Probing τ weak EDM via Spin Correlations at CEPC and FCC-ee

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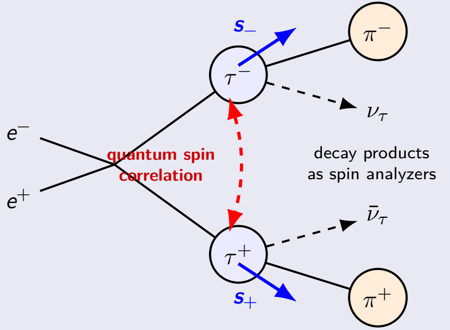
Spin Correlation as a Probe of the Weak Dipole Moment

What we measure

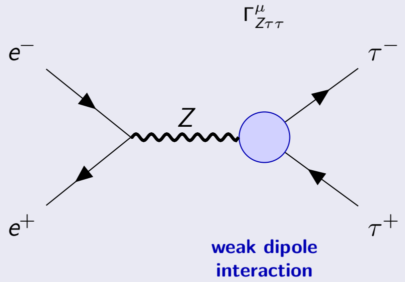


Joint decay distributions retain information about the **two-particle spin density matrix**.

Our idea

spin correlation probes

What we want to probe



At the Z pole, the process is sensitive to the $Z\tau\tau$ **weak dipole form factors**.

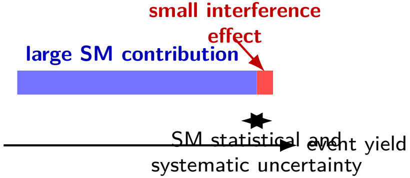
From Rate Measurements to Symmetry-Filtered Observables

Conventional strategy: total rate

$$\sigma = \sigma_{\rm SM} + d_\tau\, \sigma_{\rm int} + d_\tau^2\, \sigma_{\rm dip}$$

For a sufficiently small weak dipole moment,

$$\sigma - \sigma_{\rm SM} \simeq d_\tau\, \sigma_{\rm int}.$$



The approximate statistical significance is

$$\mathcal{Z}_\sigma \sim \frac{\mathcal{L}\, d_\tau \sigma_{\rm int}}{\sqrt{\mathcal{L}\sigma_{\rm SM}}} = d_\tau \frac{\sigma_{\rm int}}{\sqrt{\sigma_{\rm SM}}}\sqrt{\mathcal{L}}.$$

use spin correlation

Our strategy: weighted correlations

Each event is assigned a symmetry-sensitive weight $O(\Phi)$, constructed from the measured decay angles:

$$\langle O \rangle = \frac{\int d\Phi\, W(\Phi)\, O(\Phi)}{\int d\Phi\, W(\Phi)},$$

where

$$W(\Phi) = \mathrm{Tr}\left[R\left(D_- \otimes D_+\right)\right].$$

Choose $O(\Phi)$ with the appropriate CP transformation:

$$\boxed{\int d\Phi\, W_{\rm SM}(\Phi)\, O_{\text{CP-odd}}(\Phi) = 0}$$

while the dipole interference survives:

Initial State: Unpolarized Electron–Positron Beams

Spin Hilbert space

$$\mathcal{H}_{e^-e^+} = \mathcal{H}_{e^-} \otimes \mathcal{H}_{e^+}, \qquad \dim \mathcal{H}_{e^-e^+} = 4.$$

$$\{|++\rangle_e, |+-\rangle_e, |-+\rangle_e, |--\rangle_e\}.$$

Unpolarized beams

$$\rho_{e^-} = \frac{1}{2} I_2, \qquad \rho_{e^+} = \frac{1}{2} I_2.$$

$$\boxed{\rho^{\mathrm{unpol}}_{e^-e^+} = \rho_{e^-} \otimes \rho_{e^+} = \frac{1}{4} I_4}$$

Physical interpretation

Equal-probability incoherent mixture of the four initial helicity configurations.

Transition Amplitude Between Spin Spaces

Scattering amplitude as a linear map

$$\boxed{\mathcal{M} : \mathcal{H}_{e^- e^+} \longrightarrow \mathcal{H}_{\tau^- \tau^+}}$$

Initial state

$$|\lambda\rangle_e = |\lambda_- \lambda_+\rangle_e$$

$$\lambda \in \{++, +-, -+, --\}.$$

First sign: e^- helicity
Second sign: e^+ helicity

$$\xrightarrow{\mathcal{M}}$$

Final state

$$|F\rangle_\tau = |h_- h_+\rangle_\tau$$

$$F \in \{++, +-, -+, --\}.$$

First sign: τ^- helicity
Second sign: τ^+ helicity

Helicity amplitude

$$\boxed{M_{F\lambda} \equiv {}_\tau\langle F|\mathcal{M}|\lambda\rangle_e}$$

F labels the final tau-pair helicity state, while λ labels the initial electron–positron helicity state.

Production Density Matrix of the Tau Pair

The initial density matrix is propagated through the scattering amplitude according to

$$\boxed{R = M\,\rho^{\mathrm{unpol}}_{e^- e^+} M^\dagger}.$$

In components,

$$R_{FF'} = \sum_{\lambda,\lambda'} M_{F\lambda} \left(\rho^{\mathrm{unpol}}_{e^- e^+}\right)_{\lambda\lambda'} M^*_{F'\lambda'}.$$

Since

$$\left(\rho^{\mathrm{unpol}}_{e^- e^+}\right)_{\lambda\lambda'} = \frac{1}{4}\delta_{\lambda\lambda'},$$

we obtain

$$\boxed{R_{FF'} = \frac{1}{4}\sum_{\lambda} M_{F\lambda} M^*_{F'\lambda}}.$$

Matrix representation in the helicity basis

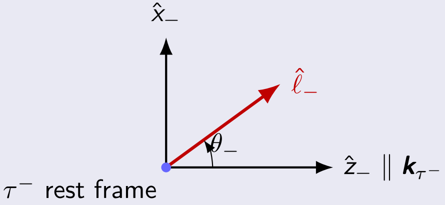
$$R = \begin{pmatrix} R_{++,++} & R_{++,+-} & R_{++,-+} & R_{++,--} \\ R_{+-,++} & R_{+-,+-} & R_{+-,-+} & R_{+-,--} \\ R_{-+,++} & R_{-+,+-} & R_{-+,-+} & R_{-+,--} \\ R_{--,++} & R_{--,+-} & R_{--,-+} & R_{--,--} \end{pmatrix}.$$

Tau Decay Products as Spin Analyzers

The tau spins are not measured directly. Instead, their spin information is reconstructed from the directions of their decay products in the corresponding tau rest frames.

τ^- analyzer

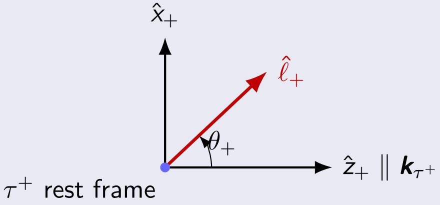
$$\boxed{D_- = \frac{1}{2}\left(I_2 + \alpha_- \, \hat{\ell}_- \cdot \boldsymbol{\sigma}\right)}$$



$$\hat{\ell}_- = (\sin\theta_- \cos\phi_-, \sin\theta_- \sin\phi_-, \cos\theta_-) \, .$$

τ^+ analyzer

$$\boxed{D_+ = \frac{1}{2}\left(I_2 - \alpha_+ \, \hat{\ell}_+ \cdot \boldsymbol{\sigma}\right)}$$



$$\hat{\ell}_+ = (\sin\theta_+ \cos\phi_+, \sin\theta_+ \sin\phi_+, \cos\theta_+) \, .$$

Joint Spin Measurement of the Tau Pair

The tau-pair helicity basis is a tensor-product basis:

$$
\begin{aligned}
|++\rangle &= |+\rangle_{\tau^-} \otimes |+\rangle_{\tau^+}, \\
|+-\rangle &= |+\rangle_{\tau^-} \otimes |-\rangle_{\tau^+}, \\
|-+\rangle &= |-\rangle_{\tau^-} \otimes |+\rangle_{\tau^+}, \\
|--\rangle &= |-\rangle_{\tau^-} \otimes |-\rangle_{\tau^+}.
\end{aligned}
$$

Joint analyzer operator

Since D_- acts only on the τ^- spin space and D_+ acts only on the τ^+ spin space,

$$\boxed{D = D_- \otimes D_+}.$$

Explicitly,

$$D = \frac{1}{4}\left(I_2 + \alpha_- \hat{\ell}_- \cdot \boldsymbol{\sigma}\right) \otimes \left(I_2 - \alpha_+ \hat{\ell}_+ \cdot \boldsymbol{\sigma}\right).$$

τ^- analyzer D_-	$\otimes$	τ^+ analyzer D_+	$=$	joint measurement $D = D_- \otimes D_+$

From Quantum Measurement to Angular Distribution

Production state R $\Longrightarrow$ **Joint analyzer** $D = D_- \otimes D_+$ $\Longrightarrow$ **Measurement** W

Quantum-mechanical prediction

$$\boxed{W(\Phi_{\mathrm{decay}}) = \mathrm{Tr}\left[R\left(D_- \otimes D_+\right)\right]}$$

Experimental interpretation

$$\boxed{\frac{d\sigma}{d\cos\theta\, d\Omega_{\mathrm{decay}}} = W(\Phi_{\mathrm{decay}})}$$

Measuring the correlated decay angles is equivalent to measuring the production

From the Angular Distribution to an Observable

For each event, we assign a measurement function

$$O(\Phi_{\rm decay}),$$

constructed from the measured decay angles.

Expectation value

$$\boxed{\langle O \rangle = \frac{\displaystyle\int_{-1}^{1} d\cos\theta \int d\Omega_{\rm decay}\; O(\Phi)\; W(\Phi)}{\displaystyle\int_{-1}^{1} d\cos\theta \int d\Omega_{\rm decay}\; W(\Phi)}}$$

measured events $\Phi_{\rm decay}$ → event weight $O(\Phi_{\rm decay})$ → sample average $\langle O \rangle$

Symmetry-Sensitive Spin-Correlation Observables

Single-tau angular projections

$$O_1 = \hat{\ell}_- \cdot \hat{k} = \cos\theta_-,$$
$$O_2 = \hat{\ell}_+ \cdot \hat{k} = -\cos\theta_+.$$

$$O_1 + O_2 \longrightarrow \mathrm{Im}(d_\tau)$$

Two-tau triple product

$$O_3 = \left(\hat{\ell}_- \times \hat{\ell}_+\right) \cdot \hat{k}.$$

$$O_3 \longrightarrow \mathrm{Re}(d_\tau)$$

Full phase-space result

$$\boxed{\langle O_1 + O_2 \rangle = f_1\ \mathrm{Im}(d_\tau)} \qquad \boxed{\langle O_3 \rangle = f_2\ \mathrm{Re}(d_\tau)}$$

From Statistical Uncertainty to Dipole Sensitivity

For an observable O, the uncertainty of the sample mean is

Statistical uncertainty

$$\boxed{\delta O = \frac{\sqrt{\langle O^2 \rangle - \langle O \rangle^2}}{\sqrt{N_{\text{eff}}}}}$$

$$\boxed{\begin{array}{c}\text{effective events}\\ N_{\text{eff}}\end{array}} \longrightarrow \boxed{\begin{array}{c}\text{observable error}\\ \delta O\end{array}} \longrightarrow \boxed{\begin{array}{c}\text{dipole sensitivity}\\ \sigma(d_\tau)\end{array}}$$

Since

$$\langle O_1 + O_2 \rangle = f_1 \ \mathrm{Im}(d_\tau), \qquad \langle O_3 \rangle = f_2 \ \mathrm{Re}(d_\tau),$$

the projected sensitivities are

Dipole uncertainty

$$\boxed{\sigma(\mathrm{Im}\, d_\tau) = \frac{\delta(O_1 + O_2)}{|f_1|}} \qquad \boxed{\sigma(\mathrm{Re}\, d_\tau) = \frac{\delta O_3}{|f_2|}}$$

Projected Sensitivity at Future Z Factories

Effective event samples

Machine	N_Z	$N_{\mathrm{eff}}^{\mp}$	N_{eff}^{+-}
FCC-ee	6.0×10^{12}	1.25×10^{10}	1.35×10^{9}
CEPC	4.1×10^{12}	8.52×10^{9}	9.22×10^{8}

$N_{\mathrm{eff}}^{\mp}$ denotes either single-tau analyzer sample, since $N_{\mathrm{eff}}^{-} = N_{\mathrm{eff}}^{+}$.

Projected 1σ sensitivities

Machine	$\sigma(\mathrm{Im}\, H_T)$	$\sigma(\mathrm{Re}\, H_T)$
FCC-ee	1.53×10^{-19}	8.10×10^{-19}
CEPC	1.85×10^{-19}	9.80×10^{-19}

Units: $e\,\mathrm{cm}$

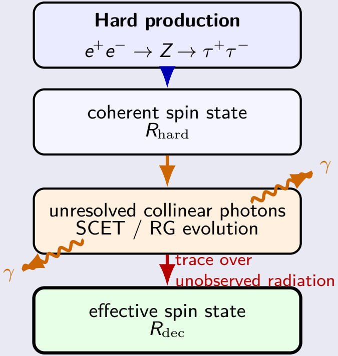
Comparison with the ALEPH bound at LEP

Component	ALEPH 95% CL	FCC-ee projection	Improvement
$\mathrm{Im}\, H_T$	$1.1 \times 10^{-17}\ e\,\mathrm{cm}$	$1.53 \times 10^{-19}\ e\,\mathrm{cm}$	~ 73
$\mathrm{Re}\, H_T$	$5.1 \times 10^{-18}\ e\,\mathrm{cm}$	$8.10 \times 10^{-19}\ e\,\mathrm{cm}$	~ 6

ALEPH bounds: $|\,\mathrm{Re}\, d_\tau\,| < 0.91 \times 10^{-3}$ and $|\,\mathrm{Im}\, d_\tau\,| < 2.01 \times 10^{-3}$, converted using $H_T = e\, d_\tau / (2 m_\tau)$.

Future Direction: Collinear Radiation and Spin Decoherence

QED collinear evolution

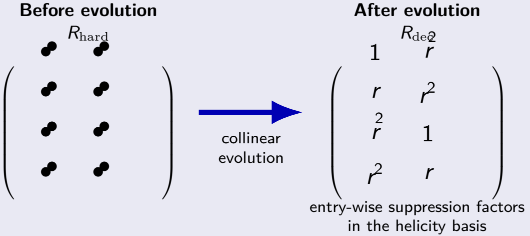


Suppression of helicity coherence

Define

$$r = \exp\left(-\frac{\alpha}{2\pi}t\right), \qquad 0 < r < 1,$$

where t is the collinear RG-evolution parameter.



Diagonal elements

$$R_{ii} \longrightarrow R_{ii}$$

Off-diagonal elements

$$R_{ij} \longrightarrow r^n R_{ij}$$