Neutrino Phenomenology

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Part 2

Looking to the Future

The Open Questions

- What is the absolute scale of neutrino mass?

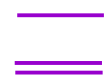
- Are neutrinos their own antiparticles?

- Are there ***more*** than 3 mass eigenstates?

- Are there non-weakly-interacting "sterile" neutrinos?

- What are the neutrino magnetic and electric dipole moments?

•How close to maximal (45^0) is θ_{23}?

•Is the spectrum like  or  ?

•Do neutrino interactions violate CP?

Is $P(\bar{\nu}_\alpha \rightarrow \bar{\nu}_\beta) \neq P(\nu_\alpha \rightarrow \nu_\beta)$?

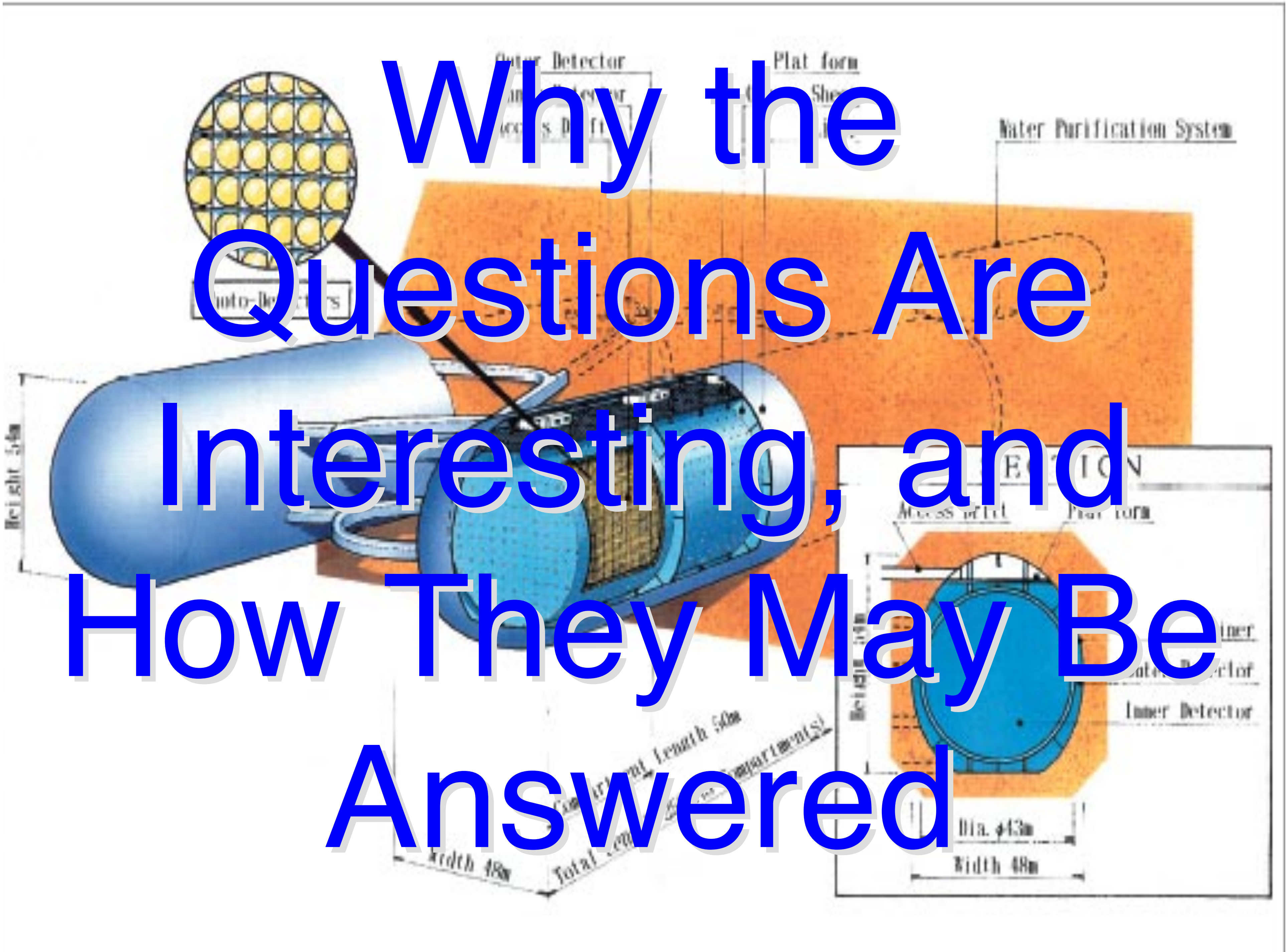
- What can neutrinos and the universe tell us about one another?

- Is CP violation involving neutrinos the key to understanding the matter – antimatter asymmetry of the universe?

- Where does neutrino mass come from?

- What surprises are in store?

Why the Questions Are Interesting, and How They May Be Answered



Does $\bar{\nu} = \nu$?

What Is the Question?

For each ***mass eigenstate*** ν_i , and ***given helicty*** h*,* does —

- $\overline{\nu_i}(h) = \nu_i(h)$ (Majorana neutrinos)

or

- $\overline{\nu_i}(h) \neq \nu_i(h)$ (Dirac neutrinos) ?

Equivalently, do neutrinos have *Majorana masses*? If they do, then the mass eigenstates are *Majorana neutrinos*.

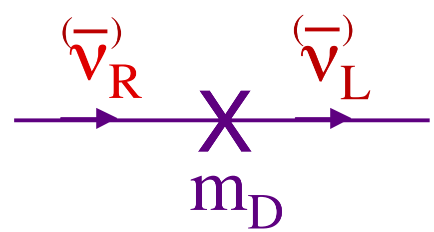
Dirac Masses

Dirac neutrino masses are the neutrino analogues of the SM quark and charged lepton masses.

To build a Dirac mass for the neutrino ν, we require not only the left-handed field ν_L in the Standard Model, but also a right-handed neutrino field ν_R.

The Dirac neutrino mass term is —

$$m_D \overline{\nu_L} \, \nu_R$$

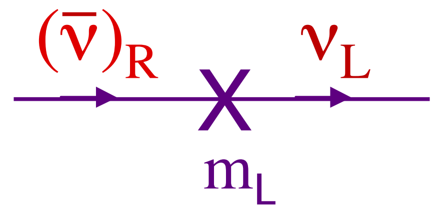


Dirac neutrino masses do not mix neutrinos and antineutrinos.

Majorana Masses

Out of, say, a left-handed neutrino field, ν_L, and its charge-conjugate, $\nu_L{}^c$, we can build a Left-Handed Majorana mass term —

$$m_L \overline{\nu_L}\, \nu_L{}^c$$



Majorana masses do mix ν and $\bar{\nu}$, so they do not conserve the Lepton Number L defined by —

$$L(\nu) = L(\ell^-) = -L(\bar{\nu}) = -L(\ell^+) = 1.$$

A Majorana mass for any fermion f causes f $\leftrightarrow$ $\bar{\mathrm{f}}$.

Quark and *charged-lepton* Majorana masses are forbidden by electric charge conservation.

***Neutrino* Majorana masses would make the neutrinos *very* distinctive.**

Majorana ν masses cannot arise via the Higgs mechanism:

$$\mathcal{L}_{SM} = fH_{SM}\bar{\nu}_L\nu_R \Rightarrow f\langle H_{SM}\rangle_0\bar{\nu}_L\nu_R \equiv m_\nu\bar{\nu}_L\nu_R$$

SM Higgs field (H_{SM})

Vacuum expectation value ($\langle H_{SM}\rangle_0$)

This, the ν analogue of the mechanism that produces the q and ℓ masses, leads only to a **Dirac** ν mass term.

Possible (Weak-Isospin-Conserving) couplings that can lead to Majorana mass terms:

$$\underbrace{H_{SM}H_{SM}\overline{\nu_L^c}\nu_L}_{\text{Not renormalizable}},\quad \underbrace{H_{I_W=1}}_{\text{This Higgs not in SM}}\overline{\nu_L^c}\nu_L,\quad \underbrace{m_R\overline{\nu_R^c}\nu_R}_{\text{No Higgs}}$$

Majorana neutrino masses must have a different origin than the masses of quarks and charged leptons.

Why Majorana Masses ➡ Majorana Neutrinos

The objects ν_L and $\nu_L{}^c$ in $m_L \overline{\nu_L} \nu_L{}^c$ are not the mass eigenstates, but just the neutrinos in terms of which the model is constructed. ν_L and $\nu_L{}^c$ are distinct.

$m_L \overline{\nu_L} \nu_L{}^c$ induces $\nu_L \leftrightarrow \nu_L{}^c$ mixing.

As a result of $K^0 \longleftrightarrow \overline{K^0}$ mixing, the neutral K mass eigenstates are —

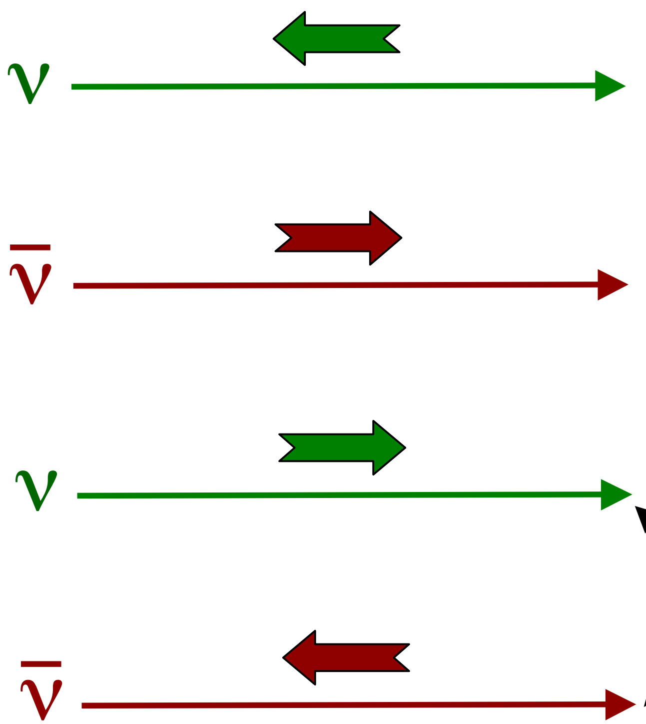
$$K_{S,L} \cong (K^0 \pm \overline{K^0})/\sqrt{2} \,. \qquad \overline{K_{S,L}} = K_{S,L} \,.$$

As a result of $\nu_L \leftrightarrow \nu_L{}^c$ mixing, the neutrino mass eigenstate is —

$$\nu_i = \nu_L + \nu_L{}^c = \text{“}\, \nu + \overline{\nu} \,\text{”}. \quad \overline{\nu_i} = \nu_i \,.$$

SM Interactions Of A Dirac Neutrino

We have 4 mass-degenerate states:

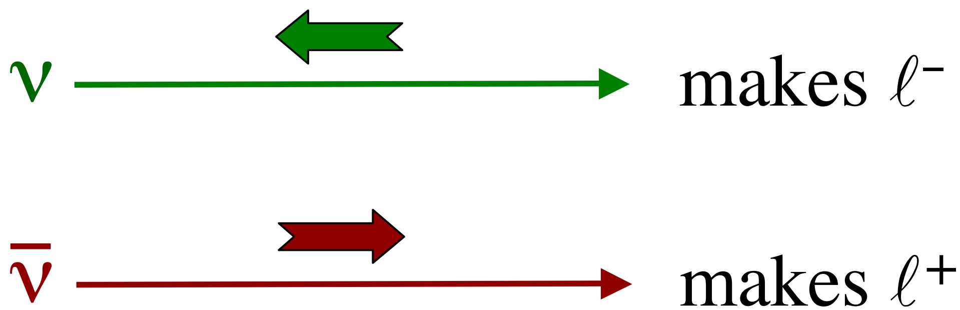


	Conserved L
makes ℓ^-	+1
makes ℓ^+	–1

These states, when Ultra Rel., do not interact.
(The weak interaction is Left Handed.)

SM Interactions Of A Majorana Neutrino

We have only 2 mass-degenerate states:



The weak interactions violate *parity*.

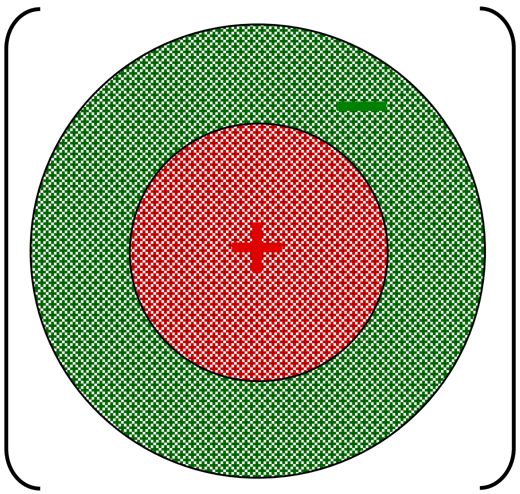
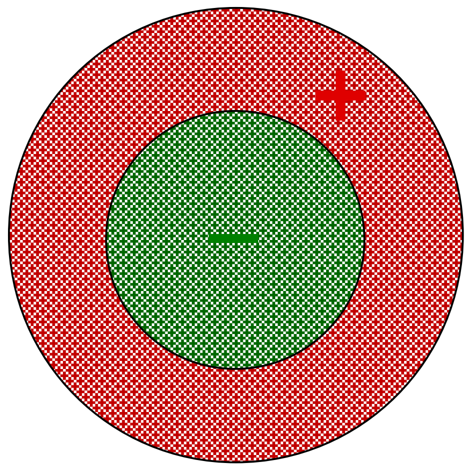
(They can tell *Left* from *Right*.)

An incoming left-handed neutral lepton makes ℓ^-.

An incoming right-handed neutral lepton makes ℓ^+.

Can a Majorana Neutrino Have an Electric Charge *Distribution*?

No!

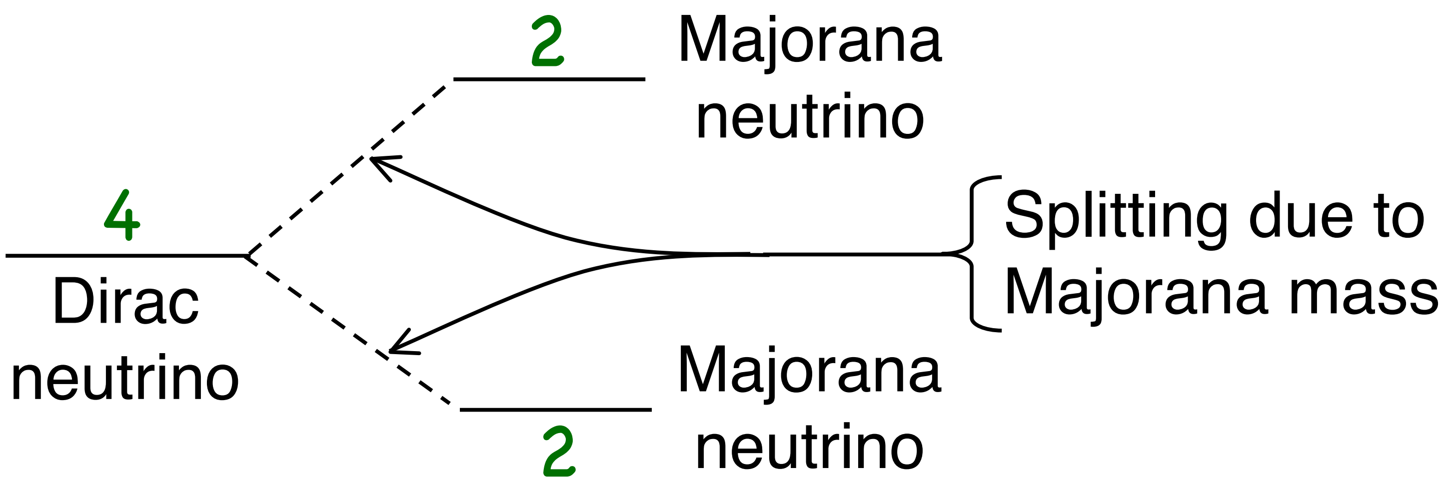
Anti  = 

But for a Majorana neutrino —

Anti (ν) = ν

Majorana Masses Split Dirac Neutrinos

A Majorana mass term splits a Dirac neutrino into two Majorana neutrinos.



Why Most Theorists Expect Majorana Masses

The Standard Model (SM) is defined by the fields it contains, its symmetries (notably weak isospin invariance), and its renormalizability.

Leaving neutrino masses aside, anything allowed by the SM symmetries occurs in nature.

Right-Handed Majorana mass terms
are allowed by the SM symmetries.

Then quite likely *Majorana masses*
occur in nature too.

To Determine Whether Majorana Masses Occur in Nature, So That $\bar{\nu} = \nu$

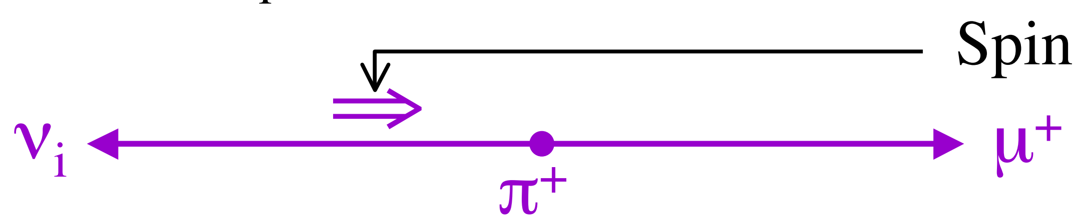
Why don't we already know whether neutrinos are their own antiparticles?

We assume neutrino interactions are correctly described by the SM. Then the interactions conserve L ($\nu \rightarrow \ell^-$; $\bar{\nu} \rightarrow \ell^+$).

It is the Majorana masses that do not conserve L.

An Idea that Does Not Work
[and illustrates why most ideas do not work]

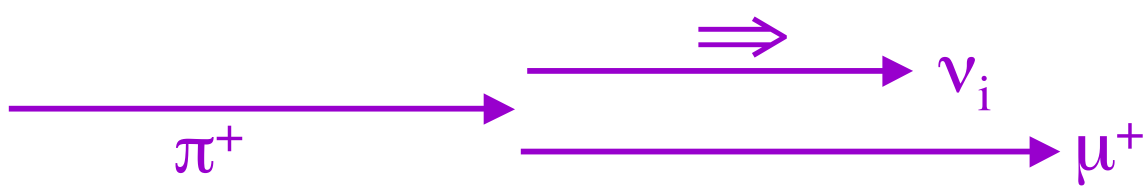
Produce a ν_i via—



Pion Rest Frame

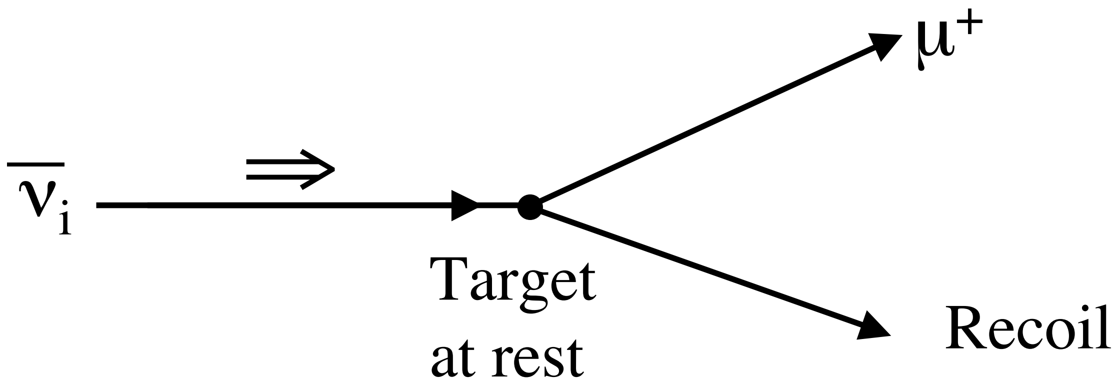
Give the neutrino a Boost:

β_π(Lab) > β_ν(π Rest Frame)



Lab. Frame

The SM weak interaction causes—



$\nu_i = \overline{\nu}_i$ means that $\nu_i(h) = \overline{\nu}_i(h)$.

helicity

If $\nu_i \xrightarrow{\Rightarrow} = \overline{\nu}_i \xrightarrow{\Rightarrow}$,

our $\nu_i \xrightarrow{\Rightarrow}$ will make μ^+ too.

Minor Technical Difficulties

$$\beta_\pi(\text{Lab}) > \beta_\nu(\pi \text{ Rest Frame})$$

$$\Rightarrow \frac{E_\pi(\text{Lab})}{m_\pi} > \frac{E_\nu(\pi \text{ Rest Frame})}{m_{\nu_i}}$$

$$\Rightarrow E_\pi(\text{Lab}) \gtrsim 10^5 \text{ TeV if } m_{\nu_i} \sim 0.05 \text{ eV}$$

Fraction of all π – decay ν_i that get helicity flipped

$$\approx \left(\frac{m_{\nu_i}}{E_\nu(\pi \text{ Rest Frame})} \right)^2 \sim 10^{-18} \text{ if } m_{\nu_i} \sim 0.05 \text{ eV}$$

Since L-violation comes only from Majorana neutrino ***masses***, any attempt to observe it will be at the mercy of the neutrino masses.

(B.K. & Stodolsky)

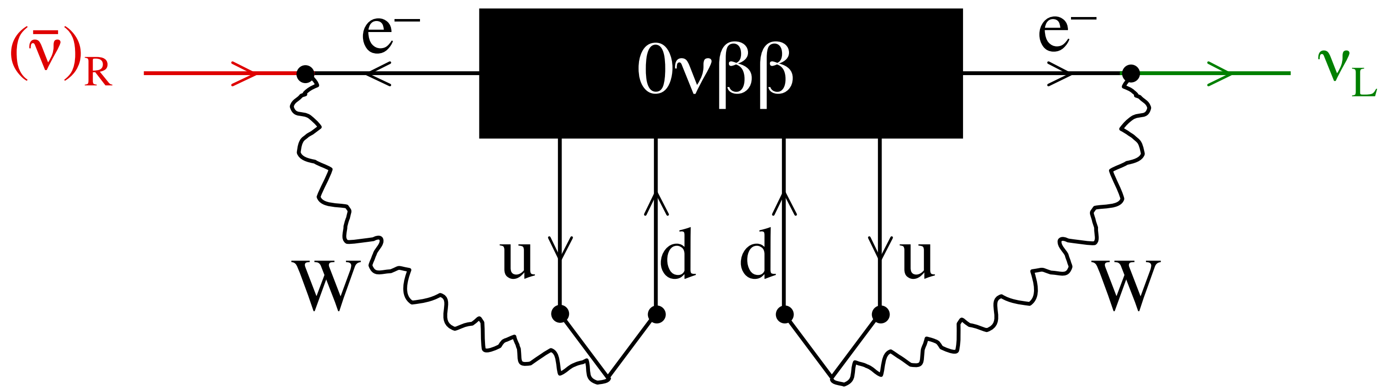
The Promising Approach — Seek

Neutrinoless Double Beta Decay [0νββ]

We are looking for a ***small*** Majorana neutrino mass. Thus, we will need ***a lot*** of parent nuclei (say, one ton of them).

Whatever diagrams cause $0\nu\beta\beta$, its observation would imply the existence of a Majorana mass term:

(Schechter and Valle)



$(\bar{\nu})_R \rightarrow \nu_L$: A (tiny) Majorana mass term

$\therefore$ $0\nu\beta\beta$ ➡ $\bar{\nu}_i = \nu_i$

What Is the Mass Ordering?

Is The Mass Spectrum  or  ?

Generically, grand unified models (GUTS) favor —

GUTS relate the Leptons to the Quarks.

However, ***Majorana masses***, with no quark analogues, could turn  into  .

If the spectrum looks like  , and neutrinos are Majorana particles, there is a lower bound on the rate for $0\nu\beta\beta$.

How To Determine If The Spectrum Is Normal Or Inverted

One way: exploit the ***matter effect***
on accelerator neutrinos.

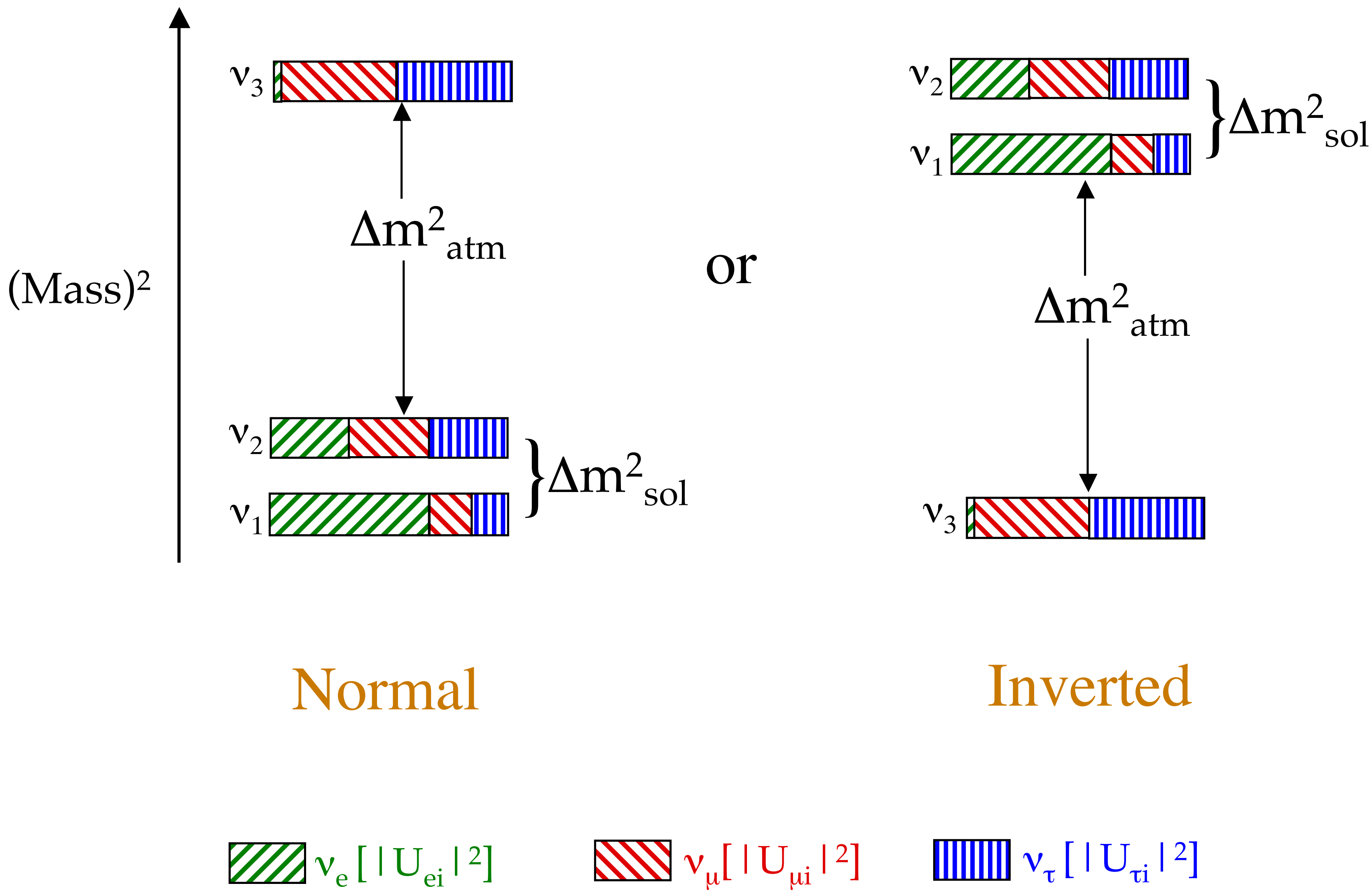
Recall that the matter effect ***raises*** the effective mass of ν_e, but ***lowers*** that of $\bar{\nu}_e$. Thus, it affects ν and $\bar{\nu}$ oscillation ***differently***, leading to:

$$\frac{P(\nu_\mu \rightarrow \nu_e)}{P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e)} \begin{cases} > 1\ ; \\ < 1\ ; \end{cases}$$

Note fake ~~CP~~

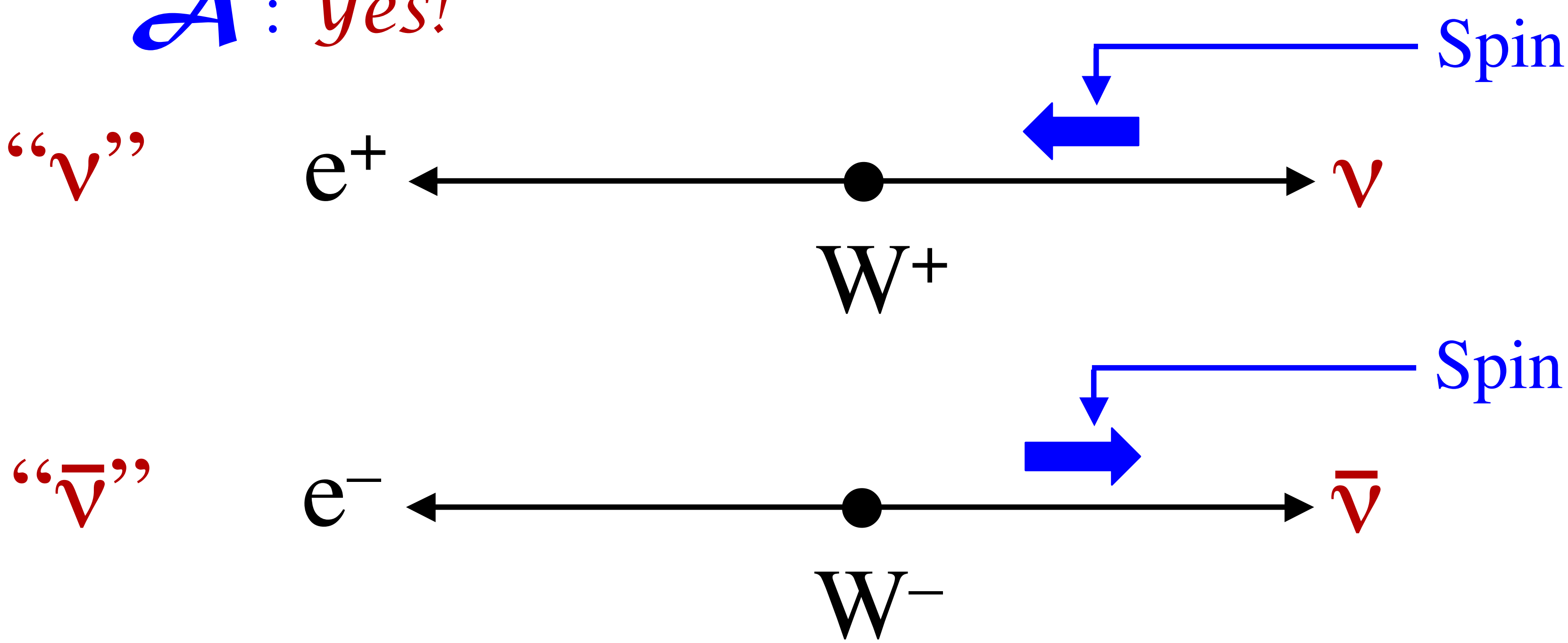
Note dependence on the mass ordering

The matter effect depends on whether the spectrum is Normal or Inverted.



Q : Does matter still affect ν *and* $\bar{\nu}$ *differently when* $\bar{\nu} = \nu$*?*

A : Yes!



The weak interactions violate ***parity***. Neutrino – matter interactions depend on the neutrino ***polarization***.

CP Violation

In General,

In Neutrino Oscillation,

and In Leptogenesis

How CP Violation Comes About

$\not{CP}$ always comes from ***phases***.

Therefore, $\not{CP}$ always requires an ***interference*** between (at least) two amplitudes.

For example, an interference between two Feynman diagrams.

Let us consider how a CP-violating rate difference between two CP-mirror-image processes, such as $B^+ \to D^0 K^+$ and $B^- \to \overline{D}^0 K^-$, arises.

Suppose some process P has the amplitude —

$$A = M_1 e^{i\theta_1} e^{i\delta_1} + M_2 e^{i\theta_2} e^{i\delta_2}$$

CP-invariant magnitude (M_1)

CP-even "strong" phase (θ_1)

CP-odd "weak" phase (δ_1)

Then the CP-mirror-image process $\overline{\mathrm{P}}$ has the amplitude —

$$\overline{A} = M_1 e^{i\theta_1} e^{-i\delta_1} + M_2 e^{i\theta_2} e^{-i\delta_2}$$

Then the rates for $\overline{\mathrm{P}}$ and P differ by —

$$\overline{\Gamma} - \Gamma = \left|\overline{A}\right|^2 - \left|A\right|^2 = 4 M_1 M_2 \sin(\theta_1 - \theta_2) \sin(\delta_1 - \delta_2)$$

Assumes **equal** amounts of
the initial states of $\overline{\text{P}}$ and P.

$$\overline{\Gamma} - \Gamma = \left|\overline{A}\right|^2 - \left|A\right|^2 = 4 M_1 M_2 \sin(\theta_1 - \theta_2) \sin(\delta_1 - \delta_2)$$

A CP-violating rate difference
requires 3 ingredients:

- Two interfering amplitudes
- These two amplitudes must have different CP-even phases
- These two amplitudes must have different CP-odd phases

Mixing Can Lead to Unequal Amounts of the CP-Mirror-Image Initial States

From K^0 only → (arrow to ℓ^+) From $\overline{K}^0$ only → (arrow to ℓ^-)

$$\Gamma\left(K_L \to \pi^- \ell^+ \nu\right) - \Gamma\left(K_L \to \pi^+ \ell^- \overline{\nu}\right) \neq 0 \text{ violates CP.}$$

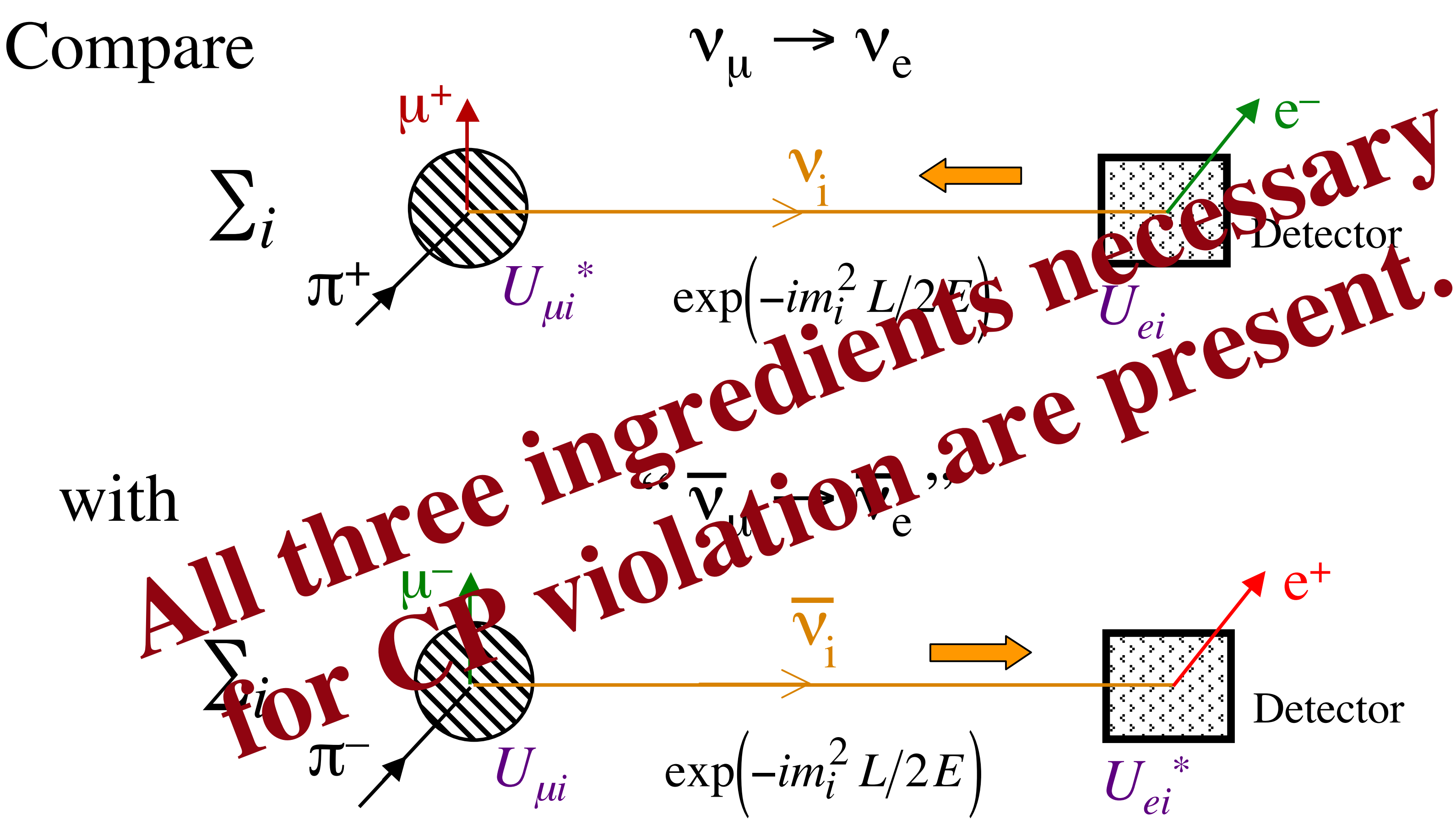
$$\text{But } K_L \propto (1+\varepsilon)K^0 - (1-\varepsilon)\overline{K}^0,$$

where ε is the usual CP-violating parameter arising from $K^0 - \overline{K}^0$ mixing, and $|1+\varepsilon|^2 \neq |1-\varepsilon|^2$.

This rate difference comes from ε, rather than the 3 ingredients just listed.

CP Violation In Neutrino Oscillation

The ingredients for CP in neutrino oscillation, even if $\bar{\nu} = \nu$



There Is Nothing Special About θ_{13}

All mixing angles must be nonzero for $\not{CP}$ in oscillation.

For example —

$$P(\bar{\nu}_\mu \to \bar{\nu}_e) - P(\nu_\mu \to \nu_e) = 2\cos\theta_{13}\sin 2\theta_{13}\sin 2\theta_{12}\sin 2\theta_{23}\sin\delta$$

$$\times \sin\left(\Delta m^2{}_{31}\frac{L}{4E}\right)\sin\left(\Delta m^2{}_{32}\frac{L}{4E}\right)\sin\left(\Delta m^2{}_{21}\frac{L}{4E}\right)$$

Remember this?

In the factored form of U, one can put δ next to θ_{12} instead of θ_{13}.

$$P\left(\bar{\nu}_\mu \to \bar{\nu}_e\right) - P\left(\nu_\mu \to \nu_e\right) = 2\cos\theta_{13}\sin 2\theta_{13}\sin 2\theta_{12}\sin 2\theta_{23}\sin\delta$$
$$\times \sin\left(\Delta m^2{}_{31}\frac{L}{4E}\right)\sin\left(\Delta m^2{}_{32}\frac{L}{4E}\right)\sin\left(\Delta m^2{}_{21}\frac{L}{4E}\right)$$

This, and everything else about neutrino oscillation, is independent of whether neutrinos are Dirac or Majorana particles.

CP Violation In Leptogenesis

Leptogenesis *explains the matter-antimatter asymmetry of the universe in terms of CP violation in the early-universe decays of super-heavy neutrinos.*

(Silvia Pascoli)

Imagine in the early universe 3 heavy neutrinos $N_i = \overline{N_i}$,
coupled to the 3 light lepton families $(\nu_\alpha, \ell_\alpha)$,
and the SM Higgs doublet $(\overline{H^0}, H^-)$,
by a "Yukawa" coupling:

$$\mathcal{L}_Y = \sum_{\substack{\alpha = e,\mu,\tau \\ i = 1,2,3}} y_{\alpha i} \left[\overline{\nu_\alpha} \overline{H^0} - \overline{\ell_\alpha} H^- \right] N_i + h.c.$$

Yukawa coupling matrix ($y_{\alpha i}$)

Imagine that this coupling is the only way the heavy neutrinos N_i interact with the rest of the world.

The Yukawa interaction —

$$\mathcal{L}_Y = \sum_{\substack{\alpha=e,\mu,\tau \\ i=1,2,3}} y_{\alpha i} \left[\overline{\nu_\alpha} \overline{H^0} - \overline{\ell_\alpha} H^- \right] N_i + h.c.$$

causes the decays —

$$N \rightarrow \ell^\mp + H^\pm \quad \text{and} \quad N \rightarrow \overset{(-)}{\nu} + \overset{(\overline{})}{H^0}$$

$\not{CP}$ phases in the matrix y will lead to —

$$\Gamma\left(N \rightarrow \ell^- + H^+\right) \neq \Gamma\left(N \rightarrow \ell^+ + H^-\right)$$

and

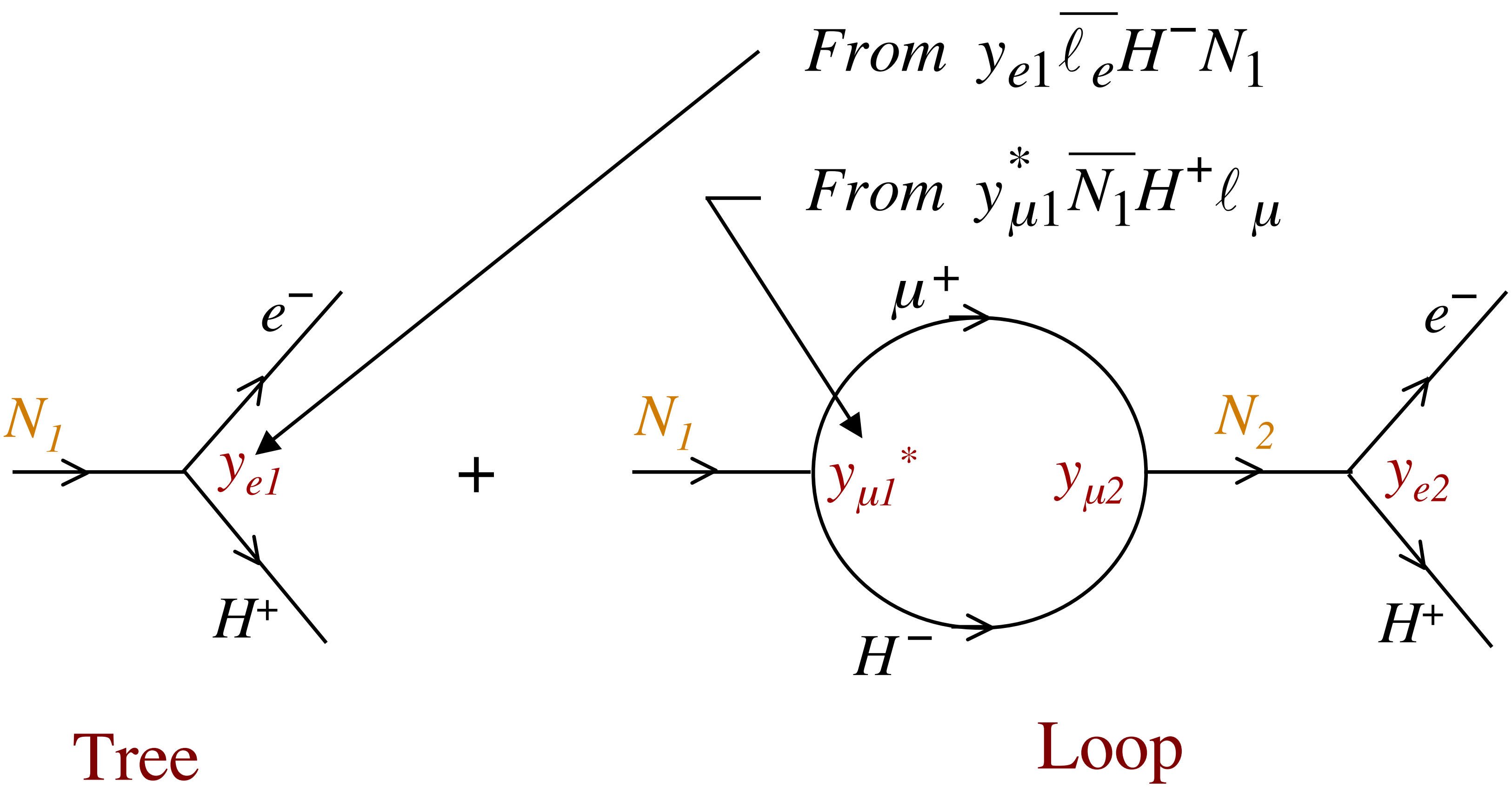
$$\Gamma\left(N \rightarrow \nu + H^0\right) \neq \Gamma\left(N \rightarrow \bar{\nu} + \overline{H^0}\right)$$

$\not{CP}$

How Do Such $\not{CP}$ Inequalities Come About?

Let us look at an example.

This example illustrates that $\not{CP}$ in ***any decay*** always involves amplitudes ***beyond*** those of lowest order in the Hamiltonian.



$$\Gamma\left(N_1 \to e^- + H^+\right) = \left|y_{e1}K_{\text{Tree}} + y^*_{\mu1}y_{\mu2}y_{e2}K_{\text{Loop}}\right|^2$$

Kinematical factors

$$\Gamma\left(N_1 \to e^- + H^+\right) = \left| y_{e1} K_{\text{Tree}} + y_{\mu 1}^* y_{\mu 2} y_{e2} K_{\text{Loop}} \right|^2$$

When we go to the CP-mirror-image decay, $N_1 \to e^+ + H^-$, all the coupling constants get complex conjugated, but the kinematical factors do not change.

$$\Gamma\left(N_1 \to e^+ + H^-\right) = \left| y_{e1}^* K_{\text{Tree}} + y_{\mu 1} y_{\mu 2}^* y_{e2}^* K_{\text{Loop}} \right|^2$$

All three ingredients needed for ~~CP~~ are present.

$$\Gamma\left(N_1 \to e^- + H^+\right) - \Gamma\left(N_1 \to e^+ + H^-\right)$$
$$= 4\,\text{Im}\left(y_{e1}^* y_{\mu 1}^* y_{e2} y_{\mu 2}\right) \text{Im}\left(K_{\text{Tree}} K_{\text{Loop}}^*\right)$$

The $\not{CP}$ inequalities —

$$\Gamma\left(N \rightarrow \ell^- + H^+\right) \neq \Gamma\left(N \rightarrow \ell^+ + H^-\right)$$

and

$$\Gamma\left(N \rightarrow \nu + H^0\right) \neq \Gamma\left(N \rightarrow \bar{\nu} + \overline{H^0}\right)$$

will produce a universe with unequal numbers of leptons (ℓ^- and ν) and antileptons (ℓ^+ and $\bar{\nu}$).

In this universe the lepton number L, defined by $L\left(\ell^-\right) = L(\nu) = -L\left(\ell^+\right) = -L(\bar{\nu}) = 1$, is not zero.

A further step in leptogenesis then converts part of this nonzero lepton number into a nonzero baryon number.

The Connection Between Leptogenesis and Light-Neutrino $\not{CP}$

This is being discussed by ***Silvia Pascoli***.

I would just like to add a comment
about the connection being generic.

Leptogenesis is an outgrowth of the
See-Saw picture of the origin of neutrino masses.

The See-Saw Relation

$$UM_{\nu}U^{T} = -v^{2}\left(y\,M_{N}^{-1}y^{T}\right)$$

- U: Leptonic mixing matrix
- M_{ν}: Light ν mass eigenvalues
- M_{N}: Heavy N mass eigenvalues
- v: The Higgs vev, a real number

$$\left(\underbrace{UM_{\nu}U^{T}}_{\text{Outputs}} = -v^{2}\underbrace{\left(y\,M_{N}^{-1}y^{T}\right)}_{\text{Inputs, in } \mathcal{L}}\right)$$

Through ***U***, the phases in ***y*** lead to $\not{CP}$ in light neutrino oscillation.

$$P(\overset{(-)}{\nu}_\alpha \to \overset{(-)}{\nu}_\beta) =$$

$$= \delta_{\alpha\beta} - 4\sum_{i>j}\Re(U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^*)\sin^2(\Delta m_{ij}^2 \frac{L}{4E})$$

$$\underset{(-)}{+}2\sum_{i>j}\Im(U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^*)\sin(\Delta m_{ij}^2 \frac{L}{2E})$$

e, μ, or τ

Distance

Neutrino (Mass)[2] splitting

Energy

Generically, leptogenesis and light-neutrino $\not{CP}$ imply each other.

Experiments to look for $\not{CP}$ in neutrino oscillation are being developed in Japan and in the US.

CP is a fundamental symmetry.

Is its nonconservation
special to quark mixing?

Or, does it occur in both
quark and lepton mixing,
as suggested by Grand Unified Theories,
which unify the quarks and the leptons?