四川大学
SICHUAN UNIVERSITY

Study of the Composition of P_c Exotic States

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王宇飞

四川大学物理学院

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Hadron structures

Part I: Dynamical Coupled-Channel Approach

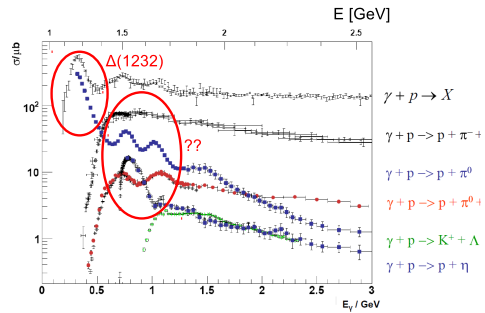
- Structures of matters $\rightarrow$ fundamental task of physics
- Structures of hadrons $\rightarrow$ extremely difficult
  - Not direct observables
  - Non-perturbative nature of QCD at low & middle energies
  - Uncertainties of the hadron spectroscopy
- A bridge between the data and the structures?
  World data $\rightarrow$??? $\rightarrow$ Spectra $\rightarrow$ Indications of the structures
- One possible option: comprehensive data-driven models
  $\rightarrow$ Jülich-Bonn Model
- Various studies:
  - Compositeness of N^* and Δ states
    [Y.F. Wang *et al.*, PRC 109, 015202 (2024)]
  - Baryon transition form factors from the proton
    [Y.F. Wang *et al.* (Jülich-Bonn-Washington Collaboration), Phys. Rev. Lett. 133, 101901 (2024)]
  - Compositeness of the P_c states
    [Y.F. Wang *et al.*, PRD 112, 074010 (2025)]



Partial Wave Analyses

Part I: Dynamical Coupled-Channel Approach

- Medium energy $\rightarrow$ involved dynamics & line-shapes
- Extraction of resonances
  $\rightarrow$ partial wave analyses (PWA)
- PW decomposition $T \sim T^{JLS} \rightarrow$ fit to the data
  $\rightarrow$ meaningful outputs
- Various methods
  - Unitary isobar models
    $\rightarrow$ unitary amplitudes + BW
    [MAID, Yerevan/JLab, KSU]
  - K-matrix Unitarization
    $\rightarrow$ on-shell intermediate states
    [GWU/SAID, BnGa, Gießen]
  - Dynamical coupled-channel (DCC) approaches
    $\rightarrow$ interaction potentials + scattering equations
    (off-shell intermediate states)
    [ANL-Osaka (EBAC), Dubna-Mainz-Taipeh]
    & Jülich-Bonn Model
  - ...



[spectra: PDG 2000. quark model calculations: Löring *et al.*, EPJA 10, 395 (2001)]

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The Jülich-Bonn Model

Part I: Dynamical Coupled-Channel Approach

Hadron dynamics

Lippmann-Schwinger-like equation

$$T_{\mu\nu}(p'', p', z) = V_{\mu\nu}(p'', p', z) + \sum_\kappa \int_0^\infty p^2 dp V_{\mu\kappa}(p'', p, z) G_\kappa(p, z) T_{\kappa\nu}(p, p', z)$$

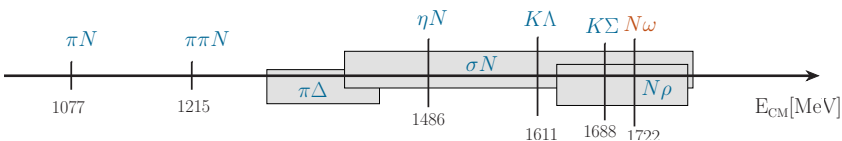
- One-dimensional: time-ordered perturbation theory + *JLS* basis [Jacob & Wick, Annals Phys. 7, 404 (1959)]
- Effective Lagrangians → flavour, chiral, CP...
- Many coupled channels & large energy region!

Photo- & electroproduction

Construction from final state interactions

$$M_{\mu\gamma^*}(Q^2) = V_{\mu\gamma^*}(Q^2) + \sum_\kappa \int p^2 dp T_{\mu\kappa} G_\kappa V_{\kappa\gamma^*}(Q^2)$$

- γ^*: the $\gamma^* N$ channel for electroproduction
- Q^2: photon virtuality
- $V_{\kappa\gamma^*}$ → phenomenologically parameterized
- Constraints: Siegert's theorem, kinematics, etc.
- Photoproduction → $Q^2 = 0$

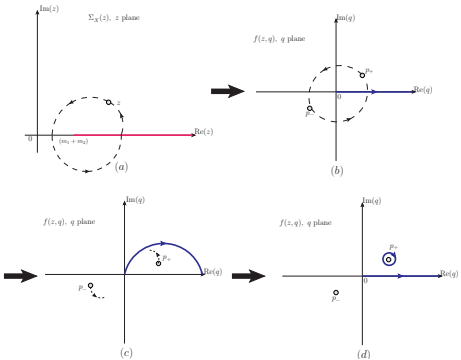


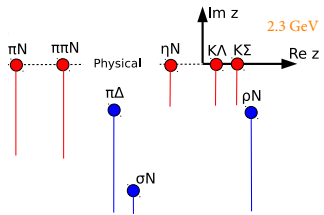
Pole searching

Part I: Dynamical Coupled-Channel Approach

- Resonances $\rightarrow$ poles on the second Riemann sheet
- Analytical continuation $\rightarrow$ contour deformation [Döring *et al.*, NPA 829, 170 (2009)]
- Pole position $z_r = M_r - i\Gamma_r/2$. Coupling strengths $\rightarrow$ normalized residues [PDG, RPP]

$$\tau^{II}_{\mu\nu} \sim \frac{R_\mu R_\nu}{z_r - z} + \cdots, NR_\mu \equiv \frac{2R_{\pi N}}{\Gamma_r} \times R_\mu$$ [Indication of the structures!]





p_+: singularities in $G = (z - E_1 - E_2)^{-1}$

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Part II: Compositeness of the P_c States

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Backgrounds

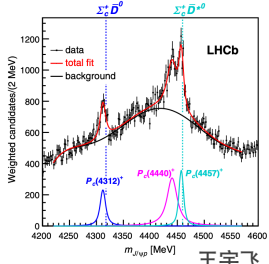
Part II: Compositeness of the P_c States

P_c as exotic states

- Exotic hadrons since $X(3872)$ [Belle, PRL 91, 262001 (2003)]
  $\rightarrow$ deviating much from conventional quark models
- Possible pictures
  - Hadronic molecules [Guo *et al.*, RMP 90, 015004 (2018)]: bound states formed by hadron-level interactions (meson exchanges etc.)
  - compact multi-quark states
  - hybrid states, kinematic cusps, etc.
- The P_c states
  - Discovered in $\Lambda_b \rightarrow J/\psi p K^-$ by LHCb [LHCb, PRL 115, 072001 (2015)]
    $P_c(4380)$ & $P_c(4450)$
  - Updated results [LHCb, PRL 122, 222001 (2019)]
    $P_c(4312)$, $P_c(4440)$, $P_c(4457)$
    $P_c(4380)$ insignificant

Interpretations

- $\bar{D}^{(*)}\Sigma_c^{(*)}$ molecules [Liu *et al.*, PRL 122, 242001 (2019)][Du *et al.*, PRL 124, 072001 (2020)][Yao *et al.*, Rept. Prog. Phys. 84, 076201 (2021)]
- Compact or cusps [Wang, Int. J. Mod. Phys. 35, 2050003 (2020)][Nakamura, PRD 103, 111503 (2021)][Burns & Swanson, PRD 106, 054029 (2022)]



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Composition of P_c

Part II: Compositeness of the P_c States

Jülich-Bonn model for P_c

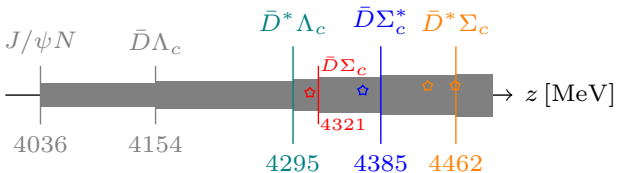
This study:

[Y.F. Wang *et al.*, PRD 112, 074010 (2025)]

- Previous studies
  [Wang *et al.*, EPJC 82, 497 (2022)][Shen *et al.*, EPJC 84, 764 (2024)]
- Three solutions → updated LHCb data (175)
- P_c as S-wave states near $\bar{D}^{(*)}\Sigma_c^{(*)}$ thresholds: $P_c(4312)\,1/2^-$, $P_c(4440)\,1/2^-$, $P_c(4380)\,3/2^-$, $P_c(4457)\,3/2^-$
- Criteria → poles, & residues

Compositeness criteria

- Dynamical generation → model dependent
- Origin of compositeness criteria: Weinberg's study of deuteron [Weinberg, PR 137, B672 (1965)]
- Modern extensions
  - Pole-counting rule[Morgan, NPA 543, 632 (1992)]
  - Spectral density functions
    [Baru *et al.*, PLB 586, 53 (2004)]
  - Gamow wave functions
    [Sekihara, PRC 95, 025206 (2017)]



Criterion I

Part II: Compositeness of the P_c States

Weinberg's criterion

- Weinberg's criterion on deuteron $|d\rangle$

$$a = -\frac{2(1-Z)}{2-Z}R + \mathcal{O}(L)$$
$$r = -\frac{Z}{1-Z}R + \mathcal{O}(L)$$

Z: "elementariness". $R \sim 4.3$ fm: deuteron radius.
$L \sim m_\pi^{-1}$ interaction range.
a: scattering length. r: effective range.

- $Z = 1 - \int d\alpha |\langle \alpha | d \rangle|^2$. α: two-nucleon continuum
- Experimental results $\rightarrow Z \simeq 0$
- Conditions: S-wave, near-threshold, stable
- Model-independent [van Kolck, Symmetry 14, 1884 (2022)]

Pole-counting rule

- Near-threshold single-channel amplitudes:
  $T \sim (rp^2/2 - ip + 1/a)^{-1}$
- One bound state p_- and one virtual state p_+:
  $|p_+| + |p_-| \geq \frac{2}{|r|}$
- A molecule $\rightarrow$ quantum mechanical potentials
  $\rightarrow |r|$ being the interaction range
  $\rightarrow$ only one pole inside $1/|r|$
- A genuine state $\rightarrow |r|$ extremely large,
  $|p_+| \simeq |p_-|$ [Castillejo *et al.*, PR 101, 453 (1956)]
  $\rightarrow$ two nearby poles
- Can be extended to coupled-channel scatterings
- Only for S-wave near threshold states
- Applications [Meng *et al.*, PRD 92, 034020 (2015)]

[Gong *et al.*, PRD 94, 114019 (2016)][Cao *et al.*, CPC 45, 103102 (2021)]

Criterion II

Part II: Compositeness of the P_c States

Spectral Density Functions

- Weinberg's Z for bound states: $Z = |\langle\psi_0|\Psi_B\rangle|^2$
- Lower decay channels: $Z \to w(E) = -\text{Im}D(E)/\pi$
  $\to$ distribution on energy E
- Narrow resonance $E = E_R - i\Gamma_R/2$:
  $w(E) \simeq \frac{Z_B}{\pi} \frac{\Gamma_R/2}{(E-E_R)^2+(\Gamma_R/2)^2}$
- S-wave nearby channels:
  $w(E) \simeq -\frac{1}{\pi}\text{Im}(E - E_0 - \sum_j ig_j^2 p_j/2)^{-1}$
  j: channels. E_0 and g_j's: from residues (couplings) and pole positions
- Elementariness $\to$ deviation of $w(E)$ from "Breit-Wigner":
  $BW(E) = \frac{1}{\pi} \frac{\Gamma_R/2}{(E-E_R)^2+(\Gamma_R/2)^2}$
- Applications [Kalashnikova and Nefediev, PRD 80, 074004 (2009)]
  [Baru *et al.*, EPJA 44, 93 (2010)][Hanhart *et al.*, EPJA 47, 101 (2011)]

Complex Compositeness

- Resonances
  - Unphysical Gamow states
    [Gamow, Zeitschrift für Physik 51, 204 (1928)]
    [Civitarese and Gadella, Physics Reports 396, 41(2004)]
  - Complex eigenstates of the Hamiltonian
    $\hat{H}|\Psi_R) = E_R|\Psi_R)$
  - Non-normalizable [Xiao and Zhou, PRD 94, 076006 (2016), J. Math. Phys. 58, 062110 (2017)]
- Complex elementariness: $Z_R = (\Psi_R^*|\psi_0\rangle\langle\psi_0|\Psi_R)$, complex-valued
- Complex compositeness $\to$ off-shell residues $r(k)$
  $T(E, p_i, p_f) \sim \frac{r(p_i)r(p_f)}{E-E_R}$
  $X = 1 - Z_R = \int_C \frac{d^3\mathbf{k}}{(2\pi)^3} \frac{r^2(k)}{[\mathcal{E}_R - k^2/(2\mu)]^2}$
  [Sekihara, PRC 95, 025206 (2017)]
- Naive measure, e.g. $\tilde{X}_j = \frac{|X_j|}{\sum_n |X_n| + |Z_R|}$

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Results I

Part II: Compositeness of the P_c States

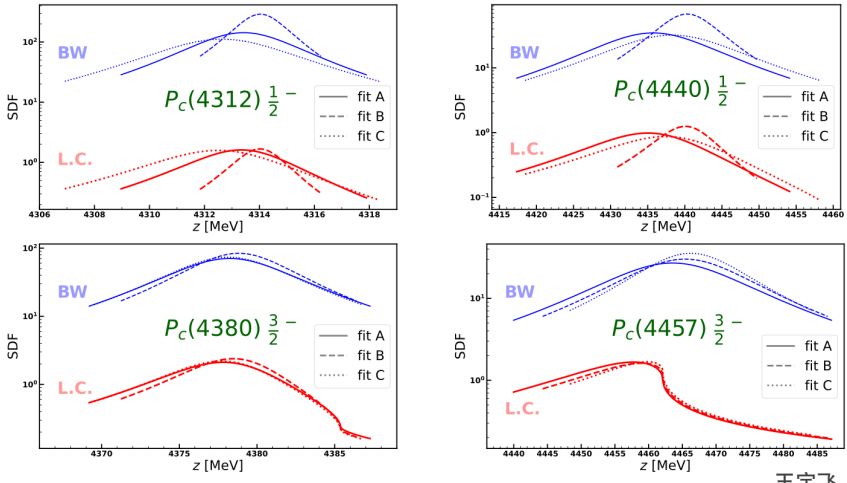
- Pole-counting rule: always only one pole
- Effective ranges: at fm level

States	Channels	a	r
$P_c(4312)\,1/2^-$	$\bar{D}\Sigma_c$	$-2.0+0.39i$	$-3.0+2.9i$
		$-2.2+0.29i$	$-3.0+2.9i$
		$-1.9+0.40i$	$-3.0+2.9i$
$P_c(4440)\,1/2^-$	$\bar{D}^*\Sigma_c$	$-1.2+0.14i$	$0.36-1.9i$
		$-1.3+0.099i$	$0.44-1.8i$
		$-1.3+0.17i$	$0.43-1.9i$
$P_c(4380)\,3/2^-$	$\bar{D}\Sigma_c^*$	$-1.9+0.29i$	$-0.84-1.4i$
		$-2.0+0.26i$	$-0.82-1.3i$
		$-1.9+0.27i$	$-0.83-1.4i$
$P_c(4457)\,3/2^-$	$\bar{D}^*\Sigma_c$	$-1.2+0.96i$	$0.78-1.6i$
		$-1.2+1.1i$	$0.77-1.6i$
		$-1.1+1.2i$	$0.76-1.7i$

Results II

Part II: Compositeness of the P_c States

Spectral density functions: one/two orders of magnitude smaller than BW!



Results III

Part II: Compositeness of the P_c States

- Gamow wave functions $\rightarrow$ fairly small elementariness
- Analyses of the compositions $\rightarrow$ the closest S-wave channels
  - $P_c(4312)\,1/2^-$: $\bar{D}\Sigma_c$ ($80\% \sim 81\%$)
  - $P_c(4440)\,1/2^-$: $\bar{D}^*\Sigma_c$ ($72\% \sim 77\%$)
  - $P_c(4380)\,3/2^-$: $\bar{D}\Sigma_c^*$ ($80\% \sim 81\%$)
  - $P_c(4457)\,3/2^-$: $\bar{D}^*\Sigma_c$ ($66\% \sim 71\%$), $\bar{D}\Sigma_c^*$ ($9\% \sim 12\%$)
- Relatively large $\bar{D}\Sigma_c^*$ configuration of $P_c(4457)$ $\rightarrow$ large coupled-channel effects
- Independent of model parameterization
- Further confirmation of the molecular interpretation!!
- A similar study on the N^* and Δ states $\rightarrow$ much more complicated
  [Y.F. Wang *et al.*, PRC 109, 015202 (2024)]

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Part III: Conclusion & Outlook

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Conclusion & Outlook

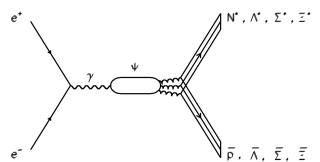
Part III: Conclusion & Outlook

Conclusions

- The Jülich-Bonn Model
  - Comprehensive dynamical coupled-channel approaches
  - Data driven PWA $\rightarrow$ resonance spectra
  - Connecting experimental observations to hadron structures!
- Compositeness of P_c
  - Three solutions $\rightarrow$ four stable P_c's
  - Generalization of Weinberg's criterion
  - Pole parameters $\rightarrow$ compositeness
  - Larger $\bar{D}^{(*)}\Sigma_c^{(*)}$ components!

Outlook

- Extensions of the JBW model
- Compositeness of mesons e.g. $\psi(3770)$, $b_1(1235)$
- Broader applications
  - BES: $J/\psi(\psi') \rightarrow \bar{N}N^*$ [BES, PRL 97, 062001 (2006)] [BES, PRL 110, 022001 (2013)] [BESIII, CPC 44, 040001 (2020)]
  - HHaS @ HIAF: π beams for πN interaction [Chen *et al.*, arXiv: 2511.22864 (2025)]




Thank
you