# Explicit Lorentz-invariant QED pair production rates from superstrings

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# The talk is based on the following papers:

- J. X. Lu,"Magnetically-enhanced open string pair production," JHEP **12**, 076 (2017);
- J. X. Lu, "Some aspects of interaction amplitudes of D branes carrying worldvolume fluxes," Nucl. Phys. B **934**, 39 (2018);
- J. X. Lu, "A possible signature of extra-dimensions: The enhanced open string pair production," Phys. Lett. B **788**, 480 (2019);
- Q. Jia and J. X. Lu,"Remark on the open string pair production enhancement," Phys. Lett. B **789**, 568 (2019);
- J. X. Lu, "A note on the open string pair production of the D3/D1 system," JHEP **10**, 238 (2019);
- Q. Jia, J. X. Lu, Z. Wu and X. Zhu, "On D-brane interaction & its related properties,"Nucl. Phys. B 953 (2020)114947;
- J. X. Lu and Nan Zhang, "More on the open string pair production" , Nucl. Phys. B **977** (2022) 115721;
- J. X. Lu, "Understanding the open string pair production of Dp/D0 system", JHEP **11**, 019(2023);
- J. X. Lu, "The open string pair production, its enhancement and the physics behind," PLB 848 (2024)138397;
- J. X. Lu, "The open string pair production revisited" PLB 872 (2026) 140125;
- J. X. Lu, "Explicit Lorentz invariant QED pair production rates from superstrings", arXiv:2608.08712 .

# Outline

- Motivation/introduction
- The general setup
- Explicit LI QED PPR from superstrings in diverse dimensions
- Conclusion and discussion

# Quantum Gravity (QG)/String Theory (ST)

QG needed to understand fundamental questions such as

the physics of cosmo. & BH singularities

&

the nature of DE and possibly DM.

Though still facing much debate, ST stands out as the most favorable one for QG among various approaches put forward.

This has further been reinforced recently by the bottom-up physical scattering amplitude bootstrap plus some modest requirement on the amplitude high energy behavior. For example, ST has recently been argued to emerge inevitably or uniquely.

1. Cheung et al, "Bootstrap Principle for the Spectrum and Scattering of Strings," PRL. **133** (204) 251601;
2. Cheung et al, "Strings from Almost Nothing," PRL. **136** (2026)251601 (2026).

# Quantum Gravity (QG)/String Theory (ST)

In this talk, I address a question, from top-down approach, on how to obtain the usual QFT results from the computations of ST.

There have been many previous examples in perturbative computations such as KKLT relation and the double-copy approach.

In particular, we focus here on how to obtain:

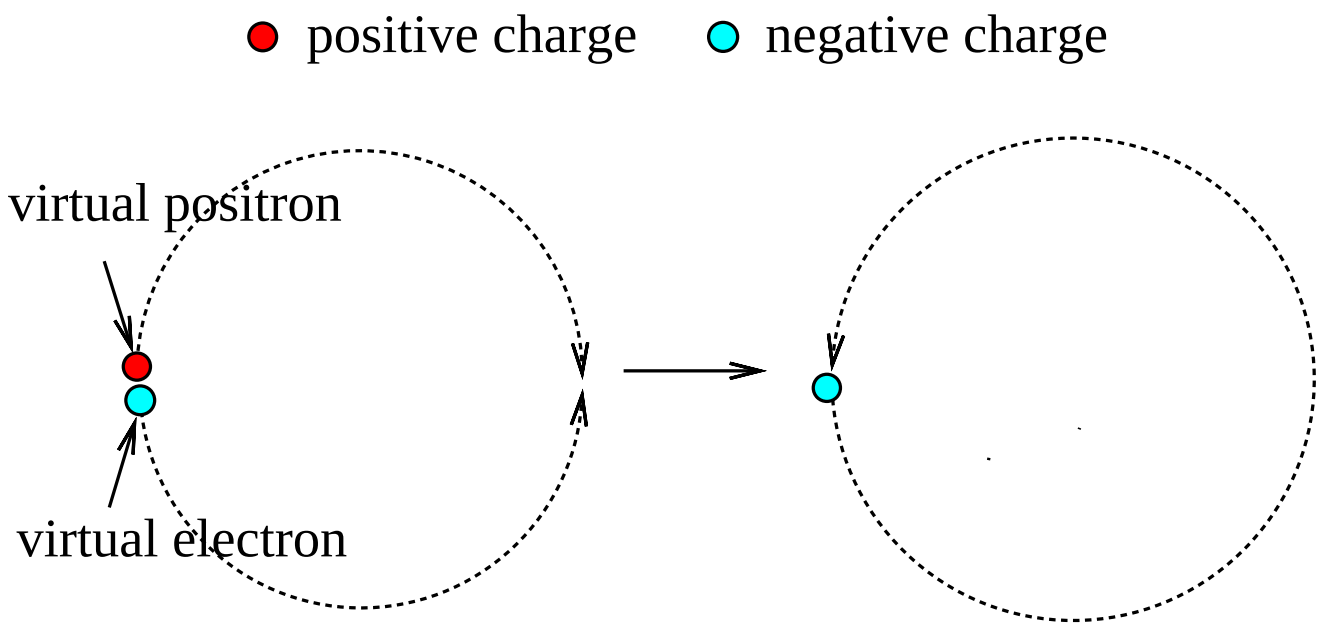
the usual non-perturbative Schwinger pair production for a pair of charged/anti-charged massive particles (scalars, spinors or vectors)

- in the most general allowed constant EM background,
- in an explicit Lorentz invariant form,
- & in diverse dimensions $2 \leq d \leq 7$.

# The QED Vacuum Picture

## VACUUM QUANTUM FLUCTUATIONS

An anti-charge moving forward in time equivalent to a charge moving backward in time




$$(\delta t \sim 10^{-21} s)$$

# D branes

For the purpose of computing PPR from superstrings in the most general allowed constant EM flux, we consider non-perturbative D-branes in oriented Type II superstring theories.

- D branes are a particular type of string solitons which were first found and classified by Duff and myself along with others in a series of publications:

  1. M. J. Duff and J. X. Lu, "The Selfdual type IIB superthreebrane," PLB273 (1991) 409-414;
  2. M. J. Duff and J. X. Lu, "Type II p-branes: The Brane scan revisited," NPB390 (1993) 276-290;
  3. M. J. Duff and J. X. Lu,"Black and super p-branes in diverse dimensions," NPB416 (1994) 301-334;
  4. M. J. Duff, R. R. Khuri and J. X. Lu, "String solitons," Phys. Rept. 259 (1995), 213-326

- They were also found in a completely different way by Polchinski et al by applying T-duality, which was found from closed strings, to open strings:

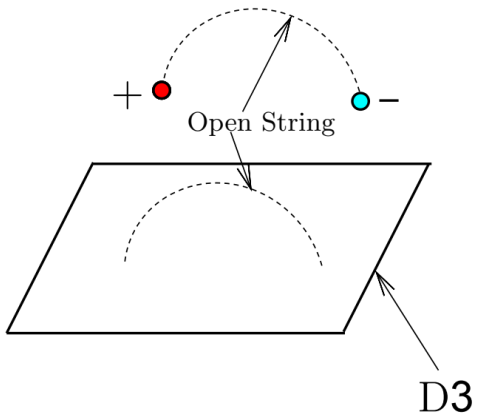
  1. J. Dai, R. G. Leigh and J. Polchinski, "New Connections Between String Theories," MPLA4(1989)2073;
  2. J. Polchinski, "Dirichlet Branes and Ramond-Ramond charges," PRL75(1995)4724

这些弦孤子发现的意义如下：

1. 纠正了早期认为的"弦与高维膜无关"的认识；
2. 为弦各种对偶性和 M-理论存在性的建立奠定了基础（弦对偶性是一种"基本弦与其高维孤子"交换的对称性，没有"孤子"的发现，无从谈起这种对偶性）；
3. 为 Polchinski 等利用从闭弦发现的 T-对偶应用到开弦的合法性以及由此（从另一个完全不同的角度）发现的弦孤子 D-膜提供了佐证；
4. 也为 AdS/CFT 对偶的发现提供了基础（不仅从 CFT 也为其引力描述提供了基础）。

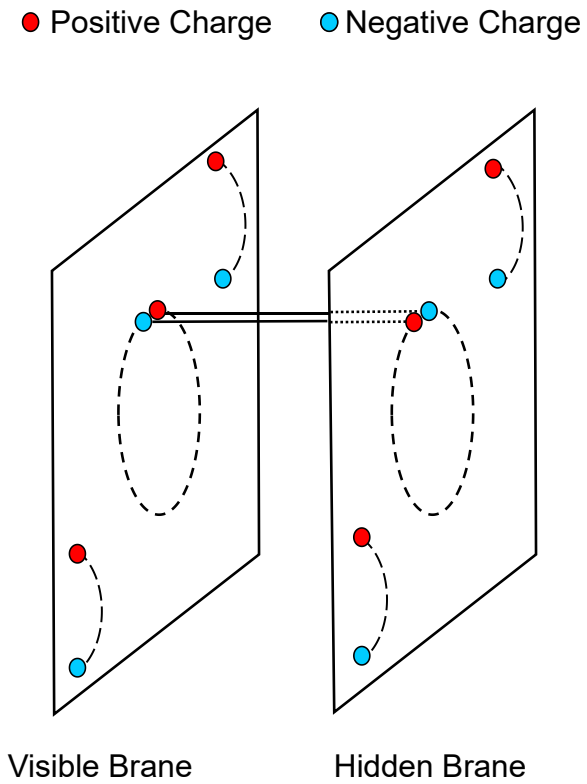
# A single isolated D-brane

Unlike QED, an isolated D-brane cannot give rise to the pair production even if one applies a constant electric field on the brane, for example, a D3 brane, mimicking our own $(1 + 3)$-dimensional world,



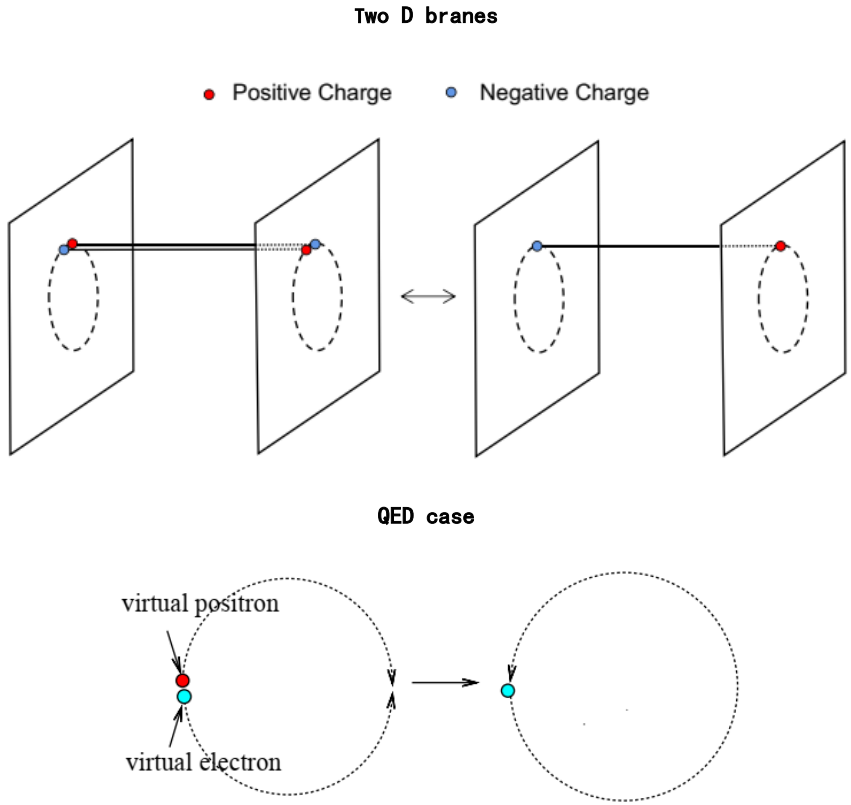
# Two D-brane system

The simplest for this purpose is to consider a system consisting of two Dp branes with $1 \leq p \leq 6$ $(d = 1 + p)$, placed parallel at a separation.



# The analogous QED vacuum

This is analogous to the QED vacuum if one looks from an observer on the visible brane.



# The OS one-loop amplitude

Similar to QED case, we need first to compute the OS one-loop annulus amplitude between the two Dp branes in the most general allowed constant EM background on the visible brane (Jia et al'20).

$$\frac{\Gamma_{p,p}}{V_{1+p}} = \frac{2^2\sqrt{\det(\mathbb{I}+\hat{F})}}{(8\pi^2\alpha')^{\frac{p+1}{2}}}\int_0^\infty \frac{dt}{t^{\frac{p-3}{2}}}\prod_{\kappa=0}^{2}\frac{\sin\pi\hat{\nu}_\kappa}{\sinh\pi\hat{\nu}_\kappa t}\left[\sum_{\kappa=0}^{2}\cosh^2\pi\hat{\nu}_\kappa t - 2\prod_{\kappa=0}^{2}\cosh\pi\hat{\nu}_\kappa t - 1\right] \times e^{-\frac{y^2 t}{2\pi\alpha'}}\prod_{n=1}^{\infty} Z_n, \tag{2.1}$$

where $\hat{\nu}_0 = i\bar{\hat{\nu}}_0$ and

$$Z_n = \frac{\prod_{a=1}^{2}\left\{\left[1-2|z|^{2n}\cos\pi\bar{\hat{\nu}}_0 t\cosh\pi c_a t + |z|^{4n}\right]^2 + 4|z|^{4n}\sin^2\pi\bar{\hat{\nu}}_0 t\sinh^2\pi c_a t\right\}}{(1-|z|^{2n})^2[1-2|z|^{2n}\cos 2\pi\bar{\hat{\nu}}_0 t + |z|^{4n}]\prod_{a=1}^{2}[1-2|z|^{2n}\cosh 2\pi\hat{\nu}_a t + |z|^{4n}]}. \tag{2.2}$$

In the above, $|z| = e^{-\pi t}$, $c_a = \hat{\nu}_1 + (-)^a\hat{\nu}_2$, $V_{1+p}$ the brane spacetime volume and $y$ the brane separation.

Q. Jia, J. X. Lu, Z. Wu and X. Zhu, "On D-brane interaction & its related properties," NPB 953 (2020)114947.

# The OS one-loop amplitude

This amplitude is completely determined by the dimensionless electromagnetic flux $\hat{F}_{\alpha\beta} \equiv 2\pi\alpha' eF_{\alpha\beta}$ with $F_{\alpha\beta}$ the standard EM one.

The parameters $\hat{\nu}_\kappa$ for $\kappa = 0, 1, \cdots [(p-1)/2]$ are determined via the relation $\lambda_\kappa + \lambda_\kappa^{-1} = 2\cos 2\pi\hat{\nu}_\kappa$ with both $\lambda_\kappa$ and $\lambda_\kappa^{-1}$ being pairwise eigenvalues of the following matrix

$$w_\alpha^{\ \beta} = \left[\left(\mathbb{I} - \hat{F}\right)\left(\mathbb{I} + \hat{F}\right)^{-1}\right]_\alpha^{\ \beta}. \tag{2.3}$$

Note one eigenvalue $\lambda = 1$ for even p ($d = 1 + p$).

These eigenvalues for $1 \leq p \leq 6$ are determined by the following respective equations:

| $p$ | Equation(s) for eigenvalue(s) |
|---|---|
| 1 | $\lambda_0 + \lambda_0^{-1} = \mathrm{tr}w$ |
| 2 | $\lambda_0 + \lambda_0^{-1} = \mathrm{tr}w - 1,\ \lambda = 1$ |
| 3 | $\sum_{\kappa=0}^{1}\left(\lambda_\kappa + \lambda_\kappa^{-1}\right) = \mathrm{tr}w,\ \sum_{\kappa=0}^{1}\left(\lambda_\kappa^2 + \lambda_\kappa^{-2}\right) = \mathrm{tr}w^2$ |
| 4 | $\sum_{\kappa=0}^{1}\left(\lambda_\kappa + \lambda_\kappa^{-1}\right) = \mathrm{tr}w - 1,\ \sum_{\kappa=0}^{1}\left(\lambda_\kappa^2 + \lambda_\kappa^{-2}\right) = \mathrm{tr}w^2 - 1,\ \lambda = 1$ |
| 5 | $\sum_{\kappa=0}^{2}\left(\lambda_\kappa + \lambda_\kappa^{-1}\right) = \mathrm{tr}w,\ \sum_{\kappa=0}^{2}\left(\lambda_\kappa^2 + \lambda_\kappa^{-2}\right) = \mathrm{tr}w^2,\ \sum_{\alpha=0}^{2}\left(\lambda_\kappa^3 + \lambda_\kappa^{-3}\right) = \mathrm{tr}w^3$ |
| 6 | $\sum_{\kappa=0}^{2}\left(\lambda_\kappa + \lambda_\kappa^{-1}\right) = \mathrm{tr}w - 1,\ \sum_{\kappa=0}^{2}\left(\lambda_\kappa^2 + \lambda_\kappa^{-2}\right) = \mathrm{tr}w^2 - 1,$ $\sum_{\kappa=0}^{2}\left(\lambda_\kappa^3 + \lambda_\kappa^{-3}\right) = \mathrm{tr}w^3 - 1,\ \lambda = 1$ |

# The OS pair production rate

The OS one-loop amplitude has an infinite number of simple poles occurring at $t_k = k/\hat{\bar{\nu}}_0$ with $k = 1, 2, \cdots$,

- signaling the decay of the system via the production of open string pairs,
- therefore having an imaginary part of the amplitude.

By the amplitude analyticity and following the standard approach, we compute the non-perturbative pair production rate for $1 \leq p \leq 6$ as

$$\mathcal{W}^{(\mathrm{String})}_{p,\,p} = \frac{2^3\sqrt{\det(\mathbb{I}+\hat{F})}\sinh\pi\bar{\hat{\nu}}_0\prod_{a=1}^{2}\sin\pi\hat{\nu}_a}{\bar{\hat{\nu}}_0(8\pi^2\alpha')^{\frac{p+1}{2}}}\bar{\hat{\nu}}_0^{\frac{p-3}{2}}\,e^{-\frac{y^2}{2\pi\bar{\hat{\nu}}_0\alpha'}}\frac{\left[\cosh\frac{\pi\hat{\nu}_1}{\bar{\hat{\nu}}_0}+\cosh\frac{\pi\hat{\nu}_2}{\bar{\hat{\nu}}_0}\right]^2}{\sinh\frac{\pi\hat{\nu}_1}{\bar{\hat{\nu}}_0}\sinh\frac{\pi\hat{\nu}_2}{\bar{\hat{\nu}}_0}}Z_1\,, \tag{2.4}$$

where, with $|z_1| = e^{-\pi/\bar{\hat{\nu}}_0}$,

$$Z_1 = \prod_{n=1}^{\infty}\frac{[1+2|z_1|^{2n}\cosh\frac{\pi(\hat{\nu}_1+\hat{\nu}_2)}{\bar{\hat{\nu}}_0}+|z_1|^{4n}]^2[1+2|z_1|^{2n}\cosh\frac{\pi(\hat{\nu}_1-\hat{\nu}_2)}{\bar{\hat{\nu}}_0}+|z_1|^{4n}]^2}{(1-|z_1|^{2n})^4[1-2|z_1|^{2n}\cosh\frac{2\pi\hat{\nu}_1}{\bar{\hat{\nu}}_0}+|z_1|^{4n}][1-2|z_1|^{2n}\cosh\frac{2\pi\hat{\nu}_2}{\bar{\hat{\nu}}_0}+|z_1|^{4n}]}\,. \tag{2.5}$$

Note the above rate is for $p = 5\,\mathrm{or}\,6$, the rate for $p = 3\,\mathrm{or}\,4$ is obtained by taking $\hat{\nu}_2 \to 0$ and the rate for $p = 1\,\mathrm{or}\,2$ is obtained by taking both $\hat{\nu}_2, \hat{\nu}_1 \to 0$.

# The OS pair production rate

Note in general $\det(\mathbb{I}+\hat{F})=(1-\hat{\alpha})(1-\hat{\beta})(1-\hat{\gamma})$ with

$$\hat{\alpha}+\hat{\beta}+\hat{\gamma}=\frac{1}{2}\mathrm{tr}\hat{F}^2,\quad \hat{\alpha}(\hat{\beta}+\hat{\gamma})+\hat{\beta}\hat{\gamma}=-\frac{1}{4}\left[\mathrm{tr}\hat{F}^4-\frac{1}{2}(\mathrm{tr}\hat{F}^2)^2\right],$$
$$\hat{\alpha}\hat{\beta}\hat{\gamma}=\frac{1}{3!}\left[\mathrm{tr}\hat{F}^6-\frac{3}{4}\mathrm{tr}\hat{F}^4\mathrm{tr}\hat{F}^2+\frac{1}{8}(\mathrm{tr}\hat{F}^2)^3\right]\geq 0. \tag{2.6}$$

One can prove in general

$$\mathrm{tr}\hat{F}^{2n}=2(\hat{\alpha}^n+\hat{\beta}^n+\hat{\gamma}^n),\quad \mathrm{tr}\hat{F}^{2n+1}=0, \tag{2.7}$$

with which we can compute $\mathrm{tr}(\mathbb{I}+\hat{F})^{-1}=\sum_{n=0}^{\infty}\mathrm{tr}\hat{F}^{2n}$, $\mathrm{tr}(\mathbb{I}+\hat{F})^{-2}=\sum_{n=0}^{\infty}(2n+1)\mathrm{tr}\hat{F}^{2n}$ and $\mathrm{tr}(\mathbb{I}+\hat{F})^{-3}=\sum_{n=0}^{\infty}(2n+1)(n+1)\mathrm{tr}\hat{F}^{2n}$.

Therefore we can compute $\mathrm{tr}w$, $\mathrm{tr}w^2$ and $\mathrm{tr}w^3$ given the following

$$\begin{aligned} w &= 2(\mathbb{I}+\hat{F})^{-1}-\mathbb{I},\quad w^2=4(\mathbb{I}+\hat{F})^{-2}-4(\mathbb{I}+\hat{F})^{-1}+\mathbb{I}, \\ w^3 &= 8(\mathbb{I}+\hat{F})^{-3}-12(\mathbb{I}+\hat{F})^{-2}+6(\mathbb{I}+\hat{F})^{-1}-\mathbb{I}. \end{aligned} \tag{2.8}$$

So we can determine the parameters $\hat{\nu}_\kappa$ given the eigenvalue equations list in the previous table as

# The OS pair production rate

$$\begin{aligned}
\cosh^2\pi\bar{\hat{\nu}}_0+\sum_{a=1}^{2}\cos^2\pi\hat{\nu}_a &= \frac{1}{1-\hat{\alpha}}+\frac{1}{1-\hat{\beta}}+\frac{1}{1-\hat{\gamma}},\\
\cosh^4\pi\bar{\hat{\nu}}_0+\sum_{a=1}^{2}\cos^4\pi\hat{\nu}_a &= \frac{1}{(1-\hat{\alpha})^2}+\frac{1}{(1-\hat{\beta})^2}+\frac{1}{(1-\hat{\gamma})^2},\\
\cosh^6\pi\bar{\hat{\nu}}_0+\sum_{a=1}^{2}\cos^6\pi\hat{\nu}_a &= \frac{1}{(1-\hat{\alpha})^3}+\frac{1}{(1-\hat{\beta})^3}+\frac{1}{(1-\hat{\gamma})^3},
\end{aligned}\tag{2.9}$$

which have the solutions

$$\sinh\pi\bar{\hat{\nu}}_0=\sqrt{\frac{\hat{\alpha}}{1-\hat{\alpha}}},\quad \sin\pi\hat{\nu}_1=\sqrt{\frac{-\hat{\beta}}{1-\hat{\beta}}},\quad \sin\pi\hat{\nu}_2=\sqrt{\frac{-\hat{\gamma}}{1-\hat{\gamma}}}.\tag{2.10}$$

where we have chosen $\hat{\alpha}>0$ and $\hat{\beta},\hat{\gamma}\leq 0$ without loss of generality.

We now take the so-called field theory limit: $|\hat{F}_{\alpha\beta}|=2\pi\alpha' e|F_{\alpha\beta}|\ll 1\rightarrow$

$$\begin{aligned}
&\hat{\alpha},|\hat{\beta}|,|\hat{\gamma}|\ll 1\rightarrow \bar{\hat{\nu}}_0,\hat{\nu}_a\ll 1(a=1,2)\rightarrow \pi\bar{\hat{\nu}}_0\approx\sqrt{\hat{\alpha}}\equiv 2\pi\alpha' e\sqrt{\alpha}\ll 1,\\
&\pi\hat{\nu}_1\approx\sqrt{-\hat{\beta}}\equiv 2\pi\alpha' e\sqrt{-\beta}\ll 1,\pi\hat{\nu}_2\approx\sqrt{-\hat{\gamma}}\equiv 2\pi\alpha' e\sqrt{-\gamma}\ll 1
\end{aligned}\tag{2.11}$$

with $\alpha>0$ and $\beta,\gamma\leq 0$, satisfying the following

# The OS pair production rate

$$\alpha\beta\gamma = \frac{1}{3!}[\mathrm{tr}F^6 - \frac{3}{4}\mathrm{tr}F^4\mathrm{tr}F^2 + \frac{1}{8}(\mathrm{tr}F^2)^3] \geq 0,$$
$$\alpha\beta + \alpha\gamma + \beta\gamma = -\frac{1}{4}[\mathrm{tr}F^4 - \frac{1}{2}(\mathrm{tr}F^2)^2],\ \alpha + \beta + \gamma = \frac{1}{2}\mathrm{tr}F^2. \quad (2.12)$$

In this limit, it is clear $|z_1| = e^{-\pi/\bar{\tilde{\nu}}_0} \to 0$ due to $\bar{\tilde{\nu}}_0 \ll 1$, therefore $Z_1 \to 1$ from (2.5), and the resulting rate counts the contribution of the 16 pairs of the lowest open/anti open string modes $(8_{\mathrm{B}} + 8_{\mathrm{F}})$. In terms of $\alpha > 0$ and $\beta, \gamma \leq 0$, we therefore have from (2.4) the rate

$$\mathcal{W}^{(\mathrm{String})}_{p,\,p} = \frac{\left(e\sqrt{\alpha}\right)^{\frac{p-3}{2}}\ e^2\sqrt{|\beta||\gamma|}}{(2\pi)^{p-2}} \frac{\left[\cosh\pi\sqrt{\frac{|\beta|}{\alpha}} + \cosh\pi\sqrt{\frac{|\gamma|}{\alpha}}\right]^2}{\sinh\pi\sqrt{\frac{|\beta|}{\alpha}}\sinh\pi\sqrt{\frac{|\gamma|}{\alpha}}} e^{-\frac{\pi m^2}{e\sqrt{\alpha}}}, \quad (2.13)$$

where $m = y/(2\pi\alpha')$ the mass for each of these modes.

From the D brane worldvolume, these 16 pairs count 8 scalar pairs and 8 spinor pairs for $p \leq 2$, and (8 - p) scalar pairs, 1 vector pair and $2^{3-[\frac{p+(-1)^{[(p+2)/4]}}{2}]}$ spinor pairs for $3 \leq p \leq 6$.

# The general relations

So we expect in general for $1 \leq p \leq 2$

$$\mathcal{W}_{p,\,p}^{(\mathrm{String})} = 8\mathcal{W}_{\mathrm{scalar}}^{(\mathrm{QED})} + 8\mathcal{W}_{\mathrm{spinor}}^{(\mathrm{QED})}, \tag{3.1}$$

while for $3 \leq p \leq 6$

$$\mathcal{W}_{p,\,p}^{(\mathrm{String})} = (8-p)\mathcal{W}_{\mathrm{scalar}}^{(\mathrm{QED})} + 2^{3-[\frac{p+(-1)^{[\frac{p+2}{4}]}}{2}]}\mathcal{W}_{\mathrm{spinor}}^{(\mathrm{QED})} + \mathcal{W}_{\mathrm{vector}}^{(\mathrm{QED})}. \tag{3.2}$$

For a pure electric field, we have

$$\begin{aligned}
\mathcal{W}_{p,\,p}^{(\mathrm{String})} &= 16\frac{(eE)^{\frac{1+p}{2}}}{(2\pi)^p}\, e^{-\frac{\pi m^2}{eE}} = 16\mathcal{W}_{\mathrm{scalar}}^{(\mathrm{QED})}, \\
\mathcal{W}_{\mathrm{spinor}}^{(\mathrm{QED})} &= 2^{[\frac{p+(-1)^{[\frac{p+2}{4}]}}{2}]}\mathcal{W}_{\mathrm{scalar}}^{(\mathrm{QED})}, \quad \mathcal{W}_{\mathrm{vector}}^{(\mathrm{QED})} = p\,\mathcal{W}_{\mathrm{scalar}}^{(\mathrm{QED})},
\end{aligned} \tag{3.3}$$

where $E = |\vec{E}|$.

The above relations uniquely determine the respective explicit Lorentz invariant QED rate for a scalar pair, or a spinor pair or a vector pair from the (2.13) as

# An explicit Example

Let us focus on the most familiar example of $p = 3$ or $d = 1 + p = 4$ case. Now we have 5 scalar pairs, 4 spinor pairs and one vector pair, all massive

$$\mathcal{W}_{3,\,3}^{(\mathrm{String})} = 5\mathcal{W}_{\mathrm{scalar}}^{(\mathrm{QED})} + 4\mathcal{W}_{\mathrm{spinor}}^{(\mathrm{QED})} + \mathcal{W}_{\mathrm{vector}}^{(\mathrm{QED})}. \tag{3.4}$$

For a pure electric field, we have

$$\begin{aligned} \mathcal{W}_{p,\,p}^{(\mathrm{String})} &= 16\frac{(eE)^2}{(2\pi)^3}\, e^{-\frac{\pi m^2}{eE}} = 16\,\mathcal{W}_{\mathrm{scalar}}^{(\mathrm{QED})}, \\ \mathcal{W}_{\mathrm{spinor}}^{(\mathrm{QED})} &= 2\,\mathcal{W}_{\mathrm{scalar}}^{(\mathrm{QED})}, \quad \mathcal{W}_{\mathrm{vector}}^{(\mathrm{QED})} = 3\,\mathcal{W}_{\mathrm{scalar}}^{(\mathrm{QED})}, \end{aligned} \tag{3.5}$$

where $E = |\vec{E}|$.

In general,

$$\begin{aligned} \mathcal{W}_{3,3}^{(\mathrm{String})} &= \frac{2\,e^2\sqrt{\alpha|\beta|}}{(2\pi)^2}\,\frac{\left[1+\cosh\pi\sqrt{\frac{|\beta|}{\alpha}}\right]^2}{\sinh\pi\sqrt{\frac{|\beta|}{\alpha}}}\, e^{-\frac{\pi m^2}{e\sqrt{\alpha}}} \\ &= \frac{e^2\sqrt{\alpha|\beta|}}{(2\pi)^2}\, e^{-\frac{\pi m^2}{e\sqrt{\alpha}}}\,\frac{\frac{5}{2}+4\cosh\pi\sqrt{\frac{|\beta|}{\alpha}}+\cosh 2\pi\sqrt{\frac{|\beta|}{\alpha}}+\frac{1}{2}}{\sinh\pi\sqrt{\frac{|\beta|}{\alpha}}}. \end{aligned} \tag{3.6}$$

$\Rightarrow$

# An explicit LI Example

$$\mathrm{W}_{\mathrm{vec.}}^{(\mathrm{QED})} = \frac{e^2\sqrt{\alpha|\beta|}}{2(2\pi)^2}\frac{1+2\cosh 2\pi\sqrt{\frac{|\beta|}{\alpha}}}{\sinh\pi\sqrt{\frac{|\beta|}{\alpha}}}\, e^{-\frac{\pi m_{\mathrm{vec.}}^2}{e\sqrt{\alpha}}},$$

$$\mathcal{W}_{\mathrm{scalar}}^{(\mathrm{QED})} = \frac{e^2\sqrt{\alpha|\beta|}}{2(2\pi)^2}\,\mathrm{csch}\pi\sqrt{\frac{|\beta|}{\alpha}}\, e^{-\frac{\pi m_{\mathrm{scalar}}^2}{e\sqrt{\alpha}}},\ \mathcal{W}_{\mathrm{spinor}}^{(\mathrm{QED})} = \frac{e^2\sqrt{\alpha|\beta|}}{(2\pi)^2}\coth\pi\sqrt{\frac{|\beta|}{\alpha}}\, e^{-\frac{\pi m_{\mathrm{spinor}}^2}{e\sqrt{\alpha}}}, \quad (3.7)$$

where

$$\alpha = \frac{1}{2}\left[\vec{E}^2-\vec{B}^2+\sqrt{\left(\vec{E}^2-\vec{B}^2\right)^2+4(\vec{E}\cdot\vec{B})^2}\right],\ \beta = \frac{1}{2}\left[\vec{E}^2-\vec{B}^2-\sqrt{\left(\vec{E}^2-\vec{B}^2\right)^2+4(\vec{E}\cdot\vec{B})^2}\right]. \quad (3.8)$$

One can check that the above rates agree with the respective known rates for a scalar pair and a spinor pair Nikishov'70 and for a vector pair Kruhlov'01 with the collinear electric field and the magnetic field, i.e., $\vec{E}\cdot\vec{B}=\pm EB$, a non-trivial test!

**The $p=1$ or $2$ case**: By the same token, for p = 1, we have the explicit Lorentz invariant scalar rate and spinor rate as

$$\mathcal{W}_{\mathrm{scalar}}^{(\mathrm{QED})} = \frac{eE}{2\pi}\, e^{-\frac{\pi m_{\mathrm{scalar}}^2}{eE}},\ \mathcal{W}_{\mathrm{spinor}}^{(\mathrm{QED})} = \frac{eE}{2\pi}\, e^{-\frac{\pi m_{\mathrm{spinor}}^2}{eE}}, \quad (3.9)$$

where $E=|F_{01}|$, being Lorentz invariant by itself.

# Explicit LI QED rates

For p = 2,

$$\mathcal{W}^{(\mathrm{QED})}_{\mathrm{scalar}} = \frac{\left(e\sqrt{\alpha}\right)^{\frac{3}{2}}}{(2\pi)^2}\, e^{-\frac{\pi m^2_{\mathrm{scalar}}}{e\sqrt{\alpha}}}, \quad \mathcal{W}^{(\mathrm{QED})}_{\mathrm{spinor}} = \frac{\left(e\sqrt{\alpha}\right)^{\frac{3}{2}}}{(2\pi)^2}\, e^{-\frac{\pi m^2_{\mathrm{spinor}}}{e\sqrt{\alpha}}}, \tag{3.10}$$

where the Lorentz invariant $\alpha = F^2/2 = E_1^2 + E_2^2 - (F_{12})^2 > 0$ with $E_a = F_{0a}$.

**The p = 3 or 4 case**: For p = 4,

$$\begin{aligned}
\mathcal{W}^{(\mathrm{QED})}_{\mathrm{vec.}} &= \frac{\left[e\sqrt{\alpha}\right]^{\frac{3}{2}}\, e|\beta|^{\frac{1}{2}}}{(2\pi)^3}\, \frac{\cosh 2\pi\sqrt{\frac{|\beta|}{\alpha}} + 1}{\sinh \pi\sqrt{\frac{|\beta|}{\alpha}}}\, e^{-\frac{\pi m^2_{\mathrm{vec.}}}{e\sqrt{\alpha}}}, \\
\mathcal{W}^{(\mathrm{QED})}_{\mathrm{spinor}} &= \frac{\left[e\sqrt{\alpha}\right]^{\frac{3}{2}}\, e|\beta|^{\frac{1}{2}}}{(2\pi)^3}\, \coth \pi\sqrt{\frac{|\beta|}{\alpha}}\, e^{-\frac{\pi m^2_{\mathrm{spinor}}}{e\sqrt{\alpha}}}, \\
\mathcal{W}^{(\mathrm{QED})}_{\mathrm{scalar}} &= \frac{\left[e\sqrt{\alpha}\right]^{\frac{3}{2}}\, e|\beta|^{\frac{1}{2}}}{2(2\pi)^3}\, \mathrm{csch}\pi\sqrt{\frac{|\beta|}{\alpha}}\, e^{-\frac{\pi m^2_{\mathrm{scalar}}}{e\sqrt{\alpha}}},
\end{aligned} \tag{3.11}$$

where

$$\alpha = \frac{1}{2}\left[\frac{1}{2}\mathrm{tr}F^2 + \sqrt{\mathrm{tr}F^4 - \frac{1}{4}(\mathrm{tr}F^2)^2}\right], \quad \beta = \frac{1}{2}\left[\frac{1}{2}\mathrm{tr}F^2 - \sqrt{\mathrm{tr}F^4 - \frac{1}{4}(\mathrm{tr}F^2)^2}\right]. \tag{3.12}$$

# Explicit LI QED rates

**p = 5 or 6 case**: For p = 5,

$$\mathcal{W}^{(\mathrm{QED})}_{\mathrm{scalar}} = \frac{e^3\sqrt{\alpha\beta\gamma}}{4(2\pi)^3}\,\mathrm{csch}\pi\sqrt{\frac{|\beta|}{\alpha}}\,\mathrm{csch}\pi\sqrt{\frac{|\gamma|}{\alpha}}\,e^{-\frac{\pi m^2_{\mathrm{scalar}}}{e\sqrt{\alpha}}},$$

$$\mathcal{W}^{(\mathrm{QED})}_{\mathrm{spinor}} = \frac{e^3\sqrt{\alpha\beta\gamma}}{(2\pi)^3}\,\coth\pi\sqrt{\frac{|\beta|}{\alpha}}\,\coth\pi\sqrt{\frac{|\gamma|}{\alpha}}\,e^{-\frac{\pi m^2_{\mathrm{spinor}}}{e\sqrt{\alpha}}},$$

$$\mathcal{W}^{(\mathrm{QED})}_{\mathrm{vec.}} = \frac{e^3\sqrt{\alpha\beta\gamma}}{4(2\pi)^3}\,\frac{1+2\left[\cosh 2\pi\sqrt{\frac{|\beta|}{\alpha}}+\cosh 2\pi\sqrt{\frac{|\gamma|}{\alpha}}\right]}{\sinh\pi\sqrt{\frac{|\beta|}{\alpha}}\,\sinh\pi\sqrt{\frac{|\gamma|}{\alpha}}}\,e^{-\frac{\pi m^2_{\mathrm{vec.}}}{e\sqrt{\alpha}}}. \qquad (3.13)$$

For p = 6,

$$\mathcal{W}^{(\mathrm{QED})}_{\mathrm{scalar}} = \frac{(e\sqrt{\alpha})^{\frac{3}{2}}\,e^2\sqrt{\beta\gamma}}{4(2\pi)^4}\,\mathrm{csch}\pi\sqrt{\frac{|\beta|}{\alpha}}\,\mathrm{csch}\pi\sqrt{\frac{|\gamma|}{\alpha}}\,e^{-\frac{\pi m^2_{\mathrm{scalar}}}{e\sqrt{\alpha}}},$$

$$\mathcal{W}^{(\mathrm{QED})}_{\mathrm{spinor}} = \frac{(e\sqrt{\alpha})^{\frac{3}{2}}\,e^2\sqrt{\beta\gamma}}{(2\pi)^4}\,\coth\pi\sqrt{\frac{|\beta|}{\alpha}}\,\coth\pi\sqrt{\frac{|\gamma|}{\alpha}}\,e^{-\frac{\pi m^2_{\mathrm{spinor}}}{e\sqrt{\alpha}}},$$

$$\mathcal{W}^{(\mathrm{QED})}_{\mathrm{vec.}} = \frac{(e\sqrt{\alpha})^{\frac{3}{2}}\,e^2\sqrt{\beta\gamma}}{2(2\pi)^4}\,\frac{1+\cosh 2\pi\sqrt{\frac{|\beta|}{\alpha}}+\cosh 2\pi\sqrt{\frac{|\gamma|}{\alpha}}}{\sinh\pi\sqrt{\frac{|\beta|}{\alpha}}\,\sinh\pi\sqrt{\frac{|\gamma|}{\alpha}}}\,e^{-\frac{\pi m^2_{\mathrm{vec.}}}{e\sqrt{\alpha}}}. \qquad (3.14)$$

For both p = 5 and 6, the Lorentz invariants $\alpha, \beta$ and $\gamma$ are given by (2.12), i.e.,

$$\alpha\beta\gamma = \frac{1}{3!}[\mathrm{tr}F^6 - \frac{3}{4}\mathrm{tr}F^4\mathrm{tr}F^2 + \frac{1}{8}(\mathrm{tr}F^2)^3] \geq 0, \alpha\beta+\alpha\gamma+\beta\gamma = -\frac{1}{4}[\mathrm{tr}F^4 - \frac{1}{2}(\mathrm{tr}F^2)^2],\ \alpha+\beta+\gamma = \frac{1}{2}\mathrm{tr}F^2. \qquad (3.15)$$

# Conclusion and discussion

- We have derived the explicit Lorentz invariant QED pair production rates, respectively, for a scalar pair, a spinor pair and a vector pair for the most general allowed constant EM field in diverse dimensions from superstring theories;
- The explicit Lorentz invariant scalar rate and the spinor rate for $p = 2$, the vector rate for $p = 3$ and all other rates for $p \geq 4$ are new and have not been given before;
- For $p \geq 4$, these results cannot be obtained from QED since the underlying QFT is not renormalizable and the one-loop computations cannot be obtained;
- D-branes say that a second brane is needed to have such rates while the charged particles originate for the open strings along the extra dimensions.

# THANK YOU!