在粲介子衰变中理解轻强子结构

Shan Cheng (程山)

Hunan University

2026 年 BESIII 粲强子物理研讨会 • 成都理工大学

Oct.17. 2026

Overview

I: 光锥分布振幅

II: 在粲介子半轻衰变中研究轻强子结构

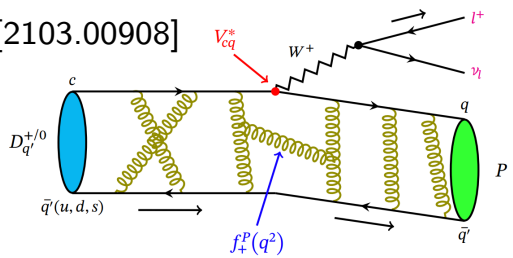
- i: η-η'-mixing 和 K^* 的精细结构
- ii: $D_s \to [\pi\pi]_S\, l\nu$ 和 $f_0(980)$ 结构探索

III: 总结和展望

Why Semileptonic $D_{(s)}$ Decays

Fundamental Parameters Hadron Matrix Elements

[2103.00908]

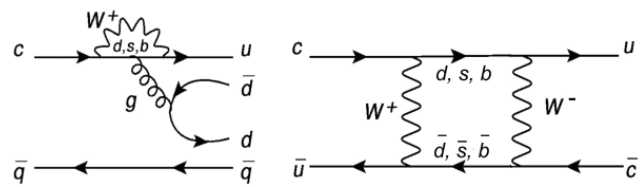


$$\frac{d\Gamma(D \to Pl^+\nu)}{dq^2} = \frac{G_F^2 |V_{cq}|^2 p_P^3}{24\pi^3} |f_+(q^2)|^2 + \mathcal{O}(\frac{m_l^2}{q^2}).$$

Up-type FCNC Process

$c \to ull$

[2011.12856]



$$\frac{d\Gamma(D \to \pi l^+ l^-)}{dq^2} \sim \frac{2}{3}\lambda \left(|F_V^{\rm SM}|^2 + |C_{10}|^2 |f_+|^2\right) +$$

Light Hadrons' Structure

$$\mathcal{A} \propto V_{cq}\bar{u}_l \Gamma_\mu v_\nu \langle P(p)|j_\mu|D_{(s)}(p+q)\rangle.$$

- QCD dynamics are emboded into **Hadron Matrix Elements**, parameterized in Form Factors
- Most measurement accuracies are already at the $\sim 1\%$ level
- QCD-based calculations (besides LQCD) remain at the $\sim 10\%$
- Mainly due to systematic theoretical errors, such as the choice of quark-hadron duality in LCSRs
- Provides opportunities for understanding light hadron structure (with effects at $\sim 10\%$ level)
- **instable particles** (ρ, f_0), **mixing of isoscalar mesons** $(\eta\text{-}\eta', \phi\text{-}\omega)$

How Does QCD Actually Work

$$\mathcal{F}_\mu = i\int d^4x e^{iq\cdot x}\langle P(p)|\,T\{\bar{q}(x)\Gamma_\mu c(x), m_c\bar{c}(0)i\gamma_5 q'(0)\}|0\rangle$$

- $q^2, (p+q)^2 \ll m_c^2$: c-quark emitted and absorbed by the currents is highly virtual and propagates at small x^2
- The light $q\bar{q}$ are emitted as **an almost lightlike separation**, **LCDAs of the light meson**
- Typically $0 \le q^2 \le m_c^2 - m_c\chi$ ($\sim$ 600 MeV), accessible with OPE
- **At large Q^2, the product of operators in $\mathcal{F}_\mu$ shrinks towards the light-cone $x^2 \sim 0$**

$$q = (q_0, 0, 0, q_3),\ p = (\mu, 0, 0, -\mu) \quad \Downarrow$$

$$p\cdot q = \mu(q_0 + q_3) \equiv \frac{Q^2\zeta}{2} \quad \Downarrow$$

$$q_0 = \frac{Q^2\zeta}{4\mu} - \frac{\mu}{\zeta},\ q_3 = \frac{Q^2\zeta}{4\mu} + \frac{\mu}{\zeta} \quad \Downarrow$$

$$iq\cdot x = -i\frac{Q^2\zeta}{4\mu}(x_0 - x_3) + i\frac{\mu}{\zeta}(x_0 + x_3)$$

$$\Downarrow \quad q\cdot x \ll \mathcal{O}(1) \text{ dominance}$$

$$x_0 - x_3 \lesssim \frac{4\mu}{Q^2\zeta},\ x_0 + x_3 \lesssim \frac{\zeta}{\mu} \quad \Downarrow$$

$$x_0^2 - x_3^2 \lesssim \frac{4}{Q^2} < \frac{4}{Q^2} \quad \Rightarrow \quad x^2 \lesssim \frac{4}{Q^2}$$

Light-cone distribution amplitudes

Hydrogen atom

Non-Relativistic Schrödinger Equation

Spherical $O(3)$ Symmetry

Separation of the angular and radial dof. in the WFs
Born—Oppenheimer approximation

Spherical Harmonics
($\propto$ an irreducible representation of $O(3)$)

Associated Laguerre Polys.
($\propto$ a trivial representation)

Rotationally Invariance
in central field approximation

Hadron LCDAs

Quantum Chromodynamics (Quantum Field Theory)

Conformal $SL(2,\mathbb{R})$ Symmetry

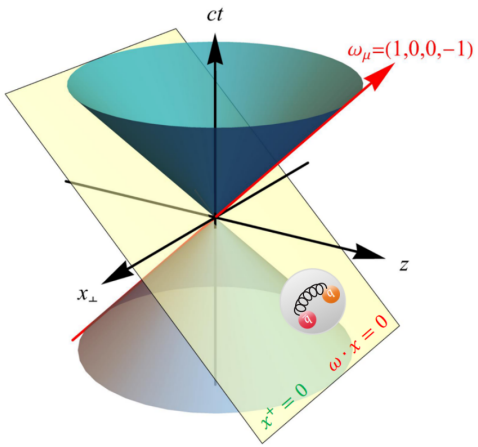
Separation of the longi. and trans. distances in the WFs
with respect to the scattering plane

Sphe.-Harm.-like Function
($\propto$ collinear subgroup of $SL(2,\mathbb{R})$)

Renornali. Group Equation
($\propto$ the scale running)

Confromal Invariance
in massless QCD

Light-cone distribution amplitudes



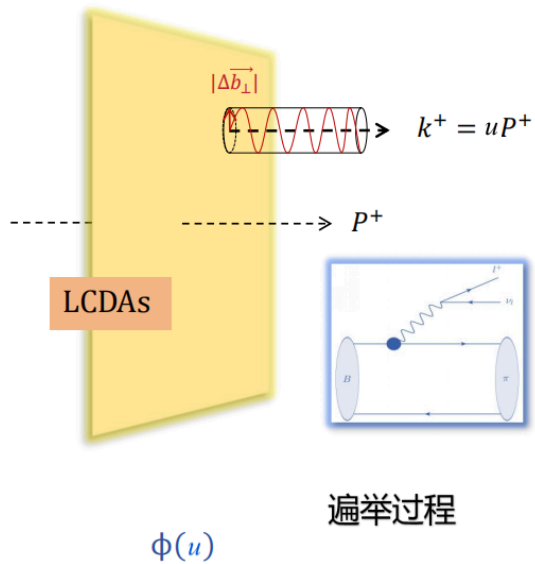
The Lorentz and gauge invariant ME

$$\langle 0|\bar{u}(x)\gamma_\mu\gamma_5 d(-x)|\pi^-(p)\rangle = f_\pi \int_0^1 du\, e^{i(2u-1)p\cdot x}\left[ip_\mu\left(\phi(u,\mu)+\frac{x^2}{4}\phi_1^4\right)+\cdots\right.$$

$$\langle 0|\bar{u}(x)\gamma_\mu\gamma_5 d(-x)|\rho^-(p)\rangle = f_\rho m_\rho \int_0^1 du\, e^{i(2u-1)p\cdot x}\left[p_\mu\frac{\epsilon^{(\lambda)}\cdot x}{p\cdot x}\left(\phi_\parallel-\phi_\perp^3\right)+\cdot\right.$$

$$\langle 0|\bar{q}(x)\gamma_\mu\gamma_5 q(-x)|f_0(p)\rangle = p_\mu \int_0^1 du\, e^{i(2u-1)p\cdot x}[\phi+\cdots]$$

- LCDAs are dimensionless functions of momentum fraction u and renorma. scale μ
- The probability amplitudes to **find the meson** in a state with minimal number of constitutes and have small transversal separation of order $1/\mu$



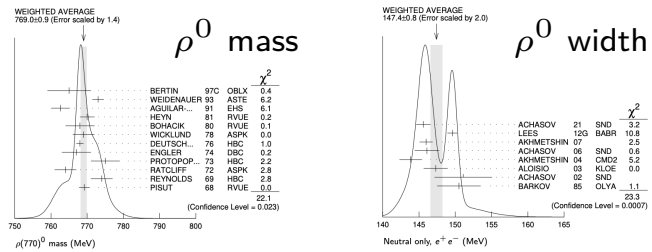
The limitations of the single particle picture

- Hadron LCDAs achieved Great Success in large Q^2 processes
  Form Factors of π, ep scattering, CPVs in $B \to \pi\pi, K\pi$ decays

- **LCDAs of V, S meson reveal certain limitations**

the $\pi\pi$ spectra in $[0.554, 0.996]$ GeV serves as the candidate region for ρ

ρ is usually treated as a stable single particle in the QCD studies of $B, D \to \rho$



- A second-best approach is the **Cascade Decay Framework**

$$\mathcal{A}(H \to \pi\pi l^+ \nu_l) = \mathcal{A}(H \to R l^+ \nu_l)\,\mathrm{BW(s)}\,\mathcal{A}(R \to \pi\pi)$$

- Model dependence, large uncertainties involved for f_0

- **Width Effects**, **Partial-Wave Interference**, **QCD Backgrounds**

Two-meson Distribution Amplitudes

- **2πDAs provide a most general description of $\pi\pi$ spectral**

  [Polyakov 1999, Diehl 1998, **SC** 2025]

$$\begin{aligned}&\langle\pi^a(k_1)\pi^b(k_2)|\overline{q}_f(zn)\gamma_\mu\tau q_{f'}(0)|0\rangle\\ =\quad &\kappa_{ab}\,k_\mu\int du\,e^{iuz(k\cdot n)}\,\Phi_\parallel^{ab,ff'}(u,\zeta,k^2,\mu)\end{aligned}$$

- Decomposed in terms of $C_n^{3/2}(2u-1)$ and $C_\ell^{1/2}(2\zeta-1)$

$$\Phi^{I=1}(u,\zeta,k^2,\mu)=6u(1-u)\sum_{n=0,\text{even}}^{\infty}\sum_{I=1,\text{odd}}^{n+1}B_{n\ell}^{I=1}(k^2,\mu)C_n^{3/2}(2u-1)C_\ell^{1/2}(2\zeta-1)$$

$$\Phi^{I=0}(u,\zeta,k^2,\mu)=6u(1-u)\sum_{n=1,\text{odd}}^{\infty}\sum_{I=0,\text{even}}^{n+1}B_{n\ell}^{I=0}(k^2,\mu)C_n^{3/2}(2u-1)C_\ell^{1/2}(2\zeta-1)$$

- **Evolution from $4m_\pi^2$ to large k^2** via the Watson theorem

$$B_{n\ell}^I(k^2)=B_{n\ell}^I(0)\,\text{Exp}\left[\sum_{m=1}^{N-1}\frac{k^{2m}}{m!}\frac{d^m}{dk^{2m}}\ln B_{n\ell}^I(0)+\frac{k^{2N}}{\pi}\int_{4m_\pi^2}^{\infty}ds\,\frac{\delta_\ell^I(s)}{s^N(s-k^2-i0)}\right]$$

在粲介子半轻衰变中理解 η-η' 混合、K^* 精细结构和 f_0 结构的能标敏感性

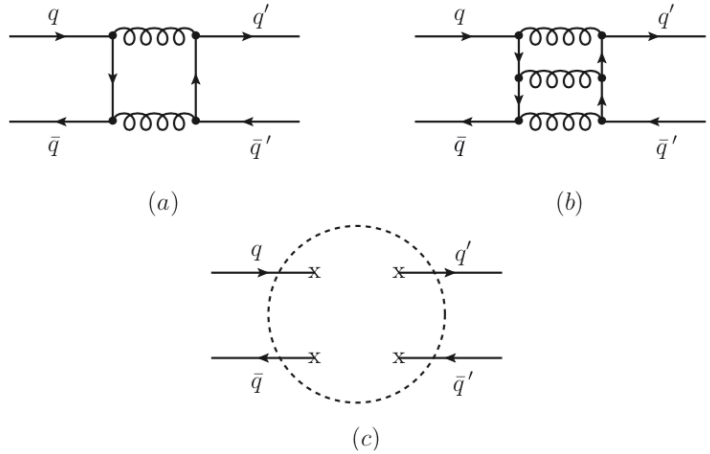
[X.E. Huang, **SC** and D.L. Yao, PRD 113 (2026) 053005]

[W. Lin, X.E. Huang, **SC** and D.L. Yao, PRD 111 (2025) 113005]

[**SC**, L.Y. Dai, J.M. Shen and S.L. Zhang, PRD 113 (2026) L031901]

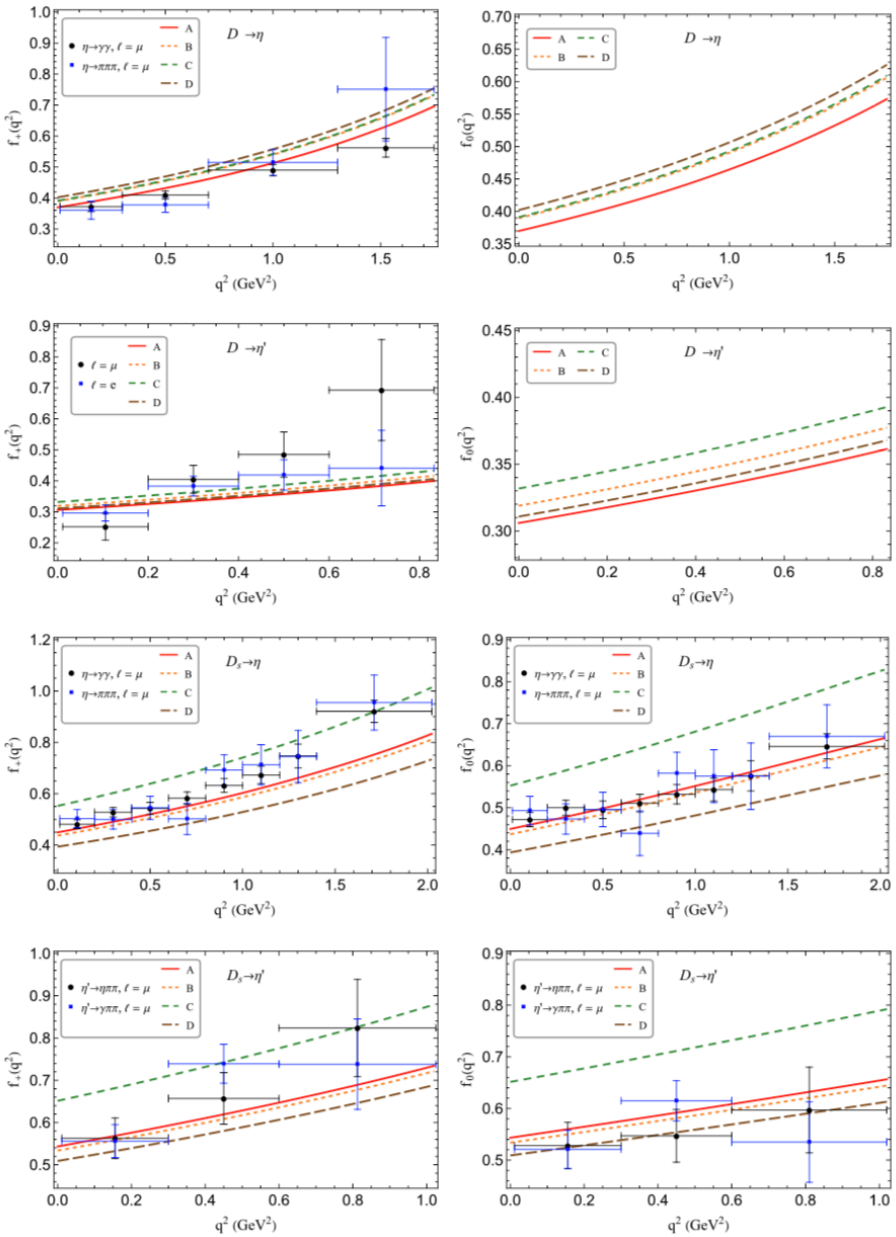
η-η'-mixing

- $q\bar{q} \to q'\bar{q}'$ **transtions** at lowest order in the pQCD theory
- One-gluon transition is not allowed due to the color charge conservation



- The spin-parties $J^{PC} = 0^{++}, 0^{-+}, 2^{++}$ are possible for a two-gluon state The gluon field has a negatice C-parity ⇓
- a $q\bar{q}$ state with $J^{PC} = 1^{--}$ can not couple to two-gluon state (a, $\mathcal{O}(\alpha_s^2)$), but can be realized with three-gluons (b, $\mathcal{O}(\alpha_s^3)$)
- The mixing is enhanced for η-η', while suppressed for ϕ-ω
- The characteristic momenta transferred via gluons $\mathcal{O}(m_b, m_c, \Lambda_{\rm QCD})$
- **The mixing intensity is, in fact, process dependent.**

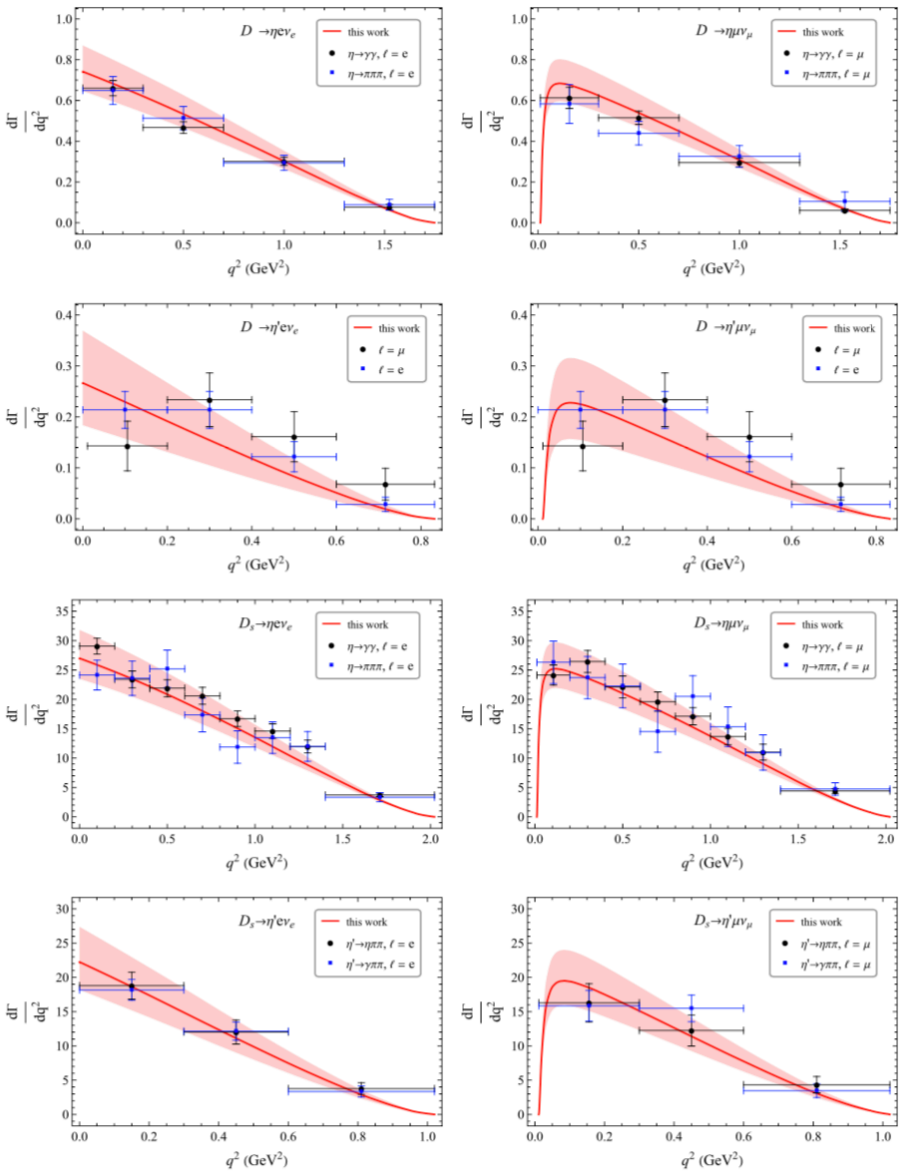
$D_{(s)} \to \eta^{(\prime)}$ Form Factors



	Twist-2 LO	Twist-2 NLO	Twist-3 LO	Twist-3 NLO	Twist-(4 + 5 + 6) LO
$f_{+,0}^{D\to\eta}(0)$	0.127	0.025	0.254	−0.035	-4.86×10^{-4}
$f_{+,0}^{D\to\eta'}(0)$	0.105	0.021	0.210	−0.029	-4.02×10^{-4}
$f_{+,0}^{D_s\to\eta}(0)$	0.153	0.031	0.317	−0.051	-1.05×10^{-4}
$f_{+,0}^{D_s\to\eta'}(0)$	0.185	0.037	0.383	−0.061	-1.27×10^{-4}

- Good OPE convergence
- Twist-3 contributions are dominate (chiral enhancement)
- NLO corrections 2%–3%
- Higher-twist and three-particle effects are negligible
- $D_{(s)} \to \eta'$ FFs show enhanced sensitivity to the **gluonic DA**

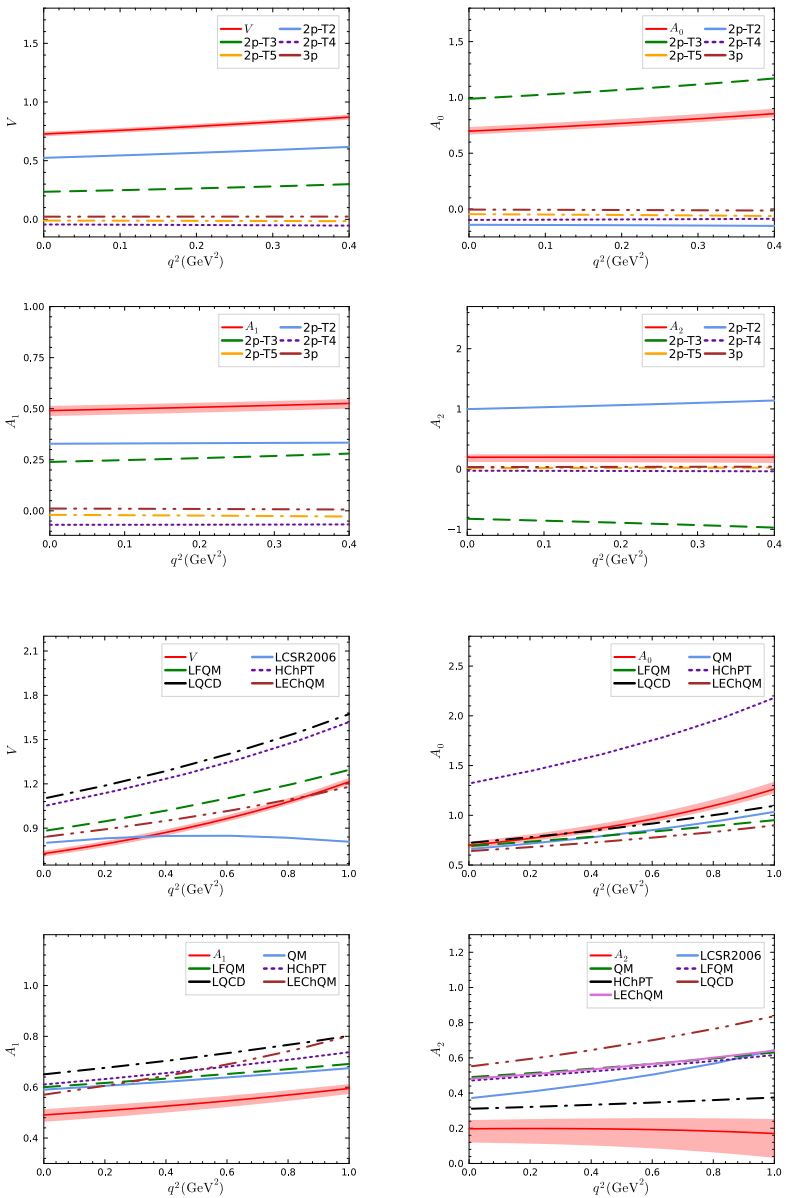
$D_{(s)} \to \eta^{(\prime)} l^+ \nu_l$ decays (10^{-3})



	$D^+ \to \eta e^+ \nu_e$	$D^+ \to \eta \mu^+ \nu_\mu$	$D^+ \to \eta' e^+ \nu_e$	$D^+ \to \eta' \mu^+ \nu_\mu$
This work	$0.99^{+0.17}_{-0.13}$	$0.97^{+0.17}_{-0.12}$	$0.16^{+0.06}_{-0.05}$	$0.15^{+0.06}_{-0.05}$
PDG [70]	1.11 ± 0.07	1.04 ± 0.11	0.20 ± 0.04	—
BESIII [61]	$0.98^{+0.29+0.28}_{-0.29-0.28}$	$0.91^{+0.35+0.23}_{-0.35-0.23}$	—	—
BESIII [63]	—	—	$0.18^{+0.02+0.01}_{-0.02-0.01}$	$0.19^{+0.03+0.01}_{-0.03-0.01}$
	$D_s^+ \to \eta e^+ \nu_e$	$D_s^+ \to \eta \mu^+ \nu_\mu$	$D_s^+ \to \eta' e^+ \nu_e$	$D_s^+ \to \eta' \mu^+ \nu_\mu$
This work	$20.35^{+3.41}_{-2.52}$	$19.96^{+3.35}_{-2.48}$	$7.80^{+1.83}_{-1.45}$	$7.45^{+1.75}_{-1.38}$
LCSR [23]	$23.46^{+4.18}_{-3.31}$	$23.20^{+4.13}_{-3.27}$	$7.92^{+1.41}_{-1.18}$	$7.73^{+1.38}_{-1.15}$
PDG [70]	22.70 ± 0.60	22.40 ± 0.70	8.10 ± 0.60	8.00 ± 0.60
BESIII [62]	—	$22.35^{+0.51+0.52}_{-0.51-0.52}$	—	$8.01^{+0.55+0.28}_{-0.55-0.28}$

- $D_s \to \eta^{(\prime)} l \nu$ decays $(c \to s)$, experimental precision exceeds tthe LCSR by a factor of 4
- $D \to \eta^{(\prime)} l \nu$ decays $(c \to d)$, LCSR predictions remain comparable to the PDG averages
- Small decay constants and large mixing angle

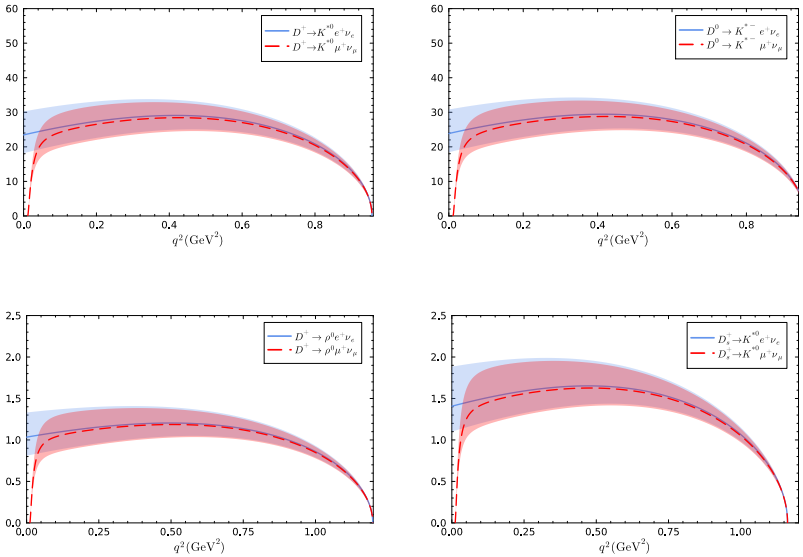
$D_{(s)} \to \rho, K^*$ Form Factors



transitions	$D^0 \to \rho^-$		$D_s^+ \to K^{*0}$		$D^+ \to K^{*0}$	
ratios	r_V	r_2	r_V	r_2	r_V	r_2
This work	$1.48^{+0.07}_{-0.05}$	$0.40^{+0.09}_{-0.15}$	$1.73^{+0.10}_{-0.07}$	$0.33^{+0.13}_{-0.25}$	$1.57^{+0.09}_{-0.06}$	$0.54^{+0.09}_{-0.16}$
LCSR2006	$1.34^{+0.16}_{-0.13}$	0.62 ± 0.08	$1.31^{+0.19}_{-0.16}$	$0.53^{+0.09}_{-0.16}$	$1.39^{+0.09}_{-0.10}$	$0.60^{+0.09}_{-0.08}$
QM	1.53	0.83	1.82	0.74	1.56	0.74
LFQM	1.47	0.78	1.55	0.82	1.36	0.82
LEChQM	1.47	0.84	1.53	0.91	1.56	0.89
HChPT	1.72	0.51	1.93	0.55	1.60	0.50
LQCD	1.69	0.85	–	–	$1.47^{+0.14}_{-0.13}$	0.60 ± 0.07
CLEO	1.48 ± 0.15	0.83 ± 0.11	–	–	–	–
BESIII	1.55 ± 0.09	0.82 ± 0.06	1.67 ± 0.38	0.77 ± 0.29	1.48 ± 0.06	0.70 ± 0.05

- V, A_1 are dominated by twist-2 LCDA, twist-3 also contribute significantly, higher-twists have minor influences
- A_2 is significantly suppressed due to the large concellation between twist-2 and twist-3, r_2 is sensitive to higher-twists
- A_0 is dominated by twist-3 DAs

$D_{(s)} \to \rho, K^* l^+ \nu_l$ decays (10^{-3})



	$D^+ \to \rho^0 \ell^+ \nu_\ell$	$D_s^+ \to K^{*0} \ell^+ \nu_\ell$	$D^0 \to K^{*-} \ell^+ \nu_\ell$	$D^+ \to \bar{K}^{*0} \ell^+ \nu_\ell$
This work	$1.96^{+0.31}_{-0.26}$	$1.25^{+0.23}_{-0.17}$	$15.0^{+2.4}_{-2.2}$	$37.5^{+6.0}_{-5.4}$
	$1.88^{+0.29}_{-0.24}$	$1.20^{+0.21}_{-0.15}$	$14.2^{+2.2}_{-2.0}$	$35.5^{+5.4}_{-4.9}$
LCSR2006	$2.29^{+0.23}_{-0.16}$	$2.33^{+0.29}_{-0.30}$	21.2 ± 0.9	$53.7^{+2.4}_{-2.3}$
	$2.20^{+0.21}_{-0.16}$	$2.24^{+0.27}_{-0.29}$	20.1 ± 0.9	$51.0^{+2.3}_{-2.1}$
CLFQM	2.32	1.90	–	73.2
	2.22	1.82		69.3
HChPT	2.50	2.20	22.0	56.0
PDG	1.90 ± 0.10	2.15 ± 0.28	21.5 ± 1.60	54.0 ± 1.00
	2.40 ± 0.40	–	18.9 ± 2.40	52.7 ± 1.50

$$\frac{d\Gamma(D \to V l\nu)}{dq^2} = \frac{G_F^2 |V_{cq}|^2 |\vec{p}| q^2}{96\pi^3 m_D^2} \left[\sum_{i=\pm,0} |H_i(q^2)|^2 + \mathcal{O}(\delta_l) \right]$$

$$-iH_\pm = -(m_D + m_V) A_1 \pm \frac{2 m_D |\vec{p}|}{m_D + m_V} V,$$

$$-iH_0 = -\frac{m_D + m_V}{2 m_V \sqrt{q^2}} (m_D^2 - m_V^2 - q^2) A_1 + \frac{m_D^2 |\vec{p}|^2}{(m_D + m_V) m_V \sqrt{q^2}} A_2.$$

- Helicity form factor H_0 is sensitive to the relative magnitudes of A_1 and A_2 (r_2)
- $H_\pm$ is sensitive to the absolute magnitudes of A_1 and V
- Good description of $D \to \rho l\nu \Downarrow$ **Well-understood ρ LCDAs**
- Discrepancies in $D_{(s)} \to K^* l\nu \Downarrow$ **Fruitfull high-twist K^* DAs**

The controversy of $f_0(980)$ structure

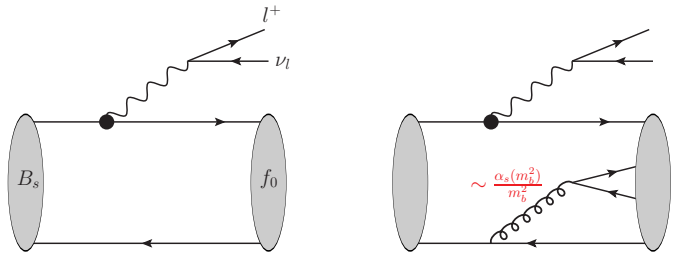
It is a quantum superposition of all possible Fock states

$$\psi_{q\bar{q}}|q\bar{q}\rangle + \psi_{q\bar{q}g}|q\bar{q}g\rangle + \psi_{q\bar{q}q\bar{q}}|q\bar{q}q\bar{q}\rangle + \cdots$$

Hadron spectroscopy provides clear evidence for the complex configuration: conventional resonant with possible gluonball component, exotic multiparticle state

The underlying partonic dynamics can be extracted directly from **Semileptonic** B, D **decays** in terms of scale-dependent LCDAs

Color transparency mechanism



High Fock states' contribution is doubly suppressed by α_s and $\mathcal{O}(1/Q^2)$

It fails in $D_s \to f_0 l^+ \nu_l$ **decays** (the produced $q\bar{q}$ is nonrelativistic)

While the energy-dependent partonic configurations is a QCD result, the cascade decay analyses of $D_s(\to f_0) \to \pi\pi e\nu$ under $q\bar{q}$ ansatz agrees with data

[EPJC 84 (2024) 379, PRD 112 (2025) 056023]

- Cascade decay analyses of $D_s(\to f_0) \to \pi\pi e\nu$ under $q\bar{q}$ ansatz

$$\frac{d\Gamma(D_s^+ \to [\pi\pi]_S\, l^+\nu)}{ds dq^2} = \frac{1}{\pi}\frac{G_F^2|V_{cs}|^2}{192\pi^3 m_{D_s}^3}|f_+(q^2)|^2 \frac{\lambda^{3/2}(m_{D_s}^2, s, q^2)\, g_1\beta_\pi(s)}{|m_S^2 - s + i(g_1\beta_\pi(s)) + g_2\beta_K(s))|^2}$$

$D_s \to f_0$ FFs have revealed the scale dependence, the seemingly agreement arises from the Flatté parameterization, with **unsound use of f_0 LCDAs under the single-meson assumption**, a fine-tuned delicate interplay ?

- Model-independent study of the signal channel $D_s \to [\pi\pi]_S e^+\nu$

$$\frac{d^2\Gamma(D_s^+ \to [\pi\pi]_S\, l^+\nu)}{ds dq^2} = \frac{G_F^2|V_{cs}|^2\, \beta_{\pi\pi}(s)\sqrt{\lambda_{D_s}}q^2}{3(4\pi)^5 m_{D_s}^3}\sum_{\ell=0}^{\infty}|F_0^{(\ell)}(q^2, s)|^2$$

- Twist-2 and twist-3 contributions to $D_s \to \pi\pi$, f_0 FFs at $q^2 = 0$

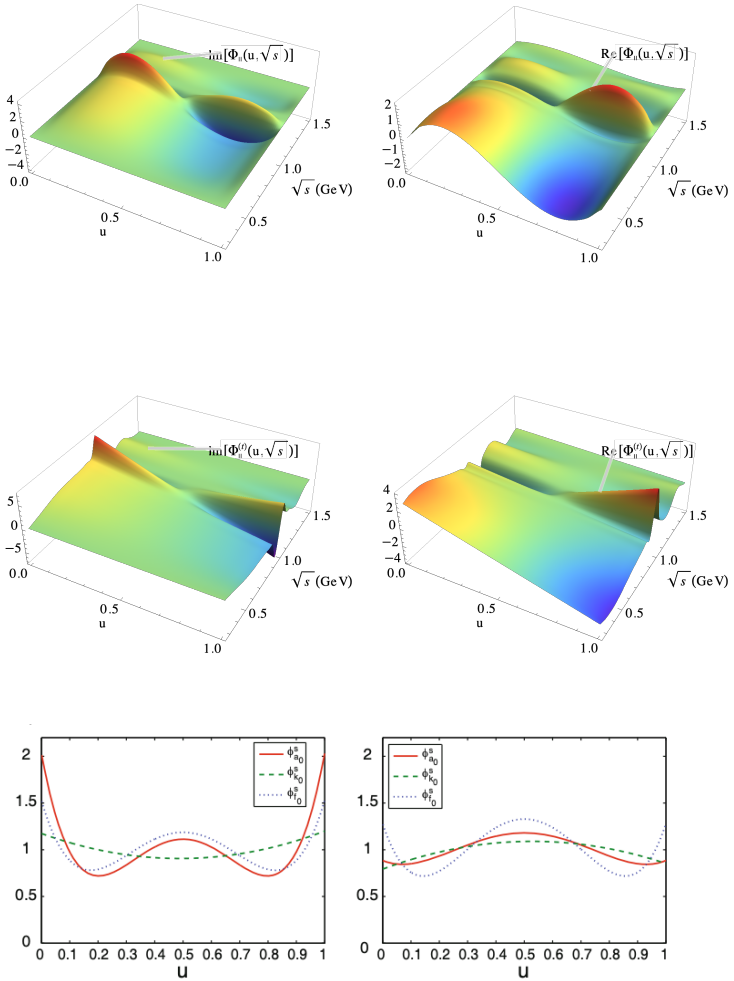
under the $q\bar{q}$ approximation

Form Factors	Twist-2	Twist-3	Total
$\sqrt{q^2}F_0^{(l=0)}(0) = \sqrt{q^2}F_t^{(l=0)}(0)$	$0.20^{+0.02}_{-0.02} - i0.24^{+0.02}_{-0.02}$	$-0.41^{+0.04}_{-0.05} + i0.51^{+0.02}_{-0.04}$	$-0.21^{+0.02}_{-0.01} + i0.27^{+0.03}_{-0.02}$
$\sqrt{q^2}F_0^{(l=2)}(0) = \sqrt{q^2}F_t^{(l=2)}(0)$	$0.27^{+0.03}_{-0.02} + i0.21^{+0.02}_{-0.01}$	$-0.55^{+0.02}_{-0.03} - i0.41^{+0.05}_{-0.04}$	$-0.28^{+0.02}_{-0.02} - i0.20^{+0.02}_{-0.01}$
$f_+(0) = f_0(0)$	$0.20^{+0.03}_{-0.05}$	$0.41^{+0.04}_{-0.06}$	$0.61^{+0.05}_{-0.07}$

destructive

constructive

Single Particle Picture or $\pi\pi$ description



[EPJA 48 (2013) 78, PRD 75 (2007) 056001]

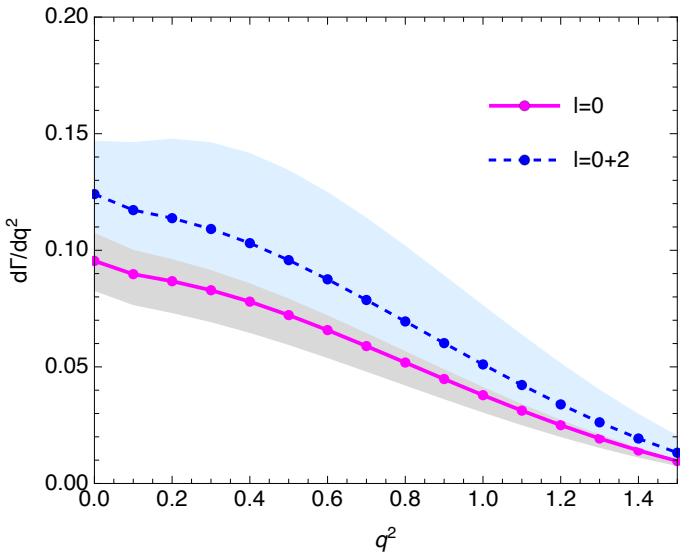
- Asymmetric twist-3 2πDAs **VS** Symmetric twist-3 f_0 LCDAs
- The single-meson picture fails to capture **the asymmetry encoded by Partial-Wave Expansion**
- Vanished asymptotic terms in twist-2 f_0 LCDAs $\Downarrow$
- Twist-3 (non-asymptotic twist-2) components dominates in CHARM (BEAUTY) decays

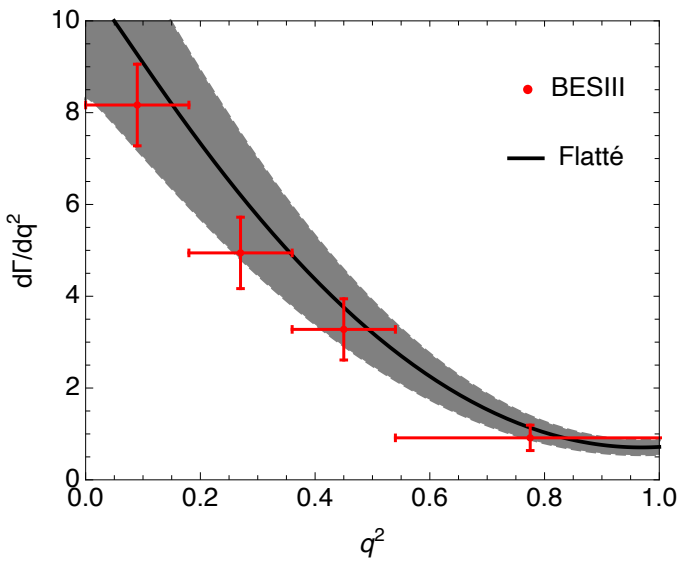
$\Updownarrow$

- Energy-dependent picture from the color transparency mecha.

$D_s \to [\pi\pi]_S\, e^+\nu$ decay

[[PRD 113 (2026) L031901]]





$D_s \to [\pi\pi]_S\, e^+\nu_e$	$D_s \to [f_0 \to \pi\pi]\, e^+\nu_e$ [23]	Data [25]
$0.81^{+0.34}_{-0.14}$	$18.8^{+4.5}_{-3.8}$	17.2 ± 1.6

- Differential widths is two-order in magnitude smaller than the data
- $q\bar{q}$ **state is not the dominate component of** $[\pi\pi]_S / f_0$ **in charm decays**, the assessment of color transparency mechanism
- Go further to multi-particle DiPion LCDAs in CHARM ($q\bar{q}g$, $q\bar{q}q\bar{q}$)
- Different in $B \to f_0$ and $D \to \rho$ decays, nonasymptotic and asymptotic twist-2 dominates respectively

Semileptonic charm and beauty decays provide an important probe of the fine structure of light hadrons in terms of LCDAs, thanks to the increasing precision of measurements and QCD calculations.

Thank you for your patience.