Quantum kinetic theory for QCD and hadron polarization in heavy ion collisions

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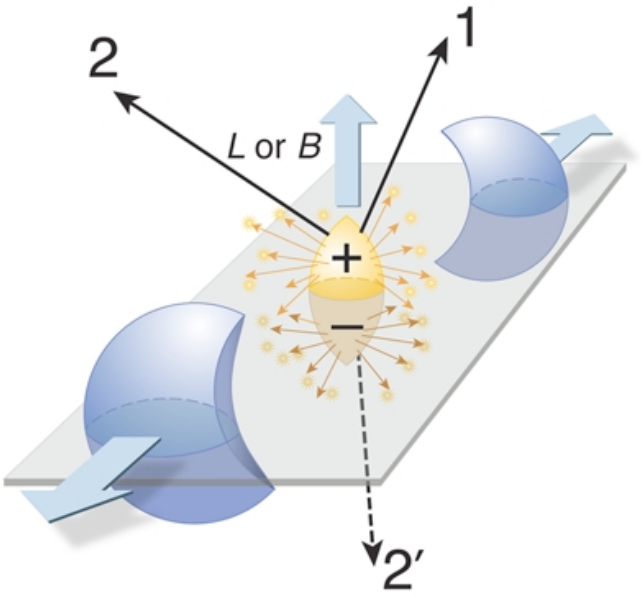
第六届量子场论及其应用研讨会 , 济南 10.16-19, 2026

SL, SciPost Phys 2026
SL, Z.y. Wang in preparation

Outline

- Spin polarization in heavy ion collisions: theoretical uncertainties
- Quantum kinetic theory for QCD
- Applications to spin polarization: toward a complete description
- Summary and outlook

Beginning of spin phenomena in HIC



$L_{ini} \sim 10^5 \hbar \rightarrow S_{final}$

Liang, Wang, PRL 2005, PLB 2005

spin-orbital coupling via parton scattering

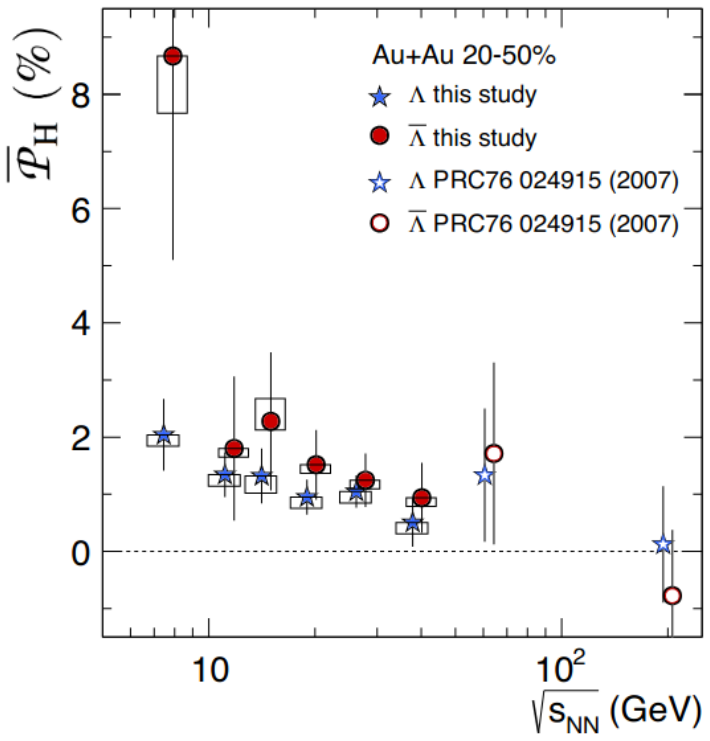
quark polarization P_q

recombination

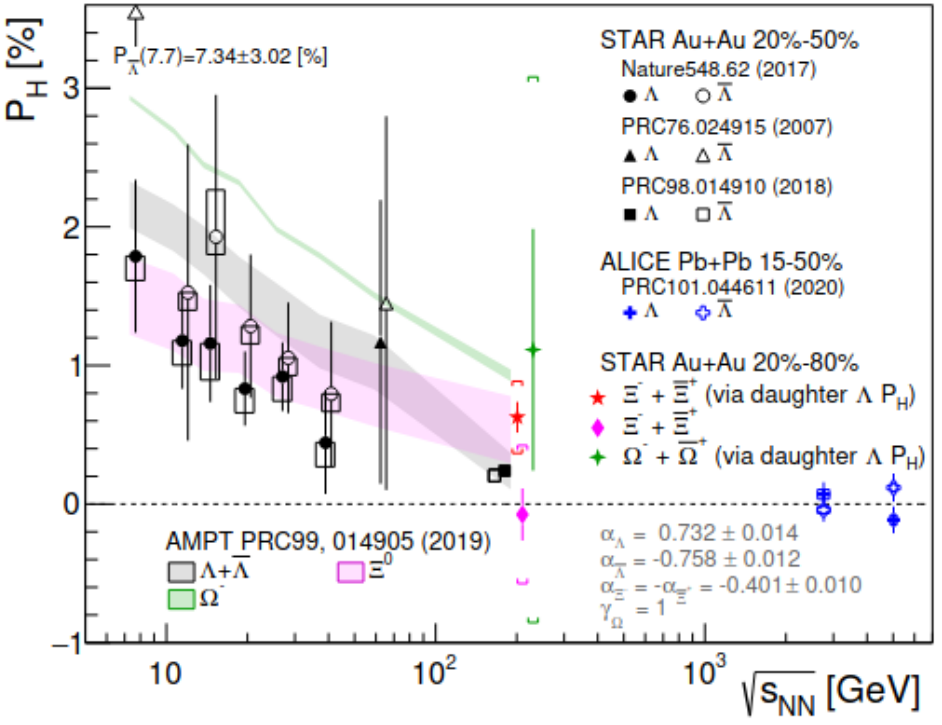
$qqq \longrightarrow B + X$ Baryon spin polarization

$q\overline{q} \longrightarrow V + X$ Vector meson spin alignment

Polarized baryons measured

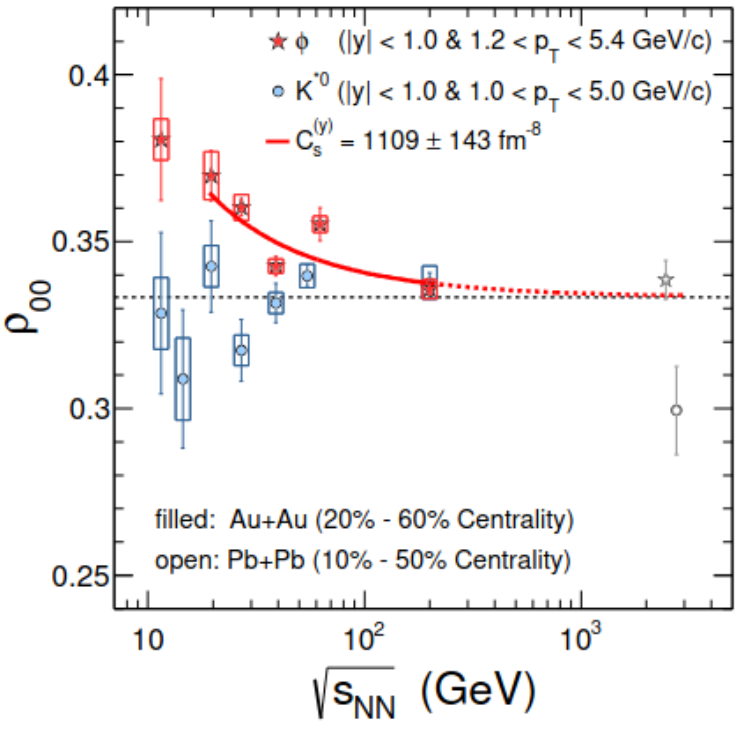


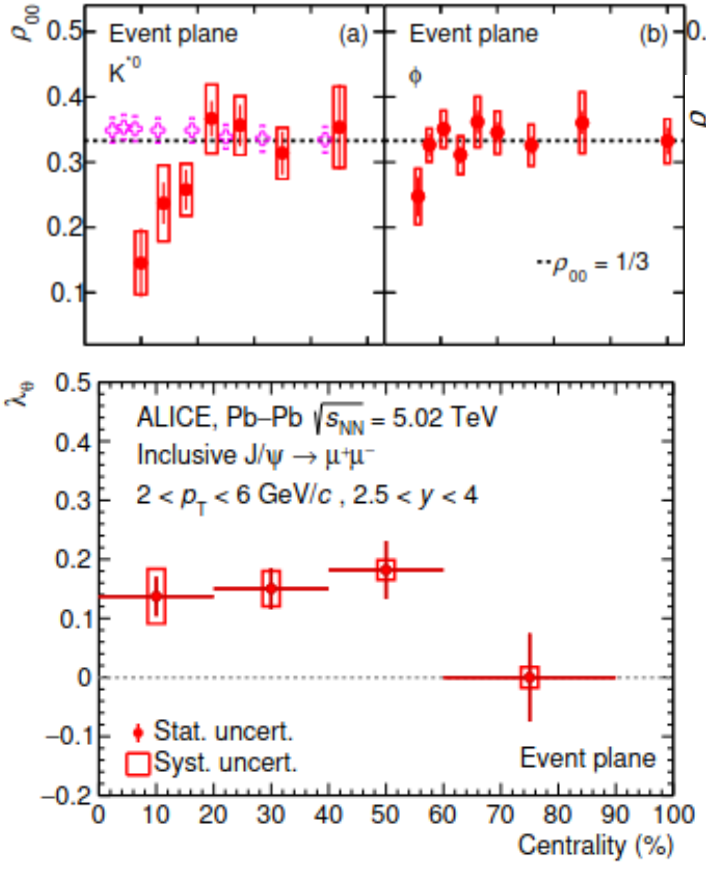
STAR, Nature 2017, PRL 2021

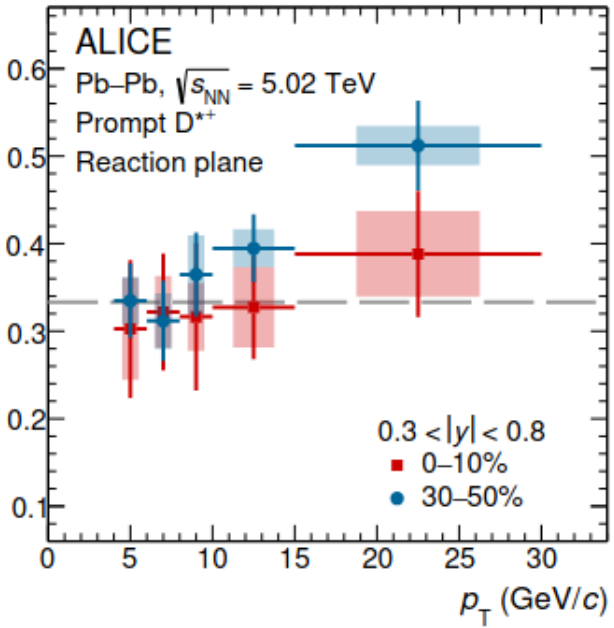


Λ, Ξ, Ω

Polarized vector meson measured







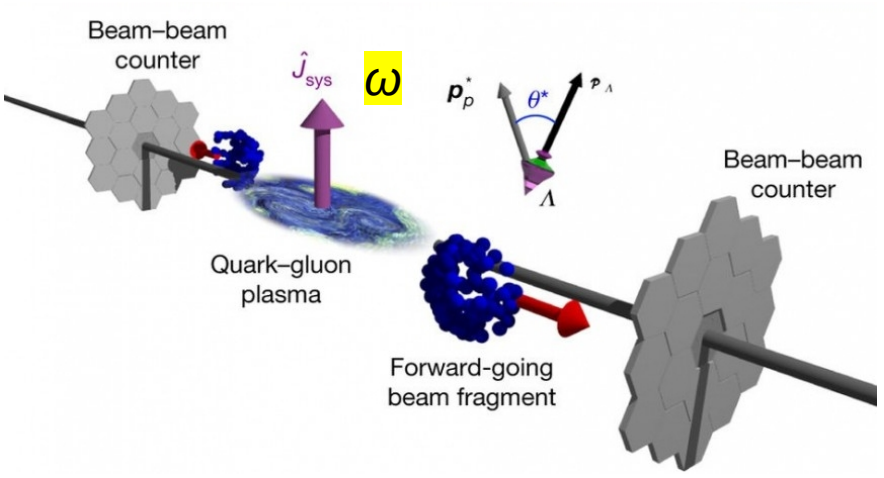
STAR, Nature 2023, ALICE PRL 2020, ALICE PRL 2023, JHEP 2025

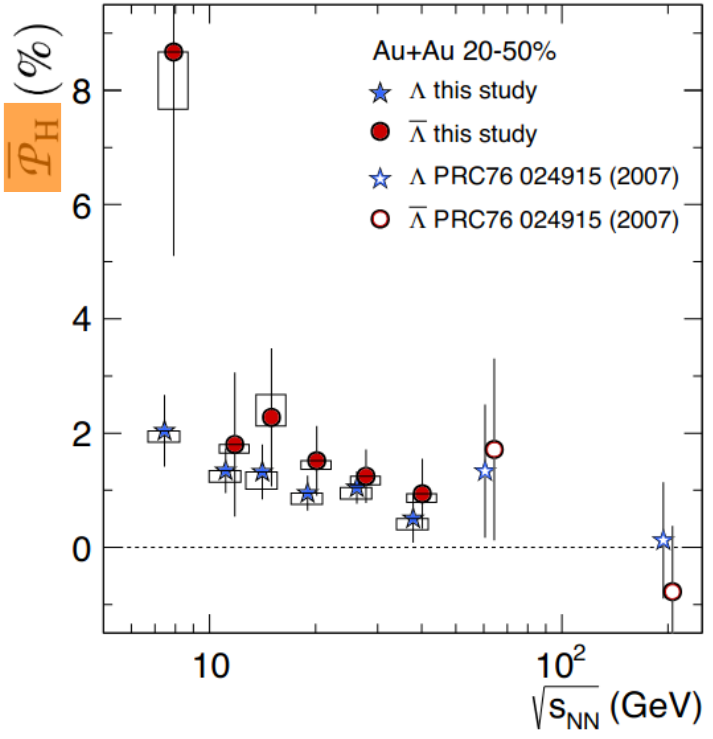
ϕ, K^{*0}, J/Ψ,D^{*+}

Global Polarization of Λ: vorticity

$$\Lambda \rightarrow p + \pi^- \quad \text{weak decay}$$

$$\frac{dN}{d\cos\theta^*} = \frac{1}{2}[1 + \alpha_H P_H \cos\theta^*]$$





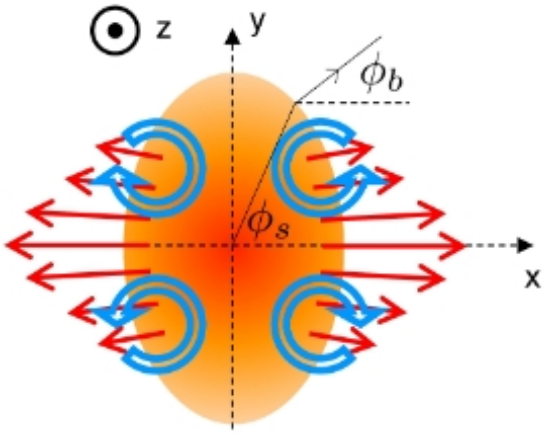
STAR, Nature 2017

Becattini et al, PRC 2017

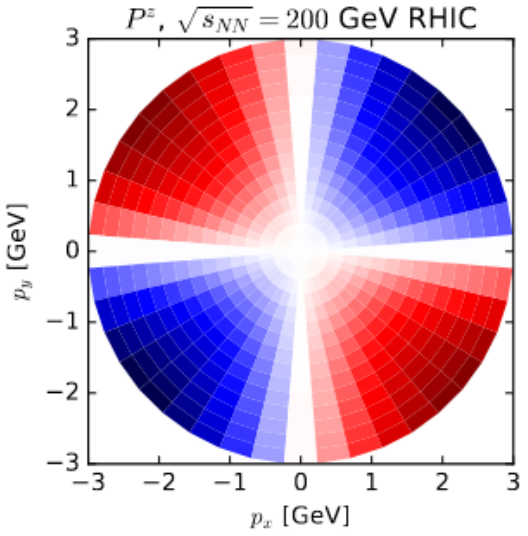
$e^{-\beta(H_0 - \mathbf{S}\cdot\omega)}$

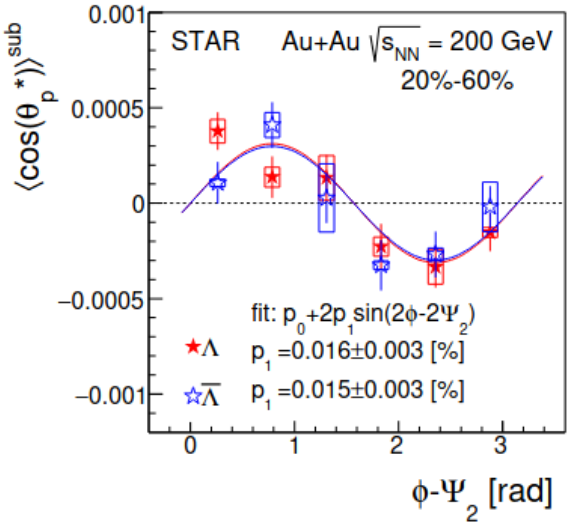
Spin couples to QGP vorticity

Local polarization of Λ: projection along beam



$$P^z \propto \omega^z$$



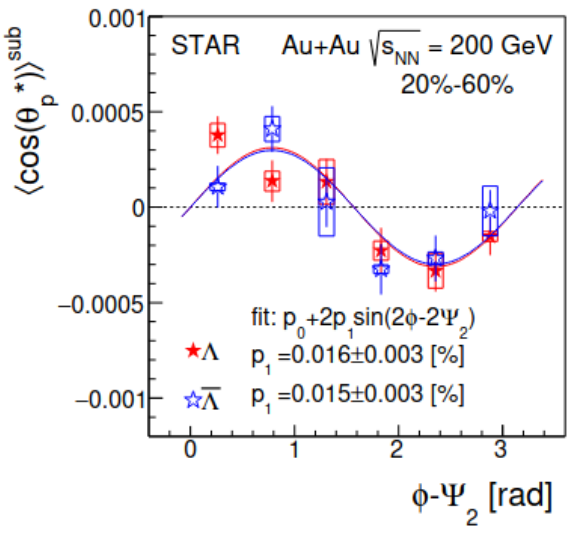


Becattini, Karpenko, PRL 2018
Wei, Deng, Huang, PRC 2019
Wu, Pang, Huang, Wang, PRR 2019
Fu, Xu, Huang, Song, PRC 2021

STAR, PRL 2019

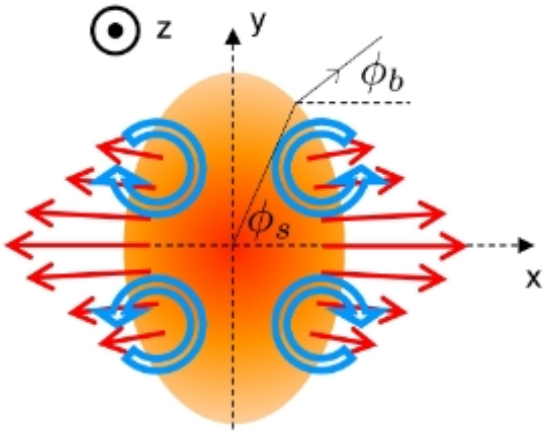
wrong sign

Local polarization of Λ: vorticity + shear



STAR, PRL 2019

vorticity



$$P^z \propto \omega^z$$

wrong sign

shear (free theory)

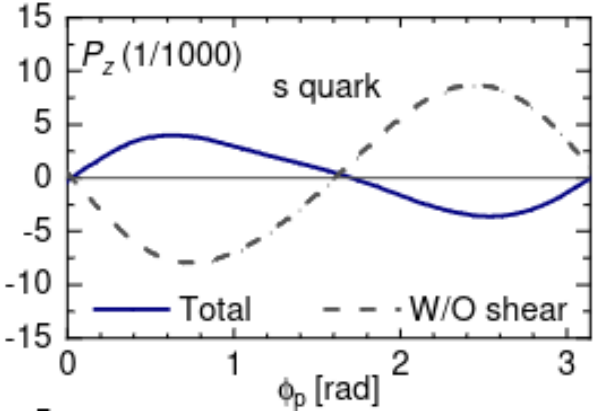
$$P^i \propto \epsilon^{ijk}\,\hat{p}_k\,\hat{p}_l\,\sigma_{jl}$$

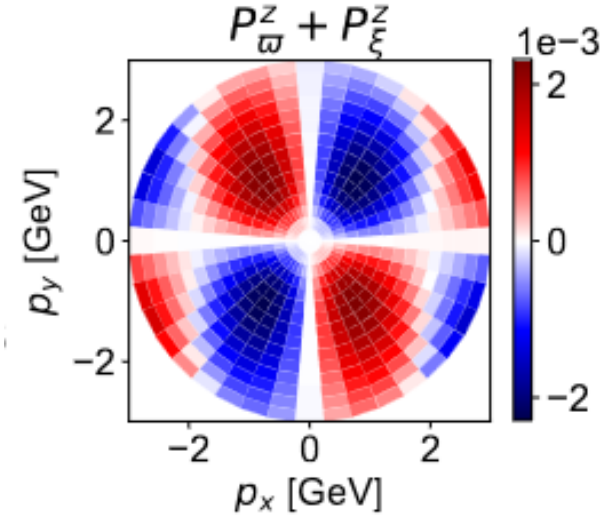
$$P^z \propto \left(\langle p_y{}^2\rangle - \langle p_x{}^2\rangle\right)\sigma_{xy}$$

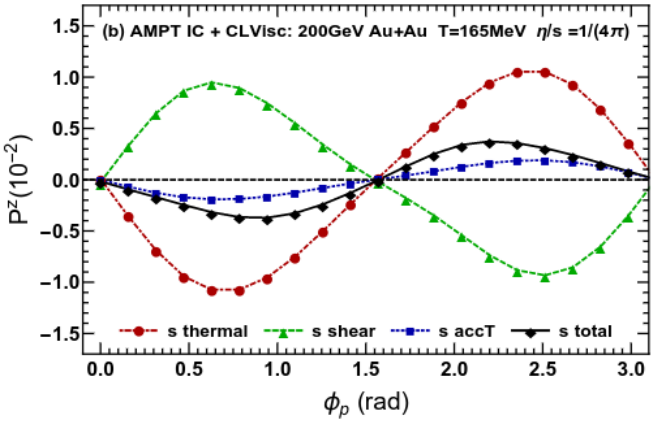
right sign

Hidaka, Pu, Yang, PRD 2018
Liu, Yin, JHEP 2021
Becattini, et al, PLB 2021

Phenomenology: vorticity + shear







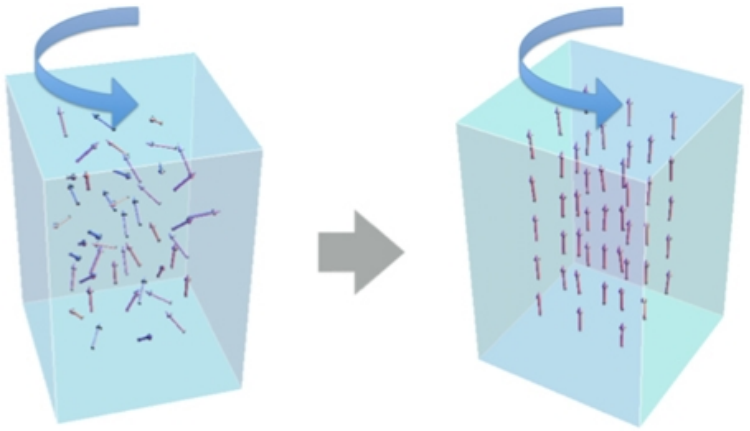
Fu, Liu, Pang, Song, Yin, PRL 2021
Becattini, et al, PRL 2021
Yi, Pu, Yang, PRC 2021

competition between shear and vorticity

shear contribution may or not reverse the sign

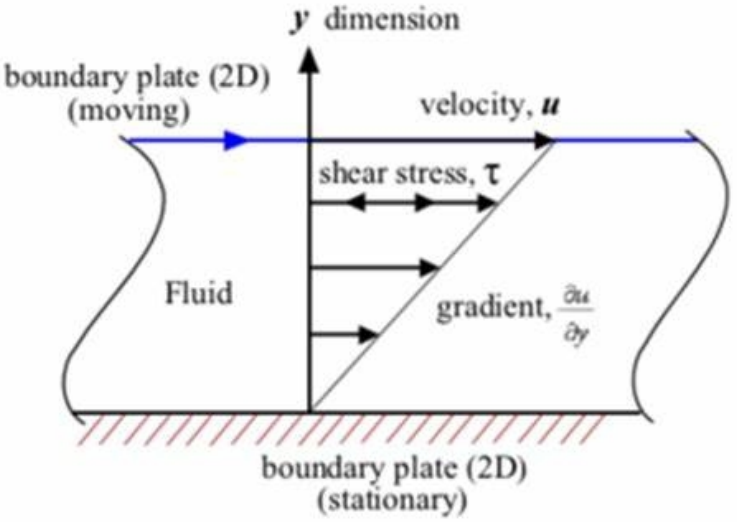
Uncertainties in local polarization: collision effect

$P^z \propto \omega^z$ ~ *B*



Equilibrium: collision vanishes

$P^i \propto \epsilon^{ijk}\, \hat{p}_k\, \hat{p}_l\, \sigma_{jl}$ ~ *E*



Off-equilibrium: collision nonvanishing

Uncertainties in local polarization: structure effect

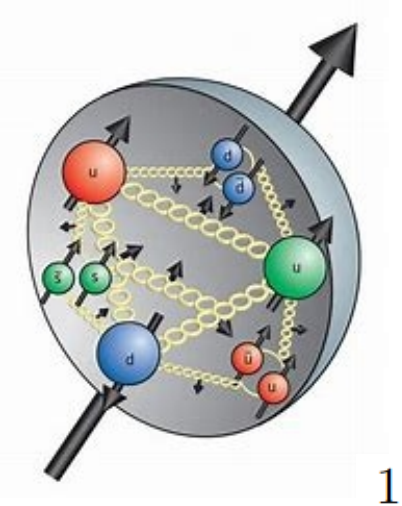
simplified structure

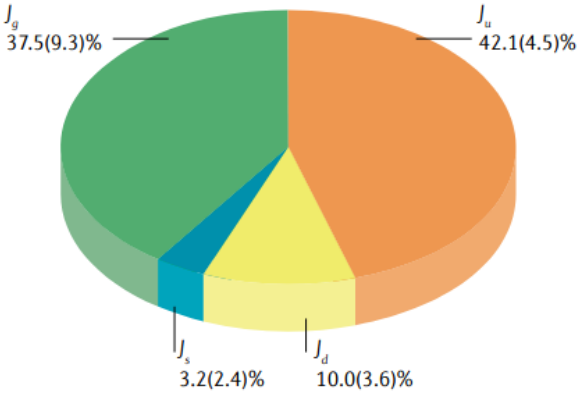
Λ: point particle

Λ: quark model



reality





$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + L_q + \Delta G + L_g$$

~40%

Alexandrou et al, PRD 2020

Hadron polarization: toward a complete description

quark polarization in QGP → (hadron structure) → hadron polarization

collisional effect

structure effect

Z. Wang, SL, JHEP 2022, PRDs 2025
Fang, Pu, Yang, PRD 2022, 2024, 2025

Need QCD based Quantum Kinetic Theory!

two effects not independent, only the sum gauge invariant

From microscopic theory to kinetic theory

N-body distribution $F_N(z_1, \cdots, z_N, t)$ $z_i = (x_i, v_i)$

reduced distribution $$f_s(z_1, \cdots, z_s, t) = \frac{N!}{(N-s)!} \int dz_{s+1} \cdots dz_N F_N(z_1, \cdots, z_N, t)$$

BBGKY

$$(\partial_t + v_1 \cdot \nabla_{x_1}) f_1(z_1) = \int dz_2 \, K_{12} f_2(z_1, z_2)$$

$$(\partial_t + v_1 \cdot \nabla_{x_1} + v_2 \cdot \nabla_{x_2} + K_{12}) f_2(z_1, z_2)$$

$$= \int dz_3 \, (K_{13} + K_{23}) f_3(z_1, z_2, z_3). \qquad f_1 \to f_2 \to f_3 \to \cdots$$

correlation suppressed (molecular chaos) $f_2 = f_1 f_1 + \cancel{g_2}$

assumed by physicists, proven by mathematicians for hard sphere model (Fields medal 2026)

Difficulties with QCD based QKT

- Elastic/inelastic collisions, IR divergences
- On-shell and off-shell DOF
- Gauge dependencies
- Generalization to spin DOF (QKT)

$$(\partial_t + \hat{\boldsymbol{p}} \cdot \boldsymbol{\nabla}_{\boldsymbol{x}})\, f_s(\boldsymbol{x}, \boldsymbol{p}, t) = -C_s^{2\leftrightarrow 2}[f] - C_s^{\text{“}1\leftrightarrow 2\text{”}}[f]$$

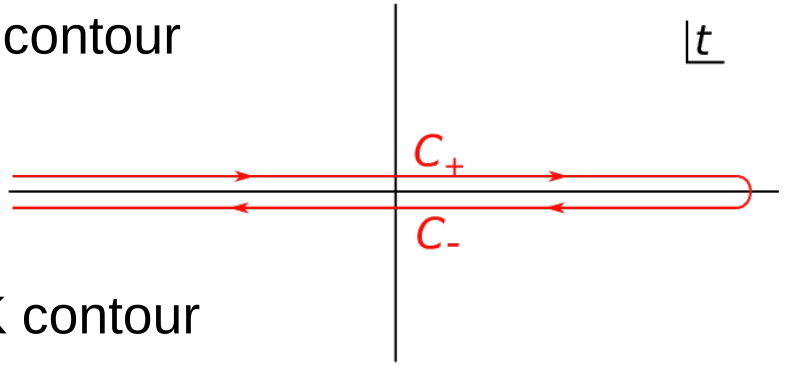
Arnold, Moore, Yaffe, early 00s

Boltzmann equation (spin-averaged kinetic theory) written down based on physical considerations, generalization to QKT difficult

Derivation of the QKT

Dyson-Schwinger equation (BBGKY) on the SK contour

truncation to
two-point function



Kadanoff-Baym equation (Boltzmann) on the SK contour

$$\left(i\partial\!\!\!/_x - m\right) S^<(x,y) = \int_z \left(\Sigma_R(x,z)S^<(z,y) + \Sigma^<(x,z)S_A(z,y)\right)$$

$$\tilde{S}^<(X = \frac{x+y}{2}, P) = \int d^4(x-y) e^{iP\cdot(x-y)} \langle S^<(x,y)\rangle \propto f(X,P)$$

Chou, Su, Hao, Yu,
Phys. Rept. 1985
Blaizot, Iancu,
Phys. Rept. 2002

- higher point functions fixed by two-point function — molecular chaos
- off-equilibrium state
- preserve conservation laws

K-B equation for quark

$$\frac{i}{2}\not{\partial} S^< + (\not{P} - m) S^< = \mathrm{Re}[\Sigma_R] S^< + \Sigma^< \mathrm{Re}[S_R] + \frac{i}{2}\left(\Sigma^> S^< - \Sigma^> S^<\right)$$

thermal mass/width

collision

real part: quasi-particle dispersion

imag part: kinetic equation

diag → $\mathrm{tr}\left(\not{\partial} S^<\right) = \mathrm{tr}\left(\Sigma^> S^< - \Sigma^> S^<\right)$ Boltzmann

off-diag → spin kinetic equation

Gradient expansion

$$\frac{i}{2}\not{\partial} S^{<} + (\not{P} - m)S^{<} = \mathrm{Re}[\Sigma_R]S^{<} + \Sigma^{<}\mathrm{Re}[S_R] + \frac{i}{2}\left(\Sigma^{>}S^{<} - \Sigma^{>}S^{<}\right)$$

$$S^{<} = S^{<(0)} + S^{<(1)} + \cdots .$$

$$S^{<}(X,P) = S^{<(0)}(X,P) + S^{<(1)}_{\mathrm{un}}(X,P) = 2\pi\epsilon(P\cdot u)\delta(P^2 - m^2)\left(\not{P} + m\right)\left(-f_q(X,P)\right)$$

$$+ S^{<(1)}_{\mathrm{pol}}(X,P) = \gamma^5\gamma_\mu\mathcal{A}^\mu + \frac{i[\gamma_\mu,\gamma_\nu]}{4}\mathcal{S}^{\mu\nu} \qquad f_q = f_q^{\mathrm{leq}} + \delta f_q.$$

$$\not{\partial} S^{<(0)} = \left(\Sigma^{>}S^{<} - \Sigma^{>}S^{<}\right)^{(1)}$$

$$f_q^{\mathrm{leq}}(X,P) = \left(\exp((P\cdot u(X) - \mu(X)/T(X)) + 1\right)^{-1}$$

hydrodynamic gradient: shear, vorticity etc

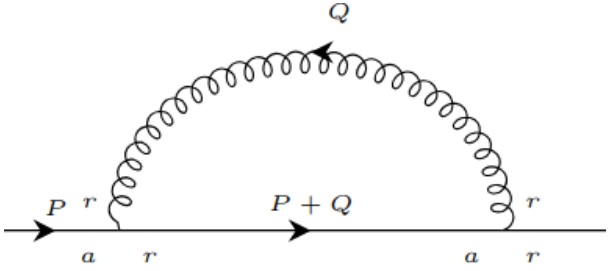
Power counting in coupling

$$\frac{i}{2}\not{\partial} S^< + (\not{P} - m) S^< = \mathrm{Re}[\Sigma_R] S^< + \Sigma^< \mathrm{Re}[S_R] + \frac{i}{2}\left(\Sigma^> S^< - \Sigma^> S^<\right)$$

thermal mass/width

collision

dispsersion correction

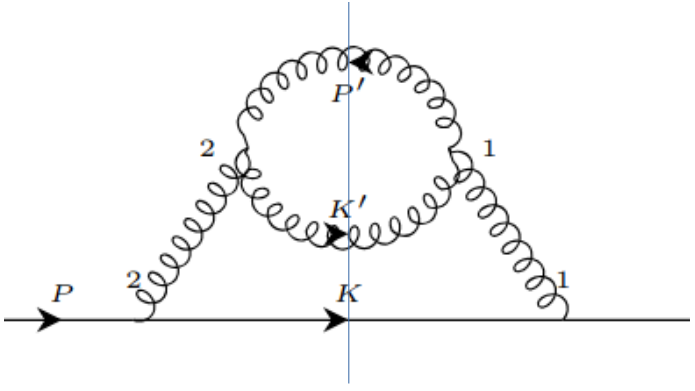


$\sim O(g^2)$

parametrically suppressed

$$\not{\partial} S^{<(0)} = \left(\Sigma^> S^< - \Sigma^> S^<\right)^{(1)} \quad \sim O(\partial\, g^0)$$

collision

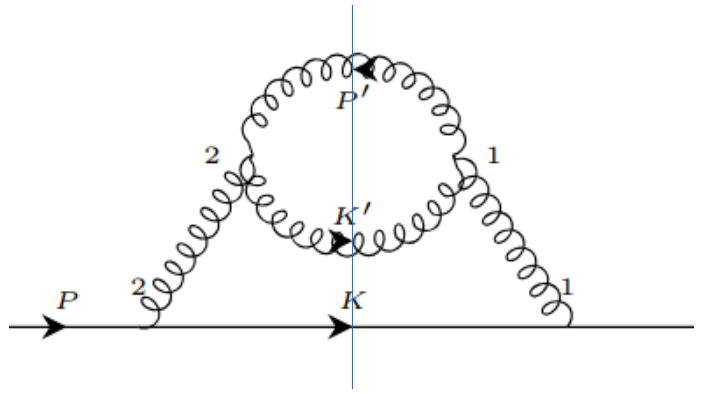


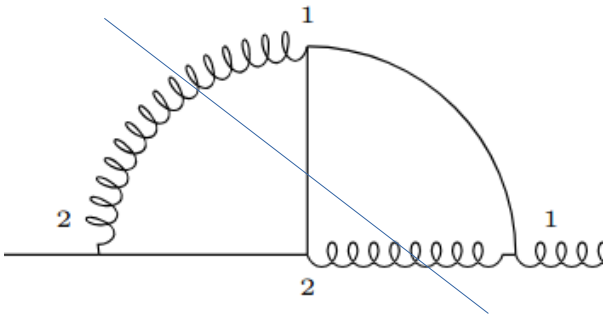
naively $\sim O(g^4)$

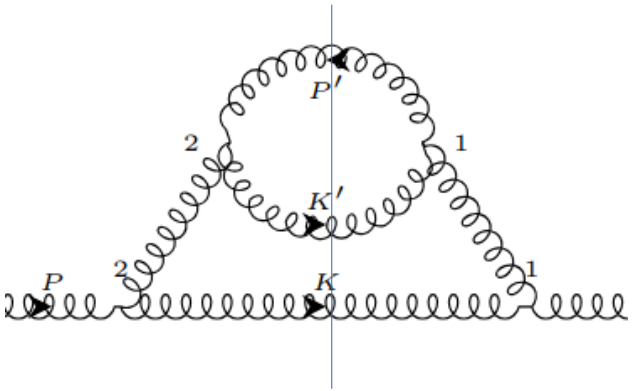
enhanced by $\delta f \sim \dfrac{\partial_X f_{\mathrm{leq}}}{g^4} \sim \tau_R \partial_X f_{\mathrm{leq}}$

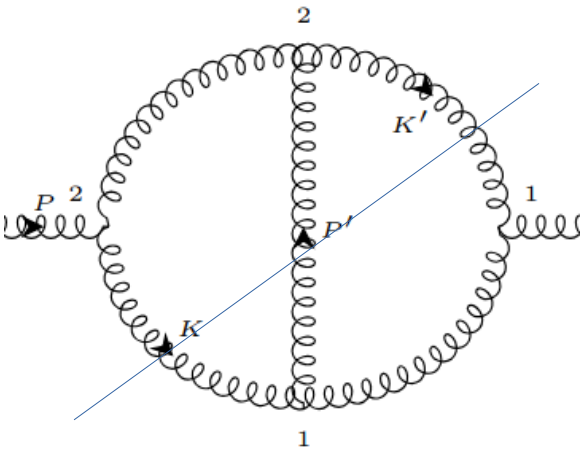
large deviation of off-equilibrium state from local equilibrium

Self-energies: elastic collisions







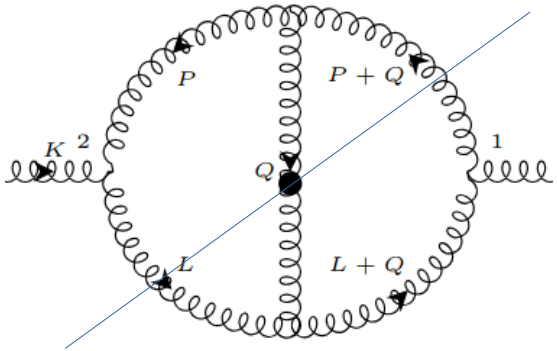


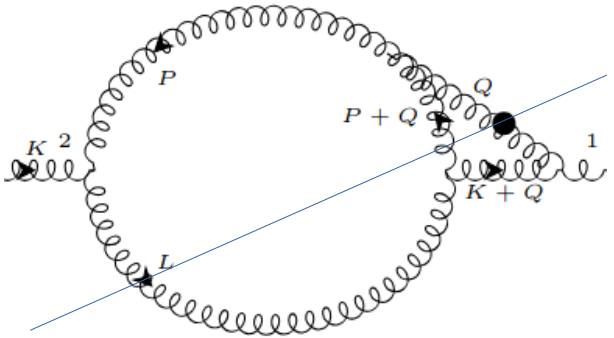
+ many other 2-loop diagrams

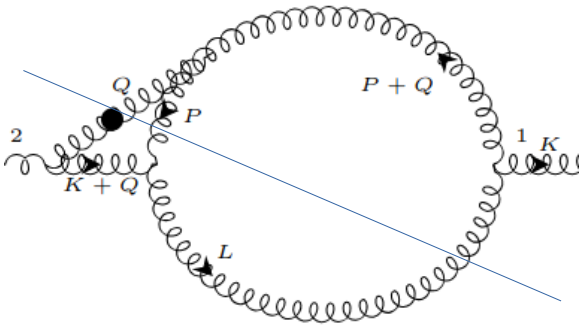


binary collisions

Self-energies: inelastic collisions





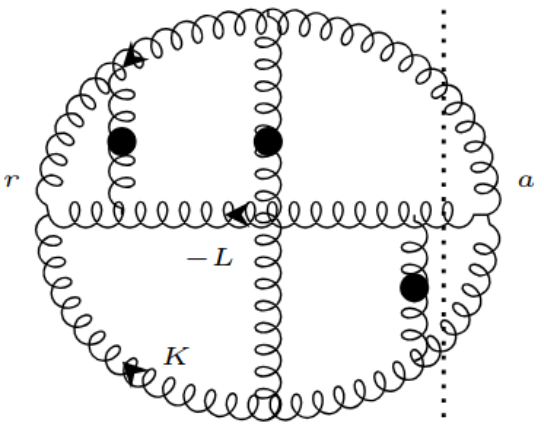


medium-induced
collinear radiation



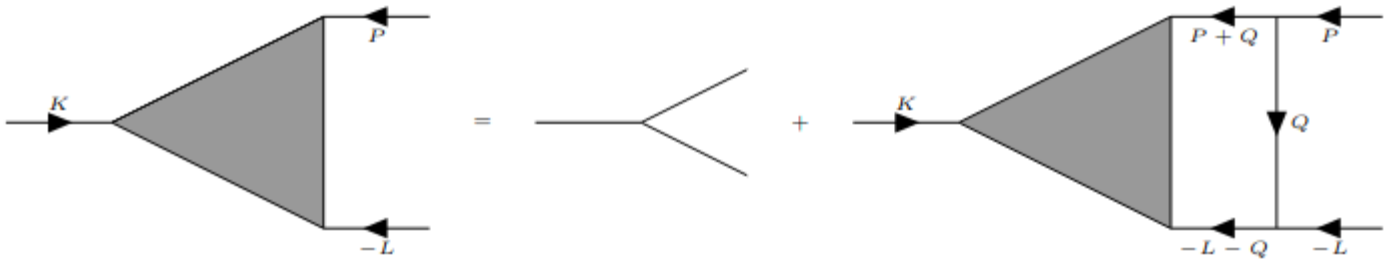
multiple interaction
with medium





3-body
“bound state”

3-g bound state interpretation



$$i\delta EF(P,-L,K) = I_0(P,-L,K) + g^2\frac{C_A}{2}\int_O 2\pi\delta(q_0-q_\parallel)D^*_{rr,\mu\nu}(Q)\hat{e}^\mu\hat{e}^\nu\Big([F(P+Q,-L-Q,K)-F(P,-L,K)]+$$

kinetic

$$[F(P+Q,-L,K-Q)-F(P,-L,K)]+[F(P,-L-Q,K+Q)-F(P,-L,K)]\Big)$$

$$\delta E = \frac{p_\perp^2+m_g^2}{2p_\parallel}+\frac{l_\perp^2+m_g^2}{-2l_\parallel}+\frac{k_\perp^2+m_g^2}{2k_\parallel}$$

3-gluon 2D “bound state”

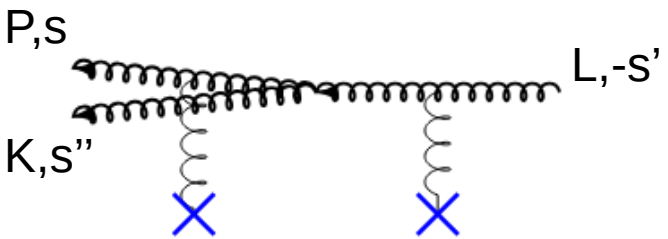
mass: $(p_\parallel, -l_\parallel, k_\parallel)$

momenta: $(\mathbf{p}_\perp, -\mathbf{l}_\perp, \mathbf{k}_\perp)$

pairwise potential

Inelastic collision rate ~ spectral density of bound state

3g-vertex in spin basis



$$\epsilon^s(L) = \frac{1}{\sqrt{2}}\left(1, is, -\frac{l^s}{l_\parallel}\right) \qquad s = \pm 1$$

$$l^s \equiv l_1 + isl_2$$

quantization axis:
collinear direction z

$$l_\parallel \sim O(1) \quad l_\perp \sim O(g)$$

$$\epsilon_i^s(P)\epsilon_j^{-s'*}(L)\epsilon_k^{s''}(K)\left[\delta_{ik}(-p+k)_j + \delta_{kj}(-k-l)_i + \delta_{ji}(l+p)_k\right]$$

$$= \delta_{s,-s''}\sqrt{2}\left[\frac{p_\parallel k_\perp - k_\parallel p_\perp}{l_\parallel}\right]^{s'} + \delta_{-s',s''}\sqrt{2}\left[\frac{k_\parallel l_\perp - l_\parallel k_\perp}{p_\parallel}\right]^{s} + \delta_{s,-s'}\sqrt{2}\left[\frac{l_\parallel p_\perp - p_\parallel l_\perp}{k_\parallel}\right]^{s''}$$

$$\Delta S = -s' \qquad \Delta L = s'$$

partial wave

$$q_1 + is'q_2 \sim Y^*_{1,-s'}$$

orbit-spin conversion
with local collisions

Application to spin polarization

$$S_{\rm pol}^{<(1)}(X,P) = \gamma^5 \gamma_\mu \mathcal{A}^\mu + \frac{i[\gamma_\mu,\gamma_\nu]}{4}\mathcal{S}^{\mu\nu}$$

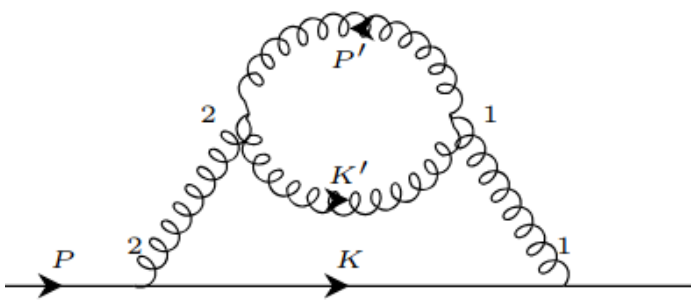
$$\mathcal{A}^\mu = -2\pi\epsilon(P\cdot u)\big[P^\mu f_q^A(P) + \frac{\epsilon^{\mu\nu\rho\sigma}P_\rho u_\sigma \mathcal{D}_\nu f_q}{2P\cdot u}\big]\delta(P^2-m^2)$$

Vorticity: same as in free theory, no collisional contribution

$$f_q^A = \frac{P\cdot\Omega}{2P\cdot u}\frac{\partial f_q}{\partial(P\cdot u)} \qquad \mathcal{D}_\nu = \partial_\nu - \Sigma_\nu^> - \Sigma_\nu^< \frac{1-f_q}{f_q}$$

Shear: collisional contribution

$$\mathcal{D}_\nu = \partial_\nu - \Sigma_\nu^> - \Sigma_\nu^< \frac{1-f_q}{f_q} \sim O(\partial\, g^0)$$



$$\delta f \sim \frac{\partial_X}{g^4}\,\hat{p}_i\hat{p}_j\sigma^{ij}$$ not suppressed by coupling

Early studies with QED:
SL, Wang, JHEP 2022, PRDs 2025
Fang, Pu, Yang, PRD 2024
Fang, Pu, PRD 2025

Gauge dependencies

QKT derived in Coulomb gauge, other gauges?

Boltzmann equation (diagonal) gauge independent

$$S_{\rm un}^{<(1)}(X,P) = 2\pi\epsilon(P\cdot u)\delta(P^2-m^2)\left(\not{P}+m\right)(-\delta f_q(X,P)),$$

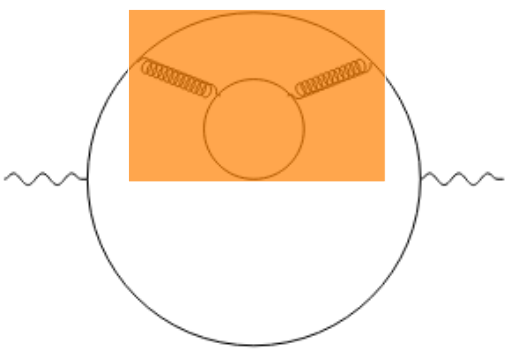
Spin polarization (off-diagonal) gauge dependent

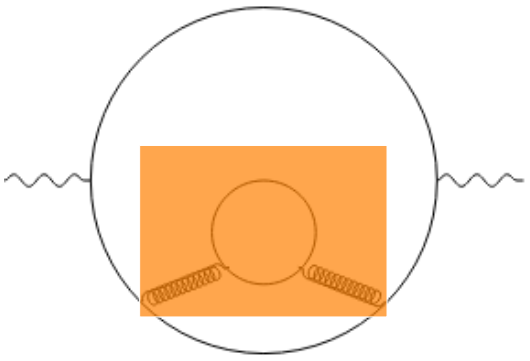
$$\mathcal{A}^\mu = -2\pi\epsilon(P\cdot u)\frac{\epsilon^{\mu\nu\rho\sigma}P_\rho u_\sigma \mathcal{D}_\nu f_q}{2(P\cdot u+m)}\delta(P^2-m^2) \qquad \mathcal{D}_\nu = \partial_\nu - \Sigma_\nu^{>} - \Sigma_\nu^{<}\frac{1-f_q}{f_q}$$

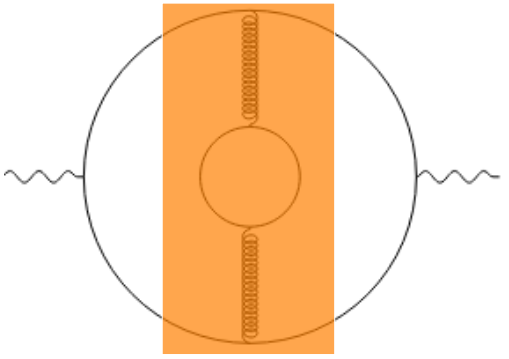
gauge dependencies expected to cancel between self-energy and vertex corrections

SL, Z.y. Wang, in preparation

From quark to vector meson polarization







Collisional: separate contributions to Q&Qbar polarization

Structure: coupled contribution to Q&Qbar polarization

$$\mathcal{P} = \mathcal{P}_Q + \mathcal{P}_{\bar{Q}}$$

Only the sum gauge invariant

SL, Z.y. Wang, in preparation

Summary and outlook

- Derived QKT for QCD. 0^{th} order reproduces Boltzmann equation, 1^{st} order gives spin kinetic equation
- Inelastic collision as a mechanism for orbit-spin conversion
- From quark spin polarization to vector meson polarization with cancelled gauge dependencies
- Extension to baryon polarization

Thank you!

qqg-vertex in spin basis

quantization axis:
collinear direction z

$$\bar{u}_s(L)\gamma^i u_s(P)\epsilon^i_s = \left(\frac{2l_\parallel}{p_\parallel}\right)^{1/2} \frac{k_\parallel p_s - p_\parallel k_s}{k_\parallel}, \quad \Delta S = -s,\ \Delta L = s$$

$$Y_{1,s}(\hat{p}) \propto p_s$$

$$\bar{u}_s(L)\gamma^i u_s(P)\epsilon^i_{-s} = \left(\frac{2l_\parallel}{p_\parallel}\right)^{1/2} \frac{k_\parallel p_{-s} - p_\parallel k_{-s}}{k_\parallel}, \quad \Delta S = s,\ \Delta L = -s$$

$$\bar{u}_s(L)\gamma^i u_{-s}(P)\epsilon^i_s = \left(\frac{1}{2l_\parallel p_\parallel}\right)^{1/2} 2mk_\parallel s. \quad \Delta S = \Delta L = 0.$$

$$Y_{1,0}(\hat{p}) \propto p_\parallel,$$

quark mass induces quark spin flip,
but conserve total spin

orbit-spin conversion
with local collisions

K-B equation for gluon

$$\left[-P^2\eta^{\mu\nu}+\frac{i}{2}\left(-2P\cdot\partial\eta^{\mu\nu}+\partial^\nu P^\mu-\frac{1}{\xi}P^{\mu\alpha}P^{\nu\beta}\partial_\beta P_\alpha\right)\right]D^{<(0)}_{\nu\rho}=\mathrm{Re}[\Pi^{T(0)}_R]P^{\mu\nu}_T D^{<(0)}_{\nu\rho}+\Pi^{<(0)}_T P^{\mu\nu}_T \mathrm{Re}[D^{R(0)}_{\nu\rho}]$$

thermal mass/width

$$+\frac{i}{2}\left(\Pi^{\mu\nu>(0)}D^{<(0)}_{\nu\rho}-\Pi^{\mu\nu<(0)}D^{>(0)}_{\nu\rho}\right)$$

collision

$$D^{<(0)}_{\mu\nu,ab}(X,P)=2\pi\epsilon(P\cdot u)\delta(P^2)P^T_{\mu\nu}f^{\mathrm{leq}}_g(X,P)\delta_{ab}$$

0^{th} order: local equilibrium

transverse

spin average $2P\cdot\partial P^{\nu\rho}_T D^{<(0)}_{\nu\rho}=\left(\Pi^{\mu\nu>}D^{<}_{\nu\mu}-\Pi^{\mu\nu<}D^{>}_{\nu\mu}\right)^{(1)}$

→ 1^{st} order: unpolarized part

$$D^{<(1)}_{\mu\nu,\mathrm{un}}(X,P)=2\pi\epsilon(p\cdot u)\delta(P^2)P^T_{\mu\nu}\delta f_g(X,P),$$