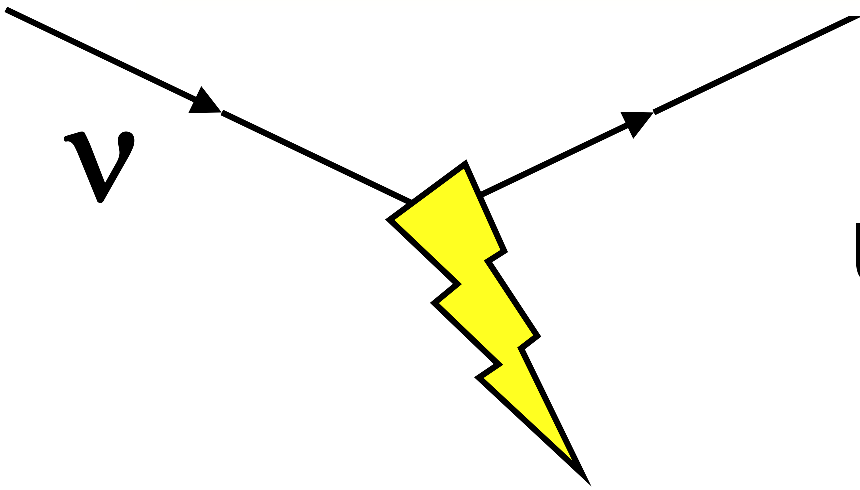
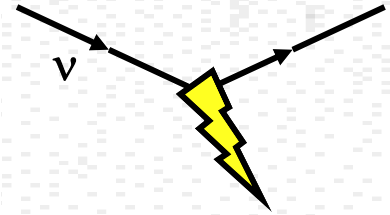
Interactions of Neutrinos



Kevin McFarland
University of Rochester
INSS 2013, Beijing
6-8 August 2013

Outline

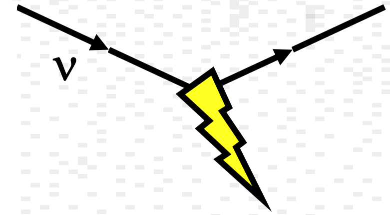


- Brief Motivation for and History of Measuring Interactions
  - Key reactions and thresholds
- Weak interactions and neutrinos
  - Elastic and quasi-elastic processes, e.g., νe scattering
  - Complication of Targets with Structure
  - Deep inelastic scattering (νq) and UHE neutrinos
  - Quasielastic and nearly elastic scattering
- Special problems at accelerator energies
  - Nuclear Effects
  - Generators, theory and experimental data
- Conclusions

Focus of These Lectures

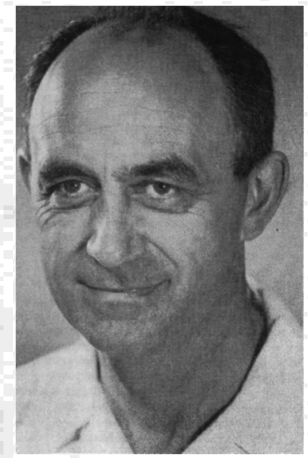
- This is not a comprehensive review of all the interesting physics associated with neutrino interactions
- Choice of topics will focus on:
  - Cross-sections useful for studying neutrino properties
  - Estimating cross-sections
  - Understanding the most important effects qualitatively or semi-quantitatively
  - Understanding how we use our knowledge of cross-sections in experiments

Weak Interactions



- Current-current interaction Fermi, Z. Physik, 88, 161 (1934)

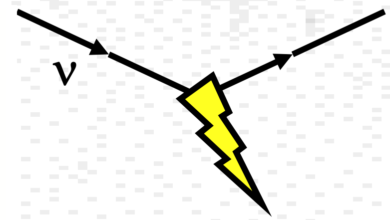
$$\mathcal{H}_w = \frac{G_F}{\sqrt{2}} \mathcal{J}^\mu \mathcal{J}_\mu$$



  - Paper famously rejected by *Nature:* *"it contains speculations too remote from reality to be of interest to the reader"*
- Prediction for neutrino interactions
  - If $n \rightarrow pe^- \bar{\nu}$, then $\bar{\nu} p \rightarrow e^+ n$
  - Better yet, it is robustly predicted by Fermi theory
    - o Bethe and Peirels, Nature 133, 532 (1934)
  - For neutrinos of a few MeV from a reactor, a typical cross-section was found to be $\sigma_{\bar{\nu} p} \sim 5\times10^{-44}\,\mathrm{cm}^2$

    This is wrong by a factor of two (parity violation)

How Weak is This?

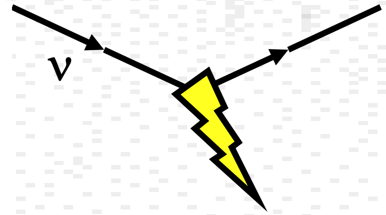


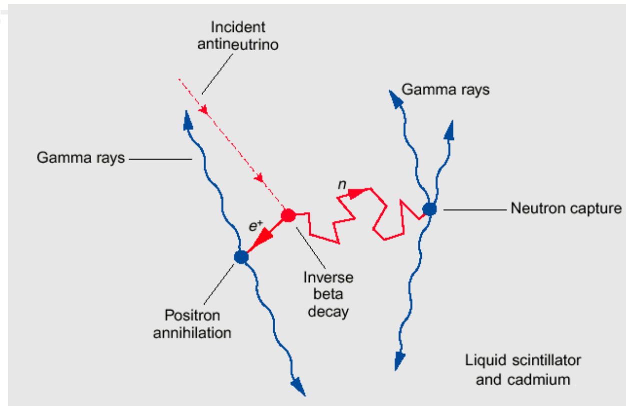
- σ~5x10^{-44}cm^2 compared with
  - $\sigma_{\gamma p}$~10^{-25} cm^2 at similar energies, for example
- The cross-section of these few MeV neutrinos is such that the mean free path in steel would be 10 light-years

"I have done something very bad today by proposing a particle that cannot be detected; it is something no theorist should ever do."

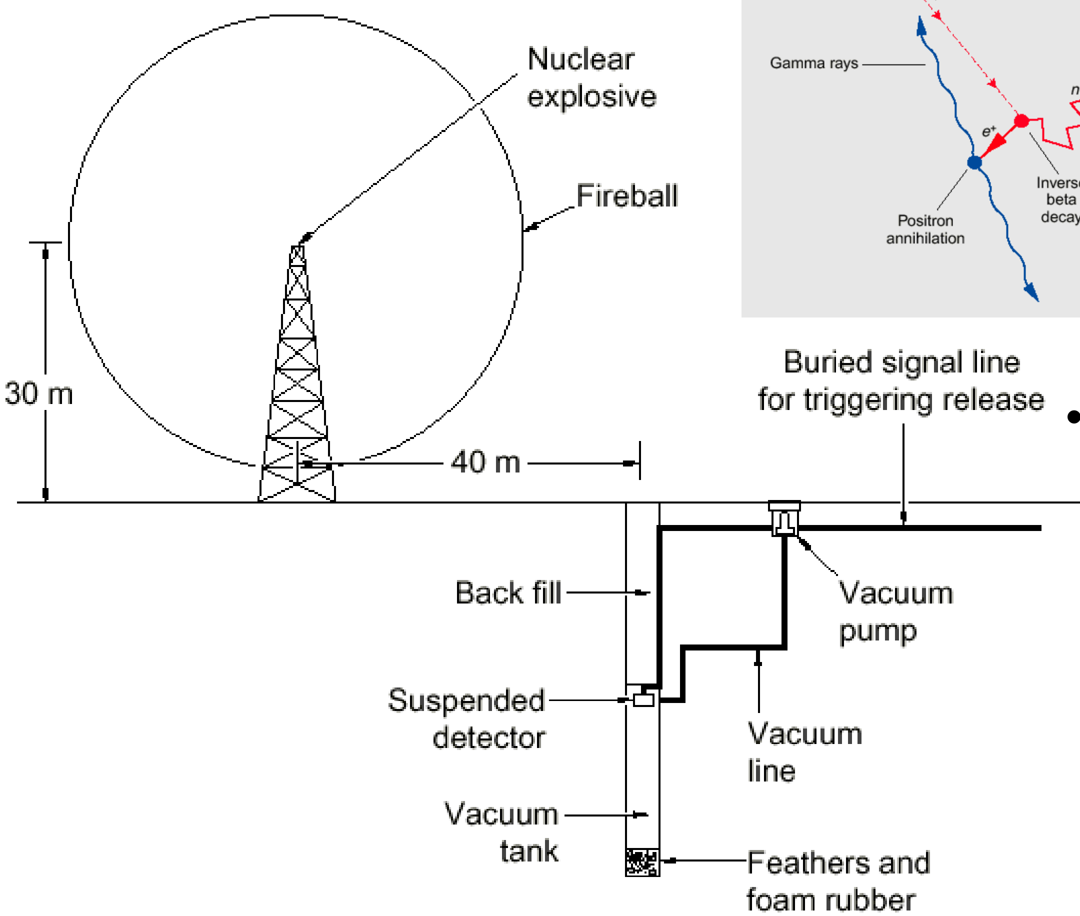
Wolfgang Pauli

Extreme Measures to Overcome Weakness (Reines and Cowan, 1946)



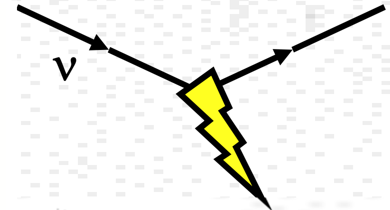


$$\bar{\nu}\, p \rightarrow e^{+} n$$

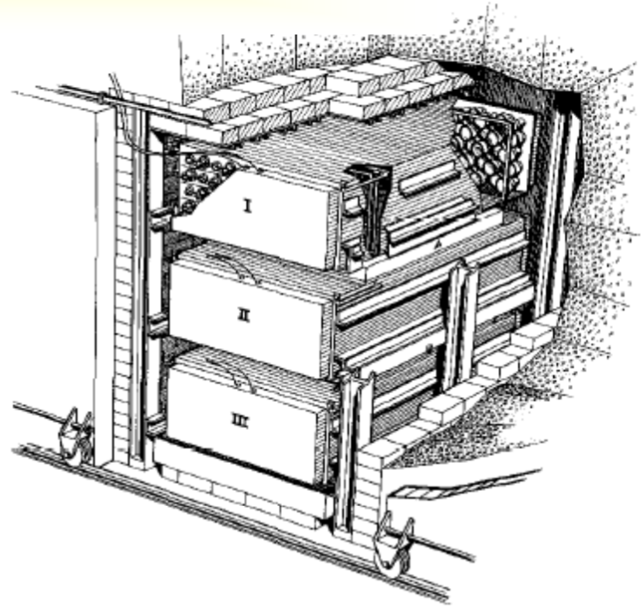


- Why inverse neutron beta decay?
  - clean prediction of Fermi weak theory
  - clean signature of prompt gammas from e^{+} plus delayed neutron signal.
    - Latter not as useful with bomb source.

Discovery of the Neutrino



- Reines and Cowan (1955)
  - Chose a constant source, nuclear reactor (Savannah River)
  - 1956 message to Pauli: ”We are happy to inform you [Pauli] that we have definitely detected neutrinos…”
  - 1995 Nobel Prize for Reines



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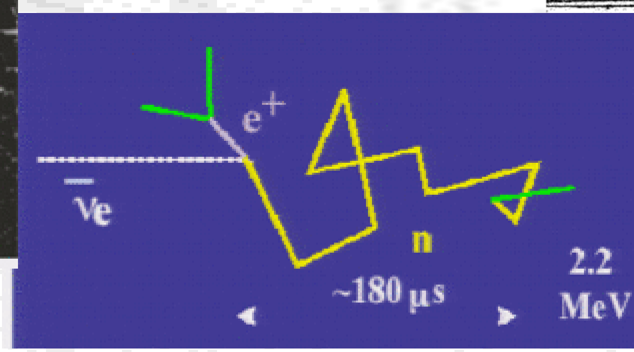
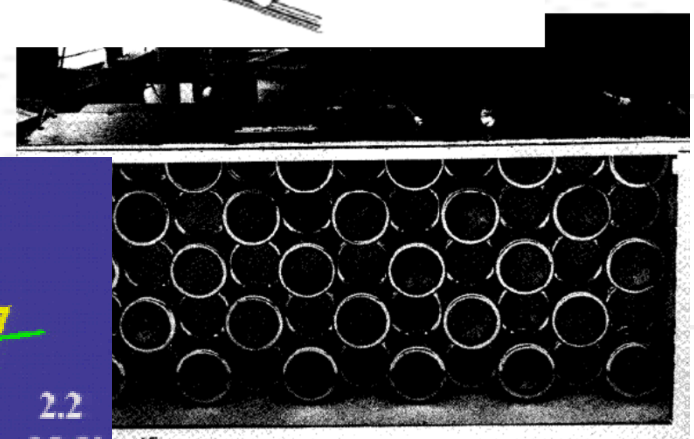
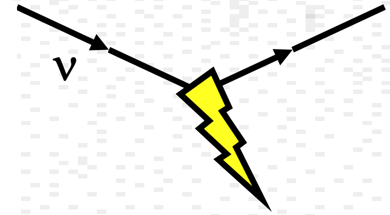
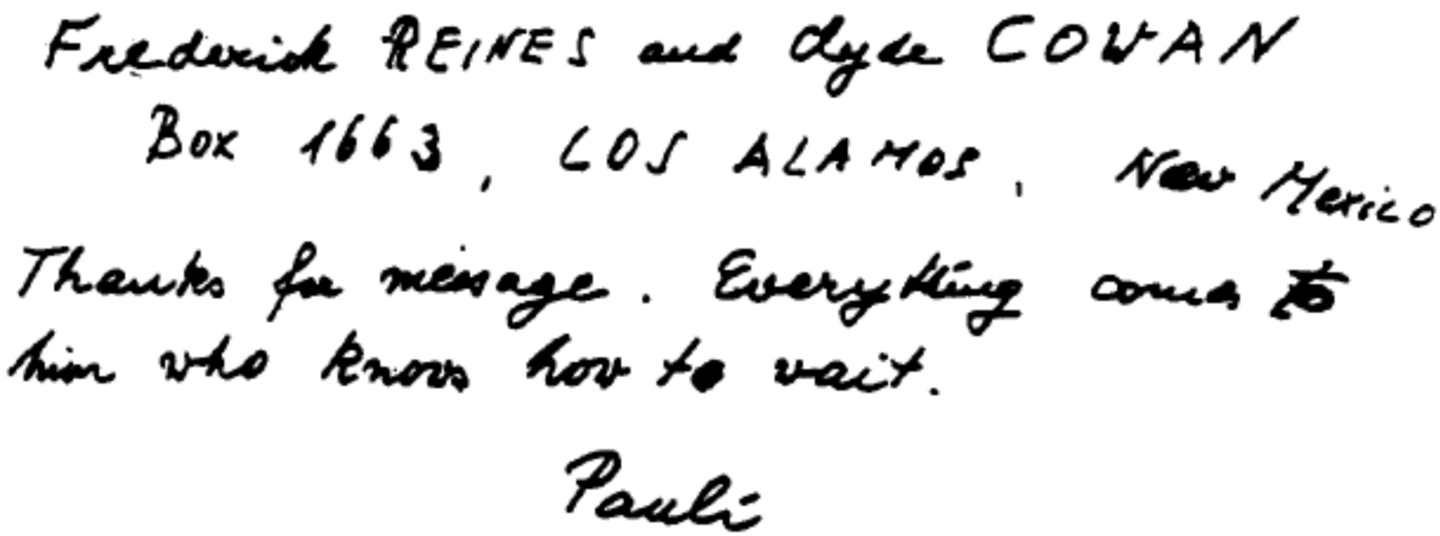




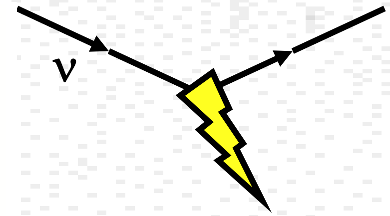
Better than the Nobel Prize?




Fredrick REINES and Clyde COWAN
Box 1663, LOS ALAMOS, New Mexico
Thanks for message. Everything comes to him who knows how to wait.
Pauli

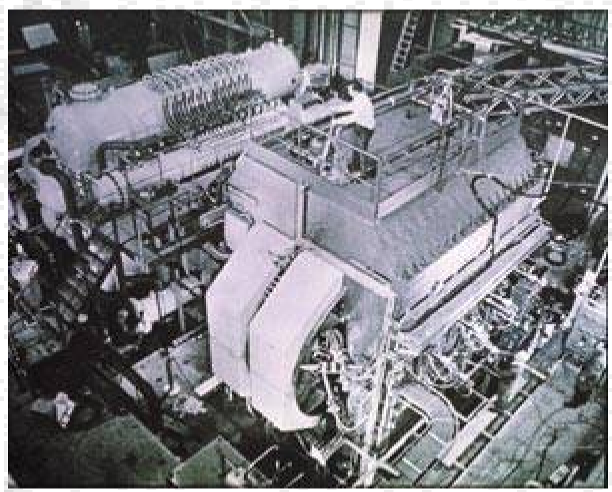
Thanks for the message. Everything comes to him who knows how to wait.

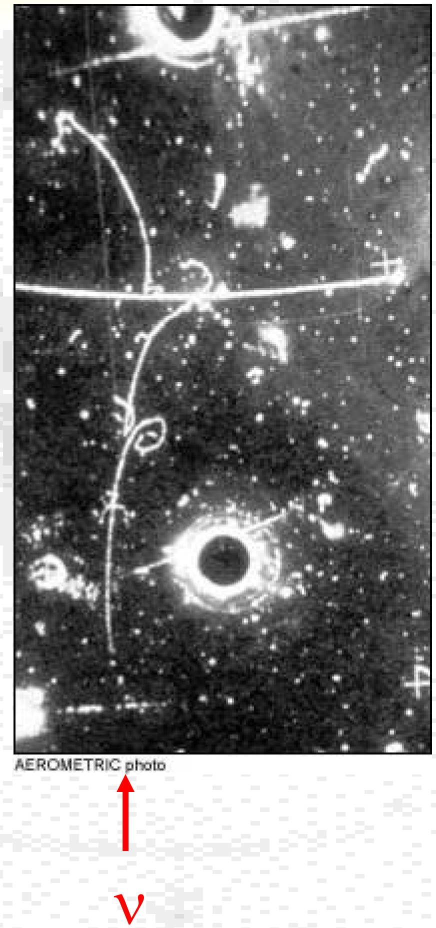
Another Neutrino Interaction Discovery



- Neutrinos only feel the weak force
  - a great way to study the weak force!
- Search for neutral current
  - arguably the most famous neutrino interaction ever observed is shown at right

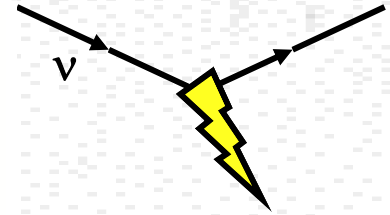
$$\bar{\nu}_\mu e^- \rightarrow \bar{\nu}_\mu e^-$$





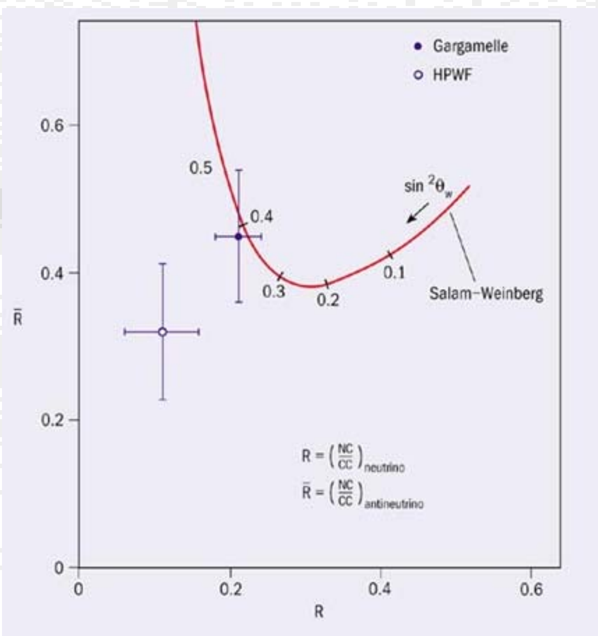
Gargamelle, event from neutral weak force

An Illuminating Aside



- The "discovery signal" for the neutral current was really neutrino scattering from nuclei
  - usually quoted as a ratio of muon-less interactions to events containing muons

$$R^{\nu} = \frac{\sigma(\nu_{\mu} N \rightarrow \nu_{\mu} X)}{\sigma(\nu_{\mu} N \rightarrow \mu^{-} X)}$$

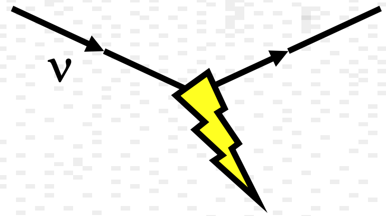


- But this discovery was complicated for 12-18 months by a lack of understanding of neutrino interactions
  - backgrounds from neutrons induced by neutrino interactions outside the detector
  - not understanding fragmentation to high energy hadrons which then "punched through" to fake muons

Great article: P. Gallison, Rev Mod Phys 55, 477 (1983)

The Future: Interactions and Oscillation Experiments

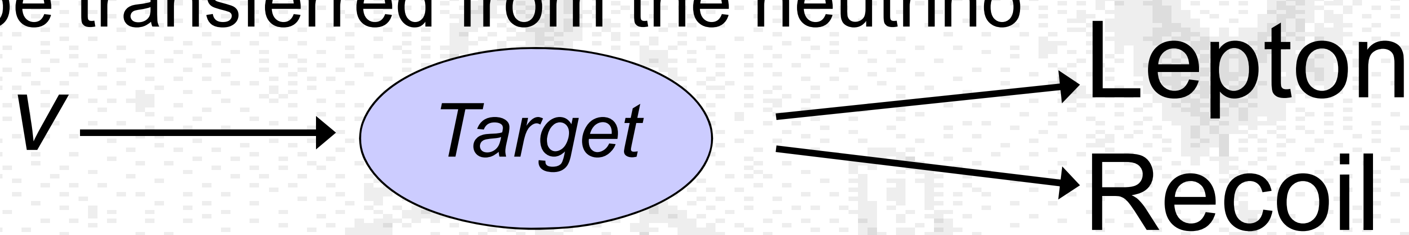
- Oscillation experiments point us to a rich physics potential at L/E~400 km/GeV *(and L/E~N·(400 km/GeV) as well)*
  - mass hierarchy, CP violation
- But there are difficulties
  - transition probabilities as a function of energy must be precisely measured for mass hierarchy and CP violation
  - the neutrinos must be at difficult energies of 1-few GeV for electron appearance experiments, few-many GeV for atmospheric neutrino and τ appearance experiments.
  - or use neutrinos from a reactor ☺
- *Our generation doesn't have neutrino flavor measurements in which distinguishing 1 from 0 or 1/3 buys a ticket to Stockholm*
  - Difficulties are akin to neutral current experiments
  - Is there a message for us here?



Kinematics of Neutrino Reactions

Thresholds and Processes

- We detect neutrino interactions only in the final state, and often with poor knowledge of the incoming neutrinos
- Creation of that final state may require energy to be transferred from the neutrino

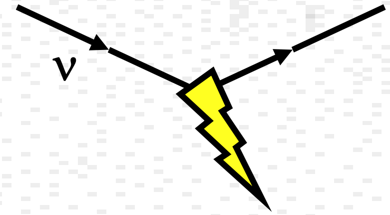


  - In charged-current reactions, where the final state lepton is charged, this lepton has mass
  - The recoil may be a higher mass object than the initial state, or it may be in an excited state

Thresholds and Processes

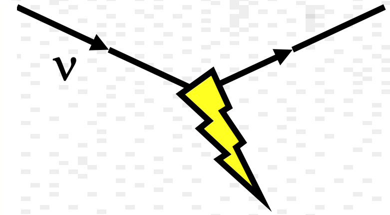
Process	Considerations	Threshold (typical)
νN→νN (elastic)	Target nucleus is often free (recoil is very small)	none
$\nu_e n \rightarrow e^- p$	In some nuclei (mostly metastable ones), this reaction is exothermic if proton not ejected	None for free neutron some others.
νe→νe (elastic)	Most targets have atomic electrons	~ 10eV – 100 keV
anti-$\nu_e p \rightarrow e^- n$	$m_n > m_p$ & m_e. Typically more to make recoil from stable nucleus.	1.8 MeV (free p). More for nuclei.
$\nu_\ell n \rightarrow \ell^- p$ (quasielastic)	Final state nucleon is ejected from nucleus. Massive lepton	~ 10s MeV for ν_e +~100 MeV for ν_μ
$\nu_\ell N \rightarrow \ell^- X$ (inelastic)	Must create additional hadrons. Massive lepton.	~ 200 MeV for ν_e +~100 MeV for ν_μ

- Energy of neutrinos determines available reactions, and therefore experimental technique

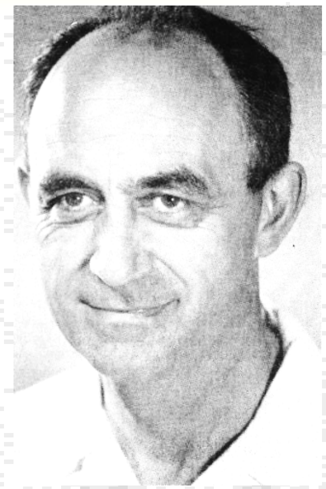


Calculating Neutrino Interactions from Electroweak Theory

Weak Interactions Revisited



- Current-current interaction (Fermi 1934) $\mathcal{H}_w = \frac{G_F}{\sqrt{2}} \mathcal{J}^\mu \mathcal{J}_\mu$

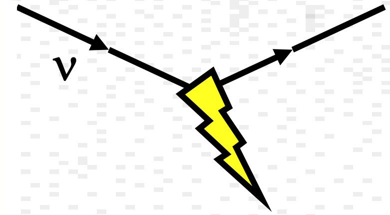


- Modern version:

$$\mathcal{H}_{weak} = \frac{G_F}{\sqrt{2}} \left[\bar{l} \gamma_\mu (1-\gamma_5) \nu \right] \left[\bar{f} \gamma^\mu (V - A\gamma_5) f \right] + h.c.$$

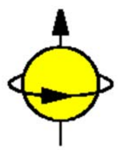
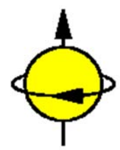
- $P_L = 1/2(1-\gamma_5)$ is a projection operator onto left-handed states for fermions and right-handed states for anti-fermions

Helicity and Chirality



- ***Helicity*** is projection of spin along the particles direction
  - Frame dependent (if massive)

The operator: $\sigma \cdot \mathbf{p}$

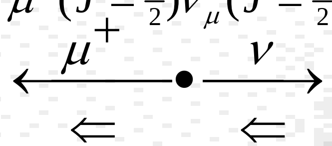
right-helicity  left-helicity 

- Neutrinos only interact weakly with a (V-A) interaction
  - All **neutrinos** are **left-handed**
  - All **antineutrinos** are **right-handed**
    - o **because of production!**
  - Weak interaction **maximally** violates parity

- However, ***chirality*** ("handedness") is Lorentz-invariant
  - – Only same as helicity for massless particles.

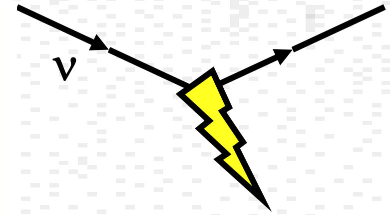
- If neutrinos have mass then left-handed neutrino is:
  - – Mainly left-helicity
  - – But also small right-helicity component $\propto$ *m/E*
- Only left-handed charged-leptons (e^-, μ^-, τ^-) interact weakly but mass brings in right-helicity:

$$\pi^+(J=0) \rightarrow \mu^+(J=\tfrac{1}{2})\nu_\mu(J=\tfrac{1}{2})$$



$$R_{theory} = \frac{\Gamma(\pi^\pm \rightarrow e^\pm \nu_e)}{\Gamma(\pi^\pm \rightarrow \mu^\pm \nu_\mu)} = \left(\frac{m_e}{m_\mu}\right)^2 \left(\frac{m_\pi{}^2 - m_e{}^2}{m_\pi{}^2 - m_\mu{}^2}\right)^2 = 1.23 \times 10^{-4}$$

Two Weak Interactions



- W exchange gives Charged-Current (CC) events and Z exchange gives Neutral-Current (NC) events

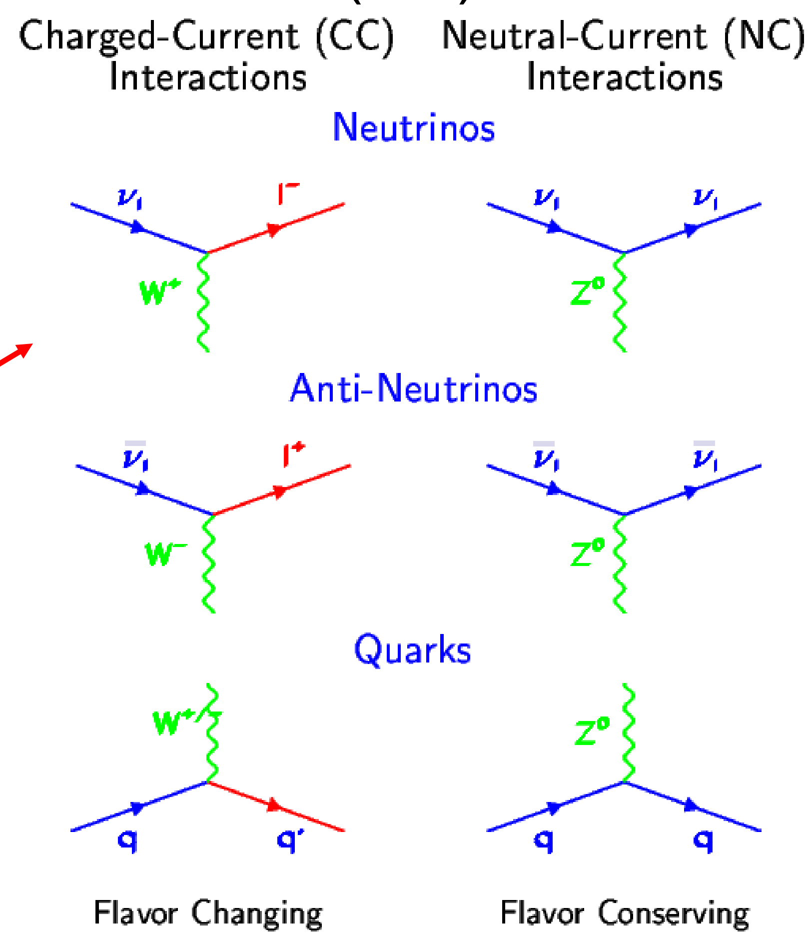
In charged-current events,

Flavor of outgoing lepton tags flavor of neutrino

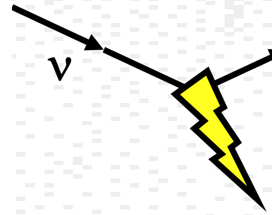
Charge of outgoing lepton determines if neutrino or antineutrino

$$l^- \Rightarrow \nu_l$$

$$l^+ \Rightarrow \bar{\nu}_l$$



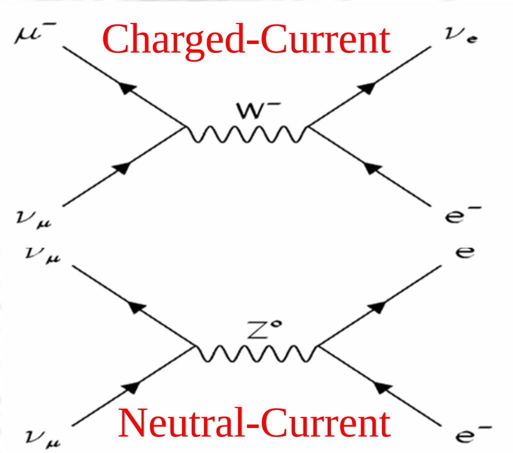
Electroweak Theory



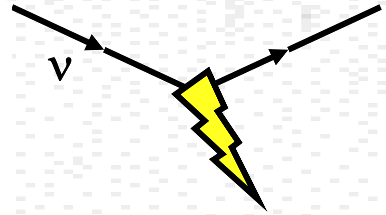
- Standard Model
  - SU(2) ⊗ U(1) gauge theory unifying weak/EM
    - ⇒ weak NC follows from EM, Weak CC
  - Physical couplings related to mixing parameter for the interactions in the high energy theory

$$\mathcal{L}_{EW}^{\text{int}} = -Q_e A_\mu \bar{e}\gamma^\mu e + \frac{g}{\sqrt{2}} W_\mu^+ \bar{\nu}_L \gamma^\mu e_L + \frac{g}{\sqrt{2}} W_\mu^- \bar{e}_L \gamma^\mu \nu_L$$

$$+\frac{g}{\cos\theta_W} Z_\mu^0 \left\{ \begin{array}{l} \frac{1}{2}\bar{\nu}_L \gamma^\mu \nu_L \\ +\left(\sin^2\theta_W - \frac{1}{2}\right)\bar{e}_L \gamma^\mu e_L \\ +\sin^2\theta_W \bar{e}_R \gamma^\mu e_R \end{array} \right\}$$



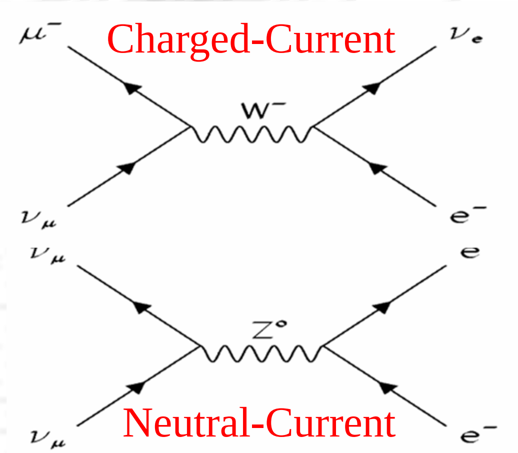
Electroweak Theory



- Standard Model
  - SU(2) ⊗ U(1) gauge theory unifying weak/EM
    ⇒ weak NC follows from EM, Weak CC
  - Measured physical parameters related to mixing parameter for the couplings.

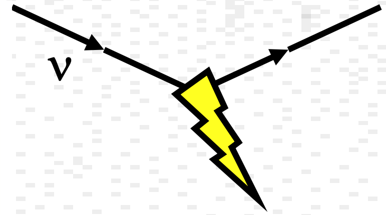
Z Couplings	g_L	g_R
ν_e , ν_μ , ν_τ	1/2	0
e , μ , τ	$-1/2 + \sin^2\theta_W$	$\sin^2\theta_W$
u , c , t	$1/2 - 2/3\ \sin^2\theta_W$	$-\ 2/3\ \sin^2\theta_W$
d , s , b	$-1/2 + 1/3\ \sin^2\theta_W$	$1/3\ \sin^2\theta_W$

$$e = g\sin\theta_W,\ G_F = \frac{g^2\sqrt{2}}{8M_W^2},\ \frac{M_W}{M_Z} = \cos\theta_W$$



- Neutrinos are special in SM
  - Right-handed neutrino has ***NO*** interactions!

Why "Weak"?

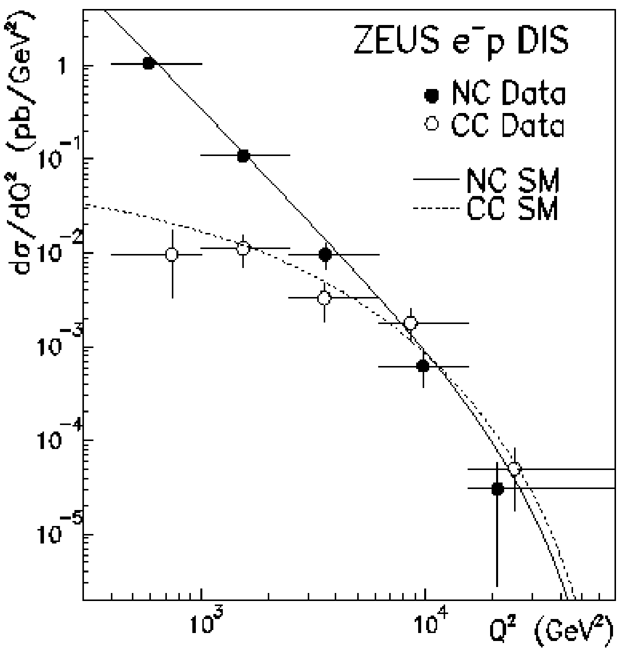


- Weak interactions are weak because of the massive W and Z bosons exchange

$$\frac{d\sigma}{dq^2} \propto \frac{1}{(q^2 - M^2)^2}$$

q is 4-momentum carried by exchange particle
M is mass of exchange particle

At HERA see W and Z propagator effects
- Also weak ~ EM strength



- Explains dimensions of Fermi "constant"

$$G_F = \frac{\sqrt{2}}{8}\left(\frac{g_W}{M_W}\right)^2$$
$$= 1.166 \times 10^{-5} / GeV^2 \quad (g_W \approx 0.7)$$

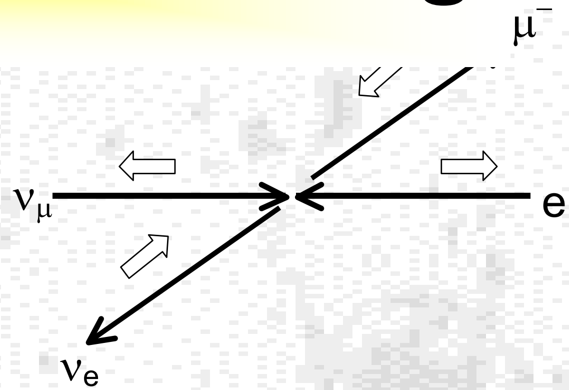
Neutrino-Electron Scattering

- Inverse μ–decay:

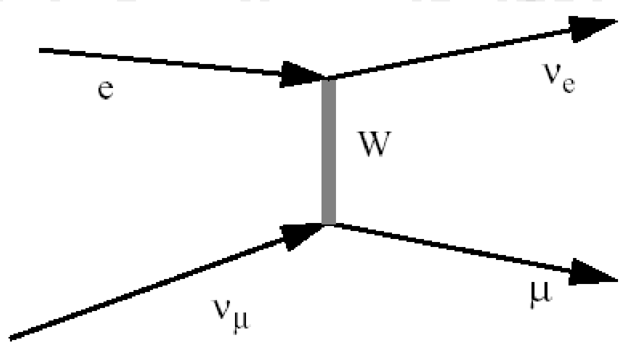
$$\nu_\mu + e^- \to \mu^- + \nu_e$$

  - Total spin J=0

    (*Assuming massless muon*, helicity=chirality)

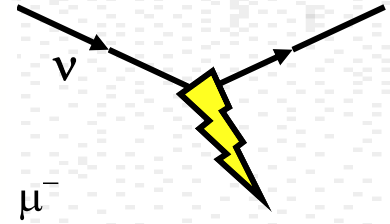


$$Q^2 \equiv -\left(\underline{e} - \underline{\nu}_e\right)^2$$

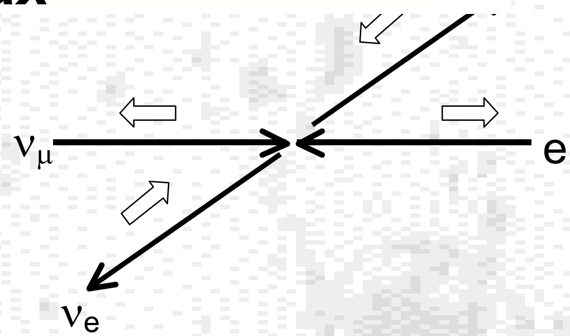


$$\sigma_{TOT} \propto \int_0^{Q^2_{\max}} dQ^2 \frac{1}{(Q^2 + M_W{}^2)^2} \approx \frac{Q^2_{\max}}{M_W{}^4}$$

Lecture Question #1
What is Q^2_{max}?



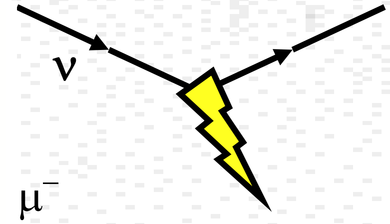
$$\nu_\mu + e^- \rightarrow \mu^- + \nu_e$$



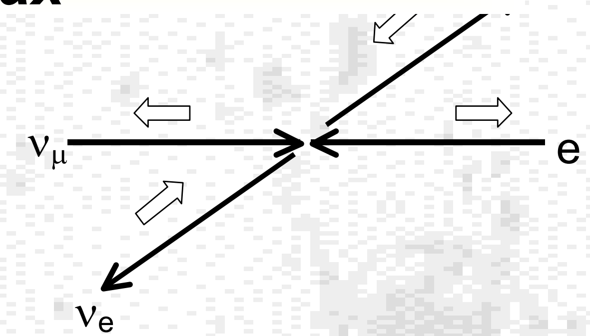
$$Q^2 \equiv -\left(\underline{e} - \underline{\nu}_e\right)^2$$

*Work in the center-of-mass frame and assume, **for now**, that we can neglect the masses.*

Lecture Question #1
What is Q^2_{max}?



$$\nu_\mu + e^- \to \mu^- + \nu_e$$



$$Q^2 \equiv -\left(\underline{e} - \underline{\nu}_e\right)^2$$

Work in the center-of-mass frame and assume, ***for now****, that we can neglect the masses.*

$$\underline{e} \approx (E_\nu^*, 0, 0, -E_\nu^*)$$

$$\underline{\nu}_e \approx (E_\nu^*, -E_\nu^* \sin\theta^*, 0, -E_\nu^* \cos\theta^*)$$

$$Q^2 = -\left(\underline{e}^2 + \underline{\nu}_e{}^2 - 2\underline{e}\bullet\underline{\nu}_e\right)^2$$

$$\approx -\left[-2E_\nu^{*2}\left(1 - \cos\theta^*\right)\right]$$

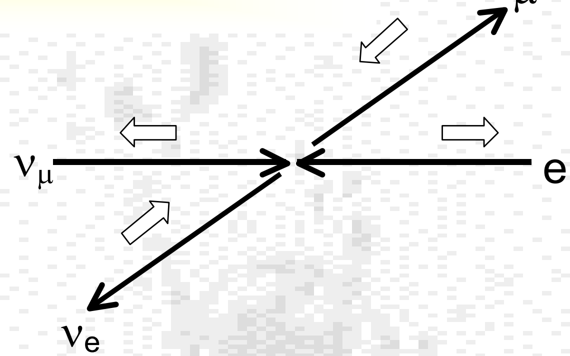
$$0 < Q^2 < \left(2E_\nu^*\right)^2 \approx \left(\underline{e} + \underline{\nu}_\mu\right)^2$$

$$0 < Q^2 < s$$

Neutrino-Electron (cont'd)

$$\sigma_{TOT} \propto Q^2_{\max} = s$$

$$\sigma_{TOT} = \frac{G_F^2 s}{\pi}$$
$$= 17.2 \times 10^{-42} cm^2 / GeV \cdot E_\nu (GeV)$$

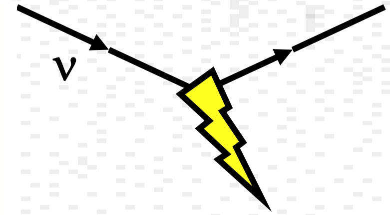


- Why is it proportional to beam energy?

$$s = (\underline{p}_{\nu_\mu} + \underline{p}_e)^2 = m_e^2 + 2 m_e E_\nu \text{ (e}^- \text{ rest frame)}$$

- Proportionality to energy is a generic feature of point-like scattering!
  - because $d\sigma/dQ^2$ is constant (at these energies)

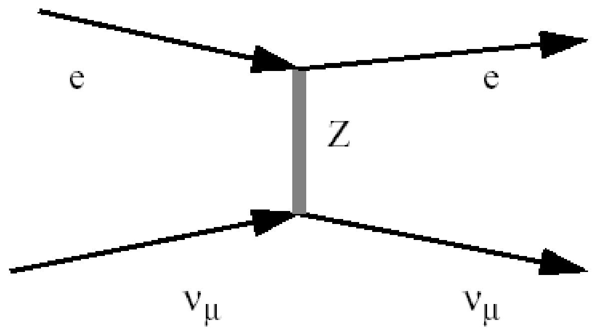
Neutrino-Electron (cont'd)



- Elastic scattering:

$$\nu_\mu + e^- \rightarrow \nu_\mu + e^-$$

  - Recall, EW theory has coupling to left *or* right-handed electron
  - Total spin, J=0,1



Z Couplings	g_L	g_R
ν_e , ν_μ , ν_τ	1/2	0
e , μ , τ	$-1/2 + \sin^2\theta_W$	$\sin^2\theta_W$
u , c , t	$1/2 - 2/3 \sin^2\theta_W$	$- 2/3 \sin^2\theta_W$
d , s , b	$-1/2 + 1/3 \sin^2\theta_W$	$1/3 \sin^2\theta_W$

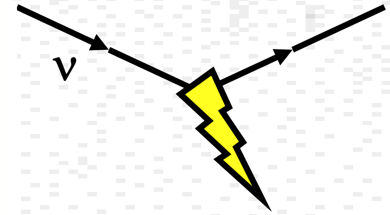
- Electron-Z^0 coupling
  - Left-handed: $-1/2 + \sin^2\theta_W$

$$\sigma \propto \frac{G_F^2 s}{\pi}\left(\frac{1}{4} - \sin^2\theta_W + \sin^4\theta_W\right)$$

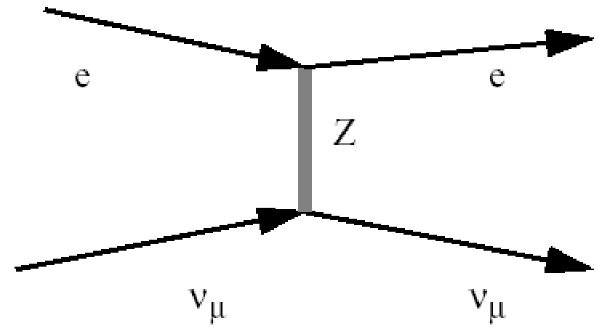
  - Right-handed: $\sin^2\theta_W$

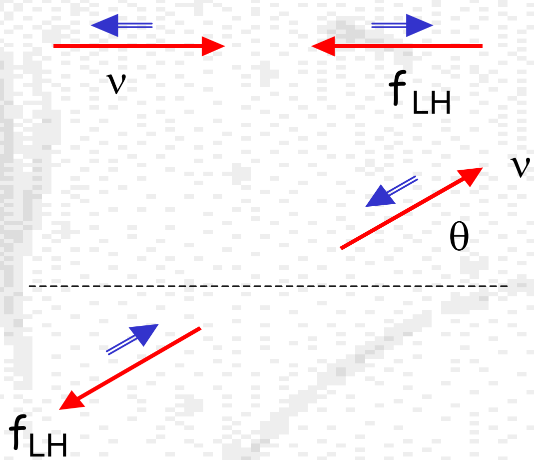
$$\sigma \propto \frac{G_F^2 s}{\pi}\left(\sin^4\theta_W\right)$$

Neutrino-Electron (cont'd)

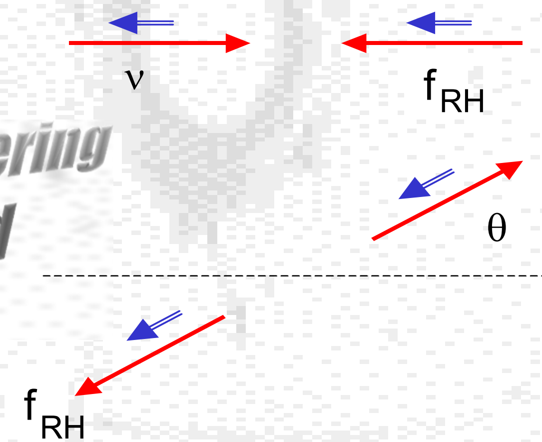


- What are relative contributions of scattering from left *and* right-handed electrons?





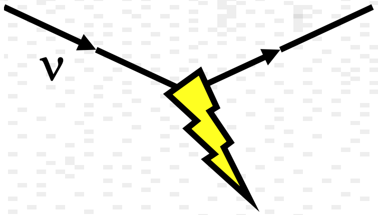
Backwards scattering is disfavored



$$\frac{d\sigma}{d\cos\theta} = \text{const}$$

$$\frac{d\sigma}{d\cos\theta} = \text{const}\times\left(\frac{1+\cos\theta}{2}\right)^2$$

Neutrino-Electron (cont'd)



- Electron-Z^0 coupling $\sigma \propto \frac{G_F^2 s}{\pi}\left(\frac{1}{4} - \sin^2\theta_W + \sin^4\theta_W\right)$
  - (LH, V-A): $-1/2 + \sin^2\theta_W$
  - (RH, V+A): $\sin^2\theta_W$ $\qquad \sigma \propto \frac{G_F^2 s}{\pi}\left(\sin^4\theta_W\right)$

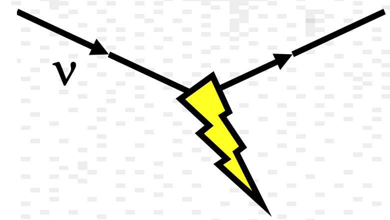
Let y denote inelasticity.
Recoil energy is related to
CM scattering angle by

$$y = \frac{E_e}{E_\nu} \approx 1 - \tfrac{1}{2}(1-\cos\theta)$$

$$\int dy \frac{d\sigma}{dy} = \begin{cases} \text{LH:} & \int dy = 1 \\ \text{RH:} & \int (1-y)^2 dy = 1/3 \end{cases}$$

$$\sigma_{TOT} = \frac{G_F^2 s}{\pi}\left(\frac{1}{4} - \sin^2\theta_W + \frac{4}{3}\sin^4\theta_W\right) = 1.4\times 10^{-42}\, cm^2/GeV \cdot E_\nu(GeV)$$

Lecture Question #2: Flavors and νe Scattering



The reaction

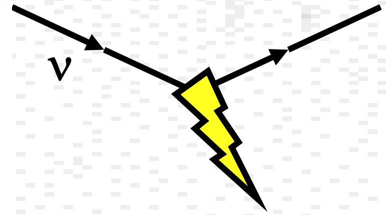
$$\nu_\mu + e^- \rightarrow \nu_\mu + e^-$$

has a much smaller cross-section than

$$\nu_e + e^- \rightarrow \nu_e + e^-$$

Why?

Lecture Question #2: Flavors and νe Scattering



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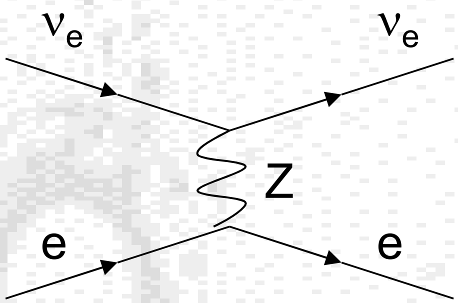
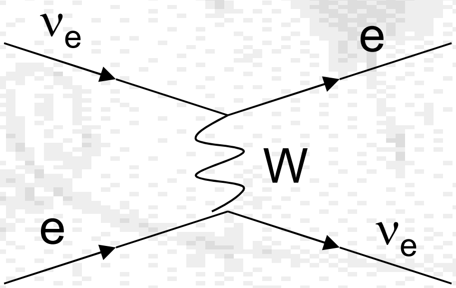
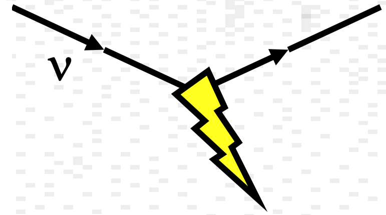


$$\nu_e + e^- \to \nu_e + e^-$$

has a second contributing reaction, charged current



Lecture Question #2: Flavors and νe Scattering



Let's show that this increases the rate

(Recall from the previous pages…

$$\sigma_{TOT} = \int dy \frac{d\sigma}{dy}$$

$$= \int dy \left[\frac{d\sigma^{LH}}{dy} + \frac{d\sigma^{RH}}{dy} \right]$$

$$= \sigma_{TOT}^{LH} + \tfrac{1}{3}\sigma_{TOT}^{RH}$$

$$\sigma_{TOT}^{LH} \propto \left| \text{total coupling}_{e^-}^{LH} \right|^2$$

For electron…	LH coupling	RH coupling
Weak NC	$-1/2+\sin^2\theta_W$	$\sin^2\theta_W$
Weak CC	-1/2	0

)

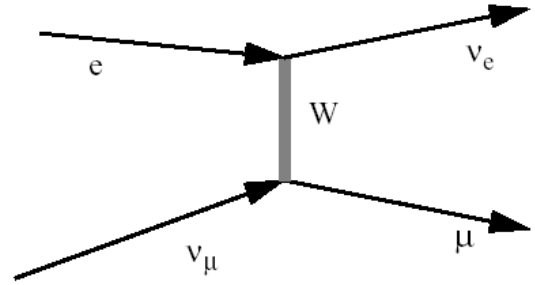
We have to show the interference between CC and NC is constructive.

The total RH coupling is unchanged by addition of CC because there is no RH weak CC coupling

There are two LH couplings: NC coupling is $-1/2+\sin^2\theta_W \approx -1/4$ and the CC coupling is -1/2. We add the associated amplitudes… and get $-1+\sin^2\theta_W \approx -3/4$

Lepton Mass Effects

- Let's return to
  Inverse μ–decay:
  $$\nu_\mu + e^- \rightarrow \mu^- + \nu_e$$
  - What changes in the presence of final state mass?
    - o pure CC so always left-handed
    - o BUT there must be finite Q^2 to create muon in final state!



$$\sigma_{TOT} \propto \int_{Q^2_{\min}}^{Q^2_{\max}} dQ^2 \frac{1}{(Q^2 + M_W{}^2)^2}$$

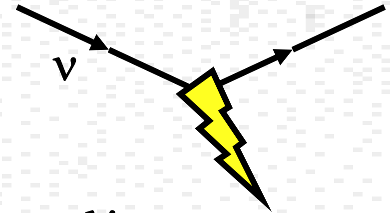
$$\approx \frac{Q^2_{\max} - Q^2_{\min}}{M_W{}^4}$$

$$Q^2_{\min} = m^2_\mu$$

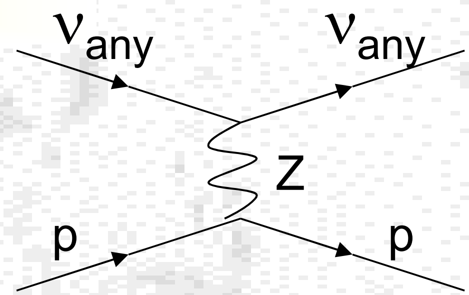
$$\sigma_{TOT} = \frac{G_F^2 (s - m^2_\mu)}{\pi}$$

$$= \left[\sigma_{TOT}{}^{\text{(massless)}}\right]\left(1 - \frac{m^2_\mu}{s}\right)$$

  - see a suppression scaling with **(mass/CM energy)^2**
    - o This can be generalized…



What about other targets?



- Imagine now a proton target
  - Neutrino-proton elastic scattering:

$$\nu_e + p \rightarrow \nu_e + p$$

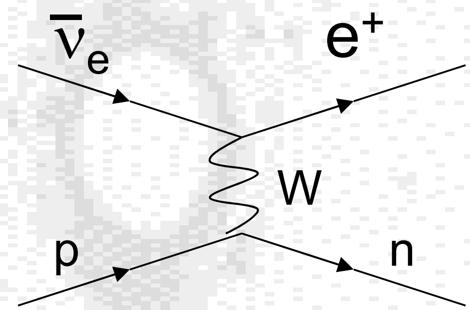
  - "Inverse beta-decay" (IBD):

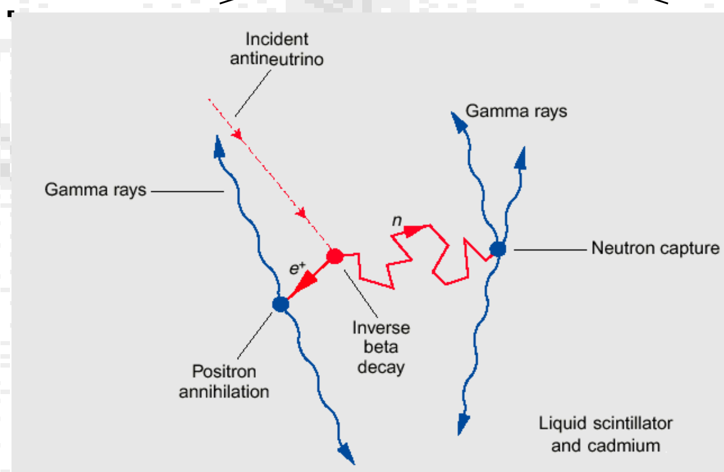
$$\overline{\nu}_e + p \rightarrow e^+ + n$$

  - and "stimulated" beta decay:

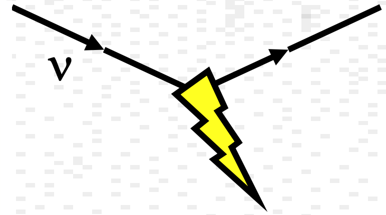
$$\nu_e + n \rightarrow e^- + p$$

  - Recall that IBD was the Reines and Cowan discovery signal

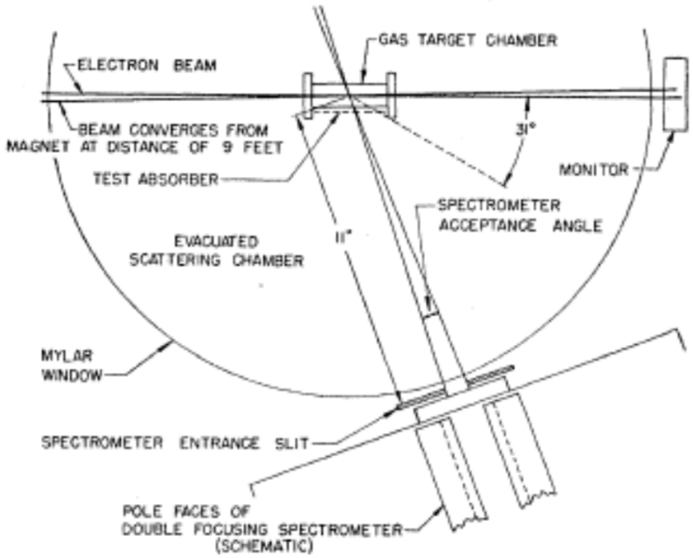




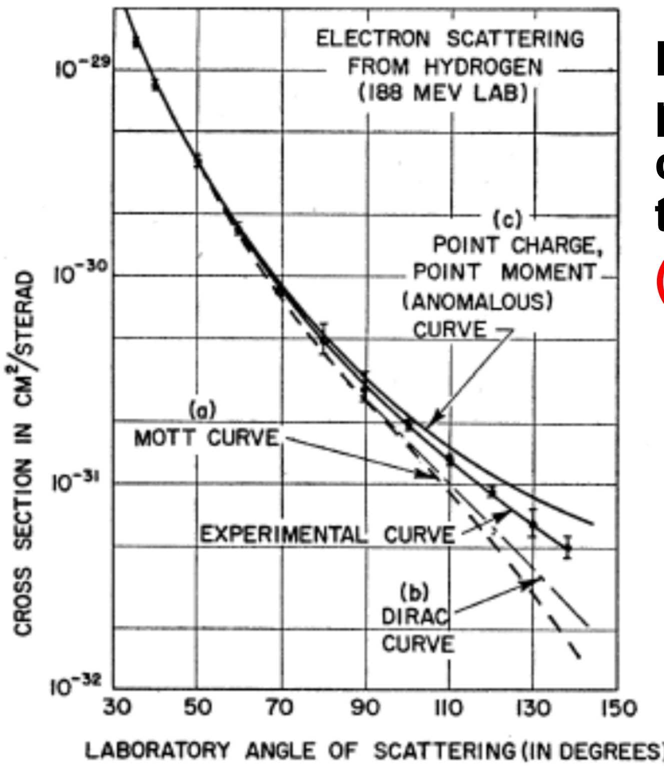
Proton Structure



- How is a proton different from an electron?
  - anomalous magnetic moment, $\kappa \equiv \frac{g-2}{2} \neq 1$
  - "form factors" related to finite size

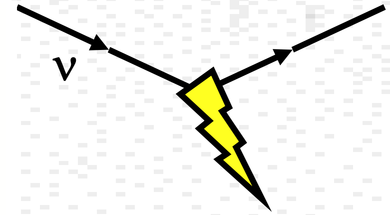


McAllister and Hofstadter 1956
188 MeV and 236 MeV electron beam from linear accelerator at Stanford

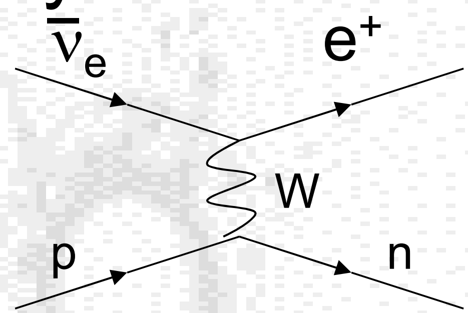


Determined proton RMS charge radius to be (0.7±0.2) x10^{-13} cm

Final State Mass Effects



- In IBD, $\bar{\nu}_e + p \rightarrow e^+ + n$, have to pay a mass penalty *twice*
  - *M_n-M_p≈1.3 MeV, M_e≈0.5 MeV*



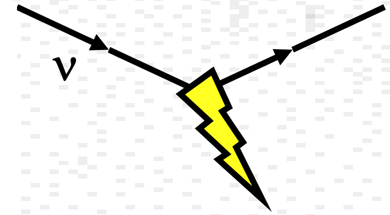
- What is the threshold?
  - kinematics are simple, at least to zeroth order in M_e/M_n → heavy nucleon kinetic energy is zero

$$s_{\text{initial}} = (\underline{p}_\nu + \underline{p}_p)^2 = M_p^2 + 2M_p E_\nu \text{ (proton rest frame)}$$

$$s_{\text{final}} = (\underline{p}_e + \underline{p}_n)^2 \approx M_n^2 + m_e^2 + 2M_n\left(E_\nu - \left(M_n - M_p\right)\right)$$

- Solving… $E_\nu^{\min} \approx \dfrac{\left(M_n + m_e\right)^2 - M_p^2}{2M_p} \approx 1.806 \text{ MeV}$

Final State Mass Effects (cont'd)



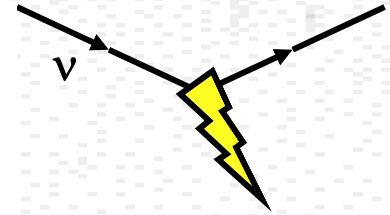
- Define δE as E_ν-E_ν^{min}, then

$$\begin{aligned} s_{\text{initial}} &= M_p^2 + 2M_p\left(\delta E + E_\nu^{\min}\right) \\ &= M_p^2 + 2\delta E \times M_p + \left(M_n + m_e\right)^2 - M_p^2 \\ &= 2\delta E \times M_p + \left(M_n + m_e\right)^2 \end{aligned}$$

- Remember the suppression generally goes as

$$\xi_{\text{mass}} = 1 - \frac{m_{\text{final}}{}^2}{s} = 1 - \frac{\left(M_n + m_e\right)^2}{\left(M_n + m_e\right)^2 + 2M_p \times \delta E}$$

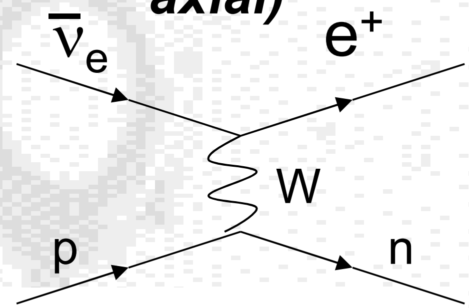
$$= \frac{2M_p \times \delta E}{\left(M_n + m_e\right)^2 + 2M_p \times \delta E} \approx \begin{cases} \delta E \times \dfrac{2M_p}{\left(M_n + m_e\right)^2} & \text{low energy} \\ 1 - \dfrac{\left(M_n + m_e\right)^2}{2M_p{}^2} \dfrac{M_p}{\delta E} & \text{high energy} \end{cases}$$



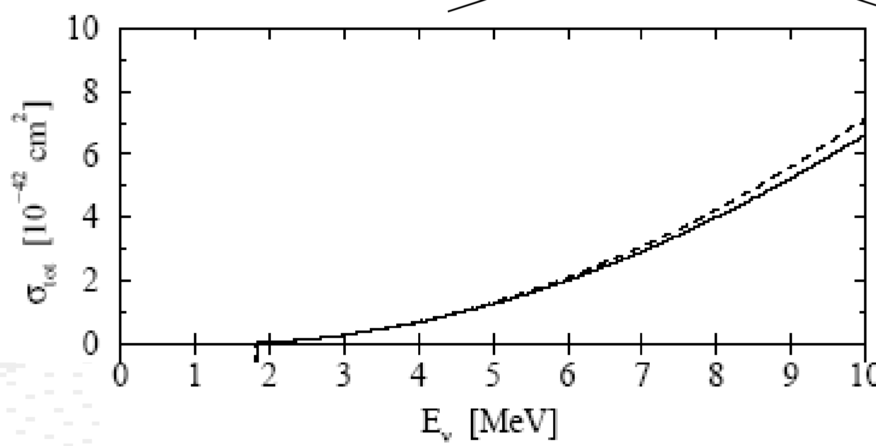
Putting it all together…

$$\sigma_{TOT} = \frac{G_F^2 s}{\pi} \times \cos^2 \vartheta_{\text{Cabibbo}} \times \left(\xi_{\text{mass}}\right) \times \left(g_V{}^2 + 3g_A{}^2\right)$$

quark mixing! ***final state mass suppression*** ***proton form factors (vector, axial)***



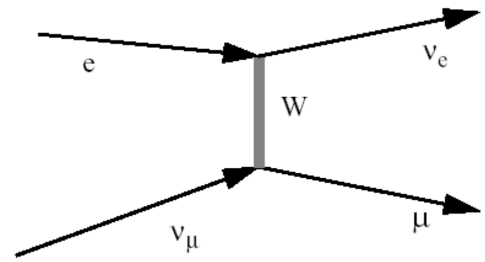
- mass suppression is proportional to δE at low E_ν, so quadratic near threshold
- vector and axial-vector form factors (for IBD usually referred to as f and g, respectively) $g_V, g_A \approx 1, 1.26$.
  - FFs, $\theta_{Cabibbo}$, best known from τ_n



Lecture Question #3: Quantitative Lepton Mass Effect

- **Which is closest to the minimum beam energy in which the reaction**

$$\nu_\mu + e^- \rightarrow \mu^- + \nu_e$$

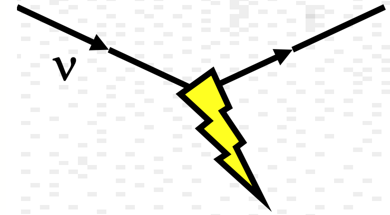


can be observed?

(a) 100 MeV **(b) 1 GeV** **(c) 10 GeV**

(It might help you to remember that $Q^2_{\min} = m^2_\mu$ or you might just want to think about the total CM energy required to produce the particles in the final state.)

Lecture Question #3: Quantitative Lepton Mass Effect



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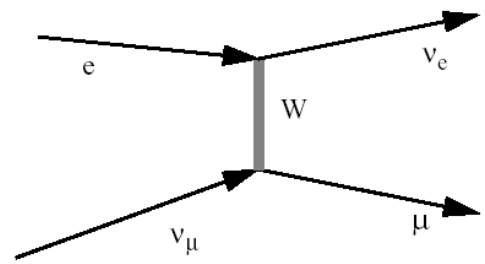


can be observed?

(a) 100 MeV **(b) 1 GeV** **(c) 10 GeV**

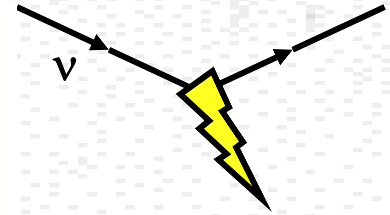
$$Q^2_{\min} = m_\mu^2$$

$$Q^2 < s = (\underline{p}_e + \underline{p}_\nu)^2$$

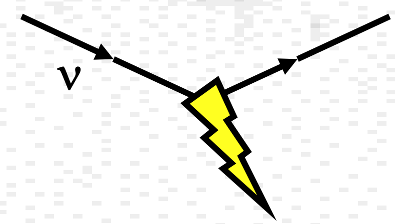
$$= (m_e + E_\nu, 0, 0, \sqrt{E_\nu^{\ 2} - m_\nu^{\ 2}})^2 \approx m_e^2 + 2m_e E_\nu$$

$$\therefore E_\nu > \frac{m_\mu^2}{2m_e} \approx 10.9 \text{ GeV}$$

Summary… and Next Topic

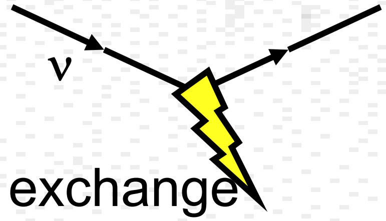


- We know νe^- scattering and IBD cross-sections!
- In point-like weak interactions, key features are:
  - $d\sigma/dQ^2$ is ≈ constant.
    - o Integrating gives $\sigma \propto E_\nu$
  - LH coupling enters w/ $d\sigma/dy \propto 1$, RH w/ $d\sigma/dy \propto (1-y)^2$
    - o Integrating these gives 1 and 1/3, respectively
  - Lepton mass effect gives minimum Q^2
    - o Integrating gives correction factor in σ of $(1-Q^2_{min}/s)$
  - Structure of target can add form factors
- Deep Inelastic Scattering is also a point-like limit where interaction is ν-quark scattering

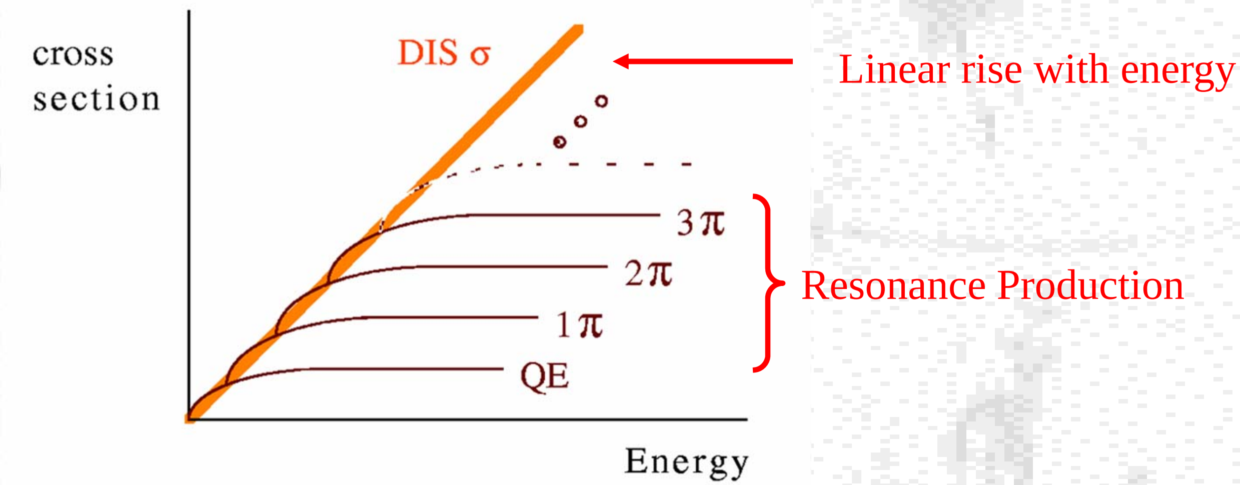


Neutrino-Nucleon Deep Inelastic Scattering

Neutrino-Nucleon Scattering



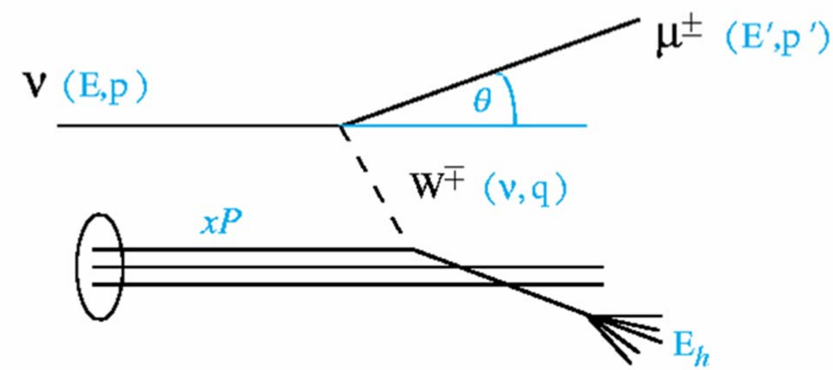
- Charged - Current: $W^{\pm}$ exchange
  - Quasi-elastic Scattering:
    (Target changes but no break up)
    $\nu_\mu + n \rightarrow \mu^- + p$
  - Nuclear Resonance Production:
    (Target goes to excited state)
    $\nu_\mu + n \rightarrow \mu^- + p + \pi^0$ (N^* or Δ)
    $n + \pi^+$
  - Deep-Inelastic Scattering:
    (Nucleon broken up)
    $\nu_\mu + \text{quark} \rightarrow \mu^- + \text{quark}'$
- Neutral - Current: Z^0 exchange
  - Elastic Scattering:
    (Target unchanged)
    $\nu_\mu + N \rightarrow \nu_\mu + N$
  - Nuclear Resonance Production:
    (Target goes to excited state)
    $\nu_\mu + N \rightarrow \nu_\mu + N + \pi$ (N^* or Δ)
  - Deep-Inelastic Scattering
    (Nucleon broken up)
    $\nu_\mu + \text{quark} \rightarrow \nu_\mu + \text{quark}$



Scattering Variables

Scattering variables given in terms of invariants

•More general than just deep inelastic (neutrino-quark) scattering, although interpretation may change.



Measured quantities: E_h , E', θ

4-momentum Transfer²: $Q^2 = -q^2 = -\left(p' - p\right)^2 \approx \left(4EE' \sin^2(\theta/2)\right)_{Lab}$

Energy Transfer: $\nu = \left(q \cdot P\right)/M_T = \left(E - E'\right)_{Lab} = \left(E_h - M_T\right)_{Lab}$

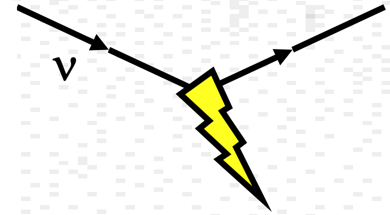
Inelasticity: $y = \left(q \cdot P\right)/\left(p \cdot P\right) = \left(E_h - M_T\right)/\left(E_h + E'\right)_{Lab}$

Fractional Momentum of Struck Quark: $x = -q^2/2\left(p \cdot q\right) = Q^2/2M_T\nu$

Recoil Mass² : $W^2 = (q+P)^2 = M_T^{\,2} + 2M_T\nu - Q^2$

CM Energy² : $s = (p+P)^2 = M_T^{\,2} + \dfrac{Q^2}{xy}$

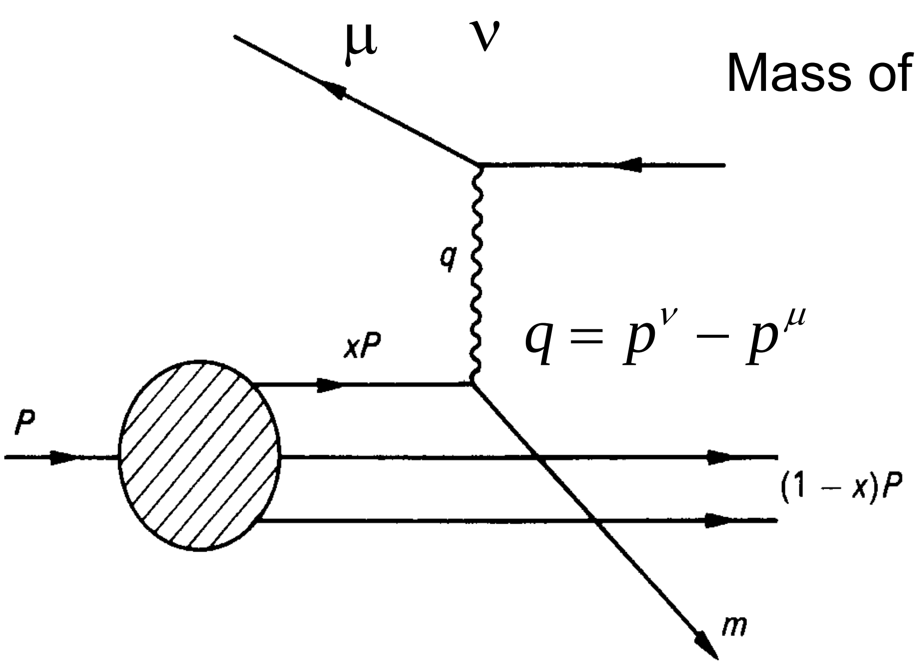
Parton Interpretation of High Energy Limit



Mass of target quark $m_q^{\,2} = x^2 P^2 = x^2 M_T^2$

Mass of final state quark

$$m_{q'}^{\,2} = (xP + q)^2$$



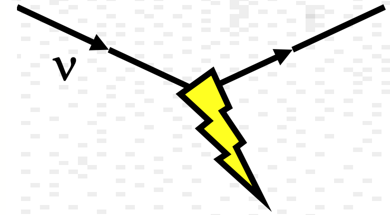
$$q = p^{\nu} - p^{\mu}$$

Neutrino scatters off a parton inside the nucleon

In "infinite momentum frame", xP is momentum of partons inside the nucleon

$$x = \frac{Q^2}{2P \cdot q} = \frac{Q^2}{2M_T \nu}$$

So why is cross-section so large?



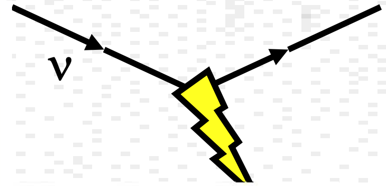
- (at least compared to νe^- scattering!)
- Recall that for neutrino beam and target at rest

$$\sigma_{TOT} \approx \frac{G_F^2}{\pi} \int_0^{Q^2_{\max} \equiv s} dQ^2 = \frac{G_F^2 s}{\pi}$$

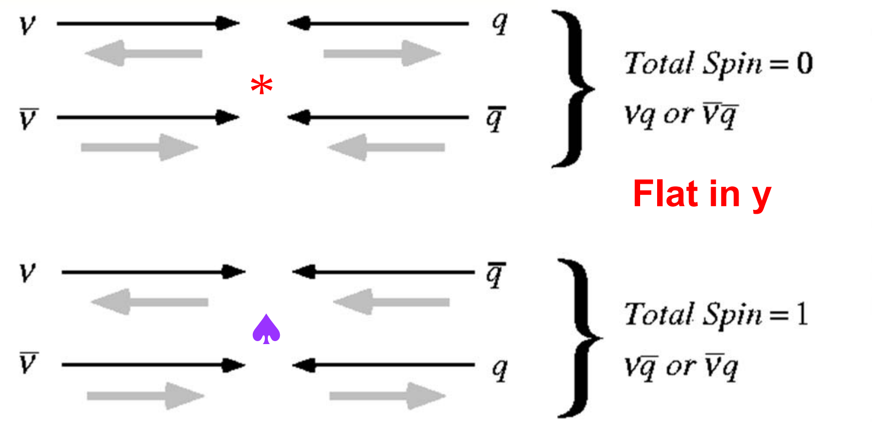
$$s = m_e^{\ 2} + 2 m_e E_\nu$$

- But we just learned for DIS that effective mass of each target quark is $m_q = x m_{\text{nucleon}}$
- So much larger target mass means larger σ_{TOT}

Chirality, Charge in CC ν-q Scattering



- Total spin determines inelasticity distribution
  - Familiar from neutrino-electron scattering



$1/4(1+\cos\theta^*)^2 = (1-y)^2$

$\int(1-y)^2dy=1/3$

point-like scattering implies linear with energy

$$\frac{d\sigma^{\nu p}}{dxdy} = \frac{G_F^2 s}{\pi}\left(x d(x) + x\bar{u}(x)(1-y)^2 \right)$$

$$\frac{d\sigma^{\bar{\nu} p}}{dxdy} = \frac{G_F^2 s}{\pi}\left(x\bar{d}(x) + x u(x)(1-y)^2 \right)$$

but what is this "q(x)"?

- Neutrino/Anti-neutrino CC each produce particular Δq in scattering

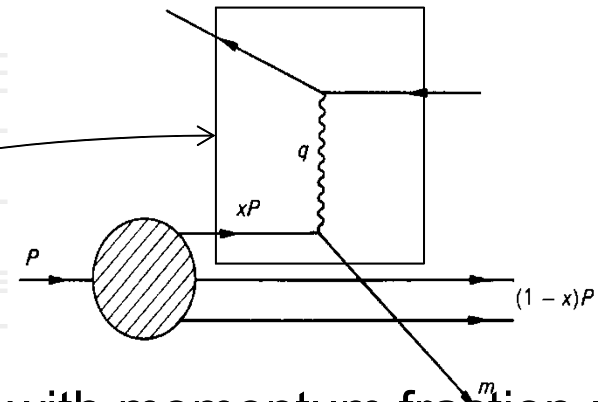
$$\nu d \rightarrow \mu^- u$$

$$\bar{\nu} u \rightarrow \mu^+ d$$

Factorization and Partons

- Factorization Theorem of QCD allows cross-sections for hadronic processes to be written as:

$$\sigma(l + h \rightarrow l + X)$$
$$= \sum_q \int dx \sigma(l + q(x) \rightarrow l + X) q_h(x)$$



  - $q_h(x)$ is the probability of finding a parton, *q*, with momentum fraction *x* inside the hadron, *h*. It is called a parton distribution function (PDF).
  - PDFs are universal
  - PDFs are not (yet) calculable from first principles in QCD
- "Scaling": parton distributions are largely independent of Q^2 scale, and depend on fractional momentum, x.