

# $\gamma\gamma$ Collider $e^-e^+$ Pair Tracking in OPALX

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### Question for the group

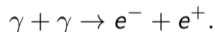
What exactly is the current OPALX model for low-energy  $e^- e^+$  pairs in a gamma-gamma collider interaction region, and what must be added before it can be treated as a production physics model?

- Recap the gamma-gamma context and existing BeamBeam machinery.
- State the implemented OPALX model.
- Review current model and manufactured-solution.
- Identify physics, numerics, and performance limitations.
- Agree on the next implementation and validation steps.

### Caveats

Most of the parameters used in this presentations are not 100% related to the proposed collider! I choose the set of parameters to conveniently do the development.

- High-energy photons produced from electron beams by inverse Compton scattering.
- Photon-photon interactions produce e.g. Breit-Wheeler  $e^-e^+$  pairs:



- The pairs can be born with keV energies compared with the MeV primary beam.
- Their trajectories are therefore
  - sensitive to fields in the interaction region
  - timing between photons and primary electrons are important
- At the moment we read in  $e^-/e^+$  pairs (from CAIN simulations by Kaoru

## OPALX task

Track generated  $e^-e^+$  witnesses through the interaction region and quantify whether/where they cross a cylindrical detector or masking aperture.

## OPALX input: three containers

```
IP1: BEAMBEAM, WITNESS={1,2} ....  
  
BEAMS = {PrimaryElectrons, GammaGammaElectrons,  
         GammaGammaPositrons};
```

- BeamBeam window length:

$$L_{\text{BB}} = 0.32 \text{ m.}$$

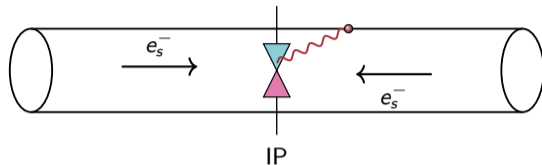
- Cylinder radius:

$$R_{\text{BB}} = 0.15 \text{ m.}$$

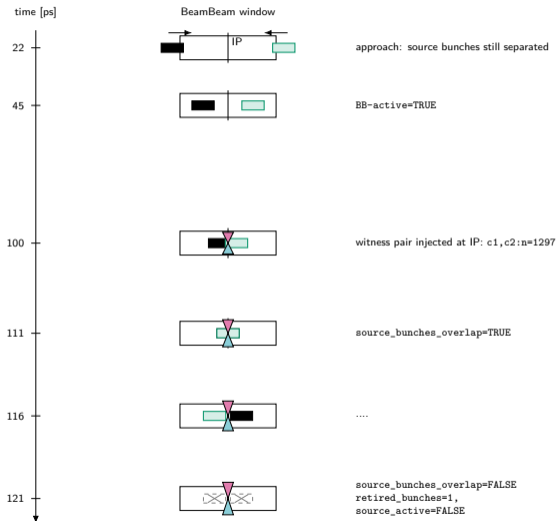
- Interaction point:

$$s_{\text{IP}} = 0.17 \text{ m.}$$

- Source kinetic energy in the manufactured model:  $K_s = 245 \text{ MeV}$ ,  $\gamma_s \simeq 480.45$ .
- Witness beams  $\approx 315 \text{ keV}$  701 electrons and 692 positrons



- Container 0 is the high-energy source bunch.
- Containers 1 and 2 are the low-energy  $e^-$  and  $e^+$  witnesses.
- Witnesses passively sample the source BeamBeam field.
- Aperture/Detector: Histograms are first trajectories crossings



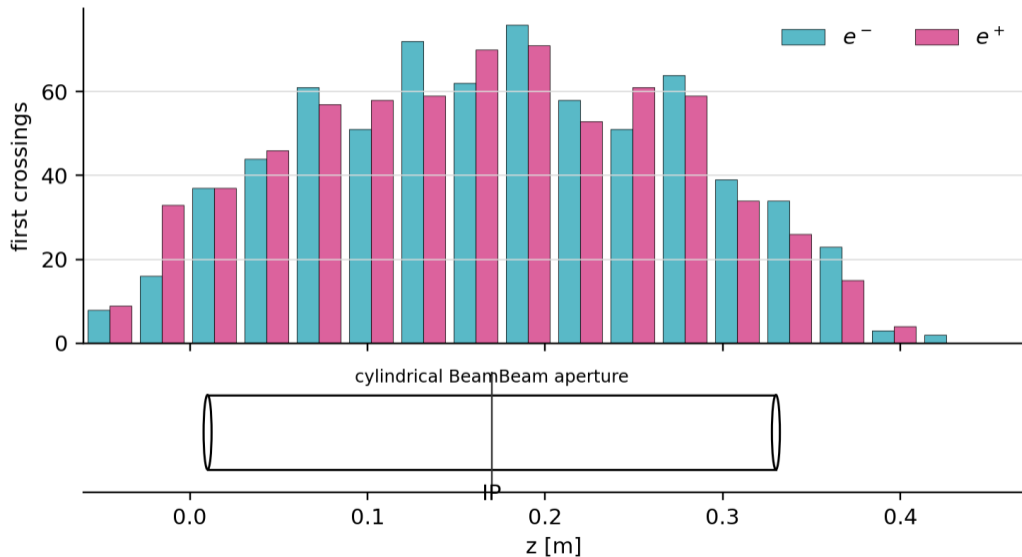
### Current high-energy source model

OPALX tracks one physical source bunch and reconstructs the counter-propagating partner as a virtual copy around the interaction point.

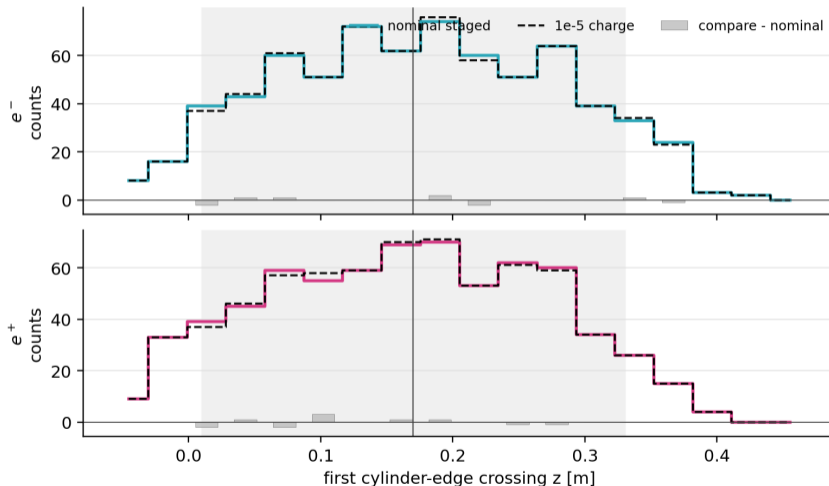
- The virtual bunch is not an independently tracked second bunch.
- The model is intended for symmetric head-on configurations.
- The mirroring is controlled by time, COPY\_TIME.

- The bunch-overlap kick is very short in lab time.
- A coarse 0.1 ps step under-resolves the transient.
- In the sandbox setup we need a time step 1 fs .

$\Delta t$ [ps]	$\Delta K$ [eV]
0.1	5.20
0.01	48.64
0.005	58.68
0.002	60.86
0.001	61.16



1000 ps retirement: nominal staged vs 1e-5 charge



$e^-$ :  $N=701/701$ ,  $|\text{bin diff}|=10$ ,  $\max |dz|=2026.5 \text{ um}$

$e^+$ :  $N=692/692$ ,  $|\text{bin diff}|=12$ ,  $\max |dz|=1847.3 \text{ um}$

### Current witness field is still an approximation

The OPALX witness gather rotates/translates witness geometry into the source frame and samples the mesh. It does not transform the witness event into each source rest frame and does not transform the full field tensor back.

For a source rest-frame field  $\mathbf{E}'$ ,  $\mathbf{B}'$ , the lab field is

$$\mathbf{E} = \gamma_s(\mathbf{E}' - \mathbf{v}_s \times \mathbf{B}') - \frac{\gamma_s^2}{\gamma_s + 1} \frac{\mathbf{v}_s}{c^2} (\mathbf{v}_s \cdot \mathbf{E}'), \quad (1)$$

$$\mathbf{B} = \gamma_s \left( \mathbf{B}' + \frac{\mathbf{v}_s \times \mathbf{E}'}{c^2} \right) - \frac{\gamma_s^2}{\gamma_s + 1} \frac{\mathbf{v}_s}{c^2} (\mathbf{v}_s \cdot \mathbf{B}'). \quad (2)$$

- No witness charge deposition back onto the source mesh.
- No independently tracked strong-strong two-bunch model.

### Numerics

- What accuracy in space do we need (detector point of view)
- Missing is a single-particle manufactured OPALX regression test.

### Goal of the appendix

Turn Section 2.2 of the note into a checklist for implementing and testing the manufactured witness solution in OPALX.

- Start from the source field OPALX already computes.
- Compare that field to the analytic boosted-Gaussian source field.
- Then compare one witness Boris kick.

### Important separation

The source field solve and the witness event evaluation are different problems. The current OPALX source field is useful, but the witness gather still needs the event-level Lorentz treatment.

The BeamBeam field calculation uses the binned quasi-static field path. This includes the case where adaptive binning has only one current bin: if a BINNING definition is attached, OPALX dispatches through `computeBinnedSelfFields()`.

For the manufactured solution we start with one current bin only. To keep the notation close to the implementation but readable, the bin index is suppressed:

- $\mathbf{P} = \beta\gamma$ : normalized mechanical momentum;
- $\bar{\mathbf{P}}$ : charge-weighted/global mean over the one source bin;
- $\gamma, \beta, \mathbf{v}$ : Lorentz factor, normalized velocity, and velocity assigned to that bin;
- primed quantities: fields and mesh quantities in the approximate source rest frame.

### Key approximation

All source particles share the same Lorentz transformation. The manufactured solution should first reproduce this single-bin assumption.

OPALX computes the global mean normalized momentum

$$\bar{\mathbf{P}} = \langle \beta \gamma \rangle. \quad (2.32)$$

This is the one velocity moment used by the field solver. From it,

$$\gamma = \sqrt{1 + \bar{\mathbf{P}}^2}, \quad \mathbf{v} = c \frac{\bar{\mathbf{P}}}{\gamma}. \quad (2.33)$$

The corresponding normalized velocity and direction are

$$\beta = \frac{\mathbf{v}}{c} = \frac{\bar{\mathbf{P}}}{\gamma}, \quad \hat{\beta} = \frac{\beta}{|\beta|}. \quad (2.34)$$

### Implementation meaning

The analytic manufactured source should be parameterized by the same  $\gamma$ ,  $\beta$ , and direction  $\hat{\beta}$  that OPALX uses for the single bin, not by each source particle separately.

Before the Poisson solve, OPALX transforms the source mesh approximately into the source rest frame:

$$\rho' = \frac{\rho}{\gamma}. \quad (2.35)$$

The longitudinal mesh spacing is stretched by the same Lorentz factor:

$$\Delta z' = \gamma \Delta z. \quad (2.36)$$

OPALX then solves an electrostatic open-boundary Poisson problem in this stretched frame and obtains a rest-frame-like field  $\mathbf{E}'$ .

#### Already available

- charge deposition on the source mesh;
- open-boundary Poisson solve;
- quasi-static Lorentz correction;
- mesh gather to source and witness containers.

#### Not yet enough

This is a source field model. The manufactured witness test must still evaluate the field at a specific witness event.

After solving in the stretched frame, OPALX transforms the electric field back to the lab:

$$\mathbf{E} = \gamma \mathbf{E}' - (\gamma - 1)(\mathbf{E}' \cdot \hat{\beta})\hat{\beta}. \quad (2.37)$$

This leaves the component parallel to  $\hat{\beta}$  unchanged and boosts the transverse component by  $\gamma$ :

$$\mathbf{E} = \mathbf{E}'_{\parallel} + \gamma \mathbf{E}'_{\perp}.$$

### For the manufactured solution

Evaluate the analytic Coulomb field of the prescribed Gaussian in the source rest frame, decompose it with respect to  $\hat{\beta}$ , and use Eq. (2.37) before comparing with the OPALX sampled field.

Eq. (2.37) is often easiest to implement as a matrix operator:

$$\mathbf{E} = \left[ \gamma I - (\gamma - 1) \hat{\beta} \hat{\beta}^T \right] \mathbf{E}'.$$

The magnetic field accumulated by OPALX is

$$\mathbf{B} = \frac{\gamma}{c} \boldsymbol{\beta} \times \mathbf{E}'.$$

With

$$[\boldsymbol{\beta}]_{\times} = \begin{pmatrix} 0 & -\beta_z & \beta_y \\ \beta_z & 0 & -\beta_x \\ -\beta_y & \beta_x & 0 \end{pmatrix}, \quad [\boldsymbol{\beta}]_{\times} \mathbf{a} = \boldsymbol{\beta} \times \mathbf{a},$$

this is equivalently

$$\mathbf{B} = \frac{\gamma}{c} [\boldsymbol{\beta}]_{\times} \mathbf{E}'.$$

- 1 Construct the manufactured source with the same  $Q$ ,  $\sigma'$ ,  $\gamma$ , and  $\beta$  as OPALX.
- 2 Evaluate the analytic rest-frame Gaussian Coulomb field  $\mathbf{E}'(\mathbf{x}'_w)$  at the witness event transformed into the source frame.
- 3 Apply Eq. (2.37) and the corresponding magnetic transform to obtain lab-frame  $(\mathbf{E}, \mathbf{B})$ .
- 4 Compare this field to the value gathered by OPALX before applying a particle push.

### Manufactured-solution use

This isolates the field model from the trajectory problem. If this test fails, a crossing histogram cannot identify whether the error comes from deposition, Lorentz transformation, gather location, or the Boris update.

Let the witness event be  $(t_w, \mathbf{x}_w)$ , and let the source reference event be  $(t_s, \mathbf{x}_s)$ . Define

$$\Delta t = t_w - t_s, \quad \Delta \mathbf{x} = \mathbf{x}_w - \mathbf{x}_s,$$

and split the displacement into

$$\Delta x_{\parallel} = \Delta \mathbf{x} \cdot \hat{\boldsymbol{\beta}}, \quad \Delta \mathbf{x}_{\perp} = \Delta \mathbf{x} - \Delta x_{\parallel} \hat{\boldsymbol{\beta}}.$$

The source-frame evaluation point is then

$$\Delta x'_{\parallel} = \gamma(\Delta x_{\parallel} - \beta c \Delta t), \quad \Delta \mathbf{x}'_{\perp} = \Delta \mathbf{x}_{\perp},$$

with source-frame time offset

$$\Delta t' = \gamma \left( \Delta t - \frac{\beta \Delta x_{\parallel}}{c} \right).$$

### Implementation point

The manufactured solution should evaluate  $\mathbf{E}'$  at the same event, not only at a rotated/transformed spatial coordinate at lab simultaneity.

- 1 Prescribe one Gaussian source bunch:

$$Q, \sigma', \beta_s, \gamma_s.$$

- 2 Choose one witness event:

$$x_w^\mu = (ct_w, \mathbf{x}_w).$$

- 3 Evaluate the analytic source rest-frame Coulomb field  $\mathbf{E}'(\mathbf{x}'_w)$ .
- 4 Transform to lab  $(\mathbf{E}, \mathbf{B})$ .
- 5 Ask OPALX for the sampled witness field at the same event.
- 6 Apply one Boris kick in Python and OPALX and compare  $\Delta\mathbf{P}$ .

### Do not start with histograms

A crossing histogram is too integrated to debug the field model. The first regression should be one source, one witness, one kick.

For the current large-cylinder parameters,

$$\gamma_s \simeq 480.45, \quad \sigma' = 0.6 \text{ mm},$$

so the lab longitudinal source scale is

$$\sigma_z = \frac{\sigma'}{\gamma_s} \simeq 1.25 \text{ } \mu\text{m}, \quad \sigma_z/c \simeq 4.16 \text{ fs.}$$

- A 0.1 ps timestep is much too coarse for the overlap kick; the note found roughly a factor-12 underestimate in  $\Delta K$ .
- Missing event Lorentz transformation and simultaneity terms are likely leading-order for off-axis or asymmetric witnesses.
- Magnetic fields cancel only in the ideal symmetric transverse launch; asymmetric cases exercise the full  $(\mathbf{E}, \mathbf{B})$  kick.
- Retarded-time effects are probably lower priority for the first test if the source bunches are prescribed as uniformly moving.
- Witness self-fields are a later strong-strong extension; first verify the weak-strong manufactured field and one-particle kick.

- 1 Add a diagnostic mode that returns the field gathered at one witness before the kick.
- 2 Store the event data needed for comparison:  $t_w, \mathbf{x}_w, \mathbf{P}_w, \beta_s, \gamma_s, Q, \sigma'$ .
- 3 Implement the manufactured analytic evaluator in the unit/regression test, not inside production tracking first.
- 4 Compare  $\mathbf{E}$ ,  $\mathbf{B}$ , and one-kick  $\Delta\mathbf{P}$  with tight, physically justified tolerances.
- 5 Only then generalize to many witnesses, multiple source bins, and detector crossing summaries.

### Success criterion

OPALX and the manufactured evaluator agree for the same event and the same Boris update before any many-particle interpretation is attempted.