



Majorana vs. Dirac Neutrino and Absolute Neutrino Mass Measurements

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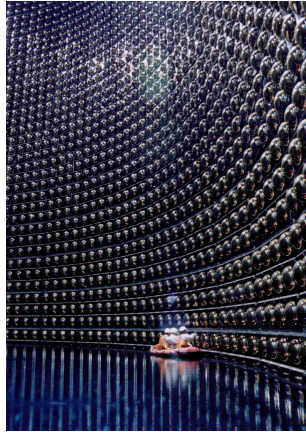
INSS 2013, Beijing, China

Lecture I, 8/13/2013

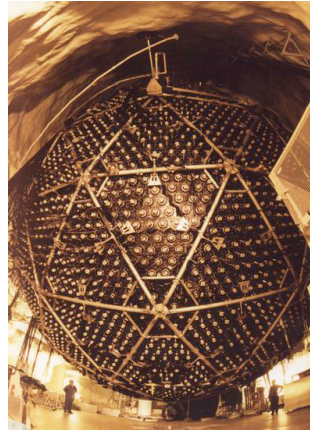
Lecture Outline

- Lecture 1: Overview
 - Brief review of theory of neutrino mass
 - Discussions on experimental ideas of how to distinguish Majorana vs. Dirac neutrinos
 - Discussions on experimental ideas of measuring absolute neutrino mass (direct mass measurement, double beta decay and cosmological constraints)
- Lecture 2: Experimental details of weak decay kinematics measurements
- Lecture 3: Experimental details of double beta decay measurements

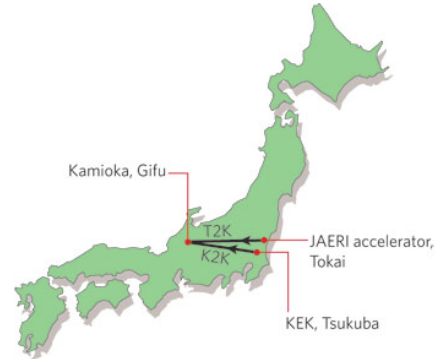
Neutrino Oscillation and Neutrino Mass



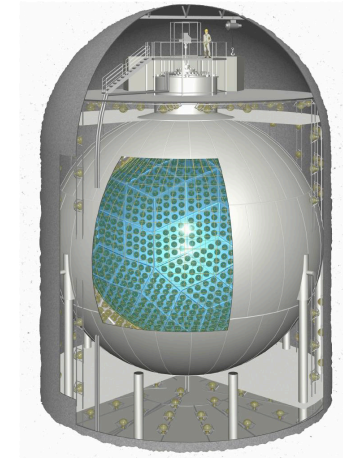
Super-K



SNO



K2K



KamLand

Super-K: atmospheric ν_μ neutrino oscillation

SNO: solar ν_e flavor transformation

K2K: accelerator ν_μ oscillation

Kamland: reactor $\bar{\nu}_e$ disappearance and oscillation

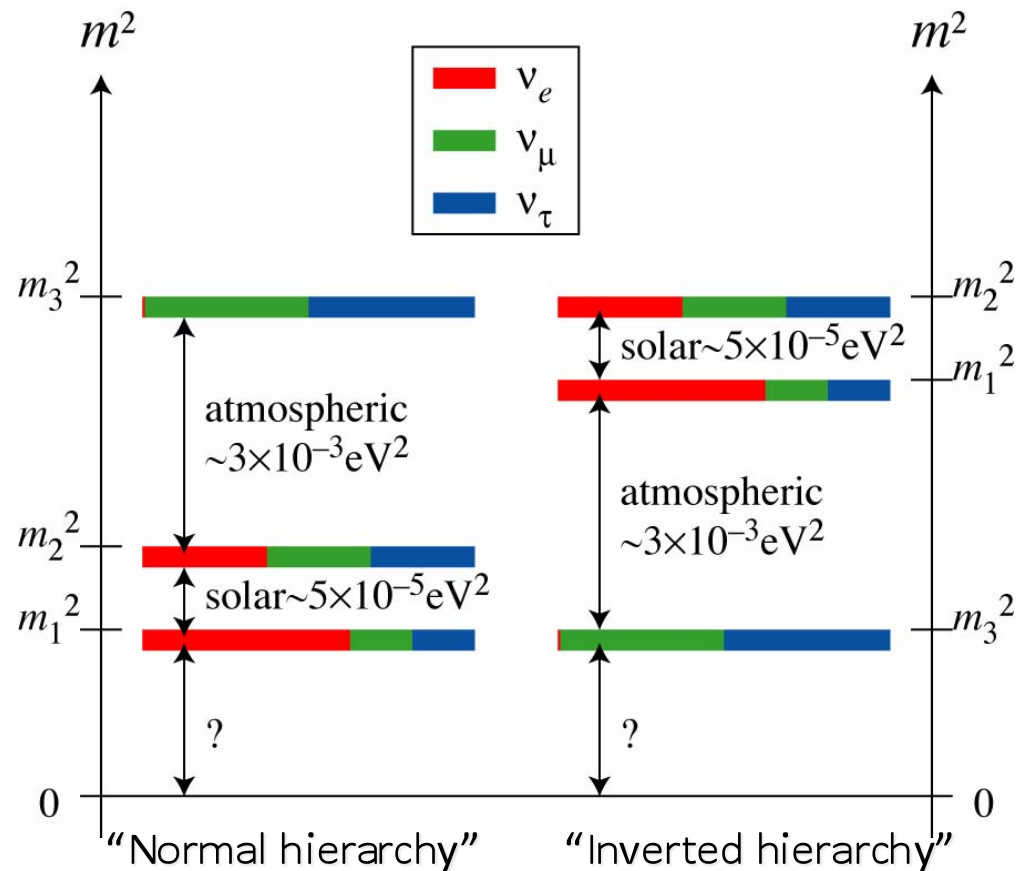
Neutrinos have Mass!!

See lectures by Boris Kayser, Takaaki Kajita and Paul Soler

Unknown Properties of Neutrinos

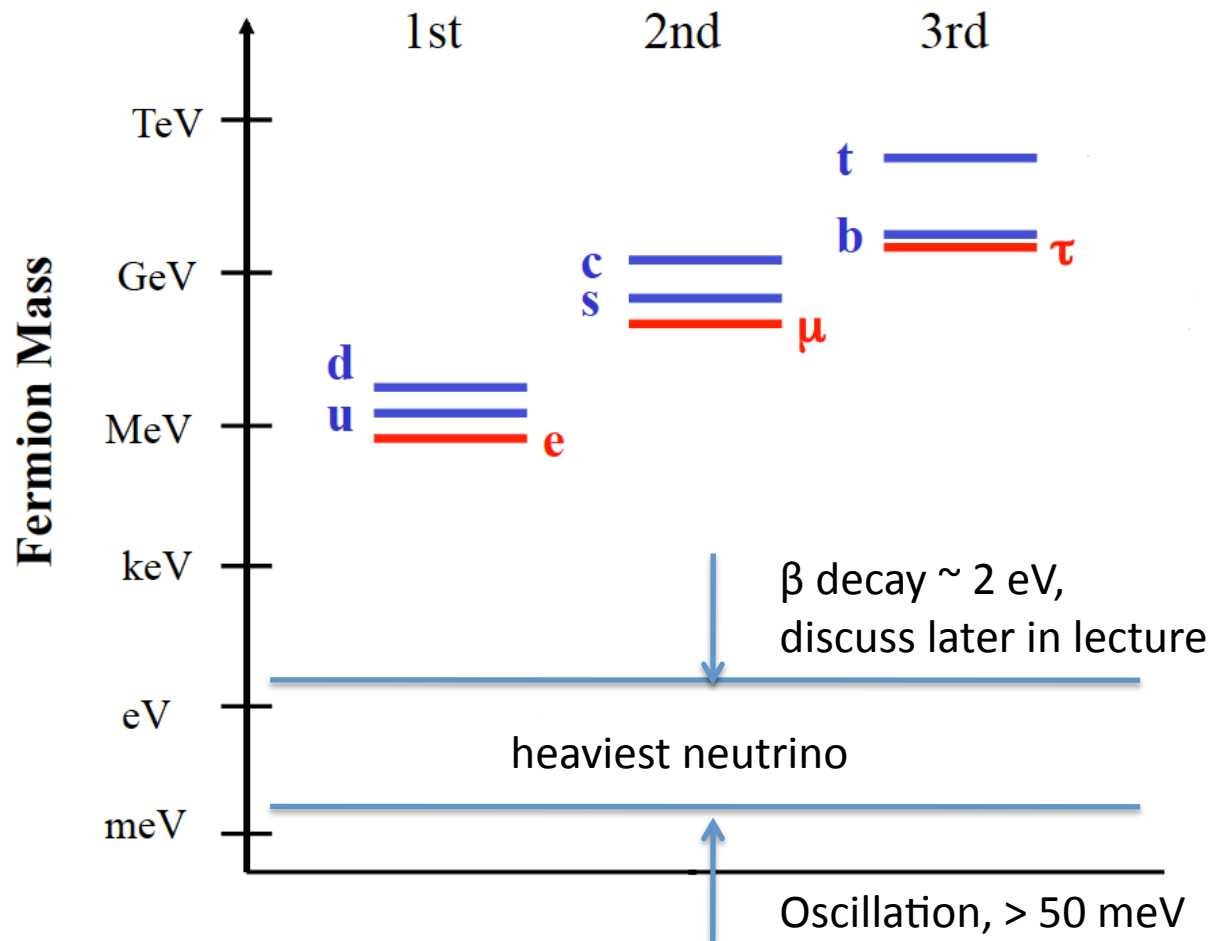
Major Questions in Neutrino Physics

- Majorana particle, (i.e. its own antiparticle)
- Absolute mass scale of neutrinos.
- Mass hierarchy
- CP violation phase
- Anomalies (Sterile neutrinos?)



In the next three lectures, we will try to tackle the first two questions. First let's try to understand why they might be related.....

Spin 1/2 Fermion Mass Spectrum



- Quark sector has ~ 1 -2 decade mass gap across doublet
- Lepton sector has 6+ decades mass gap across doublet

Why?

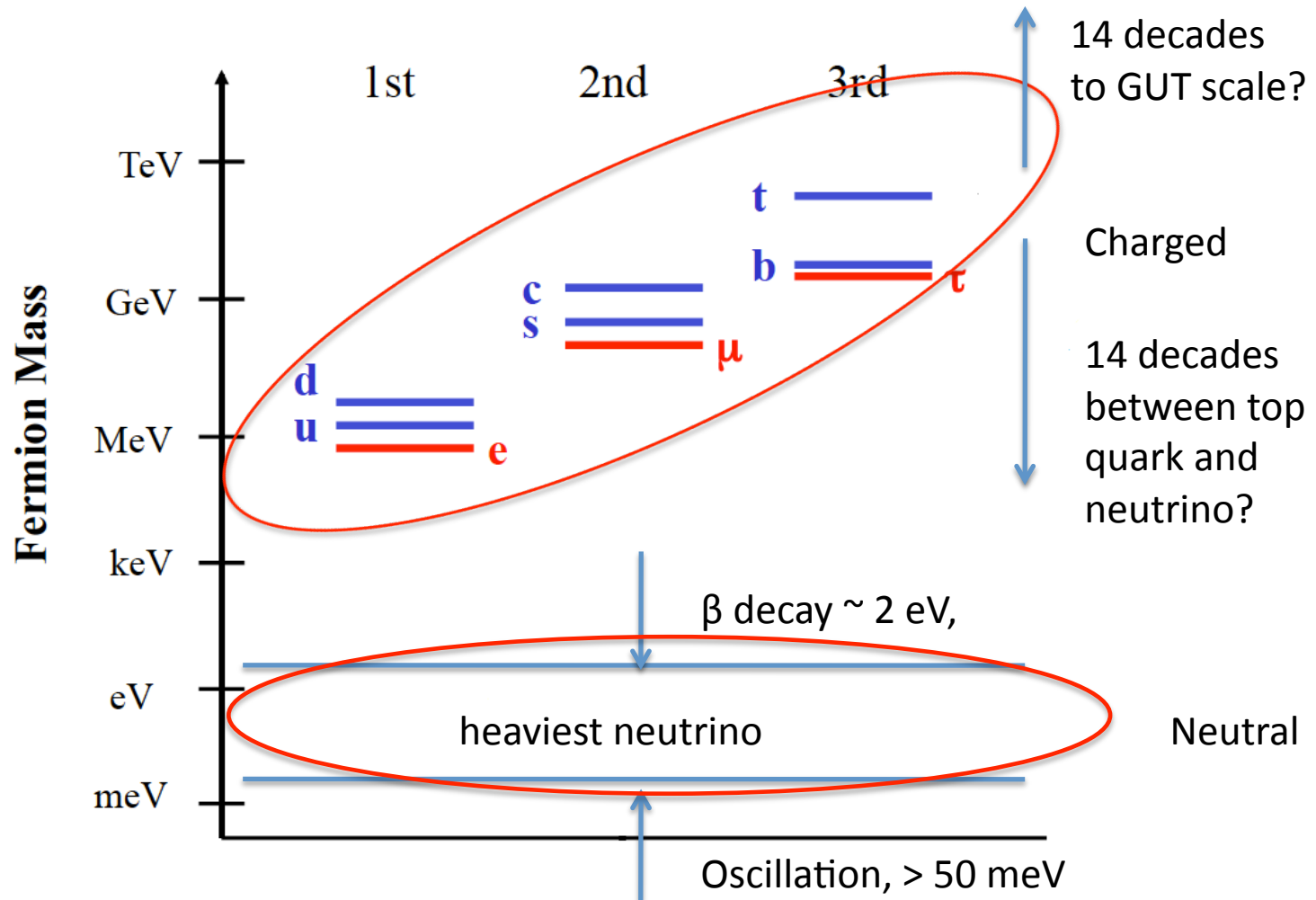
Properties of Spin 1/2 Fermions

		Generation		
		1 st	2 nd	3 rd
Charge	1	e^+	μ^+	τ^+
	2/3	u	c	t
	1/3	\bar{d}	\bar{s}	\bar{b}
	0	$\nu_e (\bar{\nu}_e?)$	$\nu_\mu (\bar{\nu}_\mu?)$	$\nu_\tau (\bar{\nu}_\tau?)$
	-1/3	d	s	b
	-2/3	\bar{u}	\bar{c}	\bar{t}
	-1	e^-	μ^-	τ^-

What about antiparticles?

Neutrinos are the only neutral spin 1/2 fermions!

Spin 1/2 Fermion Mass Spectrum



Perhaps neutrinos are very different from other fermions, such as a Majorana particle?

Dirac vs. Majorana Particles



$$(i\gamma^\mu \partial_\mu - m)\psi = 0$$

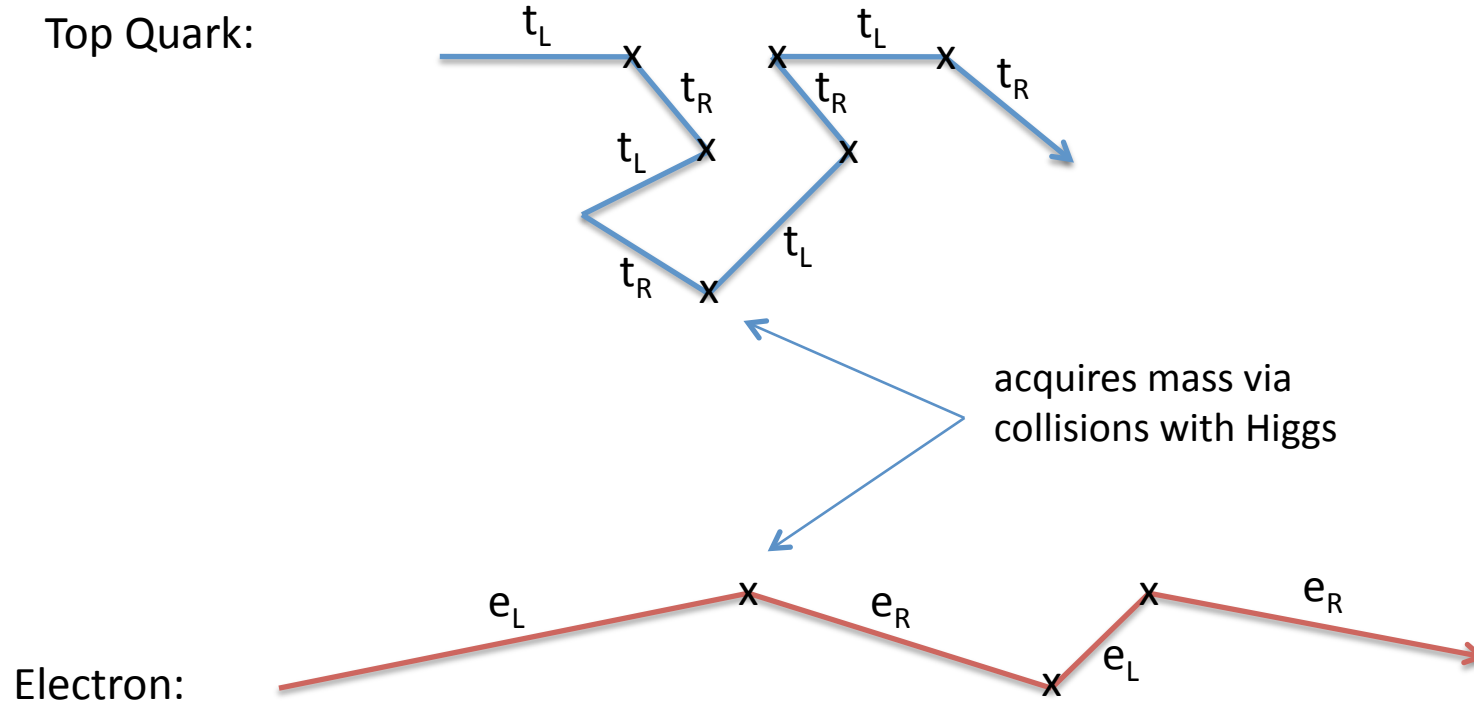
In 1928, Dirac wrote down his famous equation that describes the electrons. The solutions are complex fields, which means there are always a particle and antiparticle pairs.



$$(i\tilde{\gamma}^\mu \partial_\mu - m)\tilde{\psi} = 0$$

In 1937, Majorana found a modified version of Dirac equation with a set of γ matrices that are purely imaginary, so the solutions to the equation are real fields.

Fermion Higgs Mechanism

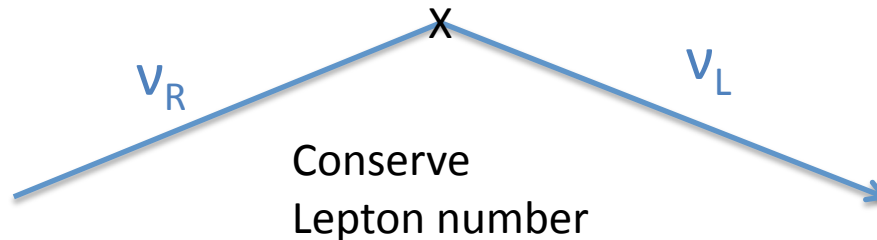


Higgs interact with left and right handed fields

Neutrino Mass Terms

Dirac Neutrino Mass:

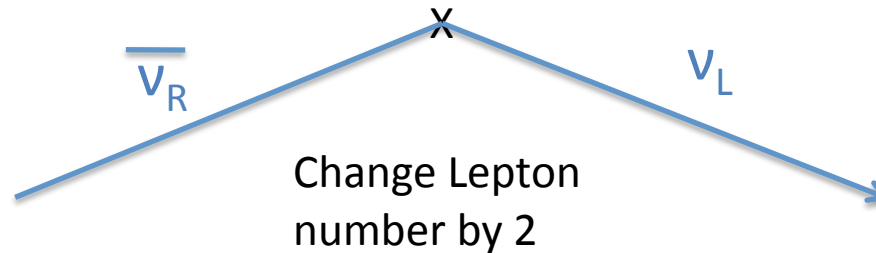
$$-m_\nu(\bar{\nu}_R\nu_L + \text{h.c.})$$



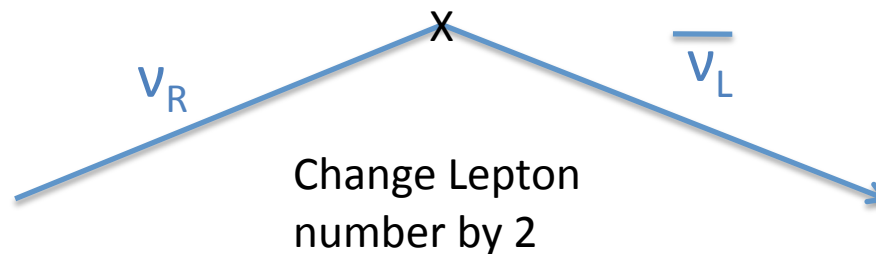
Neutrino mass can be generated by a “Yukawa” coupling to Higgs.

Majorana Neutrino Mass:

$$-\frac{1}{2}\nu_L^T M_{M,L} C^{-1} \nu_L$$



“New Physics”



$$-\frac{1}{2}\nu_R^T M_{M,R} C^{-1} \nu_R$$

See Pascoli's lecture

See-Saw Mechanism

Generating the small neutrino mass via standard Higgs mechanism will require fine-tuning of the Yukawa coupling constants, (10^{-12} compared to top quark)

See-saw mechanism can generate the light neutrino mass in a natural way:

Suppose $m_{\text{Dirac}} \sim 100 \text{ GeV}$, similar to top quark (due to Higgs)

$m_{\text{Majorana}} \sim 10^{15} \text{ GeV}$, (new physics at GUT scale)

Then we will observe two Majorana neutrinos,

$$m_1 \approx m_D^2 / M_{\text{GUT}} \approx 10^{-2} \text{ eV}$$

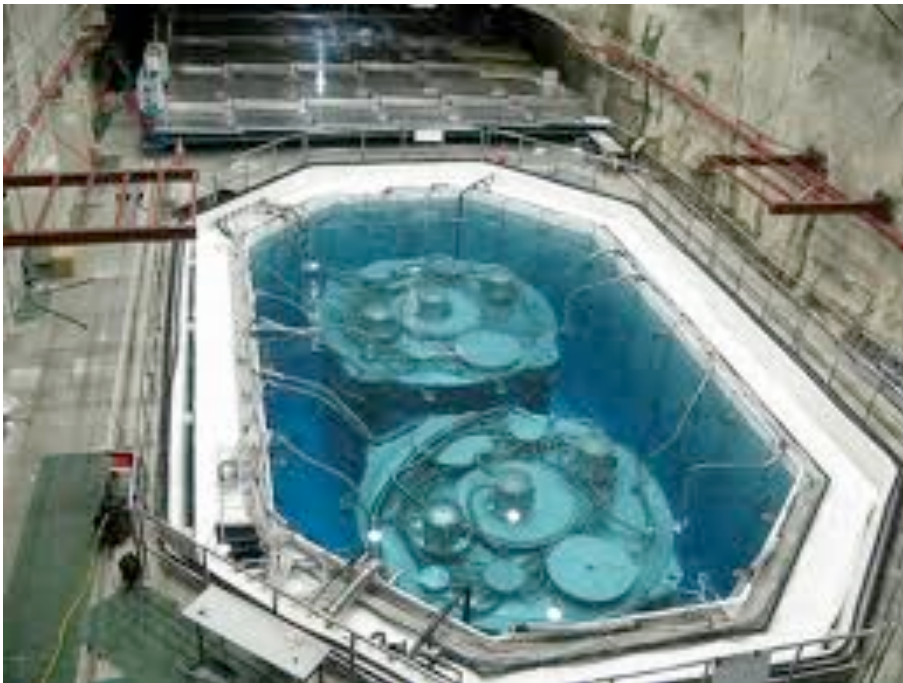
$$m_2 \approx M_{\text{GUT}}$$

See-saw Mechanism predicts that:

- Light Majorana neutrinos
- Heavy GUT scale neutrinos (possible source of leptogenesis)

Whether neutrinos are Majorana or Dirac particles needs to be answered experimentally.....

Don't we already know $\nu \neq \bar{\nu}$?



$$\bar{\nu}_e p \rightarrow e^+ n$$

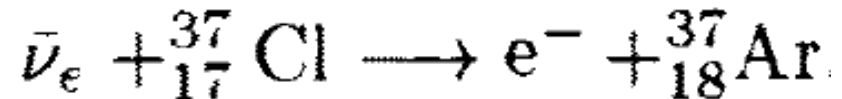
Daya Bay
Antineutrino
Detector

[See also Boris' lecture](#)

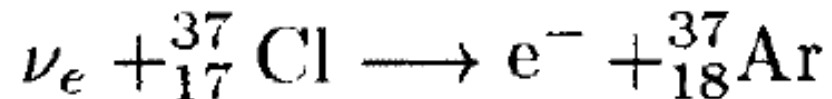
Don't we already know $\nu \neq \bar{\nu}$?

In fact this is what people thought in 1956,

In 1952, Ray Davis found no evidence that anti-neutrinos from the reactor interacted with his Cl detector,



By 1956, it is known that only neutrino can interact with ${}^{37}\text{Cl}$ and produce an electron. The reaction Davis used in his famous solar neutrino experiment.



So, it was quite obvious to almost all physicists at the time that neutrinos must be a Dirac particle, not a Majorana particle.

Not so fast

Discovery of Parity Violation

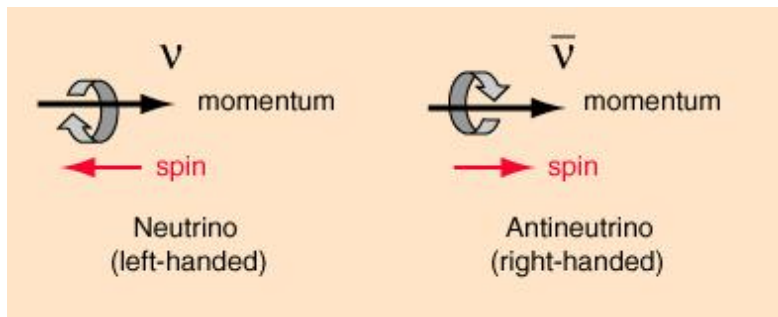
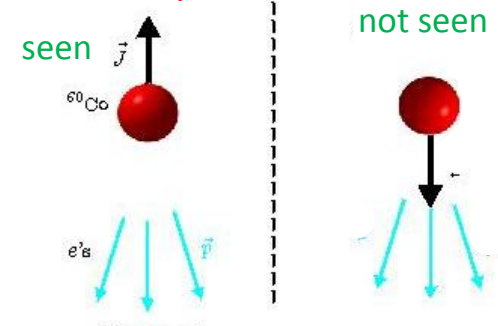


In 1956, when studying theta-tau puzzle, Lee and Yang proposed that parity could be violated in weak interaction.



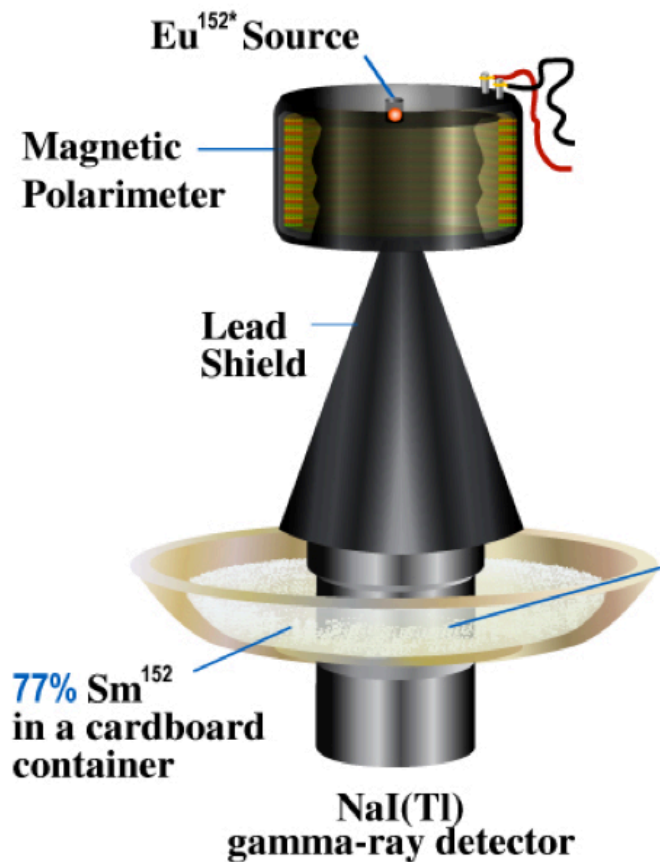
C.S. Wu's famous Co-60 experiment showed that parity is 100% violated.

Betas only emitted opposite to the nuclear spin !



It's soon realized that Davis' results could only distinguish the helicity of the particle, not whether or not they are Majorana particle.

Measurement of Neutrino Helicity (a digression)



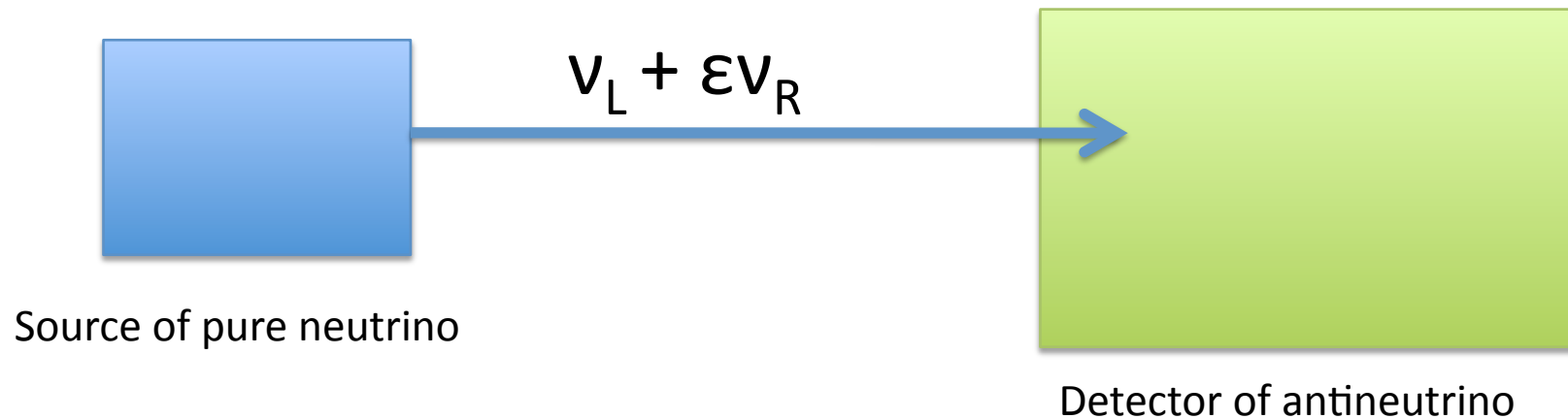
- Eu^{152} ($I=0$) undergoes electron capture. The helicity of the neutrino is transferred to the helicity of recoiling Sm^{152*} .
- Sm^{152*} decay quickly, emitting a 963 keV γ ray, and transfer the helicity.
- How do we find out the momentum of the neutrino? Using resonance absorption, only γ ray emitting in the same direction of the recoiling Sm can be resonantly absorbed.
- How do we measure the gamma ray helicity? Use an magnetic analyzer.

Dr. Grodzins' talk at Neutrino 2010

M. Goldhaber, et. al., Phys. Rev., 109, 1015, (1958)

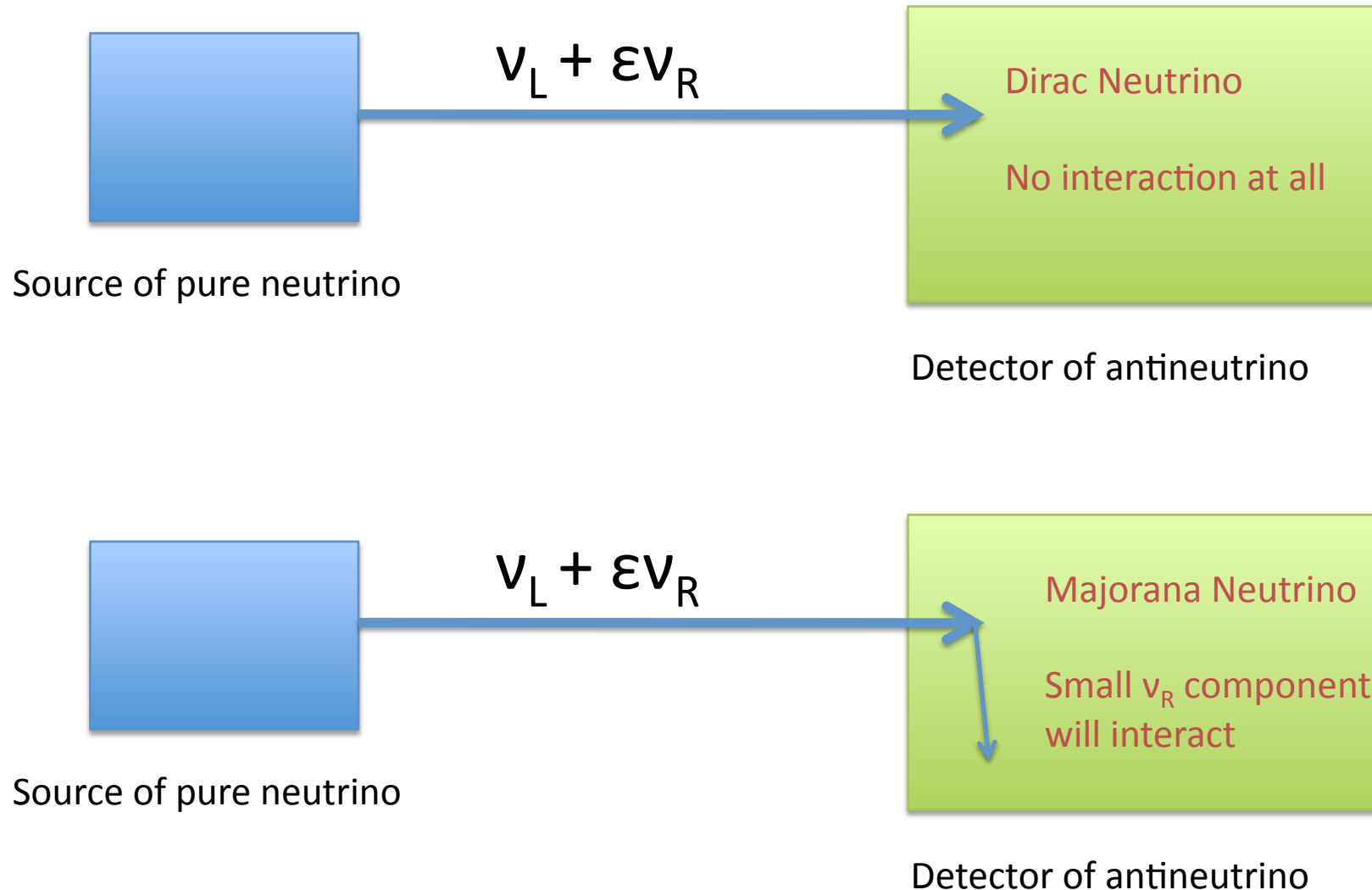
One of the most beautiful experiment in the twenties century.

Testing for Majorana Neutrino

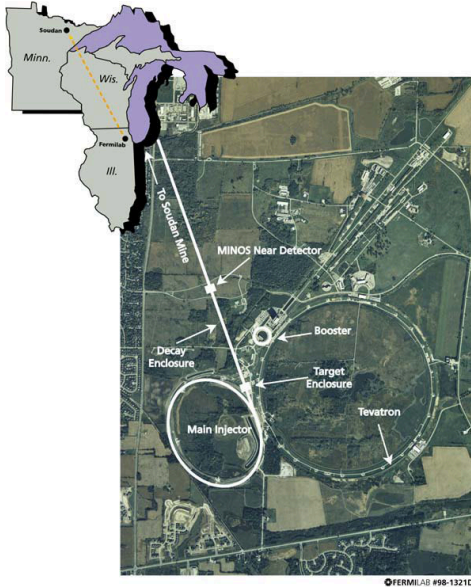


If neutrino is massless, the beam will be pure left handed neutrinos. However, we know that neutrino has mass, so a small amount of neutrinos are created with the wrong handedness (helicity).

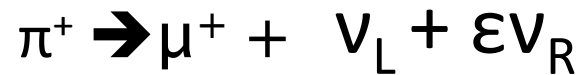
Testing for Majorana Neutrino



Testing for Majorana Neutrino



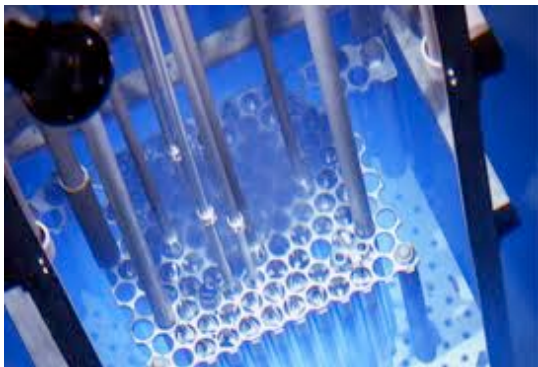
MINOS Beam



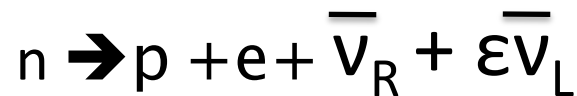
- ν_L produces μ^- , ν_R if Majorana can produce μ^+
- $\epsilon \sim m_\nu/E_\nu \sim 10^{-9}$
- μ^+ production is suppressed by $\epsilon^2 \sim 10^{-18}$
- event rate and false positive are clear limitations



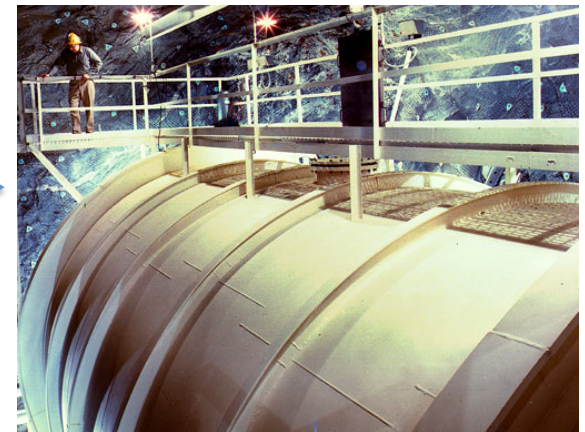
MINOS Detector, can measure muon charge



Reactor antineutrino

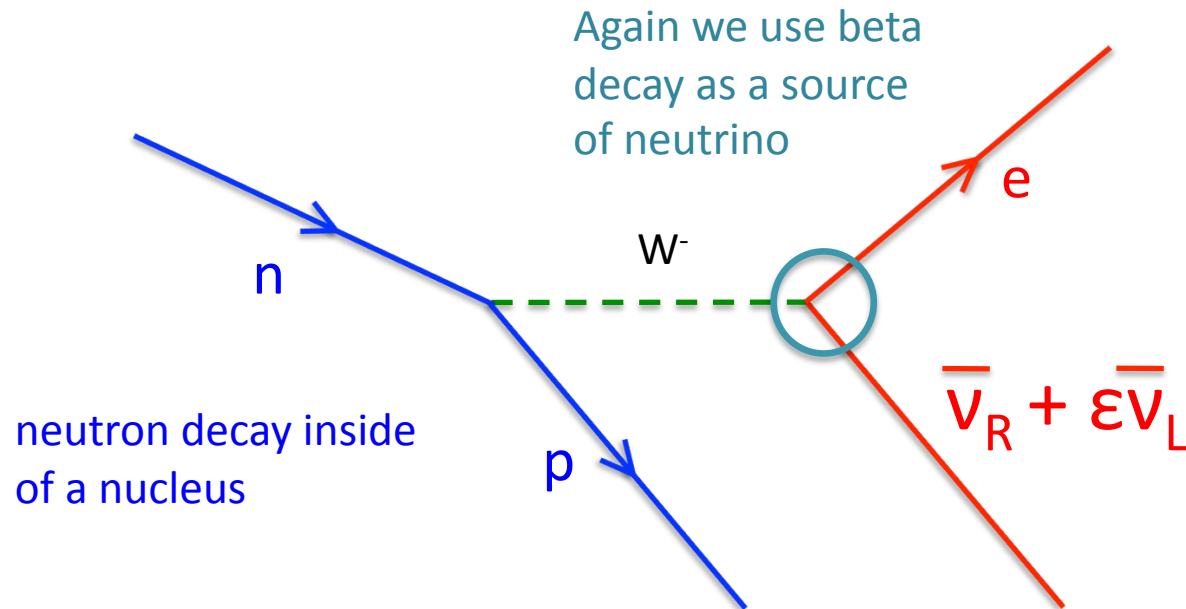


- $\epsilon \sim m_\nu/E_\nu \sim 10^{-6}$
- similar chiral suppression problems and solar neutrino background will dominate

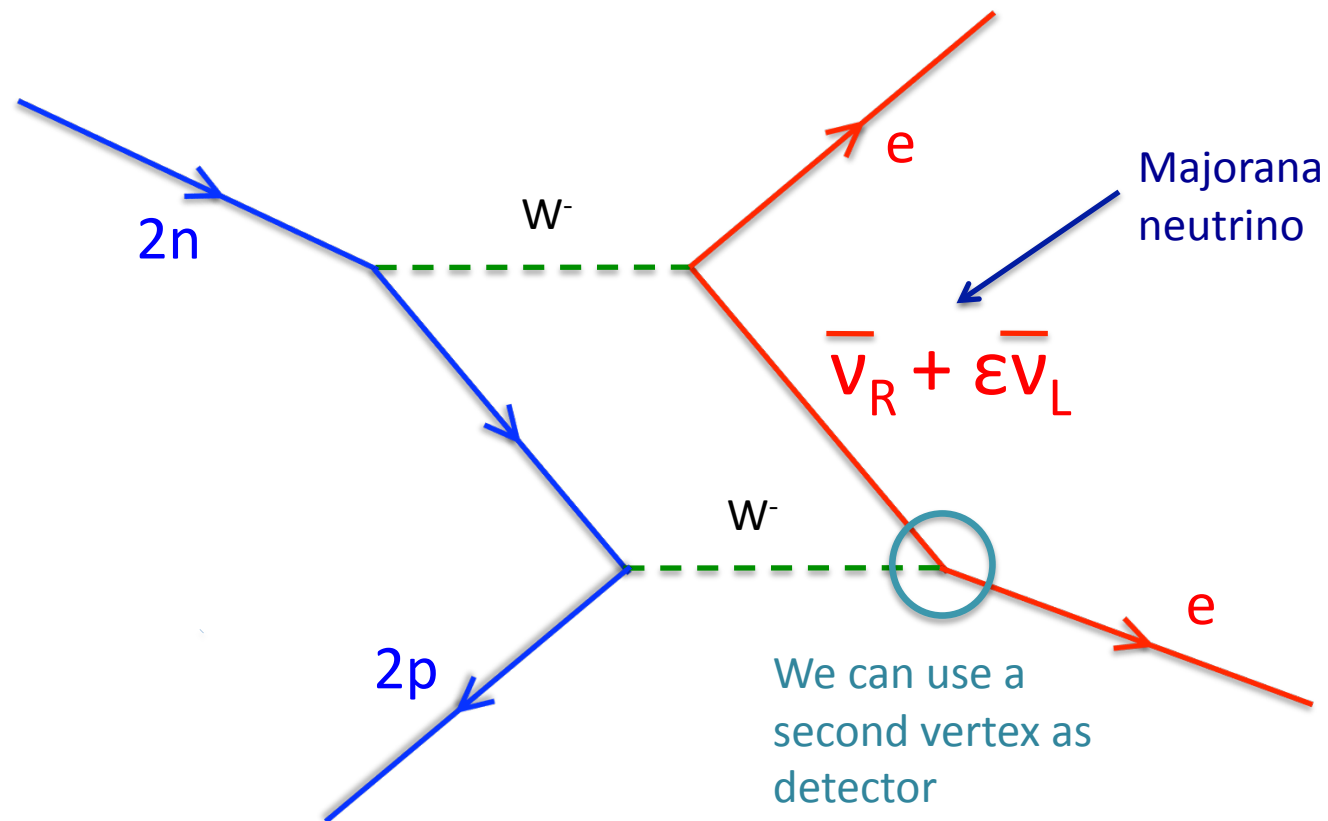


Davis' Cl detector

Neutrinoless Double Beta Decay

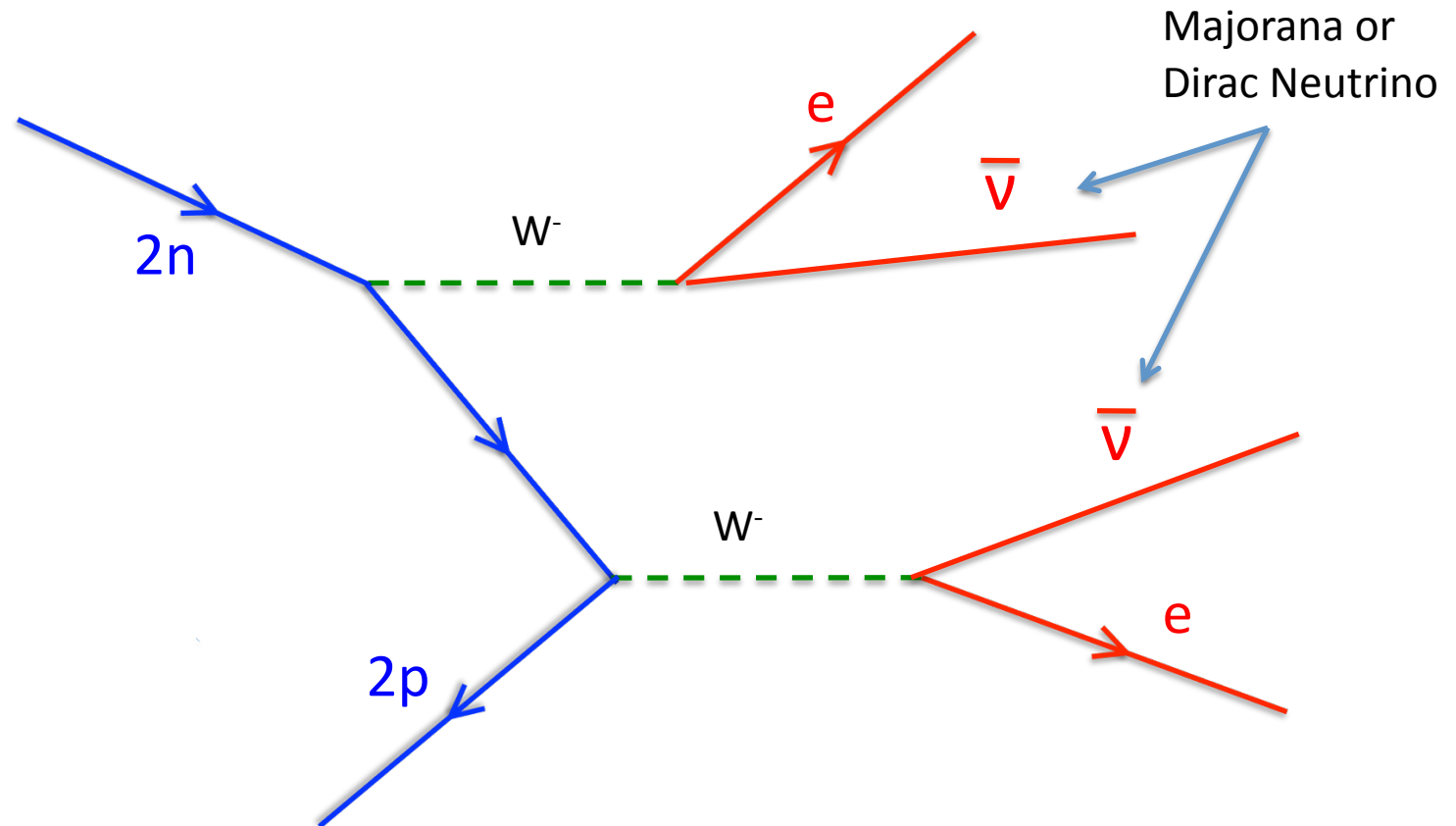


Neutrinoless Double Beta Decay



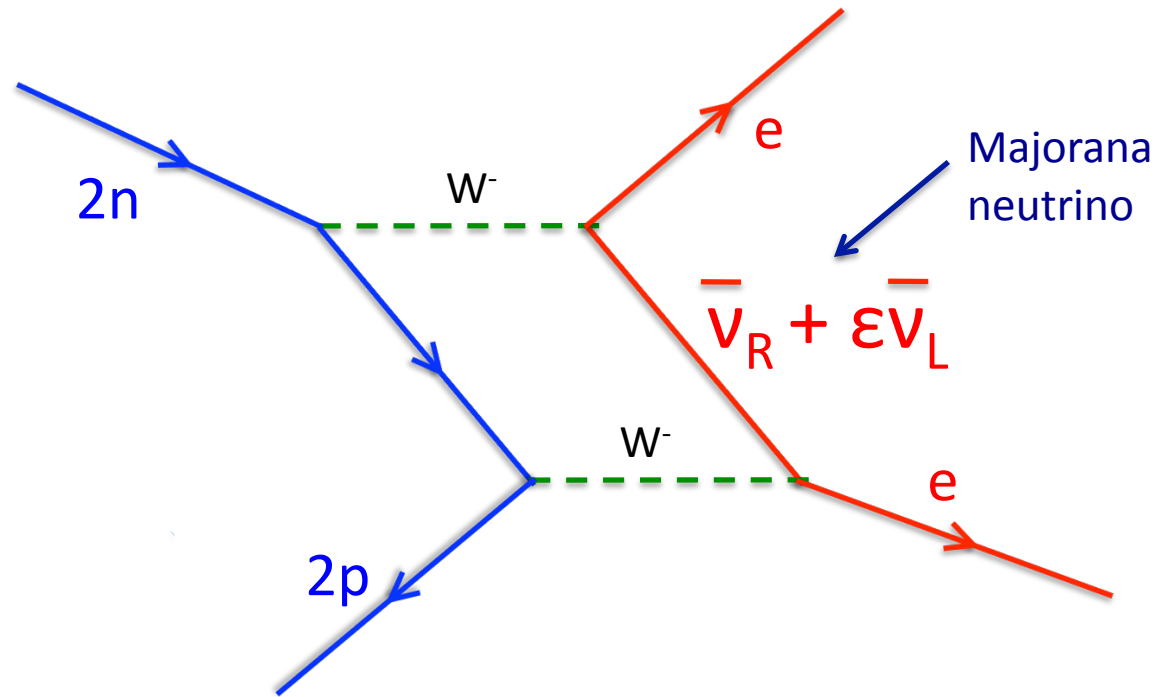
This process is only allowed if neutrino is a Majorana particle, and forbidden if neutrino is a Dirac particle.

Two Neutrino Double Beta Decay



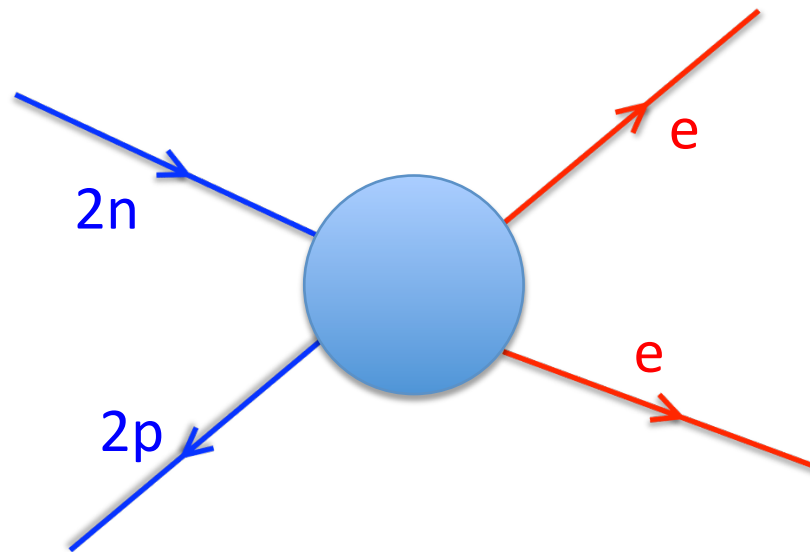
There is another kind of double beta decay process where two neutrinos gets emitted along with the two electrons. This process does not tell us whether or not neutrinos are Majorana or Dirac particles.

Why Neutrinoless Double Beta Decay Might Work?



- Eliminate the uninteresting events (10^{14}) where the neutrino would not interact with the second vertex.
- Chiral suppression still exists, and event rate is extremely low. Fortunately, Avogadro's number is large and every nucleus can both be the source and detector.
- Two neutrino double beta decay is a second order process, so rate is also low, therefore reducing background. False positive from radioactive background still an issue. (will be discussed later)

Other Mechanisms for Neutrinoless Double Beta Decay

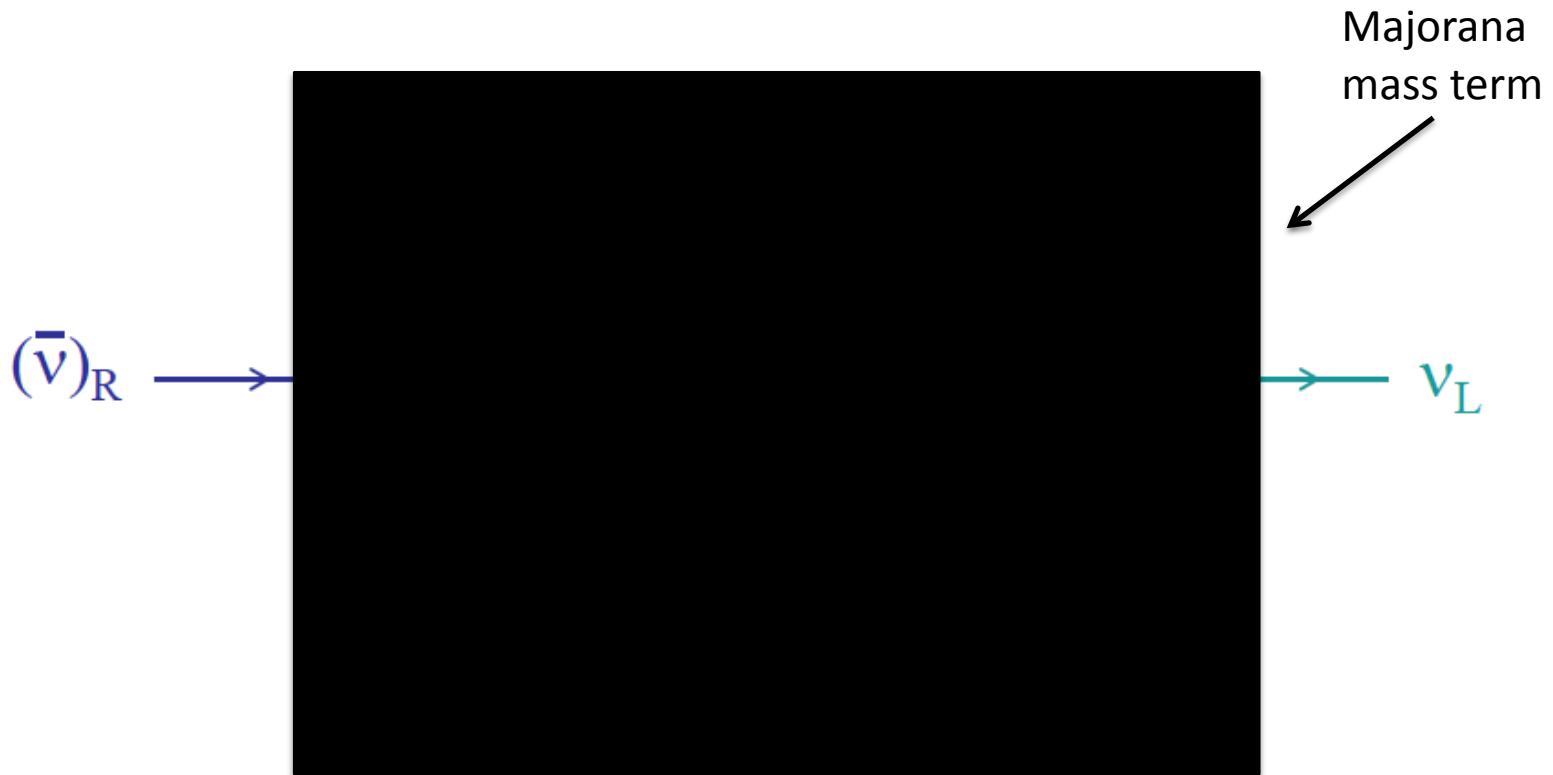


There are many possible mechanisms for neutrinoless double beta decay, such as right handed weak current, leptoquark, supersymmetry, ect ..

Does the observation neutrinoless double beta decay necessarily prove that neutrinos are Majorana particles?

Black Box Theorem

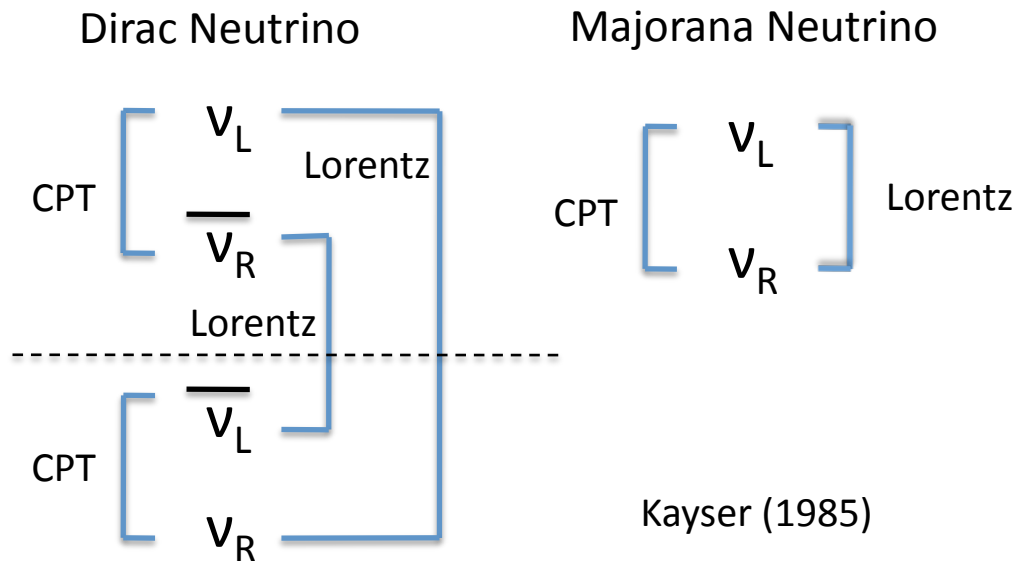
Schechter and Valle,82



No matter what underlying mechanism causes the $0\nu\beta\beta$, by adding standard model processes, we will get at Majorana mass term, therefore guaranteeing that neutrinos are massive Majorana particles.

Summary on Majorana/Dirac Measurements

- At first glance, it seems that it would be easy to distinguish Majorana and Dirac neutrinos, one has two distinct states and the other has four.
- However, parity violation and V-A weak interaction makes accessing the right hand neutrino difficult. In fact, if neutrinos are massless, the left hand and right hand neutrinos will be completely disconnected.



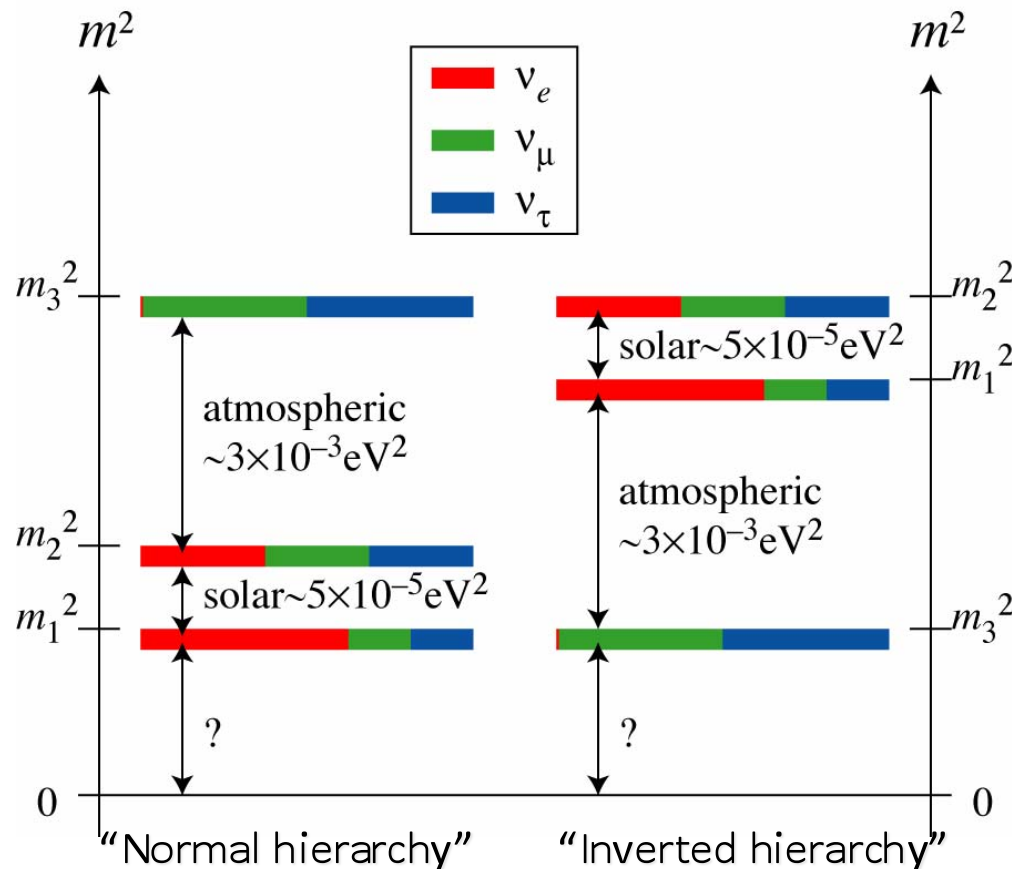
- Most experiments are unrealizable due to the chiral suppression and false positives.

- The smaller the neutrino mass, the more difficult to test Majorana/Dirac.

- Neutrinoless double beta decay is a very promising approach, other ideas?

- See-saw mechanism favors Majorana neutrino, other theoretical ideas?

Neutrino Mass Measurements



Three ways to measure absolute neutrino mass:

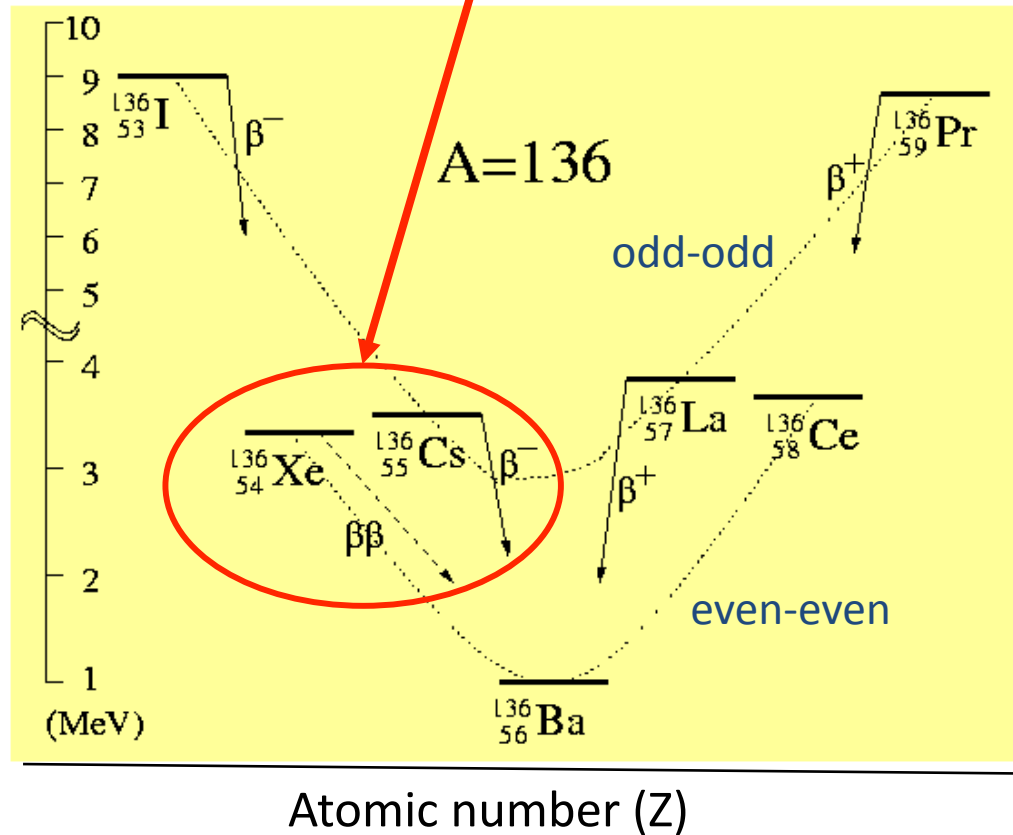
- Direct neutrino mass measurements
- **Double Beta decay**
- Cosmological constraint

Note: Due to neutrino mixing, flavor states are different from mass states.

$$\nu_e = U_{e1}\nu_1 + U_{e2}\nu_2 + U_{e3}\nu_3$$

Double-beta Decay

*a second-order process only detectable
if first order beta decay is energetically forbidden*



- For even atomic mass A, we get two mass curves, odd-odd and even-even.

- δ is a pairing term, negative for Z even and positive for Z odd.

- Double beta decay candidates are always even-even, binding energy greater than two electron mass.

- How about $0\nu\beta^+\beta^+$, or double electron capture?

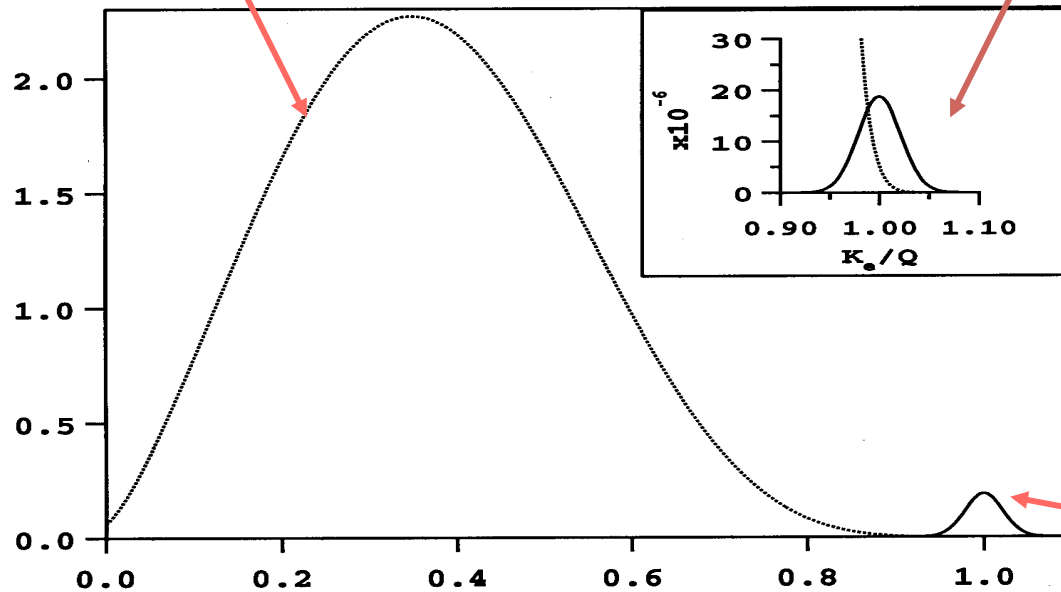
$$M_A(Z, A) = \text{const} + 2b_{\text{sym}} \frac{(A/2 - Z)^2}{A^2} + b_{\text{Coul}} \frac{Z^2}{A^{1/3}} + m_e Z + \delta$$

Semi-empirical mass formula

Double Beta Decay Energy Spectrum

$2\nu\beta\beta$ spectrum
(normalized to 1)

$0\nu\beta\beta$ peak (5% FWHM)
(normalized to 10^{-6})



$0\nu\beta\beta$ peak (5% FWHM)
(normalized to 10^{-2})

Summed electron energy in units of the kinematic endpoint (Q)

The two can be separated in a detector with good energy resolution

Only neutrinoless double beta decay tell us about Neutrino Mass!

Double Beta Decay Nuclei

Candidate	Q (MeV)	Abund.(%)
$^{48}\text{Ca} \rightarrow ^{48}\text{Ti}$	4.271	0.187
$^{76}\text{Ge} \rightarrow ^{76}\text{Se}$	2.040	7.8
$^{82}\text{Se} \rightarrow ^{82}\text{Kr}$	2.995	9.2
$^{96}\text{Zr} \rightarrow ^{96}\text{Mo}$	3.350	2.8
$^{100}\text{Mo} \rightarrow ^{100}\text{Ru}$	3.034	9.6
$^{110}\text{Pd} \rightarrow ^{110}\text{Cd}$	2.013	11.8
$^{116}\text{Cd} \rightarrow ^{116}\text{Sn}$	2.802	7.5
$^{124}\text{Sn} \rightarrow ^{124}\text{Te}$	2.228	5.64
$^{130}\text{Te} \rightarrow ^{130}\text{Xe}$	2.533	34.5
$^{136}\text{Xe} \rightarrow ^{136}\text{Ba}$	2.479	8.9
$^{150}\text{Nd} \rightarrow ^{150}\text{Sm}$	3.367	5.6

- About 35 naturally occurring isotopes, but only dozen or so good neutrinoless double beta decay candidates

- Large Q means large phase space and larger abundance usually means cheaper source material, more on these comparisons later

Candidate nuclei with $Q > 2$ MeV

$0\nu\beta\beta$ as Absolute Neutrino Mass Measurements

$$\left(T_{1/2}^{0\nu}\right)^{-1} = G^{0\nu}(Q, Z) |M^{0\nu}|^2 \langle m_{\beta\beta} \rangle^2$$

$0\nu\beta\beta$ decay rate Phase Space Matrix Element Effective Mass

$$\langle m_{\beta\beta} \rangle = \left| \sum_i U_{ei}^2 m_i e^{i\alpha_i} \right|$$

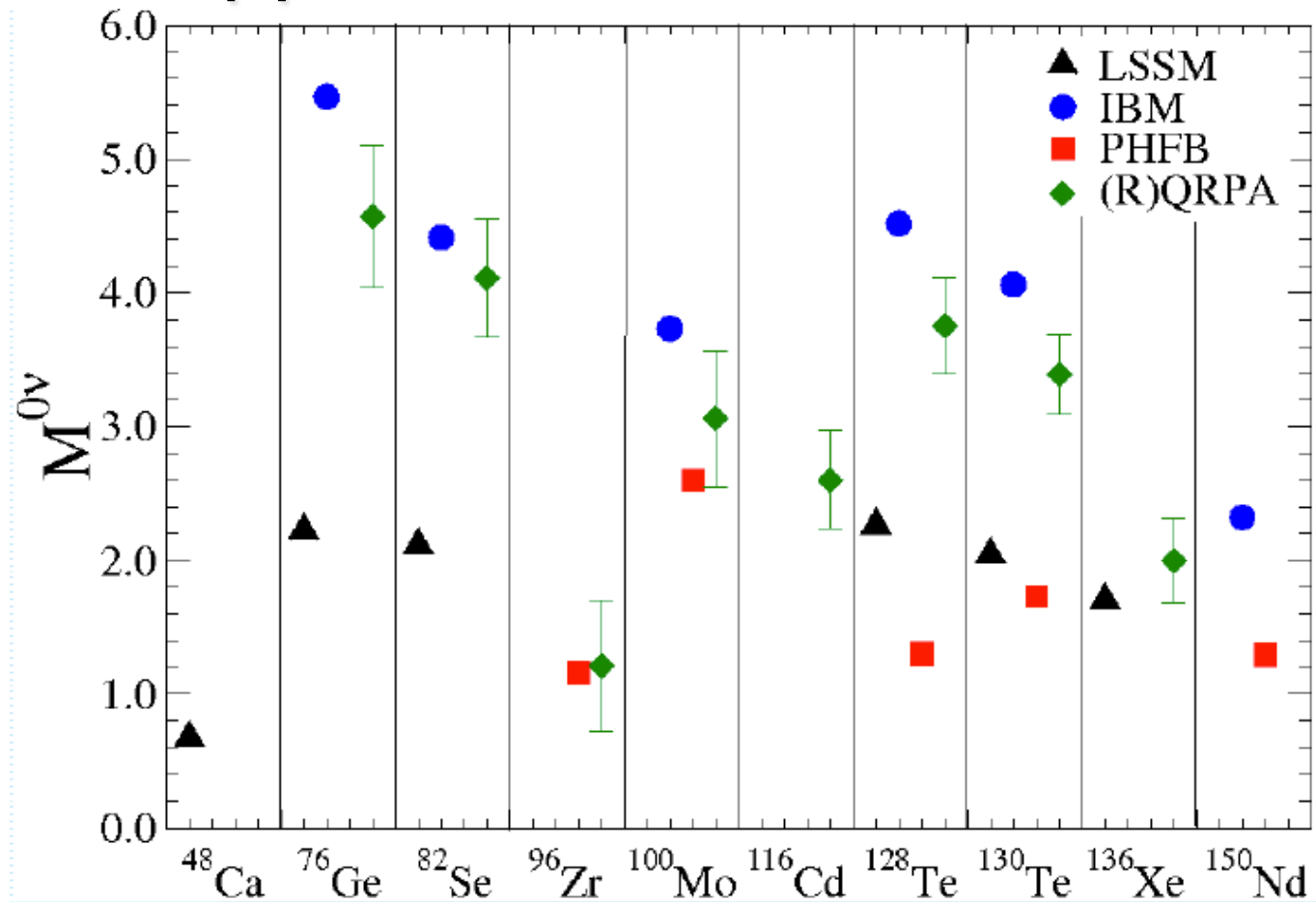
Mixing matrix mass eigenvalues Majorana phase

Three Caveats:

- Neutrino is a Majorana particle
- Light Majorana neutrino being the dominate decay mechanism
- reliable calculation of matrix elements

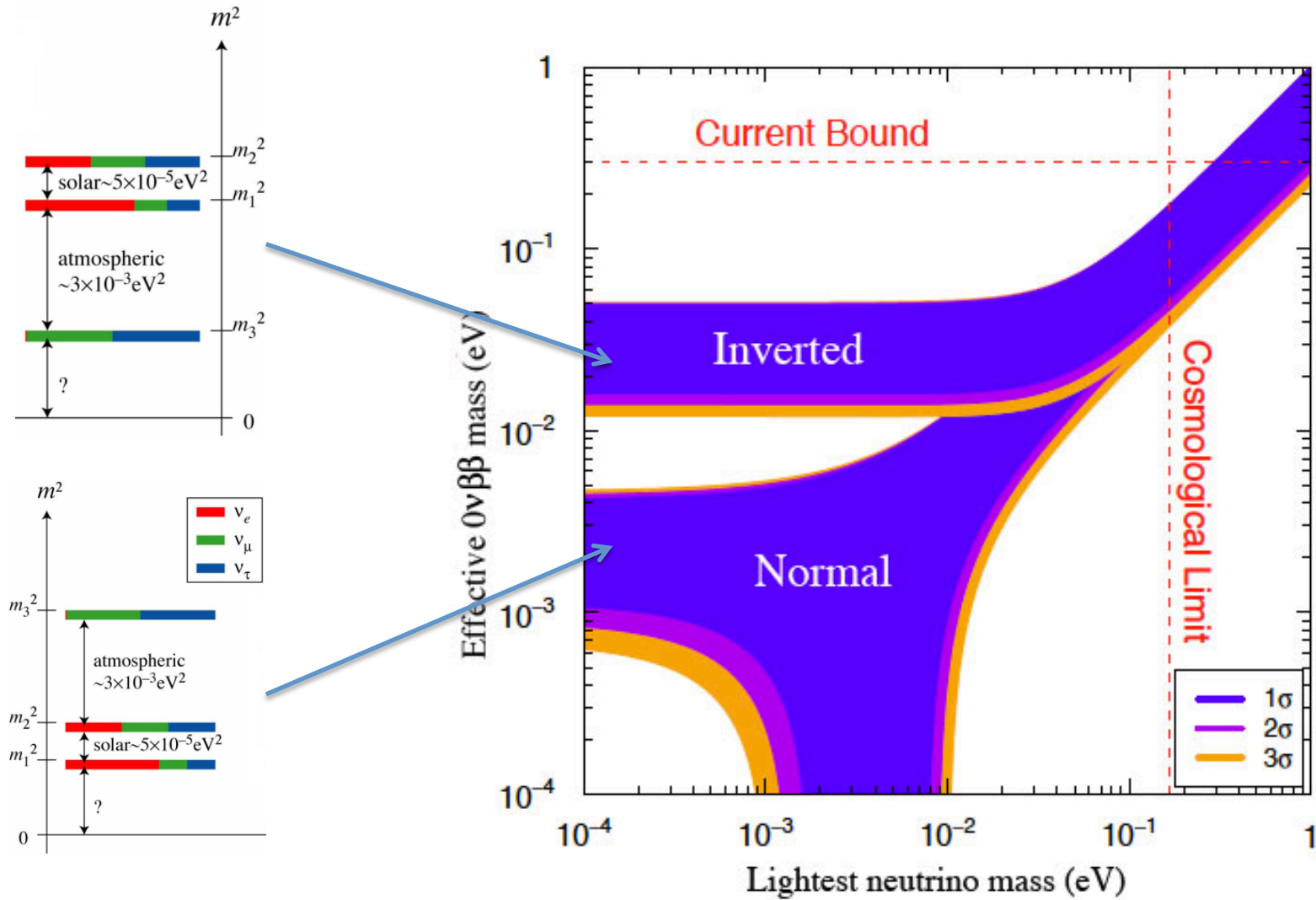
Effective Majorana mass is a coherent sum of neutrino mass eigenvalues, therefore cancellations are possible...

$0\nu\beta\beta$ Matrix Element Calculation



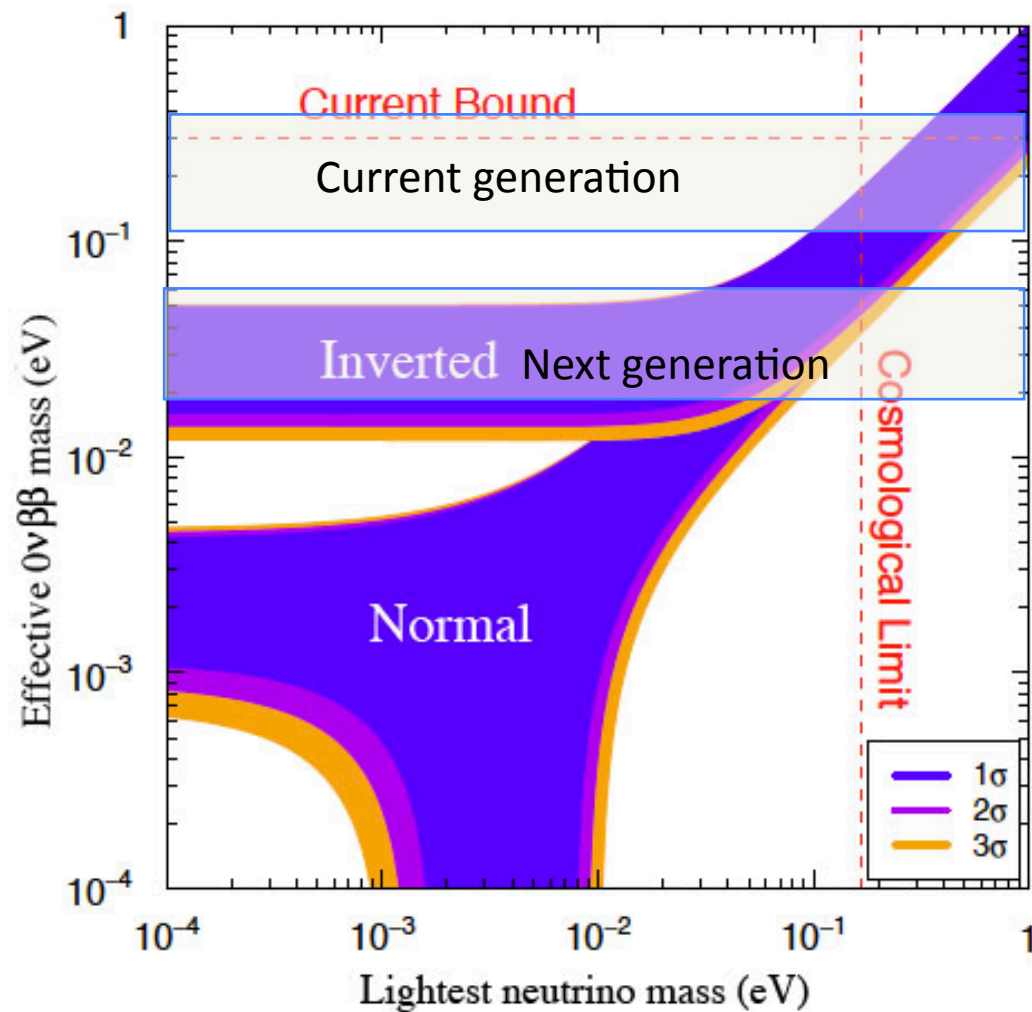
- Matrix element calculation is very difficult, in particular for $0\nu\beta\beta$.
- Recent theoretical progress has narrowed the difference between different models to within a factor of 2-3.
- Theorists need experimental input, such as study of 0^+ ground states using nucleon transfer reactions, US, Japan, India...

$0\nu\beta\beta$ as Absolute Neutrino Mass Measurements



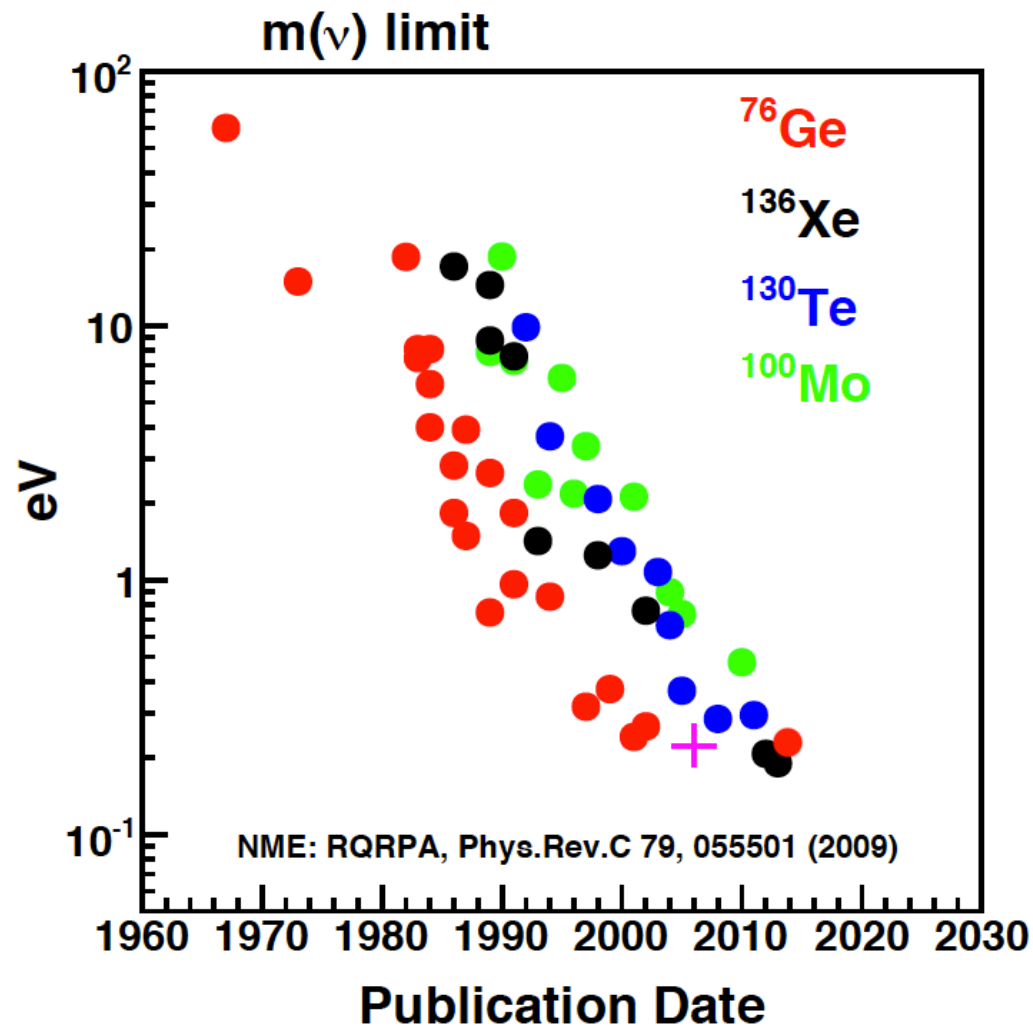
arXiv:1203.5250v1

$0\nu\beta\beta$ as Absolute Neutrino Mass Measurements



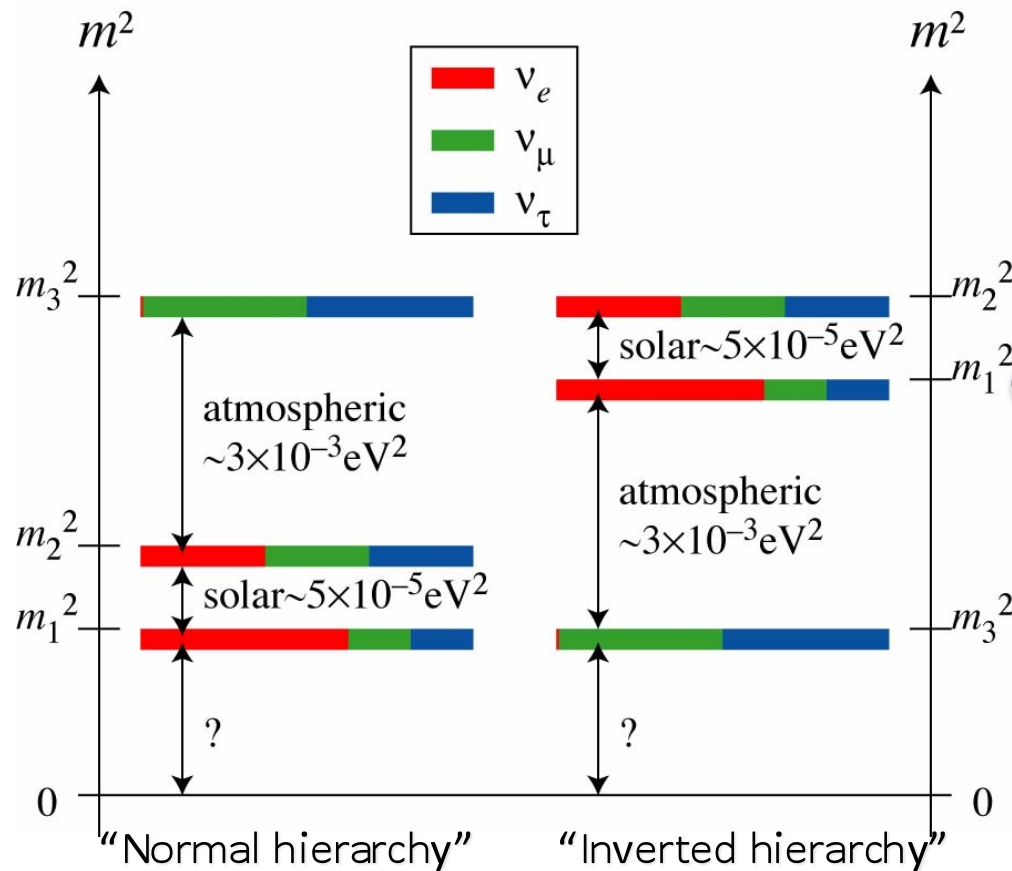
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$0\nu\beta\beta$ Historical Progress



Adapted from C. Hall - Lepton Photon, June 2013

Neutrino Mass Measurements



Three ways to measure absolute neutrino mass:

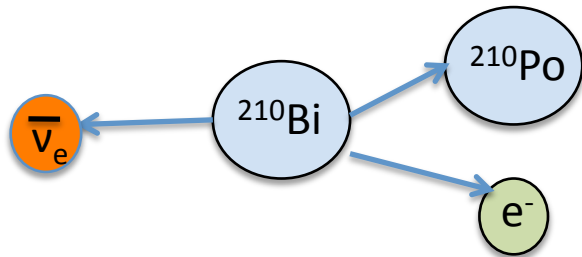
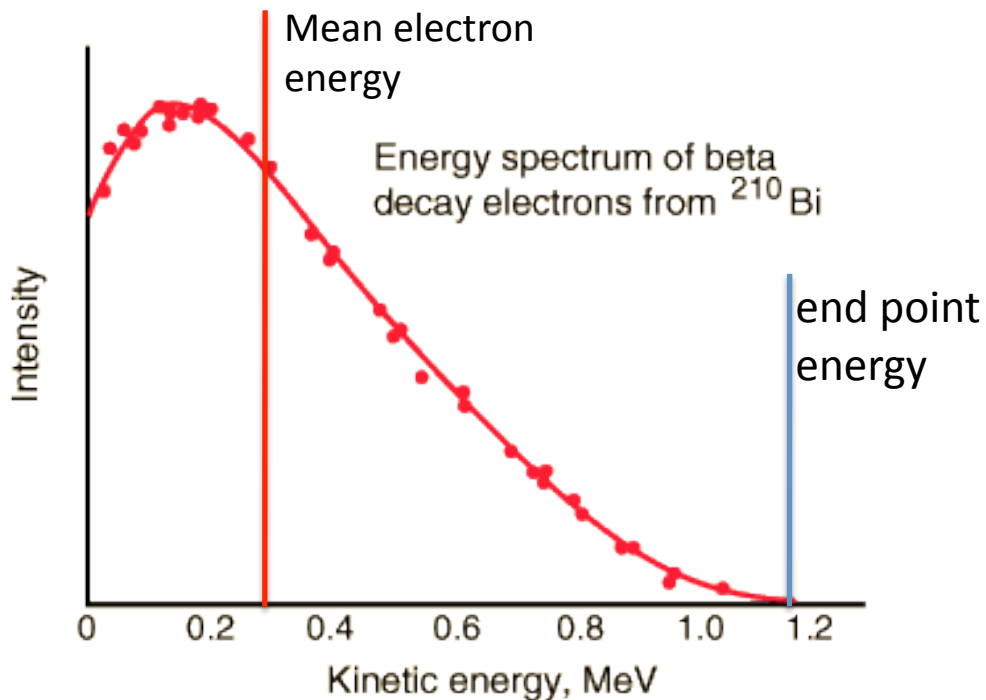
- Direct Neutrino mass measurements

- Double Beta decay

- Cosmological constraint

Beta Decay and Neutrino

β decay: $(A, Z) \rightarrow (A, Z+1)^+ + e^- + \bar{\nu}_e$

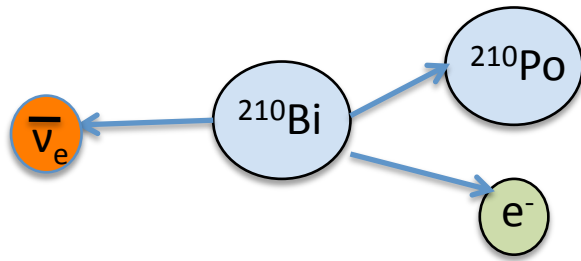


- In 1914, Chadwick first measured the continuous β spectrum with ^{214}Pb .

- In 1927, Ellis and Wooster measured total energy release during ^{210}Bi decay with calorimeter, and found it way below the decay end point.

- In 1930, Pauli proposed a “desperate” measure to solve the puzzle and first introduced neutrino.

Beta Decay and Neutrino Mass



In 1934, Fermi pointed out that spectrum shape can be used to determine neutrino mass, and concluded the neutrino mass to be either zero or very small....

In theory, the mass of neutrino can be determined using energy and momentum of conservation if full kinematics of the decay particles are known. However, experimentally, only electron energy spectrum is easily measurable.

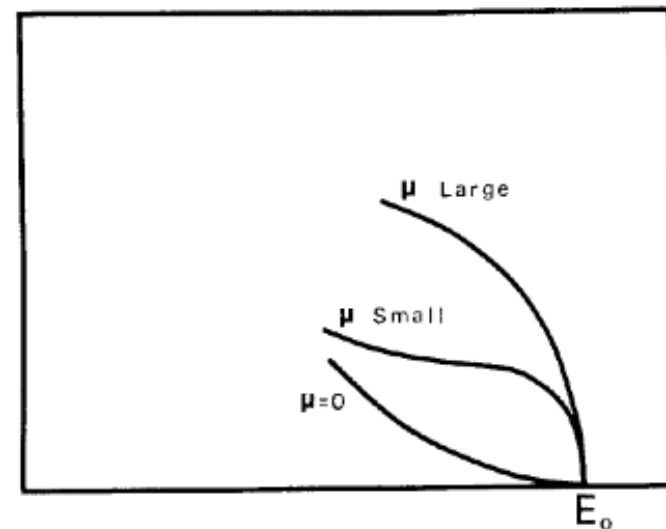


FIG. 1. The end of the distribution curve for $\mu=0$ and for large and small values of μ .

E. Fermi, Z. Physik 88, 161 (1934)

Beta Decay Theory

Differential decay rate for allowed β decay:

Matrix element

Fermi Function

Phase space factor

$$\begin{aligned}\frac{dN}{dE} &= K \times |M|^2 \times F(E, Z) \times (p_e \times E_e \times p_\nu \times E_\nu) \\ &= K \times |M|^2 \times F(E, Z) \times p_e \times (E + m_e) \times (E_0 - E) \times \sqrt{(E_0 - E)^2 - m_{\nu_e}^2}\end{aligned}$$

E_0 : Endpoint energy of the decay, E : Kinetic energy of the electron

$$m_{\nu_e}^2 = \sum_i |U_{ei}|^2 m_i^2$$

Incoherent sum of neutrino mass in “quasi-degenerate” region

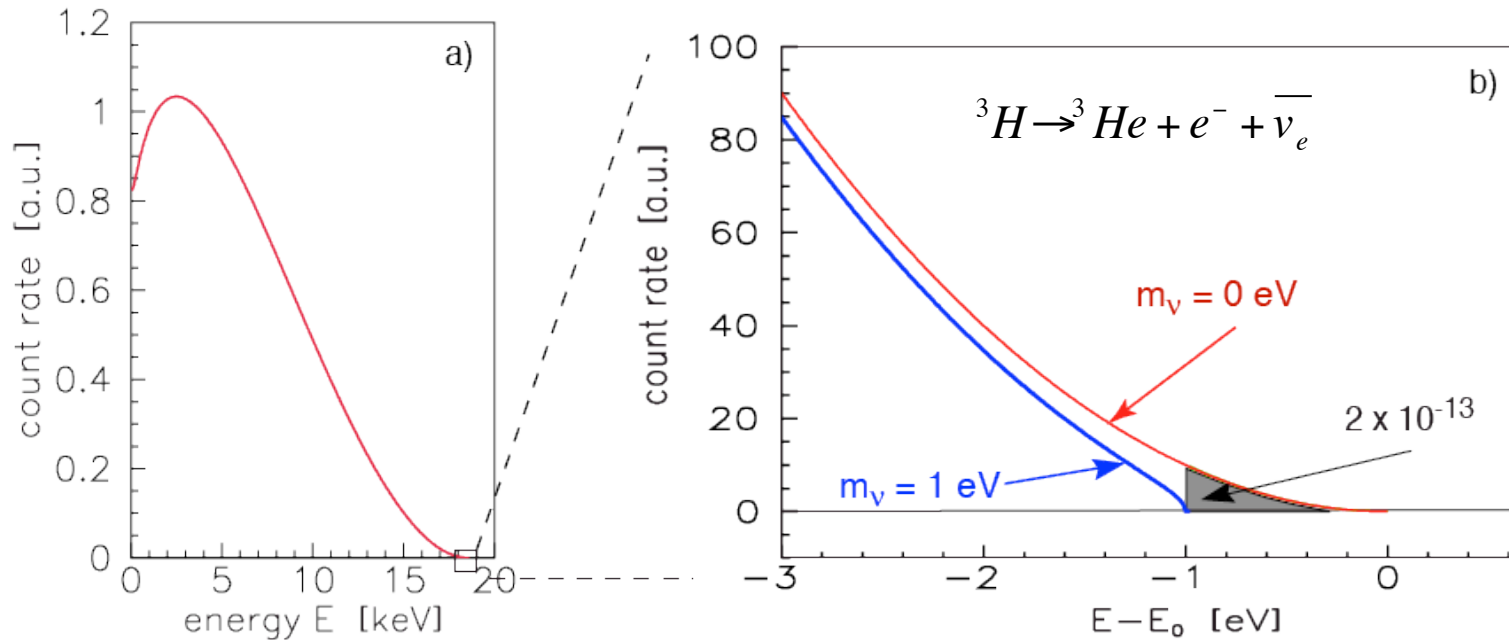
- Neutrino mass term only become important near the decay end point
- Need to consider the excited energy levels of electrons in the final atom (molecule).

$$\sum_i W^i (E_0^i - E) \sqrt{(E_0^i - E)^2 - m_{\nu_e}^2}$$

Branching ratio

Phase space factor of neutrino need to be changed to include all decay modes

Beta Decay End Point Measurements



Fractional events in ΔE energy window
just below endpoint E_0

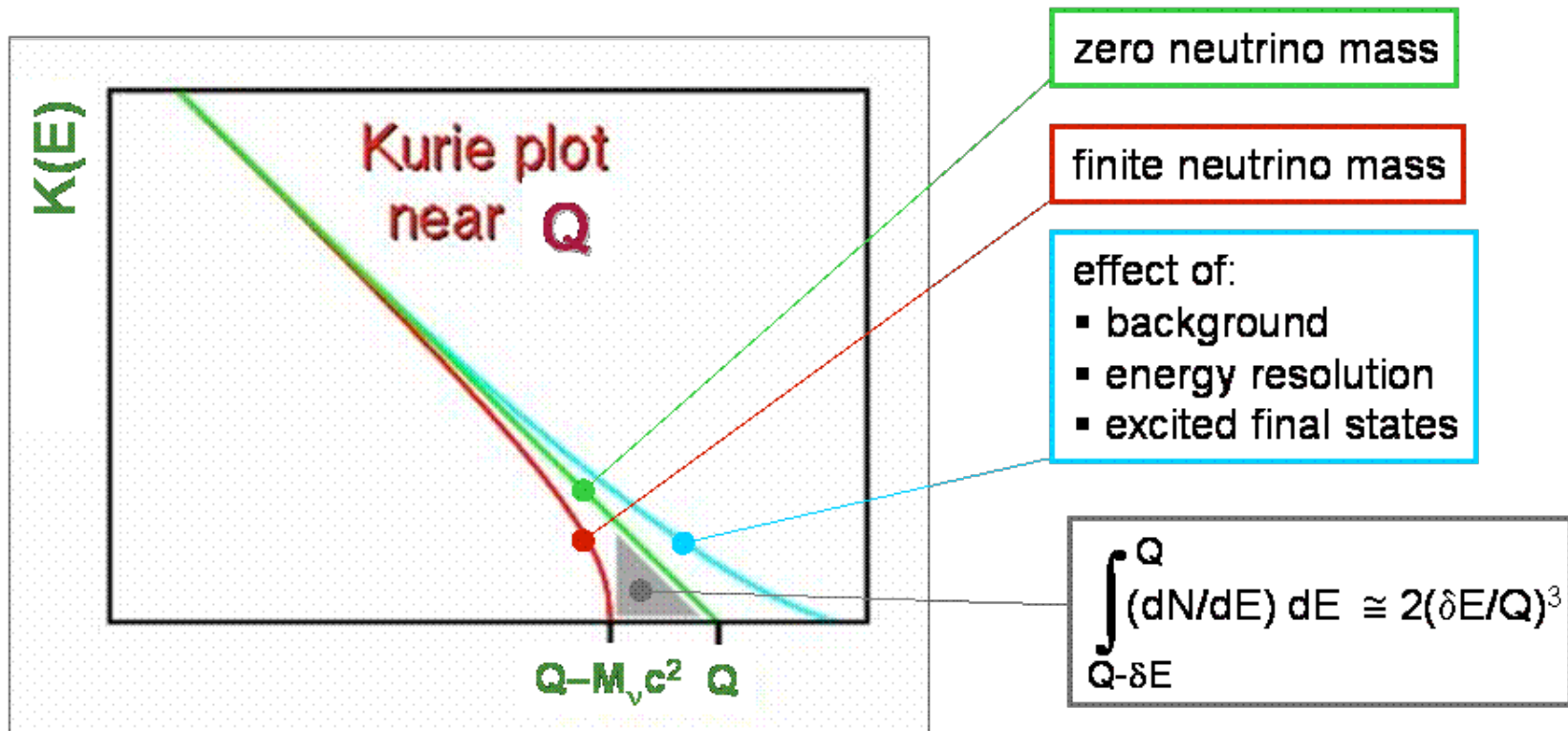
$$\frac{n(\Delta E)}{n} \propto \left(\frac{\Delta E}{E_0} \right)^3$$

- Statistics is critical \rightarrow need low end point energy, short lifetime.
- High energy resolution, low contamination
- Current best limit: $m(\nu_e) < 2.2$ eV (Mainz and Troisk)
- Why we don't talk about positron decay?

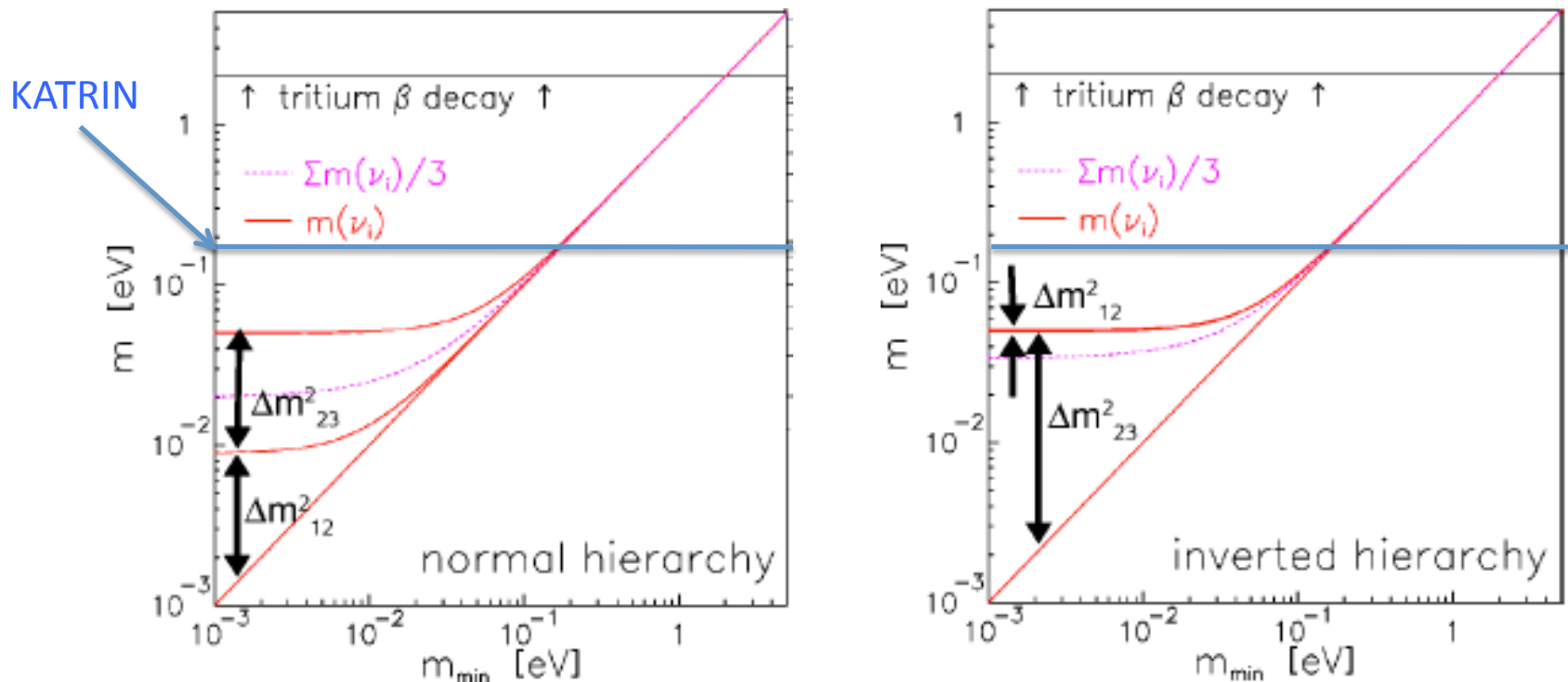
Kurie Plot

$$K(E) \equiv \left[\frac{dN/dE}{pEF(Z,E)} \right]^{\frac{1}{2}} \propto \left\{ (E_0 - E) \left[(E_0 - E)^2 - m_\nu^2 \right]^{\frac{1}{2}} \right\}^{\frac{1}{2}} \approx E_0 - E$$

The Kurie plot $K(E_e)$ is a convenient **linearization** of the beta spectrum



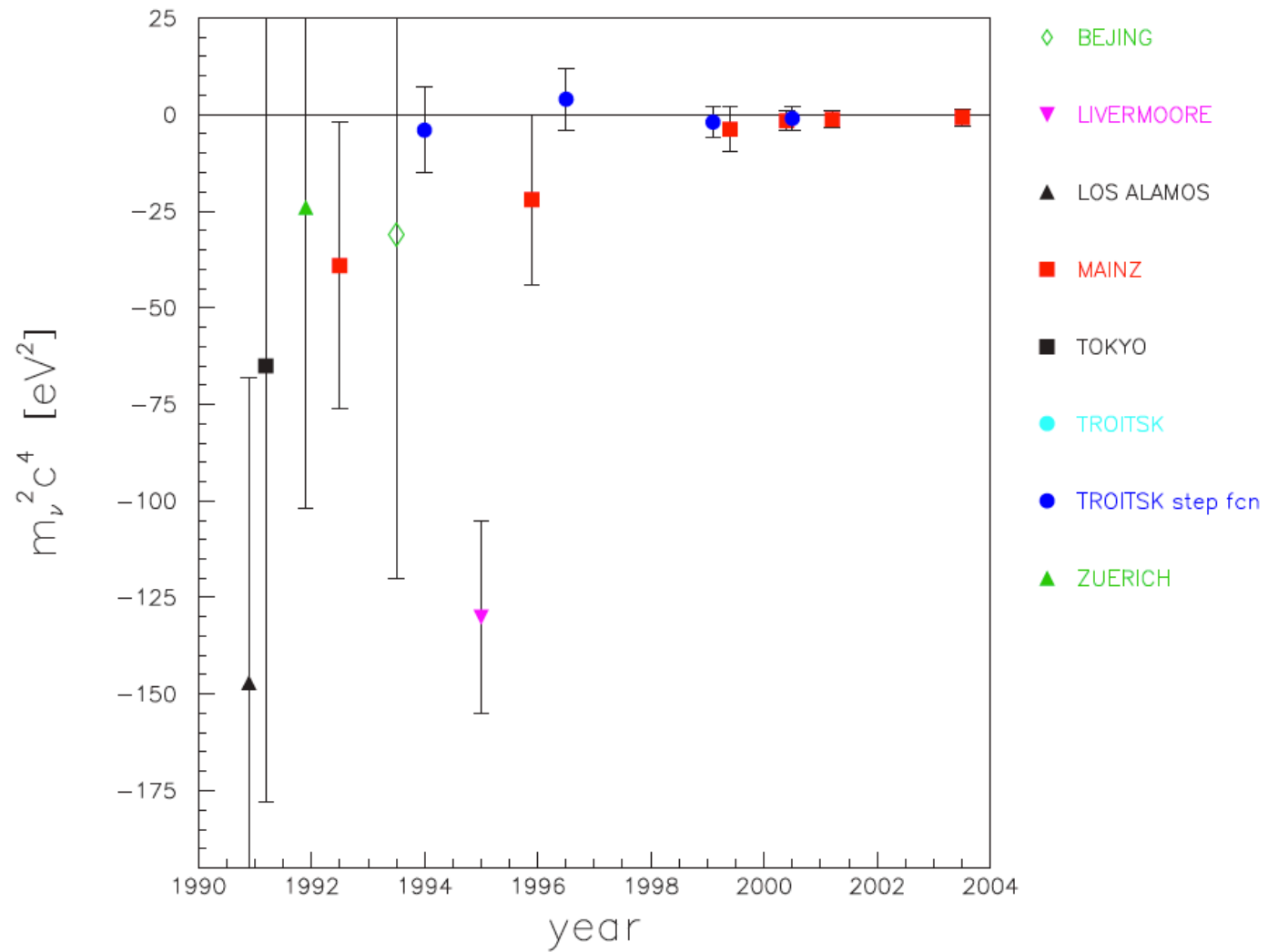
Neutrino Mass Sensitivity



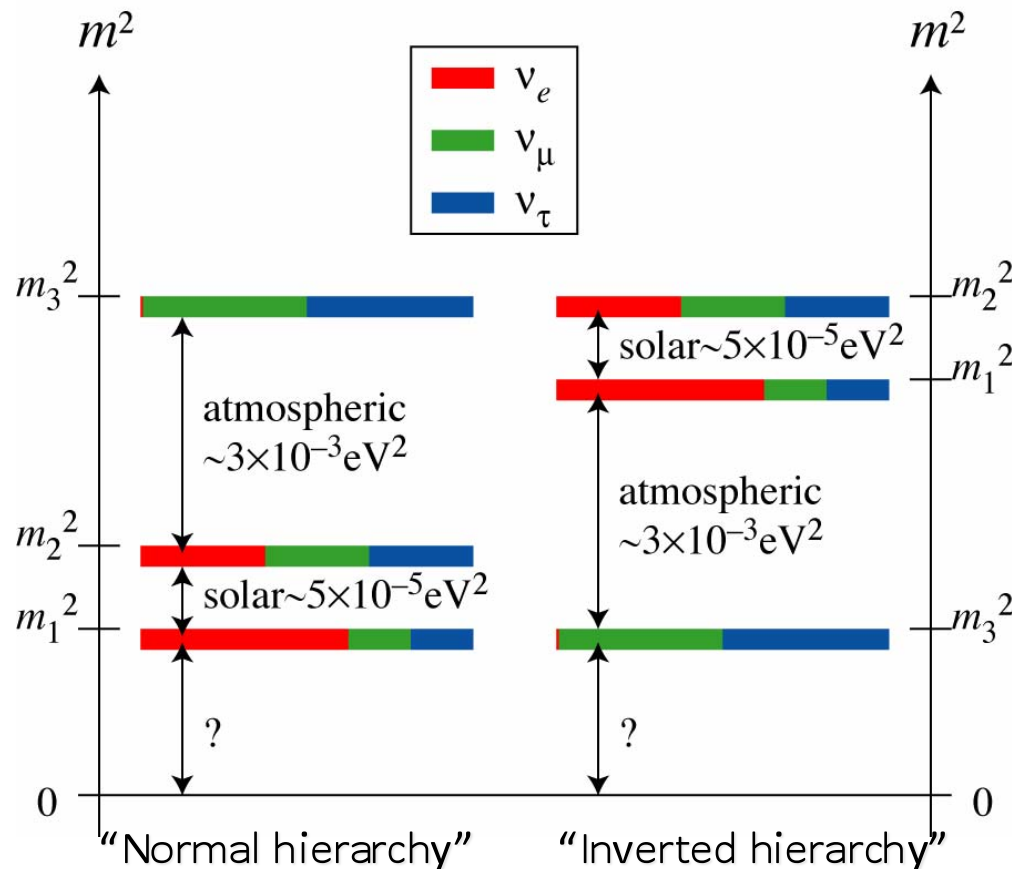
G. Drexlin et. al., arXiv:1307.0101v1

- From oscillation experiments, we know $\text{SUM}(m_i) > 55 \text{ meV}$.
- Absolute mass scale can be obtained from measuring just electron neutrinos.
- At region $> 100 \text{ meV}$, $m(\nu_e) \approx \text{SUM}(m_i)/3$

Historical Progress (Tritium)

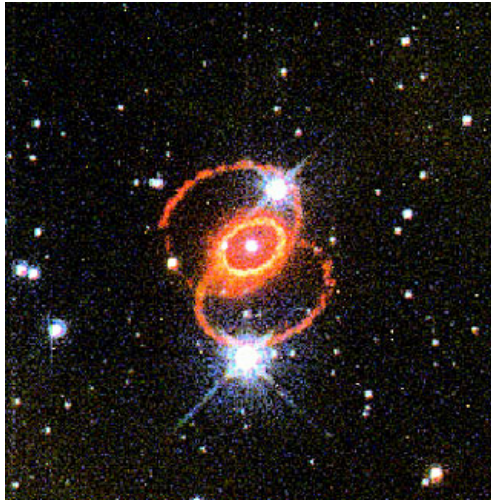


Neutrino Mass Measurements



Three ways to measure absolute neutrino mass:

- Direct neutrino mass measurements
- Double Beta decay
- Cosmological constraint



SN 1987 debris, Hubble, NASA, 1995

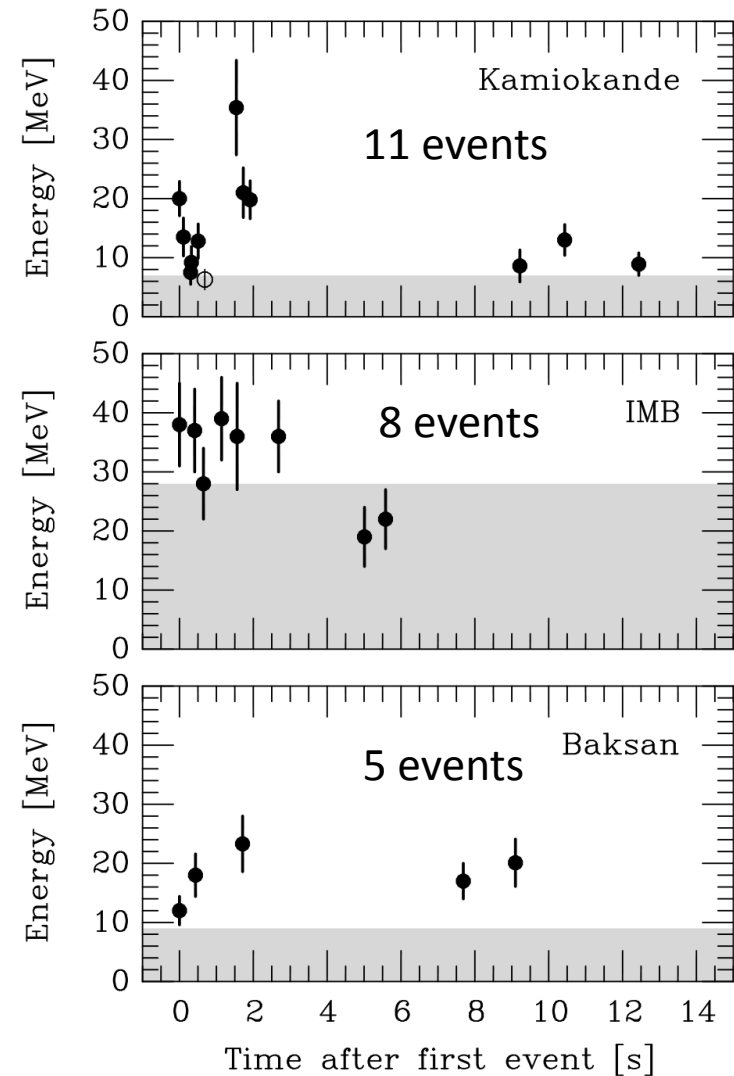
Supernova 1987

- Only SN neutrinos detected so far. Signal observed by three detectors, 2-3 hours before visible light. (1.6E5 light year away)
- All neutrinos (7.5 - 35 MeV) arrived within 10 s, no clear energy dependence.

$$\Delta t = \frac{L}{c} \left(1 - \beta_v^{-1}\right) \approx -t \frac{m_v^2}{2E_v^2} \Rightarrow m_v < 20 \text{ eV}$$

- More sophisticated analysis puts $m_v < 5.6 \text{ eV}$

Is there any fundamental limitation to the method? What about terrestrial measurements?



G. Raffelt, Ann. Rev. Nucl. Part. Sci. 49 (1999)

Relic Neutrino Density

- After big bang, neutrinos are in thermal equilibrium by frequent weak interaction, ($T > 1\text{MeV}$)
- Neutrinos decouple from the plasma before e^-/e^+ annihilation heats up the photon radiation.
- The number relic neutrinos per flavor can be calculated to be ~ 112 neutrinos per cm^2 per species.

$$f_\nu = \frac{\Omega_\nu}{\Omega_m} \approx 0.08 \frac{\sum_i m_i(\nu)}{1(\text{eV})}$$

Majorana neutrinos have two distinct states, while Dirac neutrinos have four distinct states? Shouldn't they result in different relic neutrino signature, and we can distinguish M/D neutrino with cosmological measurements?

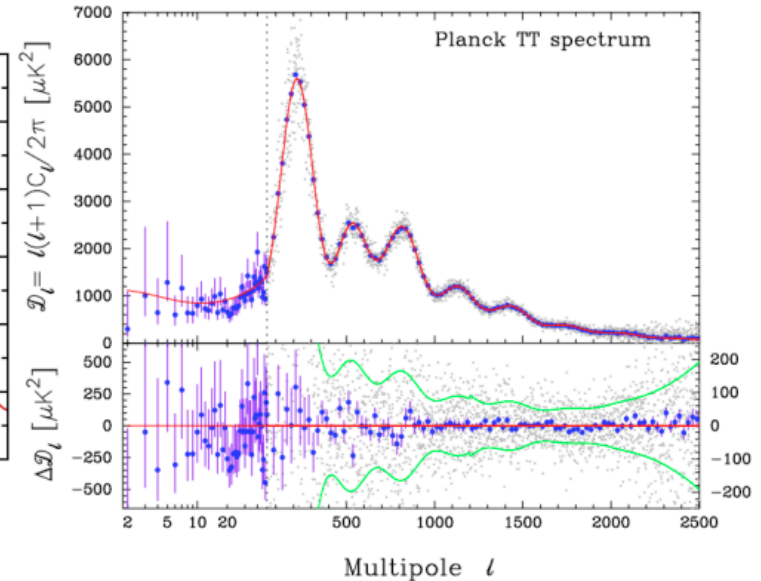
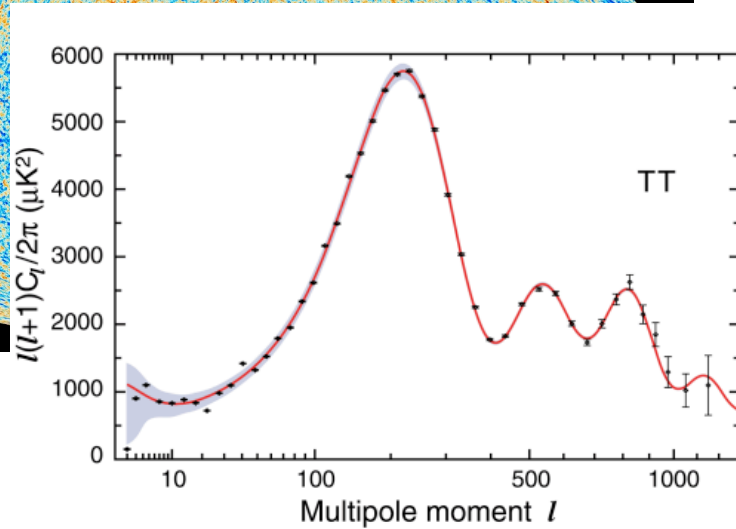
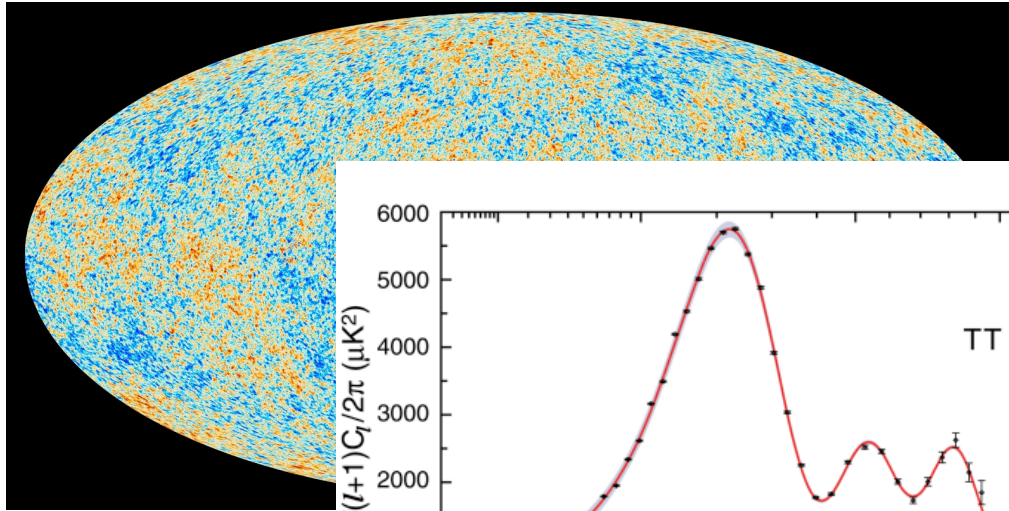
Neutrino Mass and Cosmology

- Due its small mass, neutrino plays an unique role in the evolution of the Universe.
- The large thermal velocity of the neutrino in early universe leads to smearing out of over-dense regions.
- At a later time t , neutrinos do not clump on scale much smaller than its free-streaming distance $v t$.
- They upsets the delicate balance between matter dilution due to expansion and matter accretion due to gravitational instability
- Neutrinos tend to suppress structure formation for scales $< \sim 0.1 \text{Mpc}/f_\nu$, while structures $> 100 \text{ Mpc}$ are not affected
- The effect of neutrino mass can be measured by studying matter distribution in the Universe. Experimental probes include Cosmic Microwave background (CMB), galaxy mapping and gravitational lensing.

Cosmological Constraint of $\Sigma m(\nu)$

Method	Current $\Sigma m(\nu)$ bound (eV)	Future $\Sigma m(\nu)$ sensitivity (eV)	Datasets
CMB primordial (ISW, lensing, polarization)	0.66	0.2	Planck, WMAP, SPT, ACT
CMB primordial + distance scale	0.23		Planck, WMAP, SPT, ACT + BAO & H_0
Galaxy distributions	0.6	0.1	SDSS, BOSS (DES, BigBOSS, LSST)
Lensing of galaxies	0.6	0.07	CFHT-LS, COSMOS (WFIRST, DES, LSST, EUCLID)
Lyman α	0.2	0.1	SDSS, BOSS, KECK
21 cm mapping	-	0.1 – 0.006	SKA, FFTT
Galaxy clusters	0.3	0.1	Planck, SPT, SDSS

Recent Planck Results



$$\sum m_\nu < 0.66 \text{ eV} \quad (95\%; \text{Planck}+\text{WP}+\text{highL}).$$

$$\sum m_\nu < 0.85 \text{ eV} \quad (95\%; \text{Planck}+\text{lensing}+\text{WP}+\text{highL}),$$

$$\sum m_\nu < 0.23 \text{ eV} \quad (95\%; \text{Planck}+\text{WP}+\text{highL}+\text{BAO}).$$

“Planck 2013 results. XX”,
arXiv:1303.5080

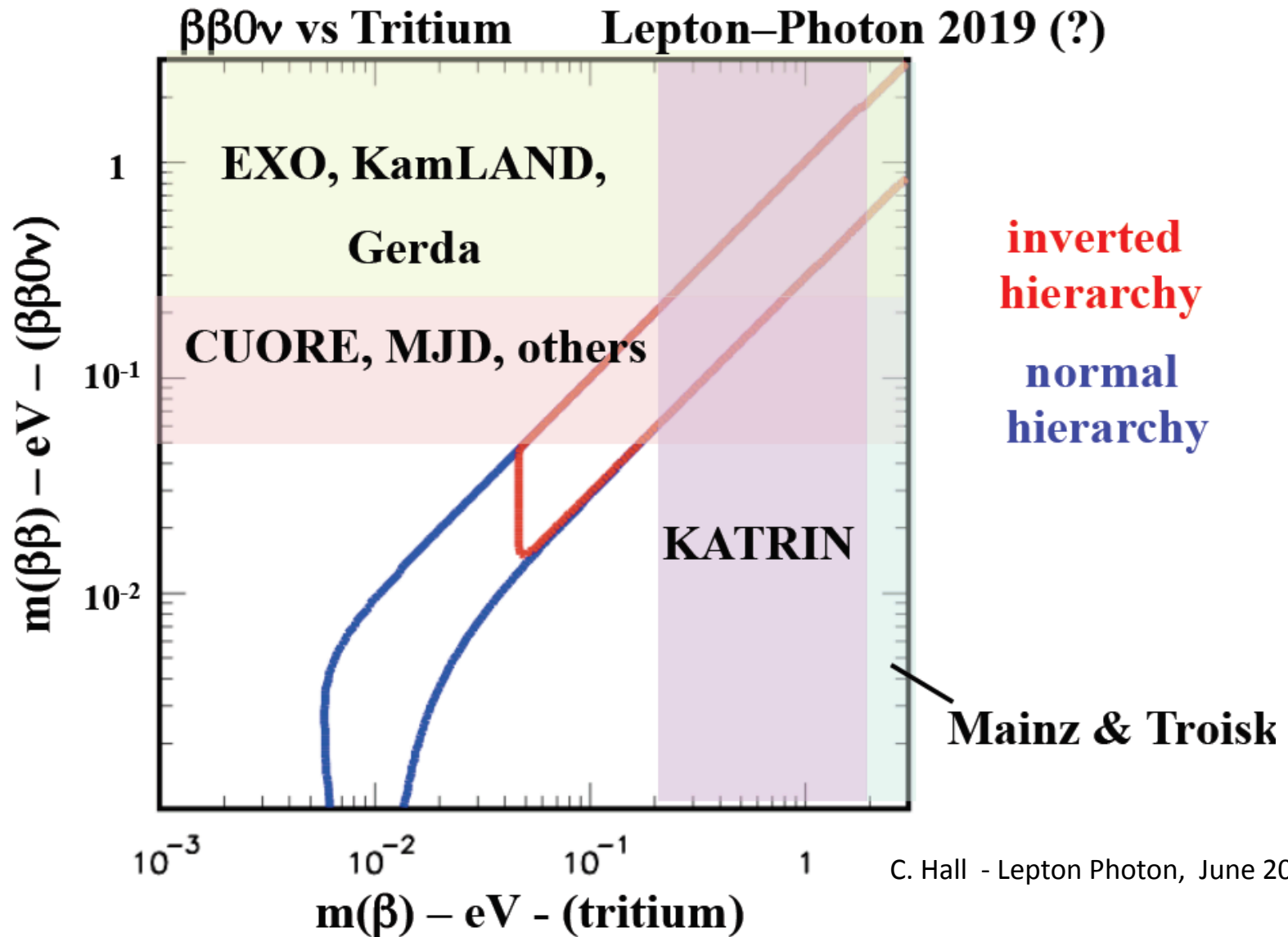
In tension with H measurements

Does the Planck result rule out 1-2 eV sterile neutrino that had been hypothesized to solve the reactor anomaly?

Summary of Neutrino Mass Measurements

	Weak decay	$0\nu\beta\beta$	Cosmological Constraint
Methods	$\sum_i U_{ei} ^2 m_i^2$	$ \sum_i U_{ei}^2 m_i e^{i\alpha} $	$\sum_i m_i$
Comparison	Direct measurement, no model dependence	Majorana neutrino, NME	Model dependent
Present limits	2.2 eV	0.2 – 0.4 eV	0.2 – 0.8 eV
Future sensitivity	0.2 eV	0.05 eV	0.05 eV

Comparison of ν mass techniques

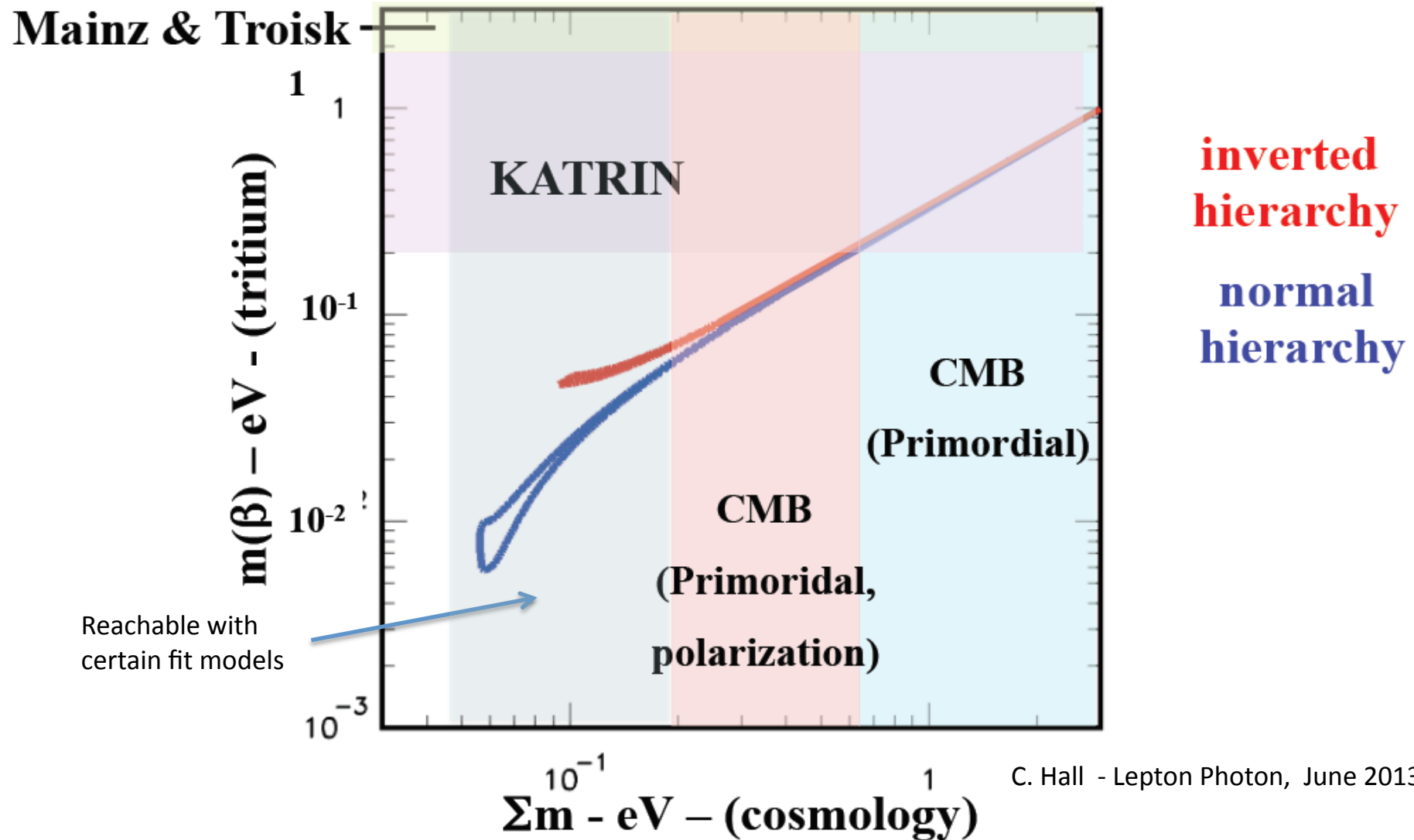


C. Hall - Lepton Photon, June 2013

Adapted from G.L Fogli, et al, PRD 78 033010 (2008)

Comparison of ν mass techniques

tritium vs cosmology Lepton-Photon 2019 (?)

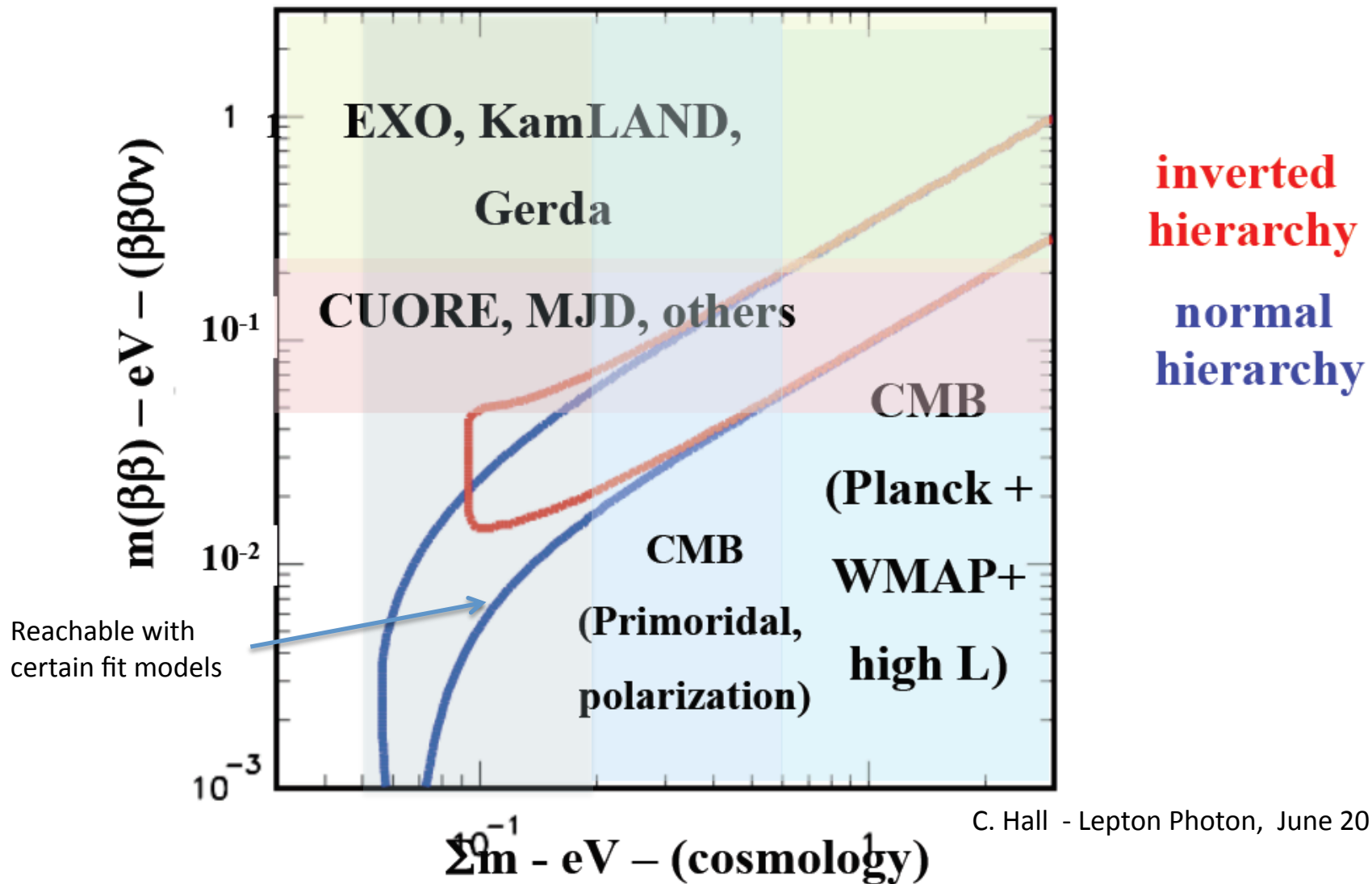


Adapted from G.L Fogli, et al, PRD 78 033010 (2008)

Comparison of ν mass techniques

$\beta\beta 0\nu$ vs cosmology

Lepton-Photon 2019 (?)



C. Hall - Lepton Photon, June 2013

Adapted from G.L Fogli, et al, PRD 78 033010 (2008)

Looking to the Future.....

This is an exciting time. Hopefully within the decade, we will find some answers.

- If DBD experiment sees signal, then neutrino is a Majorana particle, may not be able to set absolute mass scale.
- If direct mass measurements see a signal, while no signal is seen in DBD experiment, we likely can conclude that neutrino are Dirac (model dependant), and can set absolute neutrino mass scale.
- Cosmological measurement will likely be the first to see hints of a positive signal. Improve our understanding of the models necessary to set absolute mass scale.
- It is possible that none of the experiments see any signal

Nature also could have more surprises for us!