

Hadron Structure and Dynamics from Lattice QCD

Xiang Gao

极端条件下的QCD研讨会, 烟台大学, 2026年8月18日—21日

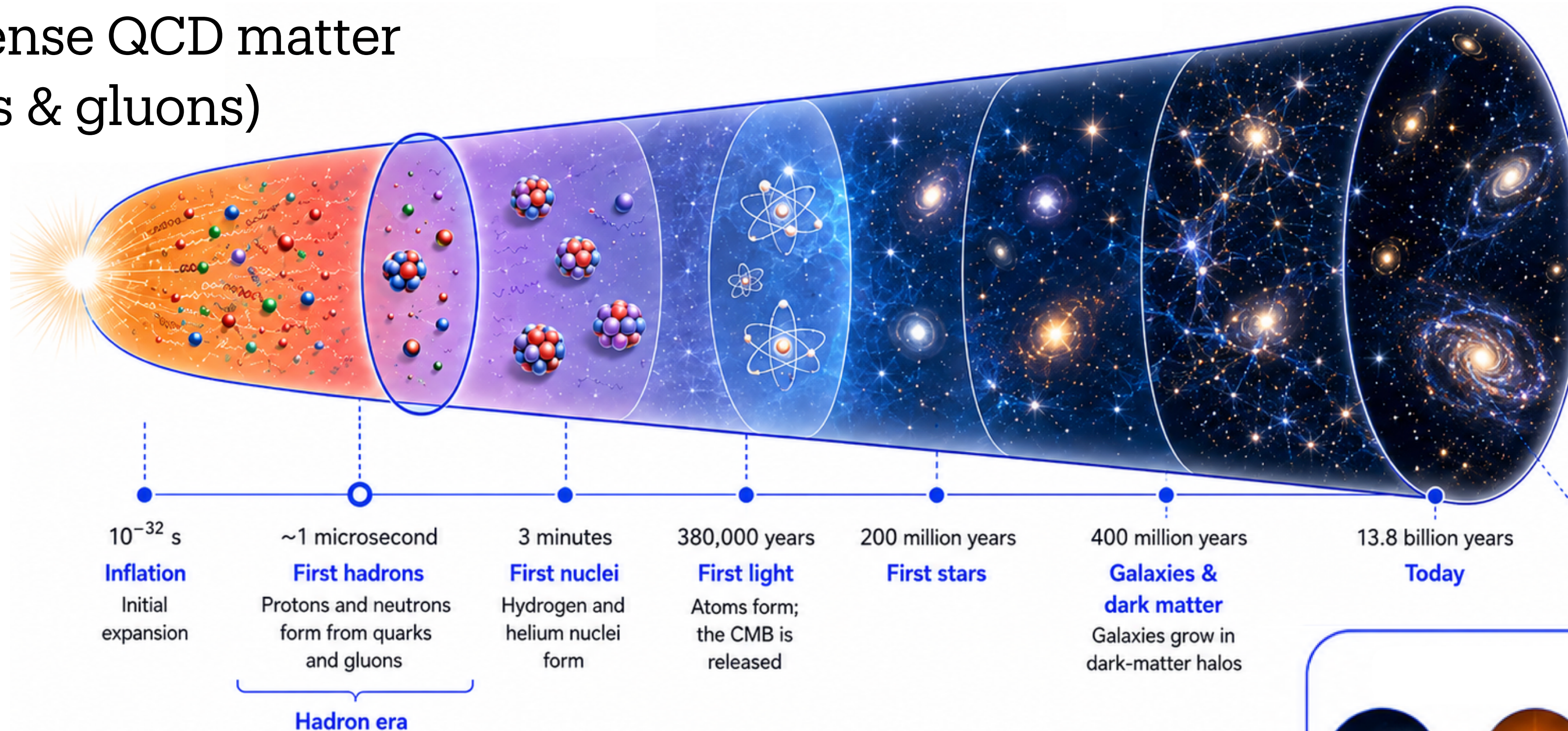
Building blocks of the visible universe

Early universe

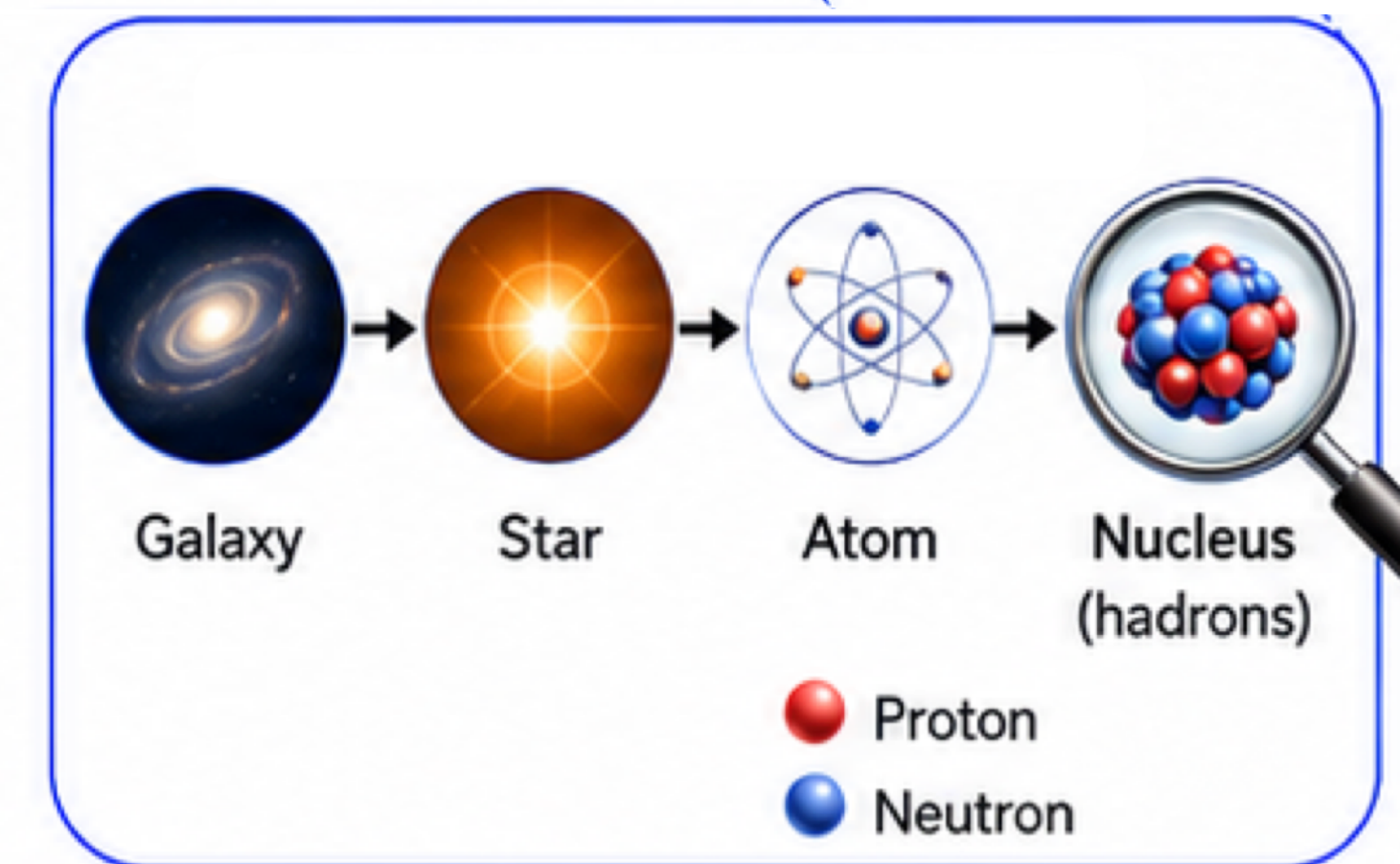
Hot, dense QCD matter
(quarks & gluons)

Today

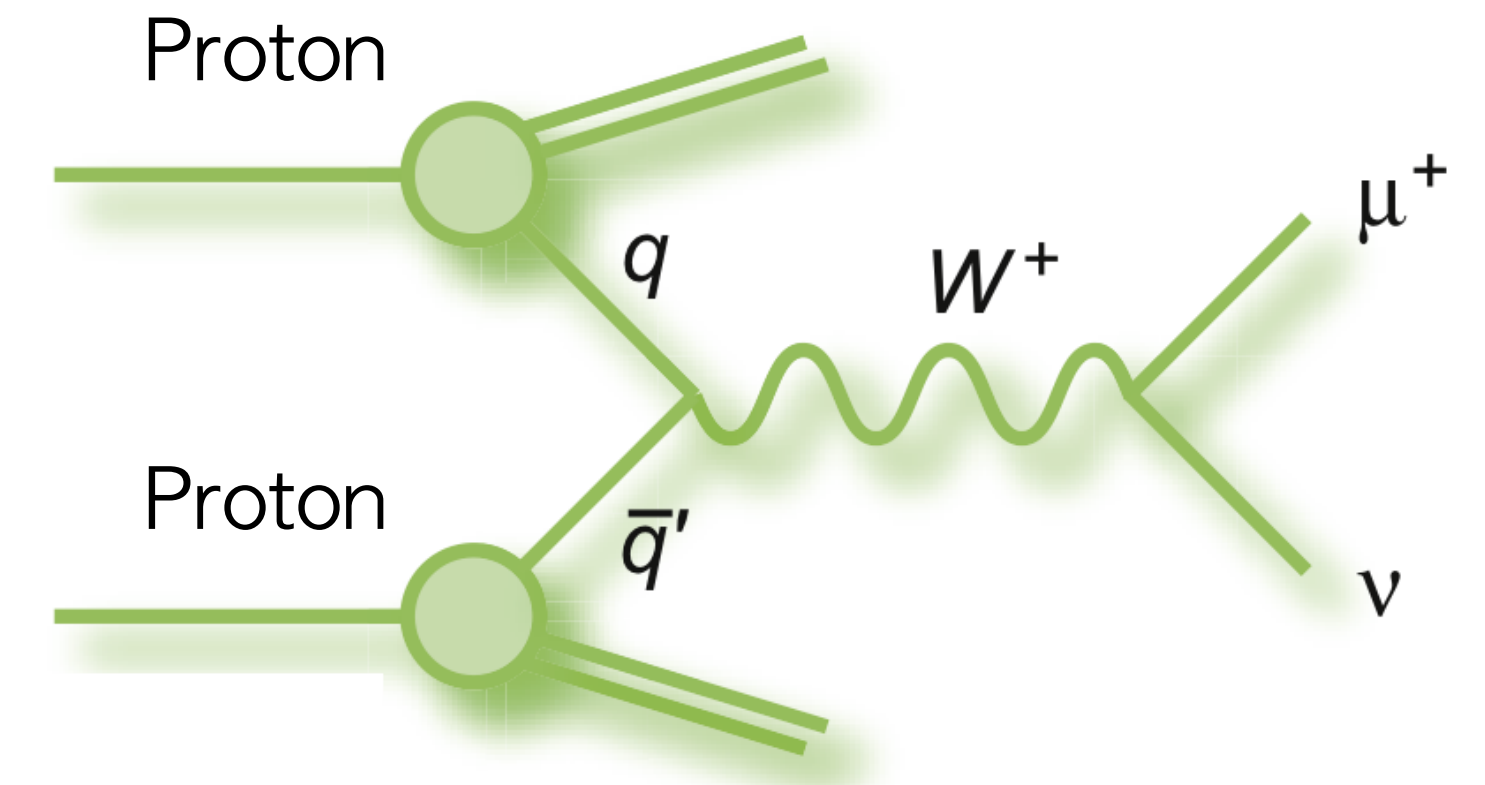
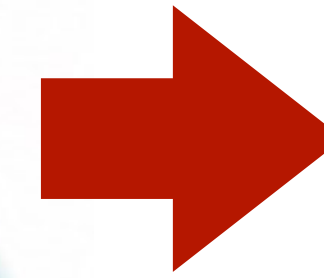
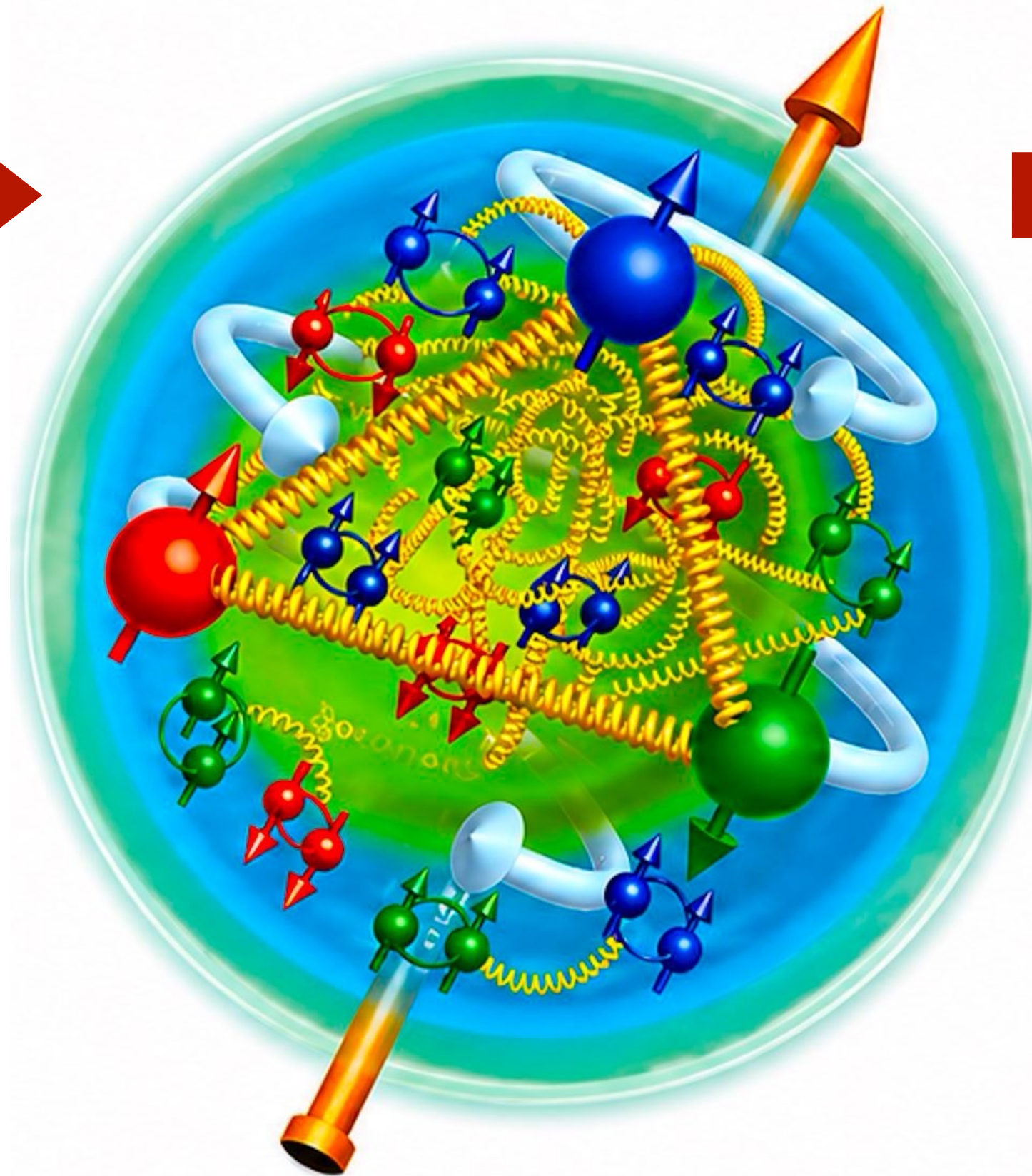
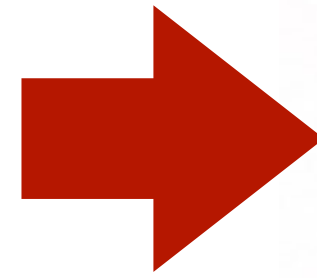
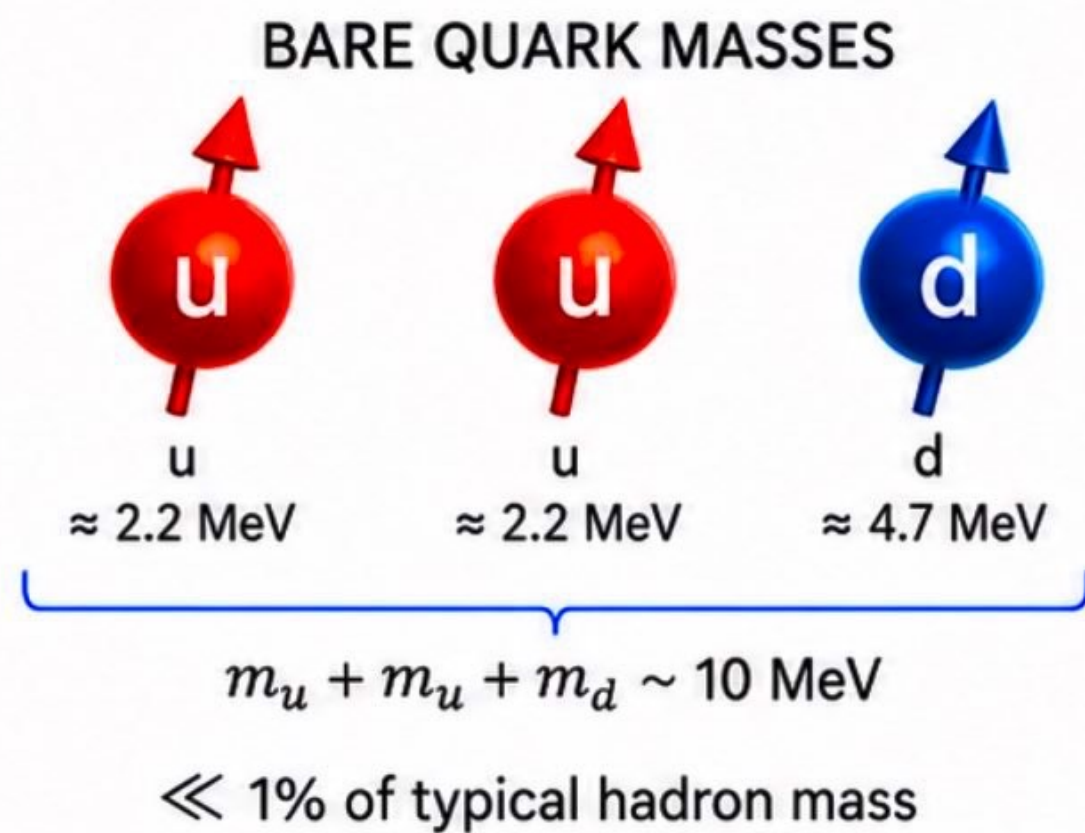
Ordinary
matter
(hadrons)



Most visible matter is built from nucleons (hadrons)



Hadron structure: from QCD emergence to precision frontier



QCD emergence

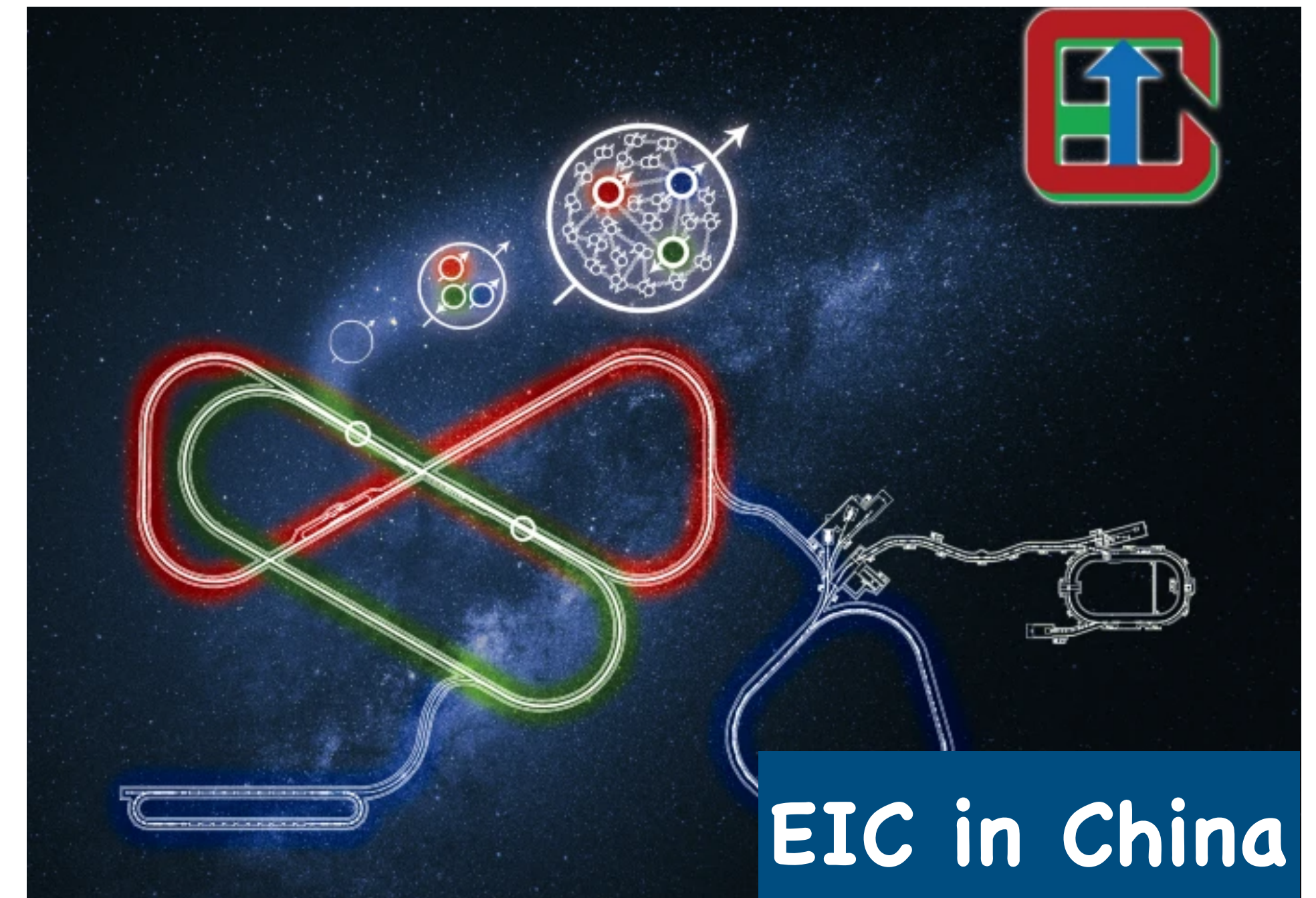
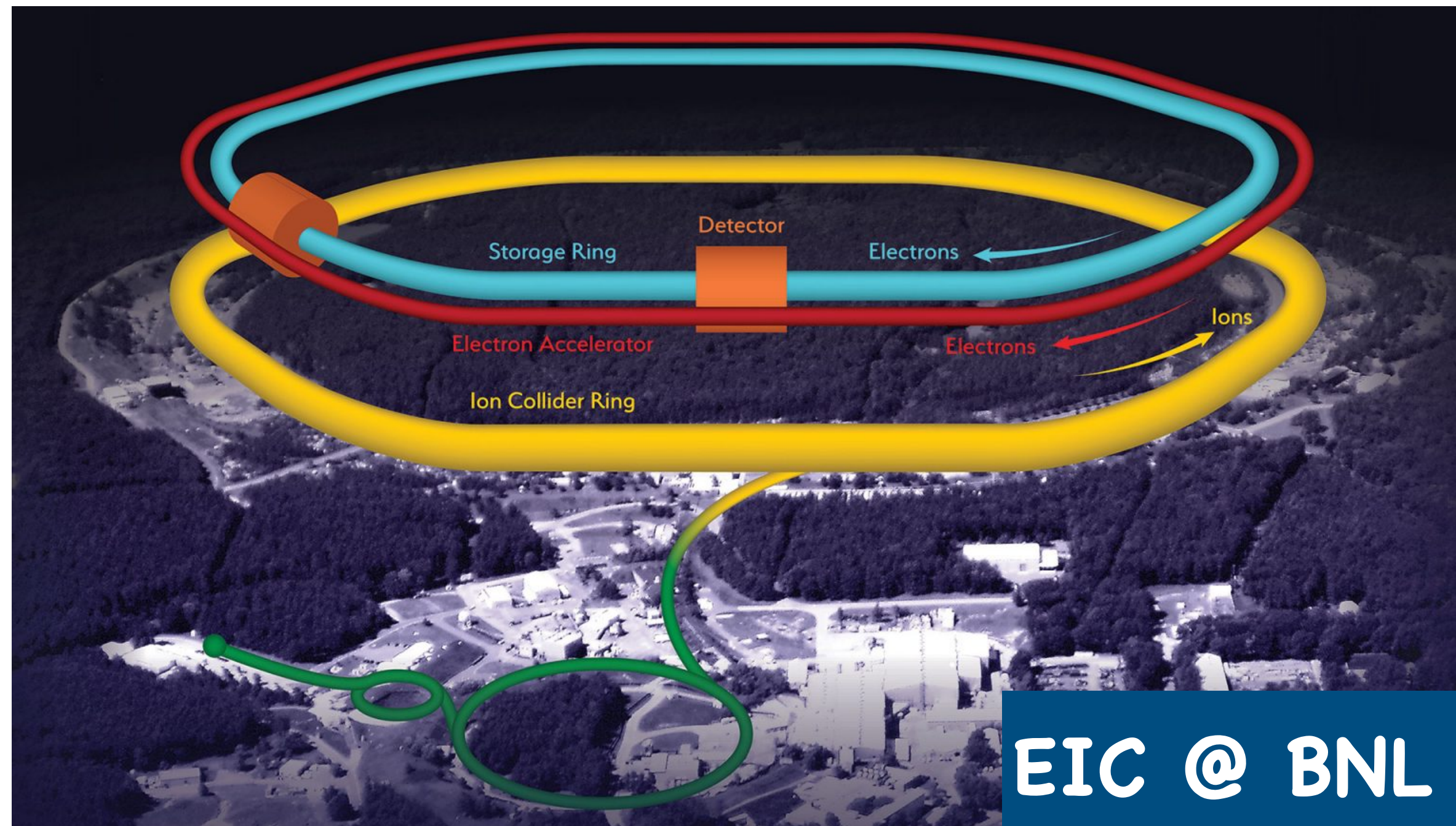
- How do hadron mass & spin arise from strong interaction?
- How are quarks and gluons distributed and moving inside hadrons?

Precision frontier

- Key nonperturbative input to precision collider predictions.
- Uncertainties propagate into precision Standard Model tests and new-physics searches.

What builds the nucleon also controls how precisely we can use it to test fundamental physics

A new era of hadron structure



Inclusive DIS

$$e p \rightarrow e' X$$

PDFs · longitudinal momentum x

SIDIS

$$e p \rightarrow e' h X$$

TMDs · transverse motion k_T

Exclusive

$$e p \rightarrow e' p' \gamma / M$$

GPDs · spatial imaging b_T

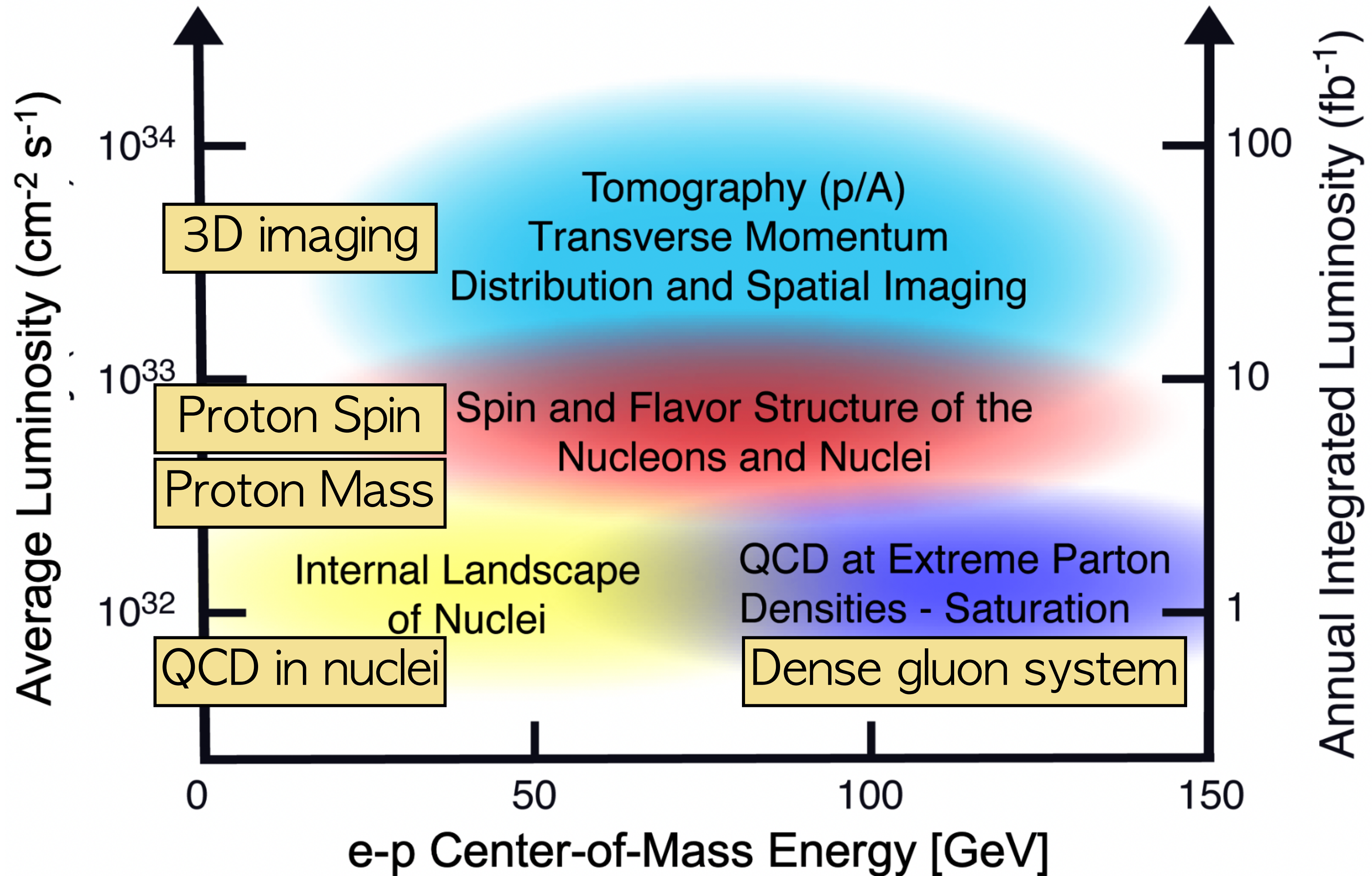
Nuclear / diffractive

$$e A \rightarrow e' X, e' A' X$$

gluons · correlations · saturation

A new generation of facilities will map multidimensional quark and gluon structure with unprecedented precision

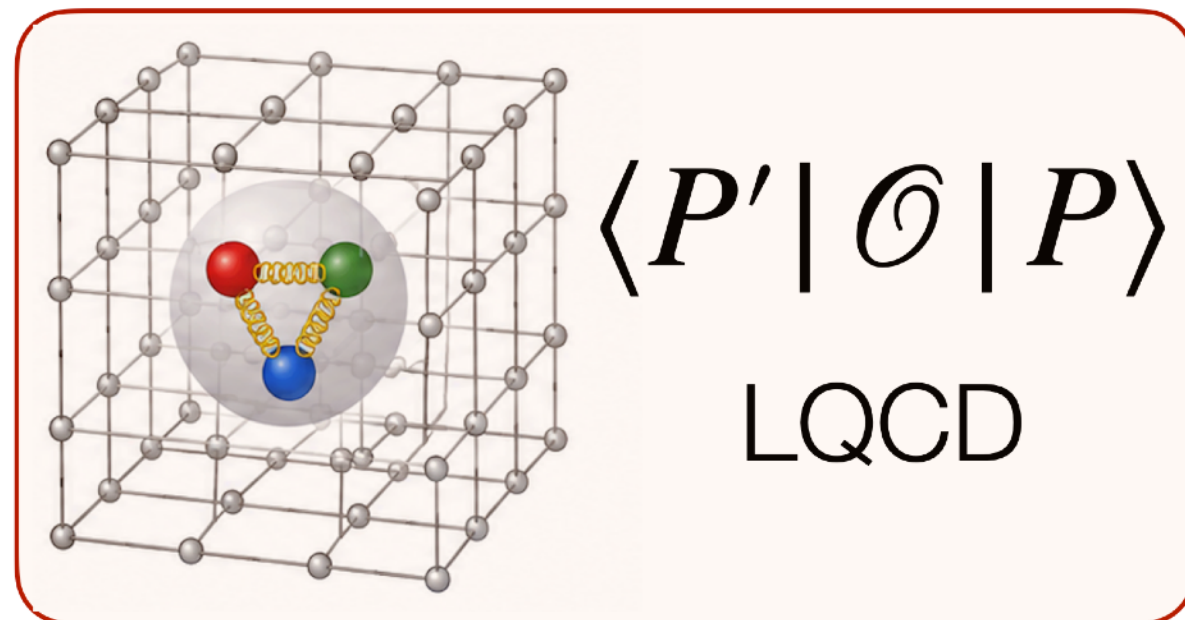
Science pillars of EIC



Lattice QCD: a first-principles complement

$$\underbrace{\sigma(\text{obs.})}_{\text{Measured in experiment}} \sim \underbrace{H(\text{hard})}_{\text{Perturbative QCD}} \otimes \underbrace{f(\text{nonpert.})}_{\text{Nonperturbative QCD}}$$

Parametrization + Global analyses



**Universal nonperturbative
hadron structure**
PDF · GPD · TMD · FFs/GFFs

Independent prediction

*Direct QCD calculation
beyond phenomenological
parametrization*

Complement to exp. data

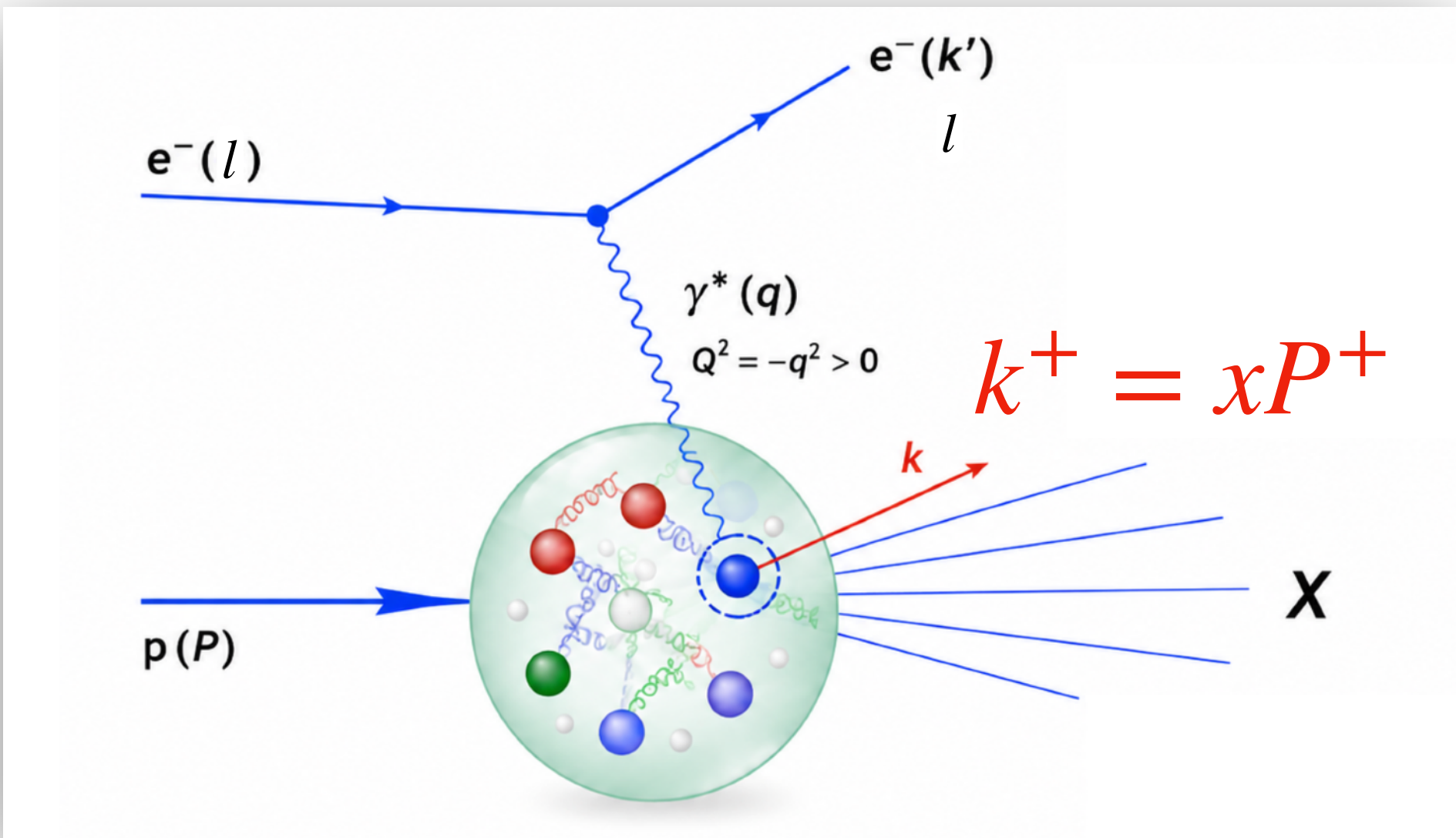
*Constrains global analysis,
especially in poorly determined
regions*

Operator-level insight

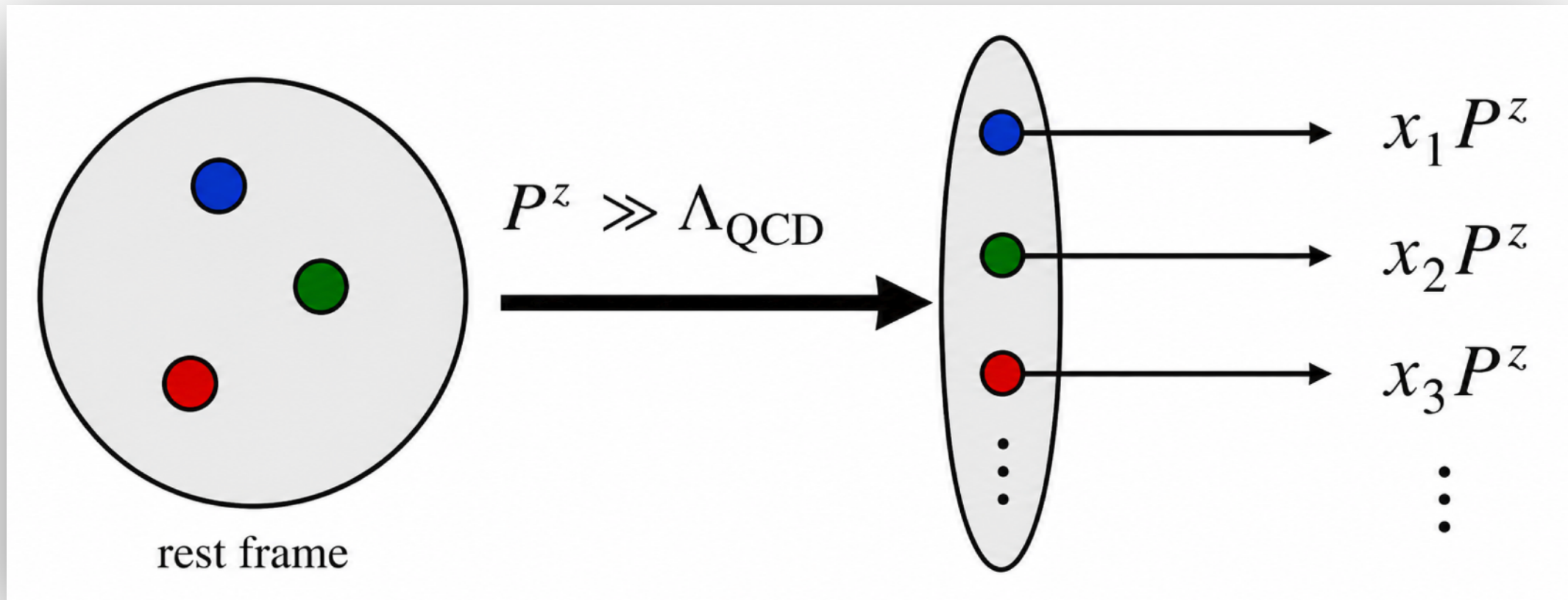
*Accesses matrix elements
difficult to isolate experimentally,
e.g., trace anomaly*

One-dimensional parton distribution functions

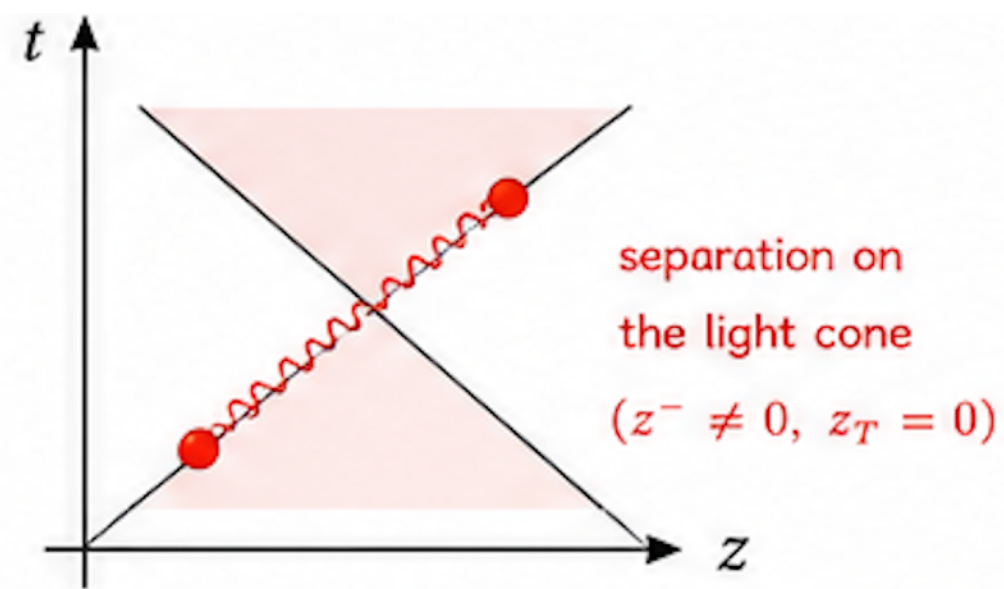
Deep inelastic scattering



Infinite-momentum picture and LaMET



$$\sigma(y) \sim H(y, x, \mu) \otimes f(x, \mu)$$

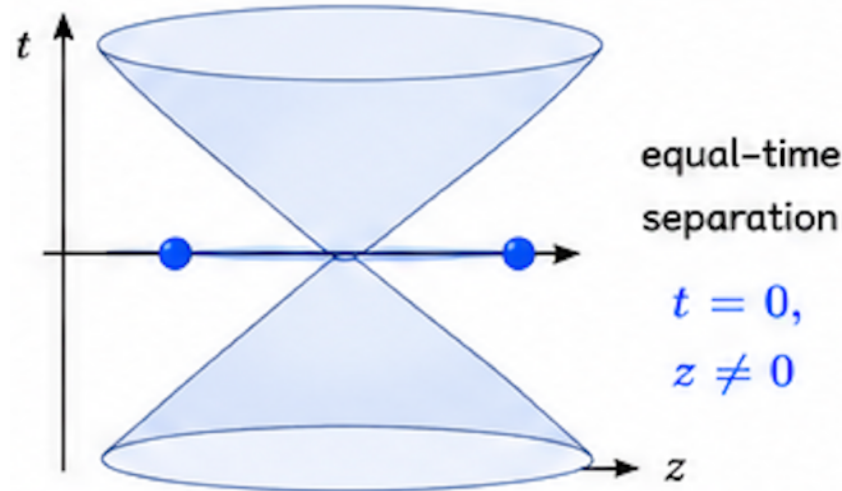


Light-cone PDF:
probability to find a
parton carrying
longitudinal *momentum*

fraction x inside a fast-
moving proton

Quasi PDF

$$\tilde{f}(x, P^z, \mu) \xrightarrow{P^z \rightarrow \infty} f(x, \mu) + \mathcal{O}\left(\frac{\Lambda_{QCD}^2}{x^2 P_z^2}, \frac{\Lambda_{QCD}^2}{(1-x)^2 P_z^2}\right)$$



*Equal-time correlator,
accessible on the lattice*

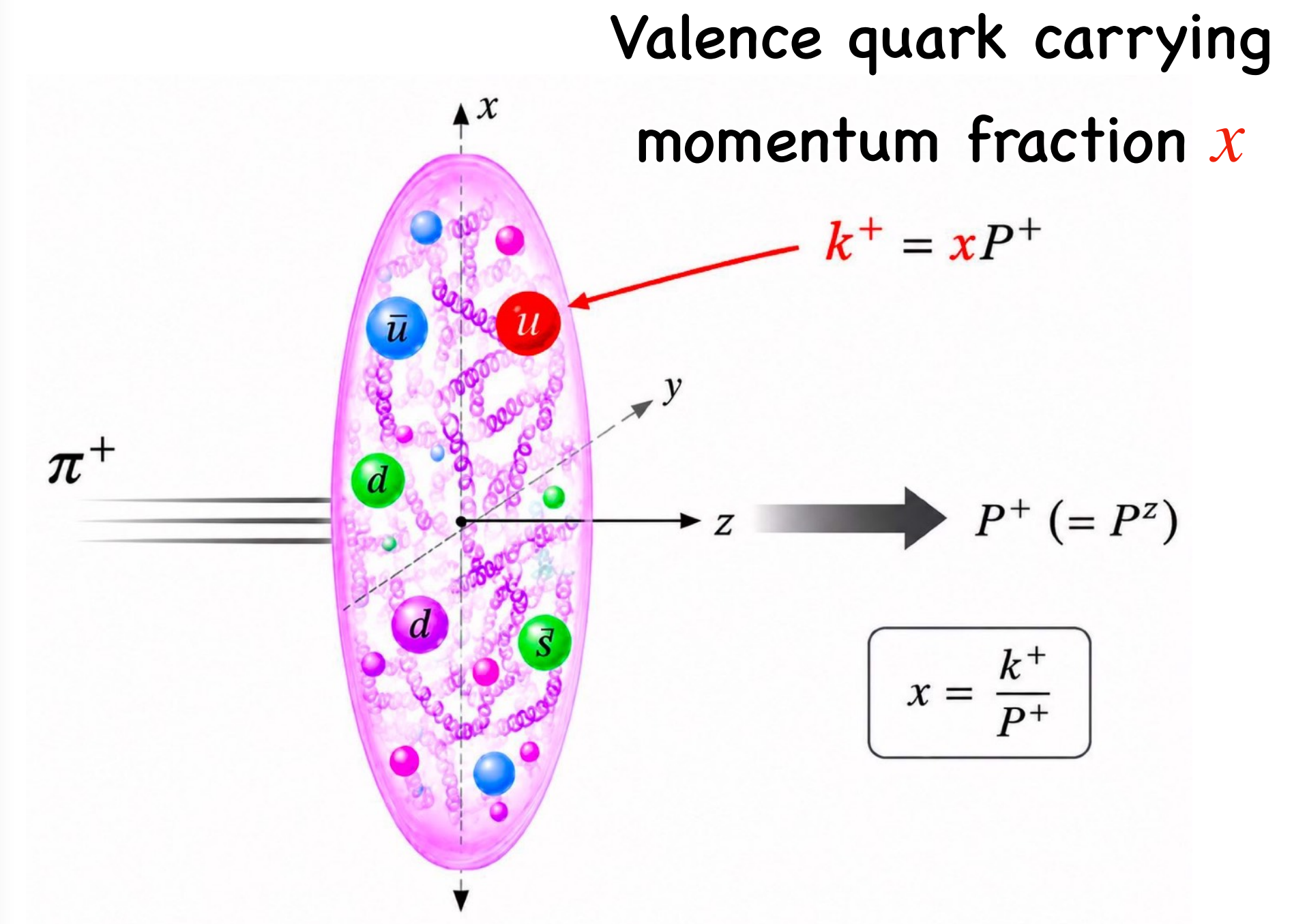
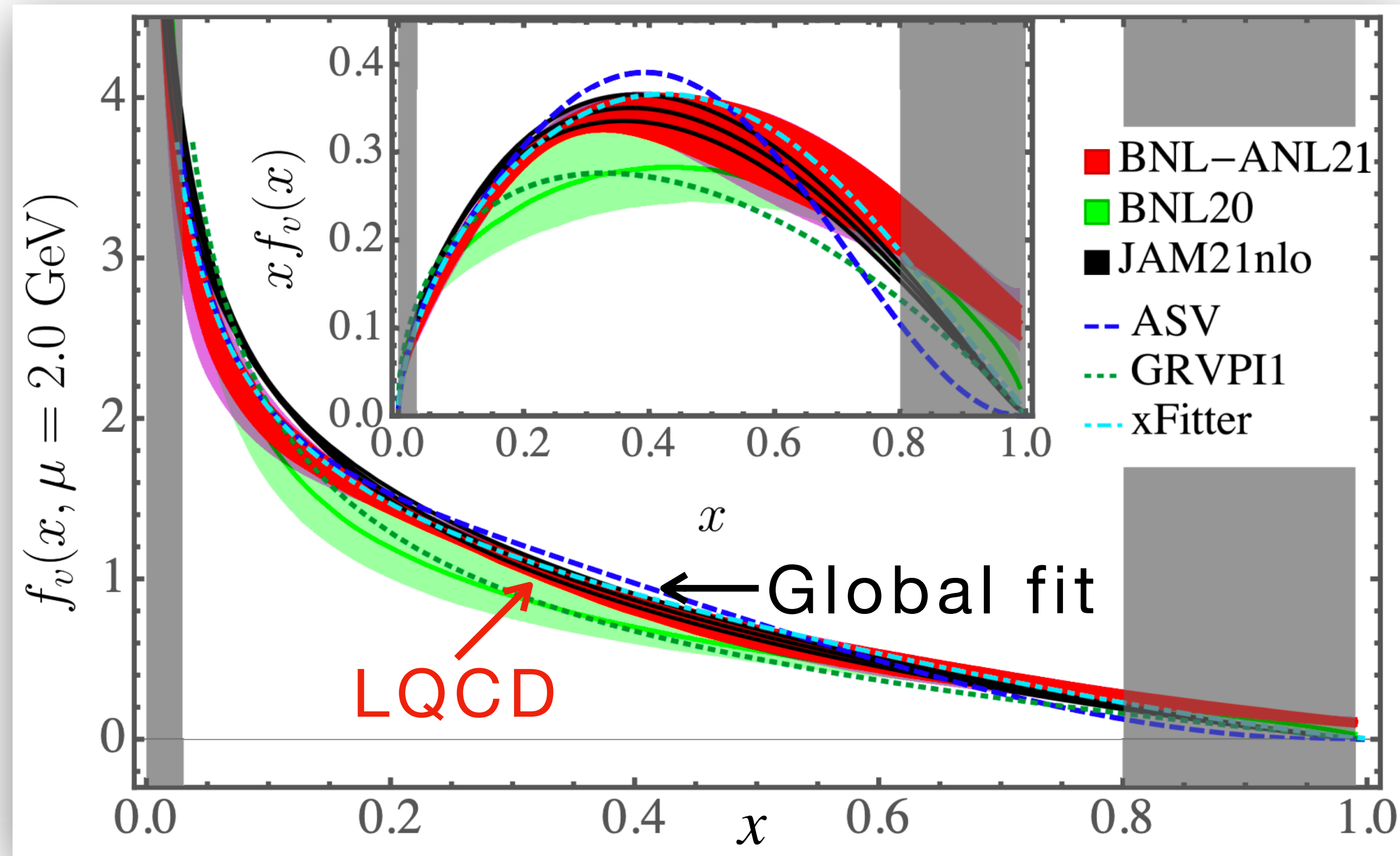
$$\tilde{h}(z) = \langle P | \bar{\psi}(0) \Gamma \mathcal{W}(0, z) \psi(z) | P \rangle$$

$$\tilde{f}(x, P_z, \mu) = \int \frac{dz}{4\pi} e^{-ixP_z z} \tilde{h}(z)$$

- X. Ji, PRL 110 (2013); SCPMA57 (2014);
- Y.-Q. Ma, et al, PRD98 (2018), PRL 120 (2018)
- X. Ji, Y. Zhao, et al, RMP 93 (2021).

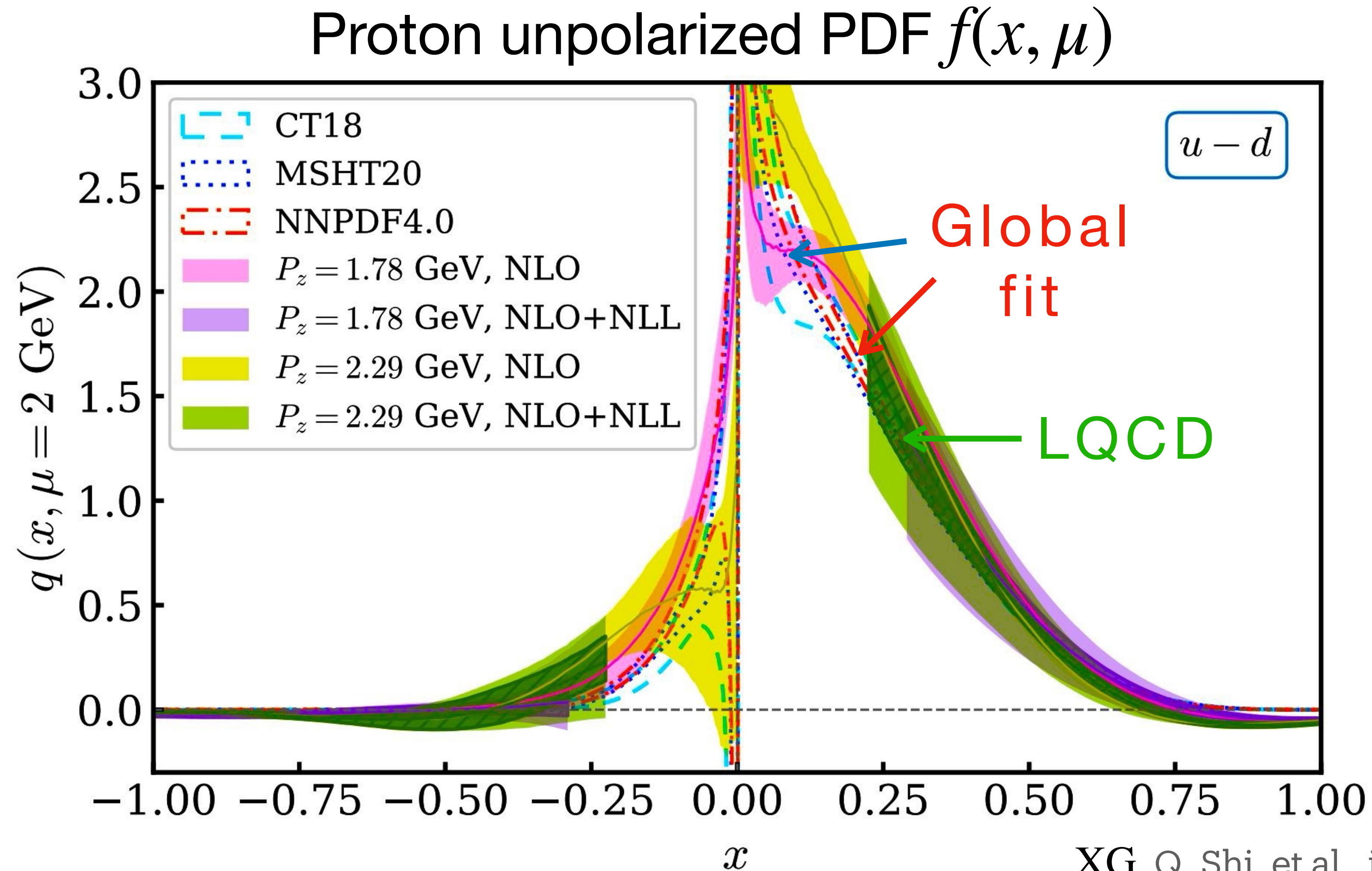
Pion valence quark PDF

Pion valence quark PDF $f(x, \mu)$



Independent QCD prediction agrees with phenomenological global fits

Proton unpolarized PDF

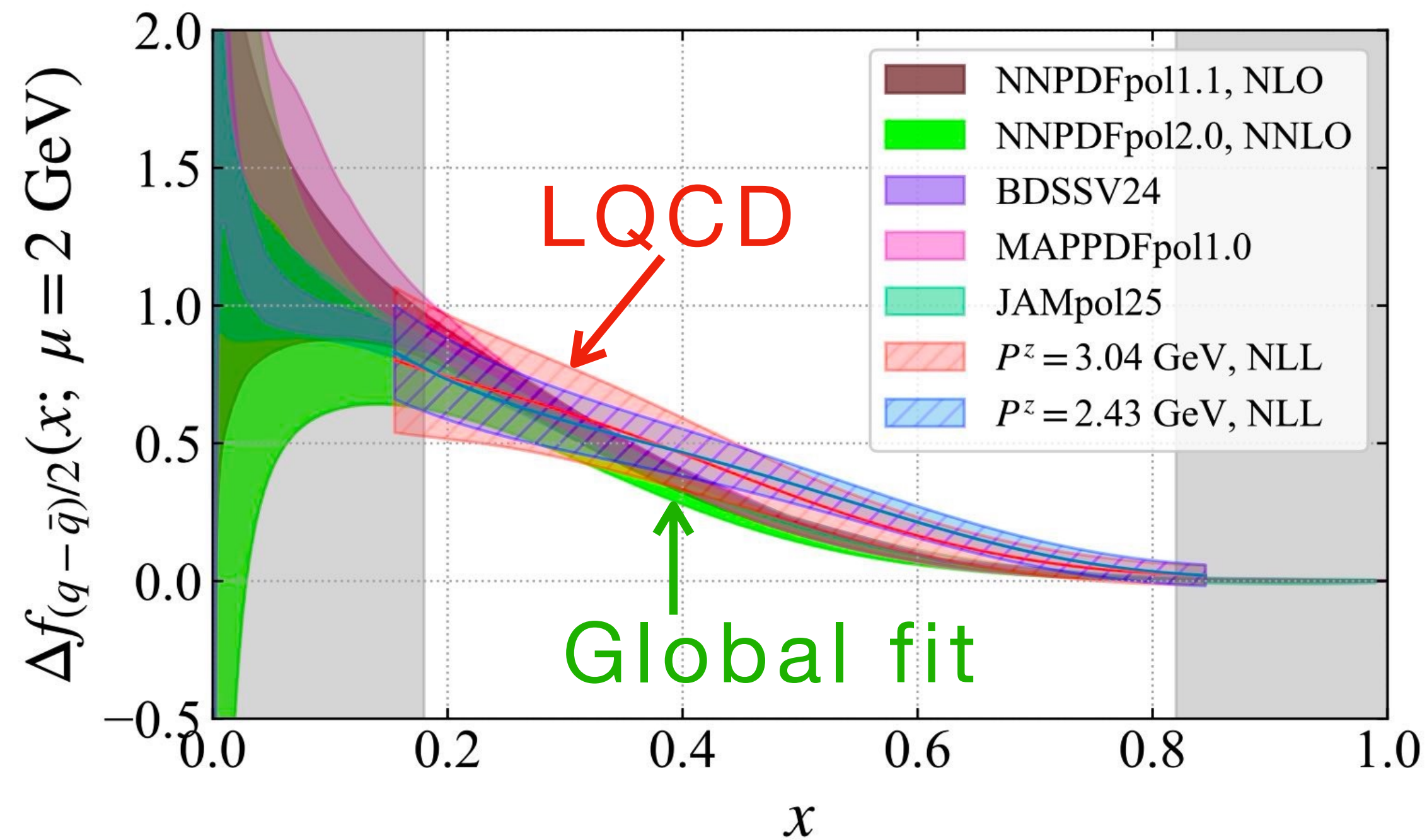


XG, Q. Shi, et al., in preparation

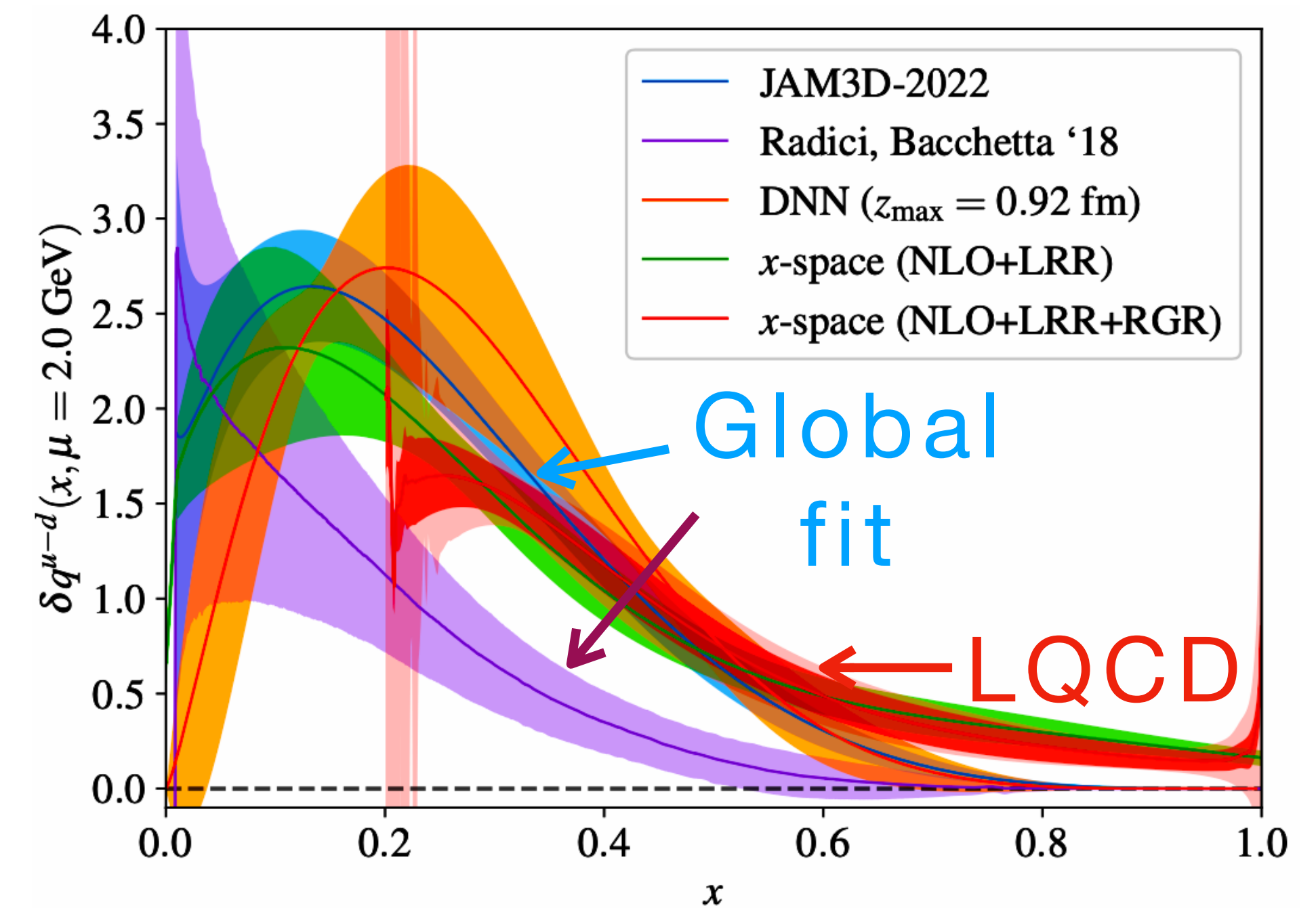
Lattice QCD as a precision-controlled tool for partonic structure from first principles

Spin-dependent proton PDFs

Proton helicity PDF $\Delta f(x, \mu)$



Proton transversity PDF $\delta f(x, \mu)$



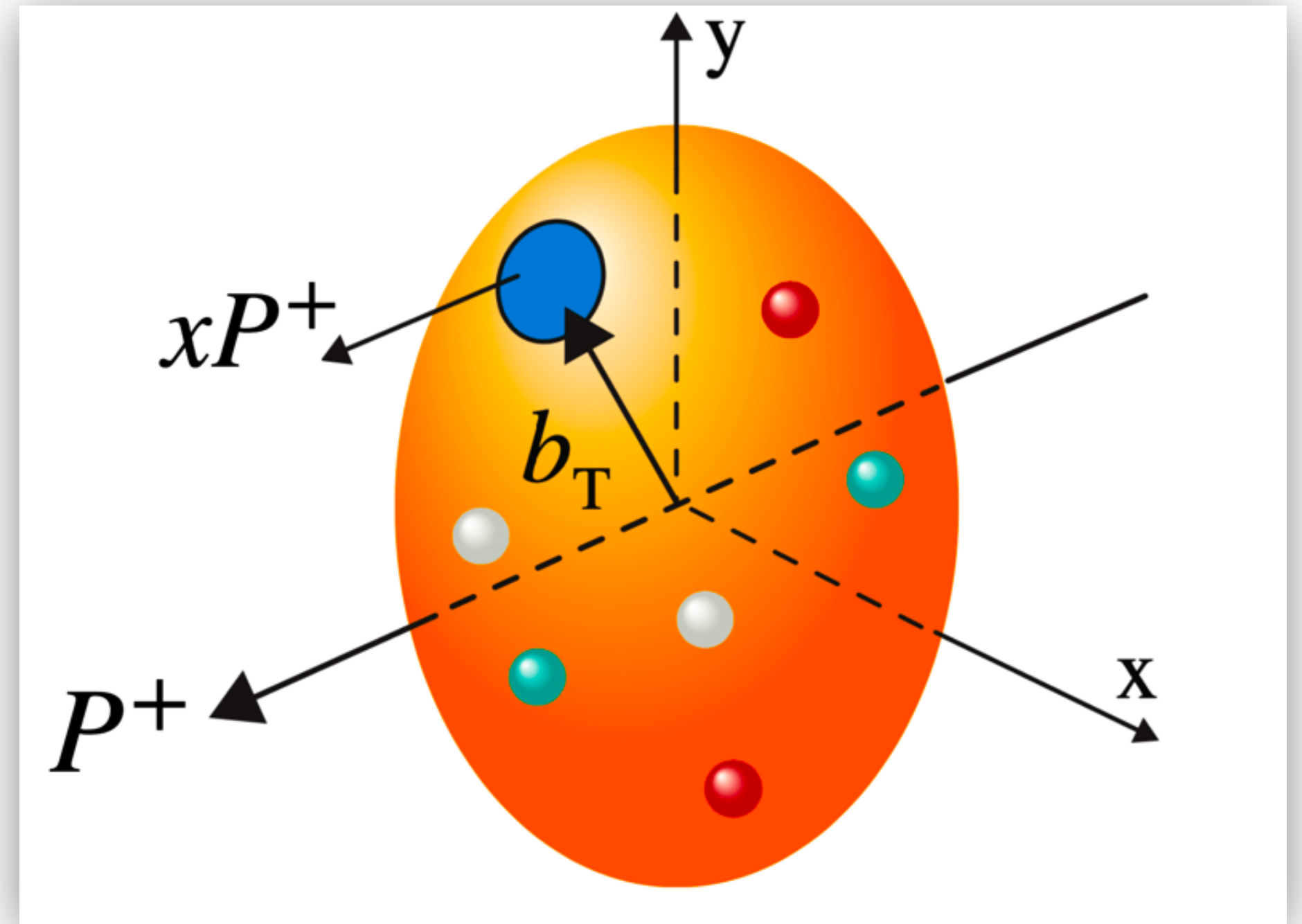
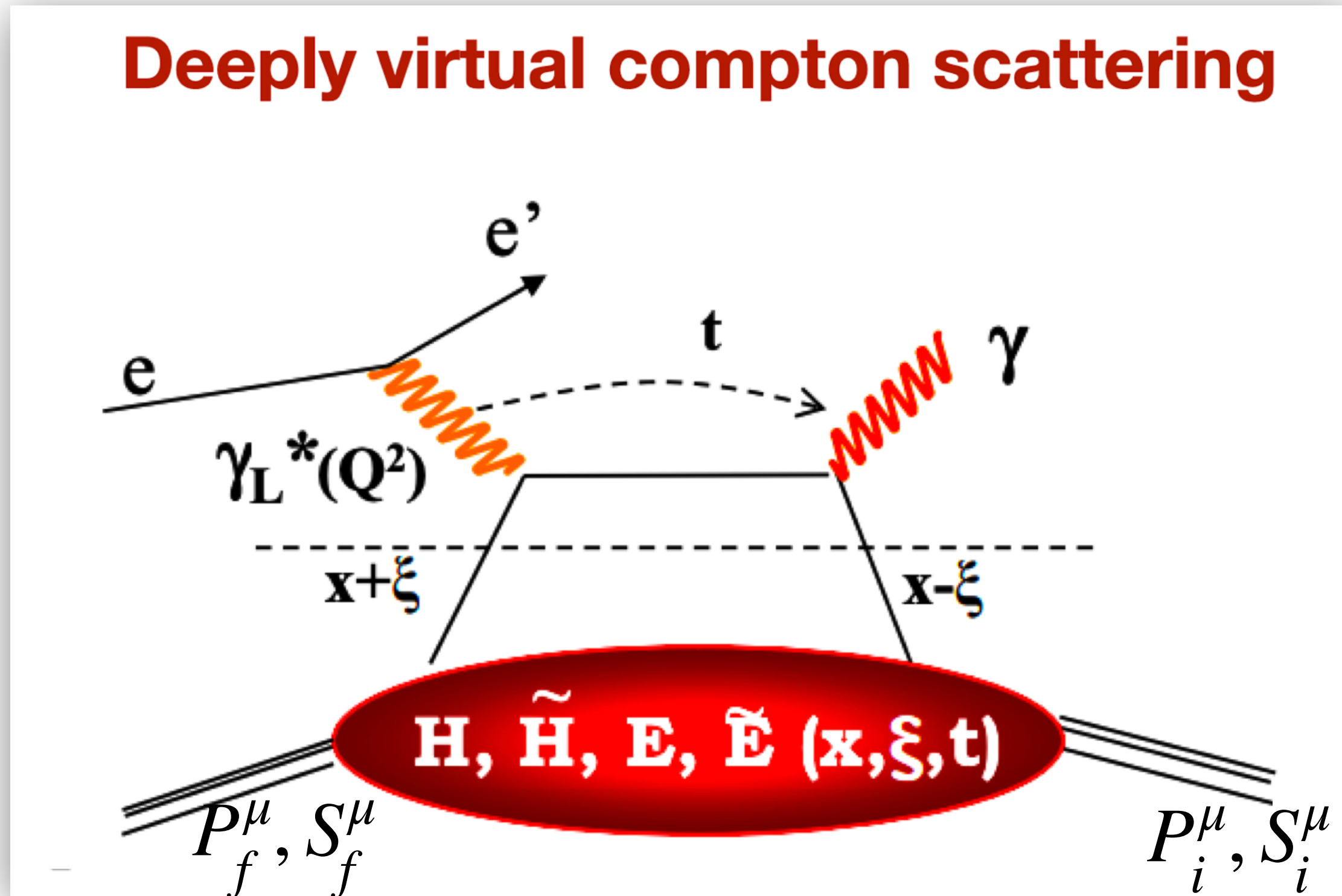
Lattice benchmarks for less-constrained spin-dependent PDFs

XG, A. Hanlon, et al., PRD 109 (2024) 5, 054506

XG, J.-C. He, et al., arXiv: 2602.11283

Generalized parton distributions: 3D hadron structure

Deeply virtual compton scattering



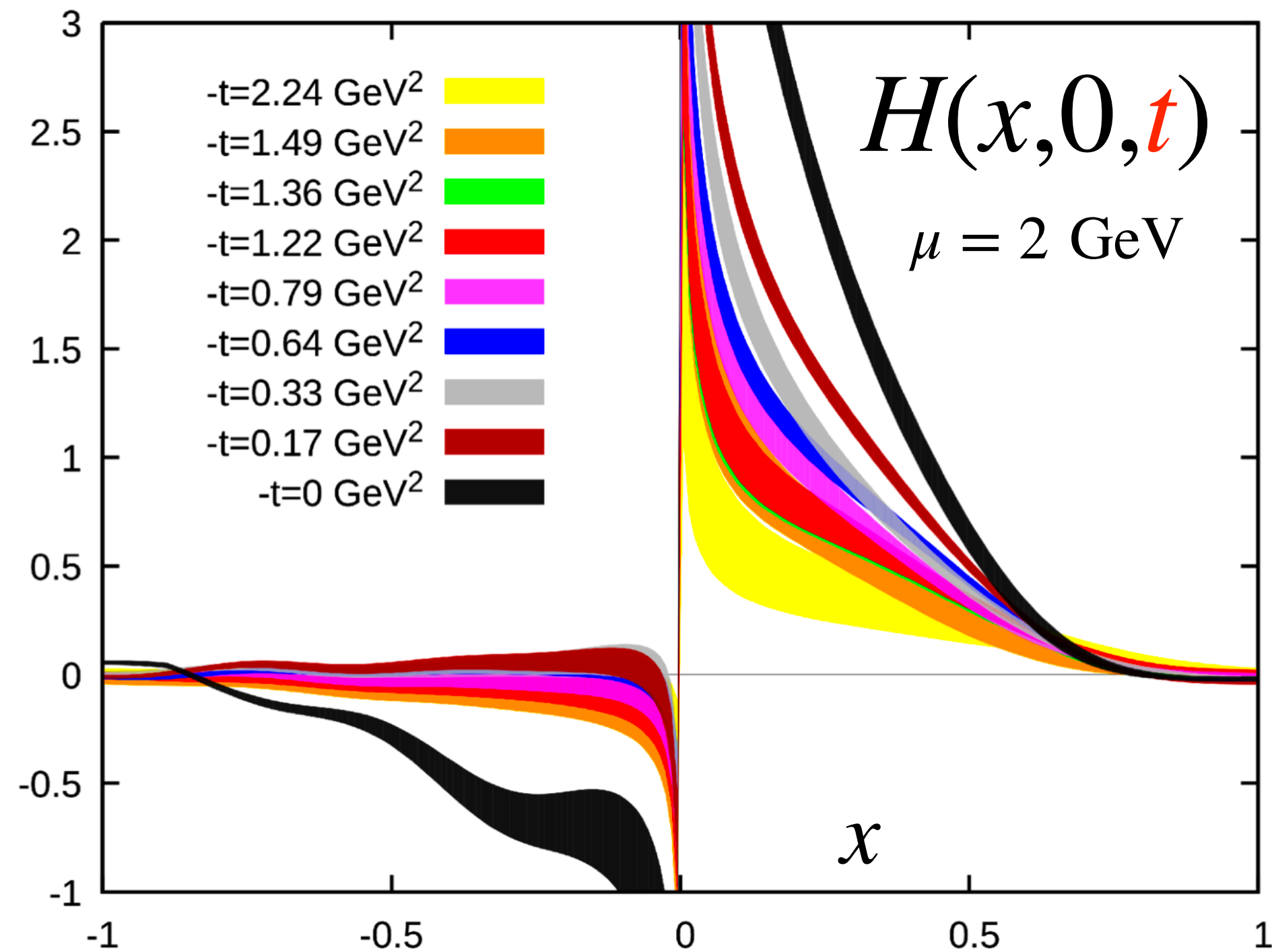
$$H(x, \xi, t) = \int \frac{dz^-}{4\pi} e^{-ixP^+z^-} \langle \mathbf{P}_f^\mu, \mathbf{S}_f^\mu | \bar{\psi}(0) \Gamma \mathcal{W}(0, z) \psi(z) | \mathbf{P}_i^\mu, \mathbf{S}_i^\mu \rangle$$

$$\Delta = P_f - P_i, \quad t = \Delta^2, \quad \xi = -\frac{\Delta^+}{2P^+}$$

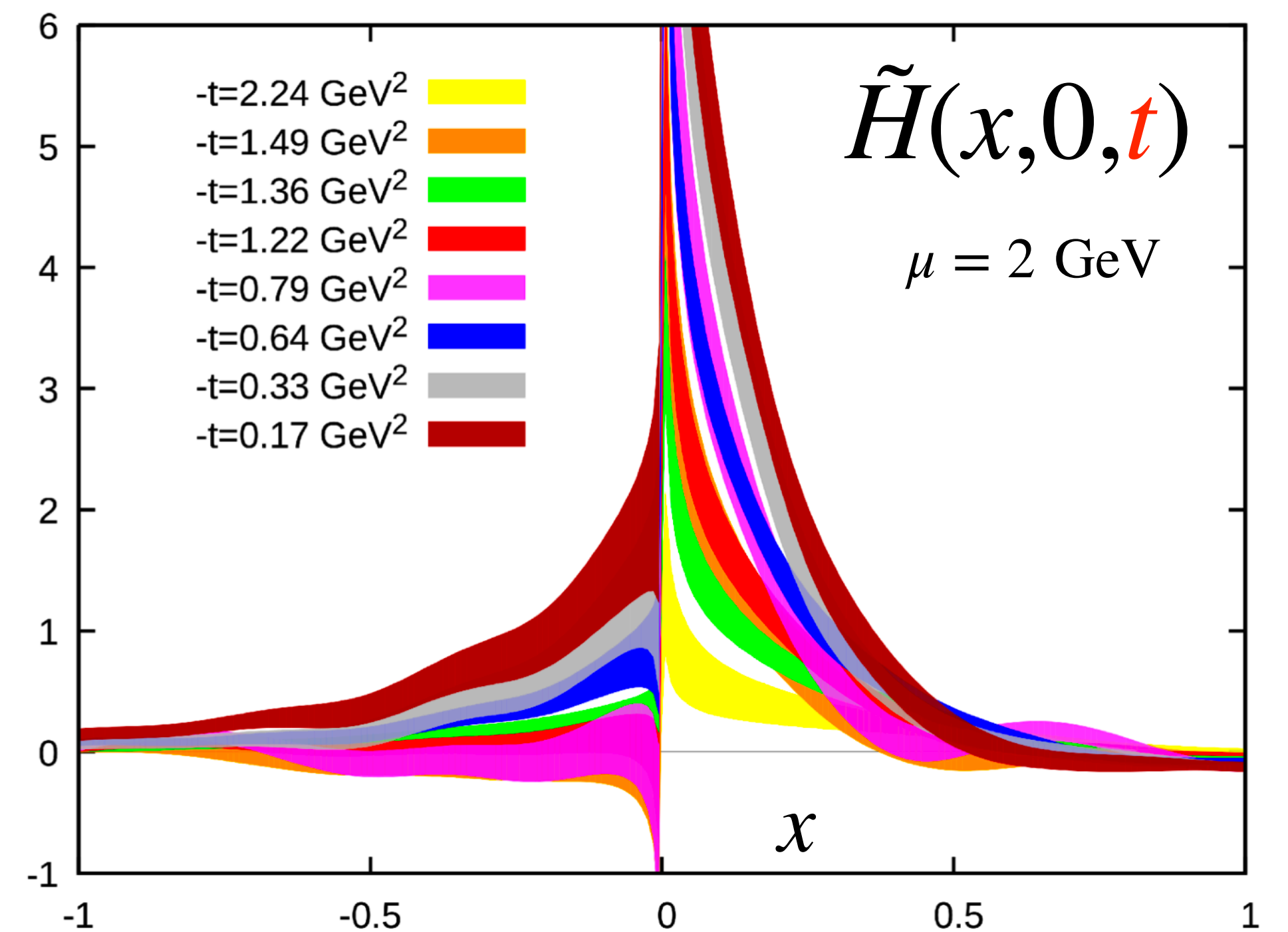
$$\text{GPD } H(x, 0, t) \xleftrightarrow{\text{F.T.}} \text{IPD } H(x, b_T)$$

Proton GPDs from lattice QCD

proton unpolarized GPD



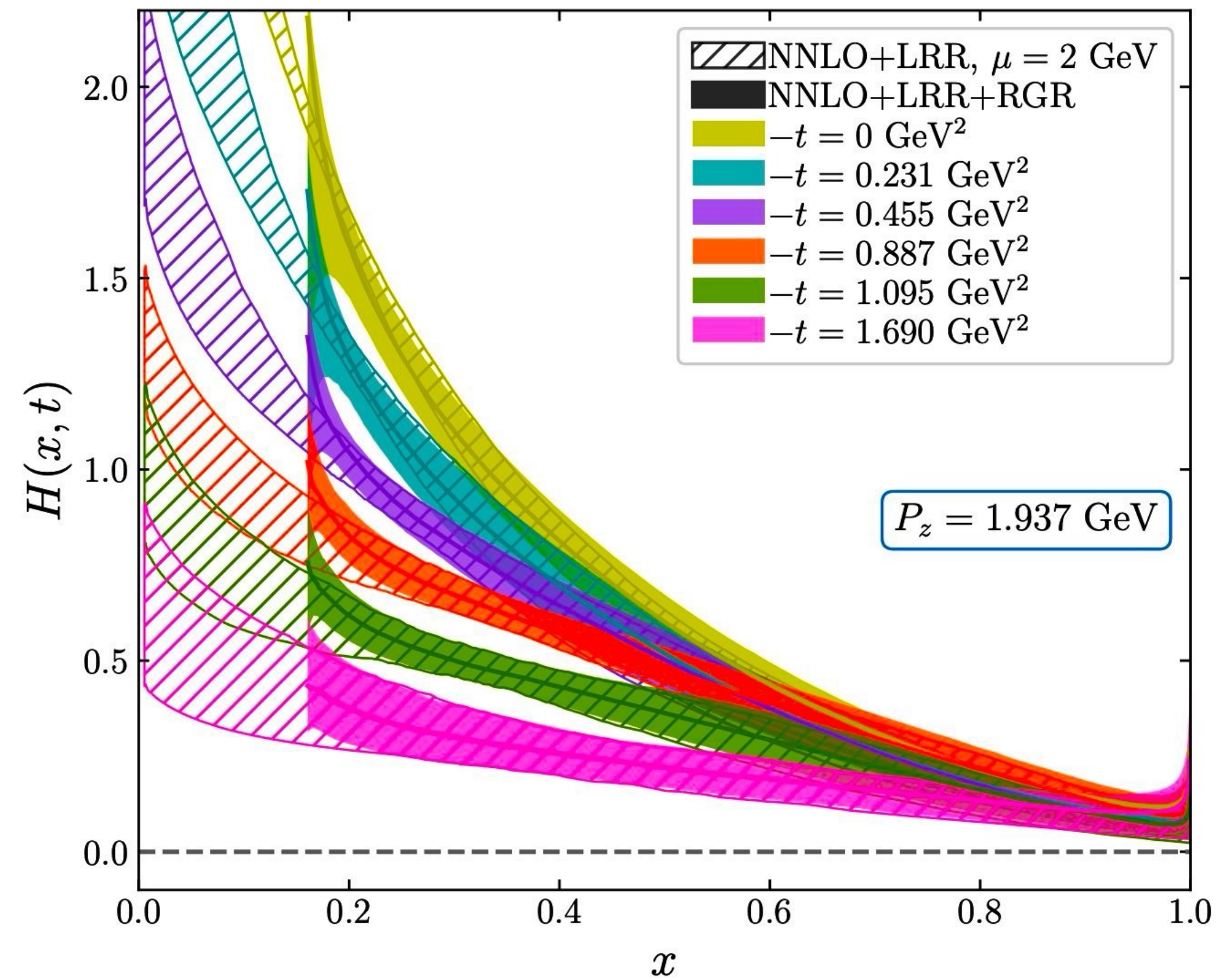
proton helicity GPD



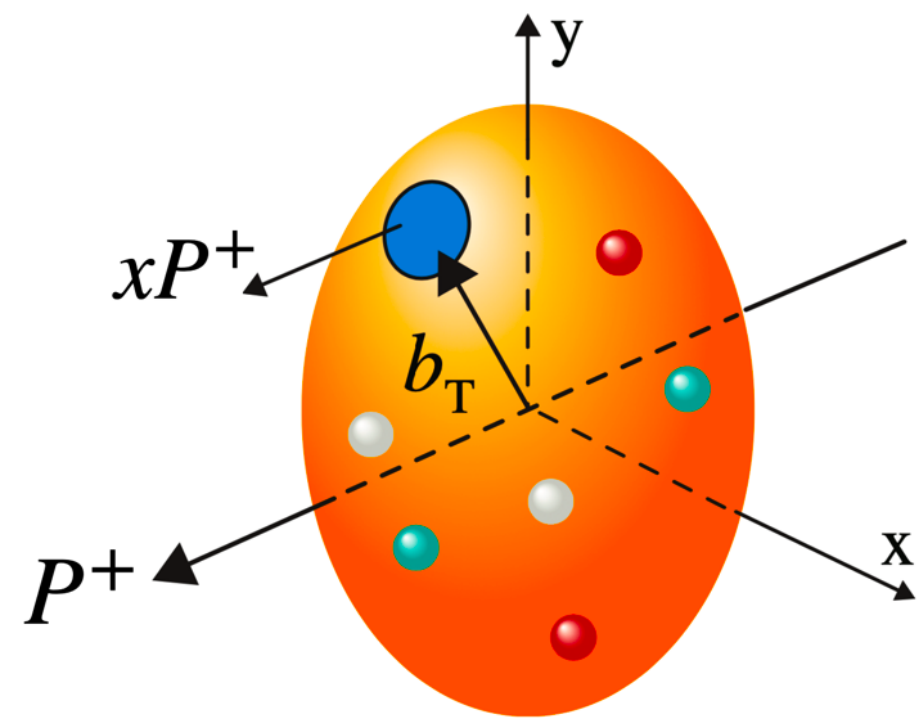
**Recent progress enables dense momentum-transfer coverage at
 $\sim 1/10$ of the conventional cost**

Pion GPDs from lattice QCD

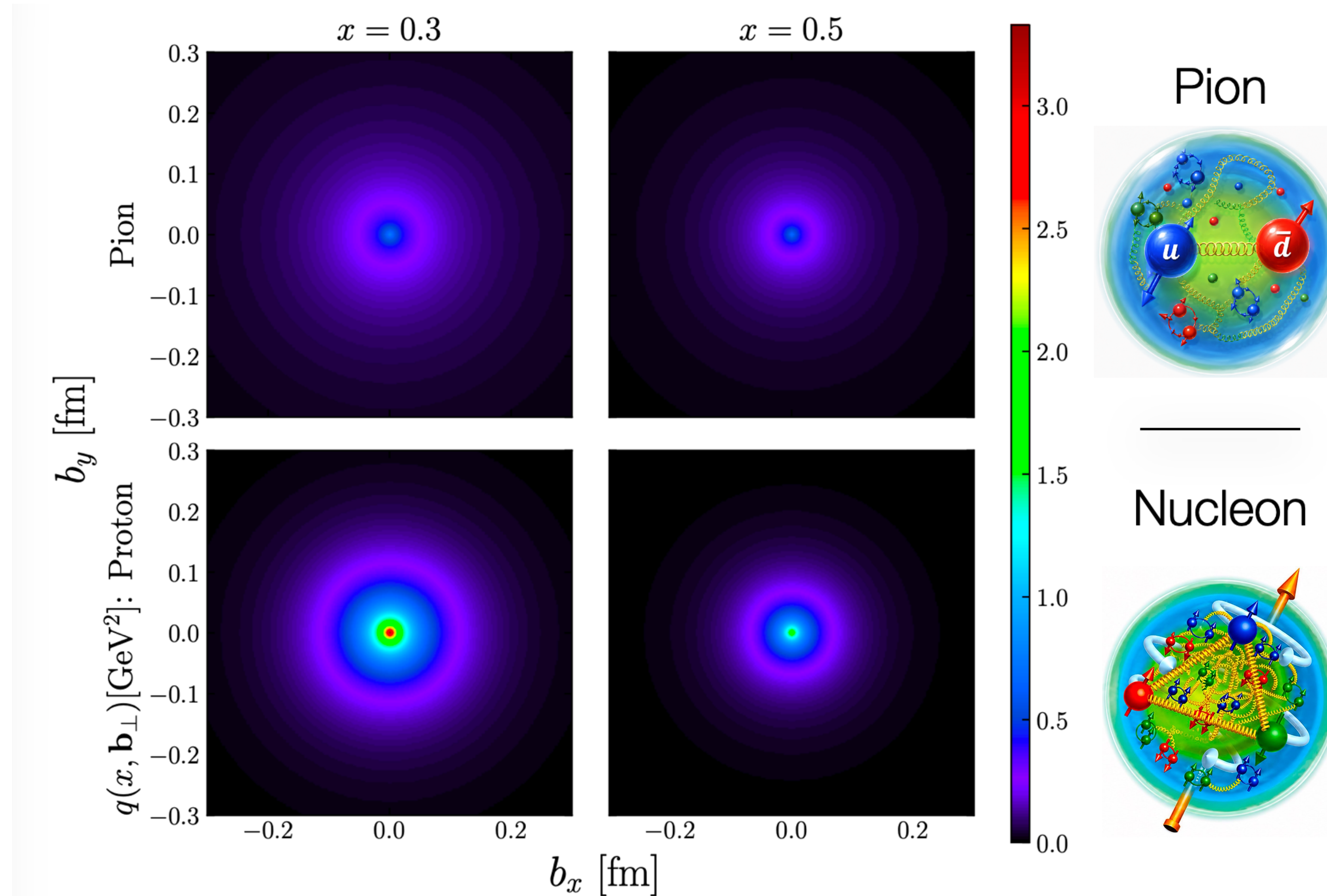
Pion valence quark GPDs $H(x, \xi, t)$



Impact-parameter distributions: transverse spatial imaging



$$H(x, b_T)$$



- Longitudinal momentum and transverse position are correlated.
- Pion and nucleon exhibit distinct transverse profiles.

Proton spin structure from GPD moments

In a **longitudinally polarized nucleon**, the contributions to the nucleon helicity $\mathbf{S}_z^N$ includes:

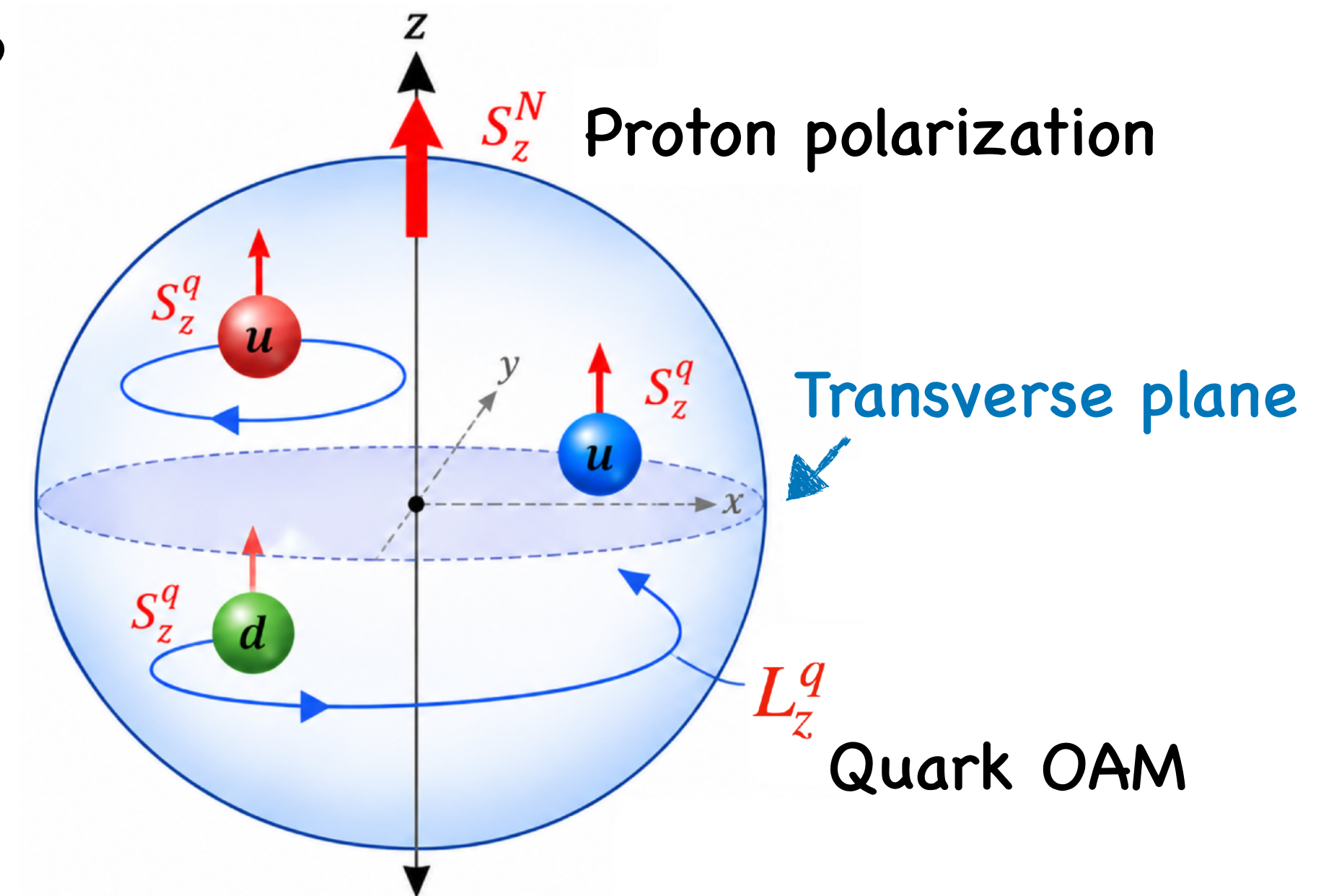
- Quark helicity $\langle \mathbf{S}_z^q \mathbf{S}_z^N \rangle$:

$$S_z^q = \frac{1}{2} \int_{-1}^1 dx \tilde{H}^q(x, \xi, t)$$

- Orbital-angular momentum (OAM) $\langle \mathbf{L}_z^q \mathbf{S}_z^N \rangle$:

$$J_z^q = \frac{1}{2} \left(\int_{-1}^1 dx x \tilde{H}^q(x, \xi, t) + \int_{-1}^1 dx x \tilde{E}^q(x, \xi, t) \right)$$

$$L_z^q = J_z^q - S_z^q$$



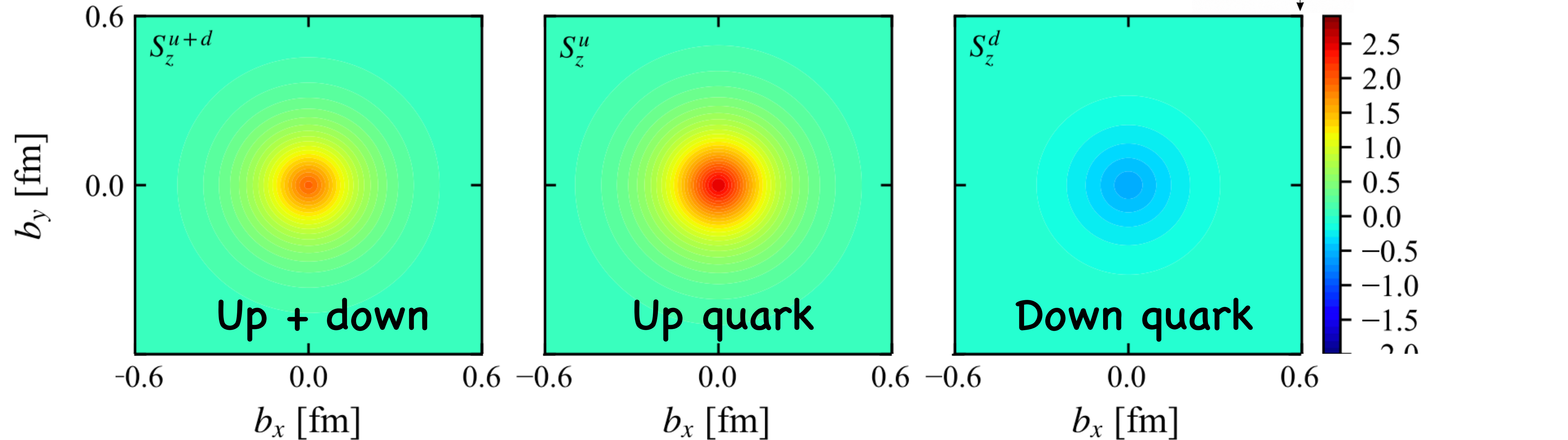
- Quark spin-orbit correlation $\langle \mathbf{S}_z^q \mathbf{L}_z^q \rangle$:

$$C_z^q \approx \frac{1}{2} \int_{-1}^1 dx x \tilde{H}^q(x, \xi, t) - \frac{1}{2} \int_{-1}^1 dx \tilde{H}^q(x, \xi, t) + \mathcal{O}\left(\frac{m_q}{2m_N}\right)$$

X.-D. Ji, PRL 78 (1997) 610-613

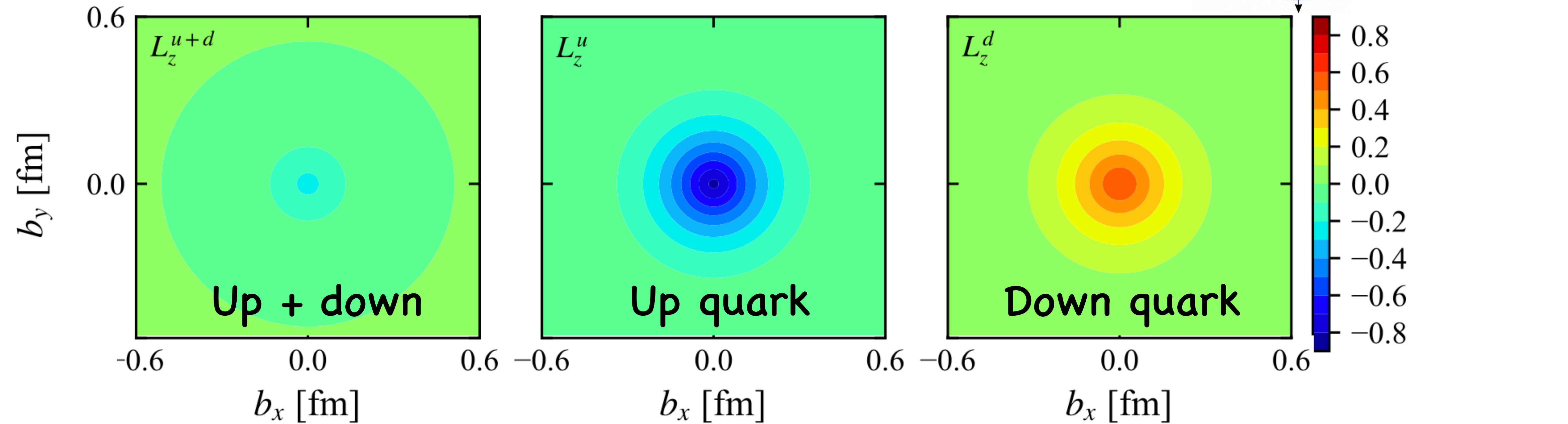
C. Lorc'e, PLB 735 (2014) 344-348

Spatial distribution of quark helicity $\langle S_z^q S_z^N \rangle$



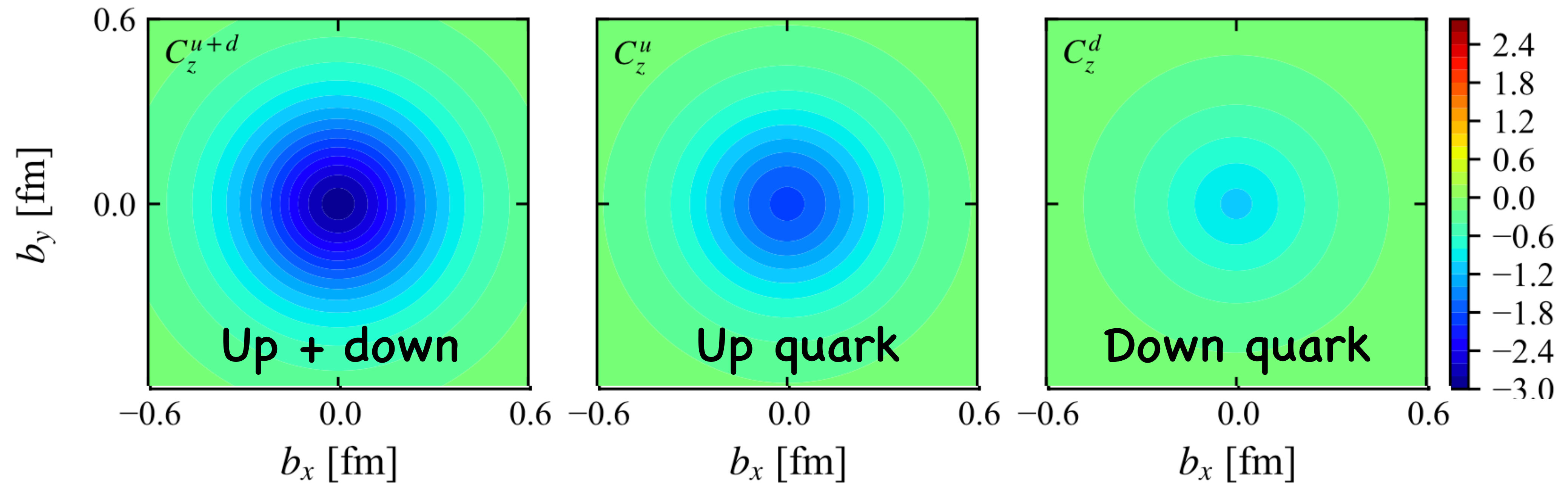
- The u- and d-quark helicity (S_z) densities have **opposite signs** and their **sum is positive**: $S_z^{u+d} > 0$

Spatial distribution of quark OAM $\langle L_z^q S_z^N \rangle$



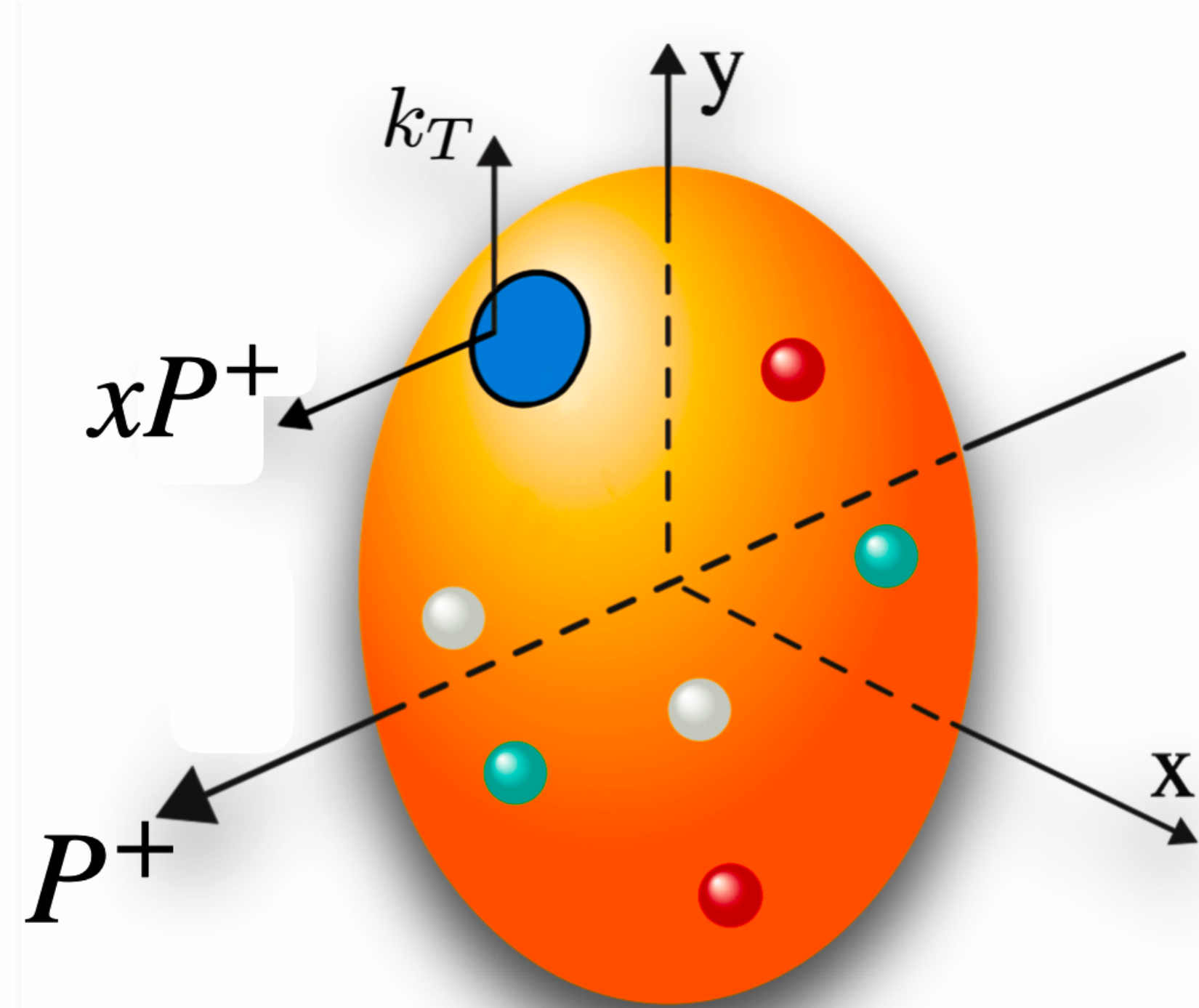
- The u - and d -quark OAM (L_z) densities have **opposite signs** and **nearly cancel** in the **u+d combination**: $L_z^{u+d} \sim 0$

Spatial distribution of quark spin-orbital correlation $\langle S_z^q L_z^q \rangle$



- The spin-orbit correlation is **negative for both light flavors**, with a larger magnitude for u, **consistent with the sign** of $S_z^q \cdot L_z^q$.

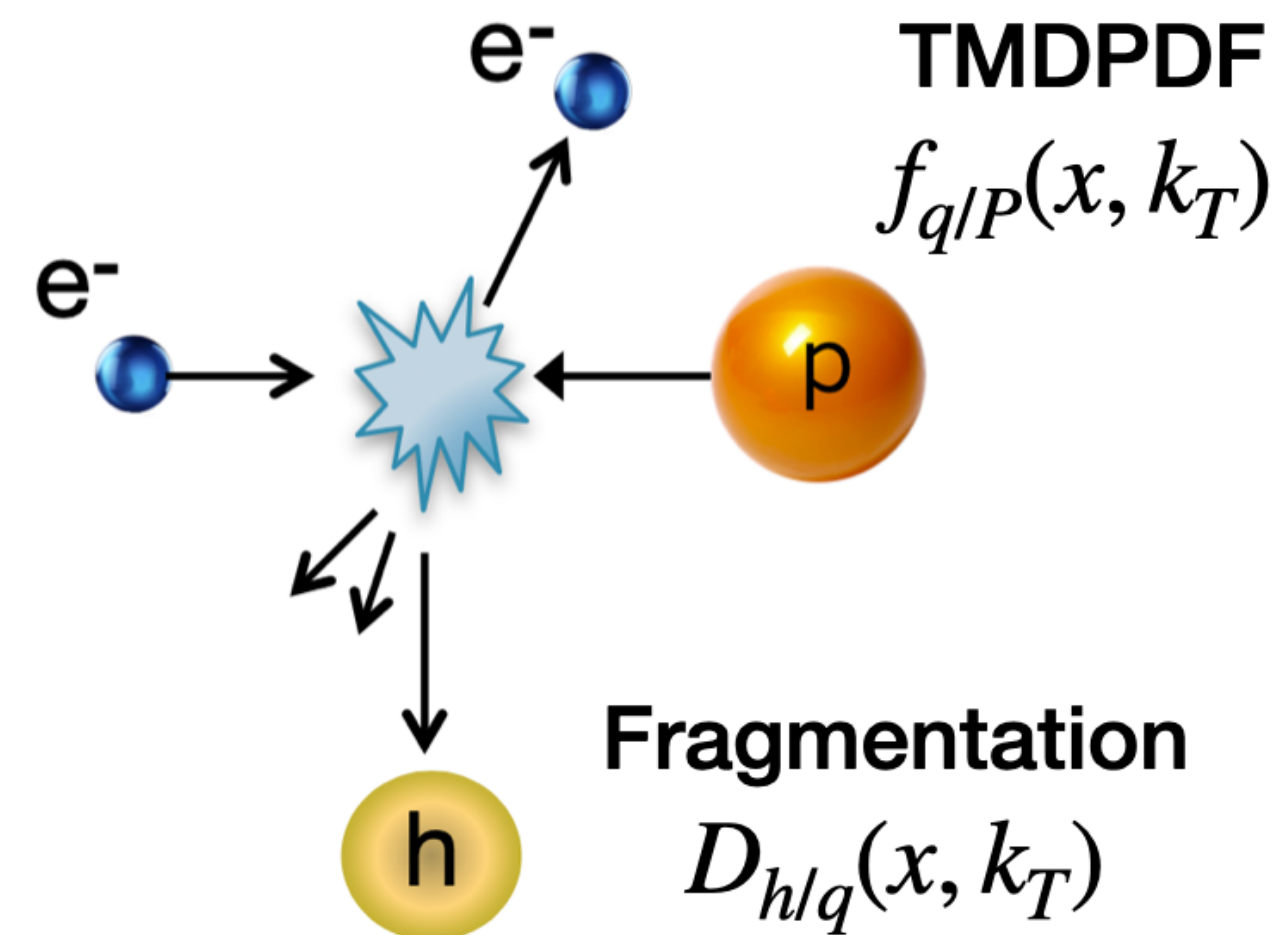
Transverse-momentum-dependent distributions: $f(x, \vec{k}_T)$



$$f(x, \vec{k}_T)$$

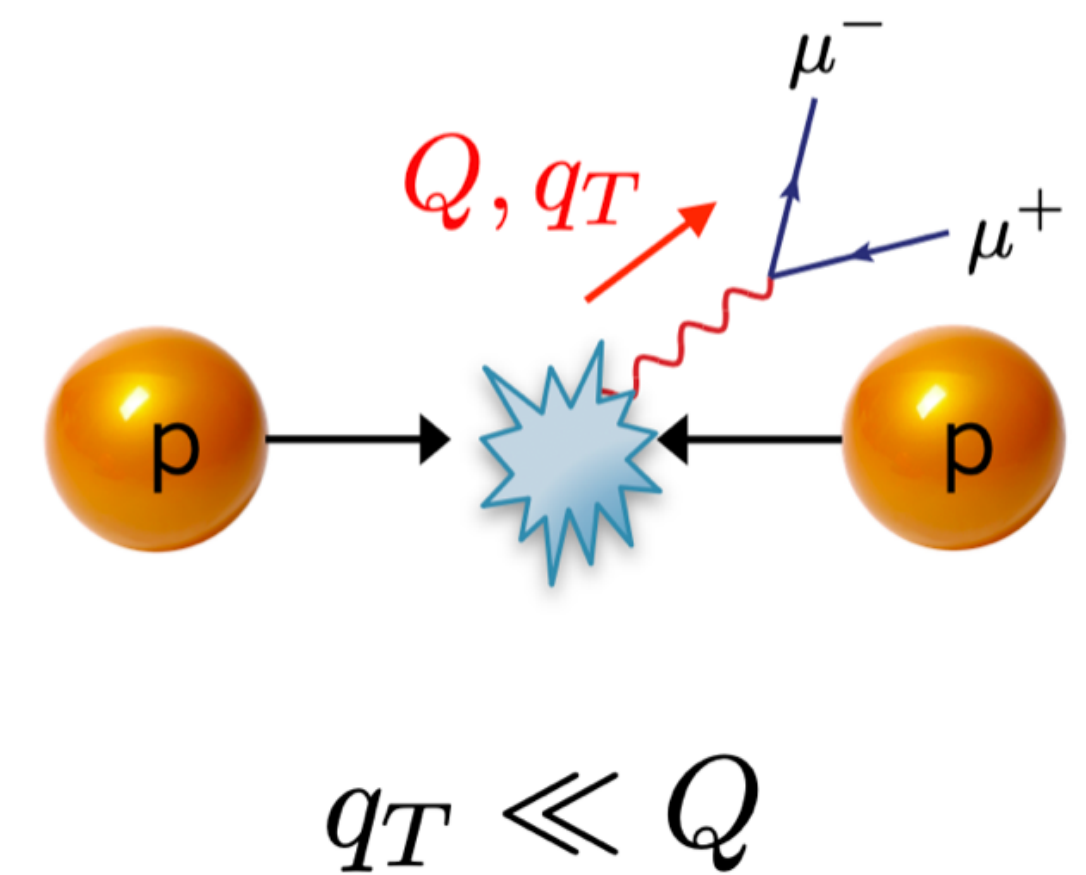
Semi-Inclusive DIS

$$\sigma \sim f_{q/P}(x, k_T) D_{h/q}(x, k_T)$$



Drell-Yan

$$\sigma \sim f_{q/P}(x, k_T) f_{q/P}(x, k_T)$$



- 3D momentum structure $f_{q/P}(x, k_T)$.
- QCD input for precision collider physics (e.g., precision W/Z phenomenology).

TMD evolution: the nonperturbative Collins-Soper kernel

- TMD evolution between energy scales:

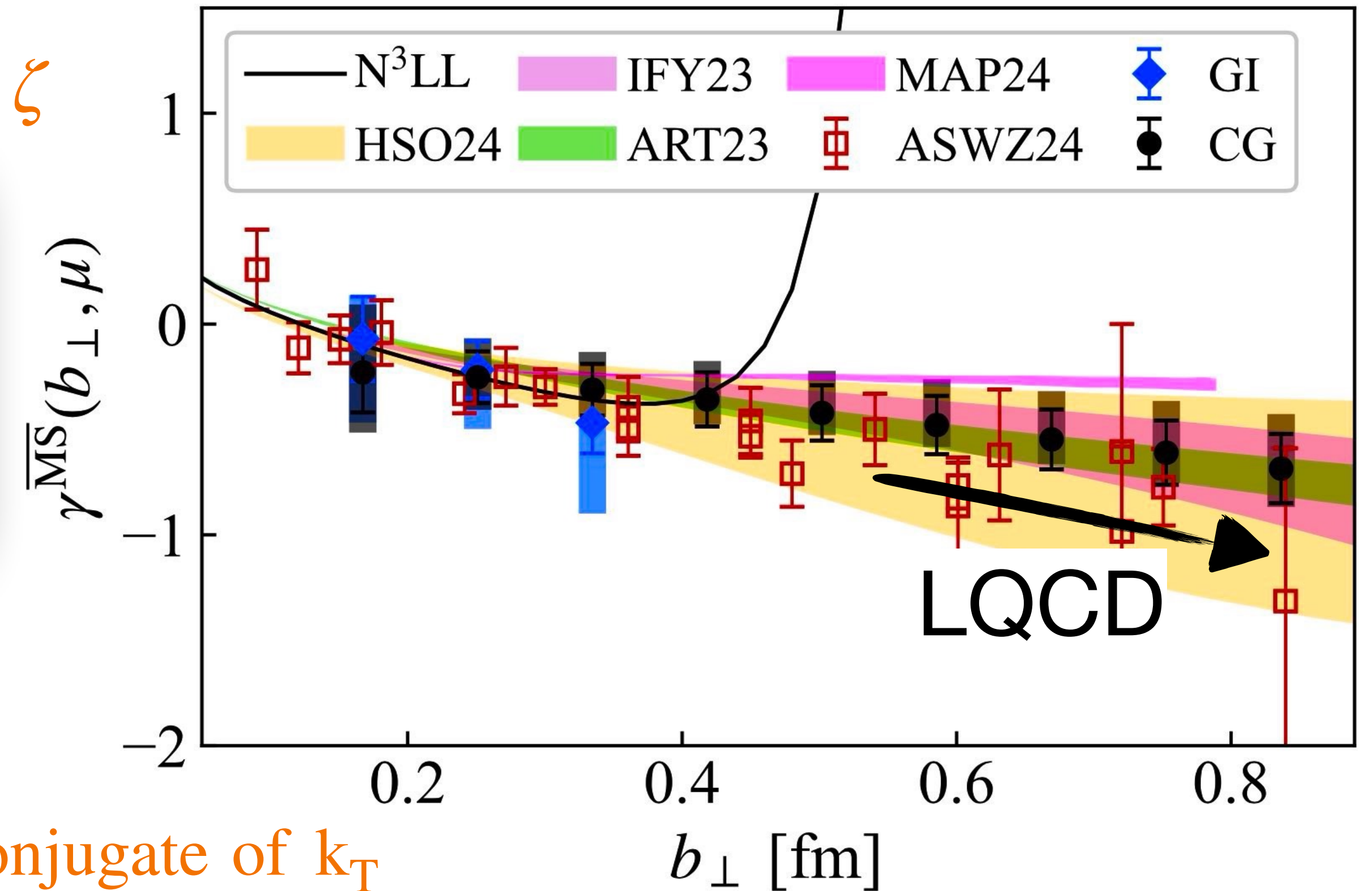
renormalization scale μ ; rapidity scale ζ

$$\mu \frac{d}{d\mu} \ln f_q(x, \vec{b}_T, \mu, \zeta) = \gamma_\mu^q(\mu, \zeta)$$
$$\zeta \frac{d}{d\zeta} \ln f_q(x, \vec{b}_T, \mu, \zeta) = \gamma_\zeta^q(\mu, b_T)$$

Collins-Soper (*rapidity evolution*) kernel:
nonperturbative when $b_T \gtrsim \Lambda_{\text{QCD}}^{-1}$

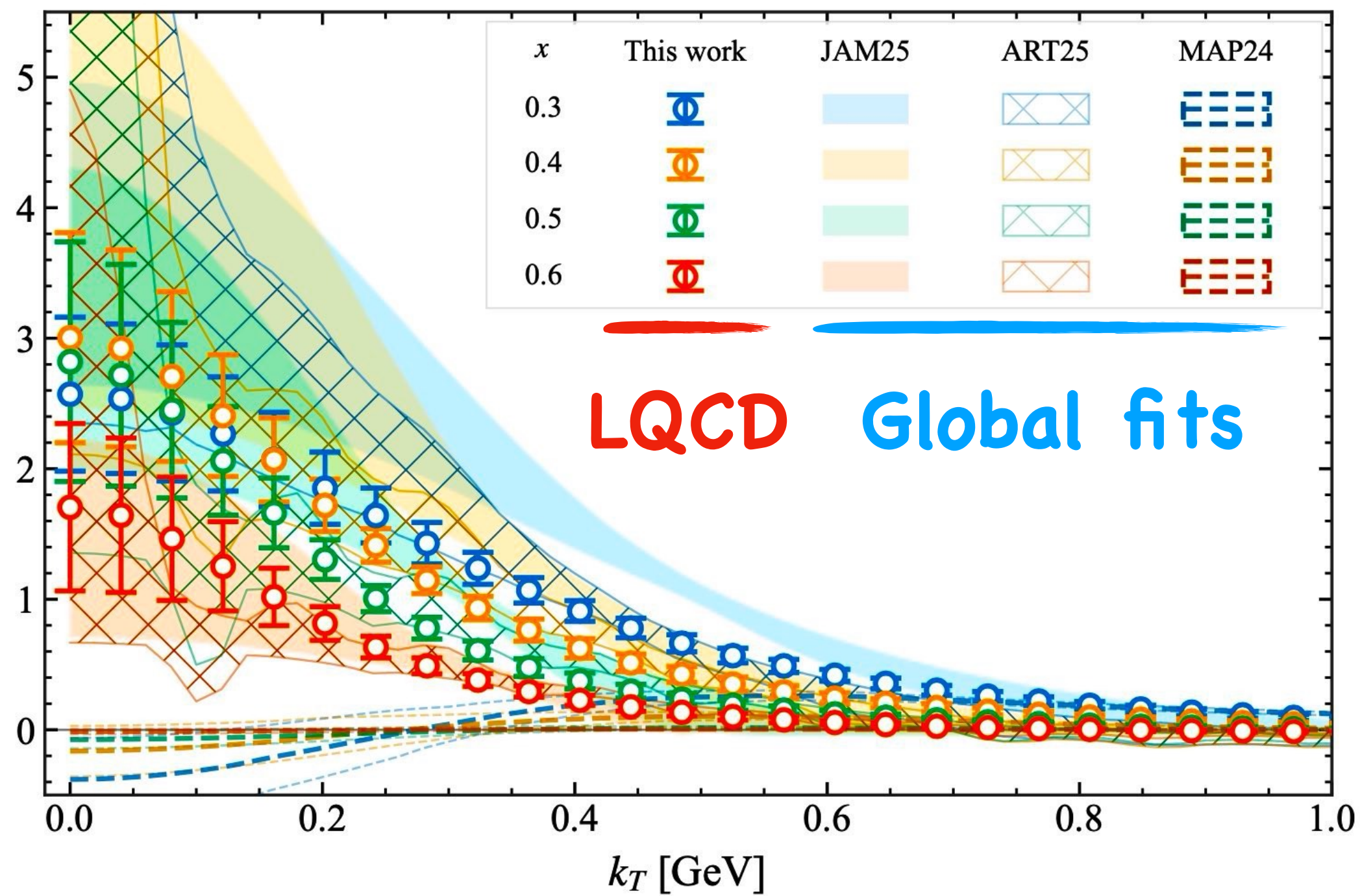
b_T : Fourier conjugate of k_T

**Lattice precision now discriminates among phenomenological models
in the nonperturbative b_T region.**



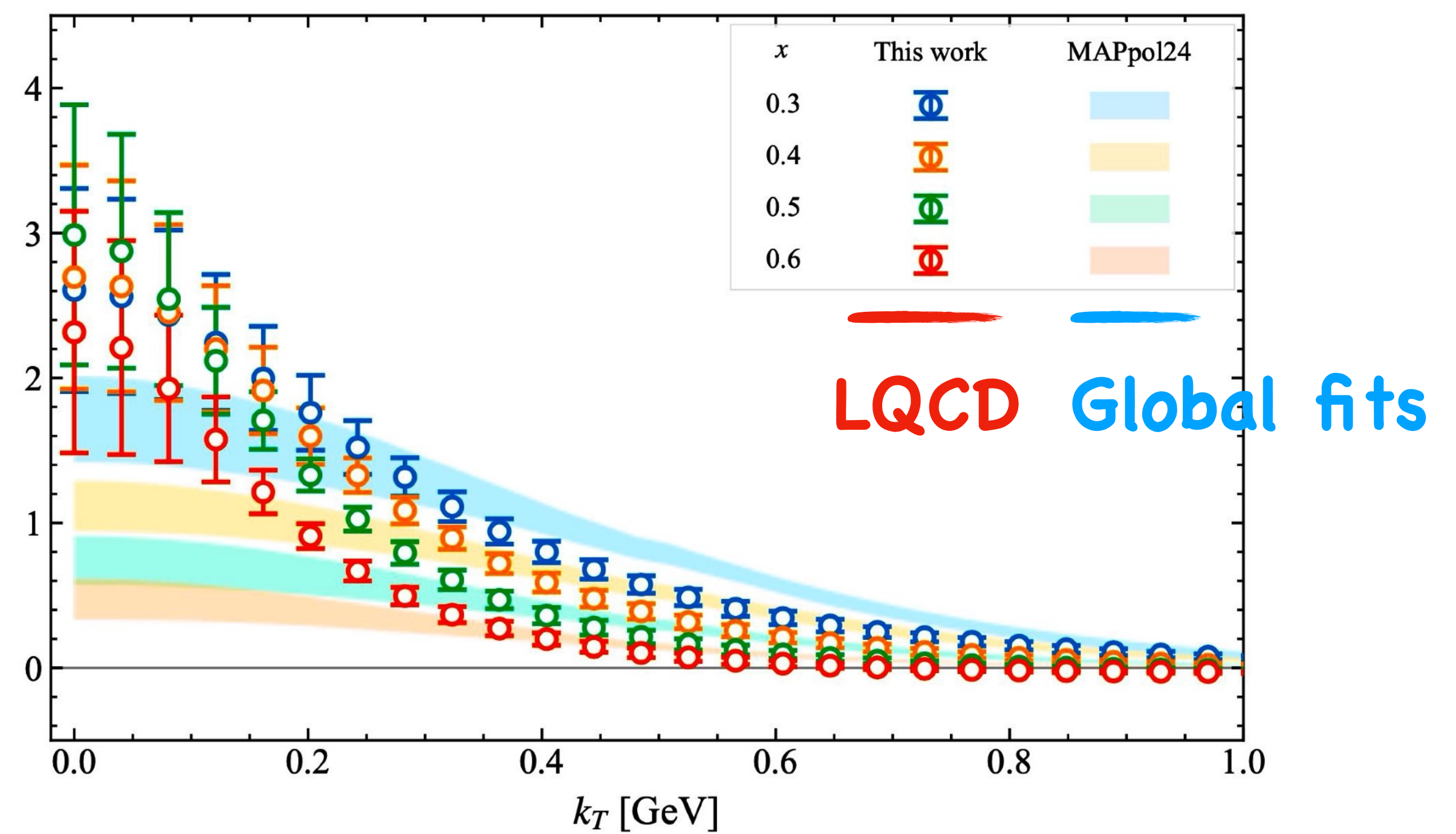
Proton TMDPDFs

Unpolarized TMDPDF $f_1^{u-d}(x, \vec{k}_T)$



$$f_1 = \text{Unpolarized}$$

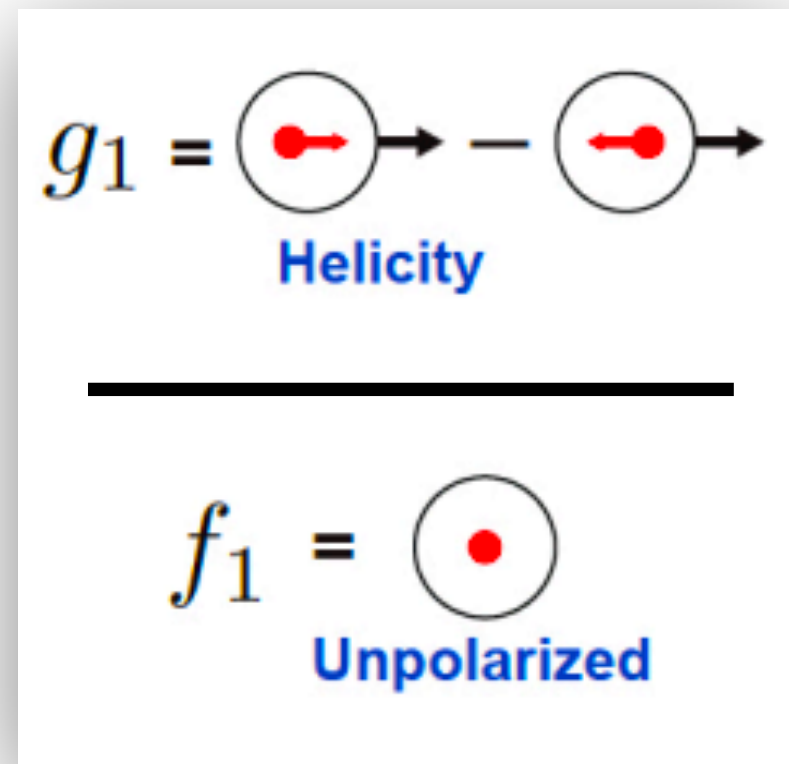
Helicity TMDPDF $g_{1L}^{u-d}(x, \vec{k}_T)$



$$g_1 = \text{Helicity}$$

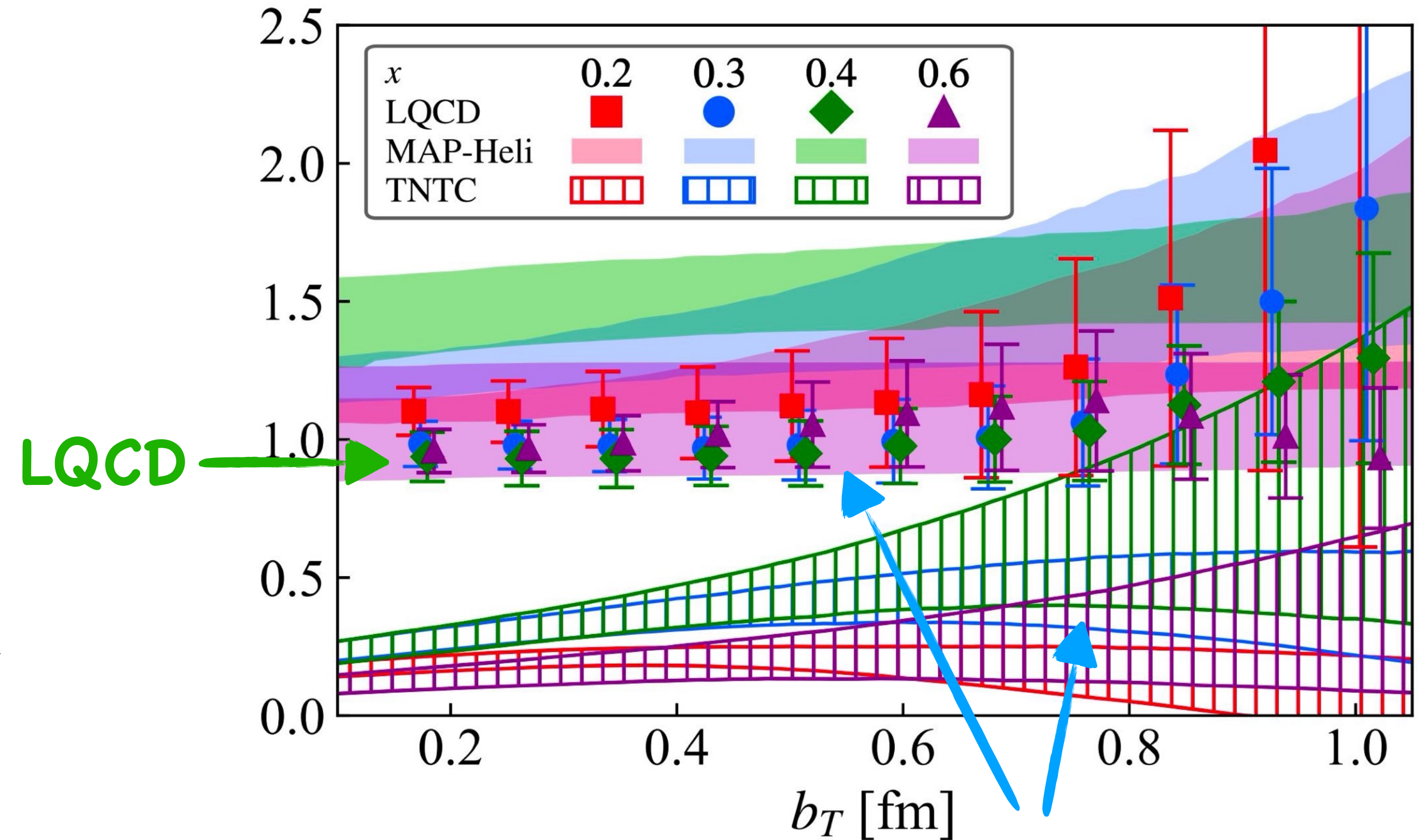
First-principles lattice benchmarks for global TMD extractions

Spin dependence of nonperturbative transverse dynamics



$$\left. \frac{g_{1L}^{\Delta u^+ - \Delta d^+}(x, b_T)}{f_1^{u_v - d_v}(x, b_T)} \frac{1}{g_A} \right|_x \approx 1$$

Helicity and unpolarized TMDPDF ratios



Within current uncertainties, no strong b_T dependence is resolved
→ approximately universal long-distance transverse dynamics?

QCD origin of hadron mass

To understand hadron mass in terms of **quarks** and **gluons**, we need the operator that measures **energy** and **momentum** in QCD.



Noether current of
spacetime translations

$$T_{\text{QCD}}^{\mu\nu} = T_{q,R}^{\mu\nu} + T_{g,R}^{\mu\nu}$$

$$P^\mu = \int d^3x T^{0\mu}$$

$$H_{\text{QCD}} = \int d^3x T^{00}$$

Hadron matrix elements

$$\langle H | T_{q,g}^{\mu\nu} | H \rangle$$

$\langle p' | T^{\mu\nu} | p \rangle \longrightarrow$ *gravitational form factors*

$\langle p | T^{00} | p \rangle \longrightarrow$ *energy / rest-frame mass*

$\langle p | T^\mu_\mu | p \rangle \longrightarrow$ *scale breaking / trace anomaly*

**The EMT connects quark-gluon dynamics to hadron mass, GFFs,
and the trace anomaly.**

Quark-gluon decomposition of hadron mass

$$M_H = \frac{1}{2M_H} \langle H | T^{00} | H \rangle$$

Rest-frame Hamiltonian definition of mass

- Ji (1995)
- Lorcé (2018)
- Metz-Pasquini-Rodini (2020)

● Two-terms

$$T^{00} = T_{q,R}^{00} + T_{g,R}^{00}$$

$$M = M_q + M_g$$

Quark/gluon energy

● Three-terms

$$\sigma \equiv \frac{1}{2M_H} \langle H | \sum_f m_f \bar{\psi}_f(x) \psi_f(x) | H \rangle$$

$$M = \sigma + (M_q - \sigma) + M_g$$

Explicit quark-mass term

● Ji' four-term decomposition

$$T^{00} = \bar{T}^{00} + \frac{1}{4} T^\mu_\mu$$

$$M = \sigma + M_q + M_g + M_a$$

Traceless vs trace sectors

$$M_H = \frac{1}{2M_H} \langle H | T^\mu_\mu | H \rangle$$

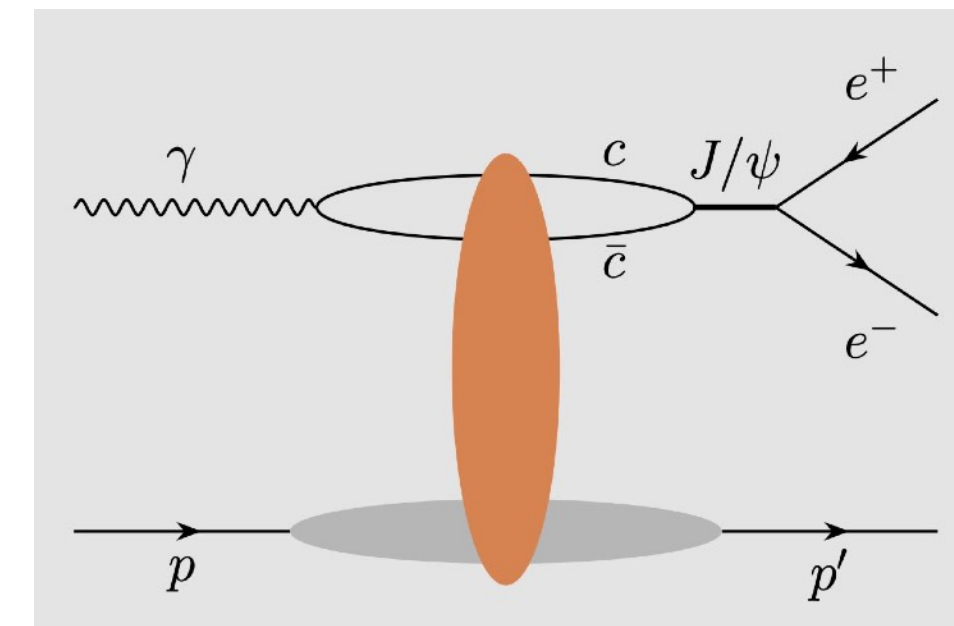
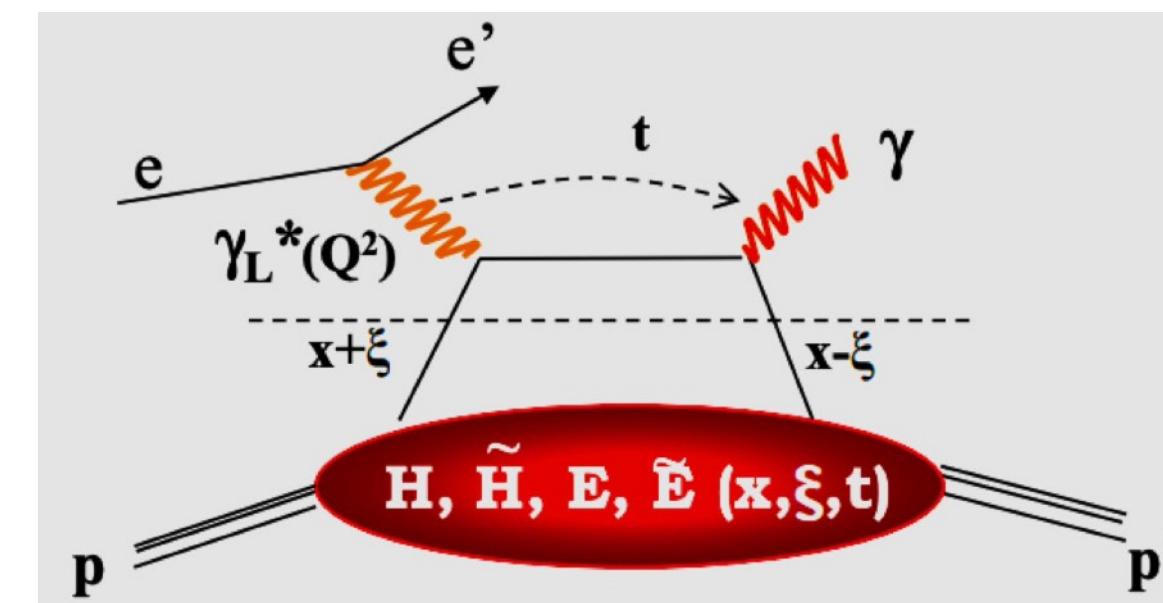
Trace encodes the hadron mass scale

● Trace decomposition

$$M = \frac{1}{2M} \left(\langle (T_{q,R})^\mu_\mu \rangle + \langle (T_{g,R})^\mu_\mu \rangle \right)$$

- Hatta-Rajan-Tanaka (2018)

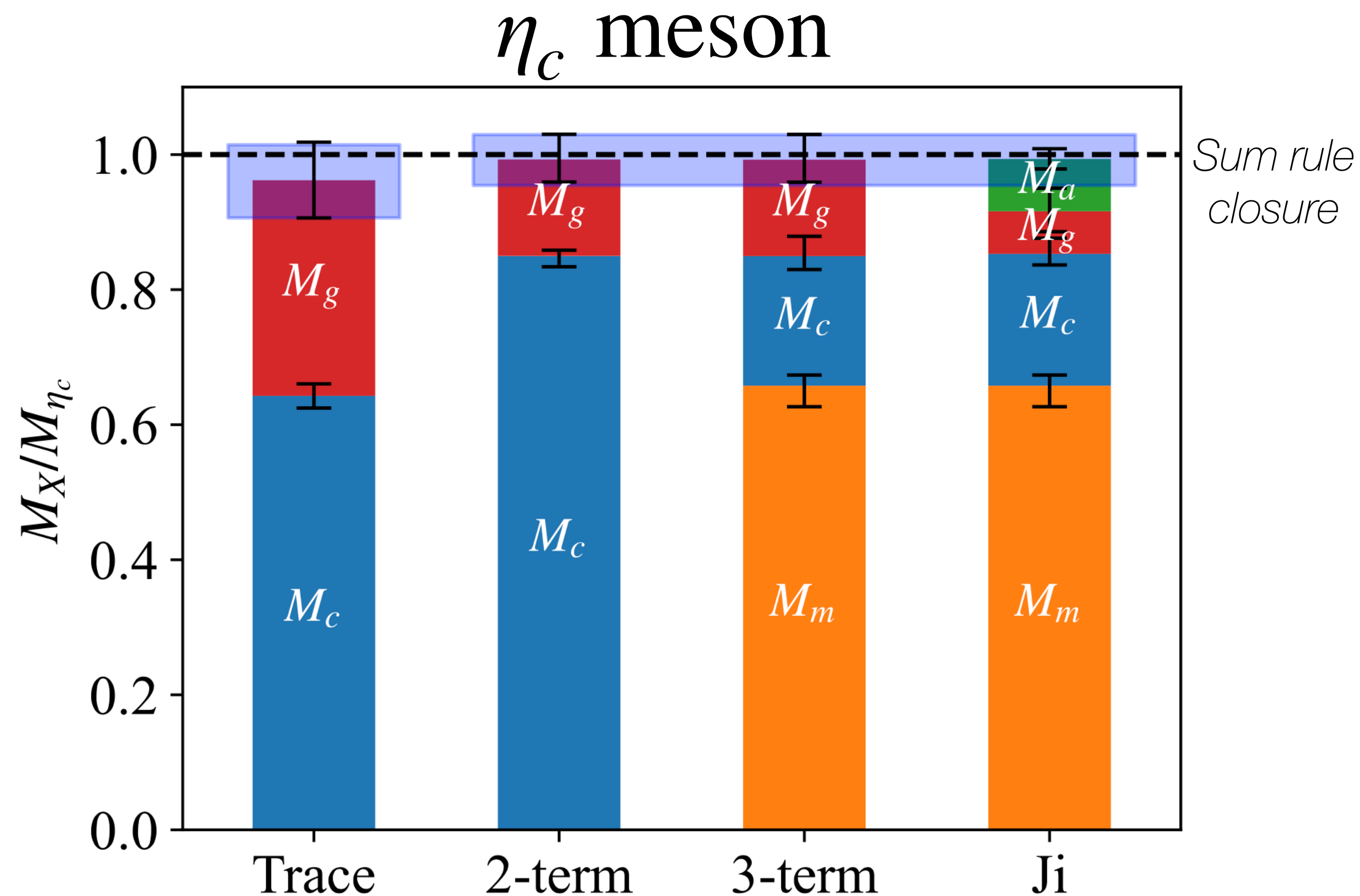
Completing the EMT with lattice QCD



$$T^{\mu\nu} = \underbrace{\left(T^{\mu\nu} - \frac{1}{4} g^{\mu\nu} T^\alpha_\alpha \right)}_{\bar{T}^{\mu\nu} : \text{spin-2 traceless, twist-2}} + \underbrace{\frac{1}{4} g^{\mu\nu} T^\alpha_\alpha}_{\hat{T}^{\mu\nu} : \text{spin-0 scalar trace, twist-4}}$$

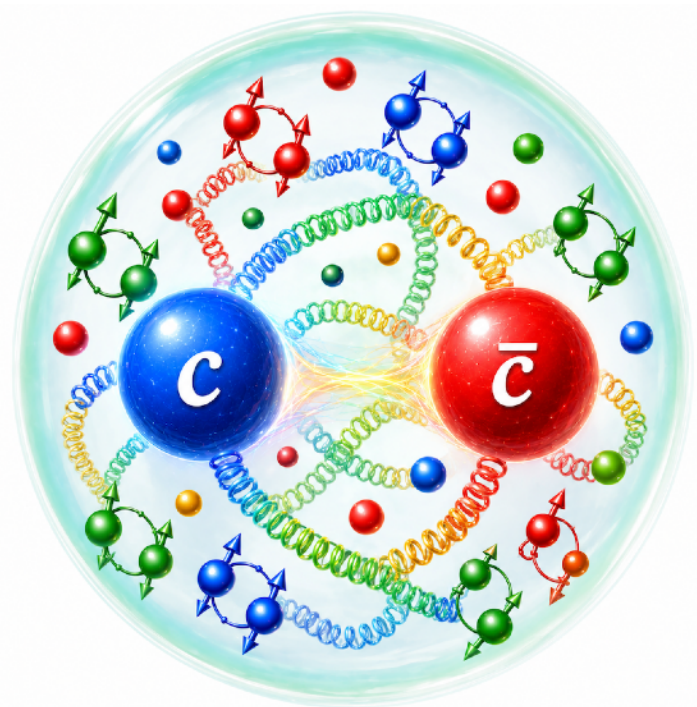
- **Hard exclusive processes** constrain the **traceless** EMT, while the scalar **trace** is **difficult** to isolate experimentally.
- Recent lattice developments enable a unified first-principles determination of **both** sectors.

Quark-gluon decomposition of hadron mass: charmonia



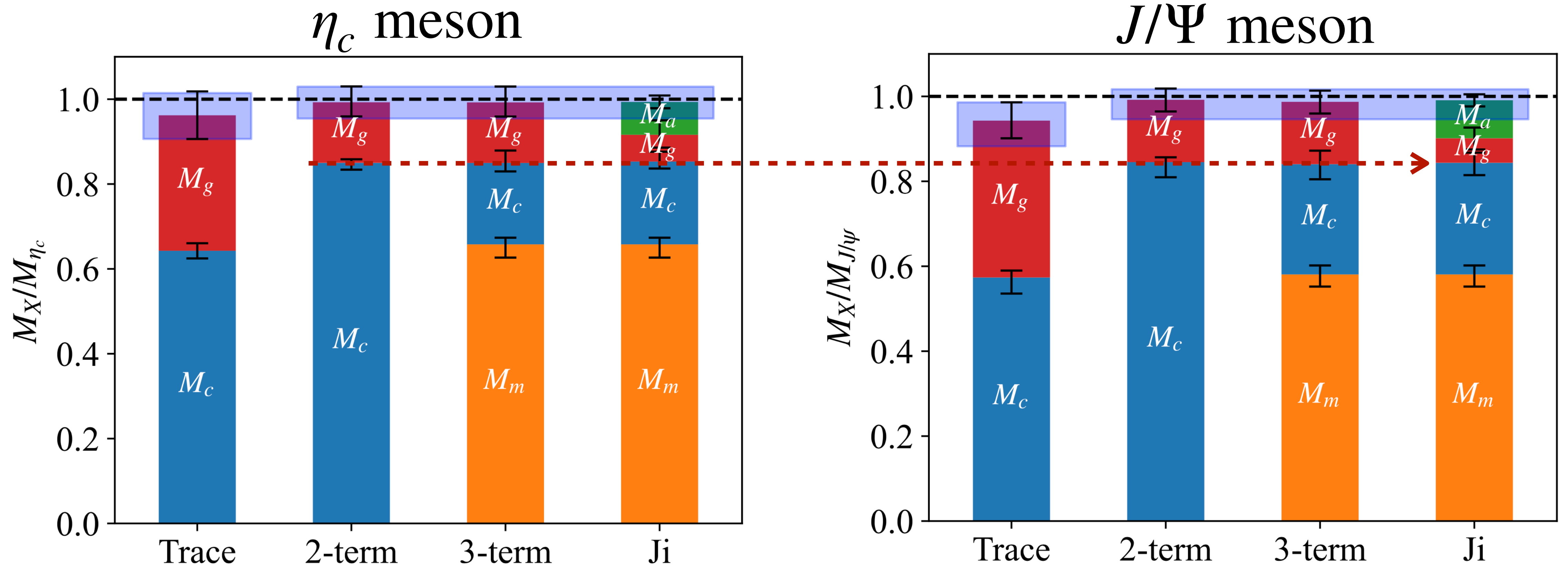
All ingredients are determined in a common $\overline{\text{MS}}$ framework.

- M_a (trace anomaly)
- M_g (gluon energy)
- M_c (charm quark energy)
- M_m (quark mass/ σ term)



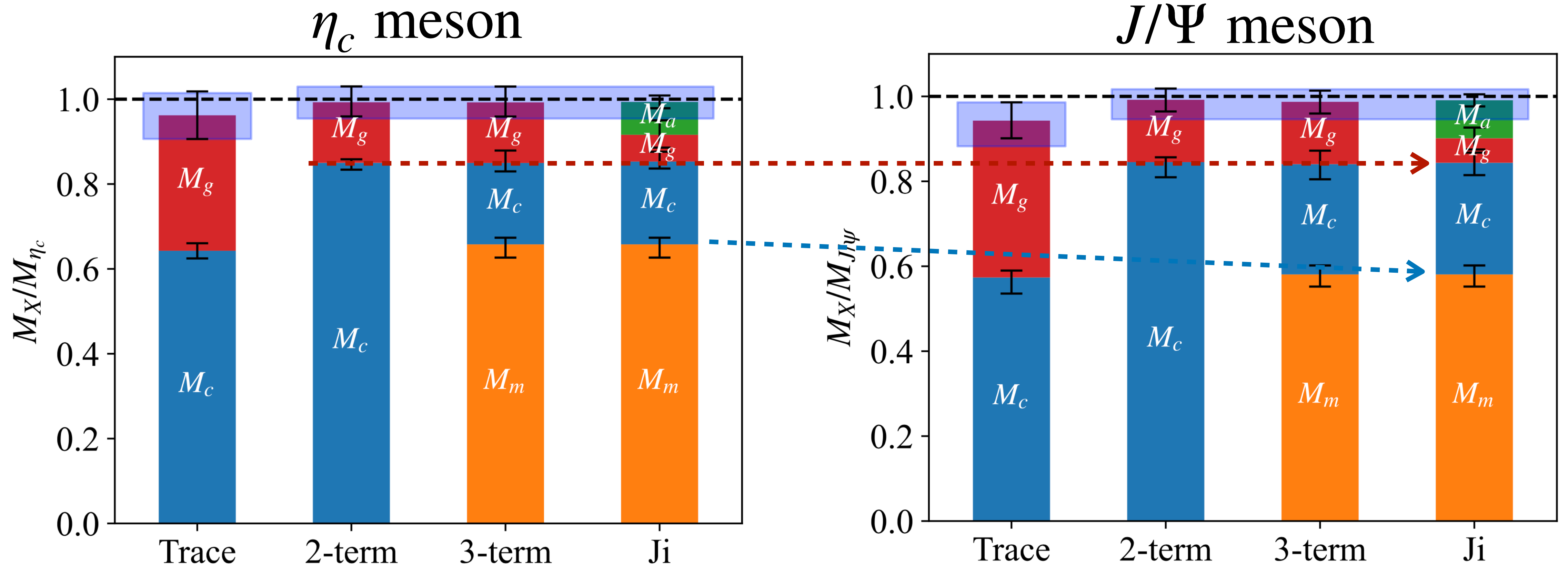
- Trace, two-term, three-term, and Ji decompositions **all close**.
- The **gluon** sector remains substantial even in heavy quarkonium.

Quark-gluon decomposition of hadron mass: charmonia



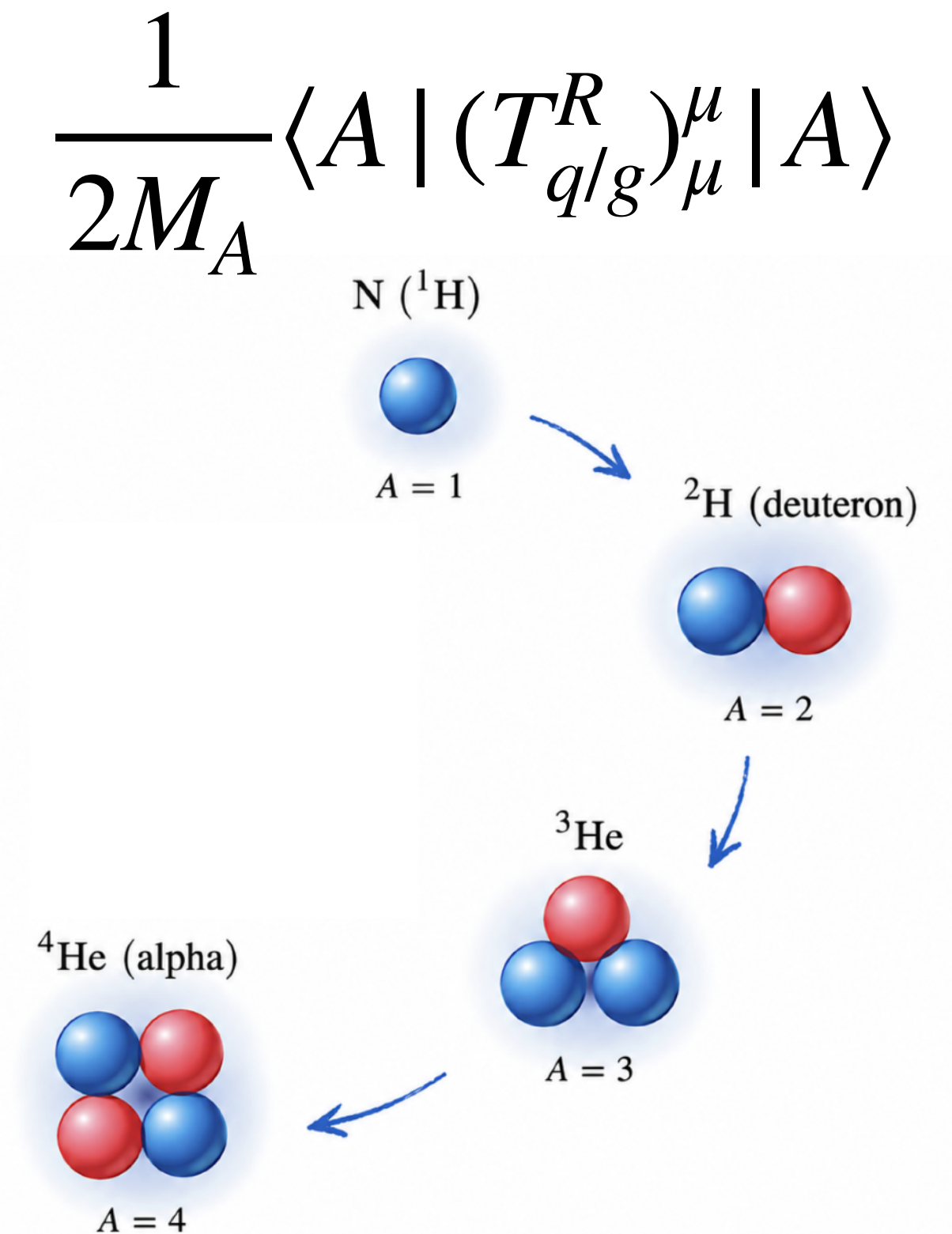
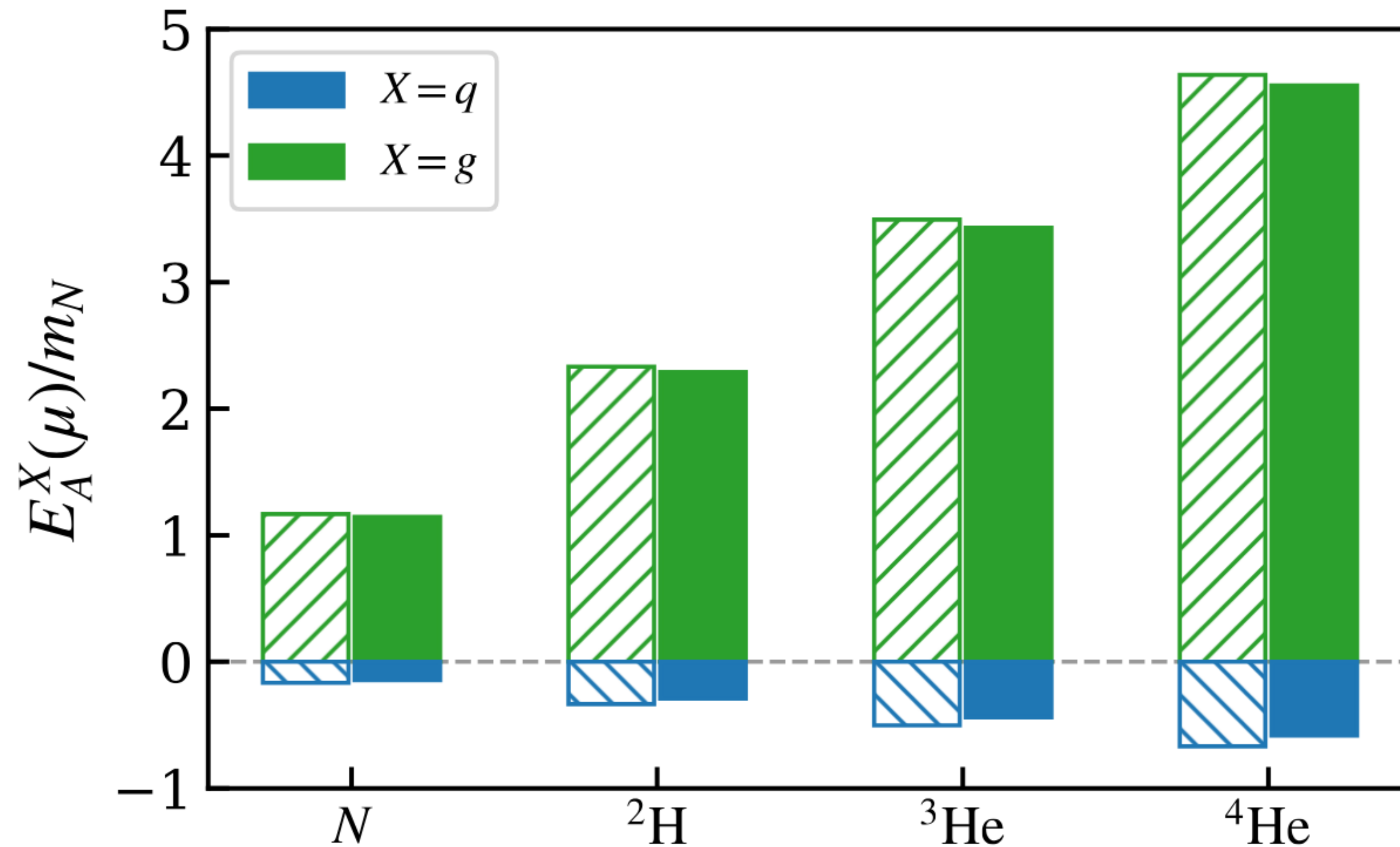
- **Gluon** contributions are **remarkably similar** between η_c and J/ψ .
- Changing heavy-quark spin primarily **reorganizes** the charm **charm energy/sigma-term contribution**.

Quark-gluon decomposition of hadron mass: charmonia



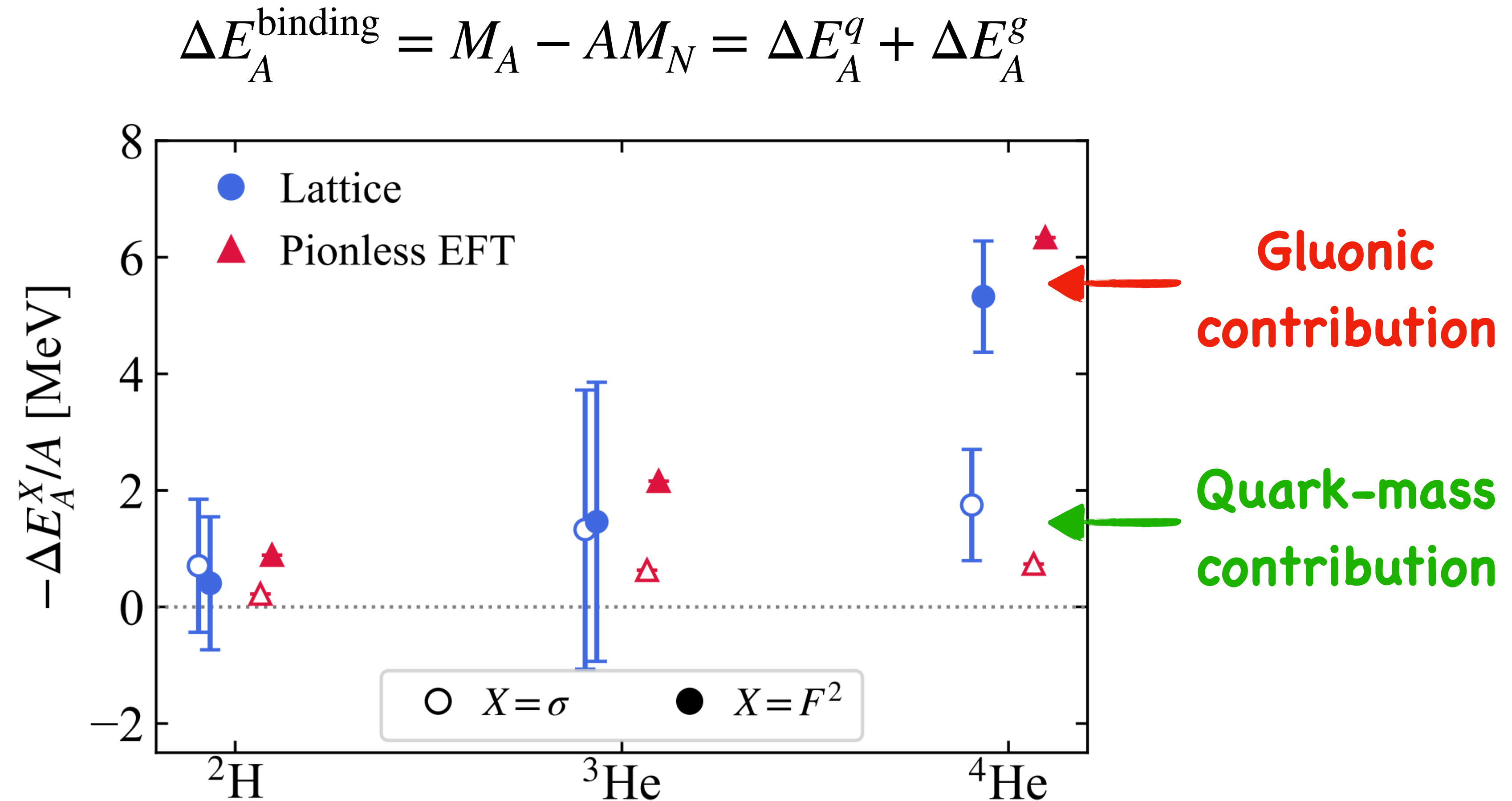
- **Gluon** contributions are **remarkably similar** between η_c and J/ψ .
- Changing heavy-quark spin primarily **reorganizes** the charm **charm energy/sigma-term contribution**.

Trace-based quark-gluon decomposition of light nuclei mass



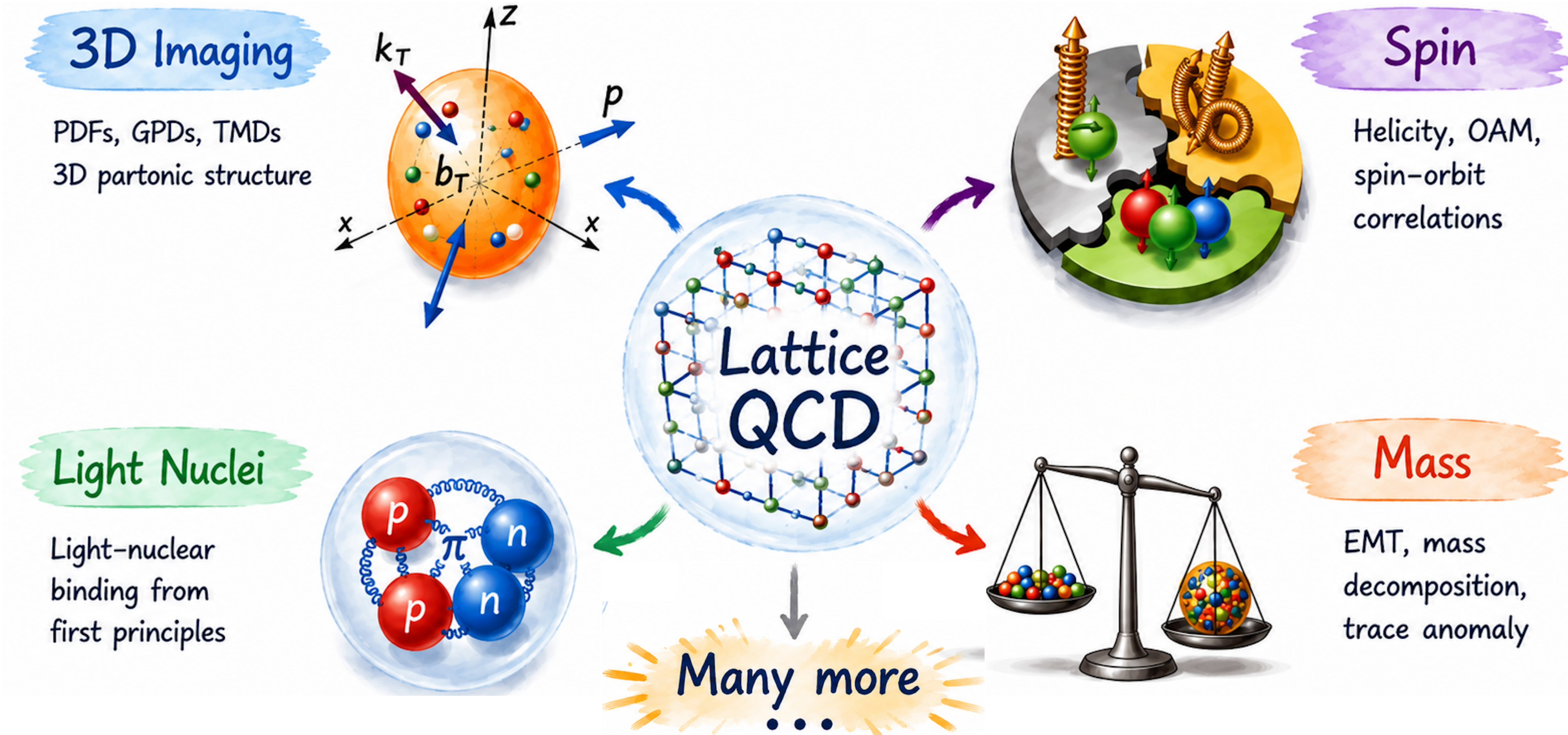
- **From nucleons to light nuclei:** resolve how quark and gluon trace contributions scale with nuclear size.

Trace-anomaly contribution to light-nuclear binding



- Within current uncertainties, light-nuclear binding is **predominantly** associated with the **gluonic trace contribution**.

Summary



From PDFs, GPDs, and TMDs to mass decomposition and nuclear binding, lattice QCD is delivering increasingly quantitative first-principles insight into hadron structure and QCD dynamics in nuclei.