

# Neutrino mass models Lecture III

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# Outline

## **1. Recap of previous lecture**

## **2. Relating masses and mixing angles**

- Mixing related to mass ratios**
- Flavour symmetries**

## **3. Sterile neutrinos and models of neutrino masses**

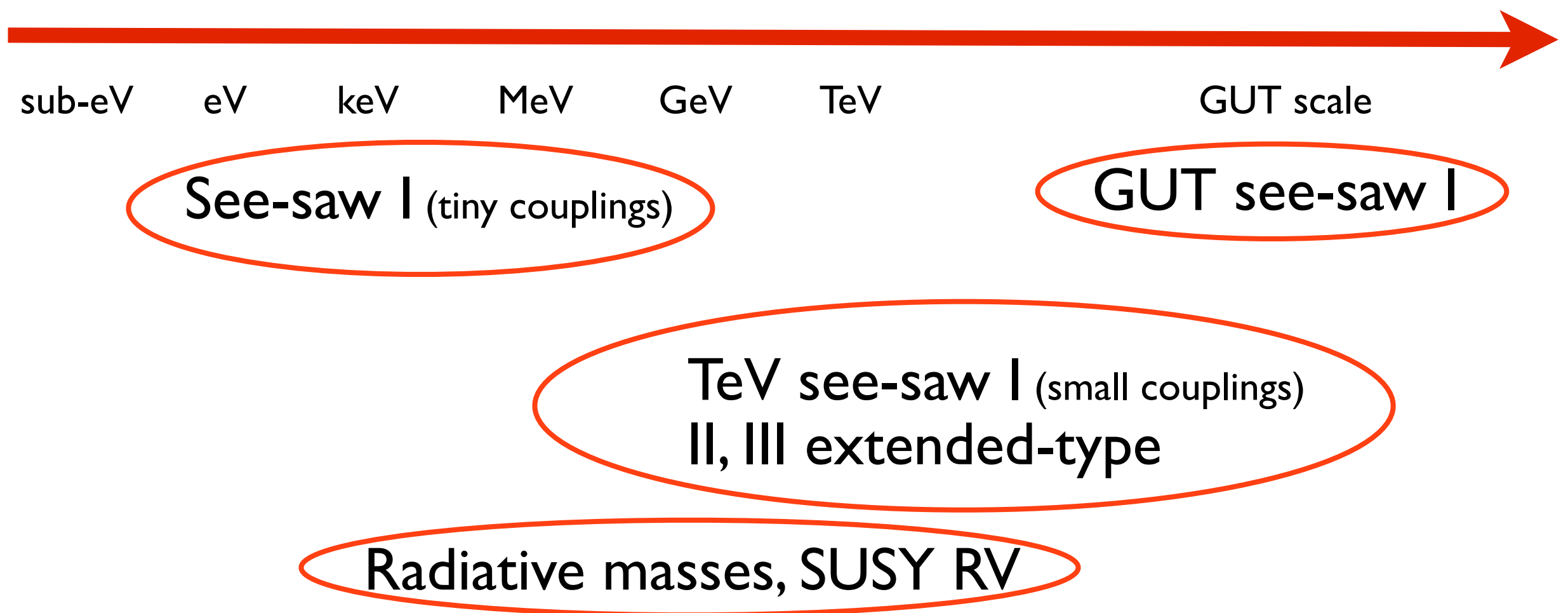
- Different approaches**
- Tests and signatures**

## **4. Conclusions and comments**

# What is the new physics scale?

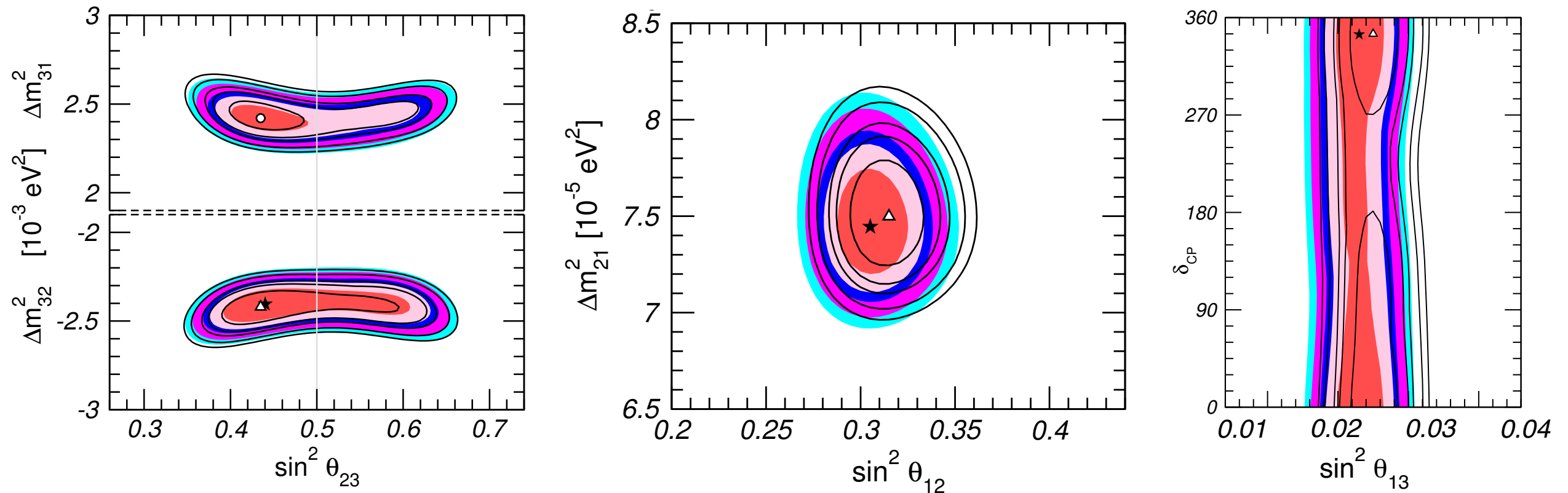
The **new Standard Model** will contain

- new particles at a new physics scale
- new interactions.



The new physics scale might be tested by looking at different signatures of the models: CLFV, leptogenesis, collider searches...

# Recap of neutrino mixing and masses



NuFit: M. C. Gonzalez-Garcia et al., 1209.3023

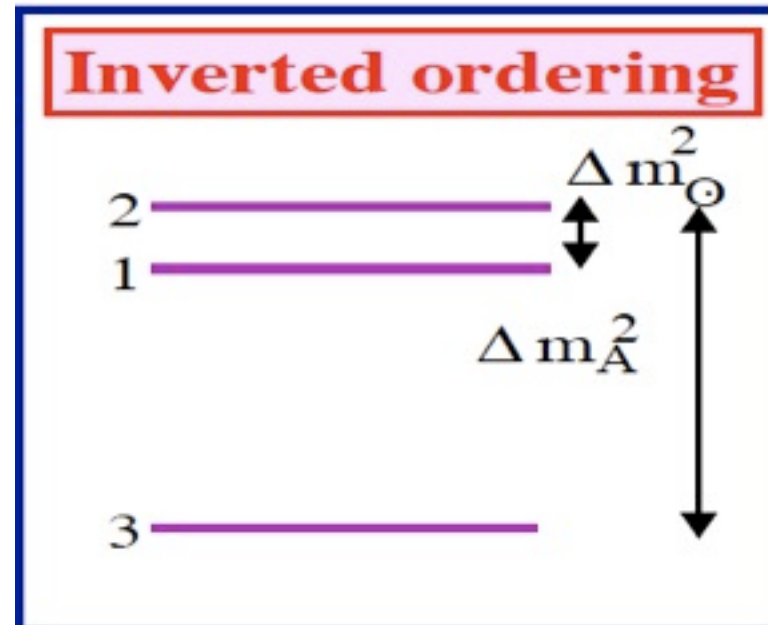
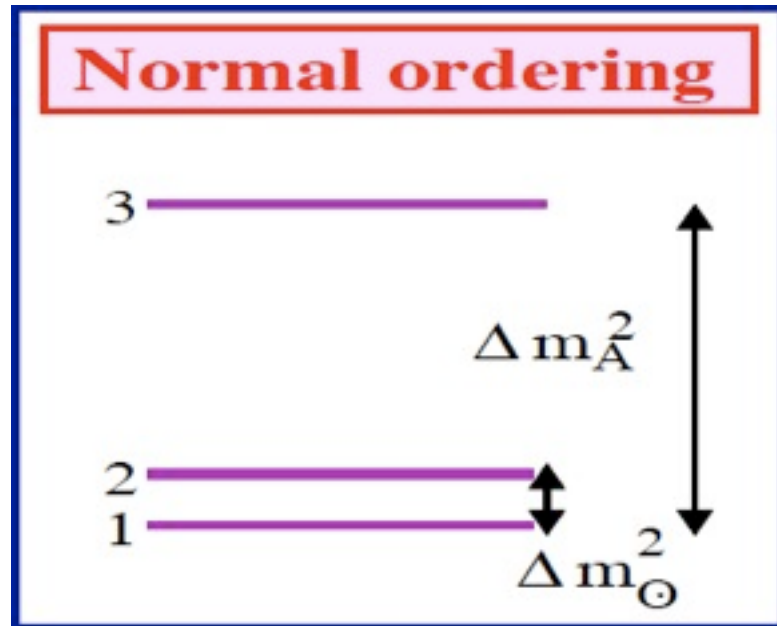
## Important aspects:

- $\theta_{23}$  maximal or close to maximal  $\theta_{23} = 36.7^\circ - 54.0^\circ$
- $\theta_{12}$  significantly different from maximal  $\theta_{12} = 31.38^\circ - 36.01^\circ$
- $\theta_{13}$  quite large. This poses some challenges for understanding the origin of the flavour structure  $\theta_{13} = 7.29^\circ - 9.96^\circ$
- Mixings very different from quark sector



# Masses have at most a mild hierarchy

$$\frac{\sqrt{\Delta m_{\odot}^2}}{\sqrt{\Delta m_A^2}} \sim 5.7^{-1}$$



Masses can have some hierarchy or be nearly degenerate.

Normal hierarchical spectrum (NH):

$$m_1 \ll m_2 \simeq \sqrt{\Delta m_{\odot}^2} \ll m_3 \simeq \sqrt{\Delta m_A^2}$$

Inverted hierarchical spectrum (IH):

$$m_1 \simeq m_2 \simeq \sqrt{\Delta m_A^2} \gg m_3$$

Quasi-degenerate spectrum (QD):

$$m_1 \simeq m_2 \simeq m_3 \gg \sqrt{\Delta m_{\odot}^2}, \sqrt{\Delta m_A^2}$$

# Neutrino masses and mixing

Recall that the mixing matrix arises from the diagonalisation of the mass matrix

$$M_M = (U^\dagger)^T m_{\text{diag}} U^\dagger$$

so the form of the mass matrix will lead to specific values of the angles.

The massive fields are related to the flavour ones as

$$n_L = U^\dagger \nu_L$$

Let's look at an example, in the diagonal basis for the leptons

$$\mathcal{M}_\nu = \begin{pmatrix} a & b \\ b & c \end{pmatrix}$$

the angle can be found to be

$$\tan 2\theta = \frac{2b}{a-c} \gg 1 \quad \text{for } a \sim c \text{ and, or } a, c \ll b$$

with this tuning of  $a$  and  $c$ , we get for the masses

$$m_{1,2} \simeq \frac{a+c \pm 2b}{2}$$

The simplest case would be to have QD neutrinos, unless  $a+c=2b$ , in which a cancellation in one of the masses lead to a NH spectrum.

Large mixing and hierarchical masses pose challenges.

In a model of flavour, both the mass matrix for leptons and neutrinos will be predicted and need to be diagonalised:

$$\begin{array}{ccc}
 (\bar{e}'_L, \bar{\mu}'_L, \bar{\tau}'_L) \mathcal{M}_\ell \begin{pmatrix} e'_R \\ \mu'_R \\ \tau'_R \end{pmatrix} & & (\bar{\nu}_{eL}^c, \bar{\nu}_{\mu L}^c, \bar{\nu}_{\tau L}^c) \mathcal{M}_\nu \begin{pmatrix} \nu_{eL} \\ \nu_{\mu L} \\ \nu_{\tau L} \end{pmatrix} \\
 \\
 (\bar{e}'_L, \bar{\mu}'_L, \bar{\tau}'_L) V_L V_L^\dagger \mathcal{M}_\ell V_R V_R^\dagger \begin{pmatrix} e'_R \\ \mu'_R \\ \tau'_R \end{pmatrix} & & (\bar{\nu}_{eL}^c, \bar{\nu}_{\mu L}^c, \bar{\nu}_{\tau L}^c) U_\nu^\dagger U_\nu^T \mathcal{M}_\nu U_\nu U_\nu^\dagger \begin{pmatrix} \nu_{eL} \\ \nu_{\mu L} \\ \nu_{\tau L} \end{pmatrix} \\
 \downarrow \quad \downarrow \quad \downarrow & & \downarrow \quad \downarrow \quad \downarrow \\
 (\bar{e}_L, \bar{\mu}_L, \bar{\tau}_L) \mathcal{M}_{\text{diag}} \begin{pmatrix} e_R \\ \mu_R \\ \tau_R \end{pmatrix} & & (\bar{\nu}_{1L}^c, \bar{\nu}_{2L}^c, \bar{\nu}_{3L}^c) \mathcal{M}_{\text{diag},\nu} \begin{pmatrix} \nu_{1L} \\ \nu_{2L} \\ \nu_{3L} \end{pmatrix}
 \end{array}$$

In a model of flavour, both the mass matrix for leptons and neutrinos will be predicted and need to be diagonalised:

$$(\bar{e}'_L, \bar{\mu}'_L, \bar{\tau}'_L) \mathcal{M}_\ell \begin{pmatrix} e'_R \\ \mu'_R \\ \tau'_R \end{pmatrix} \quad (\bar{\nu}^c_{eL}, \bar{\nu}^c_{\mu L}, \bar{\nu}^c_{\tau L}) \mathcal{M}_\nu \begin{pmatrix} \nu_{eL} \\ \nu_{\mu L} \\ \nu_{\tau L} \end{pmatrix}$$

$$(\bar{e}'_L, \bar{\mu}'_L, \bar{\tau}'_L) V_L V_L^\dagger \mathcal{M}_\ell V_R V_R^\dagger \begin{pmatrix} e'_R \\ \mu'_R \\ \tau'_R \end{pmatrix} \quad (\bar{\nu}^c_{eL}, \bar{\nu}^c_{\mu L}, \bar{\nu}^c_{\tau L}) U_\nu^* U_\nu^T \mathcal{M}_\nu U_\nu U_\nu^\dagger \begin{pmatrix} \nu_{eL} \\ \nu_{\mu L} \\ \nu_{\tau L} \end{pmatrix}$$

$$(\bar{e}_L, \bar{\mu}_L, \bar{\tau}_L) \mathcal{M}_{\text{diag}} \begin{pmatrix} e_R \\ \mu_R \\ \tau_R \end{pmatrix} \quad (\bar{\nu}^c_{1L}, \bar{\nu}^c_{2L}, \bar{\nu}^c_{3L}) \mathcal{M}_{\text{diag},\nu} \begin{pmatrix} \nu_{1L} \\ \nu_{2L} \\ \nu_{3L} \end{pmatrix}$$

in the CC interactions (and oscillations):

$$\mathcal{L}_{CC} = \frac{g}{\sqrt{2}} (\bar{e}'_L, \bar{\mu}'_L, \bar{\tau}'_L) \gamma^\mu \begin{pmatrix} \nu_{eL} \\ \nu_{\mu L} \\ \nu_{\tau L} \end{pmatrix} W_\mu \Rightarrow \frac{g}{\sqrt{2}} (\bar{e}_L, \bar{\mu}_L, \bar{\tau}_L) \gamma^\mu U_{\text{osc}} \begin{pmatrix} \nu_{1L} \\ \nu_{2L} \\ \nu_{3L} \end{pmatrix} W_\mu$$

$$U_{\text{osc}} = V_L^\dagger U_\nu$$

# Phenomenological approaches

Various strategies and ideas can be employed to understand the observed pattern (many many models!).

- Mixing related to mass ratios

$$\theta_{12,23,13} = \text{function}\left(\frac{m_e}{\underset{\text{too small}}{m_\mu}}, \dots, \frac{m_1}{m_2}\right)$$

- Flavour symmetries

- Complementarity between quarks and leptons

$$\theta_{12} + \theta_C \simeq 45^\circ$$

- Anarchy (all elements of the matrix of the same order).

# Models relating mixing and ratios of masses

If the mass matrix contains several zeros, the number of parameters is severely reduced and the models are very predictive. A useful ansatz is the Fritzsch one:

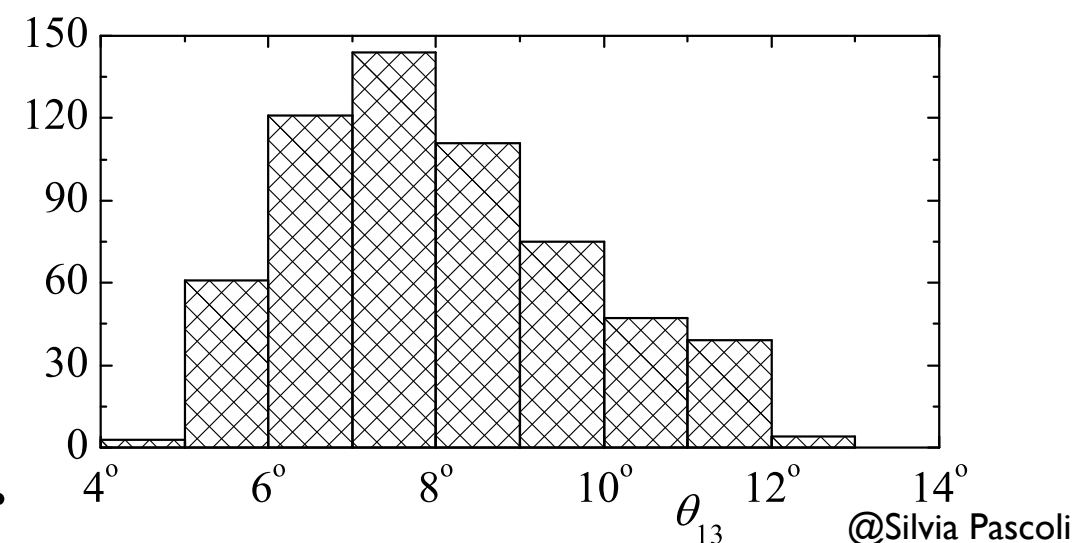
$$M_\nu = \begin{pmatrix} 0 & \dots & 0 \\ \dots & 0 & \dots \\ 0 & \dots & \dots \end{pmatrix}$$

The ratio of neutrino masses can be related to the mixing angles as

$$\frac{m_1}{m_3} \sim \tan \theta_{12} \tan \theta_{23} \sin \theta_{13}$$
$$\frac{m_2}{m_3} \sim \cot \theta_{12} \tan \theta_{23} \sin \theta_{13}$$

This predicts a mild hierarchy between  $m_1$  and  $m_2$ , no neutrinoless double beta decay and a rather large value of  $\theta_{13}$ .

Fritzsch, Xing, Zhou, 2011



# Symmetry approach

- Choose a leptonic symmetry (e.g.  $A_4$ ,  $S_4$ ,  $\mu - \tau$ )
- Use the fact that the see-saw mechanism leads to

$$U_\nu \neq V_L$$

- Obtain the zero-order matrix

$$U_0$$

- Add perturbations (coming from breaking of the symmetry or quantum corrections) to obtain the observed values.

$$U = U_0 + U_{\text{perturbations}_{\text{small}}}$$

$\theta_{13}$  poses new challenges as it is not very small.



What kind of leading matrices have been considered?

## Democratic mixing pattern

$$\mathcal{U}_0 = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & \frac{\sqrt{2}}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{pmatrix} + \begin{pmatrix} \mathcal{O}(0.1) & -\mathcal{O}(0.1) & \mathcal{O}(0.1) \\ \mathcal{O}(0.1) & \mathcal{O}(0.1) & -\mathcal{O}(0.1) \\ -\mathcal{O}(0.1) & -\mathcal{O}(0.05) & \mathcal{O}(0.1) \end{pmatrix}$$

$$\theta_{23}|_0 = 45^\circ \quad \theta_{12}|_0 = 55^\circ \quad \theta_{13}|_0 = 0$$

The perturbations are all of the same order.

## Bimaximal mixing

$$\mathcal{U}_0 = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix} + \begin{pmatrix} \mathcal{O}(0.1) & -\mathcal{O}(0.1) & \mathcal{O}(0.1) \\ \mathcal{O}(0.1) & \mathcal{O}(0.1) & -\mathcal{O}(0.01) \\ -\mathcal{O}(0.1) & -\mathcal{O}(0.1) & \mathcal{O}(0.01) \end{pmatrix}$$

$$\theta_{23}|_0 = 45^\circ \quad \theta_{12}|_0 = 45^\circ \quad \theta_{13}|_0 = 0$$

In this case,  $\theta_{23}$  requires small perturbations.

Three other patterns lead to values of  $\theta_{12}$  which are closer to the observed value: tribimaximal, golden ratio ( $\tan \theta_{12}|_0 = \frac{2}{1 + \sqrt{5}}$ ), and hexagonal ( $\theta_{12}|_0 = 30^\circ$ ) mixing patterns.

## Tribimaximal mixing pattern

$$\mathcal{U}_0 = \begin{pmatrix} \frac{\sqrt{2}}{\sqrt{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} + \begin{pmatrix} \mathcal{O}(0.001) & -\mathcal{O}(0.01) & \mathcal{O}(0.1) \\ \mathcal{O}(0.1) & \mathcal{O}(0.05) & -\mathcal{O}(0.01) \\ -\mathcal{O}(0.1) & -\mathcal{O}(0.05) & \mathcal{O}(0.01) \end{pmatrix}$$

$$\theta_{23}|_0 = 45^\circ \quad \theta_{12}|_0 \simeq 35^\circ \quad \theta_{13}|_0 = 0$$

This pattern is very popular as it might indicate an underlying symmetry. In fact the neutrino mass matrix can be written in terms of components which possess a  $S_3$  and a  $\mu$ - $\tau$  symmetry.

## Example 1: mu-tau symmetry

Large  $\theta_{23}$  motivates to consider the mu-tau symmetry.

For simplicity in 2 generations

$$\mathcal{M}_\nu = \begin{pmatrix} a & b \\ b & a \end{pmatrix}$$

We get that  $\tan 2\theta = \frac{2b}{0} = \infty \Rightarrow \theta_{23} = 45^\circ$

and  $a \simeq b \Rightarrow m_2 \ll m_3$

$$\mathcal{M}_\nu = \sqrt{\Delta m_A^2} \begin{pmatrix} 1 + \epsilon & 1 \\ 1 & 1 + \epsilon \end{pmatrix} \Rightarrow \epsilon = \sqrt{\frac{\Delta m_\odot^2}{\Delta m_A^2}} \sim \frac{1}{5.7}$$

This matrix is symmetric under the exchange of mu and tau.

In 3 generations, the following mass matrix respects the mu-tau symmetry

$$\mathcal{M}_\nu = \sqrt{\Delta m_A^2} \begin{pmatrix} \sim 0 & a\epsilon & a\epsilon \\ a\epsilon & 1 + \epsilon & 1 \\ a\epsilon & 1 & 1 + \epsilon \end{pmatrix}$$

and leads to

$$\theta_{23} = \frac{\pi}{4} - \frac{\Delta m_\odot^2}{\Delta m_A^2} \quad \theta_{13} \sim \epsilon^2 \sim \frac{\Delta m_\odot^2}{\Delta m_A^2} \sim 0.04$$

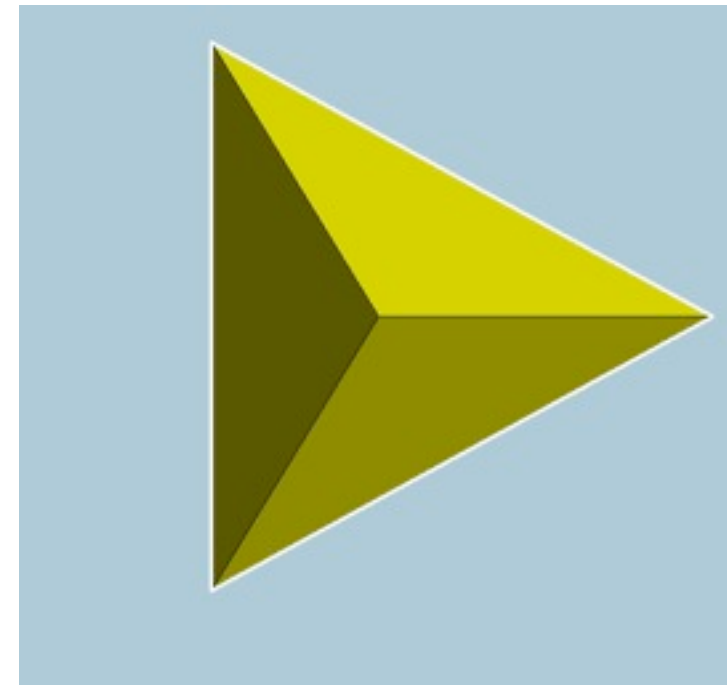
The large value of  $\theta_{13}$  is a challenge for this model as additional perturbations are required.

Typically they lead to correlations between different parameters (specifically for the deviations from special values such as 0 and 45°).

## Example 2: a discrete symmetry A4

An example of discrete symmetry:  $Z_2$  (reflections).

A4 is the group of even permutations of (1234). This is a very studied example of discrete symmetry. It is the invariant group of a tetrahedron.



There are 12 elements:  
 $I = 1234, T = 2314, S = 4321, ST, TS, STS...$   
with  $S^2 = I, T^3 = I, (ST^3) = I$ .

It has the following representations: 1, 1', 1'', and 3, distinguished by how S and T behave on it.

We need to assign fermions to the representations:

$$L \rightarrow 3$$

$$e_R \rightarrow 1$$

$$\mu_R \rightarrow 1'$$

$$\tau_R \rightarrow 1''$$

As usual, masses require the “product” of two fermions:

$$1' \times 1' = 1''$$

$$1'' \times 1'' = 1'$$

$$1' \times 1'' = 1$$

$$3 \times 3 = 1 + 1' + 1'' + 3 + 3$$

In order to break the symmetry, scalars (called ‘flavons’) are needed:

$$\phi(3), \quad \phi'(3), \quad \xi(1)$$

Requiring that the Lagrangian is invariant w.r.t. the flavour symmetry, the allowed interactions are fixed:

$$\mathcal{L} = y_e \bar{e}_R(\phi L) \frac{H_d}{\Lambda} + y_\mu \bar{\mu}_R(\phi L) \frac{H_d}{\Lambda} + y_\tau \bar{\tau}_R(\phi L) \frac{H_d}{\Lambda} + j_a \xi(LL) \frac{H_u H_u}{\Lambda^2} + j_b (\phi' LL) \frac{H_u H_u}{\Lambda^2}$$

$\mathbf{1} \quad (\mathbf{33})_{\mathbf{1}} \quad \mathbf{1}' \quad (\mathbf{33})_{\mathbf{1}'} \quad \mathbf{1}'' \quad (\mathbf{33})_{\mathbf{1}''} \quad \mathbf{1} \quad (\mathbf{33})_{\mathbf{1}} \quad (\mathbf{333})_{\mathbf{1}}$

The flavons get a vev

$$\langle \phi \rangle = (v, v, v) \quad \langle \phi' \rangle = (v', 0, 0) \quad \langle \xi \rangle = u$$

and the resulting mass matrices are

$$M_l = v \frac{v_{Hd}}{\Lambda} \begin{pmatrix} y_e & y_e & y_e \\ y_\mu & y_\mu e^{i4\pi/3} & y_\mu e^{i2\pi/3} \\ y_\tau & y_\tau e^{i2\pi/3} & y_\tau e^{i4\pi/3} \end{pmatrix} \quad M_\nu = \frac{v_u^2}{\Lambda^2} \begin{pmatrix} a & 0 & 0 \\ 0 & a & d \\ 0 & d & a \end{pmatrix}$$

Finally, the two matrices can be diagonalised and the resulting mixing matrix is the TBM one.

There are two major issues:

- **the vacuum alignment.** Without the specific choice of the vevs of the flavons, the required form of the mass matrix could not be achieved. Arranging for the potential to lead to such vevs is highly non trivial.

- **the value of  $\theta_{13}$ .**

Due to the measured value of  $\theta_{13}$ , large deviations from TBM are required and this poses some challenges to this approach. Extensions are being considered (e.g. Dirac neutrinos, additional flavons...)



## Tests of flavour models

Typically, the models considered have a reduced number of parameters, leading to **relations between the masses and/or mixing angles**.

Examples are the mixing-mass ratio relations and the so-called sumrules, e.g.:

$$\sin \theta_{23} - \frac{1}{\sqrt{2}} = \sin \theta_{13} \cos \delta$$

Two necessary ingredients for testing flavour models:

- Precision measurements of the oscillation parameters at future experiments.
- The determination of the mass hierarchy and of the neutrino mass spectrum.

# Future experimental strategy:

theta23: LBL experiments

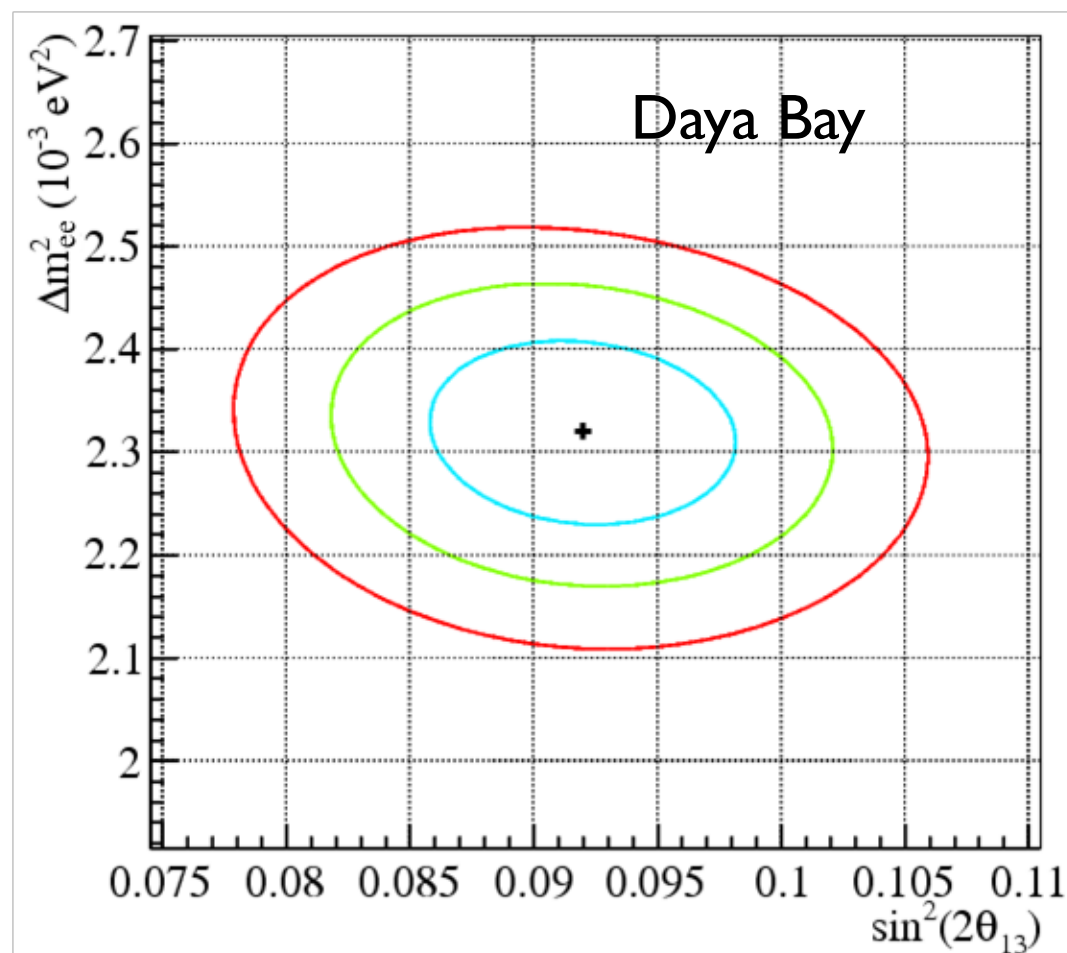
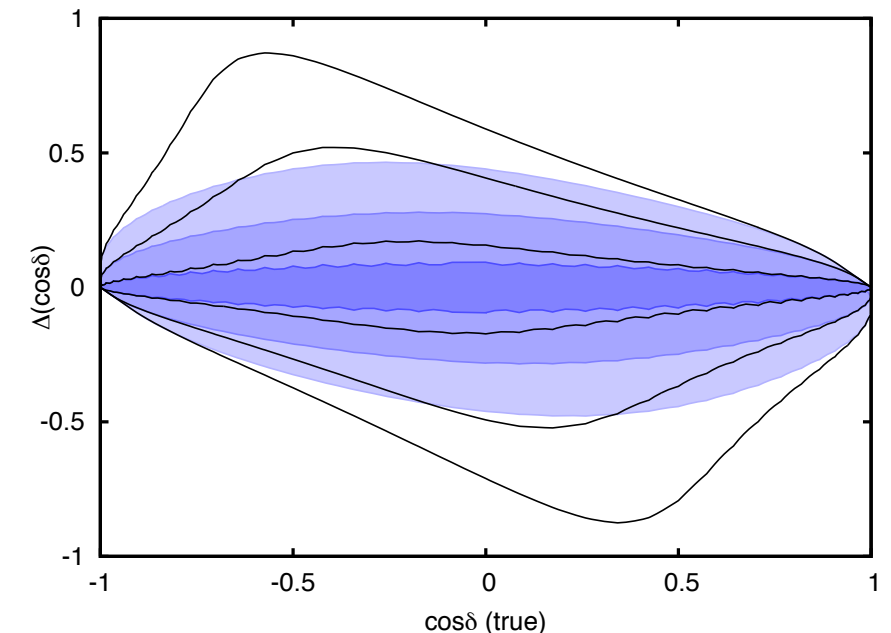
theta13: reactor experiments

theta12: reactor experiments

delta: LBL experiments

mass hierarchy: atmospheric, LBL, reactor (JUNO)  
neutrinos.

Ballet at al. preliminary



	Current	Daya Bay II
$\Delta m^2_{12}$	3%	0.6%
$\Delta m^2_{23}$	5%	0.6%
$\sin^2 \theta_{12}$	6%	0.7%
$\sin^2 \theta_{23}$	20%	N/A
$\sin^2 \theta_{13}$	14% → 4%	~15%

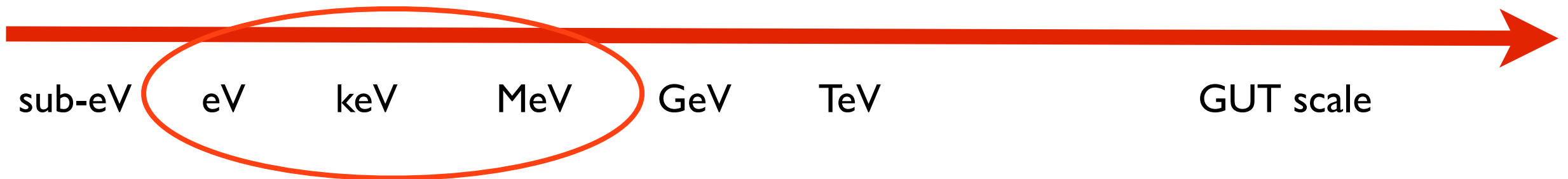
Y.Wang, LPI3

Y.Wang, LPI3

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# Sterile neutrinos (eV-GeV scale)

The scale of the new physics responsible for neutrino masses is unconstrained and could be as low as the eV scale.



On the other side, possible hints of light sterile neutrinos have been found in various experiments although their validity is under discussion.

(see [Kayser's](#) and [Soler's](#) lectures)

Two approaches:

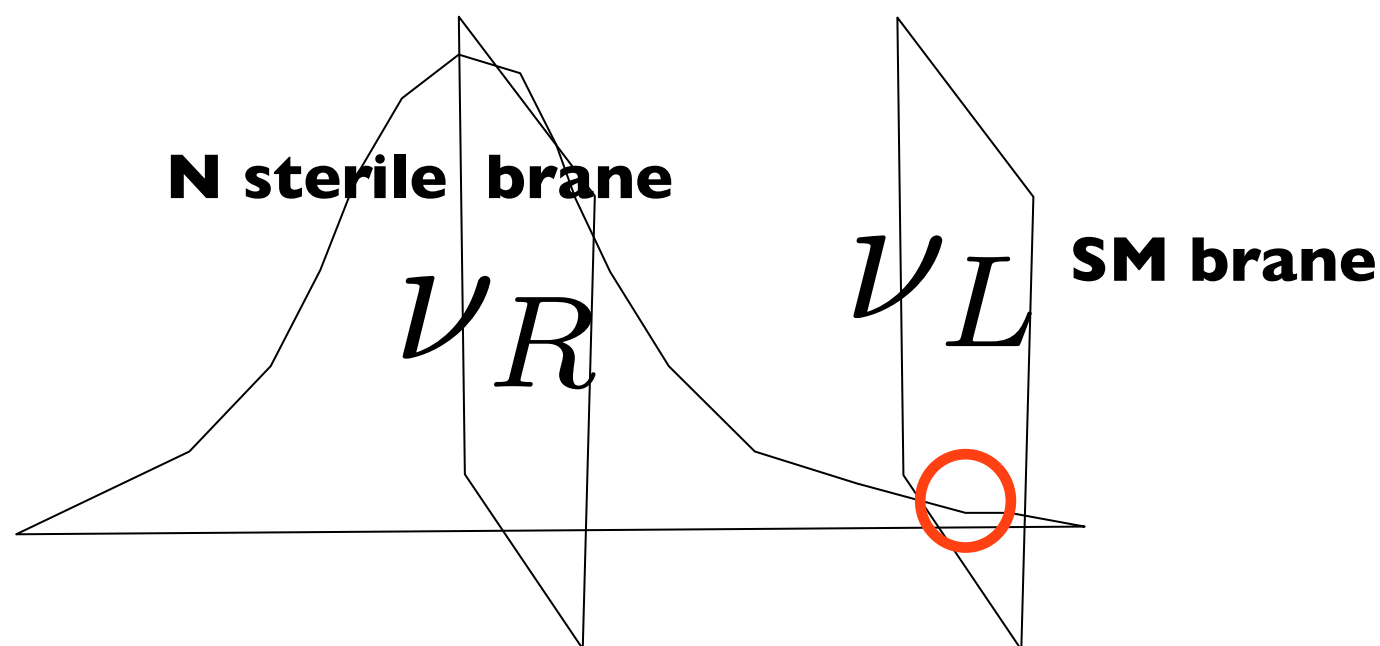
1. explain why sterile neutrinos are so light
2. use light sterile neutrinos in models to generate light neutrino masses.

At times the two approaches are related.

## Approach I

This is motivated by the phenomenology and cosmology of sterile neutrinos. The idea is to try to explain the lightness of sterile neutrinos (not necessarily in conjunction with the one of light neutrinos). These models are not as developed as the ones for active neutrinos and typically borrow similar ideas.

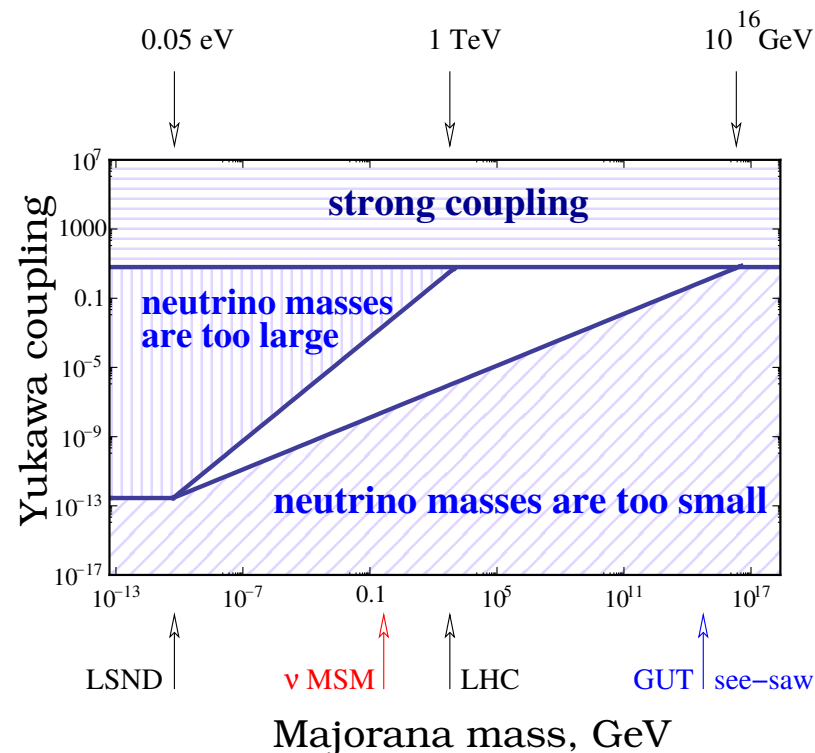
Example. The split see-saw mechanism.



In the low energy effective theory, the Yukawa coupling and the mass of the sterile neutrino are suppressed.

## Approach 2

### Example. Low energy see-saw



As the sterile mass becomes very small, tiny couplings are required.

White paper on sterile neutrinos

Models in which the new masses and mixing angles explain on one side the oscillation data (LSND...) and on the other the mass splittings and mixing angles have been considered.

It has been shown that 3+2 see-saw scheme could work and provide a good fit to the data, despite being quite constrained. But further studies are required.

## Approach 1+2

### Example 1. Use of symmetries

For active neutrinos we have seen that some symmetries lead to a massless eigenstate. A prime example is  $L_e - L_\mu - L_\tau$ .

The idea is to use the same symmetry also in the sterile neutrino sector.

$$\begin{aligned} L_e - L_\mu - L_\tau &\Rightarrow m_1 = 0, \quad m_2 = m_3 \quad (IH) \\ &\Rightarrow M_1 = 0, \quad M_2 = M_3 \end{aligned}$$

The symmetry is then slightly broken to generate both the mass of the sterile neutrino and the splitting between  $m_2$  and  $m_3$  and the correct pattern of mixing.

## Example 2. Extended see-saw

We have introduced two right-handed singlet neutrinos.

$$\begin{pmatrix} 0 & Yv & Y_2v \\ Yv & \mu' & \Lambda \\ Y_2v & \Lambda & \mu \end{pmatrix} \quad \text{with } \mu' \gg \Lambda, \mu$$

This results in a very heavy neutrino, **one light sterile** neutrino and light neutrino masses.

$$\begin{aligned} m_4 &\approx \tilde{M}_1 \approx -\Lambda^2/\mu', & U_{e4} &\approx Y_{1e}v/\sqrt{2}\Lambda \\ m_5 &\approx \tilde{M}_2 \approx \mu', & U_{e5} &\approx Y_{1e}v/\sqrt{2}\mu' \end{aligned}$$

For typical values

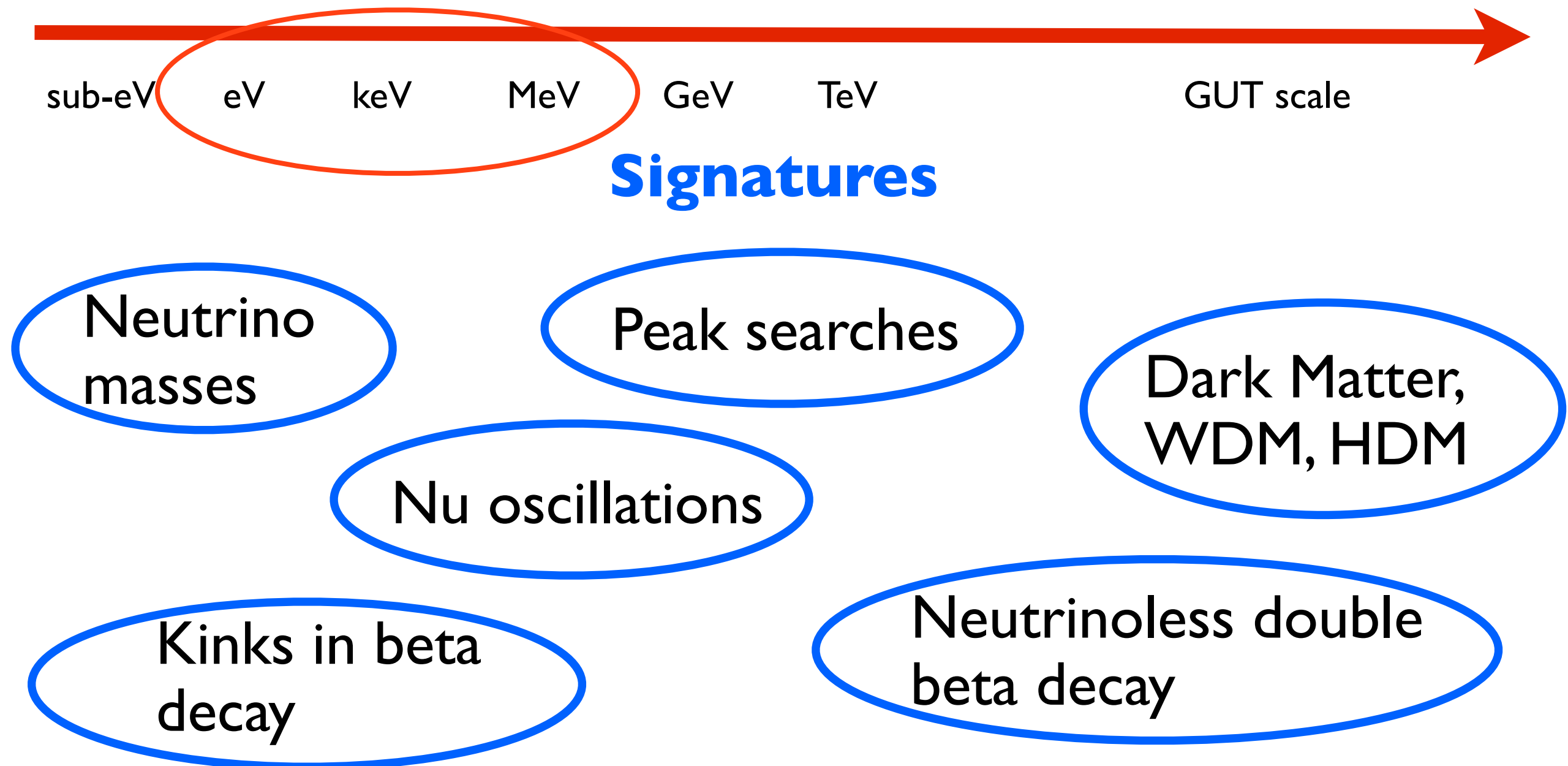
$$m_4 \sim \frac{(10^2 \text{ GeV})^2}{10^{10} \text{ GeV}} \sim 1 \text{ keV}$$

and large mixing.



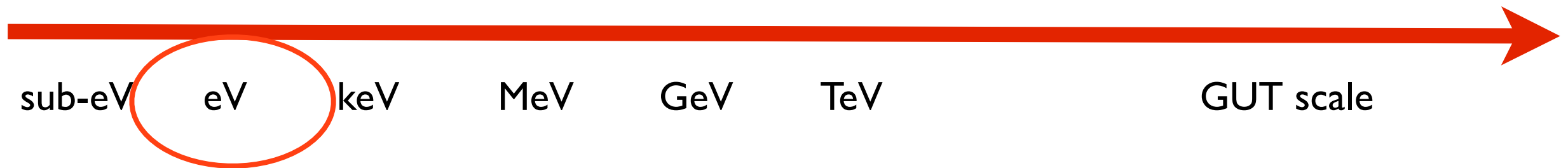
# How are we going to test these models?

We search for sterile neutrino signatures.  
As their parameters might be related to neutrino masses, the predictions can be more constrained than in general.





## *eV scale: Neutrino oscillations etc.*

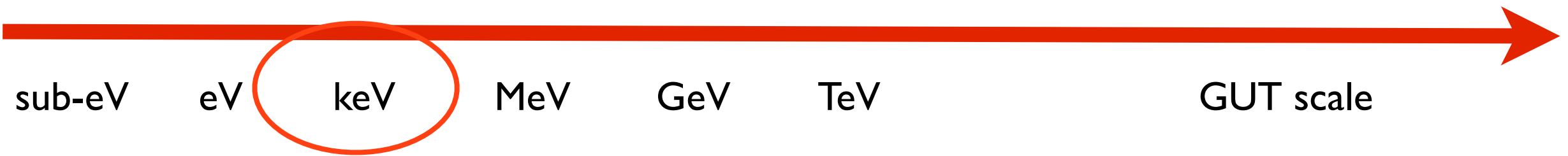


Sterile neutrinos with eV masses will induce neutrino oscillations in short baseline oscillations. This is the most sensitive search. (see [Kayser's](#) and [Soler's lectures](#))

Other signatures are present:

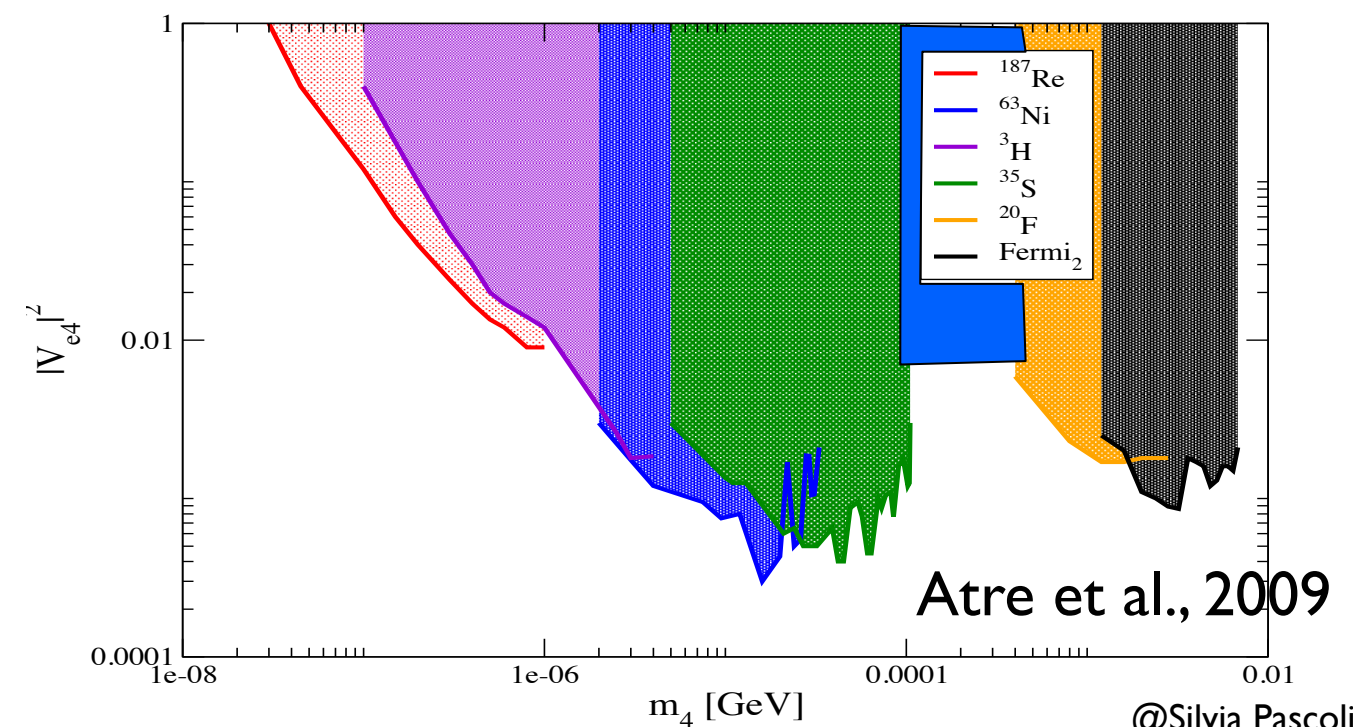
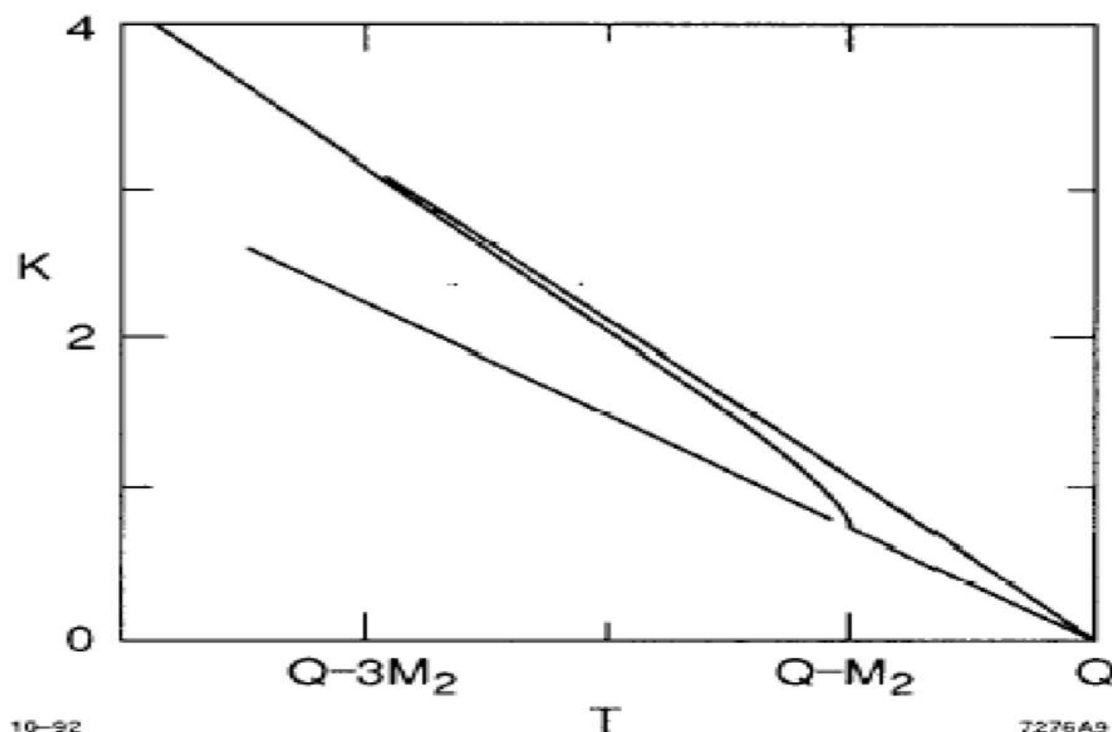
- eV sterile neutrinos, if in equilibrium in the Early Universe contribute to the HDM and to the number of relativistic degrees of freedom at the CMB. (see [Adam's lectures](#))
- they contribute to neutrinoless double beta decay (see later).
- They might be seen in beta decays.

## keV scale: Kinks and dark matter



Sterile neutrinos with keV masses have attracted a lot of attention because they constitute the favoured warm dark matter candidate. Their phenomenology depends critically on the mixing angles.

- Kinks in beta decays.



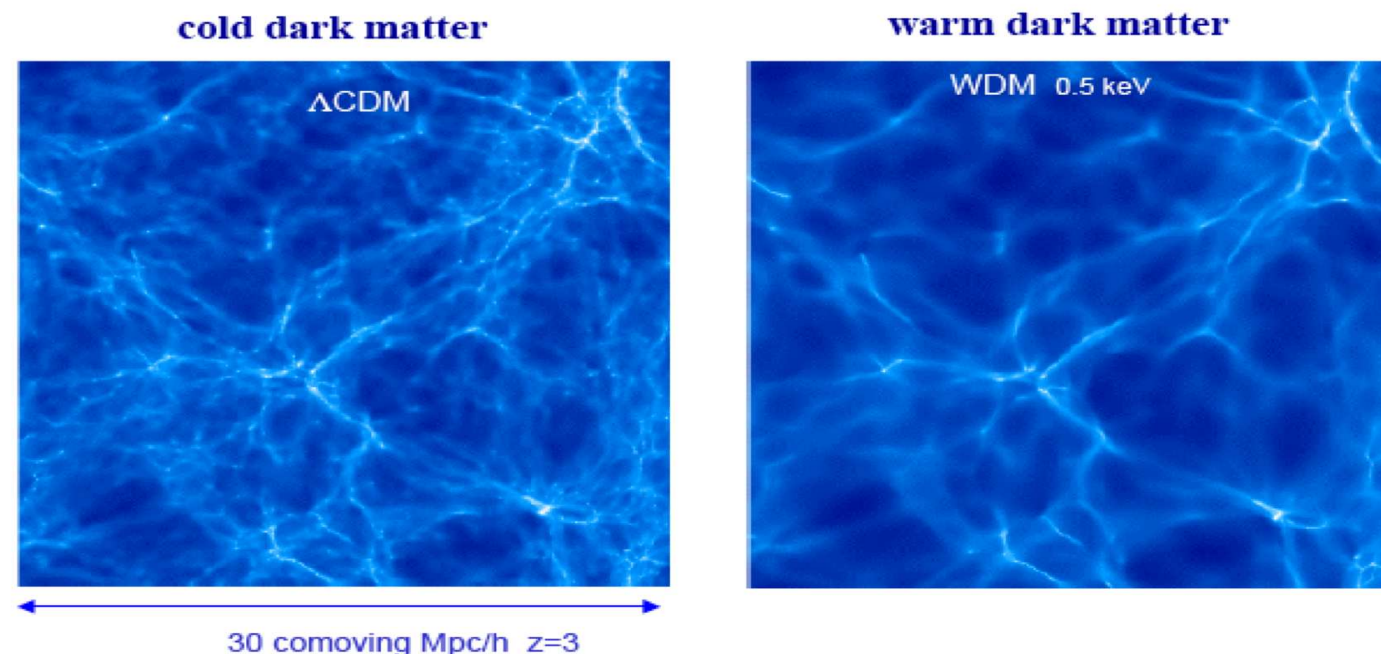
# Warm Dark Matter

In the right range of masses and mixing angles, sterile neutrinos can be “stable” on the cosmic timescales.

$$\Gamma_{3\nu} \simeq \sin^2 2\theta G_F^2 \frac{m_4^5}{768\pi^3} \sim 10^{-30} \text{s}^{-1} \frac{\sin^2 2\theta}{10^{-10}} \left( \frac{m_4}{\text{keV}} \right)^5$$

Therefore they have been considered as a DM candidate, with clustering properties intermediate between hot dark matter and cold dark matter (hence the name warm dark matter).

(see Adam's lectures)



See, e.g.  
Haehnelt, Frenk  
et al...

Their production is different from active neutrinos as they were never in equilibrium with the thermal plasma. They got produced via loss of coherence in oscillations, decays of heavier particles, etc...

Production via oscillations. Active neutrinos were kept in equilibrium with the other particles till

$$\Gamma_{\text{weak}} \sim H$$

with

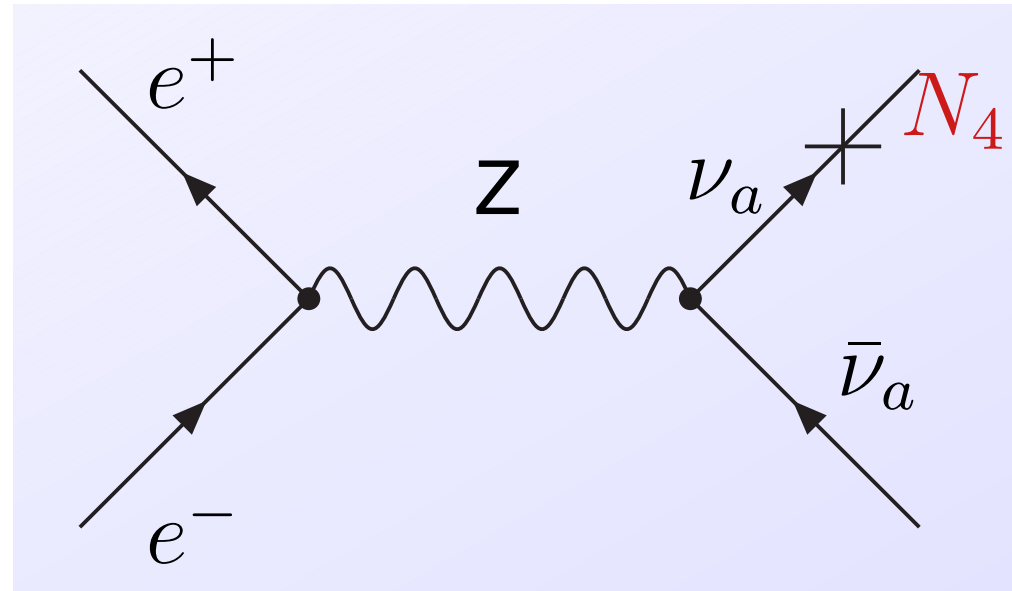
$$\Gamma_{\text{weak}} = \langle \sigma \rangle n \sim G_F^2 T^2 T^3$$
$$H = \sqrt{\frac{4\pi^3 g_*}{45}} \frac{T^2}{m_{\text{Pl}}}$$

**Exercise**

Compute the neutrino decoupling temperature

which corresponds to a decoupling temperature of MeV.

In an interaction involving active neutrinos, a heavy neutrino would be produced via loss of coherence.



These oscillations happen in the thermal plasma, so the mixing angle will be in matter.

$$\sin^2 2\theta_m = \frac{\Delta^2(p) \sin^2 2\theta}{\Delta^2(p) \sin^2 2\theta + D^2 + (\Delta(p) \cos 2\theta - V_D + |V_T|)^2}$$

Analogue to matter effects in the earth and depend on the lepton asymmetry.

Genuine thermal effects. They always suppress the oscillations.

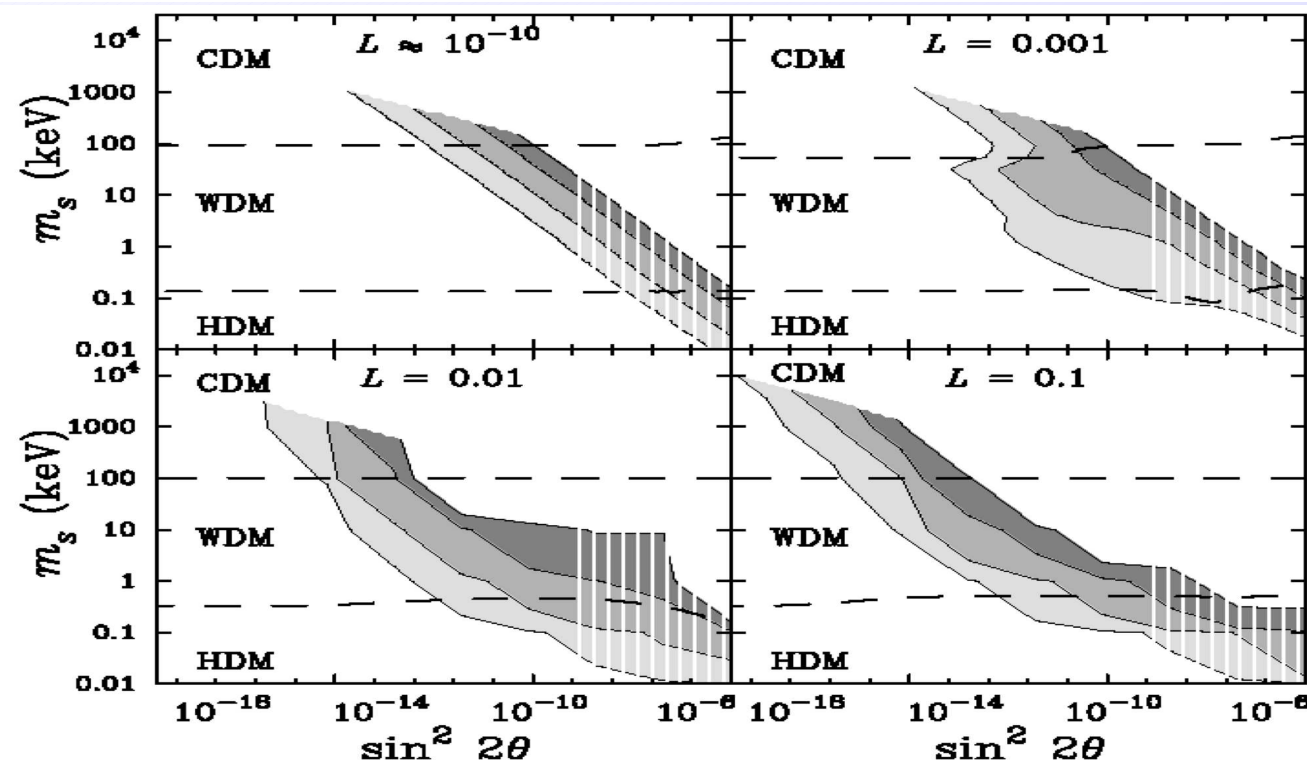


The production will depend on the mixing angle and on the interaction rate of the active neutrinos. A detailed computation requires to solve the associated Boltzmann equation for their distribution:

$$\frac{\partial}{\partial t} f_s(p, t) - H p \frac{\partial}{\partial p} f_s(p, t) \simeq \frac{\Gamma_a}{2} \langle P(\nu_a \rightarrow \nu_s; p, t) \rangle (f_a(p, t) - f_s(p, t))$$

with  $f_a(p, t) = (1 + e^{E/T})^{-1}$ .

The final abundance is  $\Omega_4 h^2 \simeq 0.3 \frac{\sin^2 2\theta}{10^{-8}} \left( \frac{m_4}{10 \text{keV}} \right)^2$



Abazajian et al., 2001

In presence of a large asymmetry, even smaller angles are required thanks to the resonant enhancement of the production.

Bounds on these DM candidates are derived from their effect on structure formation in the Early Universe and from indirect searches of their decays.

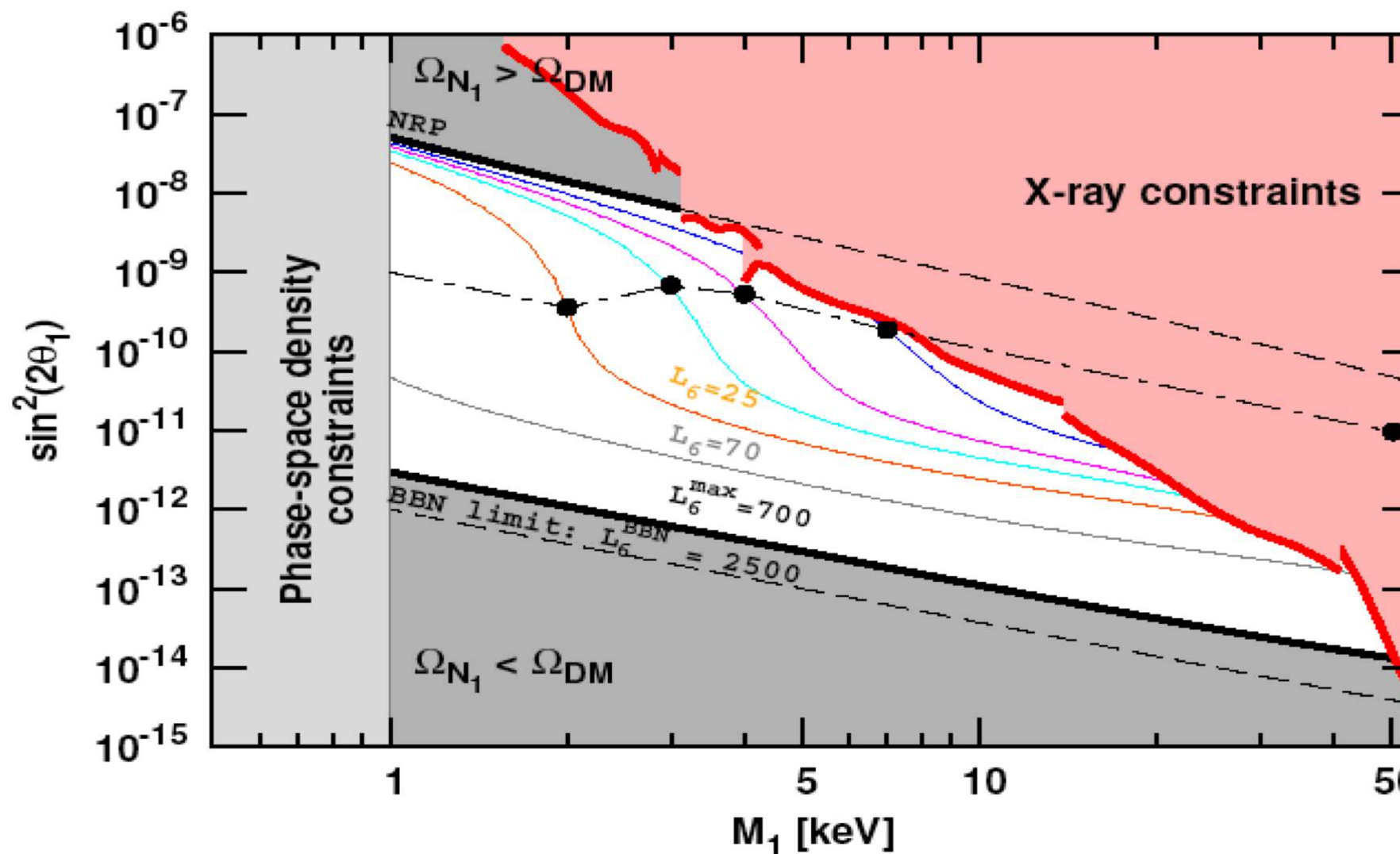
- **Structure formation.** If their mass is too low, they will behave too much as HDM erasing the structure at intermediate scales. This allows to put a bound in the  $\sim 5$  KeV range.

This bound can be relaxed if the production mechanism makes the sterile neutrinos “cooler”, eg. in resonance production or from decays of heavy particles.

- **x-ray searches.** Although nearly sterile, their small mixing with active neutrinos make them decay in photons:

$$\nu_4 \rightarrow \nu_a \gamma \quad \text{with} \quad E_\gamma = m_4/2 \quad \text{and} \quad Br(\nu\gamma) \sim 0.01$$

The resulting bounds are very stringent.



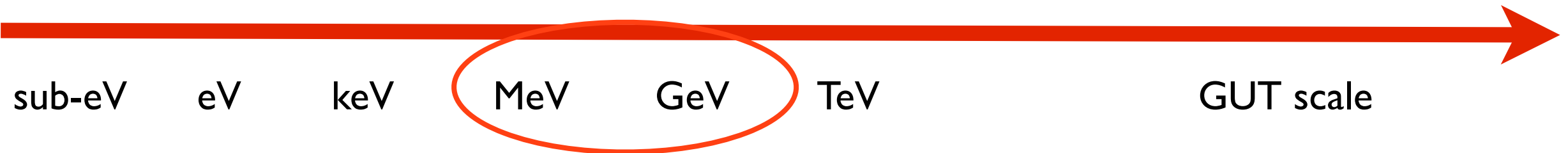
Boyarsky et al., 2009

There is some tension between x-ray searches and structure formation bounds, unless the other production mechanisms are invoked.

Recently WDM simulations of large scale structure formation have gained renewed interest.

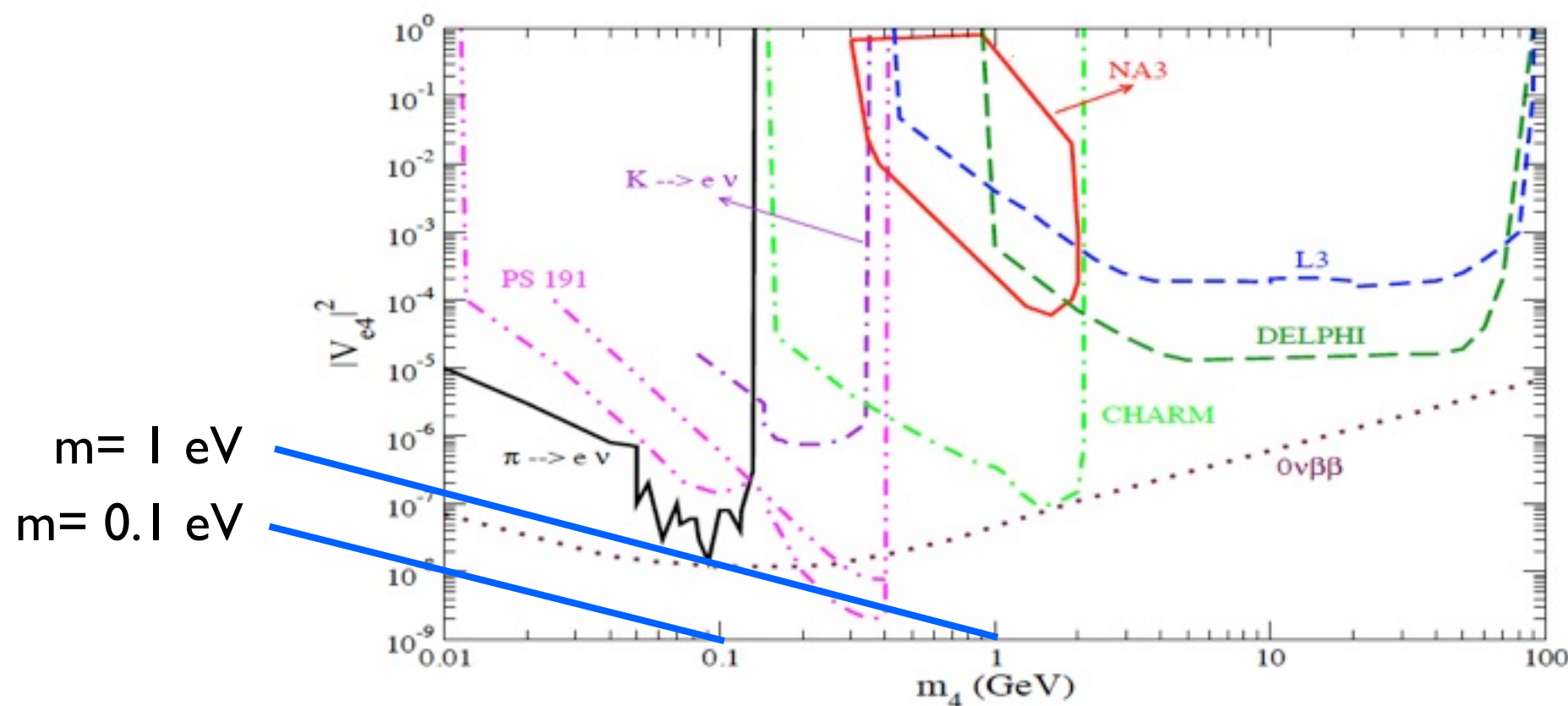


# MeV-100 GeV scale: peak searches and decays



Sterile neutrinos with MeV masses require not too Yukawa small couplings.

- **Peak searches.** In pion and kaon decays, if a heavy neutrino is emitted, the energy of the muon or electron will present a peak at smaller energy.

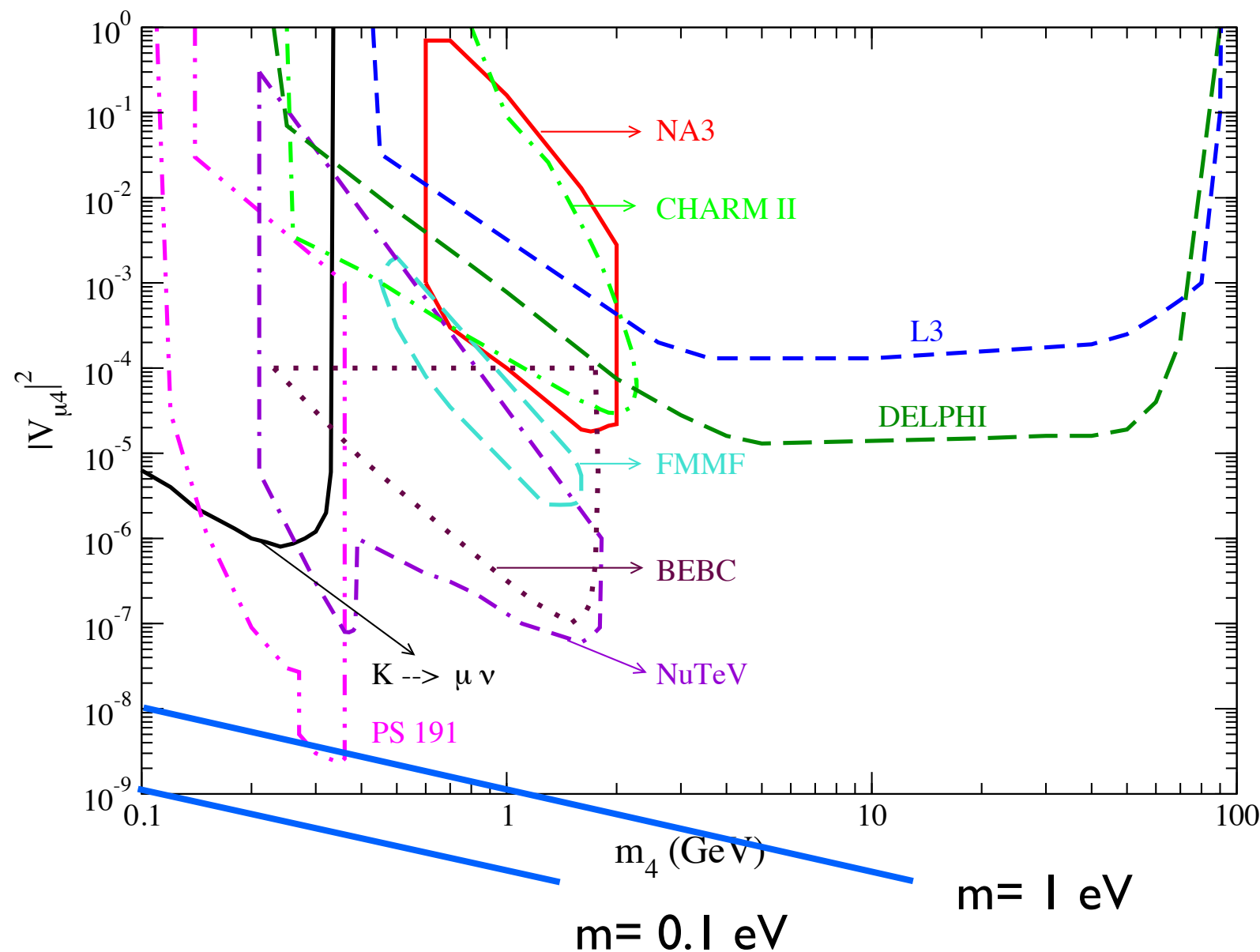


Atre et al., 2009

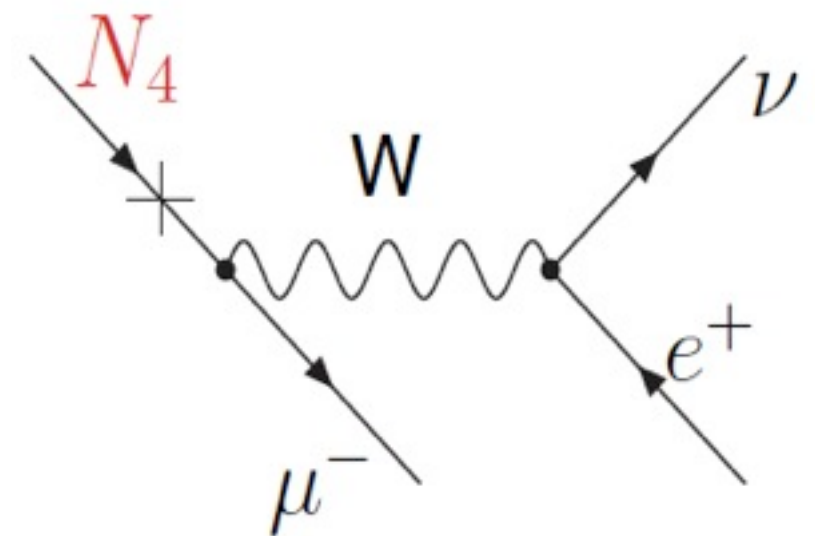
@Silvia Pascoli

## - Decays.

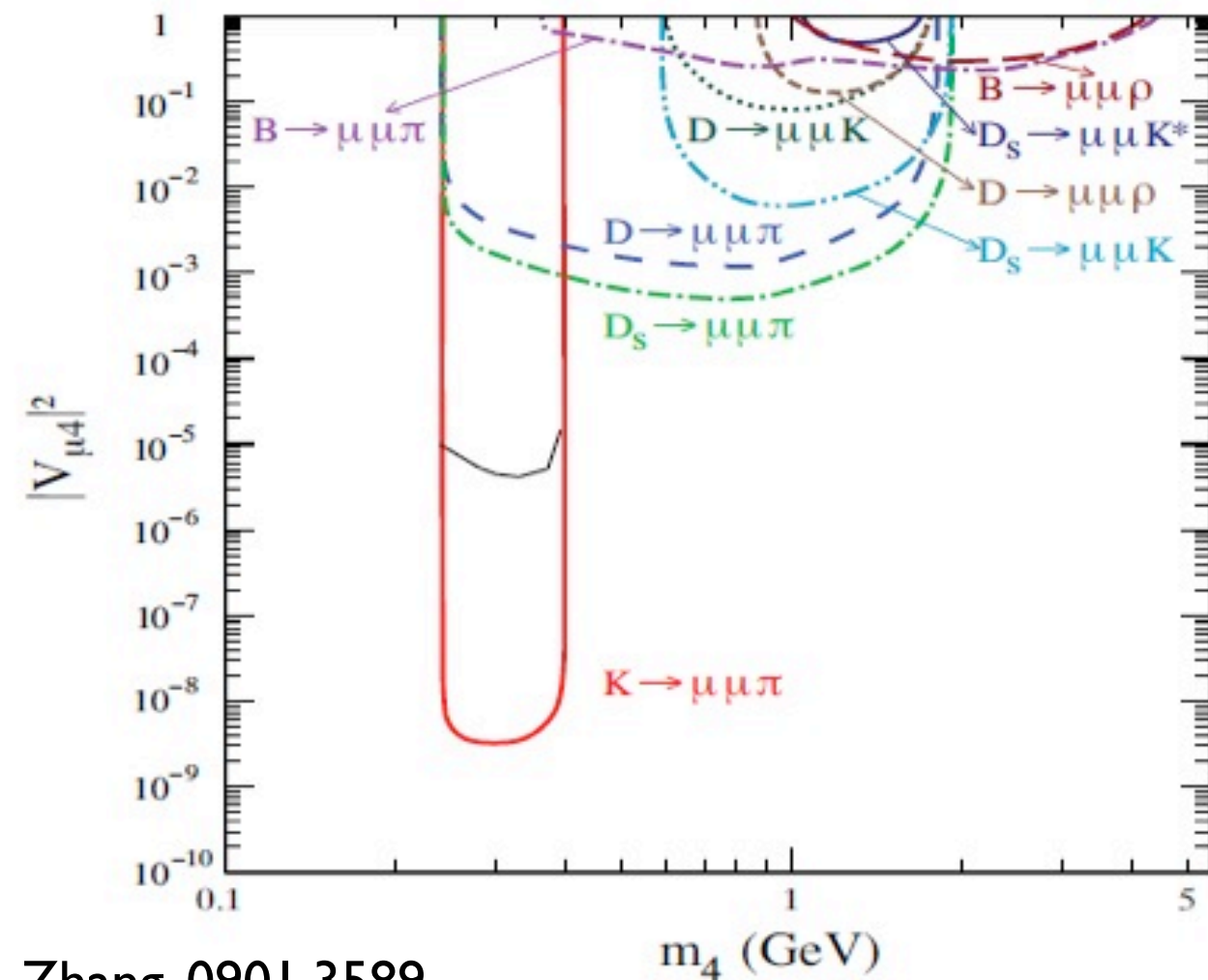
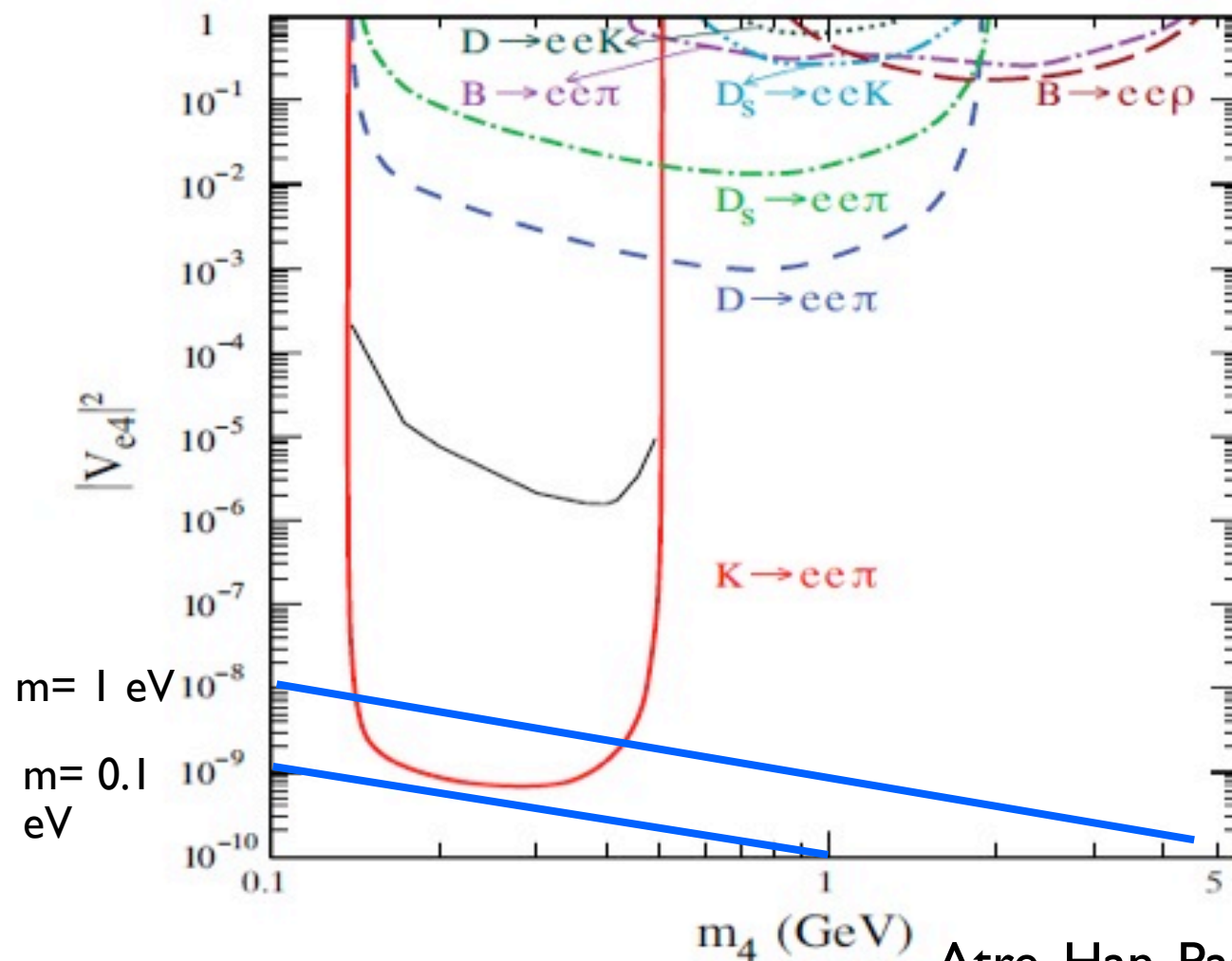
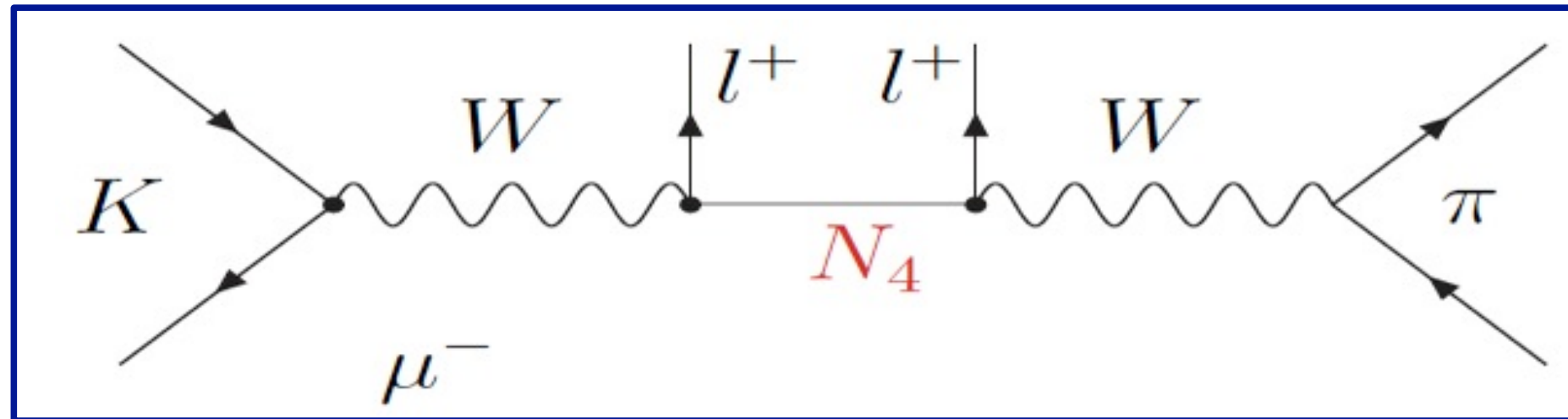
Via mixing the heavy neutrino will be produced in SM processes, e.g. pion decays, and then can decay in visible particles inside the detector (electrons, muons, pions....).



The production is controlled by the mixing angle and so is the decay length.



- **Tau and Meson LNV decays.** They get resonantly enhanced for  $M \sim 100 \text{ MeV} - 1 \text{ GeV}$ .



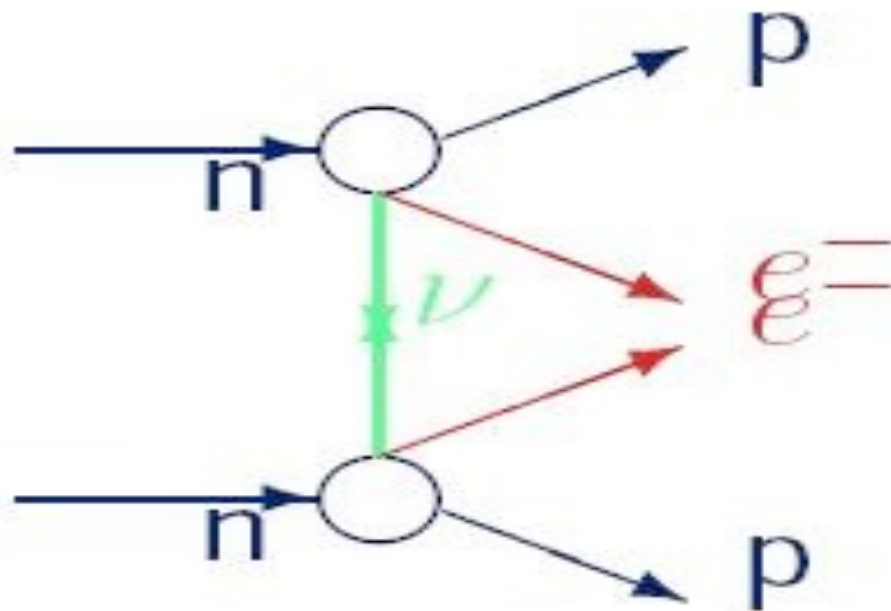
Atre, Han, Pascoli, Zhang, 0901.3589

# Neutrinoless double beta decay



As for light neutrinos, sterile neutrinos, if Majorana, will induce neutrinoless double beta decay.

(see Kayser's and Yang's lectures)

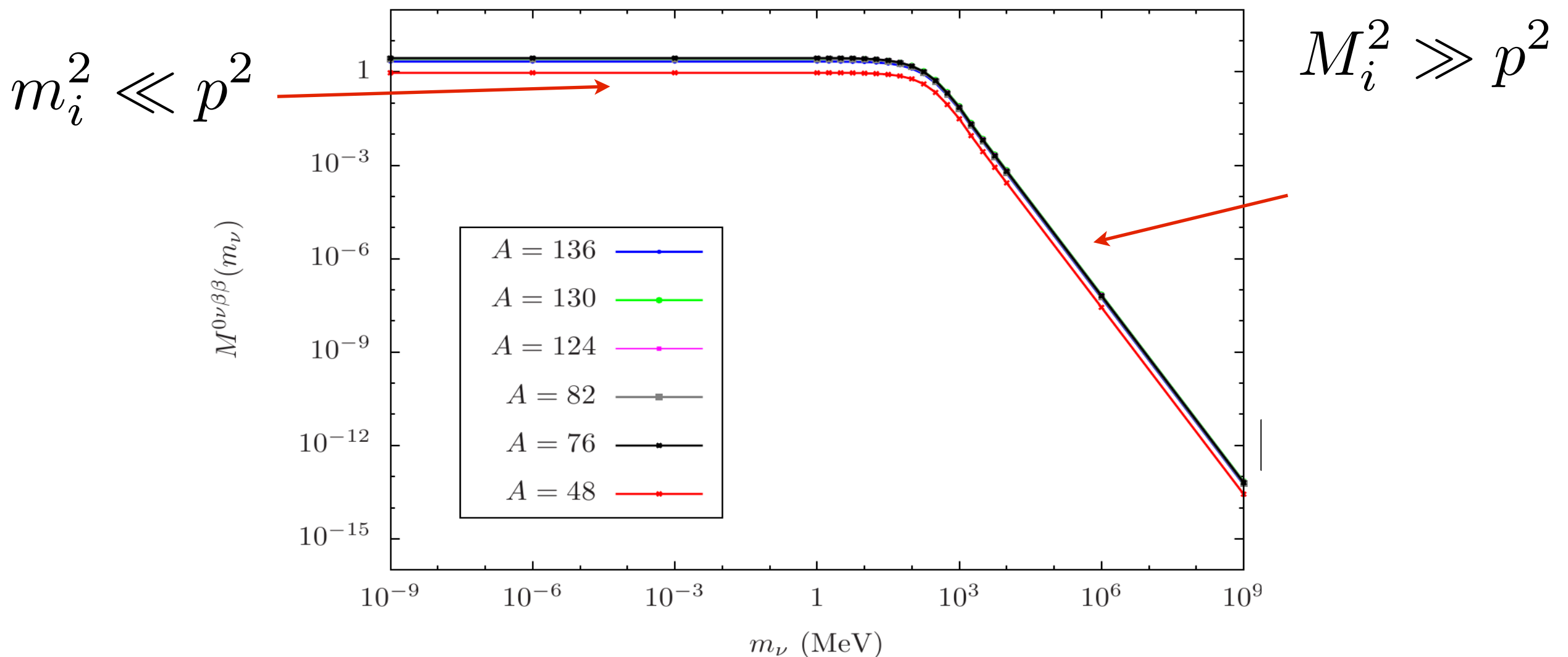


The half-life time depends on neutrino properties.

$$(T_{0\nu}^{1/2})^{-1} \propto G_{0\nu} \left| N M E_{\text{light}} \sum_i m_i U_{ei}^2 + N M E_{\text{light}} m_4 U_{e4}^2 + N M E_{\text{heavy}} m_h U_{eh}^2 \right|^2$$

The NME behaviour changes at  $p \sim 100$  MeV, the scale of the process. In most cases they are subdominant as the NME for heavy particles suppress their contribution w.r.t. the long range processes.

[http://www.th.mppmu.mpg.de/members/blennow/nme\\_mnu.dat](http://www.th.mppmu.mpg.de/members/blennow/nme_mnu.dat)



So if the new neutrinos are  $\gg 100$  MeV, typically their effects will be suppressed (there are exceptions).



If the masses are below 100 MeV, their effects are not suppressed and the effective Majorana mass is modified by the contribution due to sterile neutrinos.

$$|\langle m \rangle| \equiv \left| \sum_{\text{light}} m_i U_{ei}^2 + m_4 |U_{e4}|^2 e^{i\alpha_{41}} \right|$$

From a phenomenological point of view, the new mass, mixing angle and phase are free parameters and one can find typical bounds:

$$\sin^2 \theta \lesssim \frac{|\langle m \rangle|}{m_4} \sim 10^{-4} \frac{3 \text{ keV}}{m_4}$$

But this is not the case in specific cases. Light see-saw:

$$\begin{pmatrix} 0 & m_D \\ m_D^T & M \end{pmatrix} = U_{\text{total}} \begin{pmatrix} m_{\text{light}} & 0 \\ 0 & m_4 \end{pmatrix} U_{\text{total}}^T$$

$$0 = \sum_i U_{ei} m_i U_{ei}$$

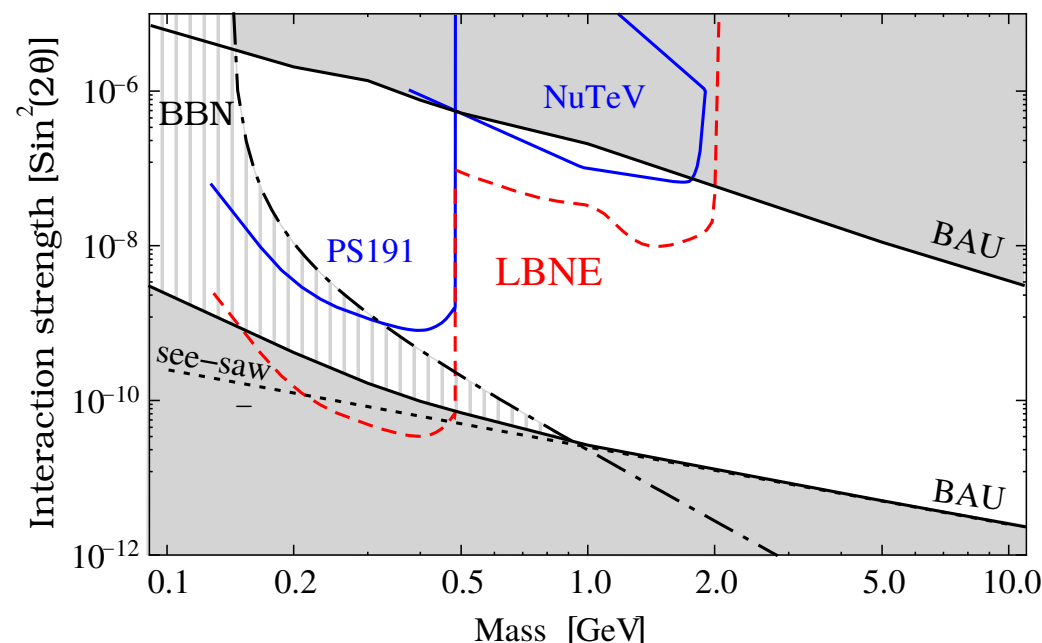
Although Majorana neutrinos, no signal in neutrinoless double beta decay!

# The nuMSM

This is the minimal extension of the SM which incorporates **neutrino masses, dark matter and the baryon asymmetry**. It requires 3 sterile neutrinos:

- $N_4$  is the DM candidate with mass  $\sim \text{keV}$ ;
- $N_5$  and  $N_6$  have masses in the GeV-100 GeV range to generate the baryon asymmetry and neutrino masses.

There is no explanation for the smallness of the mass of  $N_4$  and for the scale of  $N_5$  and  $N_6$ .



The heavier particles are testable in future experiments.

See e.g. Shaposhnikov et al.

# Conclusions (with some personal views)

1. Neutrinos have masses and mix and a wide experimental programme will measure their parameters with precision.
2. Neutrino masses cannot be accommodated in the Standard Model: extensions can lead to Dirac or Majorana neutrinos, with the latter the most studied cases. See-saw models are particularly favoured.
3. The main question concerns the energy scale of the new physics. Neutrino masses cannot pin it down by themselves and other signatures should be studied (leptogenesis, CLFV, collider LNV for TeV scale models, ...)
4. Models of flavour have typically a reduced number of parameters which can lead to relations testable in present and future experiments. Precision measurements will play a crucial role. Large  $\theta_{13}$  poses challenges to many flavour models.



# A few references

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Z.-Z. Xing, S. Zhou, Neutrinos in Particle Physics, Astronomy and Cosmology, Springer 2011

## Flavour models:

S. Luo, Z.-Z. Xing, 1211.4331

G. Altarelli, F. Feruglio, New J. Phys. 6:106, 2004 [hep-ph/0405048]

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**Sterile neutrinos:** White paper on sterile neutrinos, 1204.5379 and references therein.