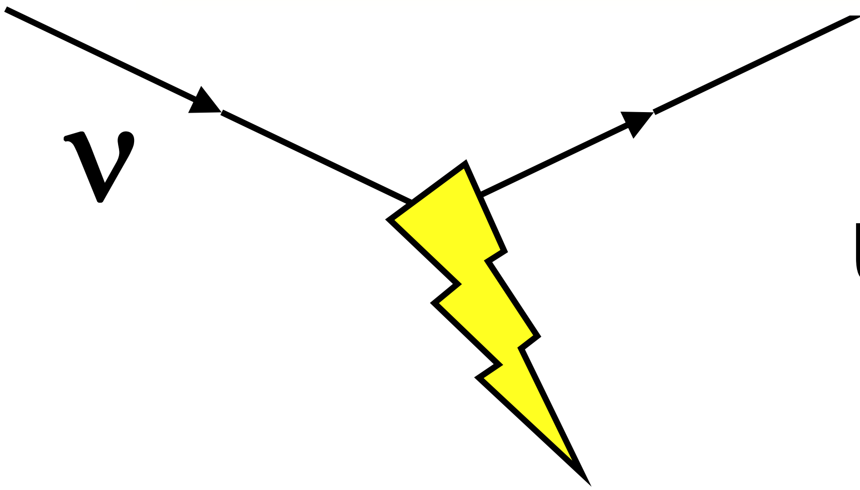
Interactions of Neutrinos

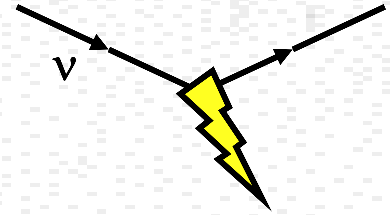


Kevin McFarland
University of Rochester
INSS 2013, Beijing
6-8 August 2013

Outline

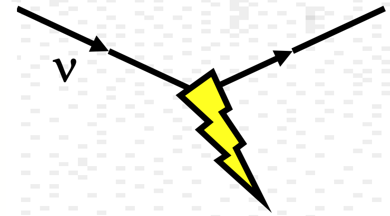
- Brief Motivation for and History of Measuring Interactions
  - Key reactions and thresholds
- Weak interactions and neutrinos
  - Elastic and quasi-elastic processes, e.g., νe scattering
  - Complication of Targets with Structure
  - Deep inelastic scattering (νq) and UHE neutrinos
  - Quasielastic and nearly elastic scattering
- Special problems at accelerator energies
  - Nuclear Effects
  - Generators, theory and experimental data
- Conclusions

Focus of These Lectures

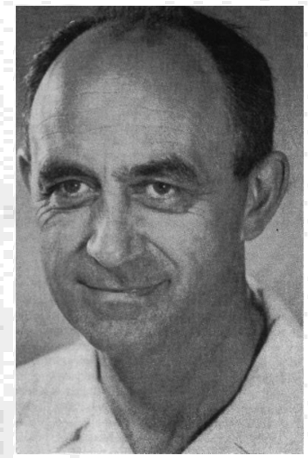


- This is not a comprehensive review of all the interesting physics associated with neutrino interactions
- Choice of topics will focus on:
  - Cross-sections useful for studying neutrino properties
  - Estimating cross-sections
  - Understanding the most important effects qualitatively or semi-quantitatively
  - Understanding how we use our knowledge of cross-sections in experiments

Weak Interactions



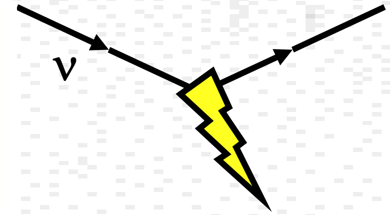
- Current-current interaction $\mathcal{H}_w = \frac{G_F}{\sqrt{2}} \mathcal{J}^\mu \mathcal{J}_\mu$
  Fermi, Z. Physik, 88, 161 (1934)
  - Paper famously rejected by *Nature:*
    "it contains speculations too remote from reality to be of interest to the reader"



- Prediction for neutrino interactions
  - If $n \to p e^- \bar{\nu}$, then $\bar{\nu} p \to e^+ n$
  - Better yet, it is robustly predicted by Fermi theory
    - o Bethe and Peirels, Nature 133, 532 (1934)
  - For neutrinos of a few MeV from a reactor, a typical cross-section was found to be $\sigma_{\bar{\nu} p} \sim 5 \times 10^{-44}\,\mathrm{cm}^2$

    This is wrong by a factor of two (parity violation)

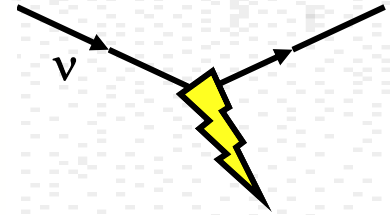
How Weak is This?

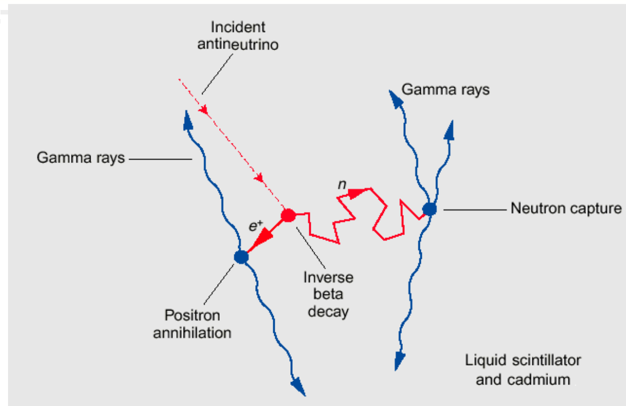


- $\sigma \sim 5 \times 10^{-44} cm^2$ compared with
    - $\sigma_{\gamma p} \sim 10^{-25}$ cm^2 at similar energies, for example
- The cross-section of these few MeV neutrinos is such that the mean free path in steel would be 10 light-years

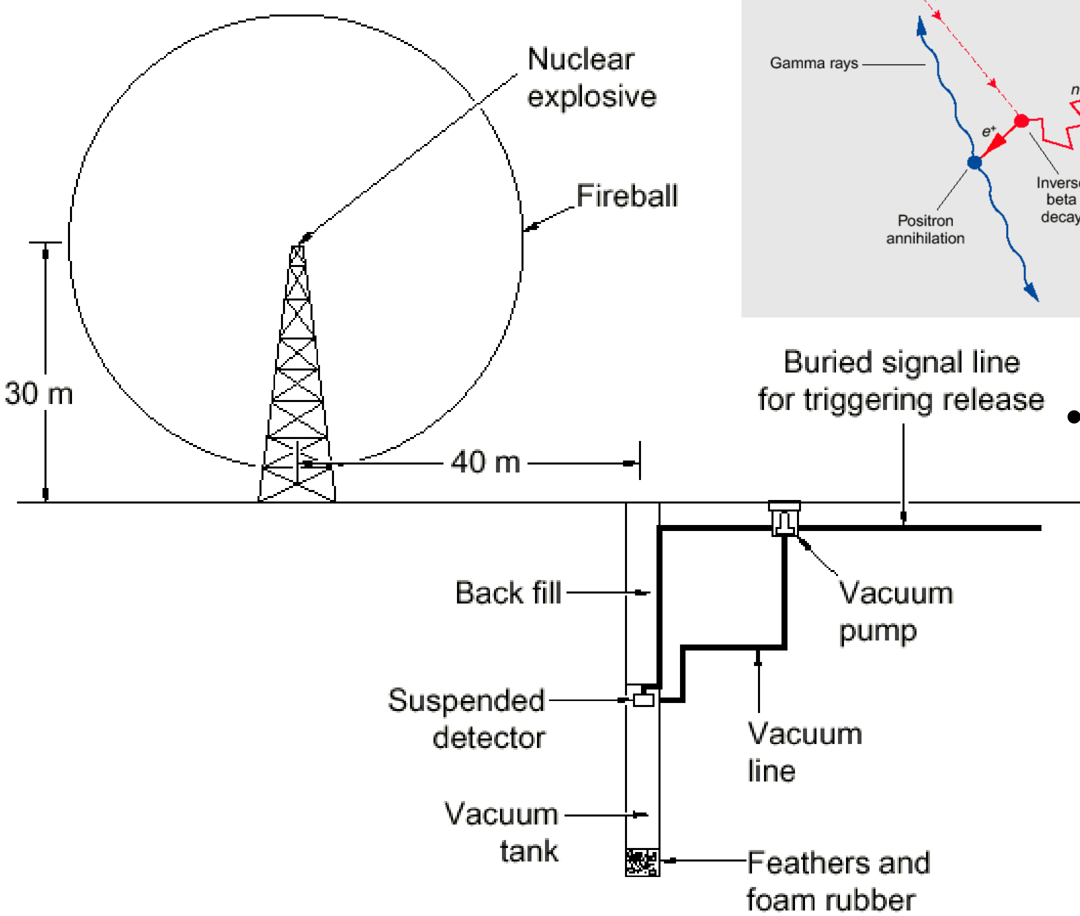
> "I have done something very bad today by proposing a particle that cannot be detected; it is something no theorist should ever do."
>
> Wolfgang Pauli

Extreme Measures to Overcome Weakness *(Reines and Cowan, 1946)*

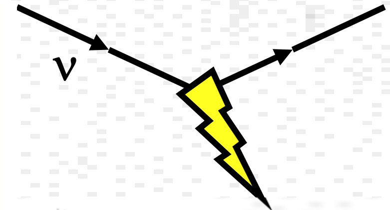




$$\bar{\nu}\, p \rightarrow e^{+} n$$

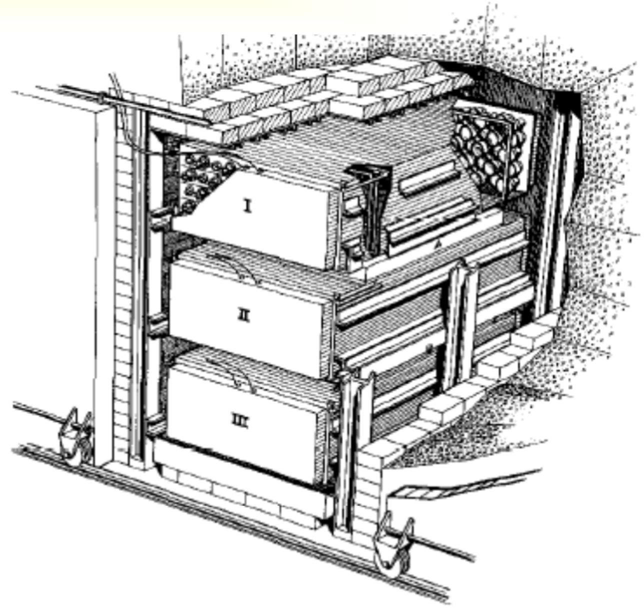


- Why inverse neutron beta decay?
  - clean prediction of Fermi weak theory
  - clean signature of prompt gammas from e^+ plus delayed neutron signal.
    - Latter not as useful with bomb source.

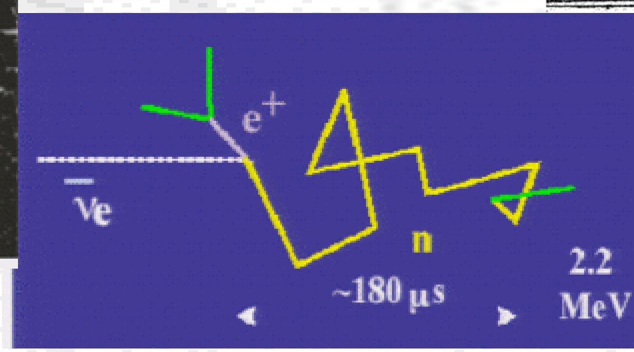


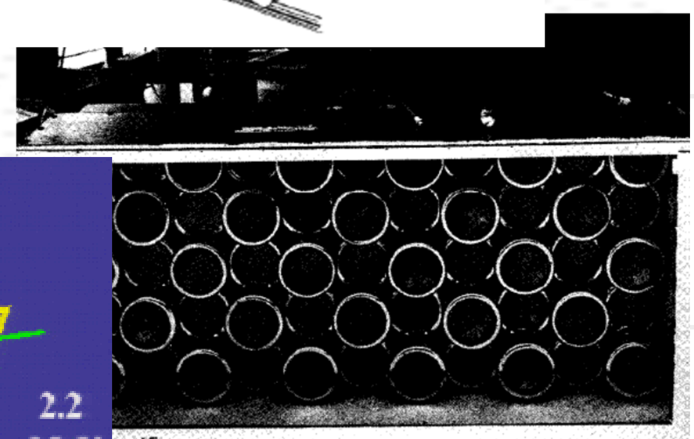
Discovery of the Neutrino

- Reines and Cowan (1955)
  - Chose a constant source, nuclear reactor (Savannah River)
  - 1956 message to Pauli: ”We are happy to inform you [Pauli] that we have definitely detected neutrinos…”
  - 1995 Nobel Prize for Reines

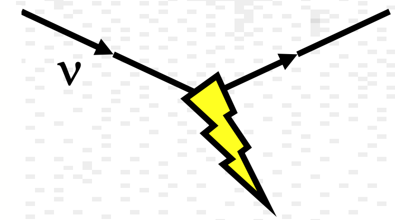


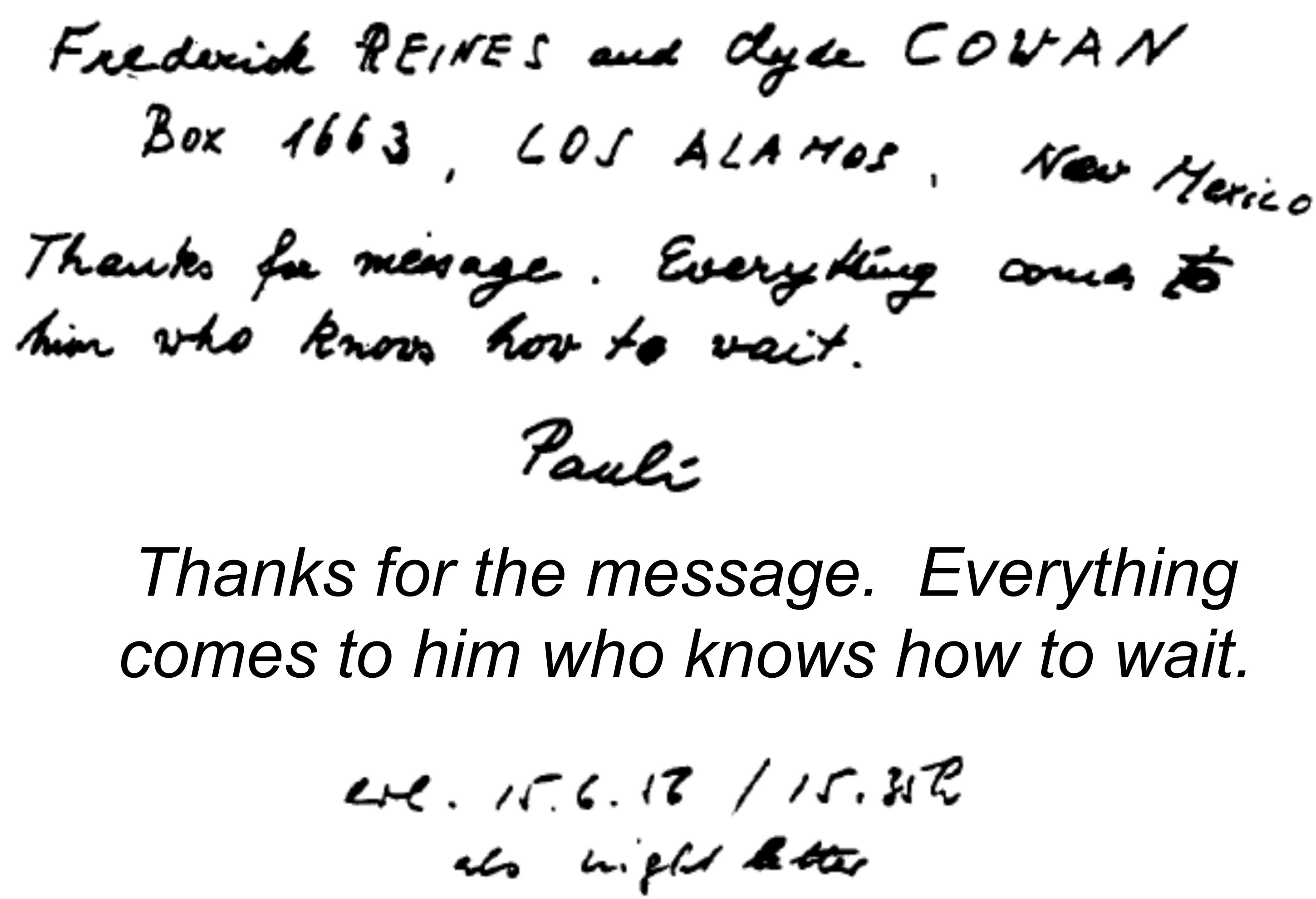
$$\bar{\nu}\, p \rightarrow e^{+} n$$





Better than the Nobel Prize?





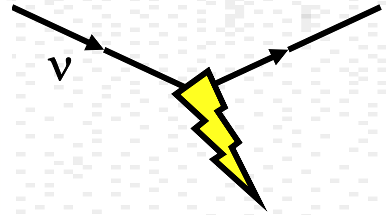
Fredrick REINES and Clyde COWAN
Box 1663, LOS ALAMOS, New Mexico

Thanks for message. Everything comes to him who knows how to wait.

Pauli

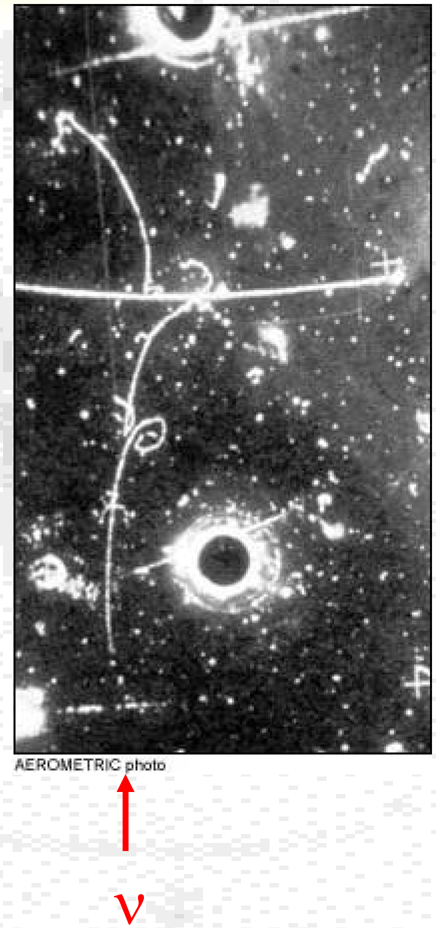
Thanks for the message. Everything comes to him who knows how to wait.

Another Neutrino Interaction Discovery

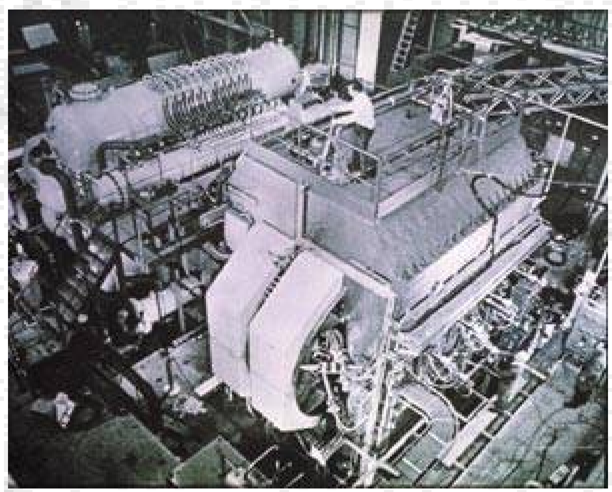


- Neutrinos only feel the weak force
  - a great way to study the weak force!
- Search for neutral current
  - arguably the most famous neutrino interaction ever observed is shown at right

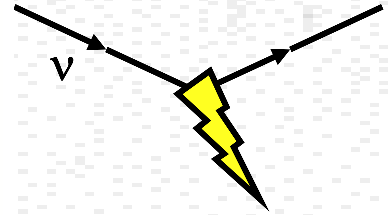
$$\bar{\nu}_\mu e^- \rightarrow \bar{\nu}_\mu e^-$$



Gargamelle, event from neutral weak force

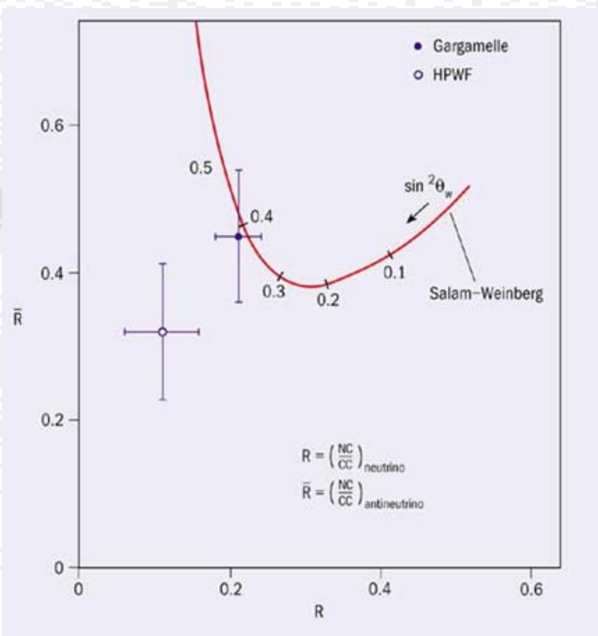


An Illuminating Aside



- The "discovery signal" for the neutral current was really neutrino scattering from nuclei
  - usually quoted as a ratio of muon-less interactions to events containing muons

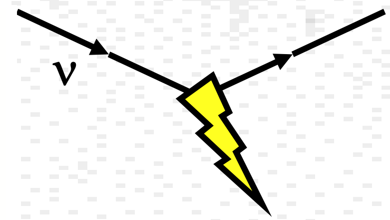
$$R^{\nu} = \frac{\sigma(\nu_{\mu} N \to \nu_{\mu} X)}{\sigma(\nu_{\mu} N \to \mu^{-} X)}$$



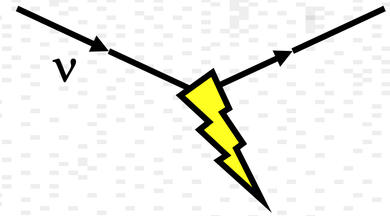
- But this discovery was complicated for 12-18 months by a lack of understanding of neutrino interactions
  - backgrounds from neutrons induced by neutrino interactions outside the detector
  - not understanding fragmentation to high energy hadrons which then "punched through" to fake muons

Great article: P. Gallison, Rev Mod Phys 55, 477 (1983)

The Future: Interactions and Oscillation Experiments



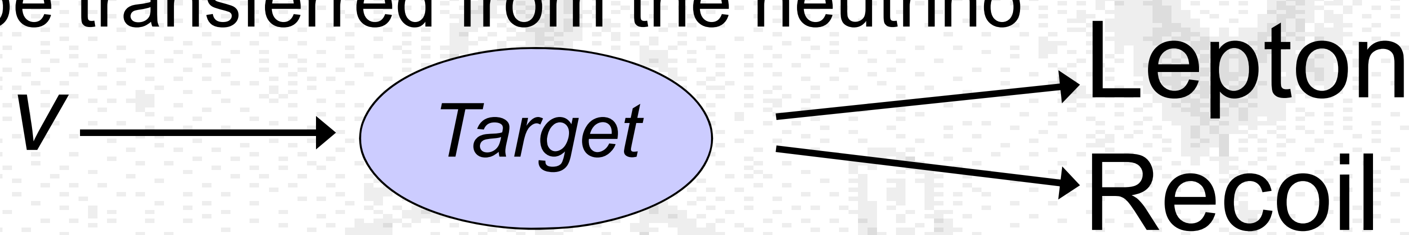
- Oscillation experiments point us to a rich physics potential at L/E~400 km/GeV *(and L/E~N·(400 km/GeV) as well)*
  - mass hierarchy, CP violation
- But there are difficulties
  - transition probabilities as a function of energy must be precisely measured for mass hierarchy and CP violation
  - the neutrinos must be at difficult energies of 1-few GeV for electron appearance experiments, few-many GeV for atmospheric neutrino and τ appearance experiments.
  - or use neutrinos from a reactor ☺
- *Our generation doesn't have neutrino flavor measurements in which distinguishing 1 from 0 or 1/3 buys a ticket to Stockholm*
  - Difficulties are akin to neutral current experiments
  - Is there a message for us here?



Kinematics of Neutrino Reactions

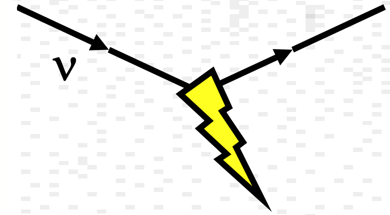
Thresholds and Processes

- We detect neutrino interactions only in the final state, and often with poor knowledge of the incoming neutrinos
- Creation of that final state may require energy to be transferred from the neutrino



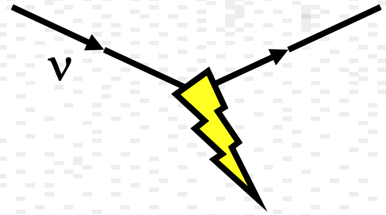
  - In charged-current reactions, where the final state lepton is charged, this lepton has mass
  - The recoil may be a higher mass object than the initial state, or it may be in an excited state

Thresholds and Processes



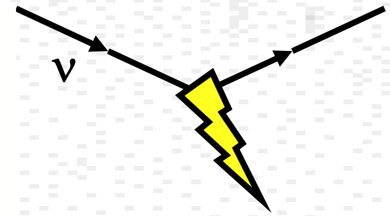
Process	Considerations	Threshold (typical)
$\nu N \to \nu N$ (elastic)	Target nucleus is often free (recoil is very small)	none
$\nu_e n \to e^- p$	In some nuclei (mostly metastable ones), this reaction is exothermic if proton not ejected	None for free neutron some others.
$\nu e \to \nu e$ (elastic)	Most targets have atomic electrons	~ 10eV – 100 keV
anti-$\nu_e p \to e^- n$	$m_n > m_p$ & m_e. Typically more to make recoil from stable nucleus.	1.8 MeV (free p). More for nuclei.
$\nu_\ell n \to \ell^- p$ (quasielastic)	Final state nucleon is ejected from nucleus. Massive lepton	~ 10s MeV for ν_e +~100 MeV for ν_μ
$\nu_\ell N \to \ell^- X$ (inelastic)	Must create additional hadrons. Massive lepton.	~ 200 MeV for ν_e +~100 MeV for ν_μ

- Energy of neutrinos determines available reactions, and therefore experimental technique



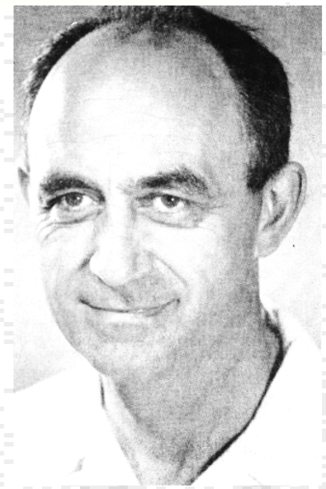
Calculating Neutrino Interactions from Electroweak Theory

Weak Interactions Revisited



- Current-current interaction (Fermi 1934)

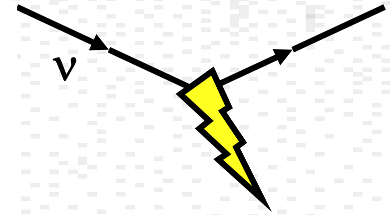
$$\mathcal{H}_w = \frac{G_F}{\sqrt{2}} \mathcal{J}^\mu \mathcal{J}_\mu$$



- Modern version:

$$\mathcal{H}_{weak} = \frac{G_F}{\sqrt{2}} \left[\bar{l} \gamma_\mu (1-\gamma_5) \nu \right] \left[\bar{f} \gamma^\mu (V - A\gamma_5) f \right] + h.c.$$

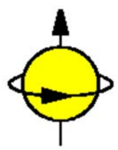
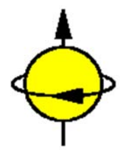
- $P_L = 1/2(1-\gamma_5)$ is a projection operator onto left-handed states for fermions and right-handed states for anti-fermions



Helicity and Chirality

- ***Helicity*** is projection of spin along the particles direction
  - Frame dependent (if massive)

The operator: $\sigma \cdot \mathbf{p}$

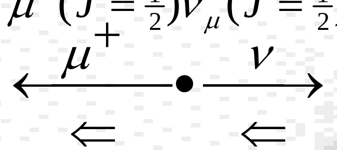
right-helicity  left-helicity 

- Neutrinos only interact weakly with a (V-A) interaction
  - All **neutrinos** are **left-handed**
  - All **antineutrinos** are **right-handed**
    - o **because of production!**
  - Weak interaction **maximally** violates parity

- However, ***chirality*** ("handedness") is Lorentz-invariant
  - Only same as helicity for massless particles.

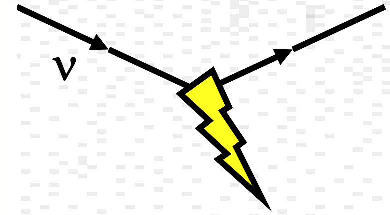
- If neutrinos have mass then left-handed neutrino is:
  - Mainly left-helicity
  - But also small right-helicity component $\propto$ *m/E*
- Only left-handed charged-leptons (e^-,μ^-,τ^-) interact weakly but mass brings in right-helicity:

$$\pi^+(J=0) \rightarrow \mu^+(J=\tfrac{1}{2})\nu_\mu(J=\tfrac{1}{2})$$



$$R_{theory} = \frac{\Gamma(\pi^\pm \to e^\pm \nu_e)}{\Gamma(\pi^\pm \to \mu^\pm \nu_\mu)} = \left(\frac{m_e}{m_\mu}\right)^2 \left(\frac{m_\pi{}^2 - m_e{}^2}{m_\pi{}^2 - m_\mu{}^2}\right)^2 = 1.23 \times 10^{-4}$$

Two Weak Interactions



- W exchange gives Charged-Current (CC) events and Z exchange gives Neutral-Current (NC) events

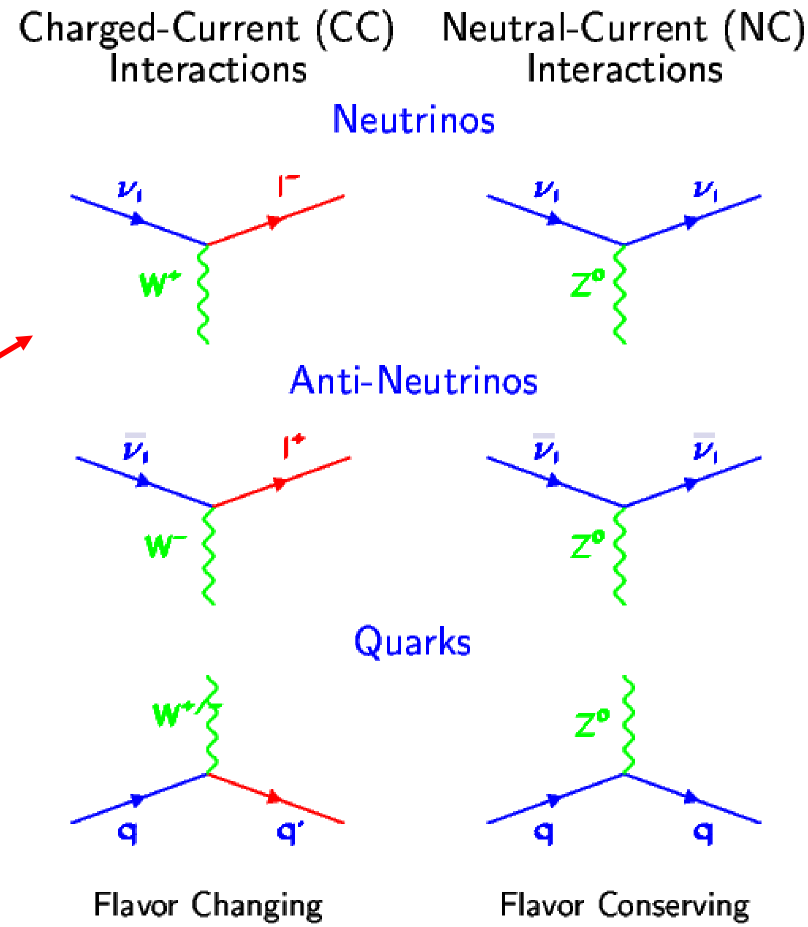
In charged-current events,

Flavor of outgoing lepton tags flavor of neutrino

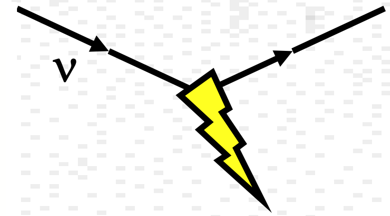
Charge of outgoing lepton determines if neutrino or antineutrino

$$l^- \Rightarrow \nu_l$$

$$l^+ \Rightarrow \bar{\nu}_l$$



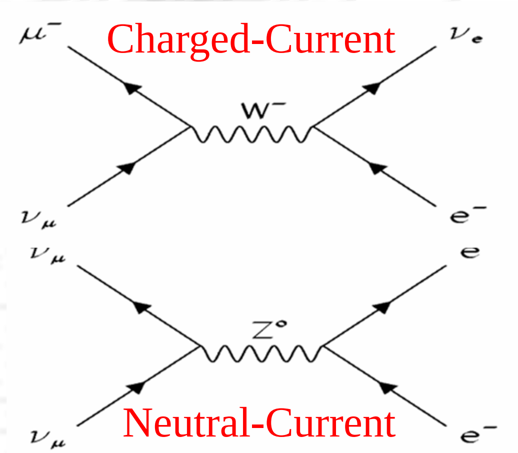
Electroweak Theory



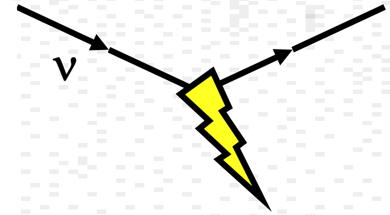
- Standard Model
  - SU(2) ⊗ U(1) gauge theory unifying weak/EM
    ⇒ weak NC follows from EM, Weak CC
  - Physical couplings related to mixing parameter for the interactions in the high energy theory

$$\mathcal{L}_{EW}^{\text{int}} = -Q_e A_\mu \bar{e}\gamma^\mu e + \frac{g}{\sqrt{2}} W_\mu^+ \bar{\nu}_L \gamma^\mu e_L + \frac{g}{\sqrt{2}} W_\mu^- \bar{e}_L \gamma^\mu \nu_L$$

$$+ \frac{g}{\cos\theta_W} Z_\mu^0 \left\{ \begin{array}{l} \frac{1}{2} \bar{\nu}_L \gamma^\mu \nu_L \\ + \left(\sin^2\theta_W - \frac{1}{2} \right) \bar{e}_L \gamma^\mu e_L \\ + \sin^2\theta_W \bar{e}_R \gamma^\mu e_R \end{array} \right\}$$



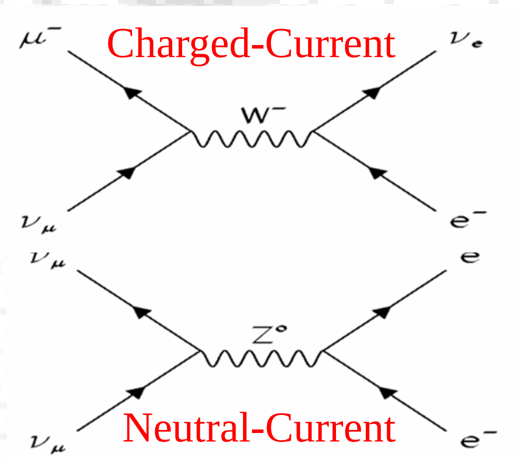
Electroweak Theory



- Standard Model
  - SU(2) ⊗ U(1) gauge theory unifying weak/EM
    - ⇒ weak NC follows from EM, Weak CC
  - Measured physical parameters related to mixing parameter for the couplings.

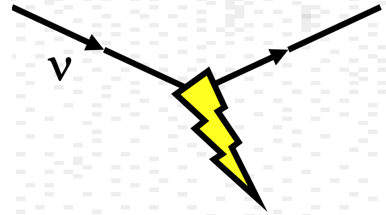
Z Couplings	g_L	g_R
ν_e , ν_μ , ν_τ	1/2	0
e , μ , τ	$-1/2 + \sin^2\theta_W$	$\sin^2\theta_W$
u , c , t	$1/2 - 2/3\ \sin^2\theta_W$	$-\ 2/3\ \sin^2\theta_W$
d , s , b	$-1/2 + 1/3\ \sin^2\theta_W$	$1/3\ \sin^2\theta_W$

$$e = g\sin\theta_W,\ G_F = \frac{g^2\sqrt{2}}{8M_W^2},\ \frac{M_W}{M_Z} = \cos\theta_W$$



- Neutrinos are special in SM
  - Right-handed neutrino has ***NO*** interactions!

Why "Weak"?

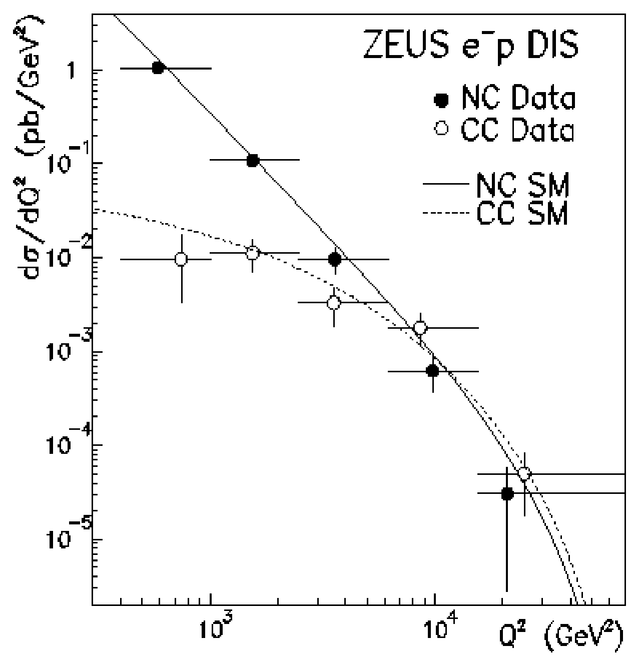


- Weak interactions are weak because of the massive W and Z bosons exchange

$$\frac{d\sigma}{dq^2} \propto \frac{1}{(q^2 - M^2)^2}$$

q is 4-momentum carried by exchange particle
M is mass of exchange particle

At HERA see W and Z propagator effects
- Also weak ~ EM strength



- Explains dimensions of Fermi "constant"

$$G_F = \frac{\sqrt{2}}{8}\left(\frac{g_W}{M_W}\right)^2$$
$$= 1.166\times 10^{-5}/GeV^2 \quad (g_W \approx 0.7)$$

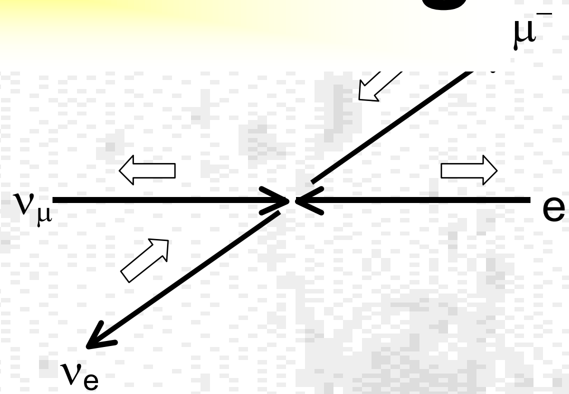
Neutrino-Electron Scattering

- Inverse μ–decay:

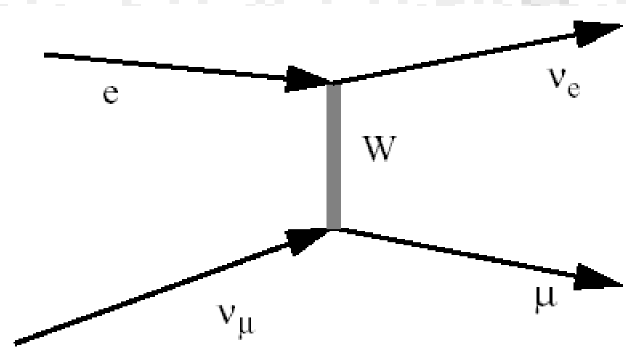
  $$\nu_\mu + e^- \to \mu^- + \nu_e$$

  - Total spin J=0

    (*Assuming massless muon*, helicity=chirality)



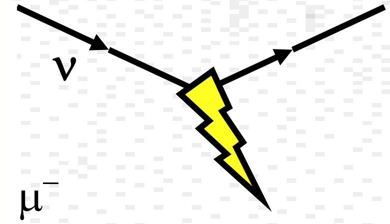
$$Q^2 \equiv -\left(\underline{e} - \underline{\nu}_e\right)^2$$

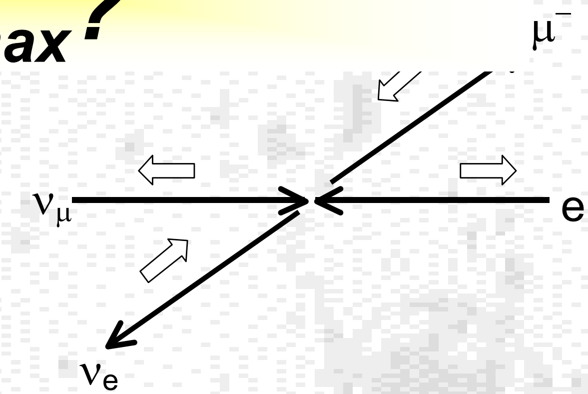


$$\sigma_{TOT} \propto \int_0^{Q^2_{\max}} dQ^2 \frac{1}{(Q^2 + M_W{}^2)^2}$$

$$\approx \frac{Q^2_{\max}}{M_W{}^4}$$

Lecture Question #1
What is Q^2_{max}?



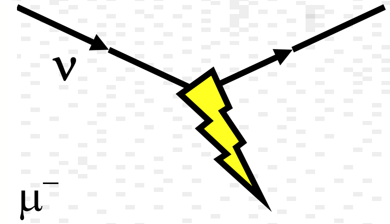


$$\nu_\mu + e^- \rightarrow \mu^- + \nu_e$$

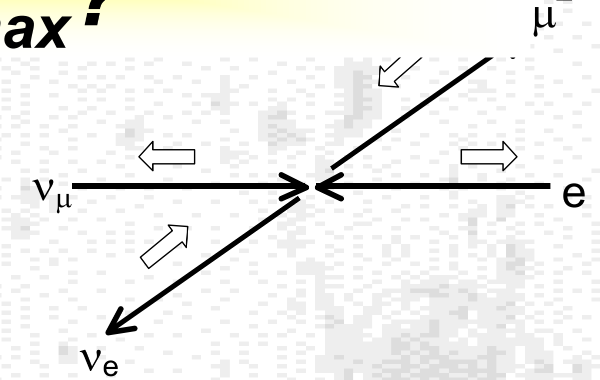
$$Q^2 \equiv -\left(\underline{e} - \underline{\nu}_e\right)^2$$

*Work in the center-of-mass frame and assume, **for now**, that we can neglect the masses.*

Lecture Question #1
What is Q^2_{max}?



$$\nu_\mu + e^- \rightarrow \mu^- + \nu_e$$



$$Q^2 \equiv -\left(\underline{e} - \underline{\nu}_e\right)^2$$

Work in the center-of-mass frame and assume, ***for now****, that we can neglect the masses.*

$$\underline{e} \approx (E_\nu^*, 0, 0, -E_\nu^*)$$

$$\underline{\nu}_\mathrm{e} \approx (E_\nu^*, -E_\nu^* \sin\theta^*, 0, -E_\nu^* \cos\theta^*)$$

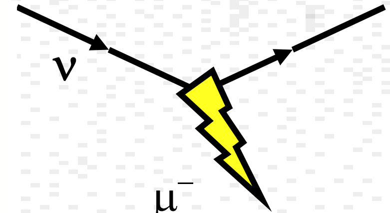
$$Q^2 = -\left(\underline{e}^2 + \underline{\nu}_e{}^2 - 2\underline{e}\bullet\underline{\nu}_e\right)^2$$

$$\approx -\left[-2E_\nu^{*2}\left(1-\cos\theta^*\right)\right]$$

$$0 < Q^2 < \left(2E_\nu^*\right)^2 \approx \left(\underline{e} + \underline{\nu}_\mu\right)^2$$

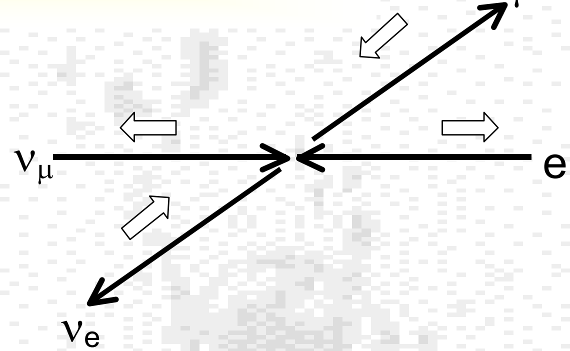
$$0 < Q^2 < s$$

Neutrino-Electron (cont'd)



$$\sigma_{TOT} \propto Q^2_{\max} = s$$

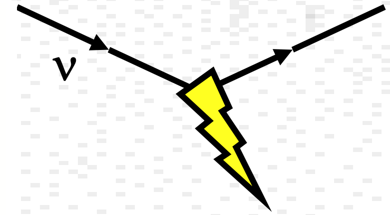
$$\sigma_{TOT} = \frac{G_F^2 s}{\pi}$$
$$= 17.2 \times 10^{-42} cm^2 / GeV \cdot E_\nu(GeV)$$



- Why is it proportional to beam energy?

$$s = (\underline{p}_{\nu_\mu} + \underline{p}_e)^2 = m_e^2 + 2m_e E_\nu \text{ (e}^- \text{ rest frame)}$$

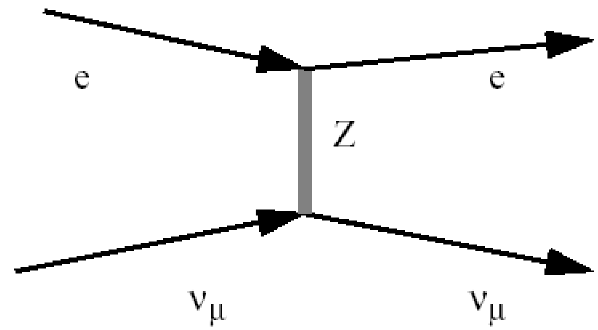
- Proportionality to energy is a generic feature of point-like scattering!
  - because $d\sigma/dQ^2$ is constant (at these energies)



Neutrino-Electron (cont'd)

- Elastic scattering:

$$\nu_\mu + e^- \rightarrow \nu_\mu + e^-$$



  - Recall, EW theory has coupling to left *or* right-handed electron
  - Total spin, J=0,1

Z Couplings	g_L	g_R
ν_e , ν_μ , ν_τ	1/2	0
e , μ , τ	$-1/2 + \sin^2\theta_W$	$\sin^2\theta_W$
u , *c* , *t*	$1/2 - 2/3 \sin^2\theta_W$	$- 2/3 \sin^2\theta_W$
d , *s* , *b*	$-1/2 + 1/3 \sin^2\theta_W$	$1/3 \sin^2\theta_W$

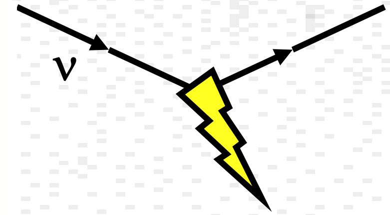
- Electron-Z^0 coupling
  - Left-handed: $-1/2 + \sin^2\theta_W$

$$\sigma \propto \frac{G_F^2 s}{\pi}\left(\frac{1}{4} - \sin^2\theta_W + \sin^4\theta_W\right)$$

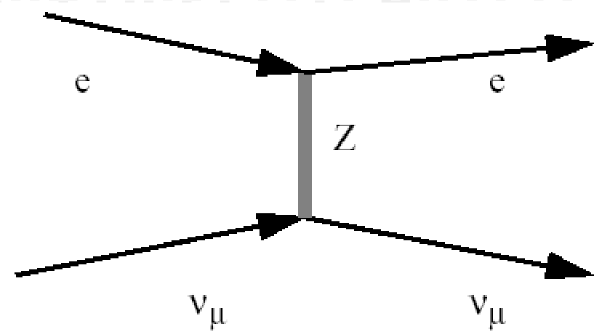
  - Right-handed: $\sin^2\theta_W$

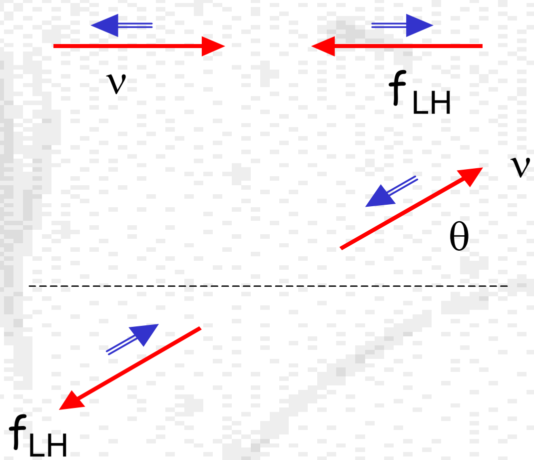
$$\sigma \propto \frac{G_F^2 s}{\pi}\left(\sin^4\theta_W\right)$$

Neutrino-Electron (cont'd)

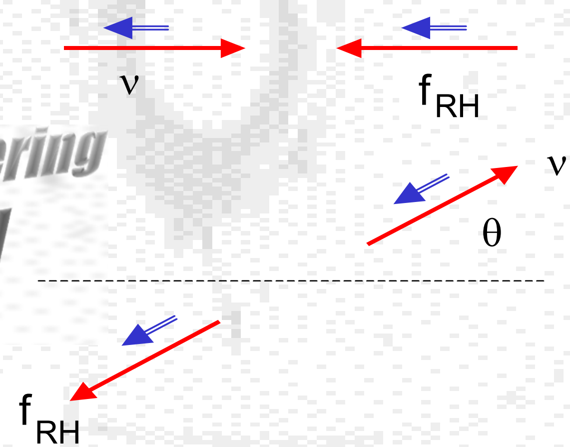


- What are relative contributions of scattering from left *and* right-handed electrons?





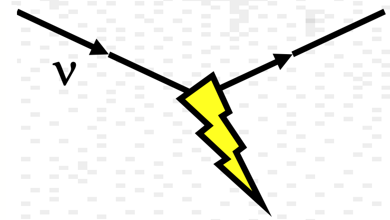
Backwards scattering is disfavored



$$\frac{d\sigma}{d\cos\theta} = \text{const}$$

$$\frac{d\sigma}{d\cos\theta} = \text{const} \times \left(\frac{1+\cos\theta}{2}\right)^2$$

Neutrino-Electron (cont'd)



- Electron-Z^0 coupling $\sigma \propto \frac{G_F^2 s}{\pi}\left(\frac{1}{4} - \sin^2\theta_W + \sin^4\theta_W\right)$
  - (LH, V-A): $-1/2 + \sin^2\theta_W$
  - (RH, V+A): $\sin^2\theta_W$ $\qquad \sigma \propto \frac{G_F^2 s}{\pi}\left(\sin^4\theta_W\right)$

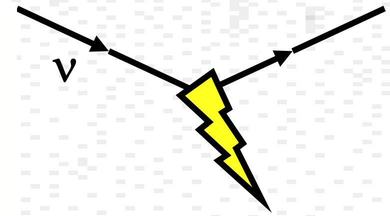
Let y denote inelasticity.
Recoil energy is related to
CM scattering angle by

$$y = \frac{E_e}{E_\nu} \approx 1 - \tfrac{1}{2}(1-\cos\theta)$$

$$\int dy \frac{d\sigma}{dy} = \begin{cases} \text{LH:} & \int dy = 1 \\ \text{RH:} & \int (1-y)^2\, dy = 1/3 \end{cases}$$

$$\sigma_{TOT} = \frac{G_F^2 s}{\pi}\left(\frac{1}{4} - \sin^2\theta_W + \frac{4}{3}\sin^4\theta_W\right) = 1.4\times 10^{-42}\, cm^2/GeV \cdot E_\nu(GeV)$$

Lecture Question #2: Flavors and νe Scattering



The reaction

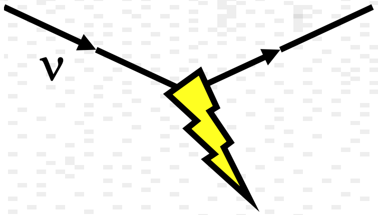
$$\nu_\mu + e^- \rightarrow \nu_\mu + e^-$$

has a much smaller cross-section than

$$\nu_e + e^- \rightarrow \nu_e + e^-$$

Why?

Lecture Question #2: Flavors and νe Scattering

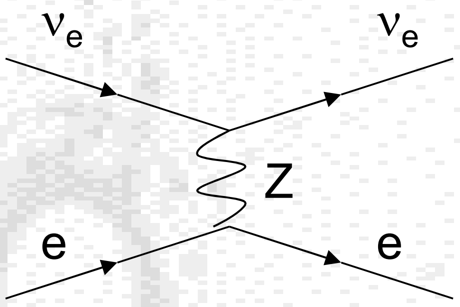


The reaction

$$\nu_\mu + e^- \rightarrow \nu_\mu + e^-$$

has a much smaller cross-section than

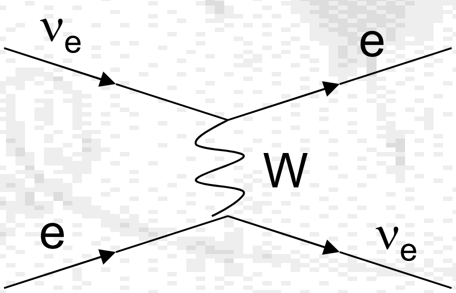
$$\nu_e + e^- \rightarrow \nu_e + e^-$$



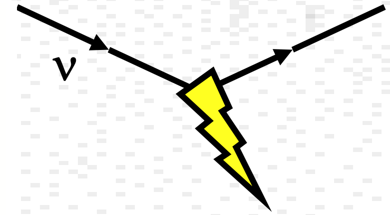
Why?

$$\nu_e + e^- \rightarrow \nu_e + e^-$$

has a second contributing reaction, charged current



Lecture Question #2: Flavors and νe Scattering



Let's show that this increases the rate

(Recall from the previous pages…

$$\sigma_{TOT} = \int dy \frac{d\sigma}{dy}$$

$$= \int dy \left[\frac{d\sigma^{LH}}{dy} + \frac{d\sigma^{RH}}{dy} \right]$$

$$= \sigma_{TOT}^{LH} + \tfrac{1}{3}\sigma_{TOT}^{RH}$$

$$\sigma_{TOT}^{LH} \propto \left| \text{total coupling}_{e^-}^{LH} \right|^2$$

For electron…	LH coupling	RH coupling
Weak NC	$-1/2+\sin^2\theta_W$	$\sin^2\theta_W$
Weak CC	-1/2	0

)

We have to show the interference between CC and NC is constructive.

The total RH coupling is unchanged by addition of CC because there is no RH weak CC coupling

There are two LH couplings: NC coupling is $-1/2+\sin^2\theta_W \approx -1/4$ and the CC coupling is -1/2. We add the associated amplitudes… and get $-1+\sin^2\theta_W \approx -3/4$

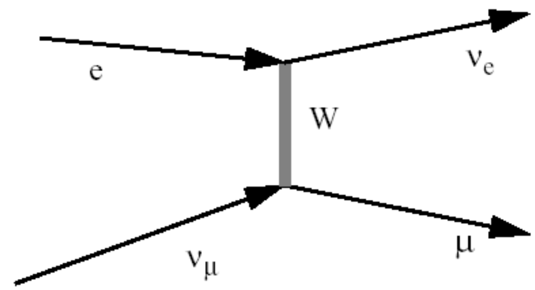
Lepton Mass Effects

- Let's return to Inverse μ–decay:

  $$\nu_\mu + e^- \rightarrow \mu^- + \nu_e$$

  - What changes in the presence of final state mass?
    - o pure CC so always left-handed
    - o BUT there must be finite Q^2 to create muon in final state!

$$Q^2_{\min} = m_\mu^2$$



$$\sigma_{TOT} \propto \int_{Q^2_{\min}}^{Q^2_{\max}} dQ^2 \frac{1}{(Q^2 + M_W{}^2)^2}$$

$$\approx \frac{Q^2_{\max} - Q^2_{\min}}{M_W{}^4}$$

$$\sigma_{TOT} = \frac{G_F^2 (s - m_\mu^2)}{\pi}$$

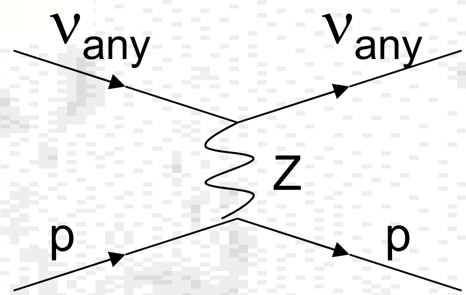
$$= \left[\sigma_{TOT}{}^{\text{(massless)}}\right]\left(1 - \frac{m_\mu^2}{s}\right)$$

  - see a suppression scaling with **(mass/CM energy)^2**
    - o This can be generalized…

What about other targets?

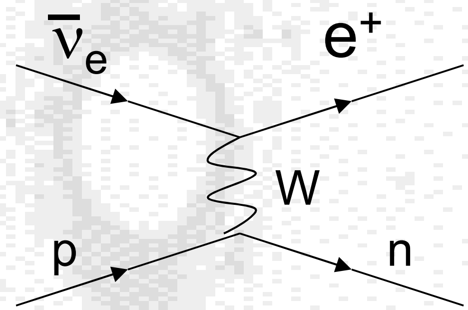
- Imagine now a proton target
  - Neutrino-proton elastic scattering:

$$\nu_e + p \rightarrow \nu_e + p$$



  - "Inverse beta-decay" (IBD):

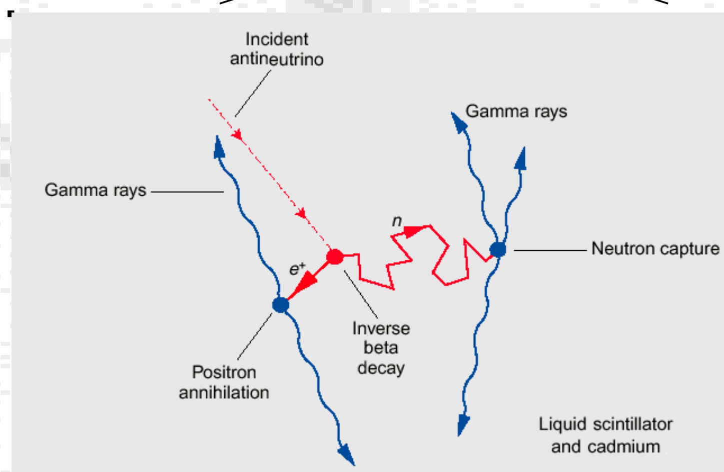
$$\bar{\nu}_e + p \rightarrow e^+ + n$$



  - and "stimulated" beta decay:

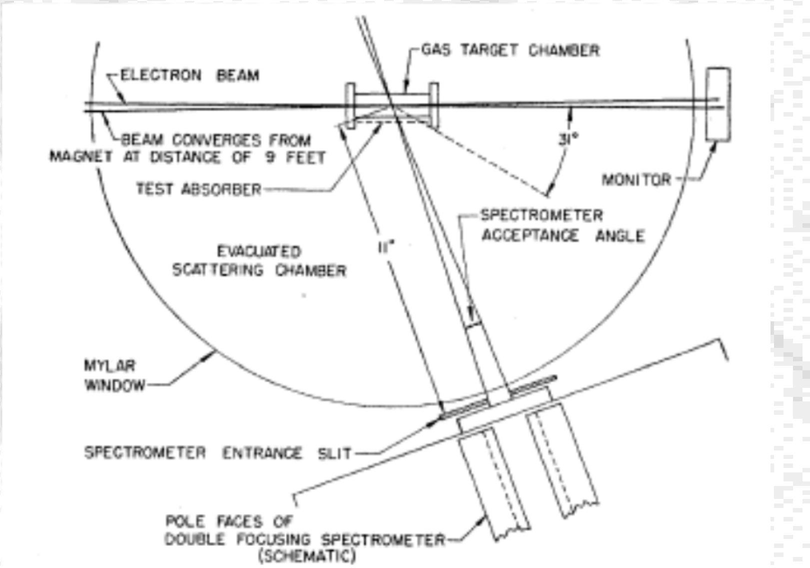
$$\nu_e + n \rightarrow e^- + p$$

  - Recall that IBD was the Reines and Cowan discovery signal

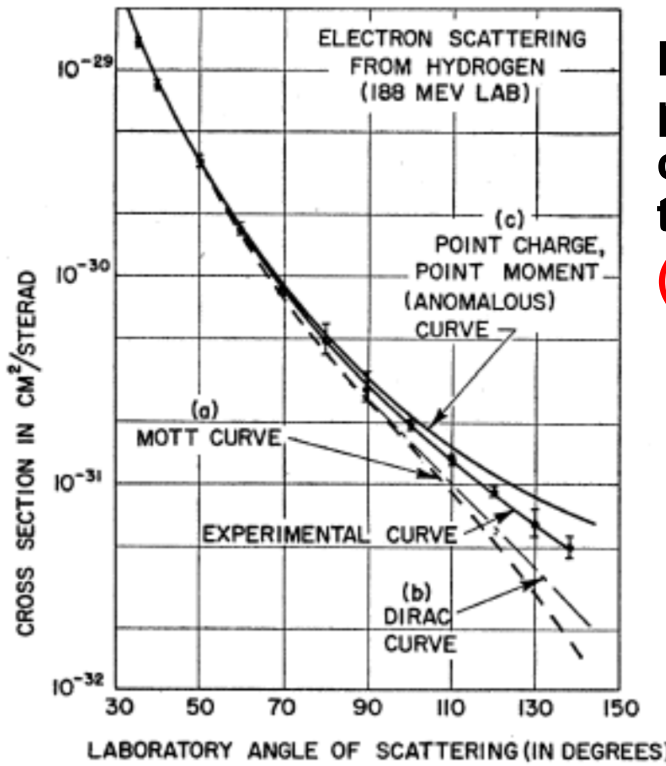


Proton Structure

- How is a proton different from an electron?
  - anomalous magnetic moment, $\kappa \equiv \frac{g-2}{2} \neq 1$
  - "form factors" related to finite size

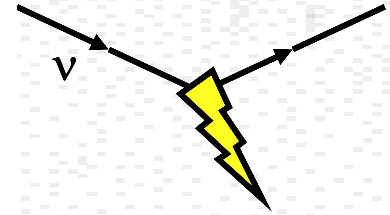


McAllister and Hofstadter 1956
188 MeV and 236 MeV electron beam from linear accelerator at Stanford

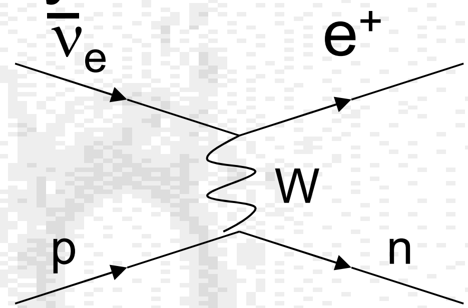


Determined proton RMS charge radius to be (0.7±0.2) x10^{-13} cm

Final State Mass Effects



- In IBD, $\bar{\nu}_e + p \rightarrow e^+ + n$, have to pay a mass penalty *twice*
  - *M_n-M_p≈1.3 MeV, M_e≈0.5 MeV*



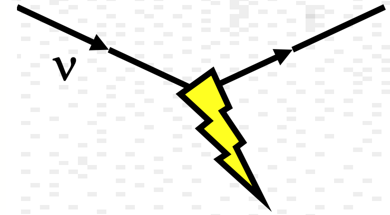
- What is the threshold?
  - kinematics are simple, at least to zeroth order in M_e/M_n → heavy nucleon kinetic energy is zero

$$s_{\text{initial}} = (\underline{p}_\nu + \underline{p}_p)^2 = M_p^2 + 2M_p E_\nu \text{ (proton rest frame)}$$

$$s_{\text{final}} = (\underline{p}_e + \underline{p}_n)^2 \approx M_n^2 + m_e^2 + 2M_n \left(E_\nu - \left(M_n - M_p\right)\right)$$

- Solving… $E_\nu^{\min} \approx \dfrac{\left(M_n + m_e\right)^2 - M_p^2}{2M_p} \approx 1.806 \text{ MeV}$

Final State Mass Effects (cont'd)



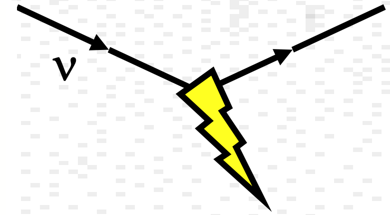
- Define δE as $E_\nu - E_\nu^{min}$, then

$$
\begin{aligned}
s_{\text{initial}} &= M_p^2 + 2M_p\left(\delta E + E_\nu^{\min}\right) \\
&= M_p^2 + 2\delta E \times M_p + \left(M_n + m_e\right)^2 - M_p^2 \\
&= 2\delta E \times M_p + \left(M_n + m_e\right)^2
\end{aligned}
$$

- Remember the suppression generally goes as

$$
\xi_{\text{mass}} = 1 - \frac{m_{\text{final}}{}^2}{s} = 1 - \frac{\left(M_n + m_e\right)^2}{\left(M_n + m_e\right)^2 + 2M_p \times \delta E}
$$

$$
= \frac{2M_p \times \delta E}{\left(M_n + m_e\right)^2 + 2M_p \times \delta E} \approx \begin{cases} \delta E \times \dfrac{2M_p}{\left(M_n + m_e\right)^2} & \text{low energy} \\ 1 - \dfrac{\left(M_n + m_e\right)^2}{2M_p{}^2}\dfrac{M_p}{\delta E} & \text{high energy} \end{cases}
$$



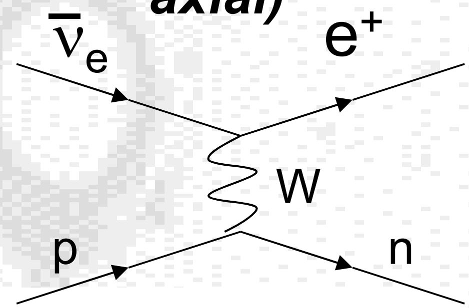
Putting it all together…

$$\sigma_{TOT} = \frac{G_F^2 s}{\pi} \times \cos^2 \vartheta_{\text{Cabibbo}} \times \left(\xi_{\text{mass}}\right) \times \left(g_V^{\ 2} + 3 g_A^{\ 2}\right)$$

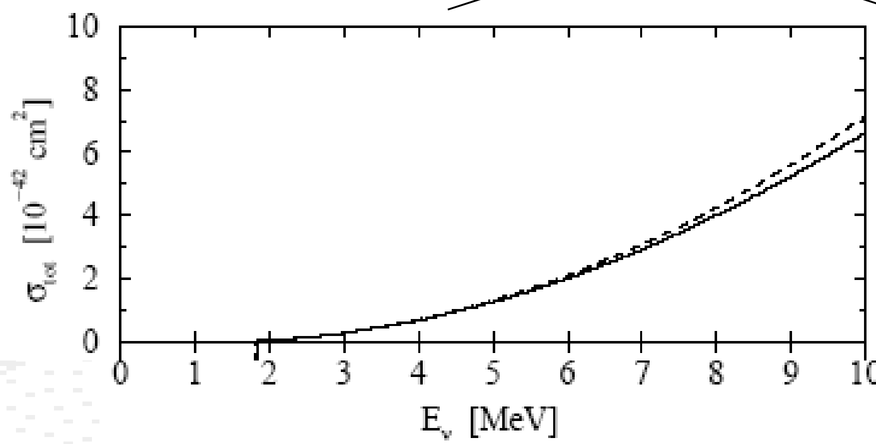
quark mixing! (→ $\cos^2 \vartheta_{\text{Cabibbo}}$)

final state mass suppression (→ ξ_{mass})

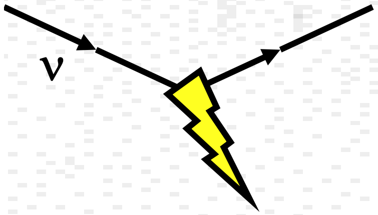
proton form factors (vector, axial) (→ $g_V^{\ 2} + 3 g_A^{\ 2}$)



- mass suppression is proportional to δE at low E_ν, so quadratic near threshold
- vector and axial-vector form factors (for IBD usually referred to as f and g, respectively)
  g_V, g_A ≈ *1, 1.26*.
  - FFs, $\theta_{Cabibbo}$, best known from τ_n

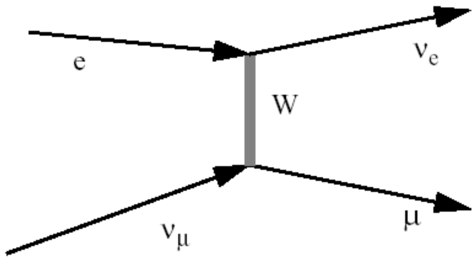


Lecture Question #3: Quantitative Lepton Mass Effect



- Which is closest to the **minimum beam energy** in which the reaction

$$\nu_\mu + e^- \rightarrow \mu^- + \nu_e$$

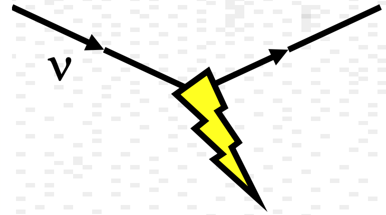


can be observed?

(a) 100 MeV (b) 1 GeV (c) 10 GeV

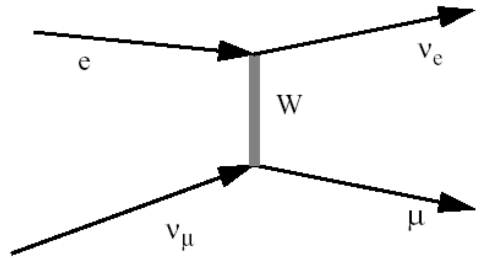
(It might help you to remember that $Q^2_{\min} = m^2_\mu$ or you might just want to think about the total CM energy required to produce the particles in the final state.)

Lecture Question #3: Quantitative Lepton Mass Effect



- Which is closest to the **minimum beam energy** in which the reaction

$$\nu_\mu + e^- \rightarrow \mu^- + \nu_e$$



can be observed?

(a) 100 MeV **(b) 1 GeV** **(c) 10 GeV**

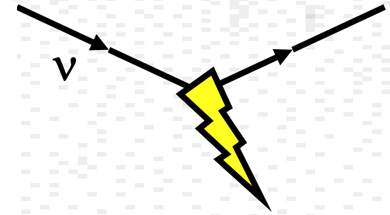
$$Q^2_{\min} = m_\mu^2$$

$$Q^2 < s = (\underline{p}_e + \underline{p}_\nu)^2$$

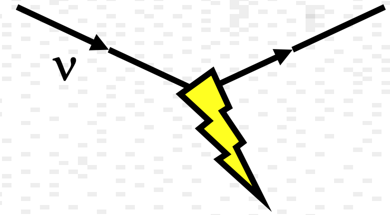
$$= (m_e + E_\nu, 0, 0, \sqrt{E_\nu^{\ 2} - m_\nu^{\ 2}})^2 \approx m_e^2 + 2m_e E_\nu$$

$$\therefore E_\nu > \frac{m_\mu^2}{2m_e} \approx 10.9 \text{ GeV}$$

Summary… and Next Topic

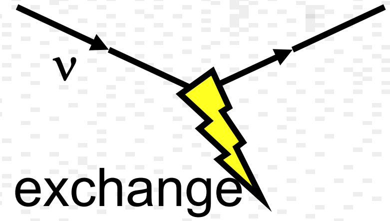


- We know νe^- scattering and IBD cross-sections!
- In point-like weak interactions, key features are:
  - $d\sigma/dQ^2$ is ≈ constant.
    - o Integrating gives $\sigma \propto E_\nu$
  - LH coupling enters w/ $d\sigma/dy \propto 1$, RH w/ $d\sigma/dy \propto (1-y)^2$
    - o Integrating these gives 1 and 1/3, respectively
  - Lepton mass effect gives minimum Q^2
    - o Integrating gives correction factor in σ of $(1-Q^2_{min}/s)$
  - Structure of target can add form factors
- Deep Inelastic Scattering is also a point-like limit where interaction is ν-quark scattering

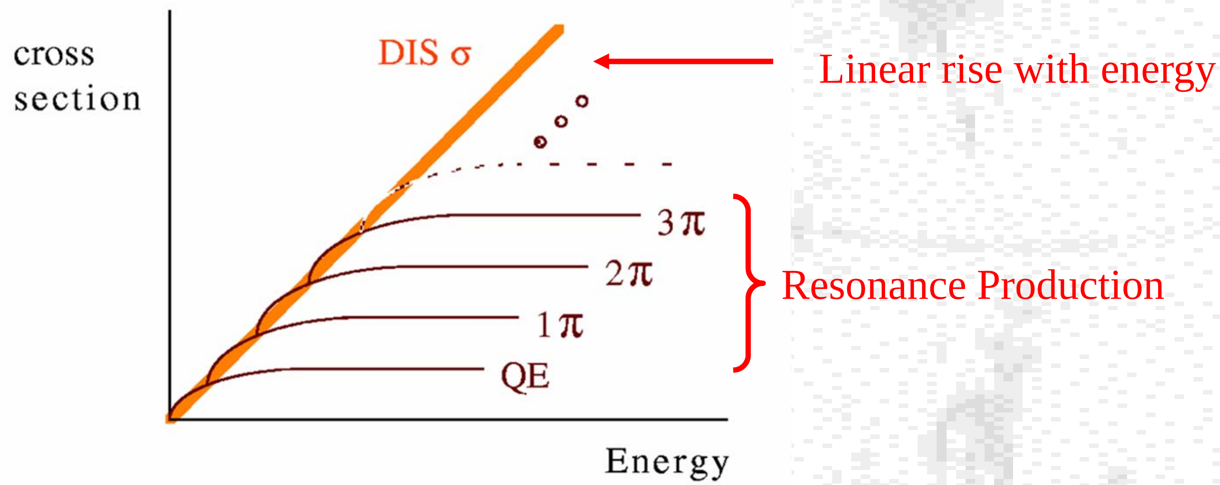


Neutrino-Nucleon Deep Inelastic Scattering

Neutrino-Nucleon Scattering



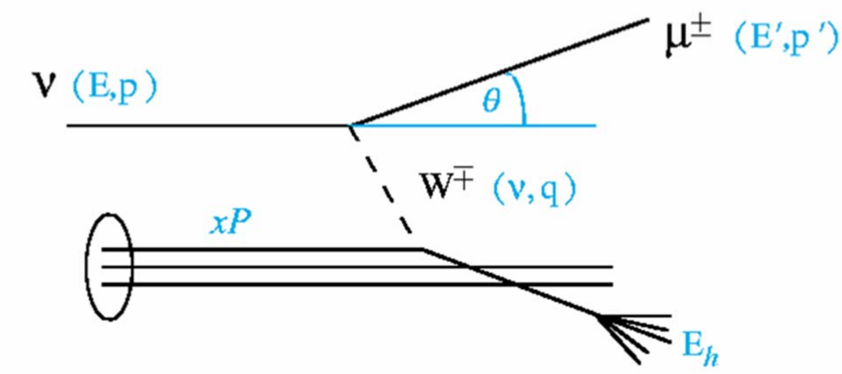
- Charged - Current: $W^{\pm}$ exchange
  - Quasi-elastic Scattering:
    (Target changes but no break up)
    $\nu_\mu + n \rightarrow \mu^- + p$
  - Nuclear Resonance Production:
    (Target goes to excited state)
    $\nu_\mu + n \rightarrow \mu^- + p + \pi^0$ $(N^*$ or $\Delta)$
    $n + \pi^+$
  - Deep-Inelastic Scattering:
    (Nucleon broken up)
    $\nu_\mu + \text{quark} \rightarrow \mu^- + \text{quark'}$
- Neutral - Current: Z^0 exchange
  - Elastic Scattering:
    (Target unchanged)
    $\nu_\mu + N \rightarrow \nu_\mu + N$
  - Nuclear Resonance Production:
    (Target goes to excited state)
    $\nu_\mu + N \rightarrow \nu_\mu + N + \pi$ $(N^*$ or $\Delta)$
  - Deep-Inelastic Scattering
    (Nucleon broken up)
    $\nu_\mu + \text{quark} \rightarrow \nu_\mu + \text{quark}$



Scattering Variables

Scattering variables given in terms of invariants

•More general than just deep inelastic (neutrino-quark) scattering, although interpretation may change.



4-momentum Transfer^2: $Q^2 = -q^2 = -\left(p^{'} - p\right)^2 \approx \left(4EE^{'}\sin^2(\theta/2)\right)_{Lab}$

Energy Transfer: $\nu = \left(q \cdot P\right)/M_T = \left(E - E^{'}\right)_{Lab} = \left(E_h - M_T\right)_{Lab}$

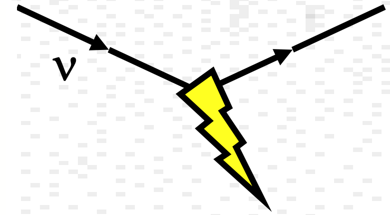
Inelasticity: $y = \left(q \cdot P\right)/\left(p \cdot P\right) = \left(E_h - M_T\right)/\left(E_h + E^{'}\right)_{Lab}$

Fractional Momentum of Struck Quark: $x = -q^2/2\left(p \cdot q\right) = Q^2/2M_T\nu$

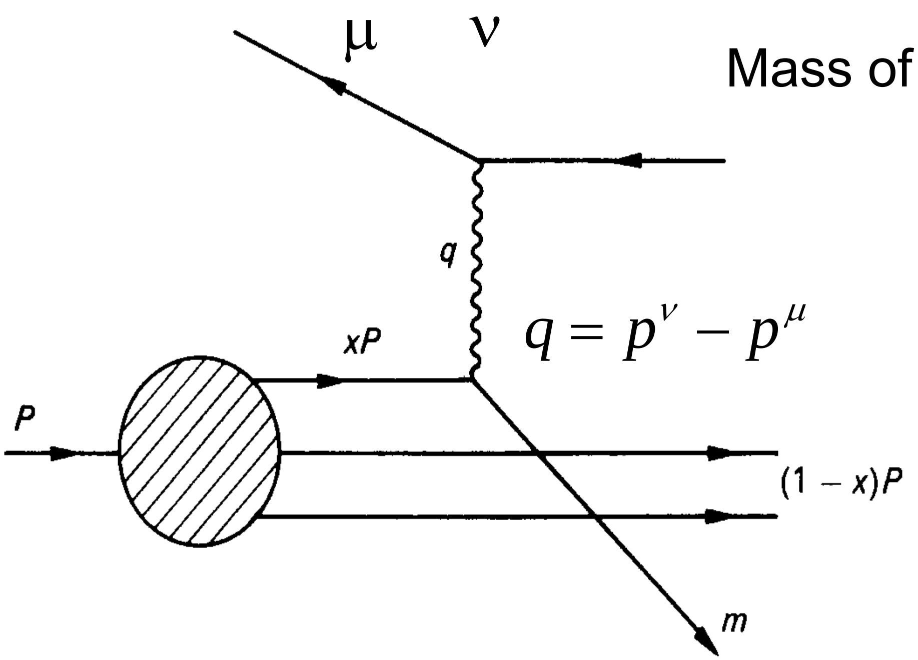
Recoil Mass^2 : $W^2 = (q+P)^2 = M_T{}^2 + 2M_T\nu - Q^2$

CM Energy^2 : $s = (p+P)^2 = M_T{}^2 + \dfrac{Q^2}{xy}$

Parton Interpretation of High Energy Limit



Mass of target quark $m_q^{\,2} = x^2 P^2 = x^2 M_T^2$



$$q = p^{\nu} - p^{\mu}$$

Mass of final state quark

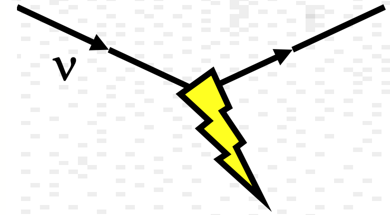
$$m_{q'}^{\,2} = (xP + q)^2$$

In "infinite momentum frame", xP is momentum of partons inside the nucleon

$$x = \frac{Q^2}{2P\cdot q} = \frac{Q^2}{2M_T\nu}$$

Neutrino scatters off a parton inside the nucleon

So why is cross-section so large?



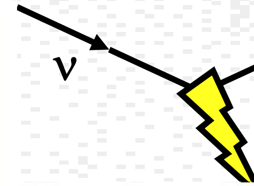
- (at least compared to νe^- scattering!)
- Recall that for neutrino beam and target at rest

$$\sigma_{TOT} \approx \frac{G_F^2}{\pi} \int_0^{Q^2_{\max} \equiv s} dQ^2 = \frac{G_F^2 s}{\pi}$$

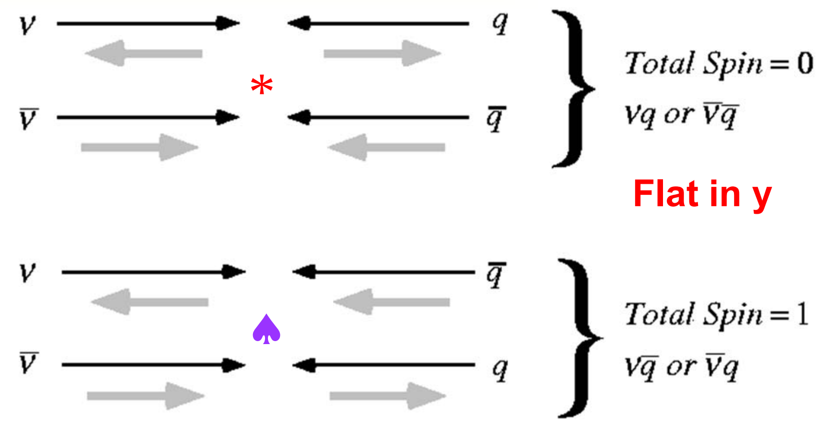
$$s = m_e^{\ 2} + 2 m_e E_\nu$$

- But we just learned for DIS that effective mass of each target quark is $m_q = x m_{\mathrm{nucleon}}$
- So much larger target mass means larger σ_{TOT}

Chirality, Charge in CC ν-q Scattering



- Total spin determines inelasticity distribution
  - Familiar from neutrino-electron scattering



$\frac{1}{4}(1+\cos\theta^*)^2 = (1-y)^2$

$\int(1-y)^2 dy = 1/3$

point-like scattering implies linear with energy

$$\frac{d\sigma^{\nu p}}{dxdy} = \frac{G_F^2 s}{\pi}\left(x\overset{*}{d}(x) + x\overset{\spadesuit}{\bar{u}}(x)(1-y)^2 \right)$$

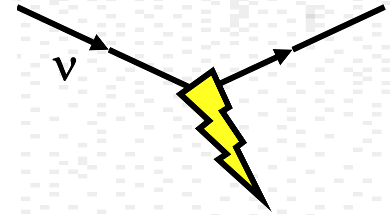
$$\frac{d\sigma^{\bar{\nu} p}}{dxdy} = \frac{G_F^2 s}{\pi}\left(x\overset{*}{\bar{d}}(x) + x\overset{\spadesuit}{u}(x)(1-y)^2 \right)$$

but what is this "q(x)"?

- Neutrino/Anti-neutrino CC each produce particular Δq in scattering

$$\nu d \rightarrow \mu^- u$$
$$\bar{\nu} u \rightarrow \mu^+ d$$

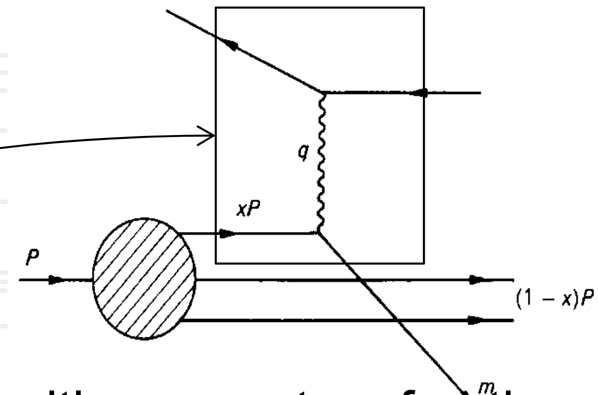
Factorization and Partons



- Factorization Theorem of QCD allows cross-sections for hadronic processes to be written as:

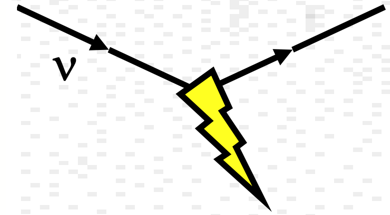
$$\sigma(l+h \to l+X)$$

$$=\sum_q \int dx \sigma(l+q(x) \to l+X) q_h(x)$$



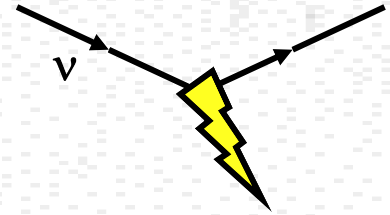
  - $q_h(x)$ is the probability of finding a parton, q, with momentum fraction x inside the hadron, h. It is called a parton distribution function (PDF).
  - PDFs are universal
  - PDFs are not (yet) calculable from first principles in QCD
- "Scaling": parton distributions are largely independent of Q^2 scale, and depend on fractional momentum, x.

Brief Summary of Neutrino-Quark Scattering so Far

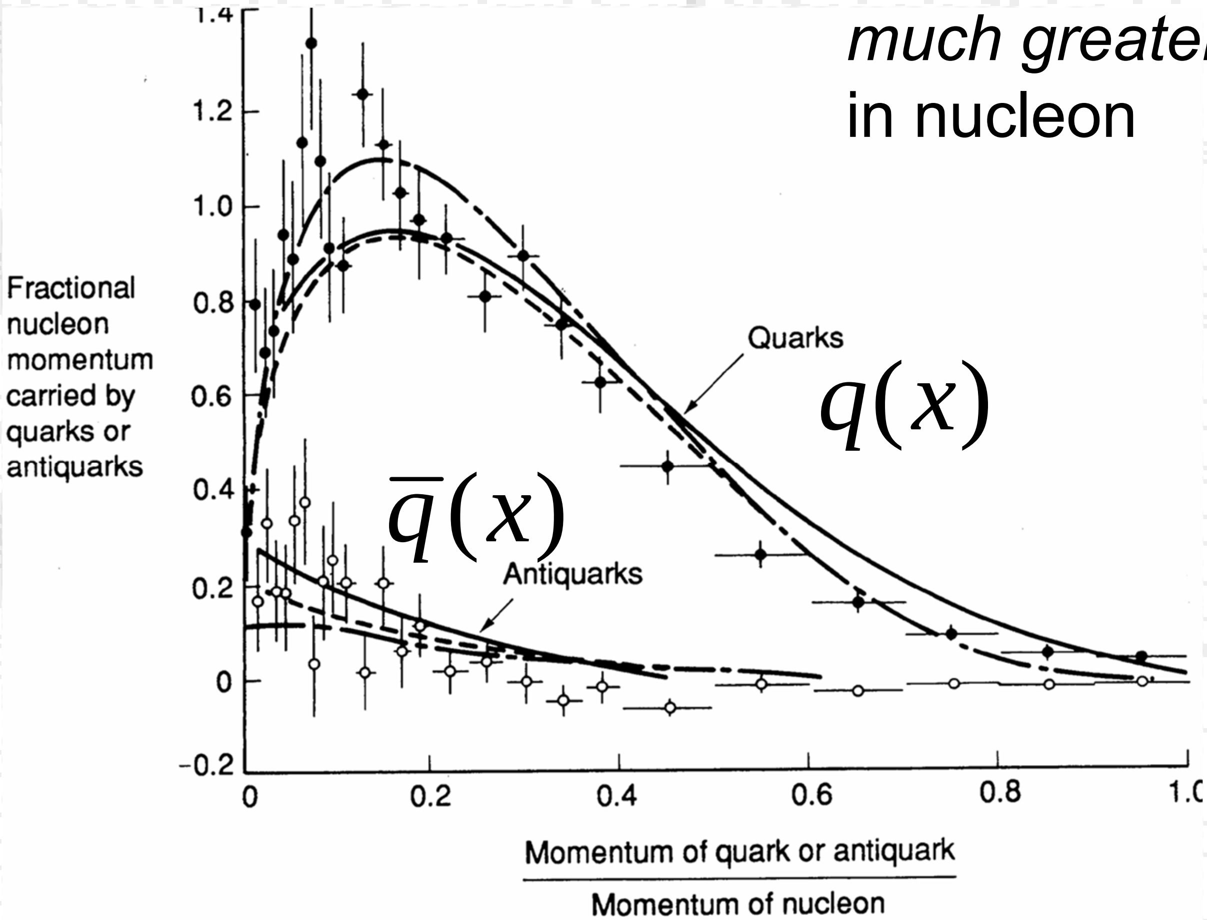


- $x \equiv Q^2/2M_T\nu$ is the fraction of the nucleon 4-momentum carried by a quark in the infinite momentum frame
  - Effective mass for struck quark, $M_q = \sqrt{(x\underline{P})^2} = xM_T$
  - Parton distribution functions, q(x), incorporate information about the “flux” of quarks inside the hadron
- Quark and anti-quark scattering from neutrinos or anti-neutrinos defines total spin
  - νq and $\overline{\nu q}$ are spin 0, isotropic
  - $\nu\bar{q}$ and $\nu\bar{q}$ are spin 1, backscattering is suppressed
- Neutrinos and anti-neutrinos pick out definite quark and anti-quark flavors (charge conservation)

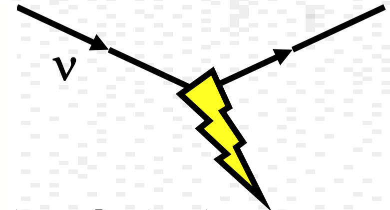
Momentum of Quarks & Antiquarks



- Momentum carried by quarks *much greater* than anti-quarks in nucleon



y distribution in Neutrino CC DIS



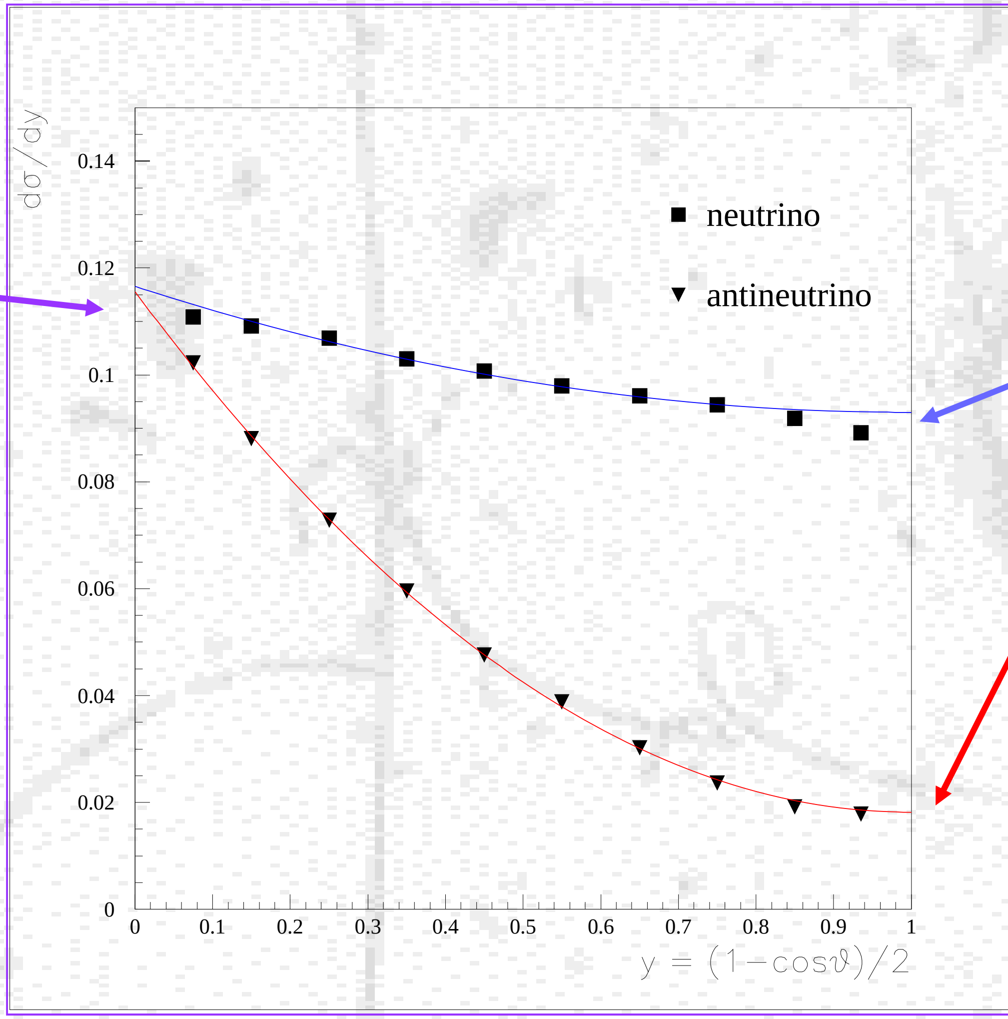
$$\frac{d\sigma(\nu q)}{dxdy} = \frac{d\sigma(\bar{\nu}\bar{q})}{dxdy} \propto 1$$

$$\frac{d\sigma(\nu \bar{q})}{dxdy} = \frac{d\sigma(\bar{\nu}q)}{dxdy} \propto (1-y)^2$$

At y=0:

Quarks & anti-quarks

Neutrino and anti-neutrino identical



At y=1:

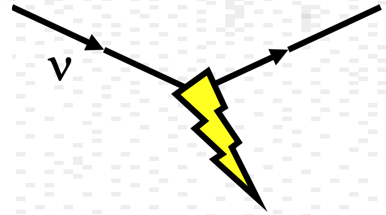
Neutrinos see only quarks.

Anti-neutrinos see only anti-quarks

Averaged over protons and neutrons,

$$\sigma^{\bar{\nu}} \approx \tfrac{1}{2}\sigma^{\nu}$$

Structure Functions (SFs)



- A model-independent picture of these interactions can also be formed in terms of nucleon "structure functions"
  - All Lorentz-invariant terms included
  - Approximate zero lepton mass (small correction)

$$\frac{d\sigma^{\nu,\bar{\nu}}}{dxdy} \propto \left[y^2 2xF_1(x,Q^2) + \left(2 - 2y - \frac{M_T xy}{E} \right) F_2(x,Q^2) \pm y(2-y)xF_3(x,Q^2) \right]$$

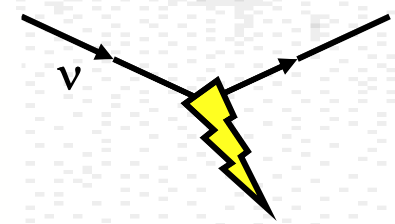
- For massless free spin-1/2 partons, one simplification…
  - Callan-Gross relationship, $2xF_1=F_2$
  - Implies intermediate bosons are completely transverse

Can parameterize longitudinal cross-section by R_L.

Callan-Gross violations result from M_T, NLO pQCD, $g \rightarrow qq$

$$R_L = \frac{\sigma_L}{\sigma_T} = \frac{F_2}{2xF_1}\left(1 + \frac{4M_T^2 x^2}{Q^2}\right)$$

SFs to PDFs



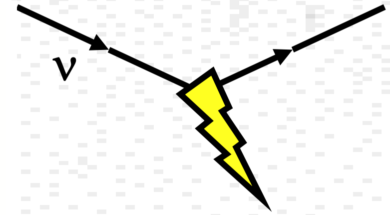
- Can relate SFs to PDFs in naïve quark-parton model by matching y dependence
  - Assuming Callan-Gross, massless targets and partons…
  - F_3: $2y-y^2=(1-y)^2-1$, $2xF_1=F_2$: $2-2y+y^2=(1-y)^2+1$

$$2xF_1^{\nu p,CC} = x\left[d_p(x)+\overline{u_p}(x)+s_p(x)+\overline{c_p}(x)\right]$$
$$xF_3^{\nu p,CC} = x\left[d_p(x)-\overline{u_p}(x)+s_p(x)-\overline{c_p}(x)\right]$$

- In analogy with neutrino-electron scattering, CC only involves left-handed quarks
- However, NC involves both chiralities (V-A and V+A)
  - Also couplings from EW Unification
  - And no selection by quark charge

$$2xF_1^{\nu p,NC} = x\left[(u_L^2+u_R^2)\left(u_p(x)+\overline{u_p}(x)+c_p(x)+\overline{c_p}(x)\right)+(d_L^2+d_R^2)\left(d_p(x)+\overline{d_p}(x)+s_p(x)+\overline{s_p}(x)\right)\right]$$
$$xF_3^{\nu p,NC} = x\left[(u_L^2-u_R^2)\left(u_p(x)-\overline{u_p}(x)+c_p(x)-\overline{c_p}(x)\right)+(d_L^2-d_R^2)\left(d_p(x)-\overline{d_p}(x)+s_p(x)-\overline{s_p}(x)\right)\right]$$

Isoscalar Targets



- Heavy nuclei are roughly neutron-proton isoscalar
- Isospin symmetry implies $u_p = d_n, d_p = u_n$
- Structure Functions have a particularly simple interpretation in quark-parton model for this case…

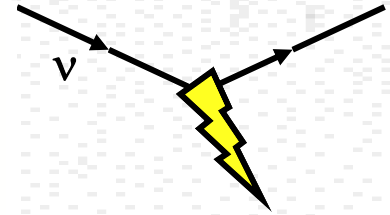
$$\frac{d^2\sigma^{\nu(\bar{\nu})N}}{dxdy} = \frac{G_F^2 s}{2\pi}\left\{ \left(1+(1-y)^2\right) F_2(x) \pm \left(1-(1-y)^2\right) xF_3^{\nu(\bar{\nu})}(x) \right\}$$

$$2xF_1^{\nu(\bar{\nu})N,CC}(x) = x(u(x)+d(x)+\bar{u}(x)+\bar{d}(x)+s(x)+\bar{s}(x)+c(x)+\bar{c}(x)) = xq(x)+x\bar{q}(x)$$

$$xF_3^{\nu(\bar{\nu})N,CC}(x) = xu_{Val}(x)+xd_{Val}(x) \pm 2x(s(x)-c(x))$$

$$\text{where } u_{Val}(x) = u(x) - \bar{u}(x)$$

Lecture Question #4: Neutrino and Anti-Neutrino $\sigma^{\nu N}$



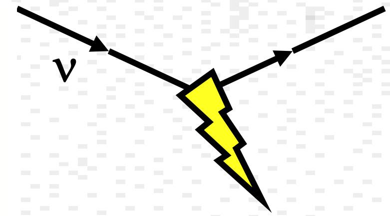
- **Given that $\sigma_{CC}^{\bar{\nu}N} \approx \frac{1}{2}\sigma_{CC}^{\nu N}$ in the DIS regime (CC)**

  and that $\frac{d\sigma(\nu q)}{dx} = \frac{d\sigma(\bar{\nu}\bar{q})}{dx} = 3\frac{d\sigma(\nu \bar{q})}{dx} = 3\frac{d\sigma(\bar{\nu} q)}{dx}$
  for CC scattering from quarks or anti-quarks of a given momentum,

  and that cross-section is proportional to parton momentum, what is the approximate ratio of anti-quark to quark momentum in the nucleon?

  (a) $\bar{q}/q \sim 1/3$ **(b)** $\bar{q}/q \sim 1/5$ **(c)** $\bar{q}/q \sim 1/8$

Lecture Question #4: Neutrino and Anti-Neutrino $\sigma^{\nu N}$



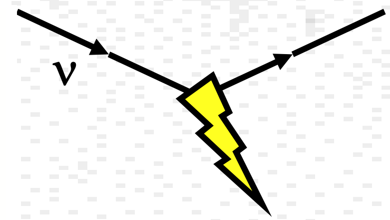
- **Given that $\sigma_{CC}^{\bar{\nu} N} \approx \frac{1}{2}\sigma_{CC}^{\nu N}$ in the DIS regime (CC)**

  and that $\frac{d\sigma(\nu q)}{dx} = \frac{d\sigma(\bar{\nu}\bar{q})}{dx} = 3\frac{d\sigma(\nu\bar{q})}{dx} = 3\frac{d\sigma(\bar{\nu} q)}{dx}$
  for CC scattering from quarks or anti-quarks of a given momentum,

  and that cross-section is proportional to parton momentum, what is the approximate ratio of anti-quark to quark momentum in the nucleon?

(a) $\bar{q}/q \sim 1/3$ (b) $\bar{q}/q \sim 1/5$ (c) $\bar{q}/q \sim 1/8$

Lecture Question #4: Neutrino and Anti-Neutrino $\sigma^{\nu N}$



- **Given:** $\sigma_{CC}^{\bar{\nu}N} \approx \frac{1}{2}\sigma_{CC}^{\nu N}$ **in the DIS regime (CC)**

and $$\frac{d\sigma(\nu q)}{dx} = \frac{d\sigma(\bar{\nu}\bar{q})}{dx} = 3\frac{d\sigma(\nu\bar{q})}{dx} = 3\frac{d\sigma(\bar{\nu}q)}{dx}$$

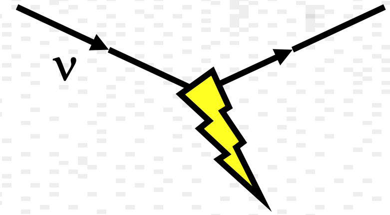
$$\sigma_\nu = \int_{q,\bar{q}} dx\left(\frac{d\sigma(\nu q)}{dx} + \frac{d\sigma(\nu\bar{q})}{dx}\right)$$

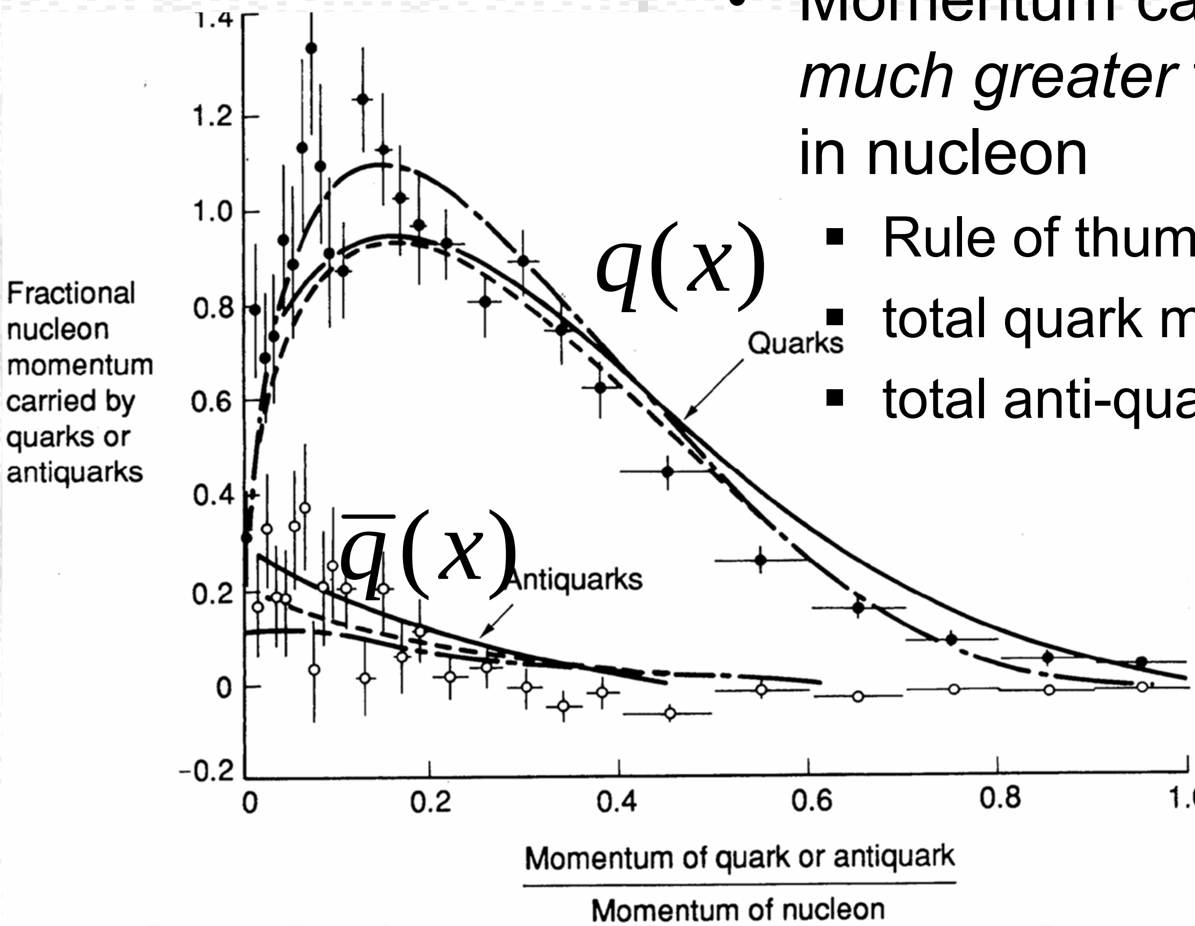
$$\sigma_{\bar{\nu}} = \int_{q,\bar{q}} dx\left(\frac{d\sigma(\bar{\nu}q)}{dx} + \frac{d\sigma(\bar{\nu}\bar{q})}{dx}\right) = \int_{q,\bar{q}} dx\left(\frac{d\sigma(\nu q)}{3dx} + \frac{3d\sigma(\nu\bar{q})}{dx}\right)$$

$$\therefore \int_{q,\bar{q}} dx\left(\frac{d\sigma(\nu q)}{dx} + \frac{d\sigma(\nu\bar{q})}{dx}\right) = 2\int_{q,\bar{q}} dx\left(\frac{d\sigma(\nu q)}{3dx} + \frac{3d\sigma(\nu\bar{q})}{dx}\right)$$

$$\frac{1}{3}\int_q dx\frac{d\sigma(\nu q)}{dx} = 5\int_{\bar{q}} dx\frac{d\sigma(\nu\bar{q})}{dx} = \frac{5}{3}\int_{\bar{q}} dx\frac{d\sigma(\bar{\nu}q)}{dx}$$

Momentum of Quarks & Antiquarks





- Momentum carried by quarks *much greater* than anti-quarks in nucleon
  - Rule of thumb: at Q^2 of 10 GeV^2:
  - total quark momentum is 1/3,
  - total anti-quark is 1/15.

From SFs to PDFs

- As you all know, there is a large industry in determining Parton Distributions for hadron collider simulations.
  - to the point where some of my colleagues on collider experiments might think of parton distributions as an annoying piece of FORTRAN code in their software package
- The purpose, of course, is to use factorization to predict cross-sections for various processes
  - combining deep inelastic scattering data from various sources together allows us to “measure” parton distributions
  - which then are applied to predict hadron-hadron processes at colliders, and can also be used in predictions for neutrino scattering, as we shall see.

From SFs to PDFs (cont'd)

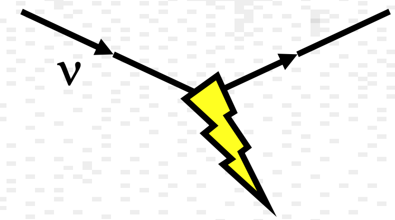
- We just learned that…

$$2xF_1^{\nu(\bar{\nu})N,CC}(x) = xq(x) + x\bar{q}(x)$$
$$xF_3^{\nu(\bar{\nu})N,CC}(x) = xu_{Val}(x) + xd_{Val}(x) \pm 2x(s(x) - c(x))$$
$$\text{where } u_{Val}(x) = u(x) - \bar{u}(x)$$

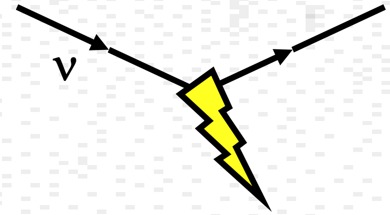
- In charged-lepton DIS

$$2xF_1^{\gamma p}(x) = \left(\tfrac{2}{3}\right)^2 \sum_{\text{up type quarks}} q(x) + \bar{q}(x)$$
$$+\left(\tfrac{1}{3}\right)^2 \sum_{\text{down type quarks}} q(x) + \bar{q}(x)$$

- So you begin to see how one can combine neutrino and charged lepton DIS and separate
  - the quark sea from valence quarks
  - up quarks from down quarks



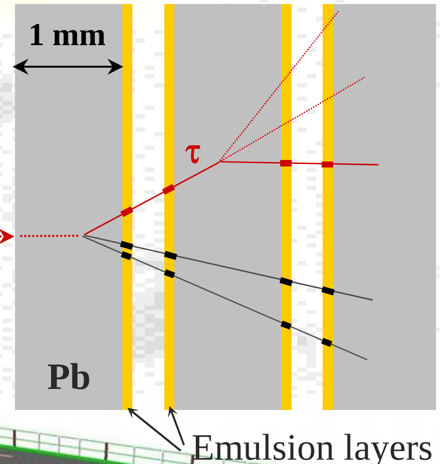
DIS: Massive Quarks and Leptons

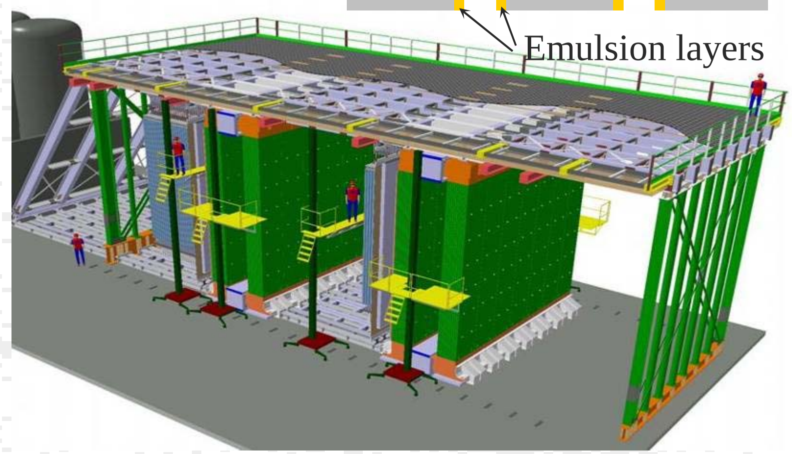


Opera at CNGS

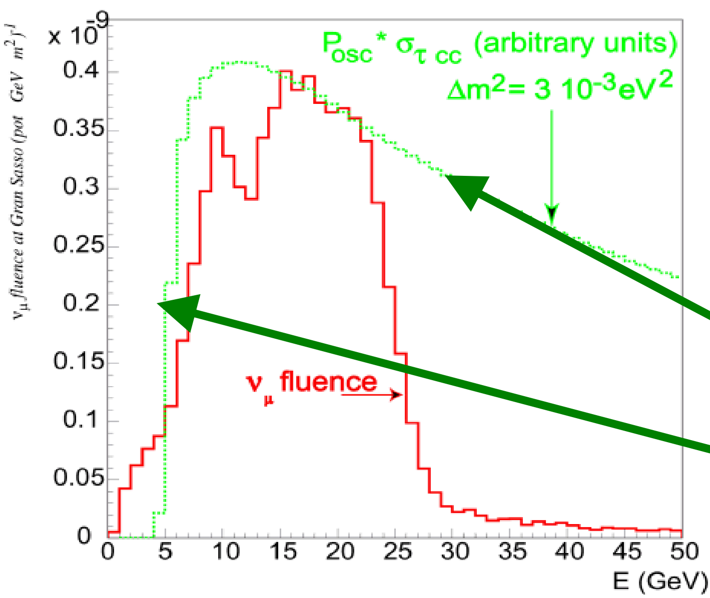
Goal: ν_τ appearance
- 0.15 MWatt source
- high energy ν_μ beam
- 732 km baseline
- handfuls of events/yr

1.8kTon





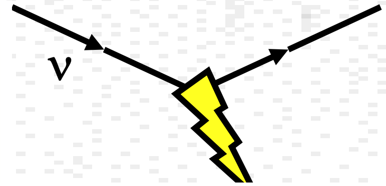
figures courtesy D. Autiero



oscillation probability

but what is this effect?

Lepton Mass Effects in DIS



- Recall that final state mass effects enter as corrections:

$$1-\frac{m^2_{\text{lepton}}}{s_{\text{point-like}}} \rightarrow 1-\frac{m^2_{\text{lepton}}}{x s_{\text{nucleon}}}$$

  - *relevant center-of-mass energy is that of the "point-like" neutrino-parton system*
  - *this is high energy approximation*
- For ν_τ charged-current, there is a threshold of

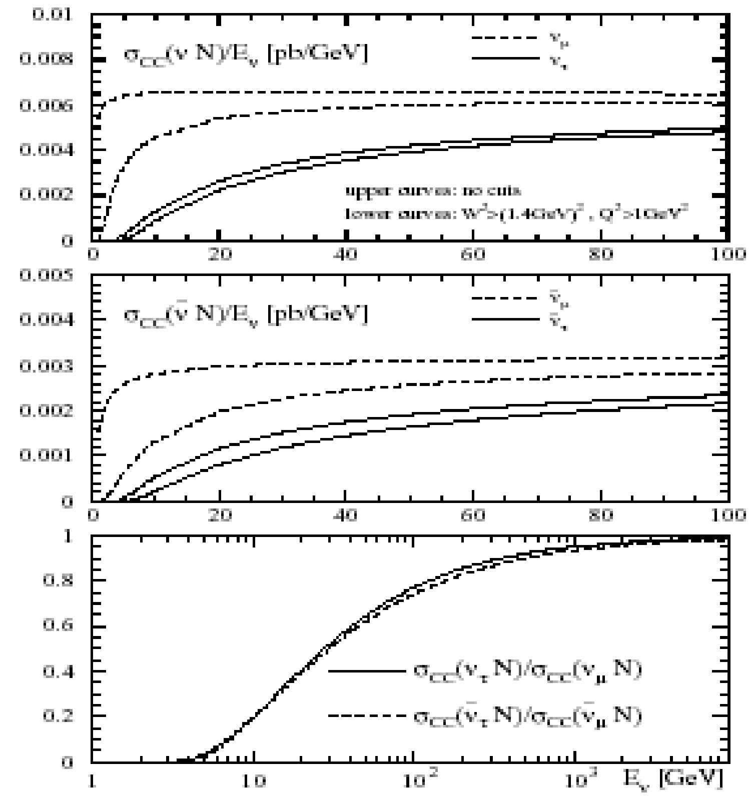
$$s_{\min} = (m_{\text{nucleon}} + m_\tau)^2$$

where

$$s_{initial} = m^2_{\text{nucleon}} + 2E_\nu m_{\text{nucleon}}$$

$$\therefore E_\nu > \frac{m_\tau^{\ 2} + 2m_\tau m_{\text{nucleon}}}{2m_{\text{nucleon}}} \approx 3.5\text{ GeV}$$

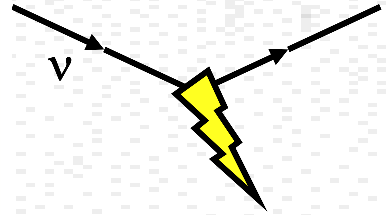
"m_{nucleon}" is M_T elsewhere, but don't want to confuse with m_τ...



(Kretzer and Reno)

- This is threshold for partons with *entire* nucleon momentum
  - effects big at higher E_ν also

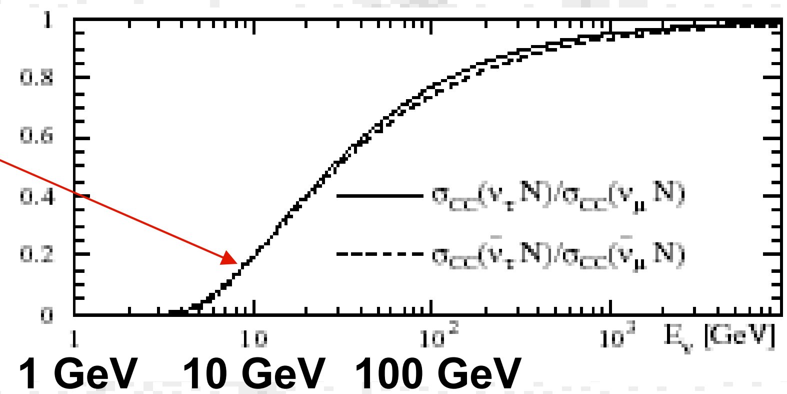
Lecture Question #5: What if Taus were Lighter?



- Imagine we lived in a universe where the tau mass was not 1.777 GeV, but was 0.888 GeV
- By how much would the tau appearance cross-section for an 8 GeV tau neutrino increase at OPERA?

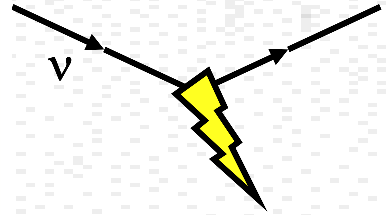
mass suppression: $1-\frac{m_{\text{lepton}}^2}{xs_{\text{nucleon}}}$

$$s_{\text{nucleon}} = m_{\text{nucleon}}^2 + 2E_\nu m_{\text{nucleon}}$$



(a) $\frac{\sigma_{\text{Light Tau}}}{\sigma_{\text{Reality}}} \sim 1.4$ **(b)** $\frac{\sigma_{\text{Light Tau}}}{\sigma_{\text{Reality}}} \sim 2$ **(c)** $\frac{\sigma_{\text{Light Tau}}}{\sigma_{\text{Reality}}} \sim 3$

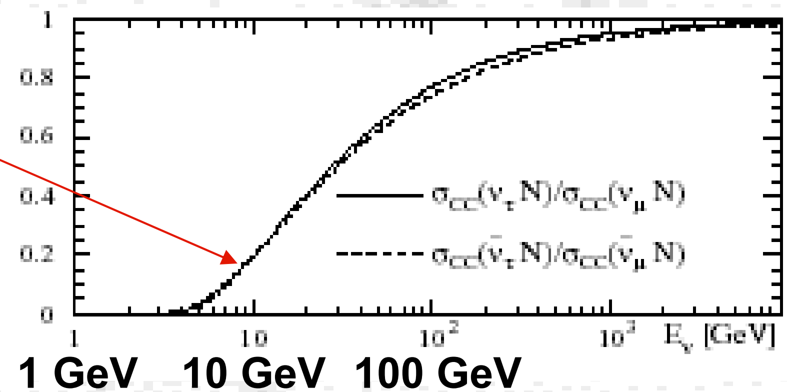
Lecture Question #5: What if Taus were Lighter?



- Imagine we lived in a universe where the tau mass was not 1.777 GeV, but was 0.888 GeV
- By how much would the tau appearance cross-section for an 8 GeV tau neutrino increase at OPERA?

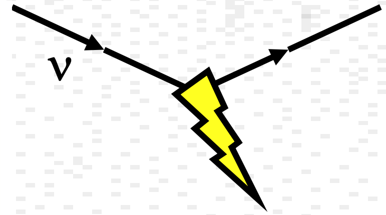
mass suppression: $1 - \frac{m^2_{\text{lepton}}}{xs_{\text{nucleon}}}$

$$s_{\text{nucleon}} = m^2_{\text{nucleon}} + 2E_\nu m_{\text{nucleon}}$$



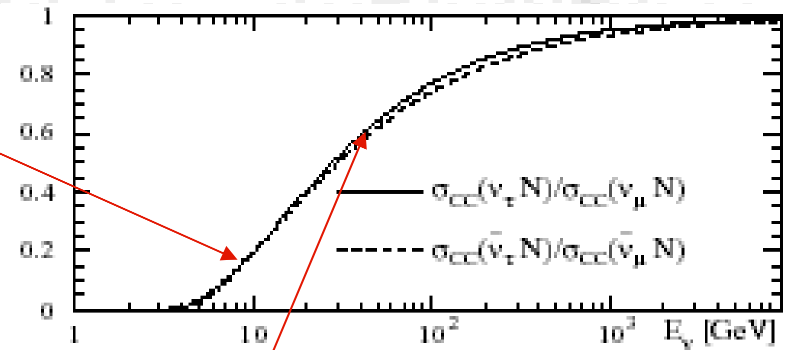
(a) $\frac{\sigma_{\text{Light Tau}}}{\sigma_{\text{Reality}}} \sim 1.4$ **(b)** $\frac{\sigma_{\text{Light Tau}}}{\sigma_{\text{Reality}}} \sim 2$ **(c)** $\frac{\sigma_{\text{Light Tau}}}{\sigma_{\text{Reality}}} \sim 3$

Lecture Question #5: What if Taus were Lighter?



- By how much would the tau appearance cross-section for an 8 GeV tau neutrino increase at OPERA?

mass suppression: $1-\frac{m^2_{\text{lepton}}}{xs_{\text{nucleon}}}$



1 GeV 10 GeV 100 GeV

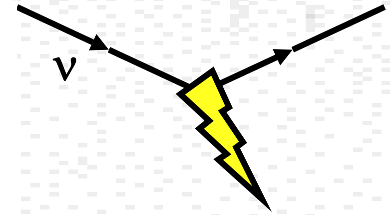
Numerator goes down by factor of four. Equivalent to denominator increasing by factor of four and tau mass unchanged…

$$s_{\text{nucleon}} = m^2_{\text{nucleon}} + 2E_\nu m_{\text{nucleon}}$$

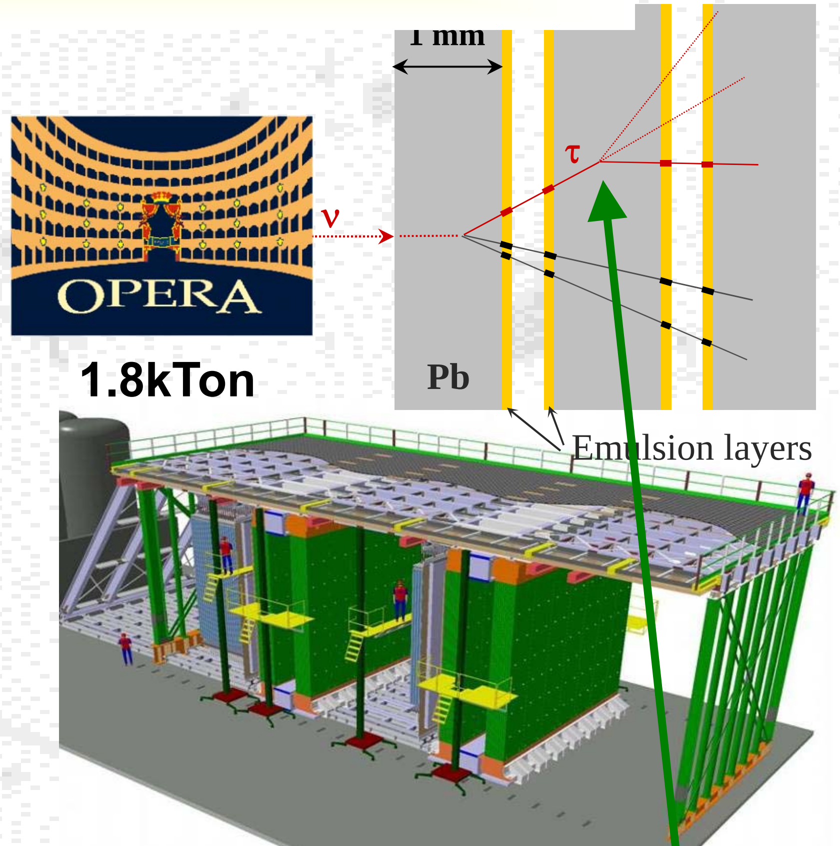
energy term dominates…
so set energy a factor of four higher

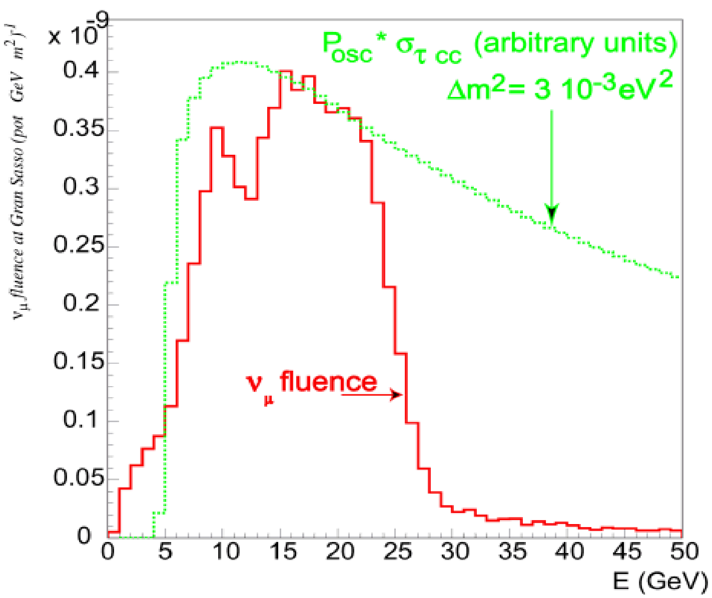
$$\frac{\sigma_{\text{Light Tau}}}{\sigma_{\text{Reality}}} \sim 3$$

Opera at CNGS



Goal: ν_τ appearance
- 0.15 MWatt source
- high energy ν_μ beam
- 732 km baseline
- handfuls of events/yr

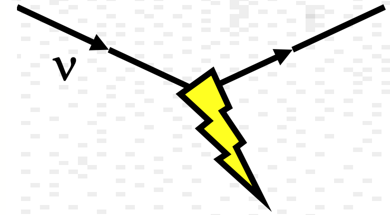




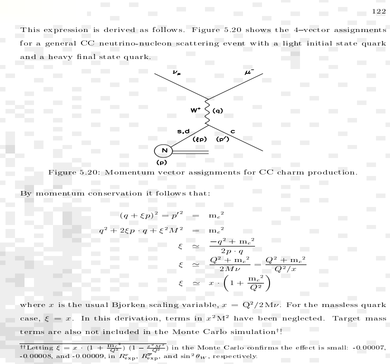
figures courtesy D. Autiero

what else is copiously produced in neutrino interactions with $c\tau$ ~ 100μm and decays to hadrons?

Heavy Quark Production



- Production of heavy quarks modifies kinematics of our earlier definition of x.
  - Charm is heavier than proton; hints that its mass is not a negligible effect…



122

This expression is derived as follows. Figure 5.20 shows the 4–vector assignments for a general CC neutrino-nucleon scattering event with a light initial state quark and a heavy final state quark.

Figure 5.20: Momentum vector assignments for CC charm production.

By momentum conservation it follows that:

$$\begin{aligned}(q+\xi p)^2 = p'^2 &= m_c^2 \\ q^2 + 2\xi p\cdot q + \xi^2 M^2 &= m_c^2 \\ \xi &\simeq \frac{-q^2+m_c^2}{2p\cdot q} \\ \xi &\simeq \frac{Q^2+m_c^2}{2M\nu} = \frac{Q^2+m_c^2}{Q^2/x} \\ \xi &\simeq x\cdot\left(1+\frac{m_c^2}{Q^2}\right)\end{aligned}$$

where x is the usual Bjorken scaling variable, $x = Q^2/2M\nu$. For the massless quark case, $\xi = x$. In this derivation, terms in x^2M^2 have been neglected. Target mass terms are also not included in the Monte Carlo simulation††

††Letting $\xi = x\cdot(1+\frac{m_c^2}{Q^2})\,(1-\frac{x^2M^2}{Q^2})$ in the Monte Carlo confirms the effect is small: -0.00007, -0.00008, and -0.00009, in $R^{\nu}_{\rm exp}$, $R^{\bar\nu}_{\rm exp}$, and $\sin^2\theta_W$, respectively.

$$(q+\zeta p)^2 = p'^2 = m_c^2$$

$$q^2 + 2\zeta p\bullet q + \zeta^2 M^2 = m_c^2$$

Note different definition of fractional momentum

$$\text{Therefore }\ \zeta \cong \frac{-q^2+m_c^2}{2p\bullet q}$$

$$\zeta \cong \frac{Q^2+m_c^2}{2M\upsilon} = \frac{Q^2+m_c^2}{Q^2/x}$$

$$\zeta \cong x\left(1+\frac{m_c^2}{Q^2}\right)$$

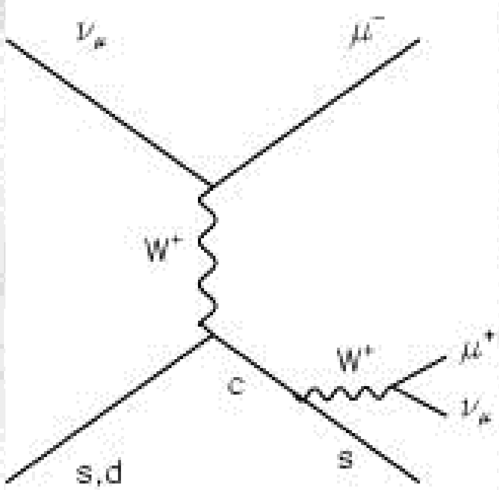
"slow rescaling" leads to kinematic suppression of charm production

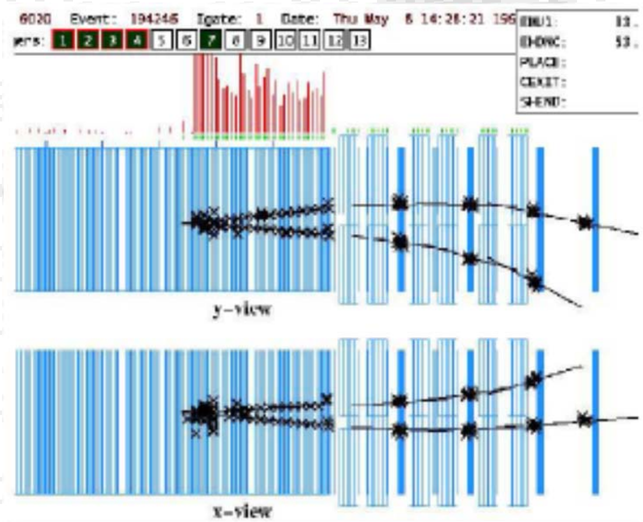
Neutrino Dilepton Events

- Neutrino induced charm production has been extensively studied
  - Emulsion/Bubble Chambers (low statistics, 10s of events). Reconstruct the charm final state, but limited by target mass.
  - “Dimuon events” (high statistics, 1000s of events)

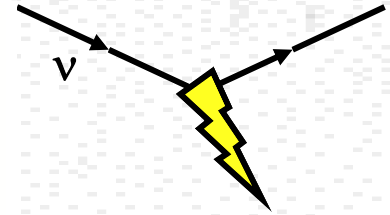
$$\nu_\mu + \begin{pmatrix} d \\ s \end{pmatrix} \rightarrow \mu^- + c + X, \qquad c \rightarrow \mu^+ + \nu_\mu + X'$$

$$\bar{\nu}_\mu + \begin{pmatrix} \bar{d} \\ \bar{s} \end{pmatrix} \rightarrow \mu^+ + \bar{c} + X, \qquad \bar{c} \rightarrow \mu^- + \bar{\nu}_\mu + X'$$





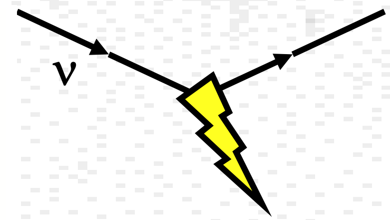
Deep Inelastic Scattering: Conclusions and Summary



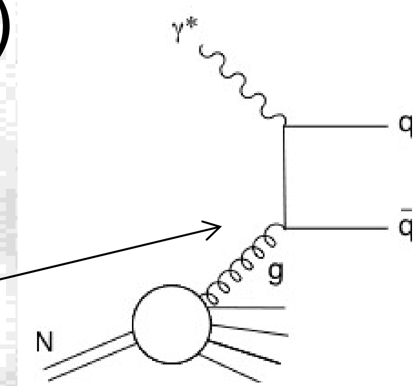
- Neutrino-quark scattering is elastic scattering!
  - complicated by fact that quarks live in nucleons
- Important lepton and quark mass effects for tau neutrino appearance experiments
- Neutrino DIS important for determining parton distributions
  - particularly valence and strange quarks

Ultra-High Energy Cross-Sections

Ultra-High Energies

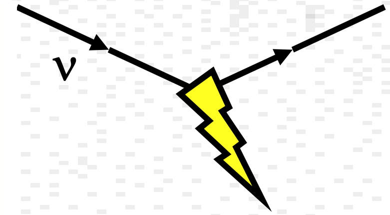


- At energies relevant for UHE Cosmic Ray studies (e.g., IceCube, Antares, ANITA)
  - ν-parton cross-section is dominated by high Q^2, since $d\sigma/dQ^2$ is constant
    - o at high Q^2, gluon radiation and splitting lead to more sea quarks at fewer high x partons (see supplemental material: scaling violations)
    - o see a rise in σ/E_ν from growth of sea at low x
    - o neutrino & anti-neutrino cross-sections nearly equal
  - *Until* $Q^2 » M_W^2$, then propagator term starts decreasing and cross-section stops growing linearly with energy



$$\frac{d\sigma}{dq^2} \propto \frac{1}{(q^2 - M^2)^2}$$

Lecture Question #6: Where does σ Level Off?



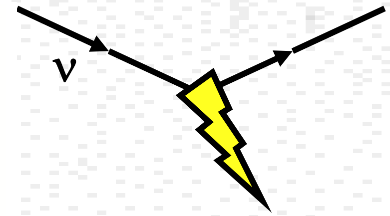
- *Until* $Q^2 \gg M_W^2$, then propagator term starts decreasing and cross-section becomes constant

$$\frac{d\sigma}{dq^2} \propto \frac{1}{(q^2 - M^2)^2}$$

- To within a few orders of magnitude, at what beam energy for a target at rest will this happen?

(a) $E_\nu \sim 10\,\mathrm{TeV}$ **(b)** $E_\nu \sim 10,000\,\mathrm{TeV}$ **(c)** $E_\nu \sim 10,000,000\,\mathrm{TeV}$

Lecture Question #6: Where does σ Level Off?



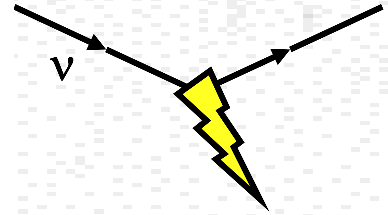
- *Until* $Q^2 » M_W^2$, then propagator term starts decreasing and cross-section becomes constant

$$\frac{d\sigma}{dq^2} \propto \frac{1}{(q^2 - M^2)^2}$$

- To within a few orders of magnitude, at what beam energy for a target at rest will this happen?

(a) $E_\nu \sim 10\text{TeV}$ **(b)** $E_\nu \sim 10,000\text{TeV}$ **(c)** $E_\nu \sim 10,000,000\text{TeV}$

Lecture Question #6: Where does σ Level Off?



- *Until* $Q^2 \gg M_W^2$, then propagator term starts decreasing and cross-section becomes constant

$$\frac{d\sigma}{dq^2} \propto \frac{1}{(q^2 - M^2)^2}$$

- At what beam energy for a target at rest will this happen?

$$Q^2 < s_{\text{nucleon}} = m_{\text{nucleon}}^2 + 2E_\nu m_{\text{nucleon}}$$

$$Q^2 < s_{\text{nucleon}} \approx 2E_\nu m_{\text{nucleon}}$$

$$\frac{M_W^2}{2m_{\text{nucleon}}} < E_\nu$$

$$\therefore E_\nu \gtrsim \frac{(80.4)^2\ \text{GeV}^2}{2(.938)\text{GeV}} \sim 3000\text{GeV}$$

Q^2 limit is s.
So won't start to plateau until $s>M_W^2$

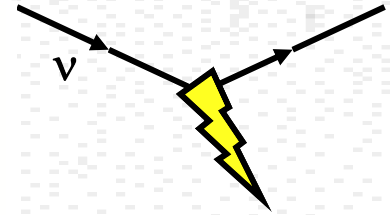
Bonus point realization…

In reality, that is only correct for a parton at x=1. Typical quark x is much less, say ~0.03

$$\frac{M_W^2}{2m_{\text{nucleon}}x} < E_\nu$$

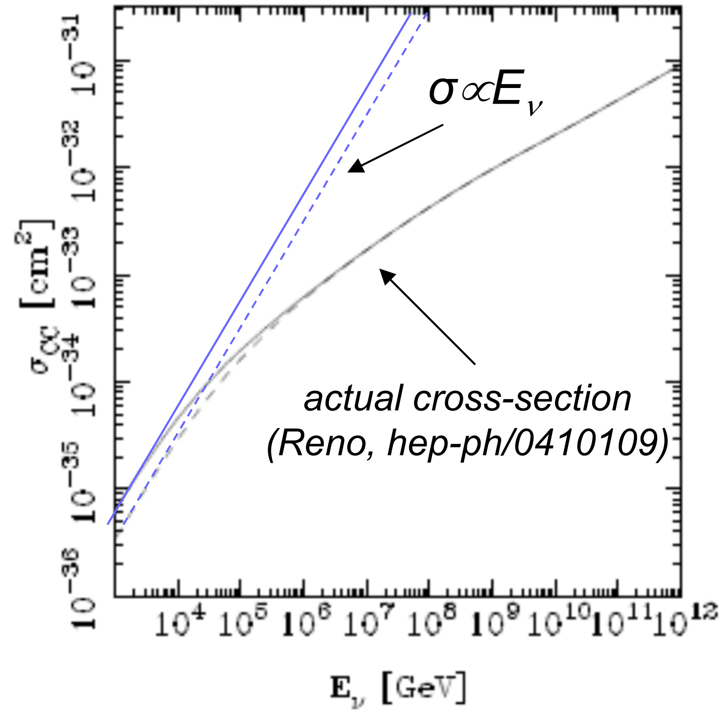
$$\therefore E_\nu \gtrsim \frac{3000\text{GeV}}{0.03} \sim 100\text{TeV}$$

Ultra-High Energies

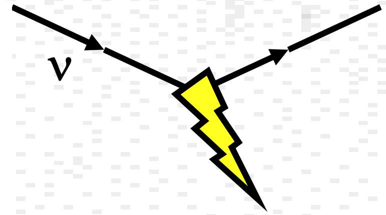


- ν-parton cross-section is dominated by high Q^2, since $d\sigma/dQ^2$ is constant
  - at high Q^2, scaling violations have made most of nucleon momentum carried by sea quarks
  - see a rise in σ/E_ν from growth of sea at low x
  - neutrino & anti-neutrino cross-sections nearly equal
- *Until* $Q^2 \gg M_W^2$, then propagator term starts decreasing and cross-section becomes constant

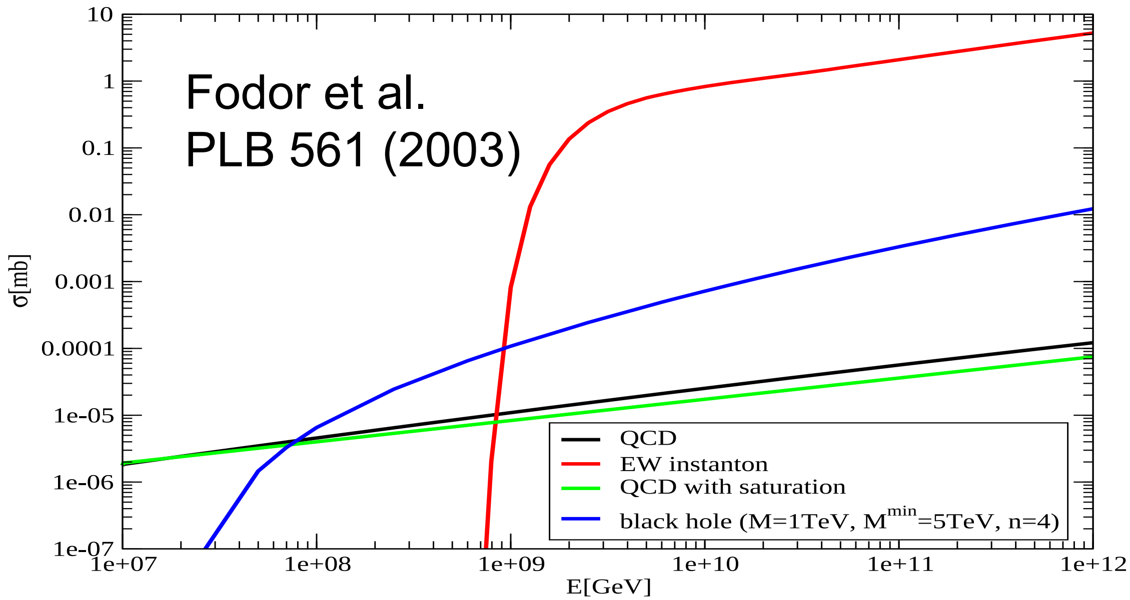
$$\frac{d\sigma}{dq^2} \propto \frac{1}{(q^2 - M^2)^2}$$

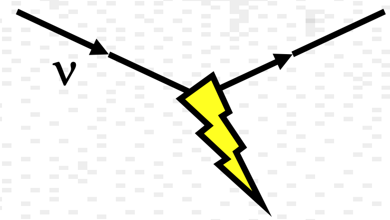


Example: Ultra-High Energies



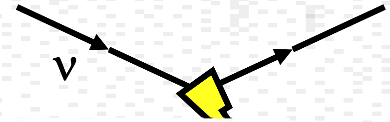
- At UHE, can we reach thresholds of non-SM processes?
  - E.g., structure of quark or leptons, black holes from extra dimensions, etc.
  - Th



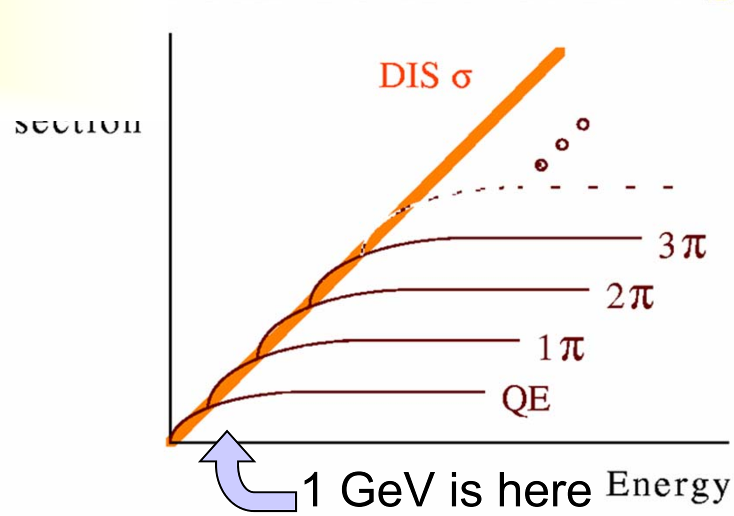


Motivation for Understanding GeV Cross-Sections

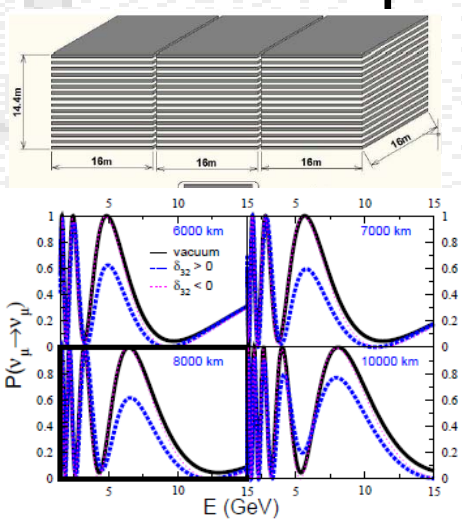
What's special about it? Why do we care?



- Remember this picture?
  - 1-few GeV is exactly where these additional processes are turning on



  - It's not DIS yet! Final states & threshold effects matter
- Why is it important? Examples from T2K, ICAL

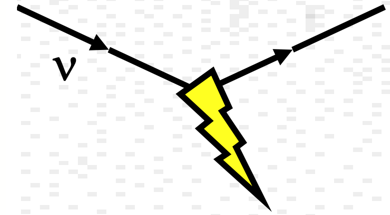


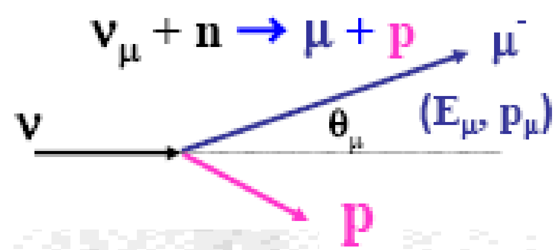
Goals:

1. $\nu_\mu \rightarrow \nu_e$
2. ν_μ disappearance

E_ν is 0.4-2.0 GeV (T2K) or 3-10 GeV (INO ICAL)

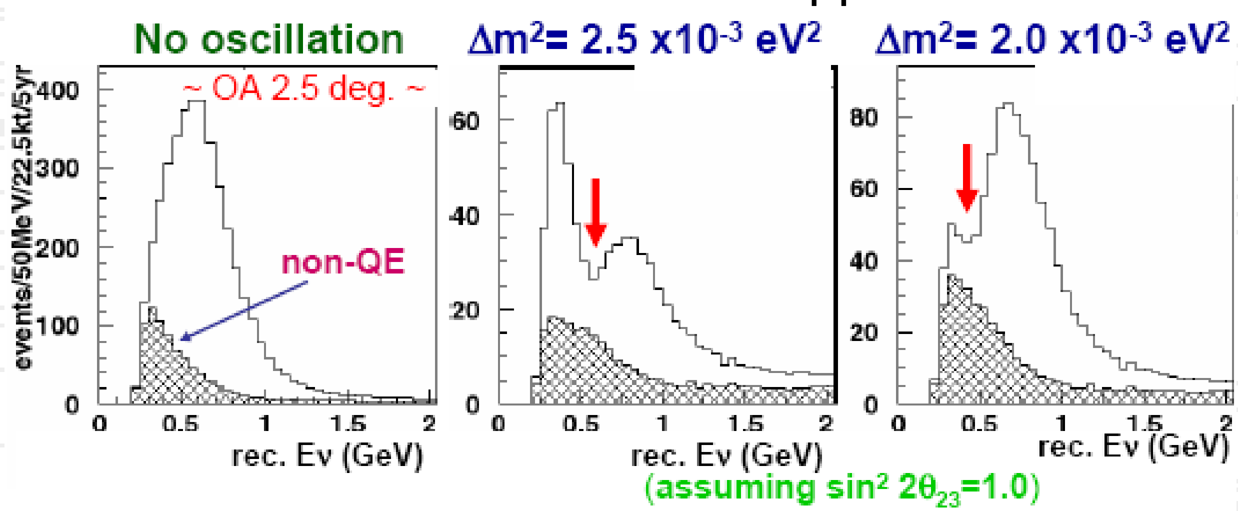
How do cross-sections effect oscillation analysis?



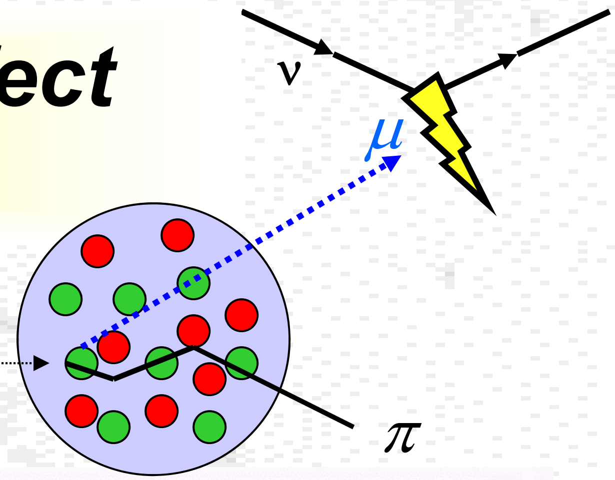


- ν_μ disappearance (low energy)
  - at Super-K reconstruct these events by muon angle and momentum (proton below Cerenkov threshold in H_2O)
  - other final states with more particles below threshold ("non-QE") will disrupt this reconstruction
- T2K must know these events at few % level to do disappearance analysis to measure Δm^2_{23}, θ_{23}

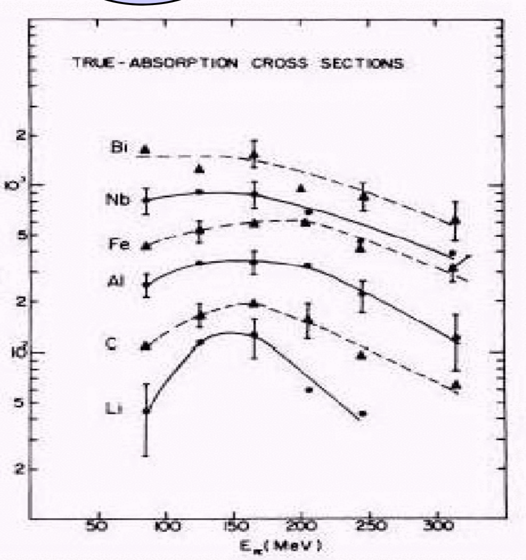
(fig. courtesy Y. Hayato)



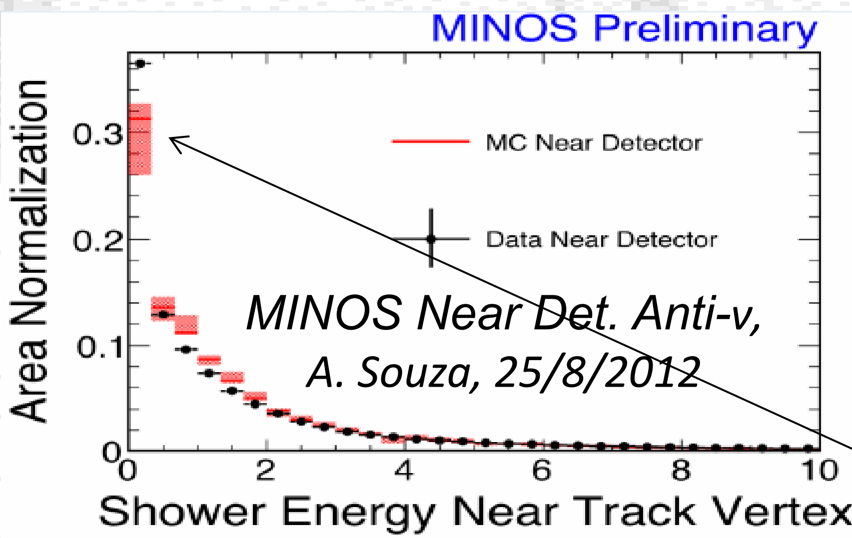
How do cross-sections effect oscillation analysis?



- ν_μ disappearance (high energy)
- Visible Energy in a calorimeter is NOT the ν energy transferred to the hadronic system
  - ➢ π absorption, π re-scattering, final state rest mass effect the calorimetric response
    - ➢ Can use external data to constrain



D. Ashery et al, PRC ***23****, 1993*

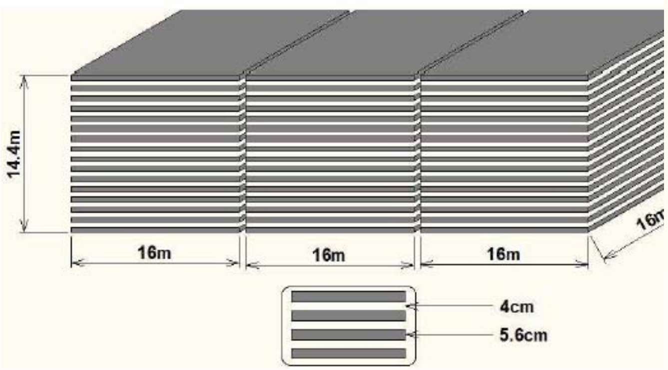


MINOS Near Det. Anti-ν, A. Souza, 25/8/2012

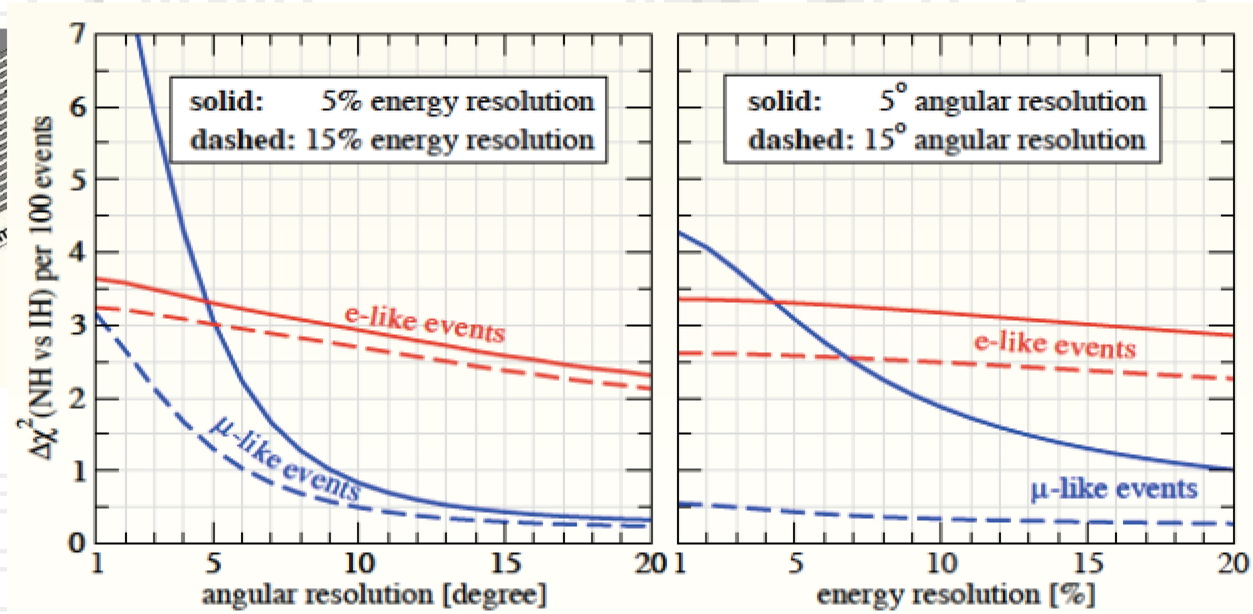
- ➢ *At very high energies, particle multiplicities are high and these effects will average out*
- ➢ *Low energy is more difficult*

How do cross-sections effect oscillation analysis?

- In the case of INO ICAL, need good energy and angle resolution to separate normal and inverted hierarchy
  - Best sensitivity requires survival probability in both E_ν and L

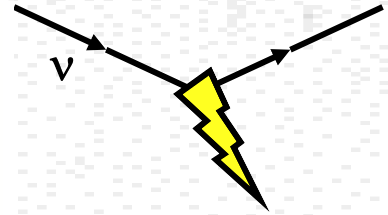


- Interaction models are understanding of detector response both needed to optimize resolution

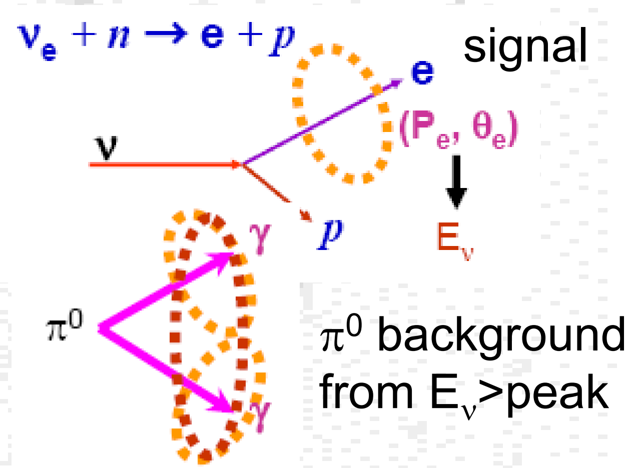


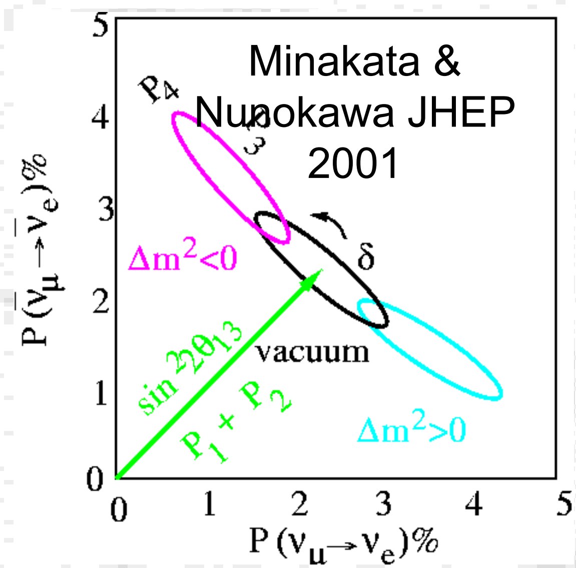
Petcov, Schwetz, hep-ph/0511277

How do cross-sections effect oscillation analysis?



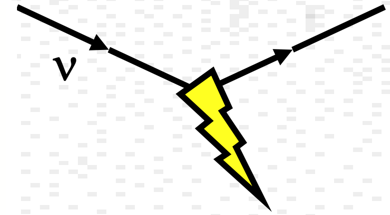
- ν_e appearance
  - different problem: signal rate is very low so even rare backgrounds contribute!
- Remember the end goal of electron neutrino appearance experiments
- Want to compare two signals with two different sets of backgrounds and signal reactions
  - with sub-percent precision
  - Requires precise knowledge of background and signal reactions





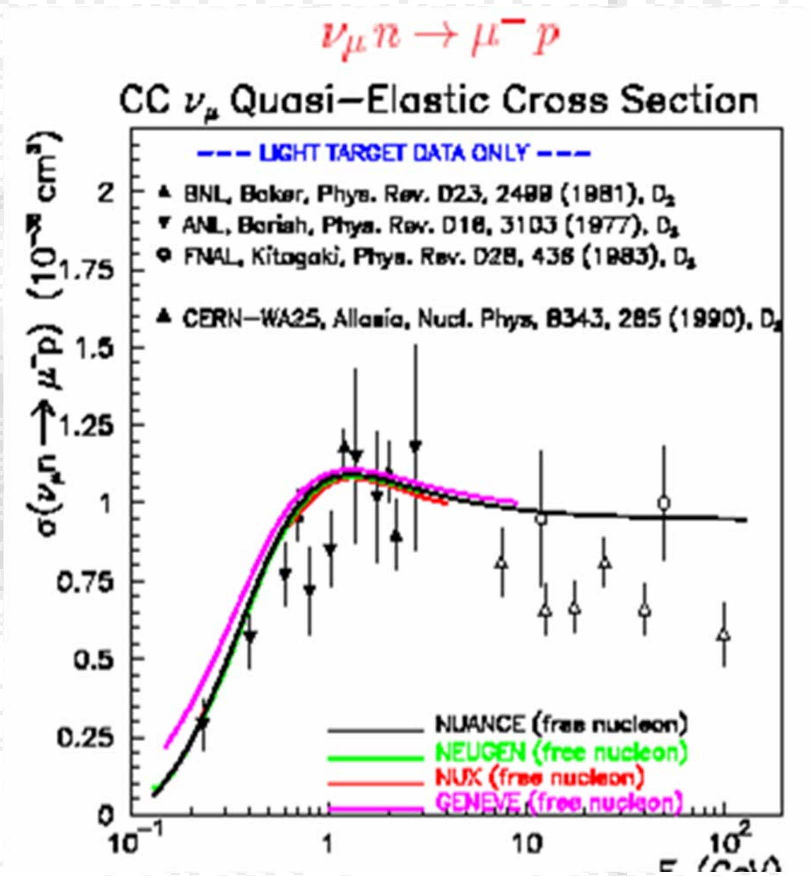
Models for GeV Cross-Sections

(Quasi-)Elastic Scattering



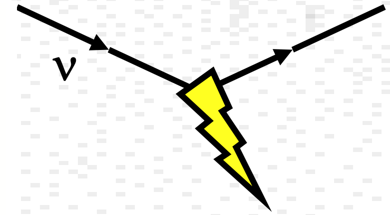
- Elastic scattering leaves a single nucleon in the final state
  - CC "quasi-elastic" easier to observe

$$\nu n \to l^- p$$
$$\bar{\nu} p \to l^+ n$$
$$\overset{(-)}{\nu} N \to \overset{(-)}{\nu} N$$



- State of data is marginal
  - No free neutrons implies nuclear corrections
  - Low energy statistics poor
- Cross-section is calculable
  - But depends on incalculable form-factors of the nucleon
- Theoretically and experimentally constant at high energy
  - 1 GeV^2 is ~ a limit in Q^2

What was that last cryptic remark?



- Theoretically and experimentally constant at high energy
  - 1 GeV^2 is ~ a limit in Q^2

- Inverse μ–decay:

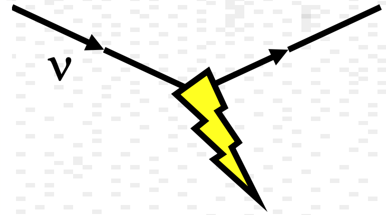
$$\nu_\mu + e^- \rightarrow \mu^- + \nu_e$$

a maximum Q^2 independent of beam energy ⇒ constant σ_{TOT}

$$\sigma_{TOT} \propto \int_0^{Q^2_{max}} dQ^2 \frac{1}{(Q^2 + M_W{}^2)^2} \approx \frac{Q^2_{max}}{M_W{}^4}$$

- OK, but why does cross-section have a Q^2_{max} limit?
  - If Q^2 is too large, then the probability for the final state nucleon to stay intact (elastic scattering) becomes low
  - This information is encoded in "form factors" of the nucleons

Elastic Scattering (cont'd)



- As with IBD, nucleon structure alters cross-section
  - Can write down in terms of all possible "form factors" of the nucleon allowed by Lorentz invariance

$$\nu n \to l^- p$$
$$\bar{\nu} p \to l^+ n$$
$$\overset{(-)}{\nu} N \to \overset{(-)}{\nu} N$$

C.H. Llewellyn Smith, Phys. Rep. 3C, 261 (1972)

$$\frac{d\sigma}{dQ^2}\left({}^{\nu n \to l^- p}_{\bar{\nu} p \to l^+ n}\right) = \left[A(Q^2) \mp B(Q^2)\frac{s-u}{M^2} + C(Q^2)\frac{(s-u)^2}{M^4}\right] \times \frac{M^2 G_F^2 \cos^2\theta_c}{8\pi E_\nu^2}$$

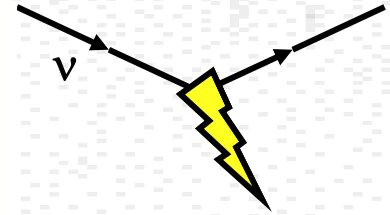
$$A(Q^2) = \frac{m^2+Q^2}{4M^2}\left[\left(4+\frac{Q^2}{M^2}\right)|F_A|^2 - \left(4-\frac{Q^2}{M^2}\right)|F_V^1|^2 + \frac{Q^2}{M^2}\xi|F_V^2|^2\left(1-\frac{Q^2}{4M^2}\right) + \frac{4Q^2 ReF_V^{1*}\xi F_V^2}{M^2}\right.$$
$$\left. -\frac{Q^2}{M^2}\left(4+\frac{Q^2}{M^2}\right)|F_A^3|^2 - \frac{m^2}{M^2}\left(|F_V^1+\xi F_V^2|^2 + |F_A+2F_P|^2 - \left(4+\frac{Q^2}{M^2}\right)(|F_V^3|^2+|F_P|^2)\right)\right],$$

$$B(Q^2) = \frac{Q^2}{M^2}ReF_A^*\left(F_V^1+\xi F_V^2\right) - \frac{m^2}{M^2}Re\left[\left(F_V^1 - \frac{Q^2}{4M^2}\xi F_V^2\right)^* F_V^3 - \left(F_A - \frac{Q^2 F_P}{2M^2}\right)^* F_A^3\right] \text{ and}$$

$$C(Q^2) = \frac{1}{4}\left(|F_A|^2 + |F_V^1|^2 + \frac{Q^2}{M^2}\left|\frac{\xi F_V^2}{2}\right|^2 + \frac{Q^2}{M^2}|F_A^3|^2\right).$$

Occupants of the form factor zoo:
F^1_V, F^2_V are vector form factors;
F_A is the axial vector form factor;
F_P is the pseudo-scalar form factor;
F^3_V and F^3_A are form factors related to currents requiring G-parity violation, small?

Elastic Scattering (cont'd)



- Form factors representing second class currents, F^3_V and F^3_A, are usually assumed to be zero
- Pseudoscalar form factor, F_P, can be calculated from F_A with reasonable assumptions (Adler's theorem and the Goldberger-Treiman relation)
- The leading form factors, F^1_V, F^2_V and F_A, are approximately dipole in form

$$F_V(q^2) \sim \frac{1}{(1-q^2/M_V^2)^2} \qquad F_A(q^2) = \frac{F_A(0)}{(1-q^2/M_A^2)^2}$$

← "dipole approximation"

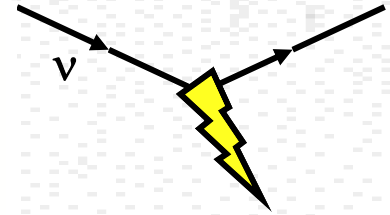
M_V ≈0.71 GeV
M_A≈1.01 GeV
$F_A(0)$ ≈-1.267
$F_V(0)$ is charge of proton

} parameters determined from data

n.b.: we've seen $F_V(0)$ and $F_A(0)$ before in IBD discussion (g_V and g_A)

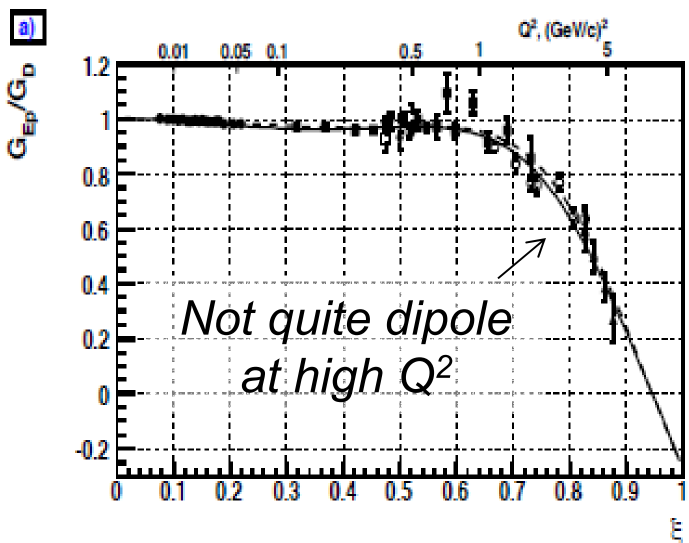
- Note that those masses which "cut off" the form factor are of order 1 GeV, so form factors are low beyond 1 GeV^2

Elastic Scattering (cont'd)



Vector form factors

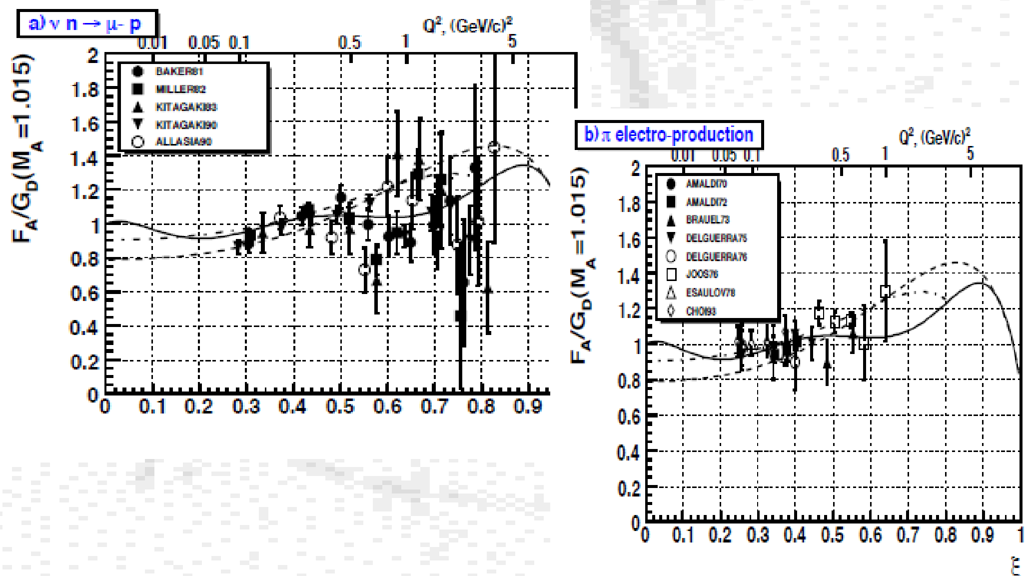
- Measured in charged lepton scattering



e.g., Bradford-Bodek-Budd-Arrington ("BBBA"), Nucl.Phys.Proc.Suppl.159:127-132,2006

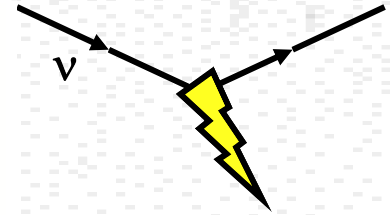
Axial vector form factors

- Measured in pion electro-production & neutrino scattering



Bodek, Avvakumov, Bradford and Budd, J. Phys. Conf. Ser. 110, 082004 (2008).

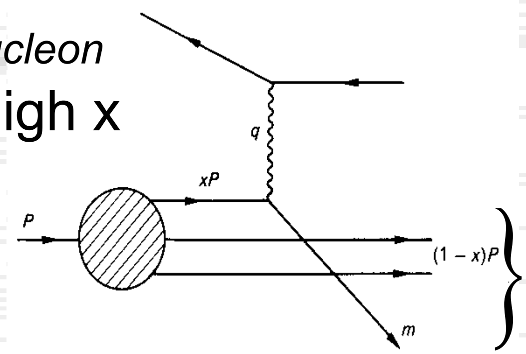
Low W, the Baryon Resonance Region



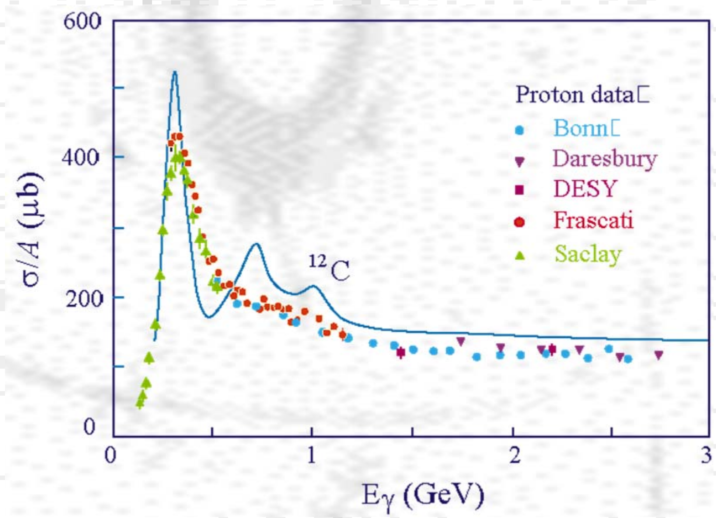
- Intermediate to elastic and DIS regions is a region of resonance production
  - Recall mass² of hadronic final state is given by

$$W^2 = M_T^2 + 2M_T\nu - Q^2 = M_T^2 + 2M_T\nu(1-x)$$

  - At low energy, nucleon-pion states dominated by N* and Δ resonances
- Leads to cross-section with significant structure in *W* just above $M_{nucleon}$
  - Low ν, high x



$$W^2$$



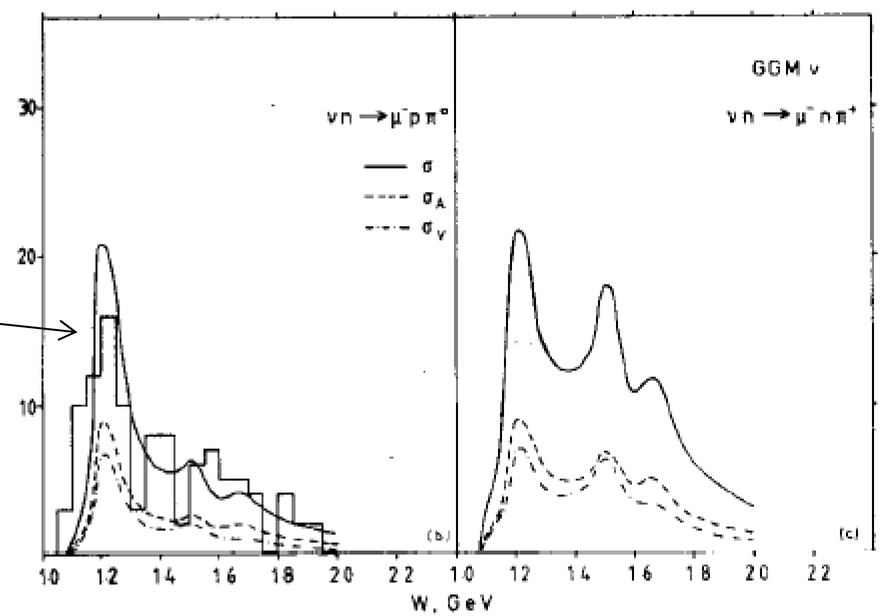
photoabsorption vs E_γ*. Line shows protons. More later…*

The Resonance Region

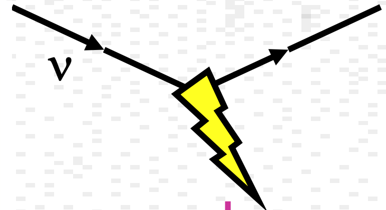
- Models of the resonance region are complicated
  - In principle, many baryon resonances can be excited in the scattering and they all can contribute
  - They de-excite mostly by radiating pions

Nucleon Resonances below 2 GeV/c² according to Ref. [4]

Resonance Symbol[a]	Central mass value M [MeV/c²]	Total with Γ_0[MeV]	Elasticity $x_E = \pi\mathcal{N}$ branching ratio	Quark-Model/ SU_6-assignment
$P_{33}(1234)$	1234	124	1	$^4(10)_{3/2}\ [56, 0^+]_0$
$P_{11}(1450)$	1450	370	0.65	$^2(8)_{1/2}\ [56, 0^+]_2$
$D_{13}(1525)$	1525	125	0.56	$^2(8)_{3/2}\ [70, 1^-]_1$
$S_{11}(1540)$	1540	270	0.45	$^2(8)_{1/2}\ [70, 1^-]_1$
$S_{31}(1620)$	1620	140	0.25	$^2(10)_{1/2}\ [70, 1^-]_1$
$S_{11}(1640)$	1640	140	0.60	$^4(8)_{1/2}\ [70, 1^-]_1$
$P_{33}(1640)$	1640	370	0.20	$^4(10)_{3/2}\ [56, 0^+]_2$
$D_{13}(1670)$	1670	80	0.10	$^4(8)_{3/2}\ [70, 1^-]_1$
$D_{15}(1680)$	1680	180	0.35	$^4(8)_{5/2}\ [70, 1^-]_1$
$F_{15}(1680)$	1680	120	0.62	$^2(8)_{5/2}\ [56, 2^+]_2$
$P_{11}(1710)$	1710	100	0.19	$^2(8)_{1/2}\ [70, 0^+]_2$
$D_{33}(1730)$	1730	300	0.12	$^2(10)_{3/2}\ [70, 1^-]_1$
$P_{13}(1740)$	1740	210	0.19	$^2(8)_{3/2}\ [56, 2^+]_2$
$P_{31}(1920)$	1920	300	0.19	$^4(10)_{1/2}\ [56, 2^+]_2$
$F_{35}(1920)$	1920	340	0.15	$^4(10)_{5/2}\ [56, 2^+]_2$
$F_{37}(1950)$	1950	340	0.40	$^4(10)_{7/2}\ [56, 2^+]_2$
$P_{33}(1960)$	1960	300	0.17	$^4(10)_{3/2}\ [56, 2^+]_2$
$F_{17}(1970)$	1970	325	0.06	$^4(8)_{7/2}\ [70, 2^+]_2$



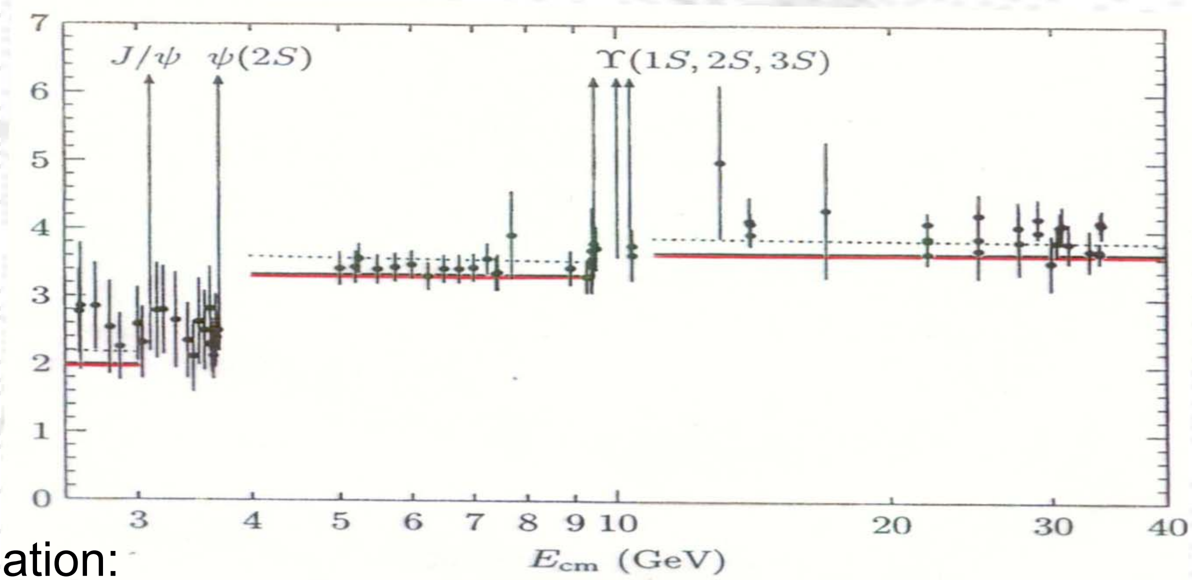
D. Rein and L. Sehgal, Ann. Phys. 133, 79 (1981)



Quark-Hadron Duality

- Bloom-Gilman Duality is the relationship between quark and hadron descriptions of reactions. It reflects:
  - link between *confinement* and *asymptotic freedom*
  - transition from *non-perturbative* to *perturbative* QCD

$$R \equiv \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

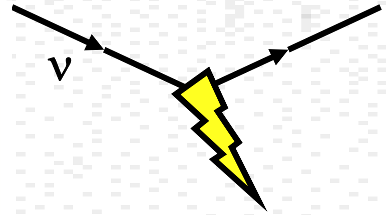


quark-parton model calculation:

$$R = N_C \sum_{q \ni 's > m_q^2} \left(Q_q^{EM}\right)^2 + O(\alpha_{EM} + \alpha_S)$$

but of course, final state is really sums over discrete hadronic systems

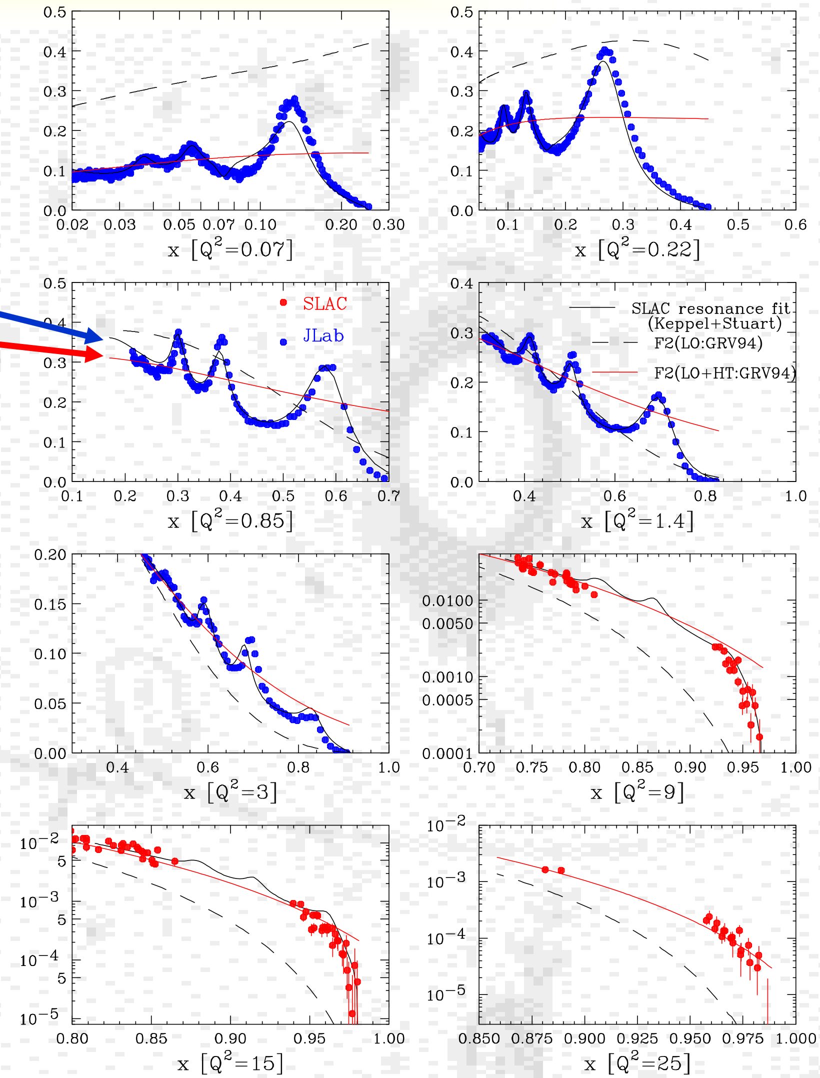
Duality and ν



$$W^2 = M_T^2 + Q^2\left(\frac{1}{x} - 1\right)$$

Low Q^2 data

DIS-Style PDF prediction



- Governs transition between resonance and DIS region
- Sums of discrete resonances approaches DIS cross-section
- Bodek-Yang: *Observe in electron scattering data; apply to ν cross-sections*

Duality's Promise

- In principle, a duality based approach can be applied over the entire kinematic region
- The problem is that duality gives "averaged" differential cross-sections, and not details of a final state

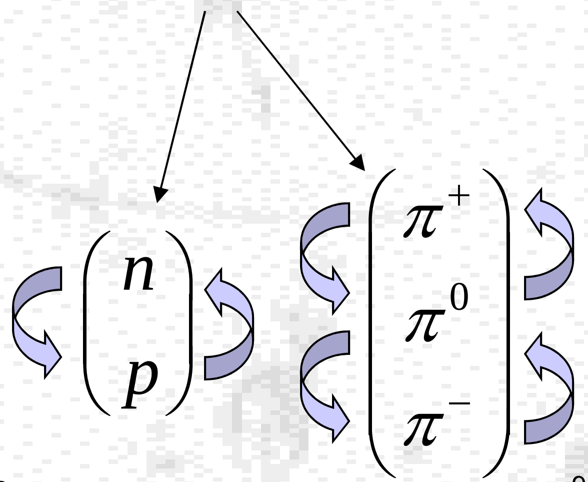
Microphysical models of exclusive processes

Duality models based on data of inclusive rates

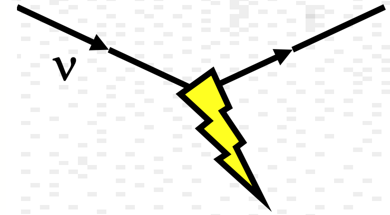
- Microphysical models may lack important physics, but duality models may not predict all we need to know
  - How to scale the mountain between the two?

Lecture Question #7: Duality meets Reality

- A difficulty in relating cross-sections of electron scattering (photon exchange) to charged-current neutrino scattering ($W^{\pm}$ exchange) is that some e-scatting reactions have imperfect ν-scattering analogues.
- Write all possible ν_μ CC reactions involving the same target particle and isospin rotations of the final state for each of the following…
  - **(a)** $e^- n \rightarrow e^- n$
  - **(b)** $e^- p \rightarrow e^- p$
  - **(c)** $e^- p \rightarrow e^- n\pi^+$
  - **(d)** $e^- n \rightarrow e^- p\pi^-$



Lecture Question #7: Duality meets Reality



Write all possible ν reactions involving the same target particle and isospin rotations of the final state for each of the following…

(a) $e^- n \rightarrow e^- n$

$$\nu_\mu n \rightarrow \mu^- p$$

(b) $e^- p \rightarrow e^- p$

there are none!

(c) $e^- p \rightarrow e^- n \pi^+$

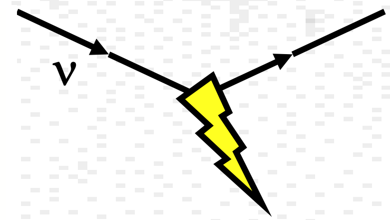
$$\nu_\mu p \rightarrow \mu^- p \pi^+$$

(d) $e^- n \rightarrow e^- p \pi^-$

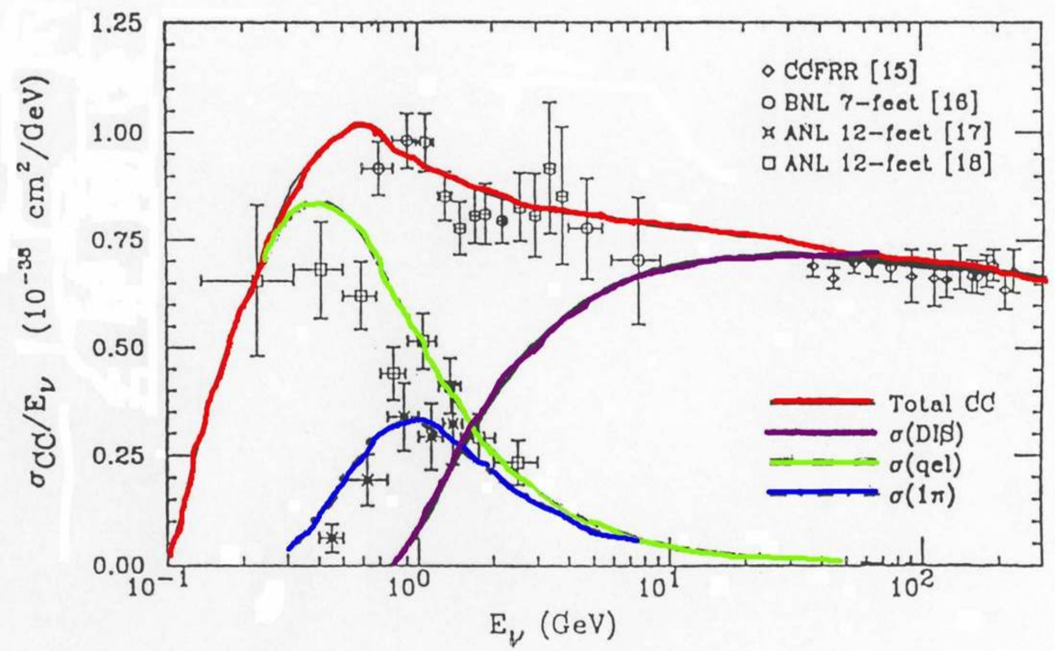
$$\nu_\mu n \rightarrow \mu^- n \pi^+$$

$$\nu_\mu n \rightarrow \mu^- p \pi^0$$

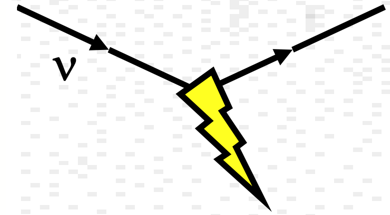
Building a Unified Model

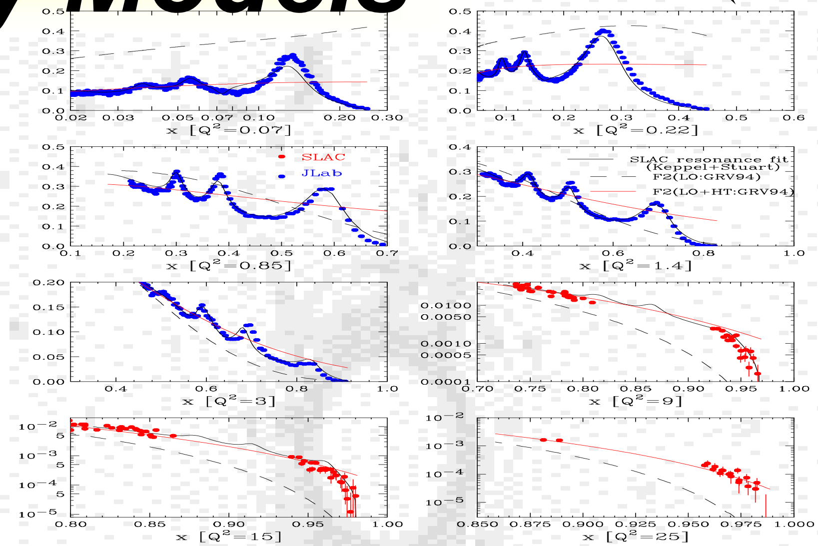


- In the relevant energy regime around 1 GeV, need a model that smoothly manages exclusive (elastic, resonance) to inclusive (DIS) transition
- Duality argues that the transition from the high W part of the resonance region (many resonances) to deep inelastic scattering should be smooth.



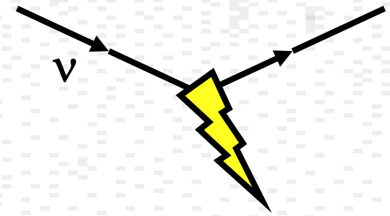
Exclusive Resonance Models and Duality Models





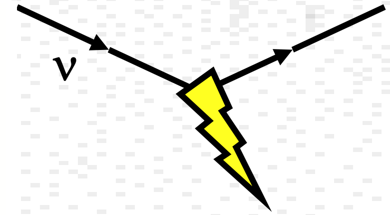
- Duality models agree with inclusive data by construction
  - However, in a generator context, have to add details of final state
- Typical approach (GENIE, NEUT and NUANCE) is to use a resonance model (Rein & Sehgal) below W<2 GeV, and duality + string fragmentation model for W>2 GeV
  - This is far from an idea solution
  - Discrete resonance model (probably) disagrees with total cross-section data below W<2 GeV and is difficult to tune
  - Average cross-section at high W does agree with data, but final state simulation is of unknown quality and difficult to tune also.

Supplemental Slides



SUPPLEMENT: Scaling Violations

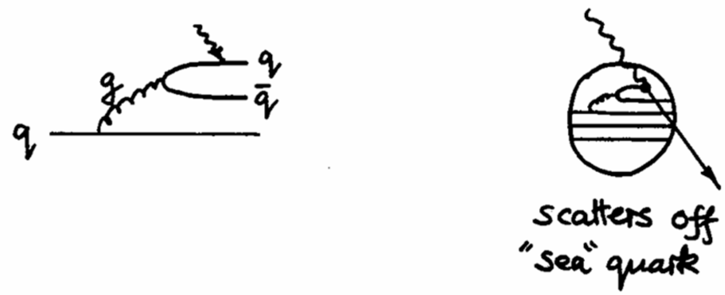
Strong Interactions among Partons

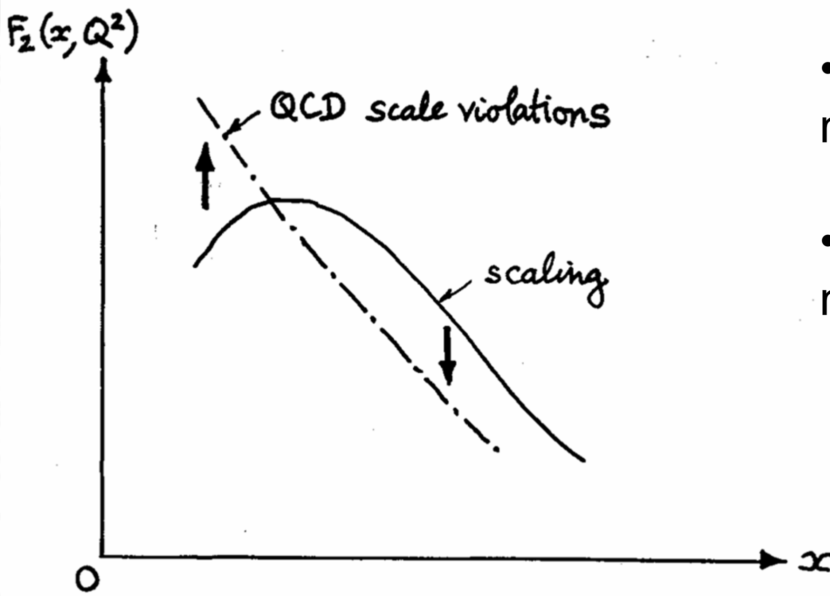


Q² Scaling fails due to these interactions

$$\frac{\partial q(x,Q^2)}{\partial \log Q^2} = \frac{\alpha_s(Q^2)}{2\pi}\int_x^1 \frac{dy}{y}$$

$$\left[P_{qq}\left(\frac{x}{y}\right) q(y,Q^2) + P_{qg}\left(\frac{x}{y}\right) g(y,Q^2) \right]$$





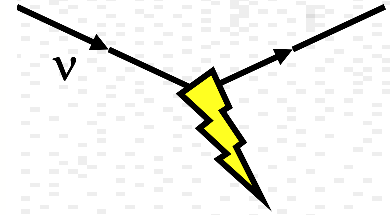
•Pqq(x/y) = probability of finding a quark with momentum x within a quark with momentum y

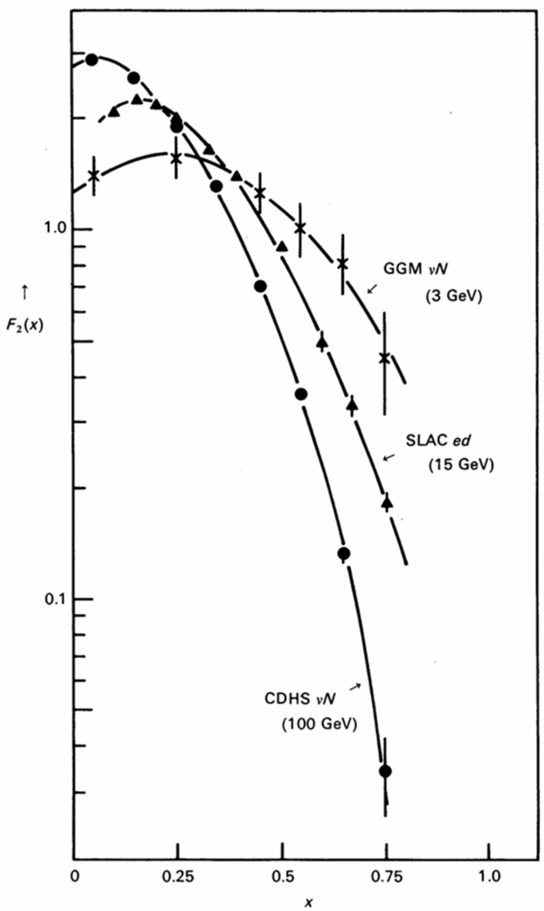
•Pqq(x/y) = probability of finding a q with momentum x within a gluon with momentum y

$$P_{qq}(z) = \frac{4}{3}\frac{1+z^2}{(1-z)} + 2\delta(1-z)$$

$$P_{gq}(z) = \frac{1}{2}\left[z^2 + (1-z)^2\right]$$

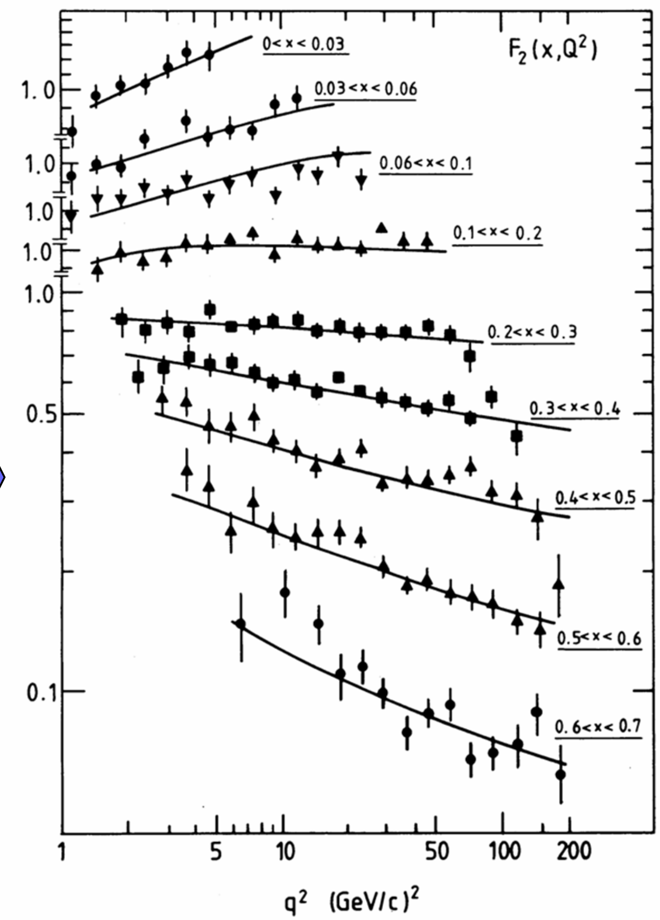
Scaling from QCD





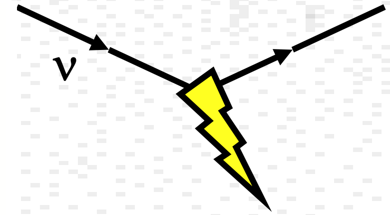
Observed quark distributions vary with Q^2

Scaling well modeled by perturbative QCD with a single free parameter (α_s)



SUPPLEMENT: NuTeV Measurement of Strange Sea

Neutrino Dilepton Events



- Rate depends on:
  - d, s quark distributions, $|V_{cd}|$
  - Semi-leptonic branching ratios of charm
  - Kinematic suppression and fragmentation

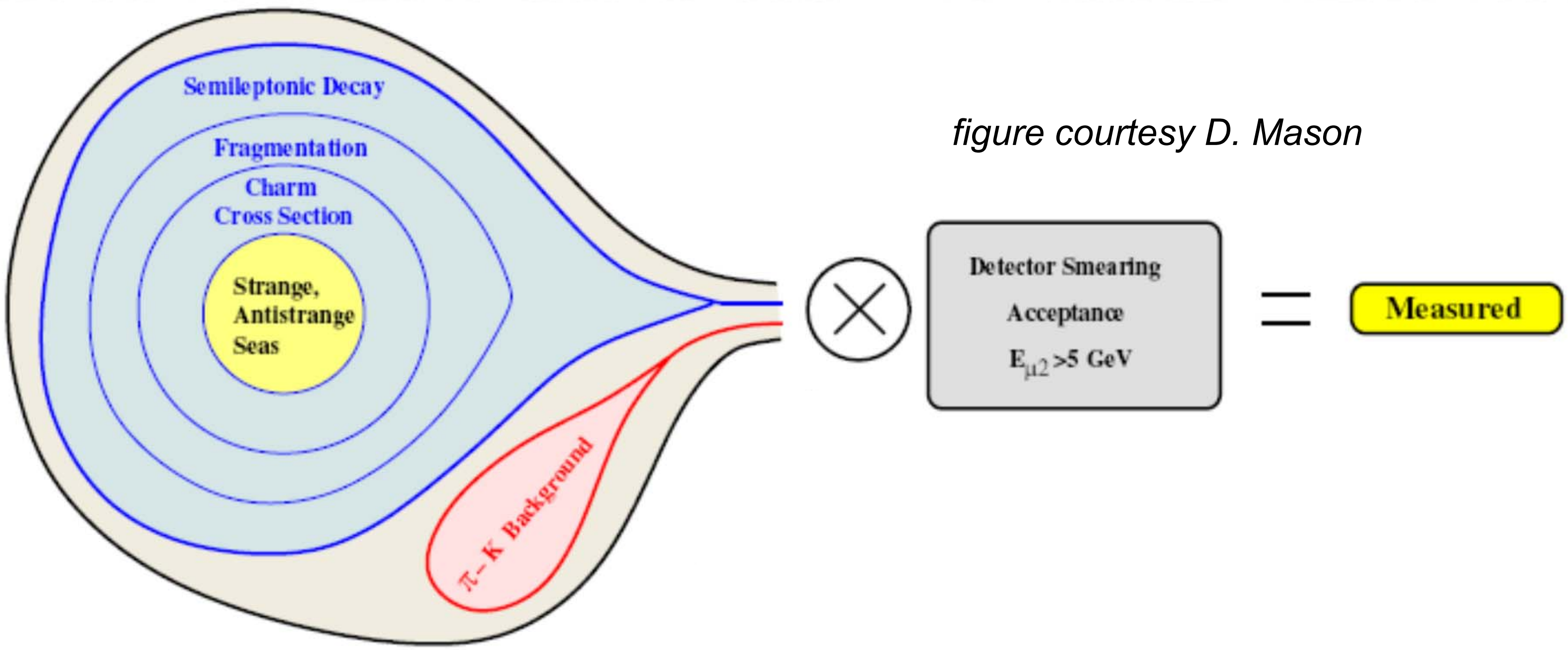
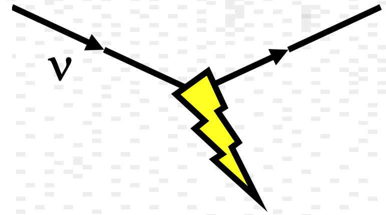
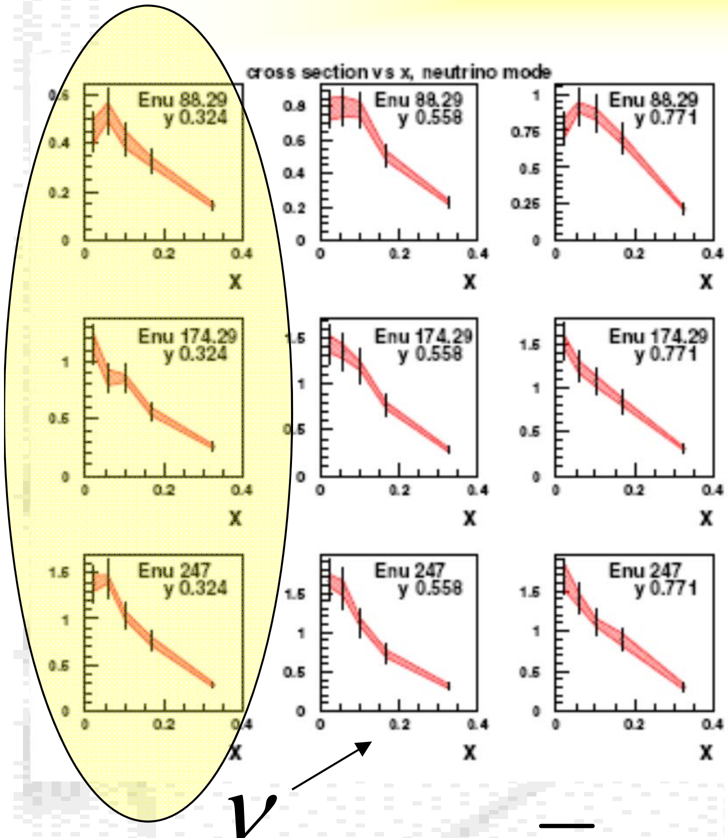


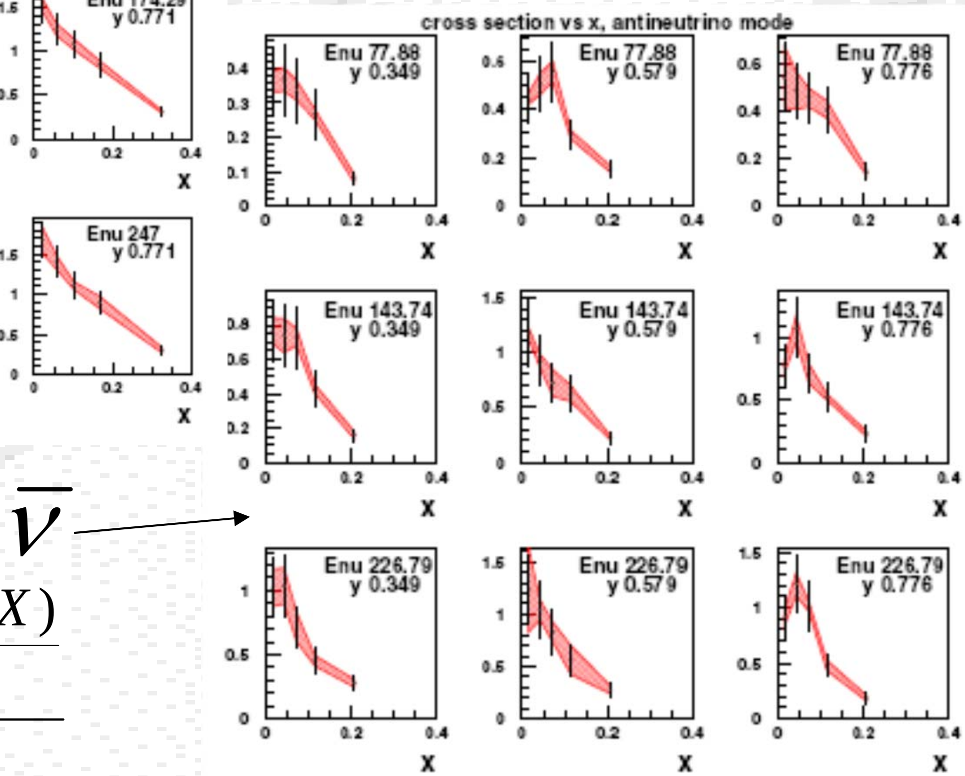
figure courtesy D. Mason

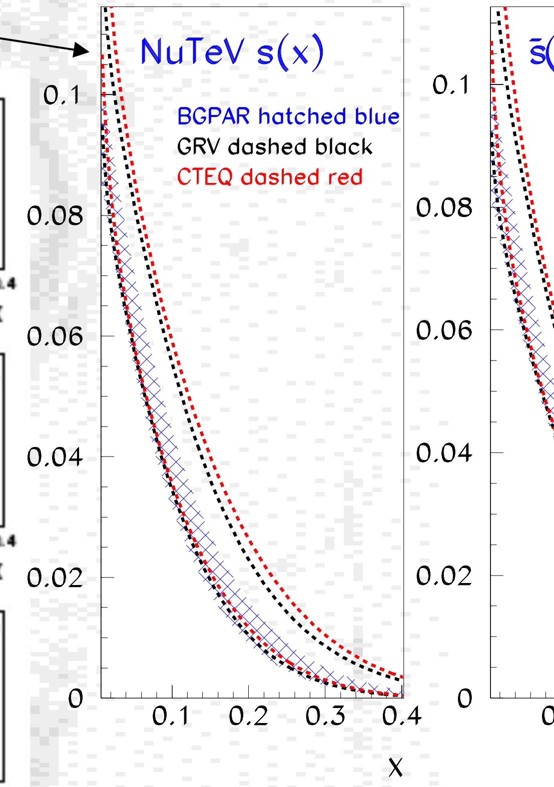
NuTeV Dimuon Sample



- Lots of data!
- Separate data in energy, x and y (inelasticity)
  - Energy important for charm threshold, m_c
  - x important for s(x)

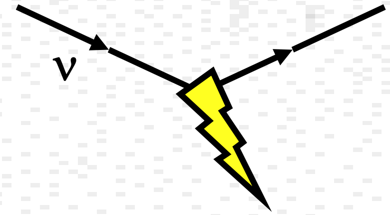




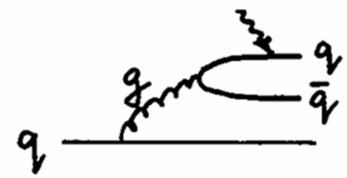


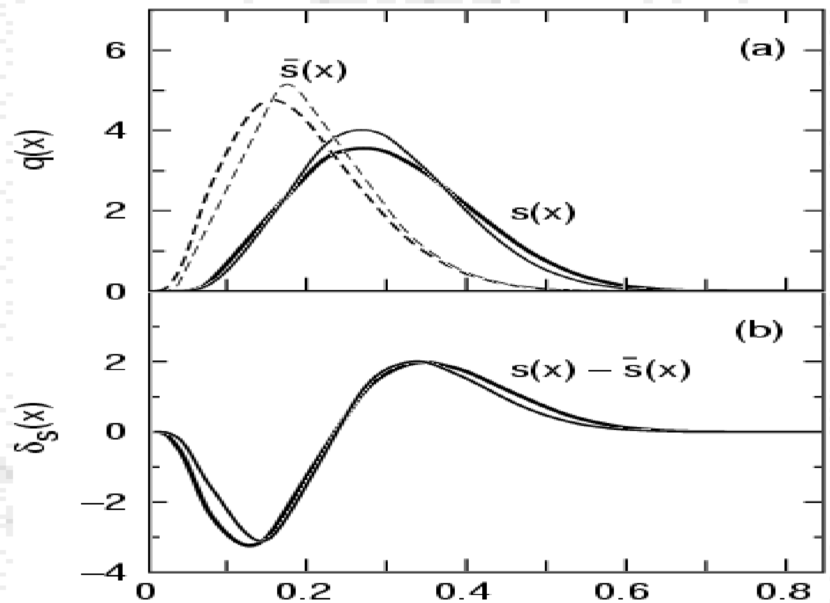
$$\frac{\pi \times \dfrac{d^2\sigma(\nu N \to \mu\mu X)}{dxdy}}{G_F^2 M_N E_\nu}$$

QCD at Work: Strange Asymmetry?



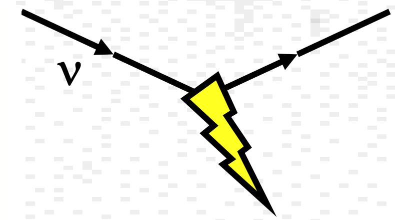
- An interesting aside…
  - The strange sea can be generated perturbatively from g→s+sbar.
  - BUT, in perturbative generation the momenta of strange and anti-strange quarks is equal
    - o well, in the leading order splitting at least. At higher order get a vanishingly small difference.
  - SO s & sbar difference probe non-perturbative ("intrinsic") strangeness
    - o Models: Signal&Thomas, Brodsky&Ma, etc.





(Brodsky & Ma, s-sbar)

NuTeV's Strange Sea



- NuTeV has tested this
  - NB: very dependent on what is assumed about non-strange sea
  - Why? Recall CKM mixing…

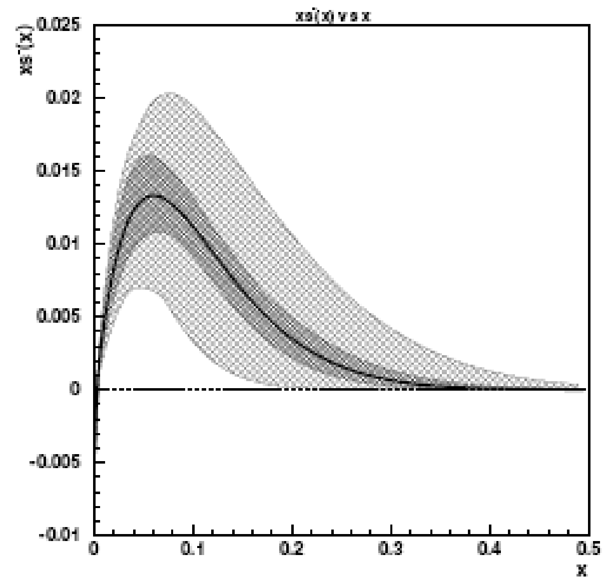
$$V_{cd}\,d(x)+V_{cs}\,s(x)\rightarrow s'(x)$$
$$V_{cd}\,\bar{d}(x)+V_{cs}\,\bar{s}(x)\rightarrow \bar{s}'(x)$$

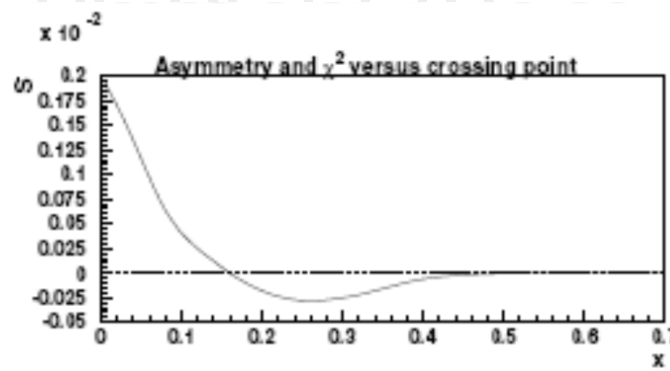
small *big*

  - Using CTEQ6 PDFs…

$$\int dx\left[x\left(s-\bar{s}\right)\right]=0.0019\pm0.0005\pm0.0014$$

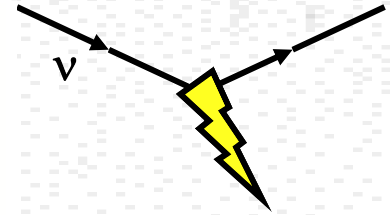
$$\text{c.f., } \int dx\left[x\left(s+\bar{s}\right)\right]\approx 0.02$$

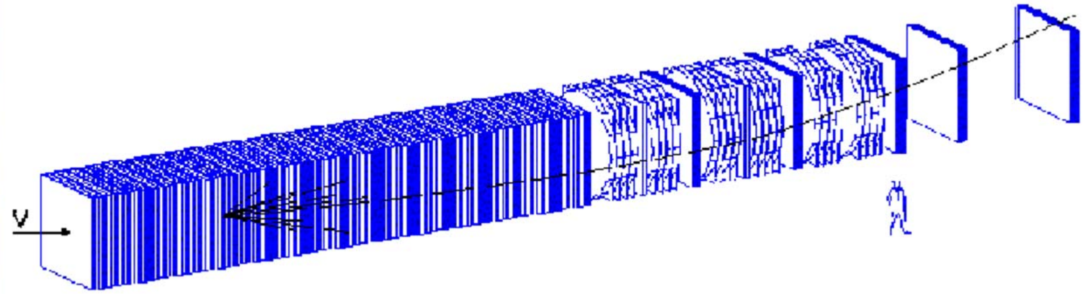


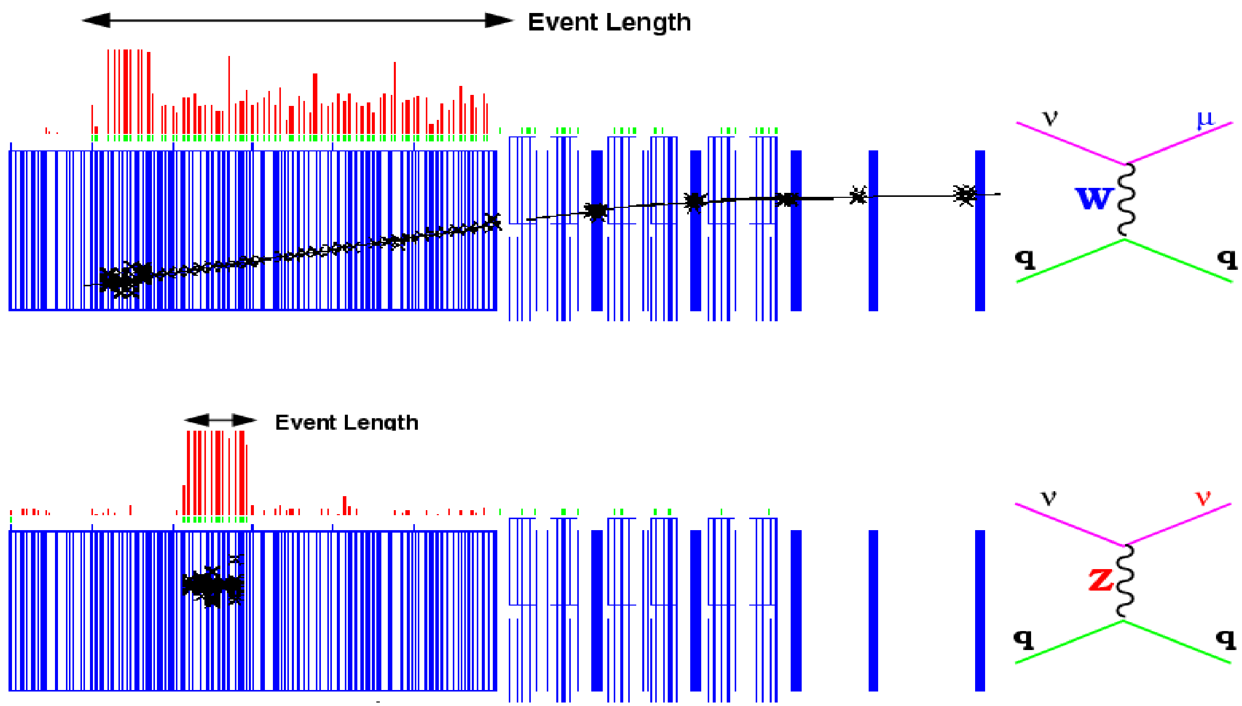


SUPPLEMENT: NuTeV $\sin^2\theta_W$

NuTeV at Work…

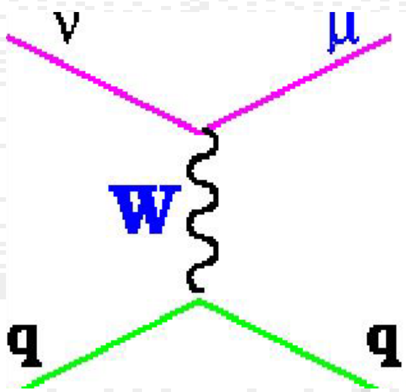




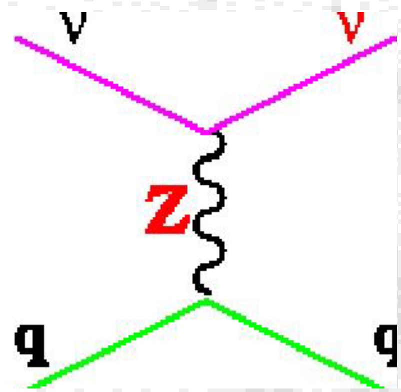


DIS NC/CC Ratio

- Experimentally, it's "simple" to measure ratios of neutral to charged current cross-sections on an isoscalar target to extract NC couplings



W-q coupling is I_3



Z-q coupling is $I_3\text{-}Q\sin^2\theta_W$

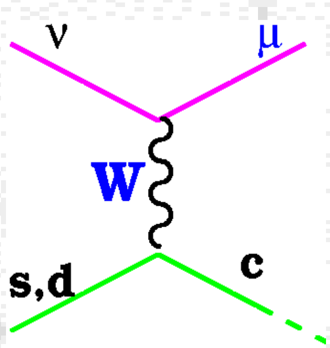
Llewellyn Smith Formulae

$$R^{\nu(\bar{\nu})} = \frac{\sigma_{NC}^{\nu(\bar{\nu})}}{\sigma_{CC}^{\nu(\bar{\nu})}} = \left(\left(u_L^2 + d_L^2\right) + \frac{\sigma_{CC}^{\bar{\nu}(\nu)}}{\sigma_{CC}^{\nu(\bar{\nu})}} \left(u_R^2 + d_R^2\right) \right)$$

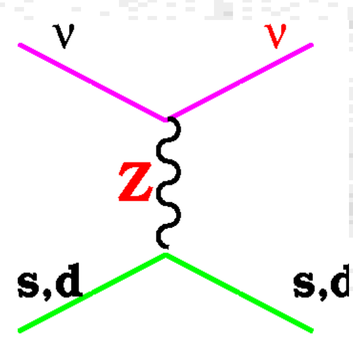
- Holds for isoscalar targets of u and d quarks only
  - Heavy quarks, differences between u and d distributions are corrections
- Isospin symmetry causes PDFs to drop out, even outside of naïve quark-parton model

Lecture Question #6: Paschos-Wolfenstein Relation

Charged-Current



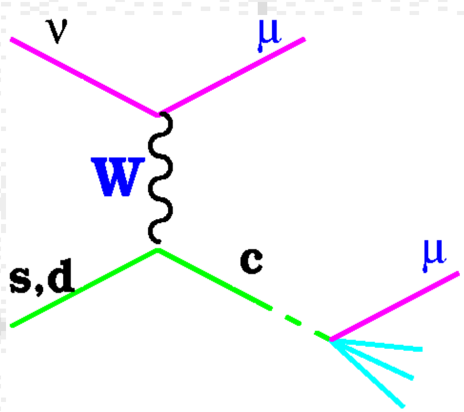
Neutral-Current



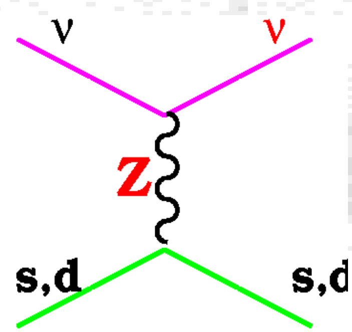
- If we want to measure electroweak parameters from the ratio of charged to neutral current cross-sections, what problem will we encounter from these processes?

Lecture Question #6: Paschos-Wolfenstein Relation

Charged-Current



Neutral-Current

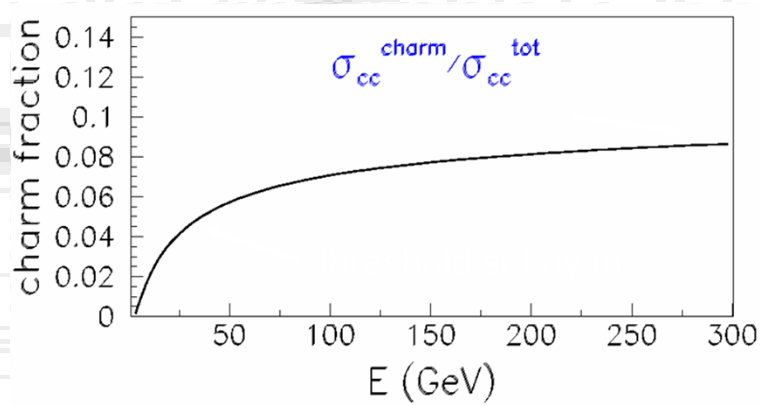


- CC is suppressed due to final state charm quark

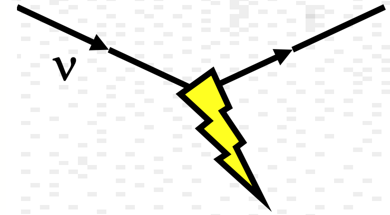
  $\Rightarrow$ Need strange sea and m_c

  - Remember heavy quark mass effect:

$$x \rightarrow \xi = x\left(1 + m_c^2 / Q^2\right)$$



Lecture Question #6: Paschos-Wolfenstein Relation



- The NuTeV experiment employed a complicated design to measure

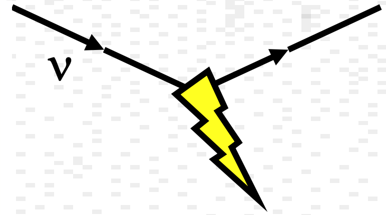
Paschos - Wolfenstein Relation

$$R^{-} = \frac{\sigma_{NC}^{\nu} - \sigma_{NC}^{\bar{\nu}}}{\sigma_{CC}^{\nu} - \sigma_{CC}^{\bar{\nu}}} = \rho^2\left(\tfrac{1}{2} - \sin^2\theta_W\right)$$

- How did this help with the heavy quark problem of the previous question?

Hint: what to you know about the relationship of: $\sigma(\nu q)$ and $\sigma(\bar{\nu}\bar{q})$

Lecture Question #6: Paschos-Wolfenstein Relation



- The NuTeV experiment employed a complicated design to measure

Paschos - Wolfenstein Relation

$$R^{-} = \frac{\sigma_{NC}^{\nu} - \sigma_{NC}^{\bar{\nu}}}{\sigma_{CC}^{\nu} - \sigma_{CC}^{\bar{\nu}}} = \rho^2 \left(\tfrac{1}{2} - \sin^2 \theta_W \right)$$

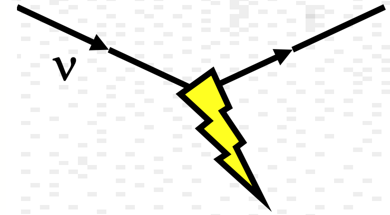
- How did this help with the heavy quark problem of the previous question?

$$\sigma(\nu q) = \sigma(\overline{\nu}\,\overline{q})$$
$$\sigma(\nu \overline{q}) = \sigma(\overline{\nu} q)$$

$$\therefore \sigma(\nu q) - \sigma(\overline{\nu}\,\overline{q}) = 0$$

So any quark-antiquark symmetric part is not in difference, e.g., strange sea.

NuTeV Fit to R^{ν} and $R^{\nu bar}$



- NuTeV result:

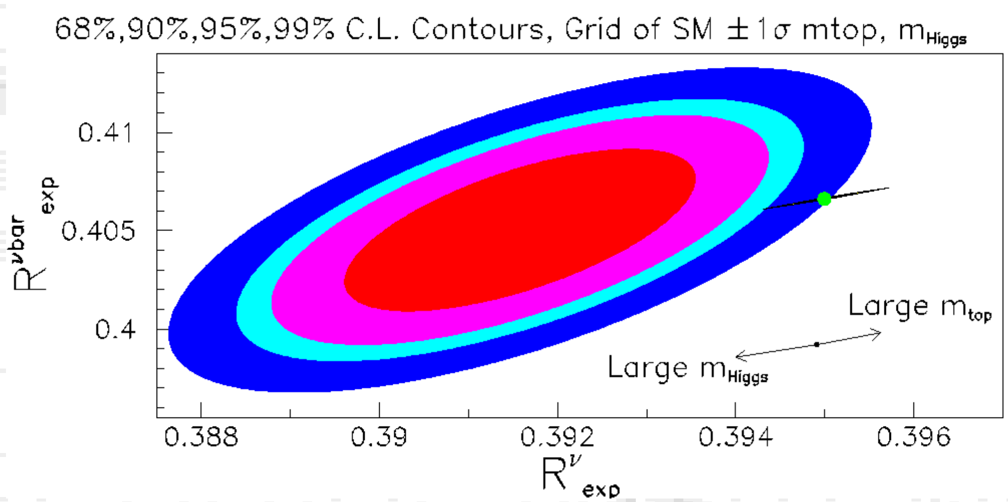
$$\sin^2\theta_W^{(on-shell)} = 0.2277 \pm \pm 0.0013(stat.) \pm 0.0009(syst.)$$
$$= 0.2277 \pm 0.0016$$

(Previous neutrino measurements gave 0.2277 ± 0.0036)

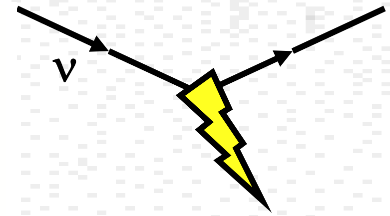
- Standard model fit (LEPEWWG): 0.2227 ± 0.00037

A 3σ discrepancy…

$$R^{\nu}_{\mathrm{exp}} = 0.3916 \pm 0.0013$$
$$(SM: 0.3950) \quad \Leftarrow 3\sigma\ difference$$
$$R^{\bar{\nu}}_{\mathrm{exp}} = 0.4050 \pm 0.0027$$
$$(SM: 0.4066) \quad \Leftarrow Good\ agreement$$



NuTeV Electroweak: What does it Mean?



- If I knew, I'd tell you.
- It could be BSM physics. Certainly there is no exclusive of a Z' that could cause this. But why?
- It could be the asymmetry of the strange sea…
  - it would contribute because the strange sea would not cancel in
  - but it's been measured; not anywhere near big enough
- It could be very large isospin violation
  - if $d_p(x) \neq u_n(x)$ at the 5% level… it would shift charge current (normalizing) cross-sections enough.
  - no data to forbid it. any reason to expect it?