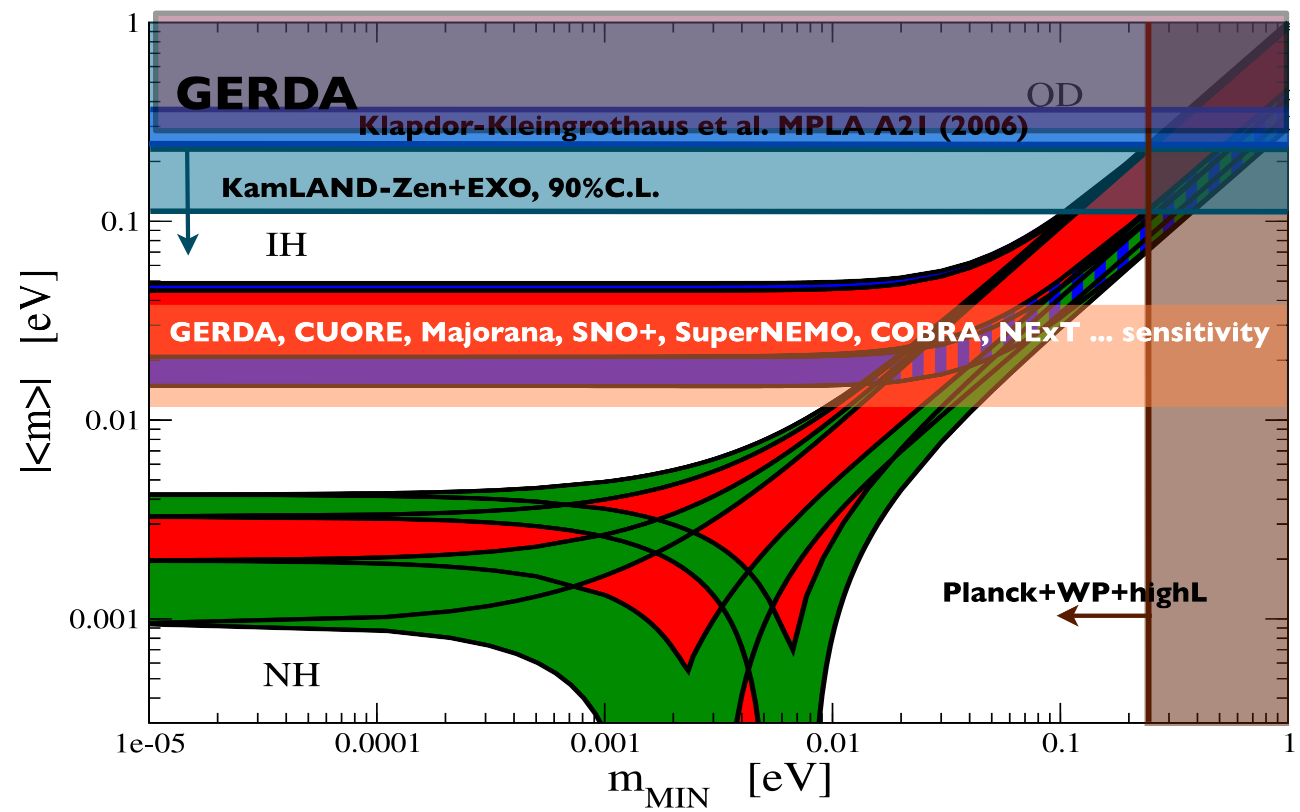
Neutrinos can be **Majorana** or **Dirac** particles. In the SM only neutrinos can be Majorana because they are neutral.

Majorana particles are indistinguishable from antiparticles.

Dirac neutrinos are labelled by the lepton number.

Testable in neutrinoless double beta decay





The **nature** of neutrinos is linked to the conservation of **Lepton number**. **This information is crucial to understand the origin of neutrino masses** and it can be linked to the existence of matter in the Universe.

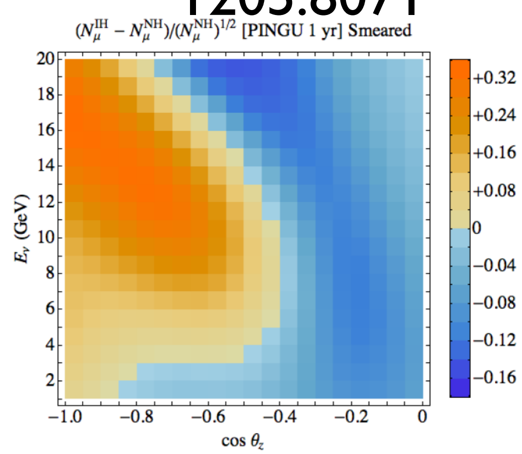
Neutrino masses

- **Neutrino mass ordering (hierarchy)**
  - use matter effects in neutrino oscillations

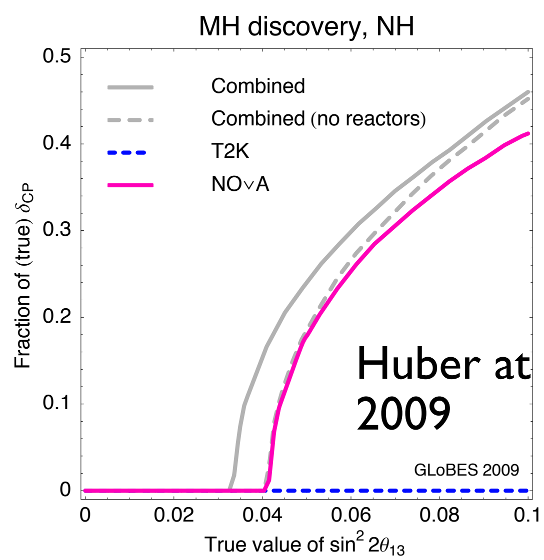
Future long baseline experiments: LBNE, LBNO, T2HK, nuFact

Atmospheric nus: ORCA, PINGU, INO

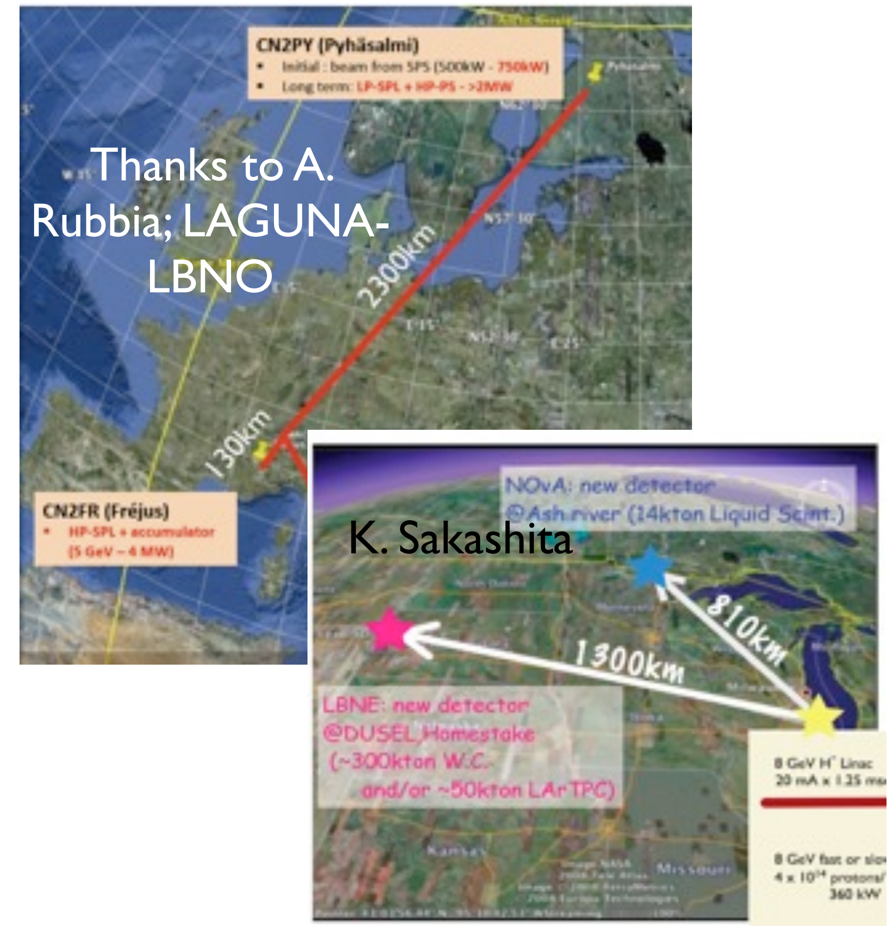
Akhmedov, Razzaque, Smirnov, 1205.8071



T2K and NOvA



Huber at al., 2009

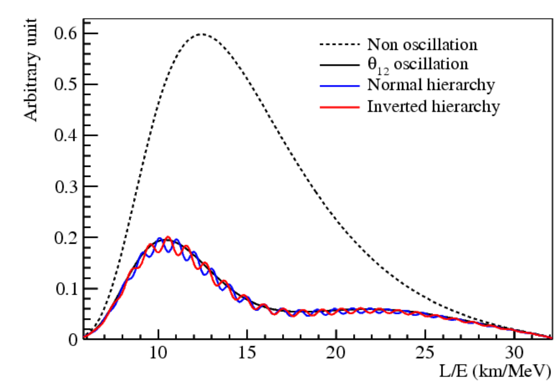


Thanks to A. Rubbia; LAGUNA-LBNO

K. Sakashita

- vacuum oscillations in reactors: **JUNO** will be able to determine the mass ordering

Petcov et al., 2001



- neutrinoless double beta decay

See Kayser's lectures

- **Absolute mass scale**

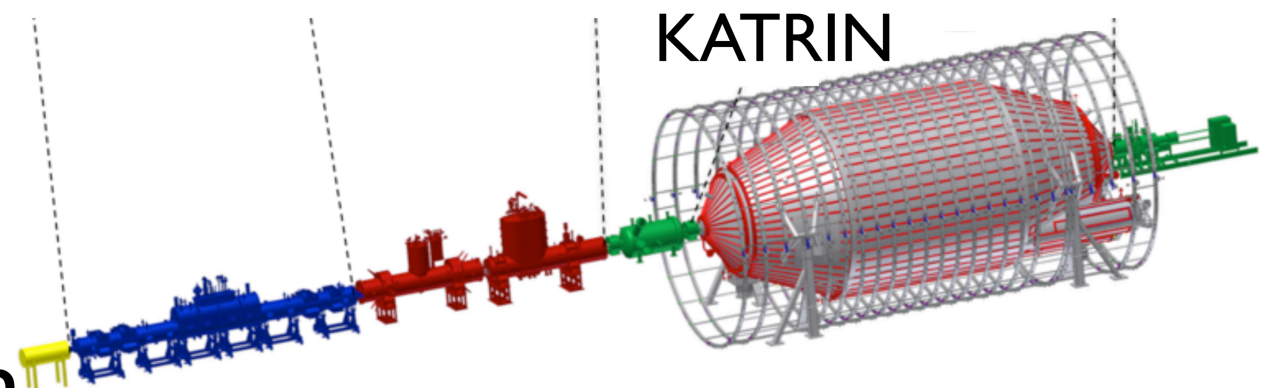
See Yang's and Adams' lectures

- KATRIN studies the end point of the beta decay spectrum

Current bound:

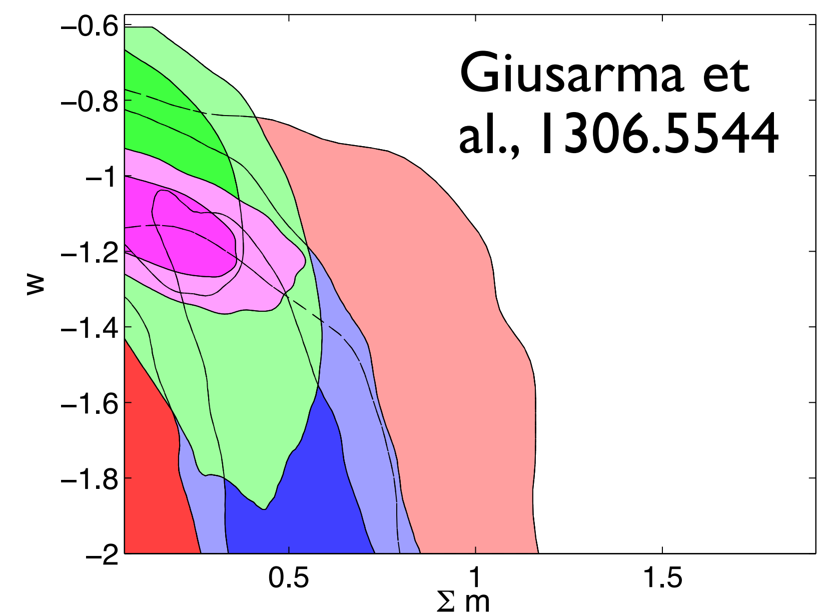
$$\sum_i m_i^2 |U_{ei}|^2 < 2.2 \text{ eV}^2$$

Troitzk and Mainz



- cosmology. The Early Universe is sensitive to the sum of neutrino masses due to their gravitational effects. They are a component of HDM and their presence smooths out the density perturbations.

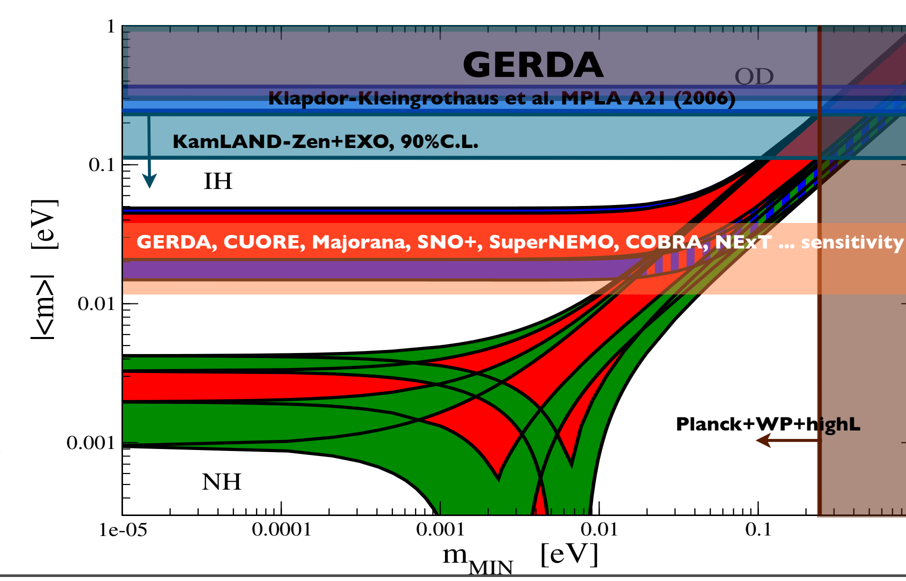
Current bound: $\sum_i m_i < \text{few } 0.1 \text{eV}$



- neutrinoless double beta decay

A measurement of |<m>| can identify a region for m.

Current bound: $\sum_i m_i U_{ei}^2 < 0.2 - 0.3 \ (0.6 - 1) \text{ eV}$

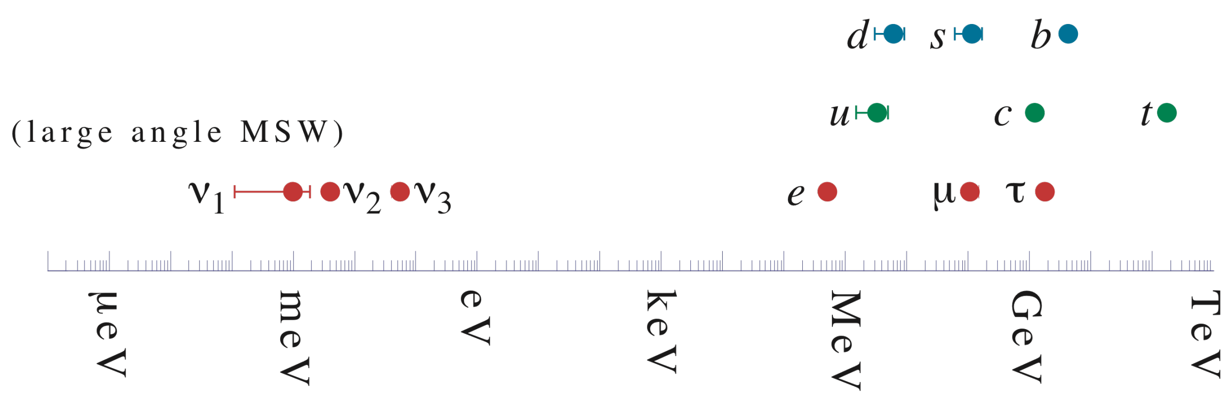


The ultimate goal is to understand
- where do neutrino masses come from?
- why there is leptonic mixing? and what is at the origin of the observed structure?

Open window on Physics beyond the SM

Neutrino physics gives a new perspective on physics BSM.

1. Origin of masses



Why neutrinos have mass?
and why are they so much lighter?
and why their hierarchy is at most mild?

2. Problem of flavour

$$\begin{pmatrix} \sim 1 & \lambda & \lambda^3 \\ \lambda & \sim 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & \sim 1 \end{pmatrix} \quad \lambda \sim 0.2$$

$$\begin{pmatrix} 0.8 & 0.5 & 0.16 \\ -0.4 & 0.5 & -0.7 \\ -0.4 & 0.5 & 0.7 \end{pmatrix}$$

Why leptonic mixing is so different from quark mixing?

This information is **complementary** with the one from flavour physics experiments and from colliders.

Useful formalism

We will use gamma matrices

$$\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}$$

The most used representations are the Pauli-Dirac, Weyl and Majorana ones. In the Weyl representation we have

$$\gamma^0 = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} \quad , \gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix}$$

$$\gamma^5 \equiv i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Then we define

See Xiangdong Ji's lectures

$$\psi_L \equiv \frac{1-\gamma^5}{2}\psi = \begin{pmatrix} 0 \\ \eta \end{pmatrix} \qquad \psi_R \equiv \frac{1+\gamma^5}{2}\psi = \begin{pmatrix} \xi \\ 0 \end{pmatrix}$$

Charge conjugation

This operation changes a field in its charge-conjugate (opposite quantum numbers but same chirality):

$$\boxed{\psi^c = C\bar{\psi}^T = i\gamma^2\psi^*}$$

Properties: $C\gamma^{\alpha T}C^\dagger = -\gamma^\alpha \quad , \quad CC^\dagger = 1 \quad , \quad C^T = -C$

In Weyl representation: $C = i\gamma^2\gamma^0$

Let's apply it to a left-handed field

$$(\psi_L)^c = i\gamma^2\psi_L^* = i\begin{pmatrix} 0 & \sigma^2 \\ -\sigma^2 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ \eta^* \end{pmatrix} = \begin{pmatrix} i\sigma^2\eta^* \\ 0 \end{pmatrix}$$

We find that it behaves as a right-handed field!

$$\boxed{(\psi_L)^c = (\psi^c)_R}$$

Exercise
using the properties of the C matrix, show that this equation is true independently of the representation of the gamma matrices.

What kind of masses can neutrinos have?

Neutrino masses

A mass term for a fermion connects a left-handed field with a right-handed one. For example the "usual" Dirac mass

$$m_\psi(\bar{\psi}_R \psi_L + \mathrm{h.c.}) = m_\psi \bar{\psi}\psi$$

Exercise
check this formula

Dirac masses

This is the simplest case. We assume that we have two independent Weyl fields: ν_L , ν_R
and we can write down the term as above.

$$\mathcal{L}_{mD} = -m_\nu(\bar{\nu}_R \nu_L + \mathrm{h.c.})$$

Does it conserve lepton number?

$$\nu_L \to e^{i(+1)\alpha} \nu_L$$
$$\nu_R \to e^{i(?)\alpha} \nu_R$$

@Silvia Pascoli

Friday, 9 August 13

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This conserves lepton number!

$$\nu_L \to e^{i\alpha}\nu_L$$
$$\nu_R \to e^{i\alpha}\nu_R$$

$$\mathcal{L}_{mD} \to \mathcal{L}_{mD}$$

Diagonalize a Dirac mass term

If there are several fields, there will be a Dirac mass matrix.

$$\mathcal{L}_{mD} = -\bar{\nu}_{Ra} \, (m_D)_{ab} \, \nu_{Lb} + \text{h.c.}$$

This requires two unitary mixing matrices to diagonalise it

$$m_D = V m_{\text{diag}} U^\dagger$$

and the massive states are

$$n_L = U^\dagger \nu_L \quad n_R = V^\dagger \nu_R$$

This is the mixing matrix which enters in neutrino oscillations. So the form of the mass matrix determines the mixing pattern.

Majorana masses

If we have only the left-handed field, we can still write down a mass term, called Majorana mass term. We use the fact that

$$(\psi_L)^c = (\psi^c)_R$$

then the mass term is

$$\mathcal{L}_{mM} \propto -M_M \bar{\nu_L^c} \nu_L + \text{h.c.} = M_M \nu_L^T C^{-1} \nu_L$$

Exercise
Show that these two formulations are equivalent.

Hint:

$$\bar{\nu_L^c} \nu_L = (C \bar{\nu}_L^T)^\dagger \gamma^0 \nu_L = \bar{\nu}_L^* C^\dagger \gamma^0 \nu_L$$
$$= \nu_L^T \gamma^{0*} C^\dagger \gamma^0 \nu_L = -\nu_L^T C^{-1} \nu_L$$

This breaks lepton number!

$$\nu_L \to e^{i\alpha} \nu_L \qquad \mathcal{L}_{mM} \to e^{2i\alpha} \mathcal{L}_{mM}$$

Diagonalize a Majorana mass term

If there are several fields, there will be a Majorana mass matrix. We can show that it is symmetric.

$$\boxed{M_M = M_M^T}$$

In fact:

$$\nu_L^T M_M C^{-1} \nu_L = (\nu_L^T M_M C^{-1} \nu_L)^T$$
$$= -\nu_L^T M_M^T C^{-1,T} \nu_L = \nu_L^T M_M^T C^{-1} \nu_L$$

This implies that only one unitary mixing matrix is required to diagonalise it

$$\boxed{M_M = (U^\dagger)^T m_{\text{diag}} U^\dagger}$$

The massive fields are related to the flavour ones as

$$\boxed{n_L = U^\dagger \nu_L}$$

and the Lagrangian can be rewritten in terms of a Majorana field

$$\mathcal{L}_M = -\frac{1}{2}\overline{n_L^c} m_{\rm diag} n_L - \frac{1}{2}\overline{n_L} m_{\rm diag} n_L^c = -\frac{1}{2}\bar{\chi} m_{\rm diag} \chi$$

with $$\boxed{\chi \equiv n_L + n_L^c \Rightarrow \chi = \chi^c}$$

A Majorana mass term (breaks L) leads to Majorana neutrinos (breaks L).

Dirac + Majorana masses

If we have both the left-handed and right-handed fields, we can write down three mass terms:

- a Dirac mass term
- a Majorana mass term for the left-handed field and
- a Majorana mass term for the right-handed field.

$$\mathcal{L}_{mD+M} = -m_\nu \bar{\nu}_R \nu_L - \frac{1}{2}\nu_L^T M_{M,L} C^{-1} \nu_L - \frac{1}{2}\nu_R^T M_{M,R} C^{-1} \nu_R + \text{h.c.}$$

What do we expect the massive neutrinos to be?
Dirac, Majorana, both?

Dirac + Majorana masses

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This breaks lepton number, in both the Majorana mass terms.

The expectation is that, as lepton number is not conserved, neutrinos will be Majorana particles. Let's prove it.

We start by rewriting $\mathcal{L}_{mD+M} = -\frac{1}{2}\bar{\psi^c_L}\mathcal{M}\psi_L + \text{h.c.}$

with $\psi_L \equiv \begin{pmatrix} \nu_L \\ \nu^c_R \end{pmatrix}$ and $\mathcal{M} \equiv \begin{pmatrix} M_{M,L} & m_D^T \\ m_D & M_{M,R} \end{pmatrix}$

In fact

$$\mathcal{L}_{mD+M} = -\frac{1}{2}\bar{\nu^c_L} M_{M,L}\nu_L - \frac{1}{2}\bar{\nu_R} M_{M,R}\nu_R - \bar{\nu}_R m_D \nu_L + \text{h.c.}$$

and one can use $\bar{\nu^c_L} m_D^T \nu^c_R = \bar{\nu}_R m_D \nu_L$

Exercise
Show that these two formulations are equivalent.

Then, we need to diagonalise the full mass matrix, and we find the Majorana massive states, in analogy to what we have done for the Majorana mass case.

$$\chi \equiv n_L + n_L^c \Rightarrow \chi = \chi^c$$

The difference is that $n_L = U_j \nu_L + U_k \nu^c_R$

Not unitary

Mixing between mass states and sterile neutrinos

@Silvia Pascoli

Summary of neutrino mass terms

Dirac masses

$$\mathcal{L}_{mD} = -m_\nu(\bar{\nu}_R \nu_L + \text{h.c.})$$

This term conserves lepton number.

Majorana masses

$$\mathcal{L}_{mM} \propto -M_M \bar{\nu^c_L} \nu_L + \text{h.c.} = M_M \nu_L^T C^{-1} \nu_L$$

This term breaks lepton number.

Dirac + Majorana masses

$$\mathcal{L}_{mD+M} = -m_\nu \bar{\nu}_R \nu_L - \frac{1}{2}\nu_L^T M_{M,L} C^{-1} \nu_L - \frac{1}{2}\nu_R^T M_{M,R} C^{-1} \nu_R + \text{h.c.}$$

Lepton number is broken -> Majorana neutrinos.

Can neutrino masses arise in the SM? and if not, how can we extend the SM to generate them?

Neutrino Masses in the SM and beyond

In the SM, neutrinos do not acquire mass and mixing:

- like the other fermions as there are no right-handed neutrinos.

$$m_e \bar{e}_L e_R \qquad\qquad m_\nu \bar{\nu}_L \cancel{\nu_R}$$

Solution: Introduce ν_R for Dirac masses

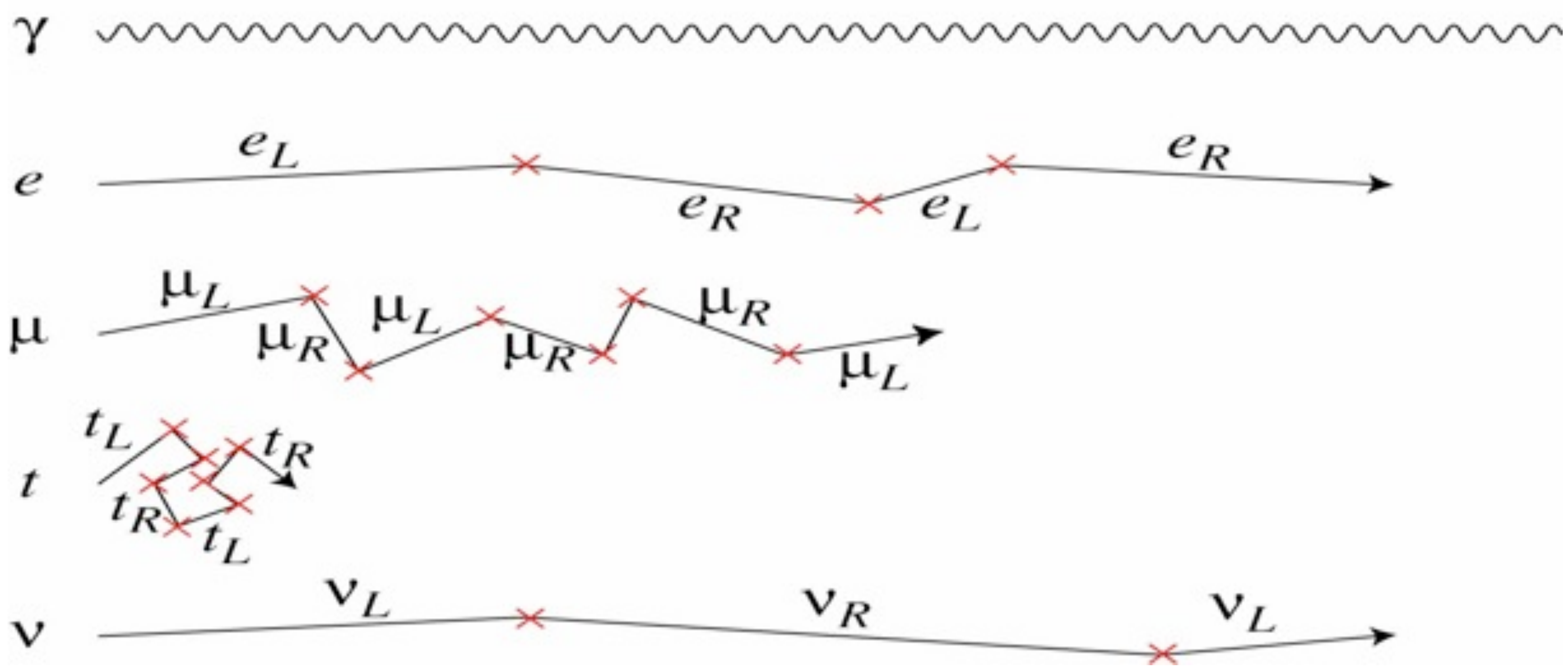
- they do not have a Majorana mass term

$$M \nu_L^T C \nu_L$$

as this term breaks the SU(2) gauge symmetry.
Solution: Introduce an SU(2) scalar triplet or gauge invariant non-renormalisable terms (D>4). This term breaks Lepton Number.

Dirac Masses

If we introduce a right-handed neutrino, then a lepton-number conserving interaction with the Higgs boson emerges.



Thanks to
H. Murayama

$$\mathcal{L} = -y_\nu \bar{L} \cdot \tilde{H} \nu_R + \text{h.c.}$$

See also Xiangdong Ji's lectures

with

$$L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \quad \text{and} \quad \tilde{H} = \begin{pmatrix} H^{0,*} \\ -H^- \end{pmatrix}$$

This term is
- SU(2) invariant and
- respects lepton number

When the neutral component of the Higgs field gets a vev, a Dirac mass term for neutrinos is generated.

$$
\begin{aligned}
\mathcal{L}_{\nu H} &= -y_\nu (\bar{\nu}_L, \bar{\ell}_L) \cdot \begin{pmatrix} H^{0*} \\ -H^- \end{pmatrix} \nu_R + \text{h.c.} \\
&= -y_\nu (\bar{\nu}_L H^{0*} - \bar{\ell}_L H^-) \nu_R + \text{h.c.} \\
&= -y_\nu \frac{v_H}{\sqrt{2}} \bar{\nu}_L \nu_R + \text{h.c.} + \ldots
\end{aligned}
$$

$H^0 \to \frac{v_H}{\sqrt{2}} + h^0$

It follows that

$$
y_\nu \sim \frac{\sqrt{2} m_\nu}{v_H} \sim \frac{0.2 \text{ eV}}{200 \text{ GeV}} \sim 10^{-12}
$$

Tiny couplings!

Many theorists consider this explanation of neutrino masses not satisfactory. We would expect this Yukawa couplings to be similar to the ones in the quark sector:

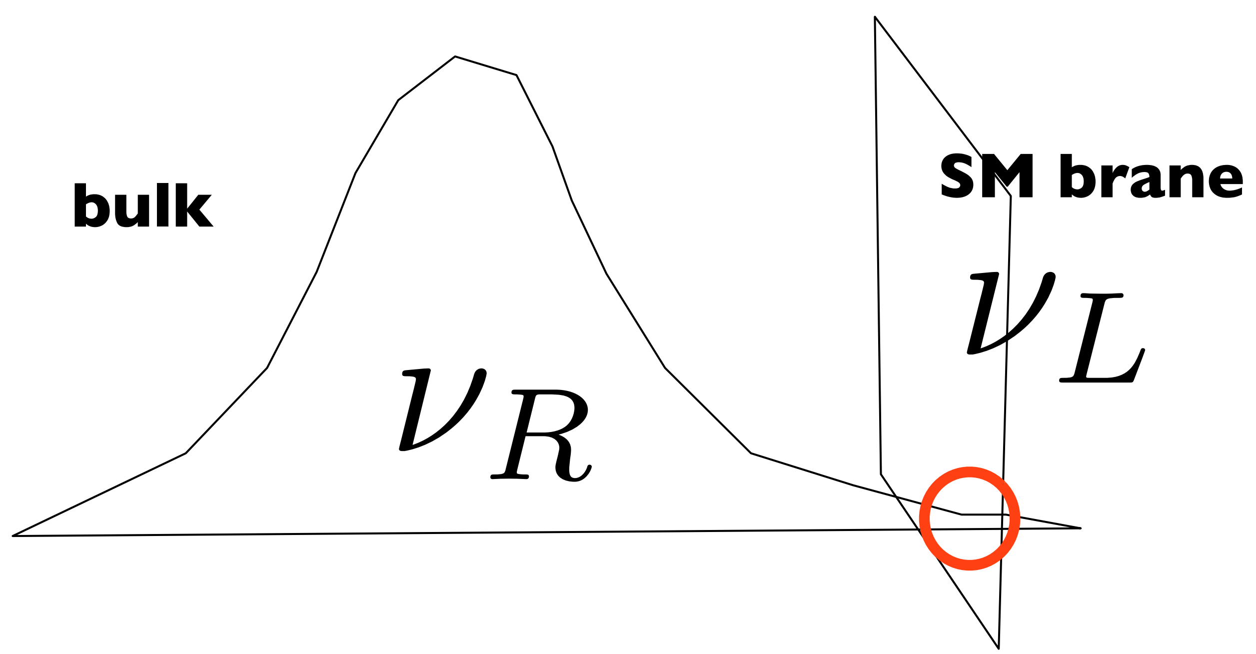
1. why the coupling is so small????
2. why the mixings are large? (instead of small as in the quark sector)
3. why neutrino masses have at most a mild hierarchy if they are not quasi-degenerate? instead of what happens to quarks?

Dirac masses are strictly linked to lepton number conservation. But this is an accidental global symmetry. Should it be conserved at high scales?

There are models which address the problem of the smallness of the couplings.

Extra-D models

In these models all gauge-interacting fields are in the SM brane. Right-handed neutrinos are singlets and therefore will be in the bulk.



The overlap of the wavefunctions (which are normalised) of the left-handed and right-handed neutrinos leads to a small Yukawa coupling.

See e.g. Arkani-Hamed et al., 2002; Grossman and Neubert, 2000. Models with warped extra-D....

Majorana Masses

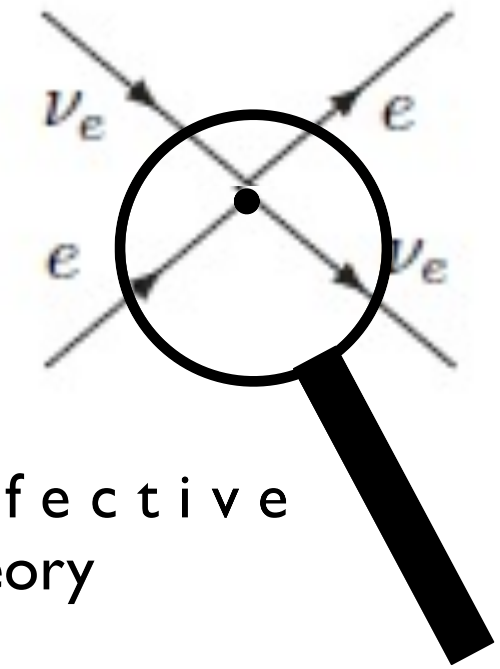
In order to have an SU(2) invariant mass term for neutrinos, it is necessary to introduce a Dimension 5 operator (or to allow for new scalar fields, e.g. a scalar triplet):

$$-\mathcal{L} = \lambda \frac{\nu_L H \, \nu_L H}{M} = \frac{\lambda v^2}{M} \nu_L^T C \nu_L$$

D=5 term

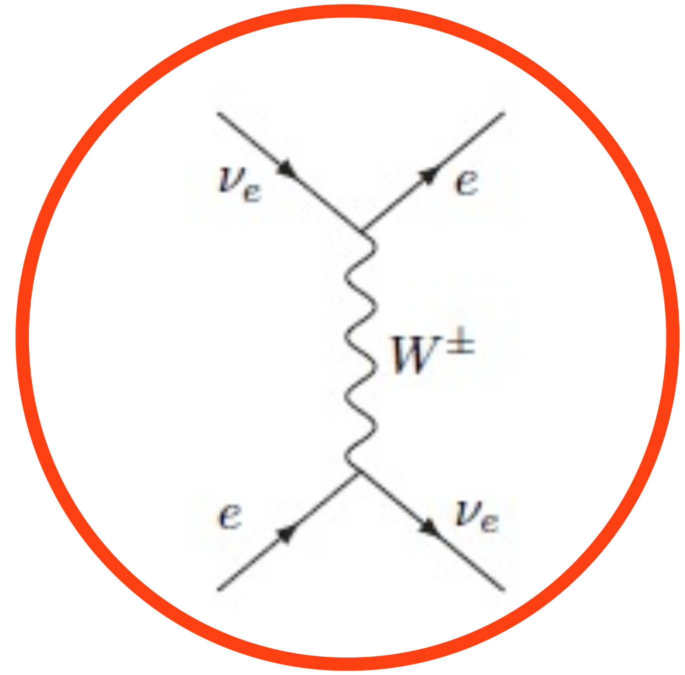
Lepton number violation!

If neutrino are Majorana particles, a **Majorana mass** can arise as the **low energy realisation of a higher energy theory (new mass scale!)**.



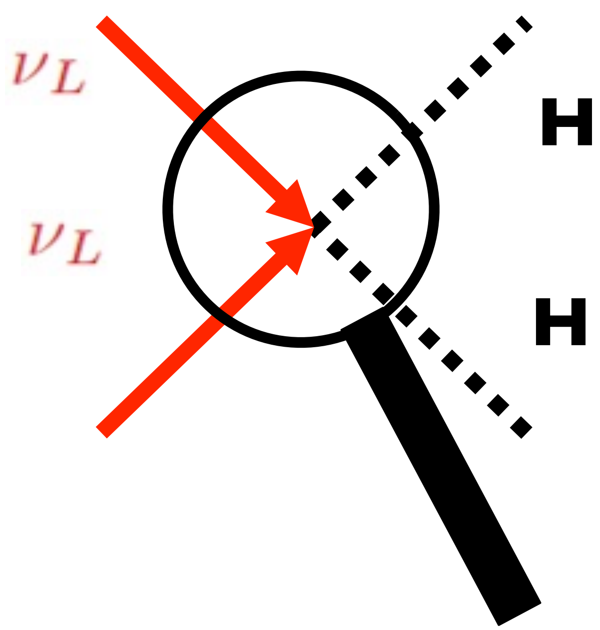
effective theory

$$\mathcal{L} \propto G_F(\bar{e}_L \gamma_\mu \nu_L)(\bar{\nu}_L \gamma^\mu e_L)$$



Standard Model:
W exchange

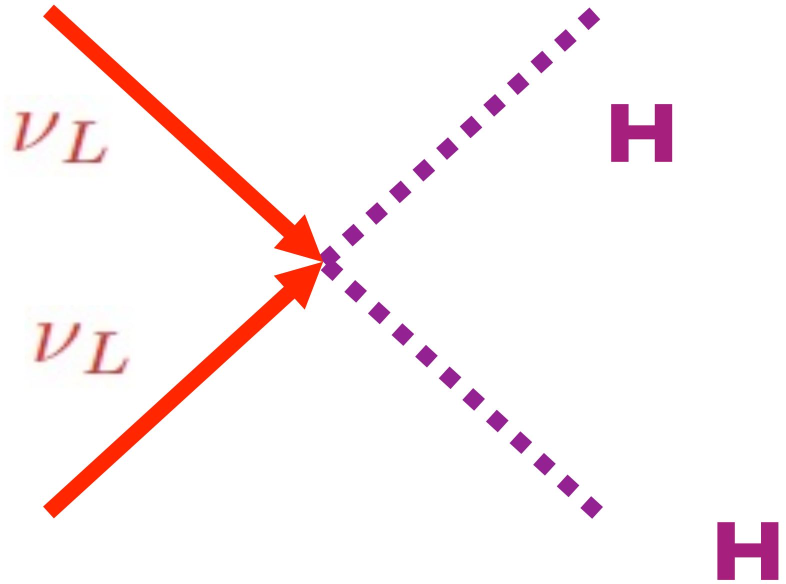
$$\mathcal{L}_{SM} \propto g \bar{\nu}_L \gamma^\mu e_L W_\mu \Rightarrow G_F \propto \frac{g^2}{m_W^2}$$



Neutrino mass

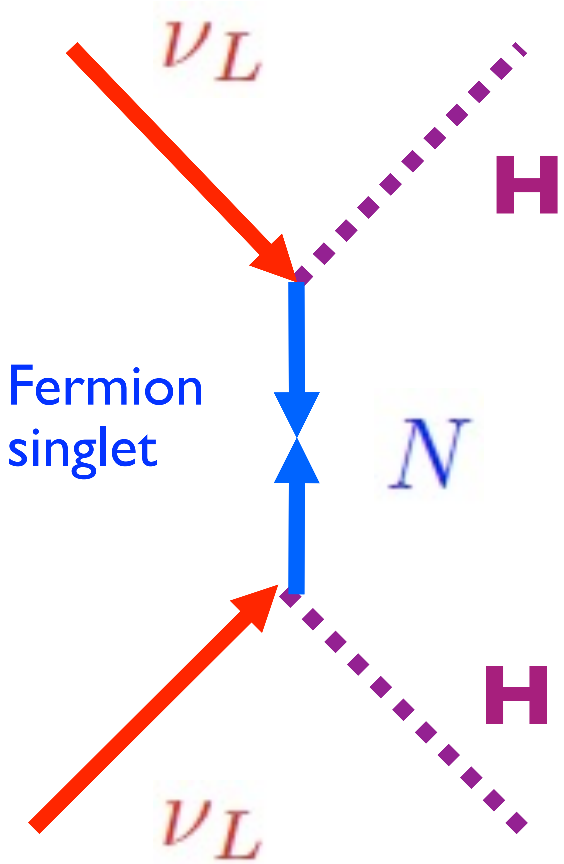
$$-\mathcal{L} = \lambda \frac{\nu_L H \, \nu_L H}{M} = \frac{\lambda v^2}{M} \nu_L^T C \nu_L$$

New theory:
new particle exchange
with mass M



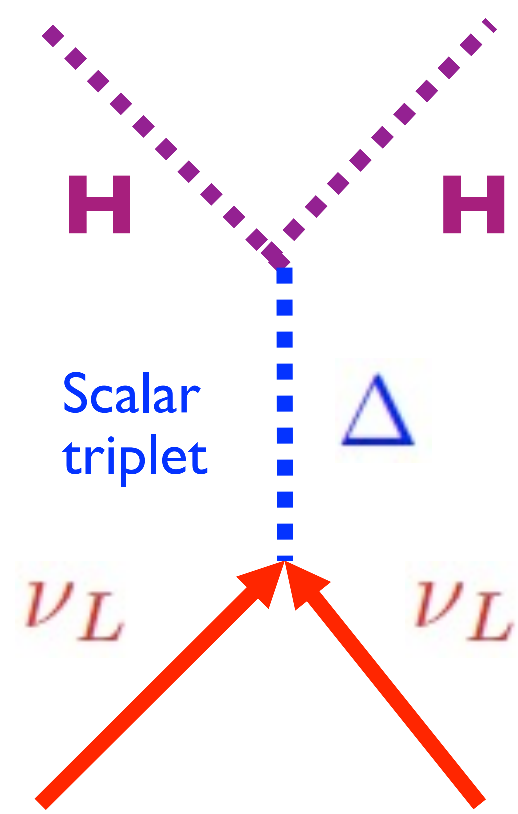
$$\frac{\nu_L H \, \nu_L H}{\Lambda}$$

See-saw Type I



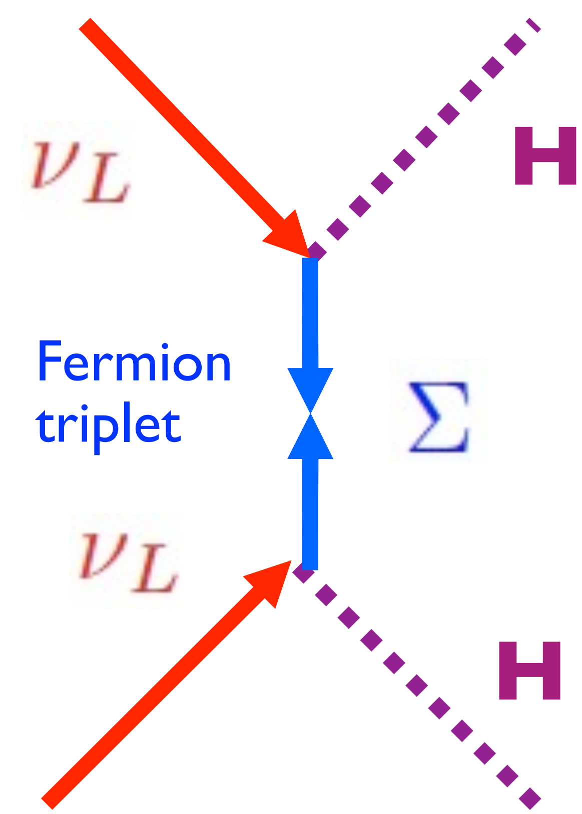
Minkowski, Yanagida, Glashow, Gell-Mann, Ramond, Slansky, Mohapatra, Senjanovic

See-saw Type II



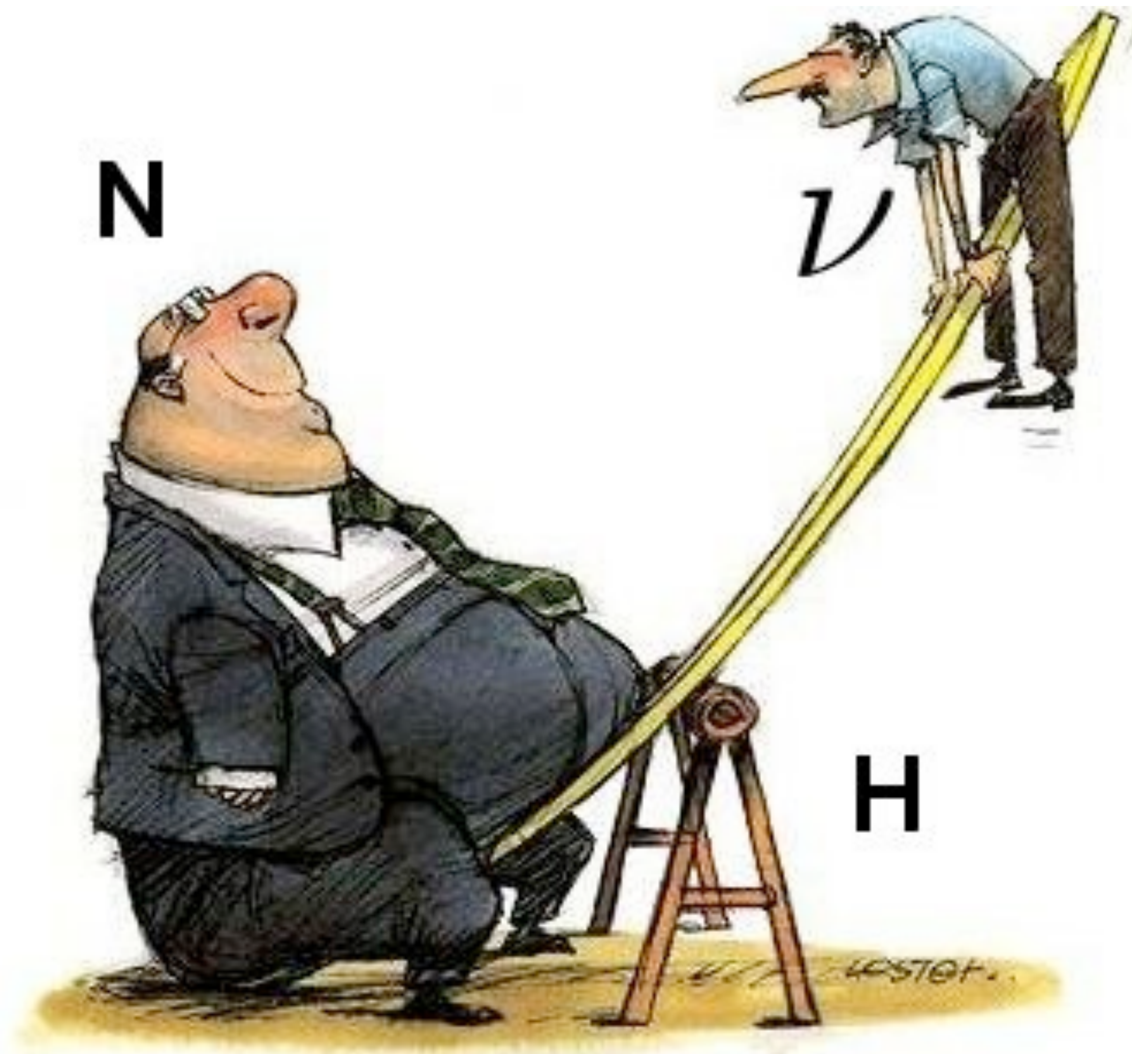
Magg, Wetterich, Lazarides, Shafi. Mohapatra, Senjanovic, Schecter, Valle

See-saw Type III



Ma, Roy, Senjanovic, Hambye

The simplest see saw mechanism: type I



- Introduce a right handed neutrino **N (sterile neutrino)**
- Couple it to the Higgs and left handed neutrinos

The Lagrangian is

breaks lepton number

$$\mathcal{L} = -Y_\nu \bar{N} L \cdot H - 1/2 \bar{N}^c M_R N$$

When the Higgs boson gets a vev, Dirac masses will be generated and the mass matrix will be (for one generation)

$$\mathcal{L} = \left(\nu_L^T N^T\right) \begin{pmatrix} 0 & m_D \\ m_D^T & M \end{pmatrix} \begin{pmatrix} \nu_L \\ N \end{pmatrix}$$

This is of the Dirac+Majorana type we discussed earlier. So we know that the massive states are found by diagonalising the mass matrix and the massive states will be Majorana neutrinos.

$$\begin{vmatrix} -\lambda & m_D \\ m_D & M - \lambda \end{vmatrix} = 0$$

$$\lambda^2 - M\lambda - m_D^2 = 0$$

$$\lambda_{1,2} = \frac{M \pm \sqrt{M^2 + 4m_D^2}}{2} \simeq \begin{matrix} M \\ \frac{M-M}{2} - \frac{4m_D^2}{4M} = -\frac{m_D^2}{M} \end{matrix}$$

One massive state remains very heavy, the light neutrino masses acquires a tiny mass!

$$m_\nu \simeq \frac{m_D^2}{M} \sim \frac{1\,\mathrm{GeV}^2}{10^{10}\,\mathrm{GeV}} \sim 0.1\,\mathrm{eV}$$

Mixing between active neutrinos and heavy neutrinos will emerge but it will be typically very small

$$\tan 2\theta = \frac{2m_D}{M}$$

Relevant for tomorrow's discussion.

and can be related to neutrino masses

$$m_\nu \simeq \frac{m_D^2}{M} \simeq \sin^2\theta M$$

It is necessary to introduce more than one sterile neutrino. (With one only, two masses will be zero)

$$m_\nu = U^* d_m U^\dagger \simeq -y_\nu^T M_R^{-1} y_\nu v_H^2$$

It is possible to invert this matrix and express the Yukawa couplings in terms of the low energy measurable parameters.

$$U^* d_m^{1/2} d_m^{1/2} U^\dagger \simeq -y_\nu^T d_M^{-1/2} d_M^{-1/2} y_\nu v_H^2$$

$$U^* d_m^{1/2} R^T R d_m^{1/2} U^\dagger \simeq -y_\nu^T d_M^{-1/2} d_M^{-1/2} y_\nu v_H^2$$

$$y_\nu \simeq \frac{1}{v_H} d_M^{1/2} \underbrace{R}_{\text{Unknown}} \underbrace{d_m^{1/2}}_{\text{Measurable}} \underbrace{U^\dagger}_{\text{Measurable}}$$

Casas, Ibarra, 2001

Can we determine the high energy parameters from low energy measurements in experiments?
Parameter counting.

High energy parameters			Low energy parameters		
M_R	3	0	d_m	3	0
λ	9	6	U	3	3

At high energy there are 6 extra real parameters and 3 phases. So in a model independent way there is no direct connection.

However, in specific models of neutrino masses and flavour structure, the number of parameters will be reduced and a direct connection can be there.
See other lectures.

Pros and cons of type I see-saw models

Pros:

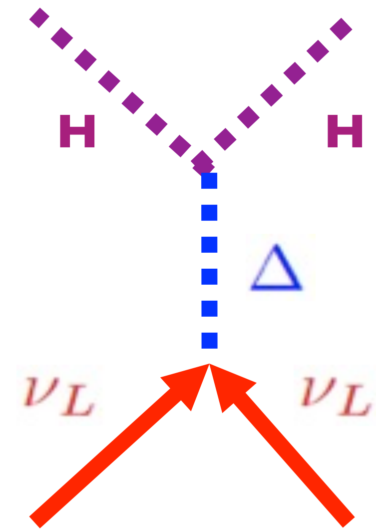
- they explain "naturally" the smallness of neutrino masses.
- can be embedded in GUT theories!
- neutrino masses are a indirect test of GUT theories
- have several phenomenological consequences (depending on the mass scale), e.g. leptogenesis, LFV

Cons:

- the new particles are typically too heavy to be produced at colliders (but TeV scale see-saws)
- the mixing with the new states are tiny
- in general: difficult to test

See saw type II

We introduce a Higgs triplet which couples to the Higgs and left handed neutrinos. It has hypercharge 2.



$$\mathcal{L}_\Delta \propto y_\Delta L^T C^{-1} \sigma_i \Delta_i L + \text{h.c.}$$

with $$\Delta_i = \begin{pmatrix} \Delta^{++} \\ \Delta^{+} \\ \Delta^{0} \end{pmatrix}$$

Once the Higgs triplet gets a vev, Majorana neutrino masses arise: $\boxed{m_\nu \sim y_\Delta v_\Delta}$

Cons: why the vev is very small?
Pros: the component of the Higgs triplet could tested directly at the LHC.

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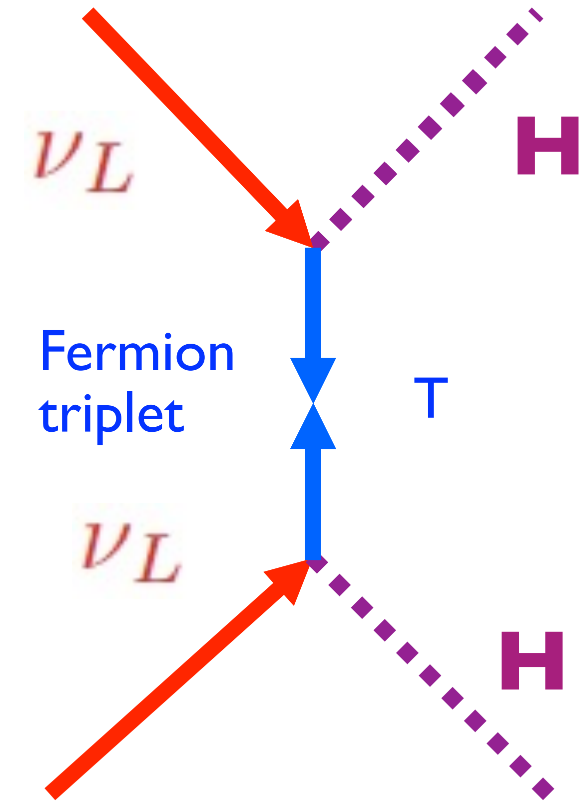
See saw type III

We introduce a fermionic triplet which has hypercharge 0.

$$\mathcal{L}_T \propto y_T \bar{L} \sigma H \cdot T + \text{h.c.}$$

with $T = \begin{pmatrix} T^0 & T^+ \\ T^- & -T^0 \end{pmatrix}$



Majorana neutrino masses are generated as in see-saw type I: $\boxed{m_\nu \simeq -y_T^T M_T^{-1} y_T v_H^2}$

Pros: the component of the fermionic triplet have gauge interactions and can be produced at the LHC
Cons: why the mass of T is very large?

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Extensions of see-saw models

In see-saw type I typically the smallness of neutrino masses is related to a very heavy mass scale and/or small Yukawa couplings. Models in which it is possible to lower the mass scale, keeping large Yukawa couplings have been studied.

Let's introduce two right-handed singlet neutrinos.

$$\mathcal{L} = Y \bar{L} \cdot H N_1 + Y_2 \bar{L} \cdot H N_2^c + \Lambda \bar{N}_1 N_2 + \mu' N_1^T C N_1 + \mu N_2^T C N_2$$

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$$\begin{pmatrix} 0 & Yv & Y_2 v \\ Yv & \mu' & \Lambda \\ Y_2 v & \Lambda & \mu \end{pmatrix}$$

Depending on the assignment one can have different lepton numbers:

N1=1, N2=1:

$$\begin{pmatrix} 0 & Yv & Y_2 v \\ Yv & \mu' & \Lambda \\ Y_2 v & \Lambda & \mu \end{pmatrix}$$

N1=0, N2=-1:

$$\begin{pmatrix} 0 & Yv & Y_2 v \\ Yv & \mu' & \Lambda \\ Y_2 v & \Lambda & \mu \end{pmatrix}$$

N1=1, N2=0:

$$\begin{pmatrix} 0 & Yv & Y_2 v \\ Yv & \mu' & \Lambda \\ Y_2 v & \Lambda & \mu \end{pmatrix}$$

This implies that neutrino masses require

- $Yv, \quad \mu'$ (= standard see-saw plus light sterile neutrino)
- $Yv, \quad \Lambda, \quad \mu, \quad \text{and/or } Y_2 v \quad \text{and/or } \mu'$

$$m_{tree} \simeq -m_D^T M^{-1} m_D \simeq \frac{v^2}{2(\Lambda^2 - \mu'\mu)} \left(\mu Y_1^T Y_1 + \mu' Y_2^T Y_2 - \Lambda \left(Y_2^T Y_1 + Y_1^T Y_2\right)\right)$$

Small neutrino masses associated to small breaking of L.

Three interesting limits:

Gavela et al., 0906.1461; Ibarra, Molinaro, Petcov, 1103.6217

- Inverse see-saw: $\Lambda \gg \mu, \mu'$

Two quasi-Dirac neutrinos with large mixing

$$m_4 \approx -m_5 \approx \tilde{M}_1 \approx -\tilde{M}_2 \approx \Lambda, \qquad U_{e4} \approx U_{e5} \approx Y_{1e} v / 2\Lambda,$$

$$\Delta\tilde{M} \equiv |\tilde{M}_2| - |\tilde{M}_1| \approx \mu', \qquad m_\nu \simeq \frac{v_H^2}{\Lambda^2} \mu Y_1^T Y_1$$

Small parameter

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- Extended see-saw: $\mu' \gg \Lambda, \mu$

Kang, Kim, 2007; Majee et al., 2008; Mitra, Senjanovic, Vissani, 1108.0004

$$m_4 \approx \tilde{M}_1 \approx -\Lambda^2/\mu', \qquad U_{e4} \approx Y_{1e} v/\sqrt{2}\Lambda$$
$$m_5 \approx \tilde{M}_2 \approx \mu', \qquad U_{e5} \approx Y_{1e} v/\sqrt{2}\mu'$$

One very heavy neutrino, one light sterile neutrino and light neutrino masses. Differently from the standard see-saw, the mixing with the light sterile neutrino can induce cancellations in the neutrino masses. For example, for Y2=0, no neutrino masses at leading order.

- Linear see-saw: $\mu = \mu' = 0, \quad Y_2 v \;\; \text{small}$

Malinsky, Romao, Valle, 2005

Two quasi-Dirac neutrinos with large mixing. Neutrino masses are controlled by $Y_2 v$.
Large mixing between sterile neutrinos and light neutrinos and TeV scales are allowed.

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What is the new physics scale?

The **new Standard Model** will contain

- new particles at a new physics scale
- new interactions.

sub-eV eV keV MeV GeV TeV GUT scale

See-saw I (tiny couplings)

GUT see-saw I

TeV see-saw I (small couplings)
see-saw II
see-saw III
extended-type seesaws

Coupling with the dark sector. Neutrinos can be a portal to new physics: $\mathcal{L}_\nu = y\, \bar{L} \cdot H \ \text{new}$

What is the new physics scale?

Tomorrow we will look at how to test and distinguish different models.

sub-eV eV keV MeV GeV TeV GUT scale

Peak searches

Neutrino masses

Signatures

Indirect signals (proton decay)

Nu oscillations

Direct signals in colliders

Charged lepton flavour violation

Kinks in beta decay

Leptogenesis

Neutrinoless double beta decay

Summary

1. Neutrinos have masses and a wide experimental programme will measure them with precision.

2. Neutrino masses beyond the Standard Model: Dirac, Majorana and Dirac+Majorana masses

3. We have looked at models of masses BSM:
   - Dirac masses
   - see saw type I
   - see-saw type II
   - see-saw type III
   - extended-type see-saw models