小x event generator

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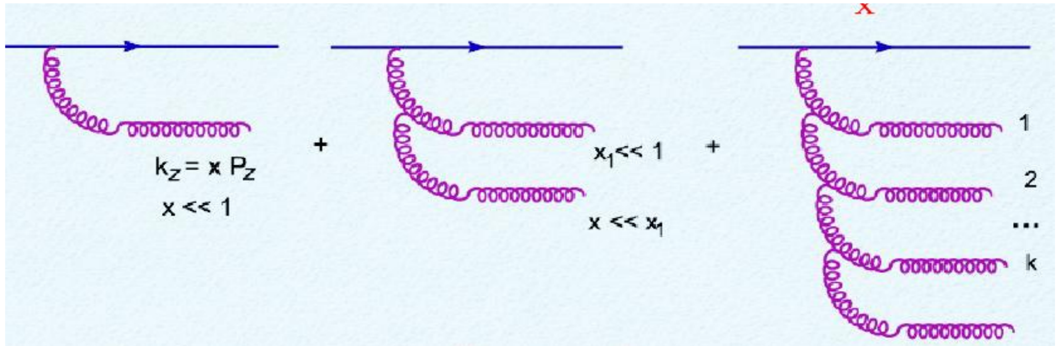
Outline:

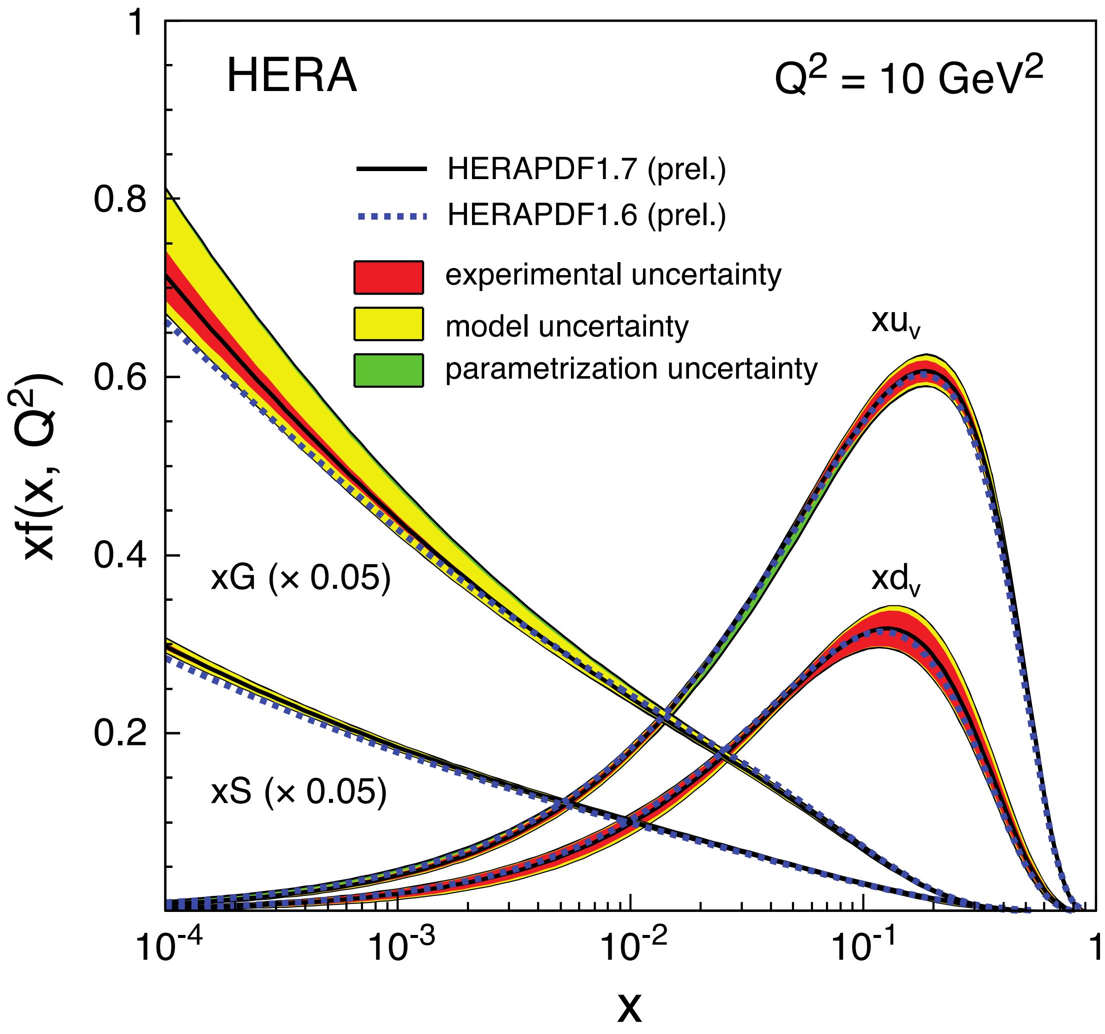
- A pedagogical introduction to small x physics
- Observables sensitive to small x physics
- Small x event generator
- Summary

Glouns at small x

DGLAP splitting function

$$P_{gg}(x) \sim \frac{1}{x} \text{ for } x \to 0$$

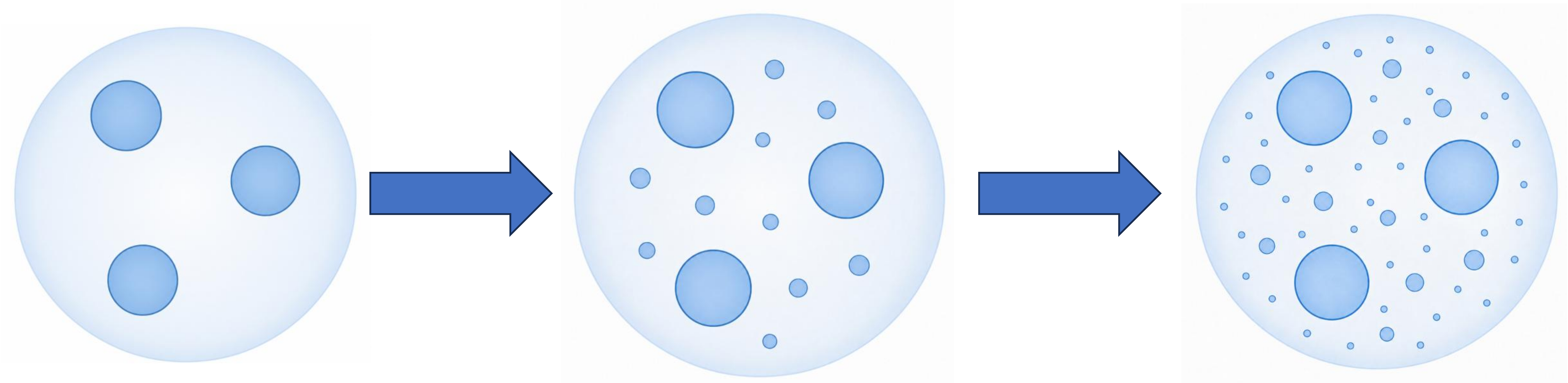




多小的x算小x:　　x=0.01

Why small x physics interesting?

- Transverse size of gluon is reversely proportional to its kt
- First partons are produced with relatively large size(small kt).
- When some critical density is reached no more partons of given size can fit in the wave function. Smaller gluon(larger kt) is produced to fit the rest space.
- Average kt increase with increasing number density.



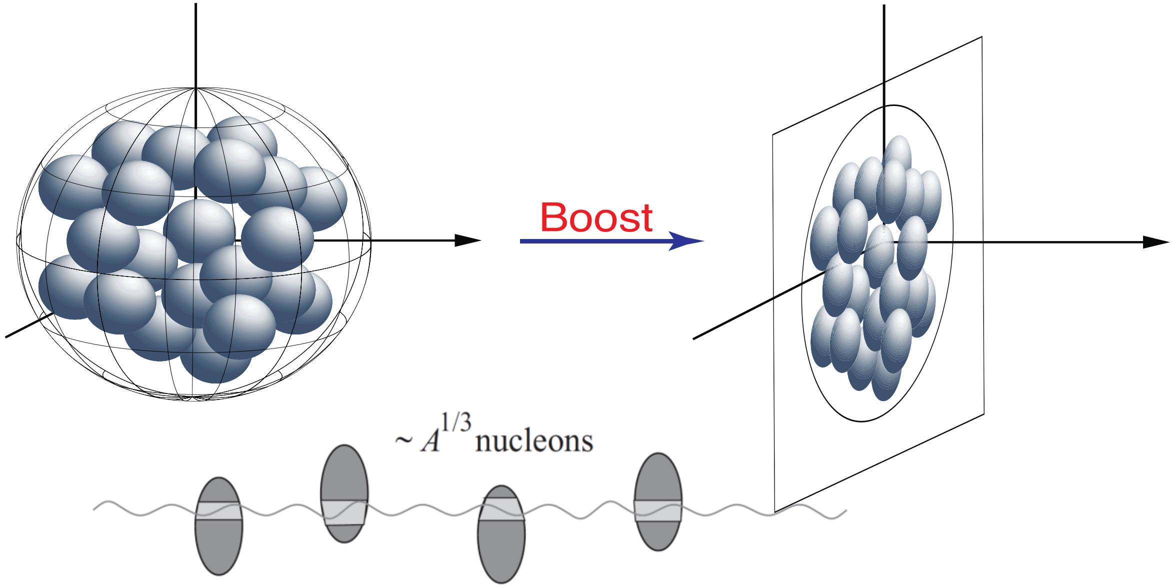
Perturbative calculable!

Why do we need a nucleus target

- ◆ A semi-hard scale Qs emerge when gluon density is very high

Q_s^2 ∞ gluons per unit transverse area

- ☐ **To reach high gluon density: very small x & large nucleus**



Small x gluons(with long wave length) from different nucleons overlap with each other!

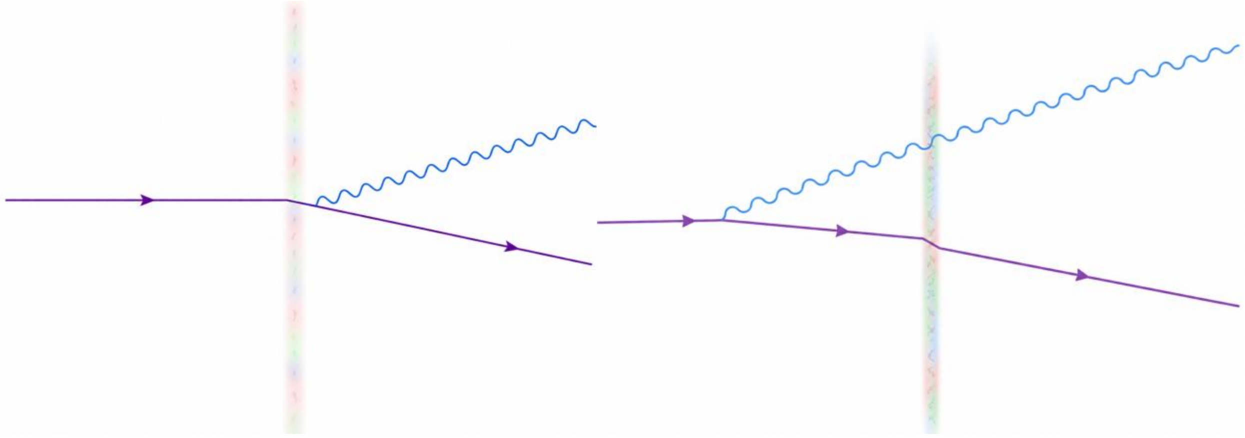
$$Q_s^2(x) \sim \left(\frac{A}{x}\right)^{1/3}$$

rcBK

质子： x=0.00001
金核： x=0.01

How to probe gluon distribution

Dipole gluon TMD：



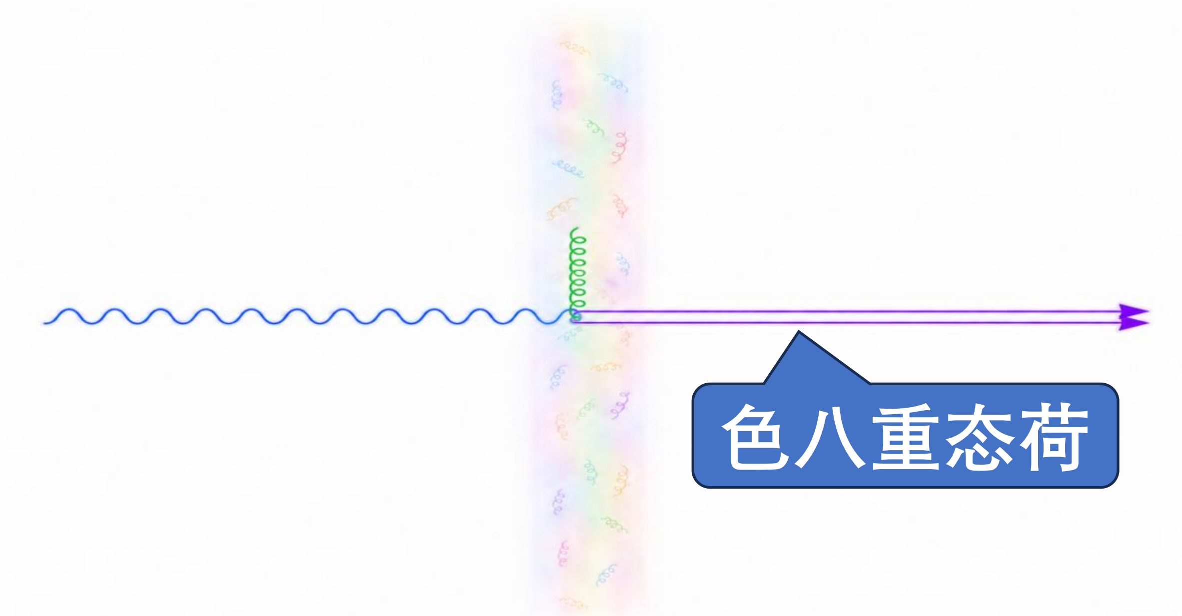
初态多重散射

末态多重散射

Low qt: q_t^2

high qt: $1/q_t^2$

WW gluon TMD：



多重散射

Low qt: $\ln(Q_s^2/q_t^2)$

high qt: $1/q_t^2$

F. Dominguez, B-W. Xiao, F. Yuan 2011
F. Dominguez, C. Marquet, B-W. Xiao, F. Yuan 2011

Background II

MV model: Classical gluon fields:

- For $Q_s \gg \Lambda_{QCD}$, $\alpha_s(Q_s^2) \ll 1$

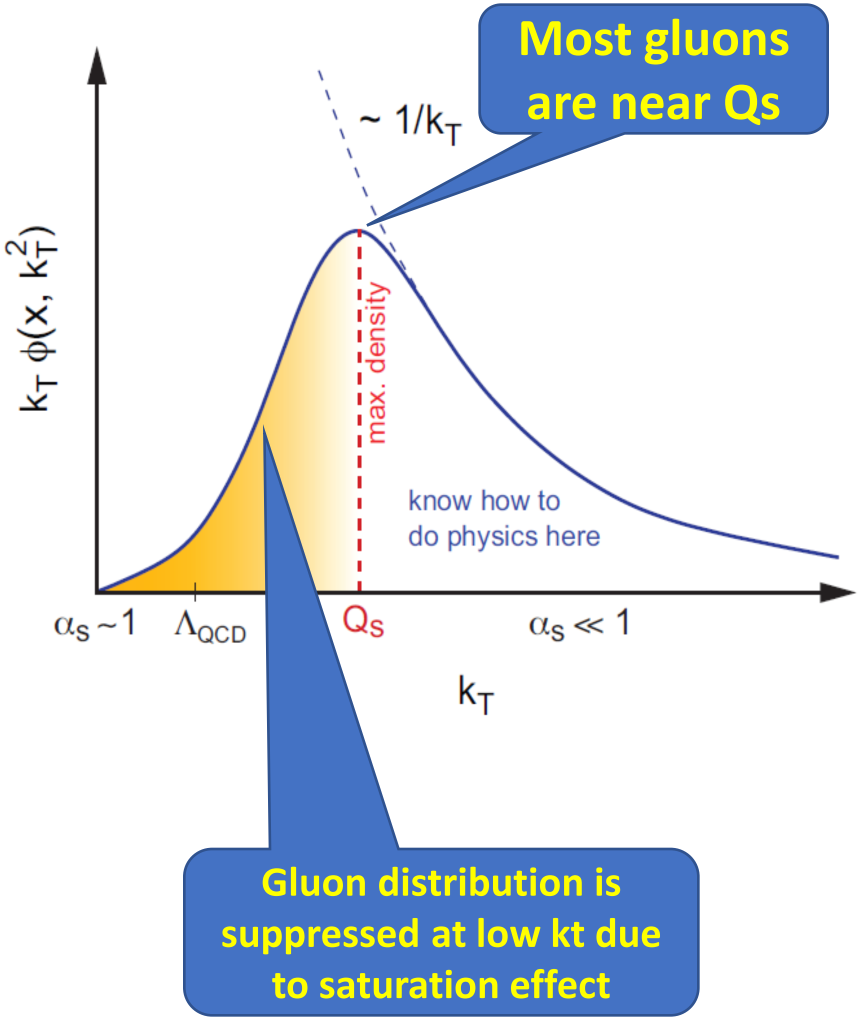
Perturbative treatment is justified!

- In the small x limit, high occupation number

A semi-classical treatment is justified

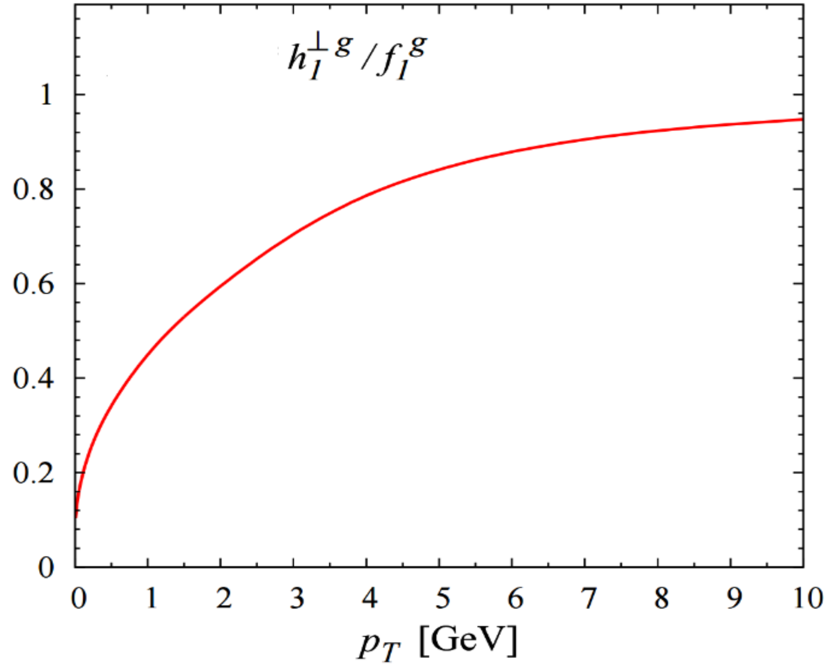
MV model & Glauble-Mueller model applied at x=0.01

Dipole distribution amplitude



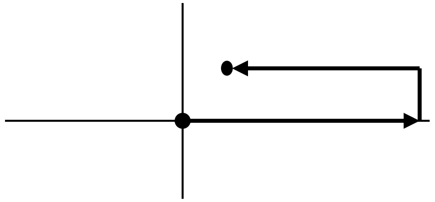
Gluon TMDs in the MV model

- The linearly polarized gluon TMDs in the MV model,



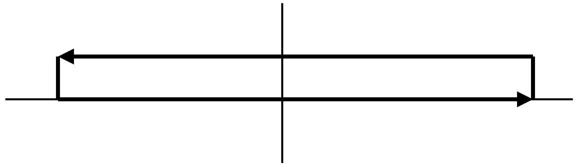
Metz & ZJ, 2011

Weizsäcker-Williams(WW) distribution:



$$xh_{1,WW}^{\perp g}(x,k_\perp) = \frac{N_c^2-1}{8\pi^3} S_\perp \int d\xi_\perp \frac{K_2(k_\perp\xi_\perp)}{\frac{1}{4\mu_A}\xi_\perp Q_s^2}\left(1-e^{-\frac{\xi_\perp^2 Q_s^2}{4}}\right)$$

Dipole distribution:



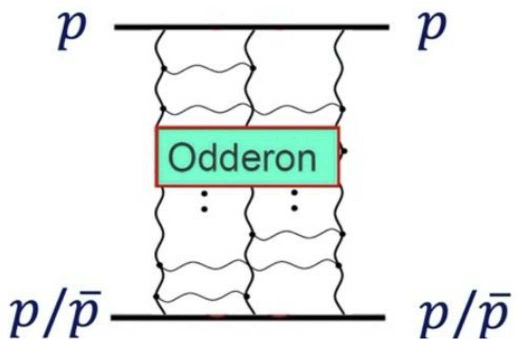
$$xh_{1,DP}^{\perp g}(x,k_\perp) = xG_{DP}^g(x,k_\perp) = \frac{k_\perp^2 N_c}{2\pi^2\alpha_s} S_\perp \int \frac{d^2\xi_\perp}{(2\pi)^2} e^{-i\vec{k}_\perp\cdot\vec{\xi}_\perp} e^{-\frac{Q_{sq}^2\xi_\perp^2}{4}}$$

positivity bound saturated for any value of k_t

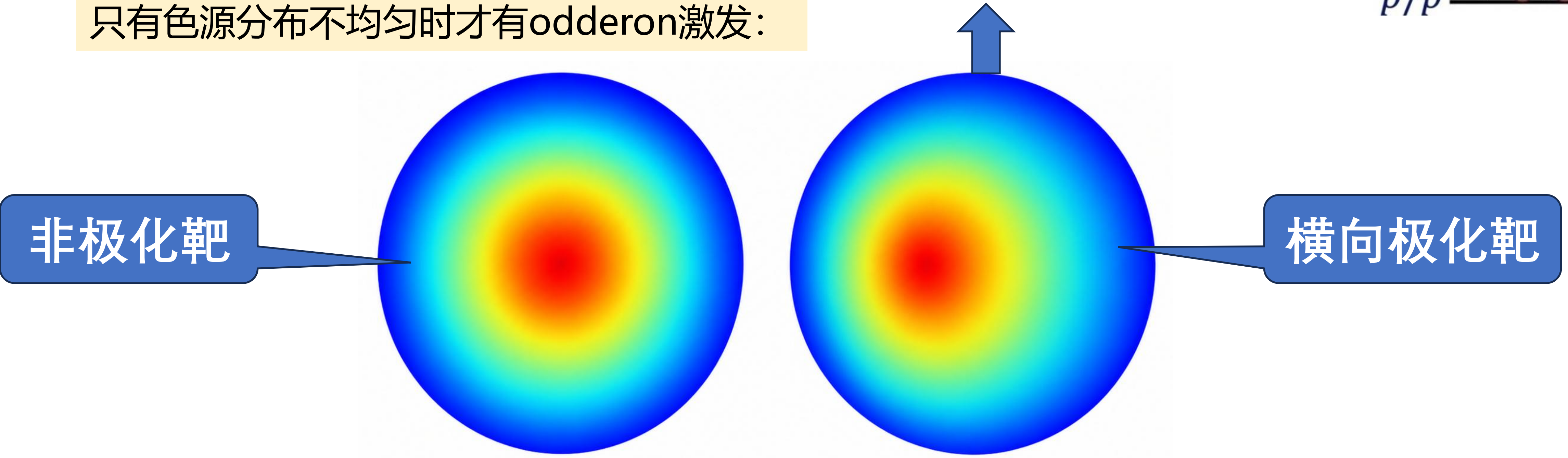
Odderon and asymmetric color source distribution

Odderon, a colorless exchange with C=-1 Łukaszuk, L.; Nicolescu, 1973

3胶子交换，色对称态（反色对称态不是odderon)



只有色源分布不均匀时才有odderon激发：



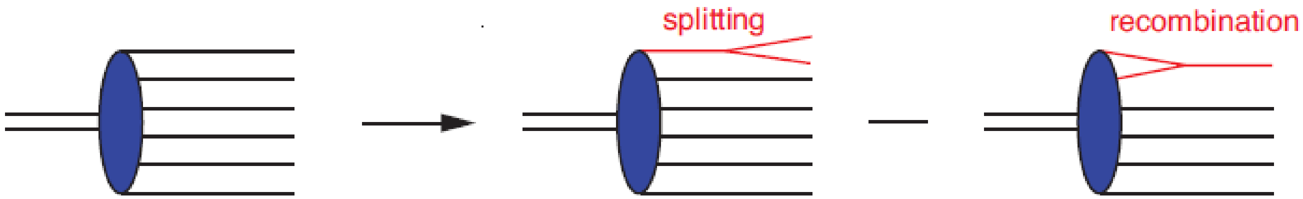
激发odderon

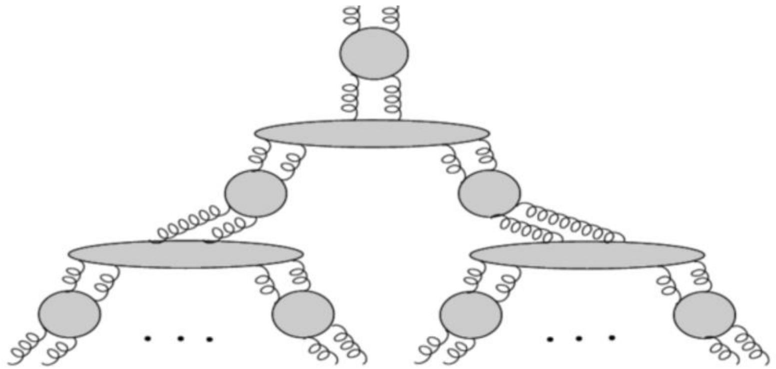
激发spin dependent odderon ZJ 2013

模型计算表明spin dependent odderon 更大

Small x evolution equations

必须要考虑胶子融合/重散射：





- 胶子数密度随x增长行为：$\frac{1}{x^{1+\delta}}$ 量子修正
- 胶子平均横动量随x增长行为：$Q_s^2(x)$

BFKL → GLR/MQ → BK → JIMWLK

线性方程，无胶子融合

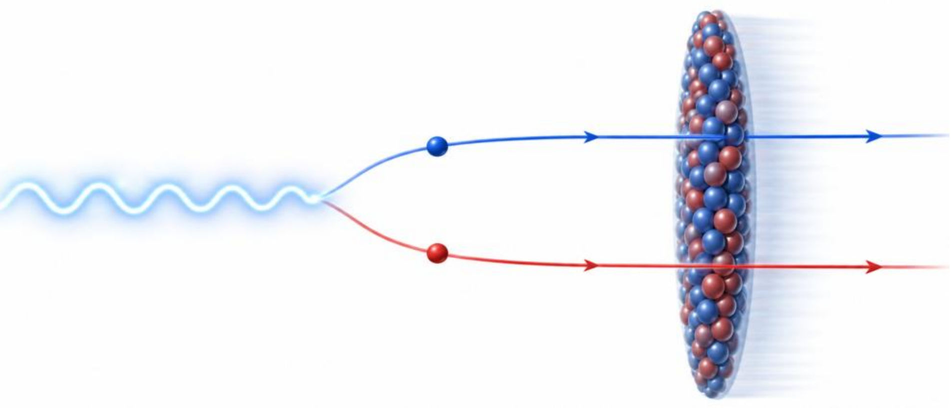
2->1 胶子融合

胶子融合（重散射)求和到无穷阶，large N_C 近似，闭合方程

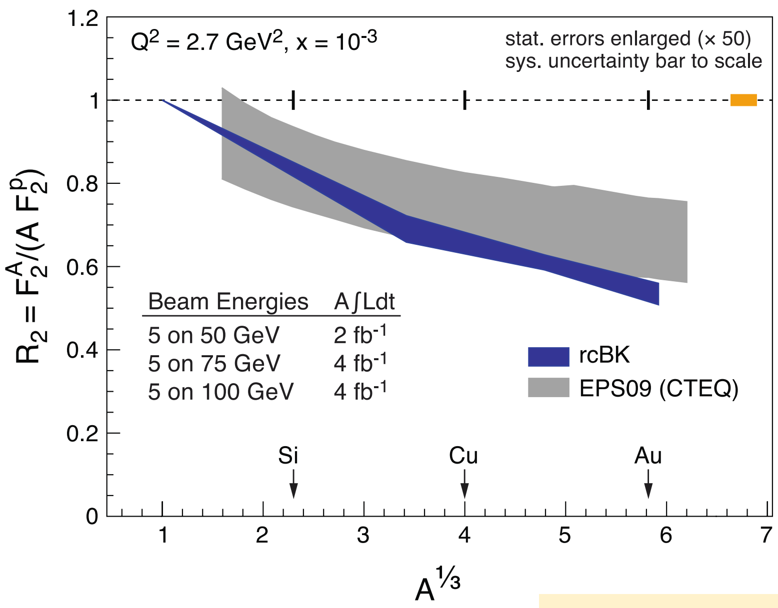
完整的小x演化方程，非闭合

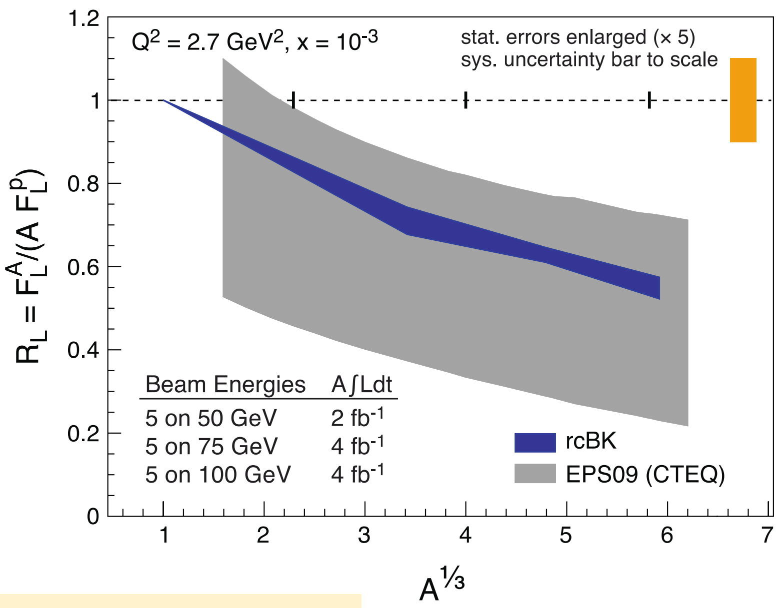
Nuclear suppression

DIS过程的dipole图像：



在相同的运动学区，对比ep, eA inclusive cross section:

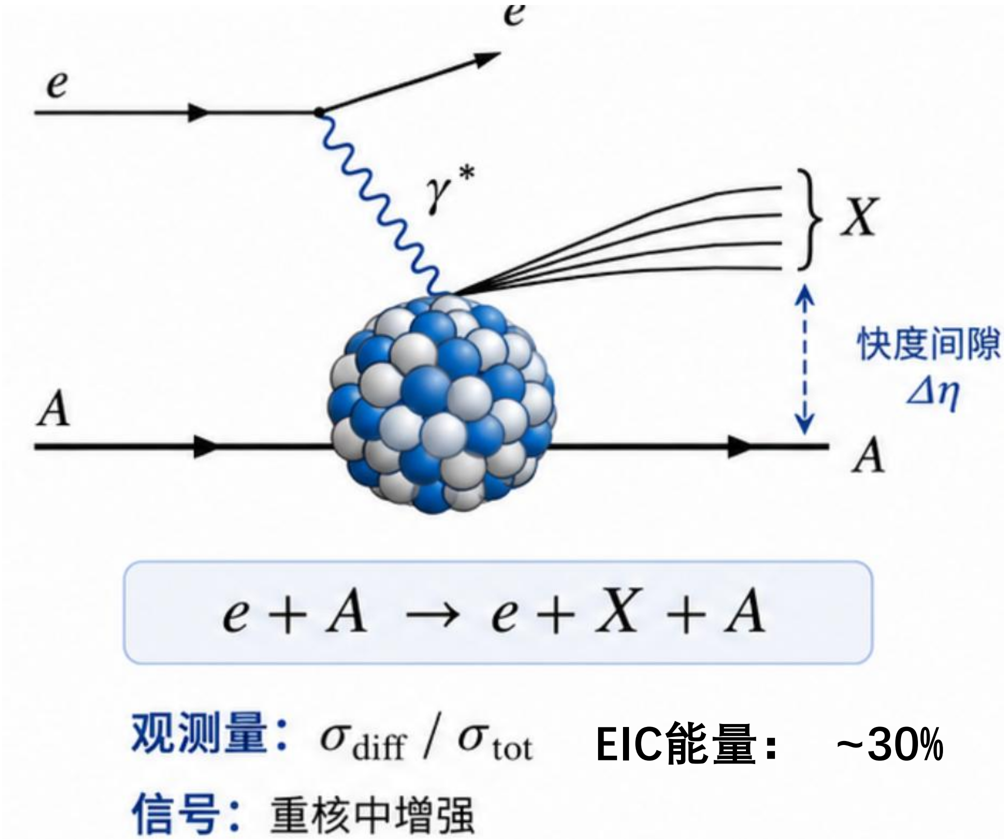




核越重，总截面压低越厉害

Saturation 探针

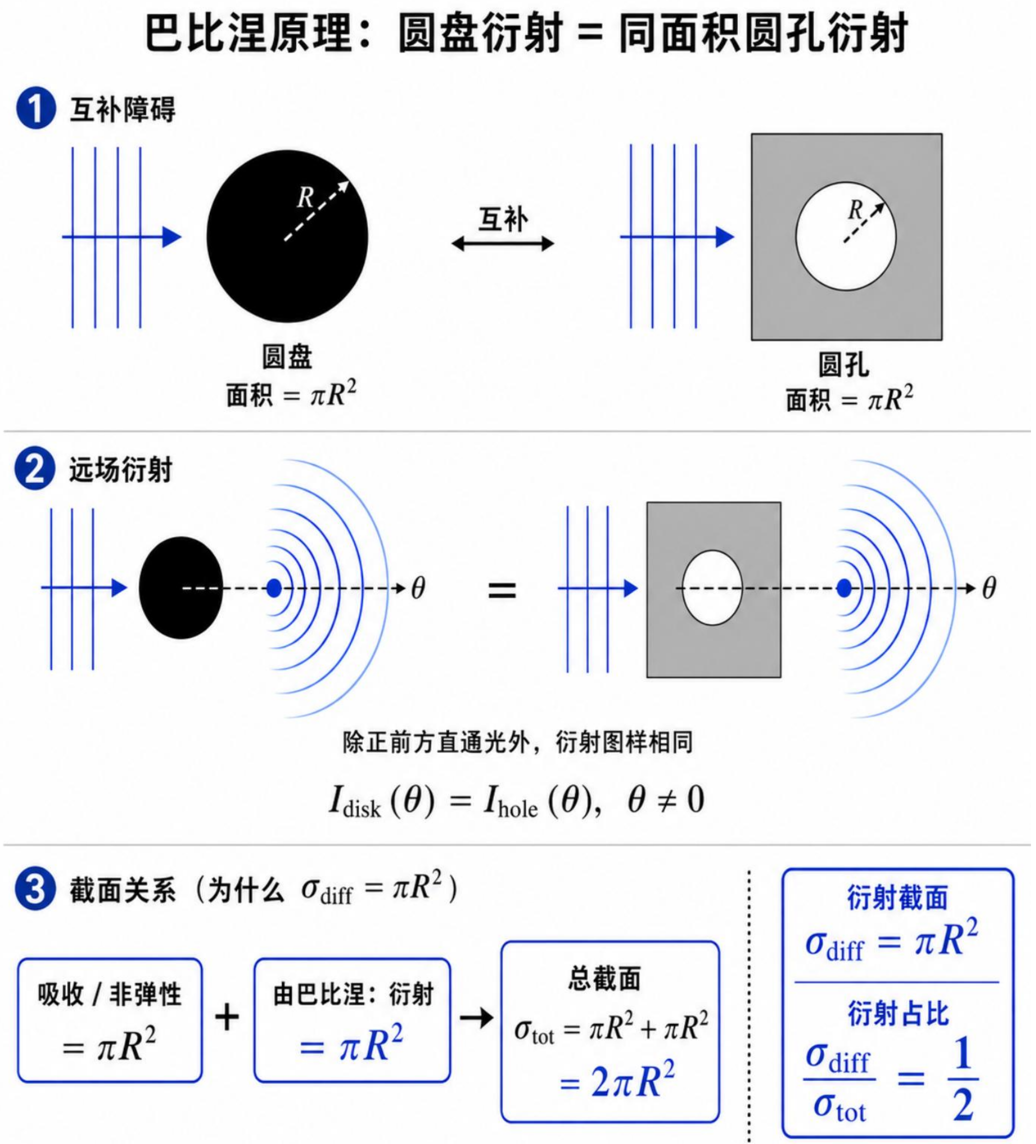
相干衍射：



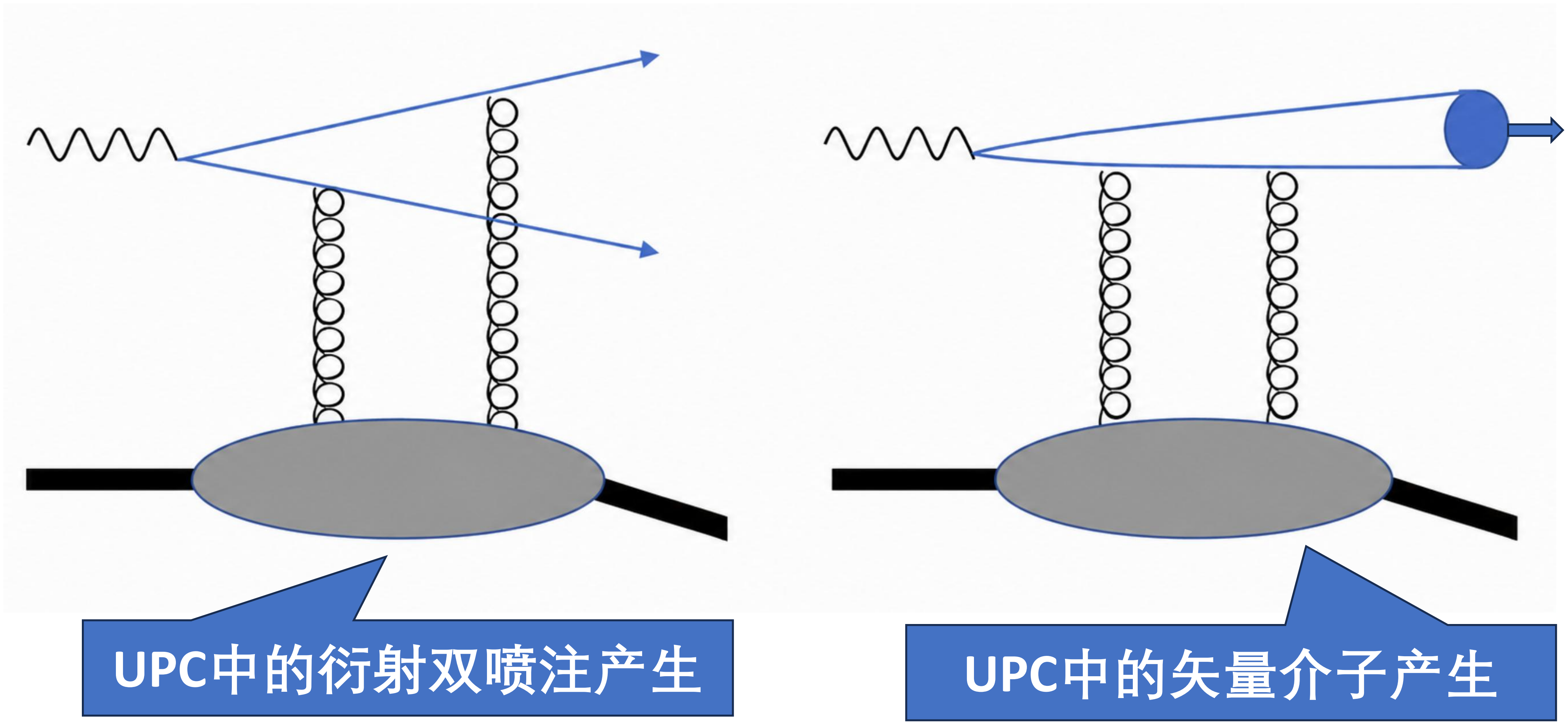
EIC能量： ~30%

能量越高，原子核相对被撞碎的几率越低： 50%

光学类比：

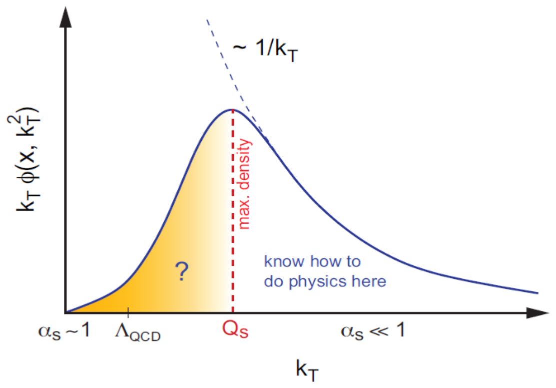


LHC上的相干衍射过程



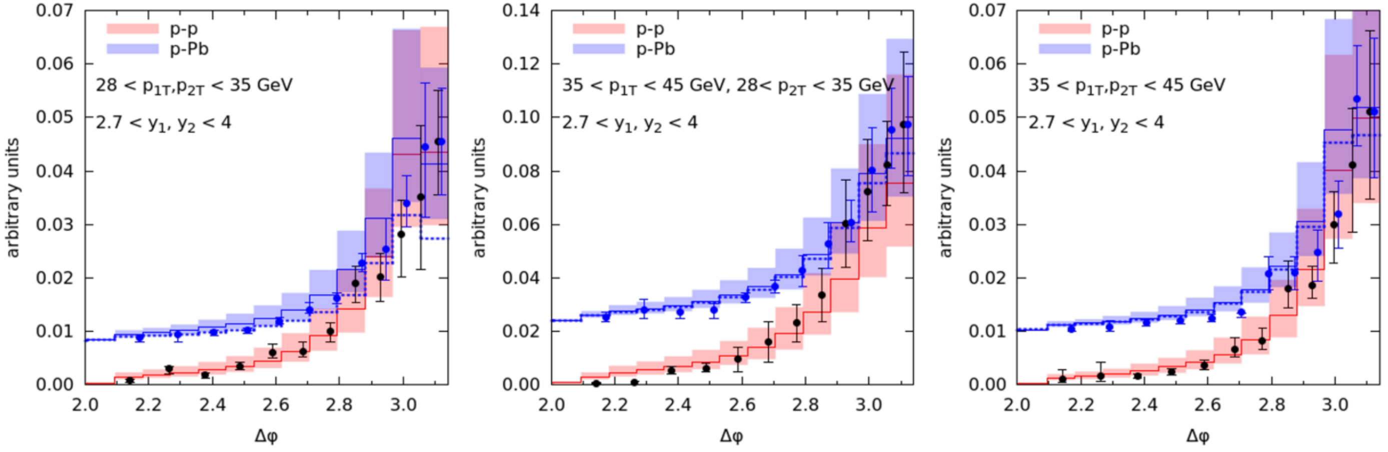
Di-jet/hadron de-correlation

更大的胶子横动量导致更大的imbalance:



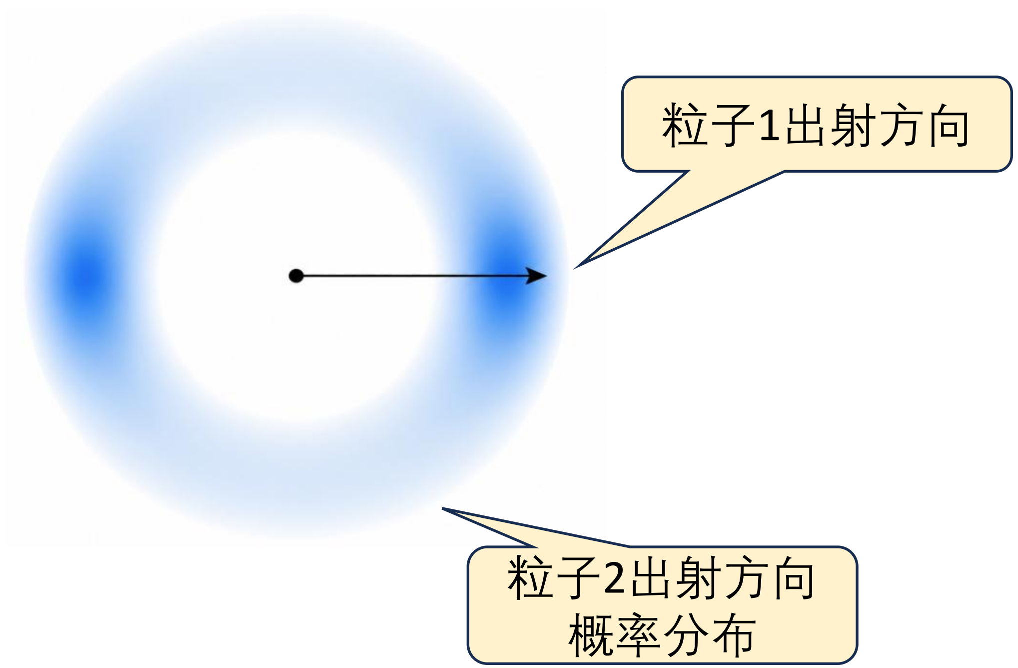
需要用pp对撞做为基线

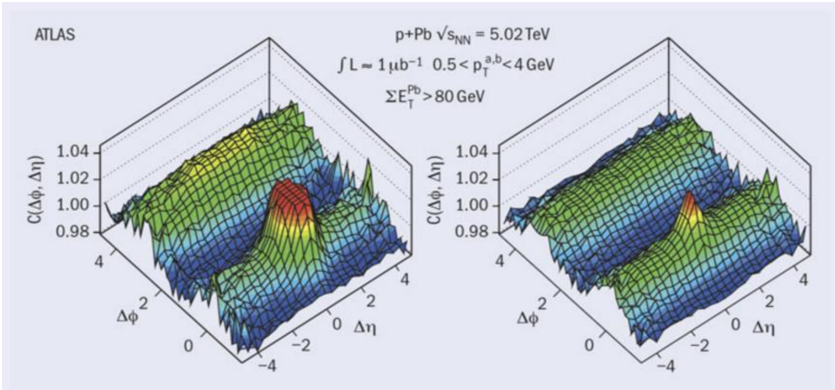
Forward dijets at the LHC:



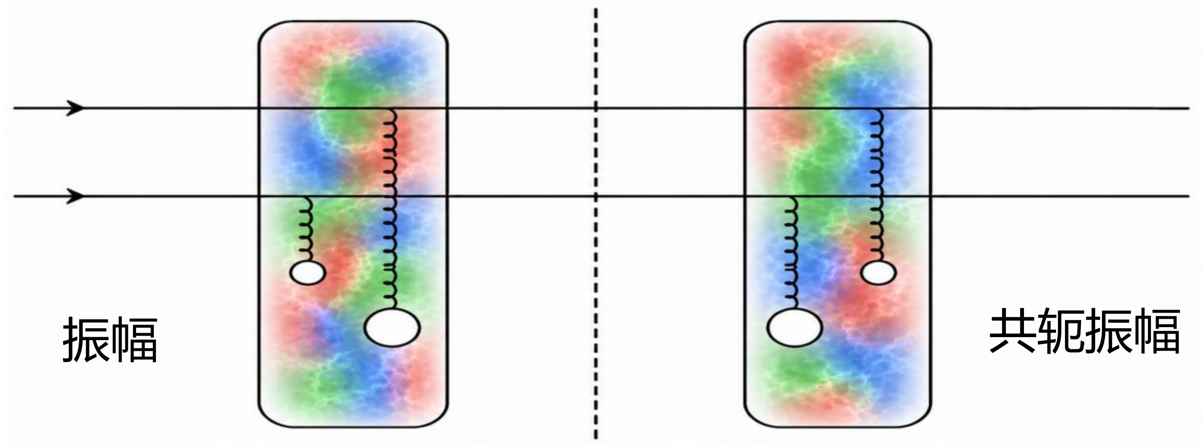
Ridge effect

2粒子倾向于沿相同方向出射：

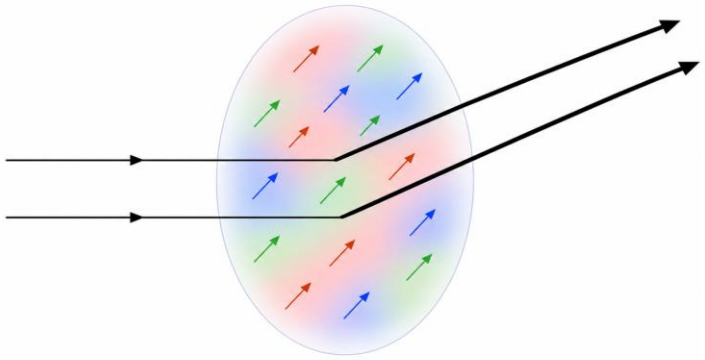




量子相干多重散射：



直觉物理解释：

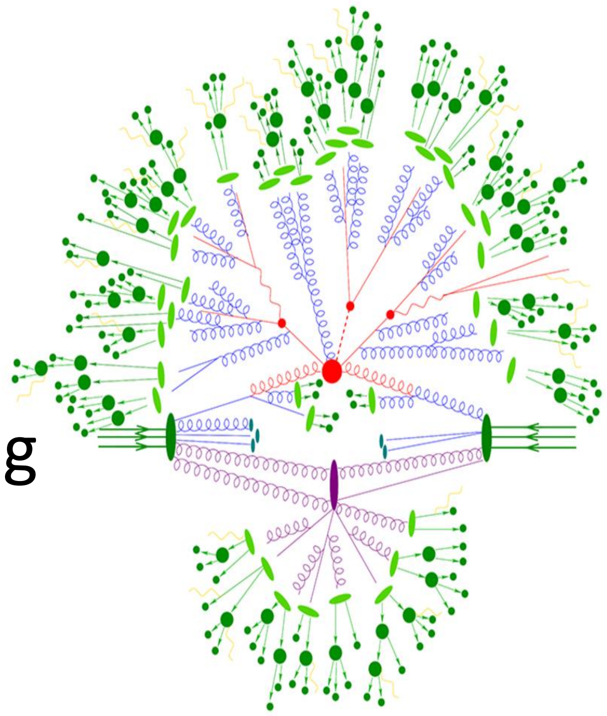


被一个局域均匀色电场散射

Why parton shower generator

- Describe fully exclusive hadronic state
- Coherent branching
- Keep four momentum conservation in each branching
- Impact studies for future experiments

….

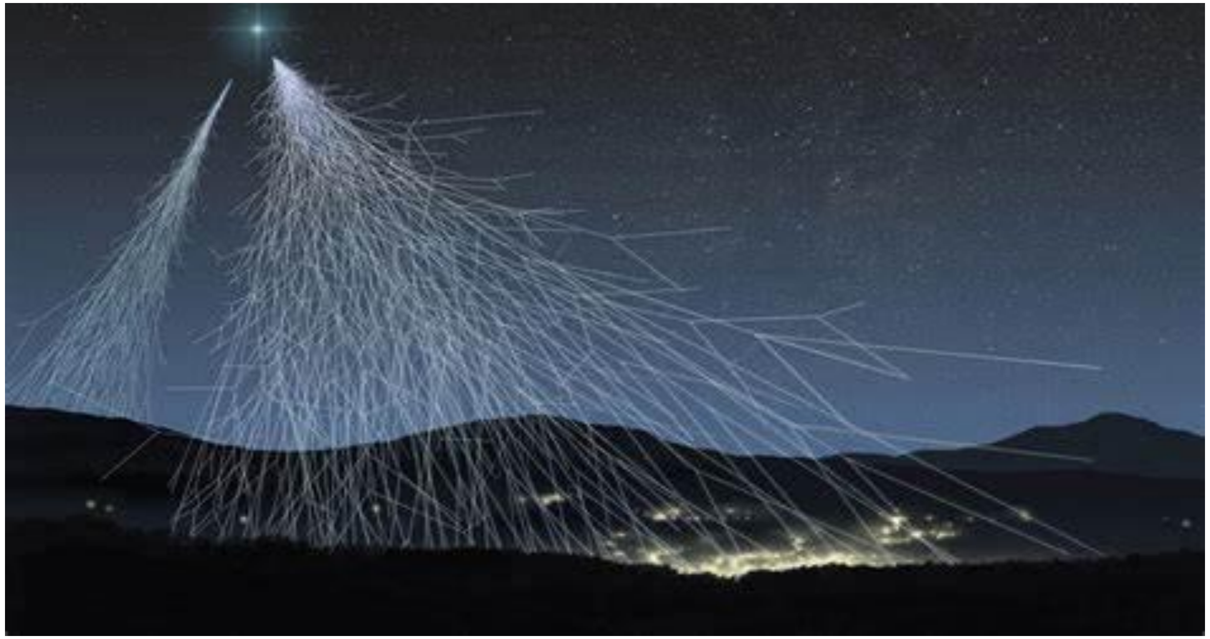


Why small *x* parton shower generator

- Saturation effect is absent in all existing generators

- Aim at developing a PS algorithm to be used:

- Phenomology in eA collisions @EIC
- Forward physics in pA collisions @LHC
- Cosmic ray event generator



Folded and unfolded GLR equation

- The standard GLR equation(unfolded one)

$$\frac{\partial N(\eta,k_\perp)}{\partial\eta}=\frac{\bar{\alpha}_s}{\pi}\left[\int\frac{\mathrm{d}^2l_\perp}{l_\perp^2}N(\eta,k_\perp+l_\perp)-\int\frac{\mathrm{d}^2l_\perp}{(k_\perp-l_\perp)^2}\frac{k_\perp^2}{2l_\perp^2}N(\eta,k_\perp)\right]-\bar{\alpha}_sN^2(\eta,k_\perp)$$

- Resolved and unresolved branching:

$$\int\frac{\mathrm{d}^2l_\perp}{l_\perp^2}N(\eta,k_\perp+l_\perp)\approx\int_\mu\frac{\mathrm{d}^2l_\perp}{l_\perp^2}N(\eta,k_\perp+l_\perp)+\int_0^\mu\frac{\mathrm{d}^2l_\perp}{l_\perp^2}N(\eta,k_\perp)$$

- Folded GLR equation: $\boxed{\text{virtual correction is manifestly resumed to all orders}}$

$$\frac{\partial}{\partial\eta}\frac{N(\eta,k_\perp)}{\Delta(\eta,k_\perp)}=\frac{\bar{\alpha}_s}{\pi}\int_\mu\frac{\mathrm{d}^2l_\perp}{l_\perp^2}\frac{N(\eta,l_\perp+k_\perp)}{\Delta(\eta,k_\perp)}$$

◆ Non-Sudakov form factor

$$\Delta(\eta,k_\perp)=\exp\left\{-\bar{\alpha}_s\int_{\eta_0}^{\eta}d\eta'\left[\ln\frac{k_\perp^2}{\mu^2}+N(\eta',k_\perp)\right]\right\}$$

Shi-Wei-ZJ, 2022

Parton shower algorithms

GLR	v.s.	DGLAP/CCFM
$$\Delta(\eta, k_\perp) = \exp\left\{-\bar{\alpha}_s \int_{\eta_0}^{\eta} d\eta' \left[\ln\frac{k_\perp^2}{\mu^2} + N(\eta', k_\perp)\right]\right\}$$		$$\Delta_a(t, t') = \exp\left\{-\sum_{b\in\{q,g\}} \int_t^{t'} \frac{\mathrm{d}\bar{t}}{\bar{t}} \int_{z_{\min}}^{z_{\max}} \mathrm{d}z\, \frac{\alpha_s}{2\pi}\, \frac{1}{2}\, P_{ab}(z)\right\}$$
gluon splitting gluon fusion		parton splitting

The evolution variable:

$\eta = \ln(1/x)$	Q

The generated event:

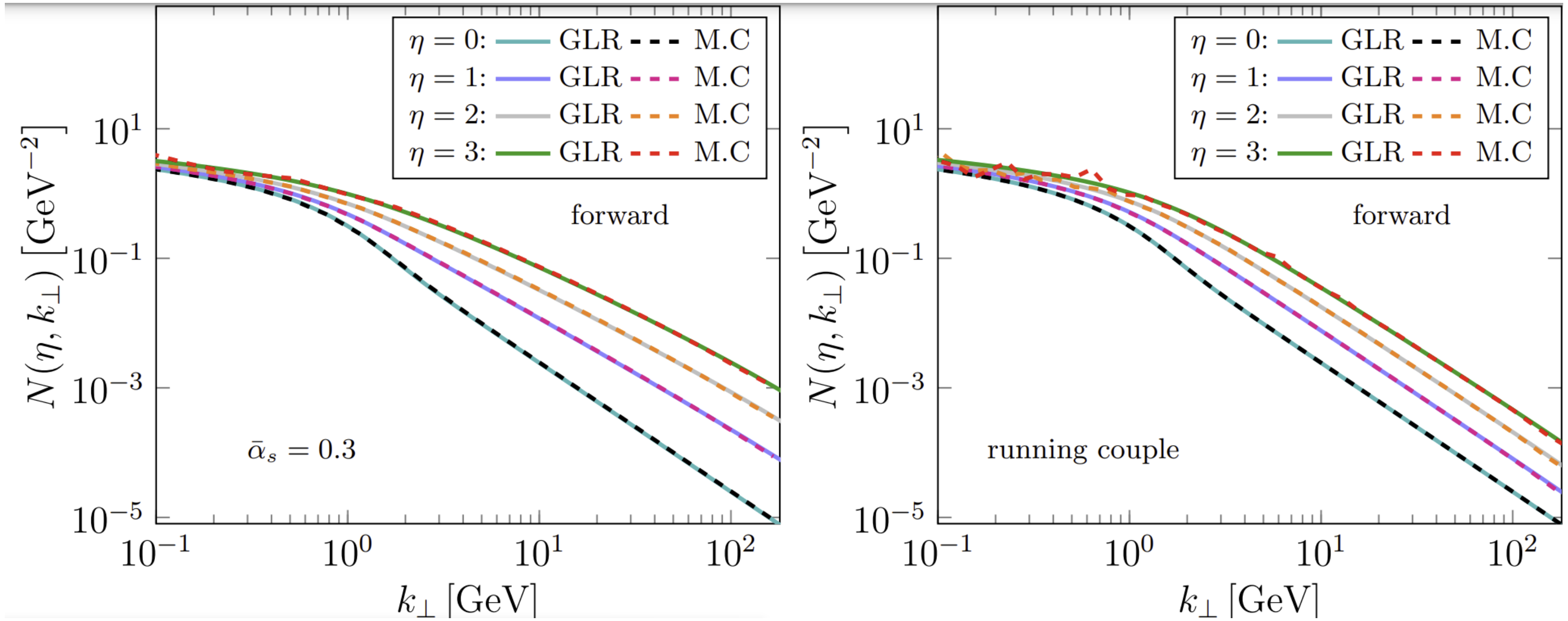
reweight	Unitary

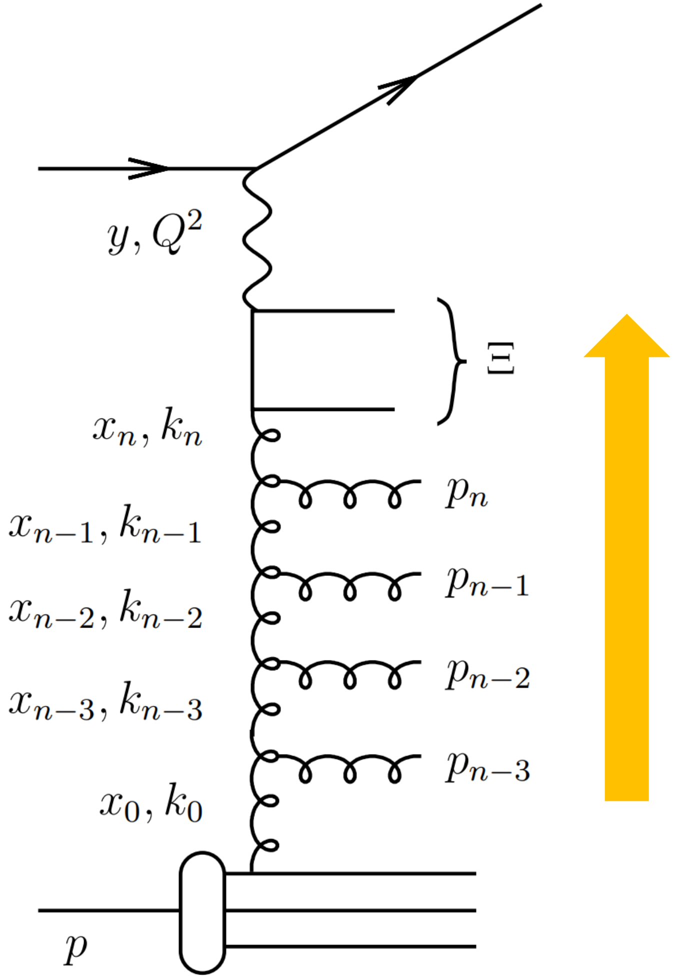
Forward evolution

- MV model as the initial condition at x_0=0.01:

$$N(\eta=0,k_\perp)=\int\frac{d^2r_\perp}{2\pi}e^{-ik_\perp\cdot r_\perp}\frac{1}{r_\perp^2}\left(1-\exp[-\frac{1}{4}Q_{s0}^2r_\perp^2\ln(e+\frac{1}{\Lambda r_\perp})]\right)$$

- Test the algorithm



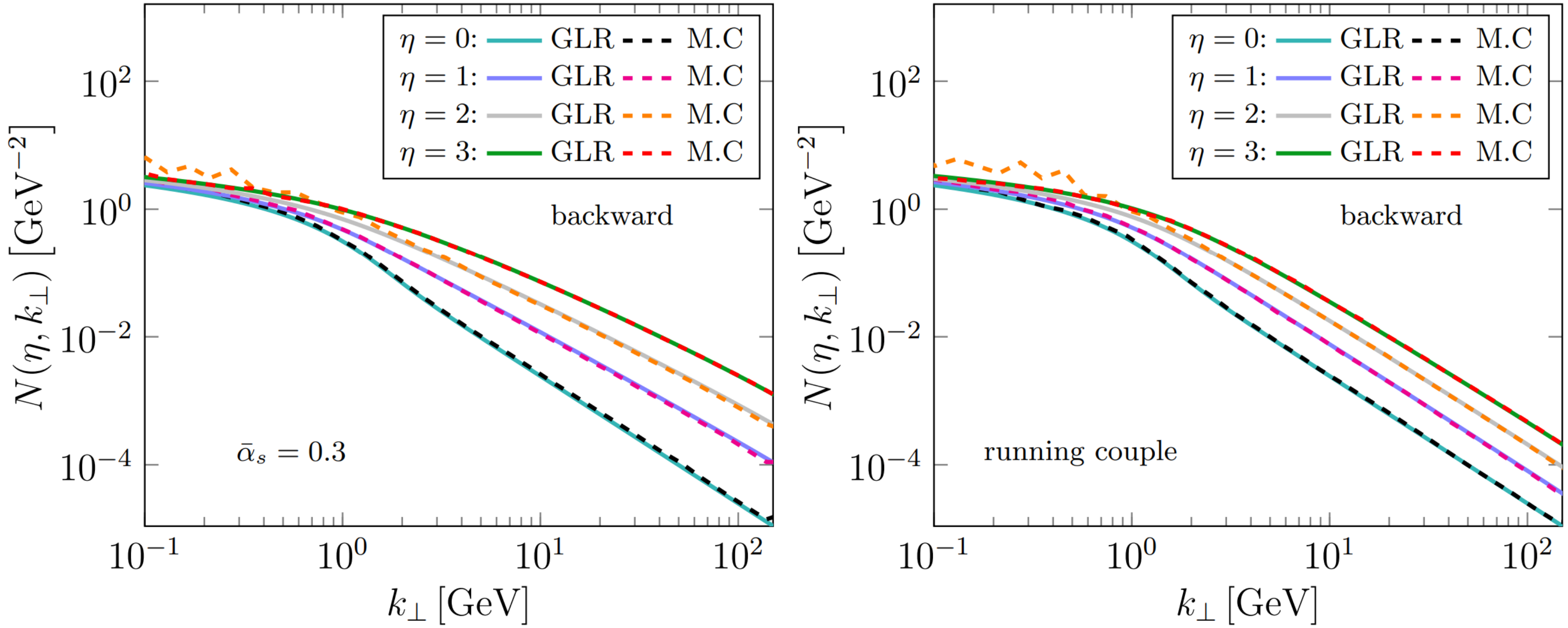


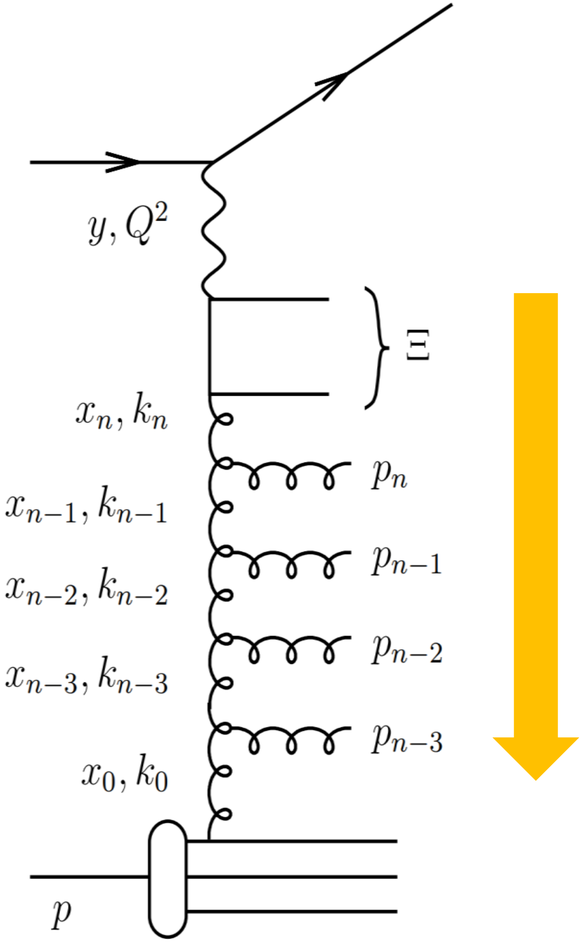
Backward evolution

➢ Non-Sudakov form factor:

$$\frac{\Delta(\eta_{i+1}, k_{\perp,i+1}) N(\eta_i, k_{\perp,i+1})}{\Delta(\eta_i, k_{\perp,i+1}) N(\eta_{i+1}, k_{\perp,i+1})} = \exp\left[-\frac{\bar{\alpha}_s}{\pi} \int_{\eta_i}^{\eta_{i+1}} \mathrm{d}\eta \int_{\mu} \frac{\mathrm{d}^2 l_\perp}{l_\perp^2} \frac{N(\eta, k_{\perp,i+1} + l_\perp)}{N(\eta, k_{\perp,i+1})}\right]$$

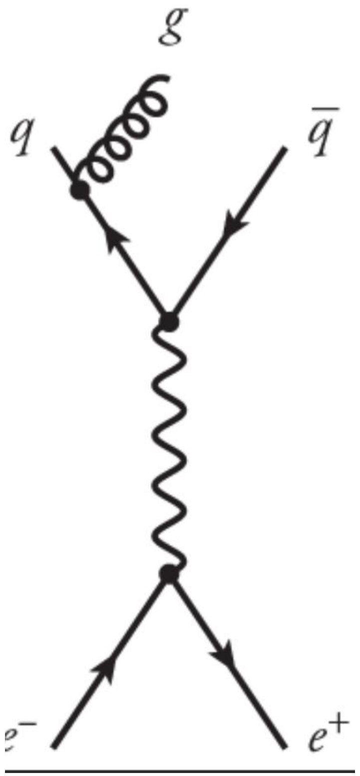
◆ $l_\perp$ **is also sampled in a different way!**



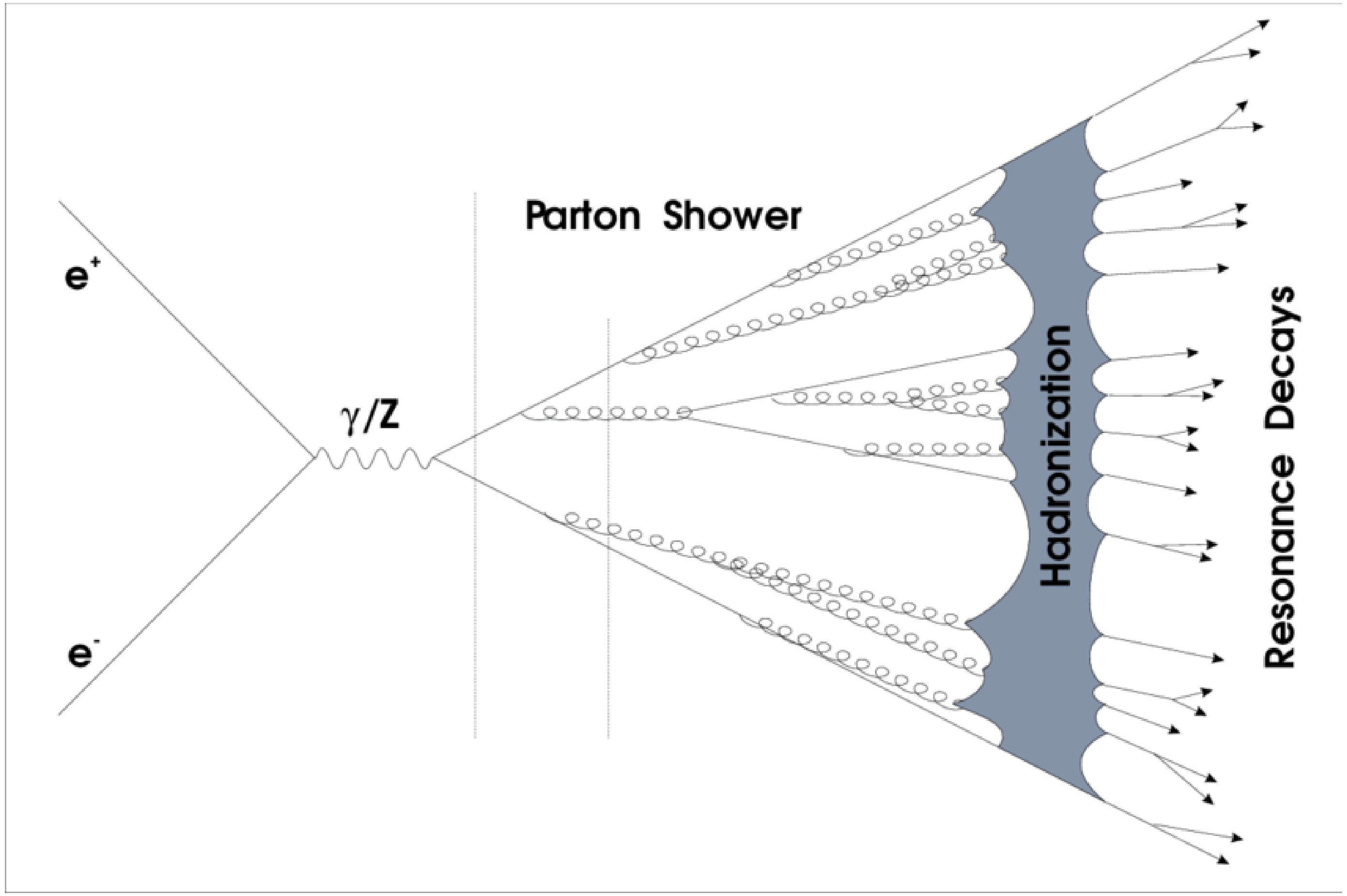


Kinematic constraint & Coherent branching

- The Chudakov effect



- Angular ordering



➢ Wide angle radiation suppressed!

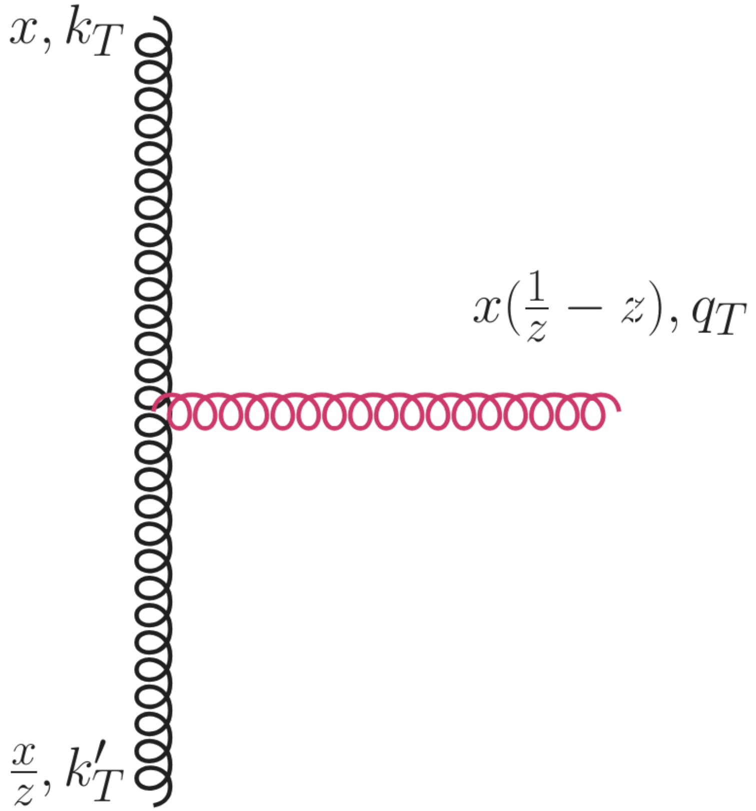
Kinematical constraint in the GLR evolution equation

[Kwiecinski, Martin, Sutton, Z. Phys. C, 96; Deak, Kutak, Li, Stasto, EPJC, 19]

- **The virtuality of t-channel gluon should be dominated by transverse components**

$$k_T^2 > |k^+ k^-| \qquad k^- = k'^- - q^- \simeq -q^- = -q_T^2/q^+$$

$$k^+ k^- \simeq -\frac{k^+}{q^+} q_T^2 = -\frac{k^+}{k'^+ - k^+} q_T^2 = -\frac{z}{1-z} q_T^2$$



- **The on shell condition for s-channel gluon leads to the constraint**

$$q_T^2 < \frac{1-z}{z} k_T^2 \qquad \eta \longrightarrow \eta + \boxed{\ln \frac{k_\perp^2}{k_\perp^2 + l_\perp^2}}$$

- **Results in a non-local GLR equation**

$$\frac{\partial N(\eta, k_\perp)}{\partial \eta} = \frac{\bar{\alpha}_s}{\pi} \int \frac{\mathrm{d}^2 l_\perp}{l_\perp^2} N\left(\boxed{\eta + \ln \frac{k_\perp^2}{k_\perp^2 + l_\perp^2}}, l_\perp + k_\perp\right) - \frac{\bar{\alpha}_s}{\pi} \int_0^{k_\perp} \frac{\mathrm{d}^2 l_\perp}{l_\perp^2} N(\eta, k_\perp) - \bar{\alpha}_s N^2(\eta, k_\perp)$$

Coherent branching in the GLR evolution

- Kinematic constrained GLR equation

$$\frac{\partial}{\partial\eta}\frac{N(x,k_\perp)}{\Delta(\eta,k_\perp)}=\frac{\bar{\alpha}_s}{\pi}\int_\mu\frac{\mathrm{d}^2l_\perp}{l_\perp^2}\frac{N\left(\eta+\ln\left[\frac{k_\perp^2}{k_\perp^2+l_\perp^2}\right],l_\perp+k_\perp\right)}{\Delta(\eta,k_\perp)}$$

Shi-Wei-ZJ, 2022

- Weighting factor for forward evolution:

$$\mathcal{W}_{kc}(\eta_i,\eta_{i+1};k_\perp)=\frac{(\eta_{i+1}-\eta_i)\int_\mu^{\min\left[P_\perp,\sqrt{(k_\perp-l_\perp)^2\frac{1-z}{z}}\right]}\frac{\mathrm{d}^2l_\perp}{l_\perp^2}e^{-\bar{\alpha}_s\int_{\eta_{i+1}}^{\eta_{i+1}+\ln\frac{(k_\perp-l_\perp)^2}{(k_\perp-l_\perp)^2+l_\perp^2}}\mathrm{d}\eta\left[\ln\frac{k_\perp^2}{\mu^2}+N(\eta,k_\perp)\right]}}{(\eta_{i+1}-\eta_i)\ln\frac{k_\perp^2}{\mu^2}+\int_{\eta_i}^{\eta_{i+1}}d\eta N(\eta,k_\perp)}$$

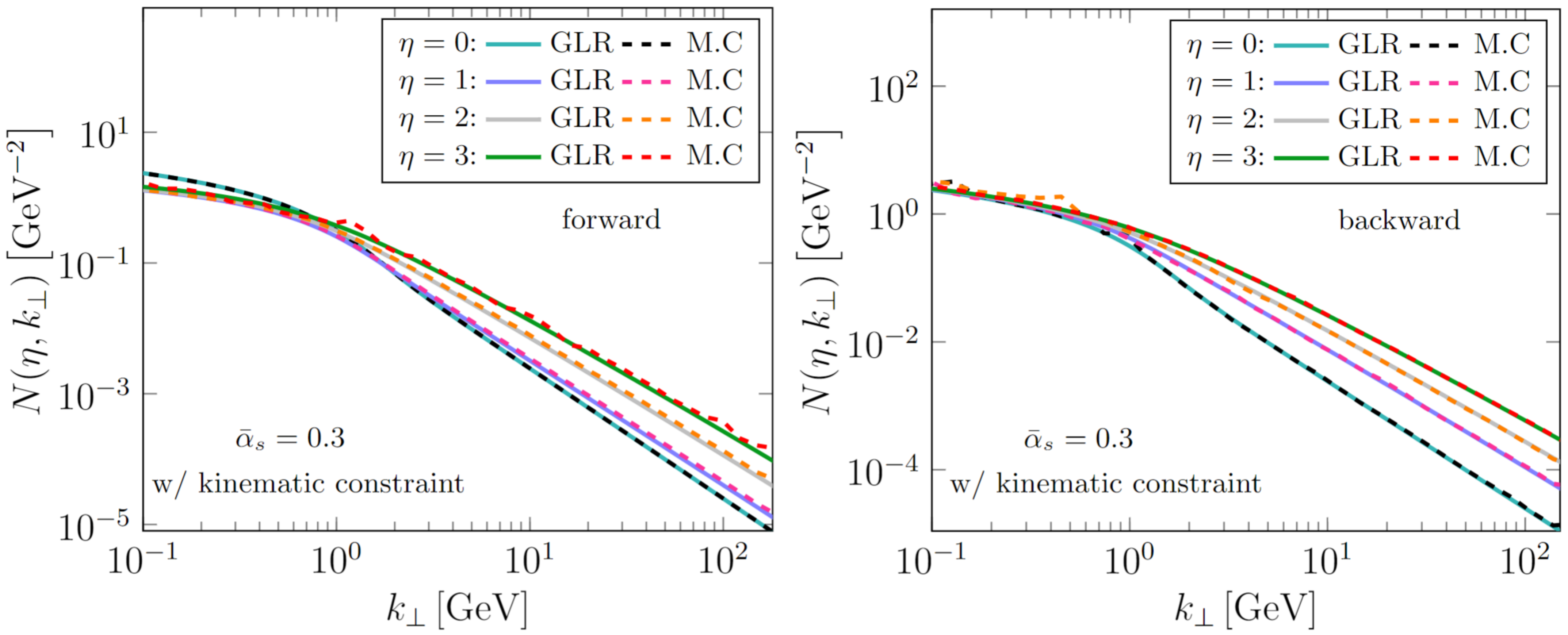
- Weighting factor for backward evolution:

$$\mathcal{W}(\eta_{i+1},\eta_i;k_{\perp,i+1})=\frac{(\eta_{i+1}-\eta_i)\ln\frac{k_{\perp,i}^2}{\mu^2}+\int_{\eta_i}^{\eta_{i+1}}d\eta N(\eta,k_{\perp,i})}{(\eta_{i+1}-\eta_i)\ln\frac{P_\perp^2}{\mu^2}}\frac{N(\eta_i,k_{\perp,i})}{N\left(\eta_i+\ln\left[\frac{k_{\perp,i+1}^2}{k_{\perp,i+1}^2+l_\perp^2}\right],k_{\perp,i}\right)}$$

- Non-Sudakov factor for backward evolution

$$\frac{\Delta(\eta_{i+1},k_{\perp,i+1})N(\eta_i,k_{\perp,i+1})}{\Delta(\eta_i,k_{\perp,i+1})N(\eta_{i+1},k_{\perp,i+1})}=\exp\left[-\frac{\bar{\alpha}_s}{\pi}\int_{\eta_i}^{\eta_{i+1}}\mathrm{d}\eta\int_\mu^{\min\left[P_\perp,\sqrt{\frac{1-z}{z}k_{\perp,i+1}^2}\right]}\frac{\mathrm{d}^2l_\perp}{l_\perp^2}\frac{N\left(\eta+\ln\left[\frac{k_{\perp,i+1}^2}{k_{\perp,i+1}^2+l_\perp^2}\right],k_{\perp,i+1}+l_\perp\right)}{N(\eta,k_{\perp,i+1})}\right]$$

Coherent branching: test against the numerical results



Shi-Wei-ZJ, 2022

Joint small x and kt resummation

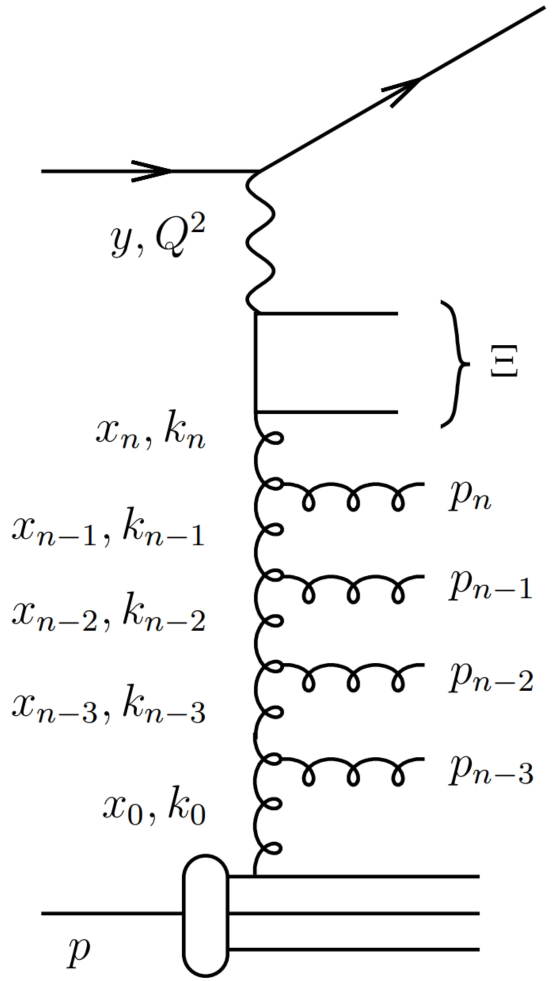
- When $S >> Q^2 >> k_T^2$, resume two type large logs

$$\ln \frac{S}{Q^2} \qquad \ln^2 \frac{Q^2}{k_\perp^2}$$

- Starting point:

$$xG(x, l_\perp, x\zeta) = \int \frac{dy^- d^2 y_\perp}{(2\pi)^3 P^+} e^{-ixP^+ y^- + il_\perp \cdot y_\perp} \langle P | F_\mu^+(y^-, y_\perp) \mathcal{L}_{\tilde{n}}^\dagger(y^-, y_\perp) \mathcal{L}_{\tilde{n}}(0, 0_\perp) F^{\mu +}(0) | P \rangle$$

Collins, Soper, 81; Collins, Soper, Sterman, 85



Gloun---->gloun splitting kernel; $\mathcal{P}_{gg}(z) = 2C_A \frac{(z^2 - z + 1)^2}{z(1-z)}$

When z(or x) --->0, large logarithm $\ln \frac{1}{x}$ summed by BFKL

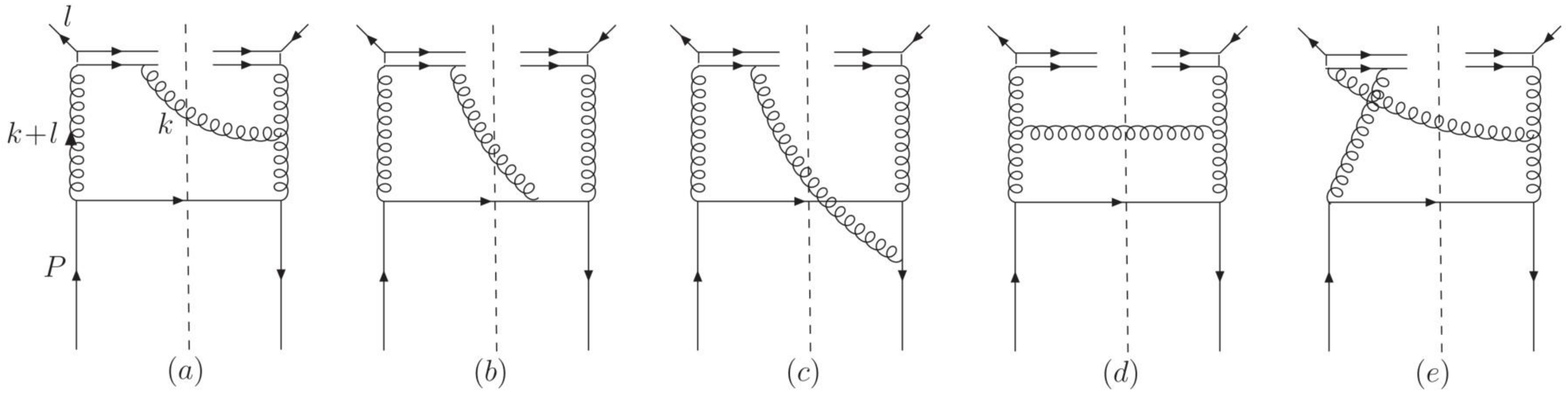
When z--->1, light cone divergence, introduce ζ to regularize

large logarithm $\ln \frac{x^2 \zeta^2}{k_T^2}$ summed by CS

Mueller, Xiao, Yuan 2012; Xiao, Yuan, Zhou, 2017; Caucal, Salazar, Schenke, Venugopalan, 2022-2023; Taels, Altinoluk, Beuf, Marquet, 2022; Mukherjee, Skokov, Tarasov, Tiwari, 2023….

Formulation in terms of gluon TMD(dilute limit)

- Sample real diagrams



- The resulting gluon TMD indeed simultaneously satisfies the both

$$\int_0^\infty \frac{dk^+}{k^+} = \int_{l^+}^\infty \frac{dk^+}{k^+} + \int_0^{l^+} \frac{dk^+}{k^+}$$

2016, ZJ

BFKL equation:

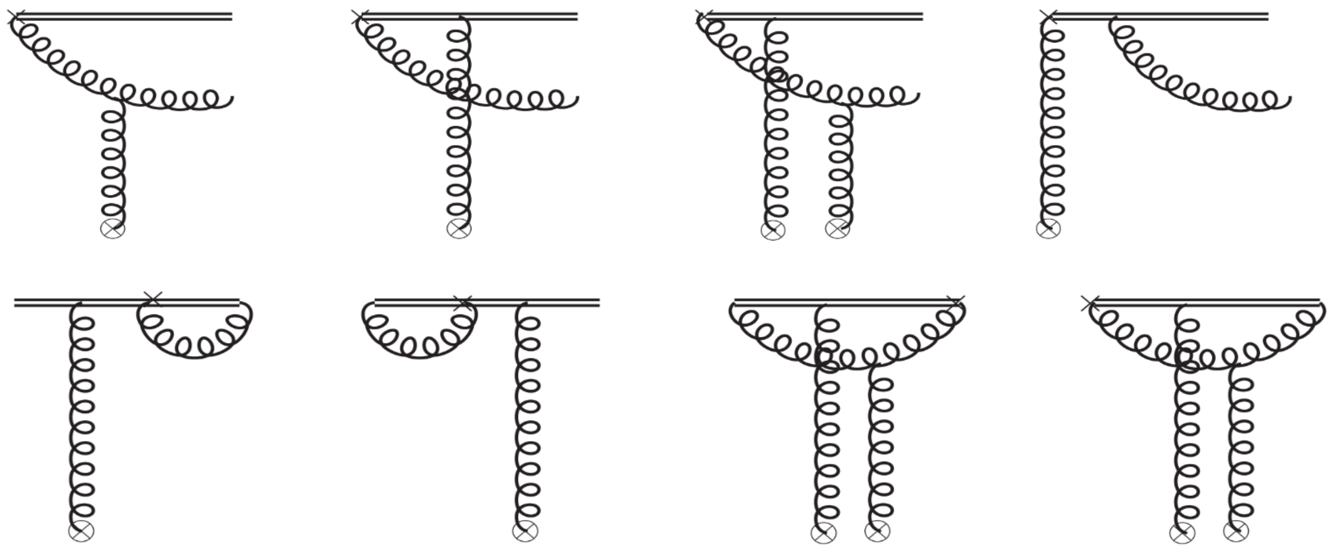
$$\frac{\partial\left[xG(x,l_\perp,x\zeta)\right]}{\partial \ln(1/x)} = \frac{\alpha_s N_c}{\pi^2}\int \frac{d^2k_\perp}{k_\perp^2}\left\{xG(x,k_\perp+l_\perp,x\zeta) - \frac{l_\perp^2}{2(l_\perp+k_\perp)^2}xG(x,l_\perp,x\zeta)\right\}$$

CS equation:

$$\frac{\partial\left[G(x,b_\perp,x\zeta)\right]}{\partial \ln\zeta} = -\frac{\alpha_s N_c}{\pi}\ln\left[\frac{x^2\zeta^2 b_\perp^2}{4}e^{2\gamma_E-\frac{1}{2}}\right]G(x,b_\perp,x\zeta)$$

Small x TMDs in CGC at NLO(double log)

Sample diagrams **(Collins-2011 scheme)**



Xiao, Yuan and ZJ, 2017.

The basic nonperturbative ingredient:

$$U(x_\perp) = \langle \mathcal{P} e^{-ig\int_{-\infty}^{+\infty} dx^- A^+(x^-,\, x_\perp)} \rangle$$

Multiple gluon rescattering is treated in an equal way.

Collinear approach V.S. CGC

- **Collinear factorization:**

$$\tilde{f}_g^{(sub.)}(x, r_\perp, \zeta_c{}') = e^{-S^g_{pert}(Q, r_\perp)} \sum_i C_{g/i}(\mu_r/\mu) \otimes f_i(x, \mu)$$

Sudakov factor

Hard coefficient

Colliner PDF

- **CGC** (Colliner divergence absent)

$$xG^{(1)}(x, k_\perp, \zeta) = -\frac{2}{\alpha_S} \int \frac{d^2x_\perp d^2y_\perp}{(2\pi)^4} e^{ik_\perp \cdot r_\perp} \mathcal{H}^{WW}(\alpha_s(Q)) e^{-S_{sud}(Q^2, r_\perp^2)} \mathcal{F}^{WW}_{Y=\ln 1/x}(x_\perp, y_\perp)$$

Hard coefficient

Sudakov factor

Two point function

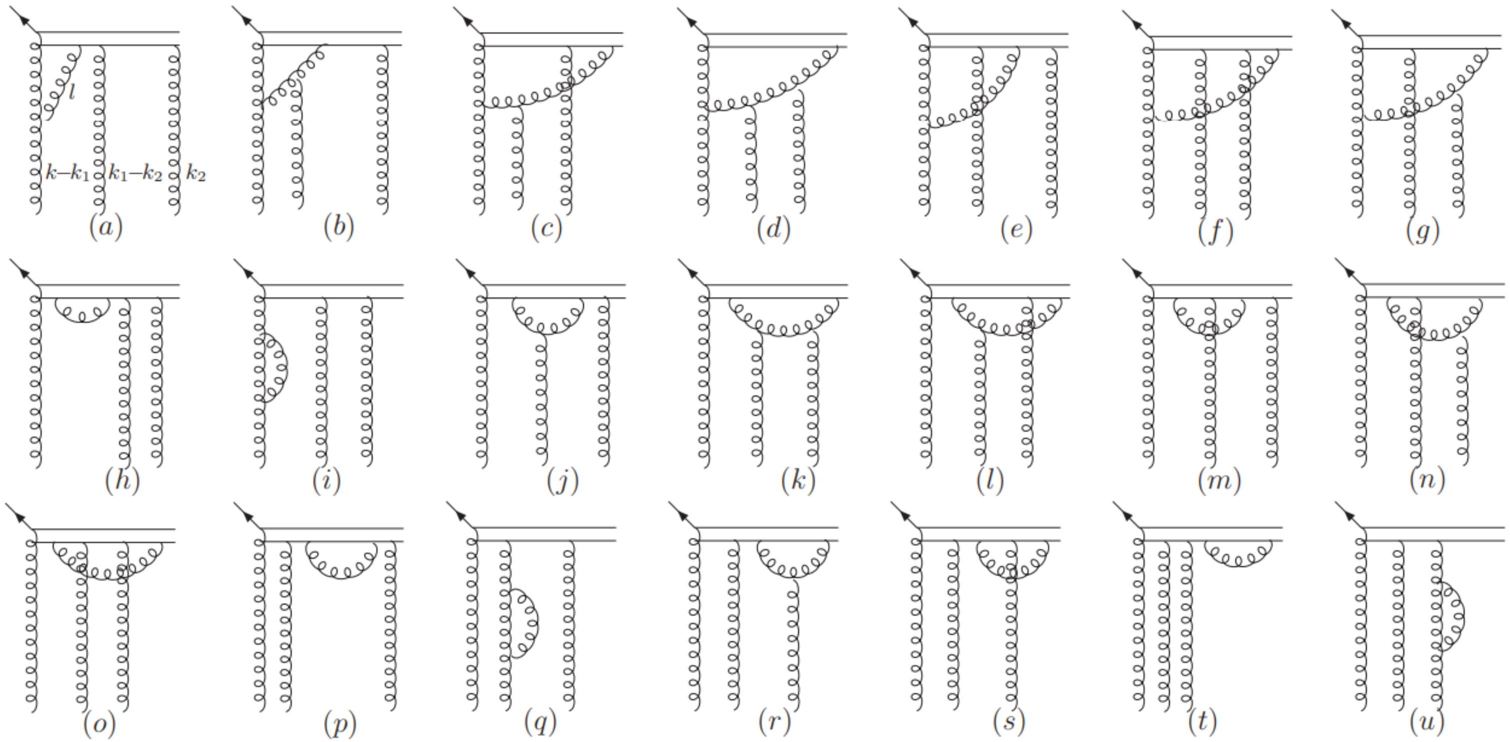
Two step evolution: $x_0 \rightarrow x$ $k_t \rightarrow Q$

Sudakov Single log at small x

The anomalous dimension at small x?

$$\frac{\mathrm{d}\ \ln G(x, b_\perp, \mu^2, \zeta_c^2)}{\mathrm{d}\ \ln\mu} = \gamma_G\left(g(\mu), \zeta_c^2/\mu^2\right)$$

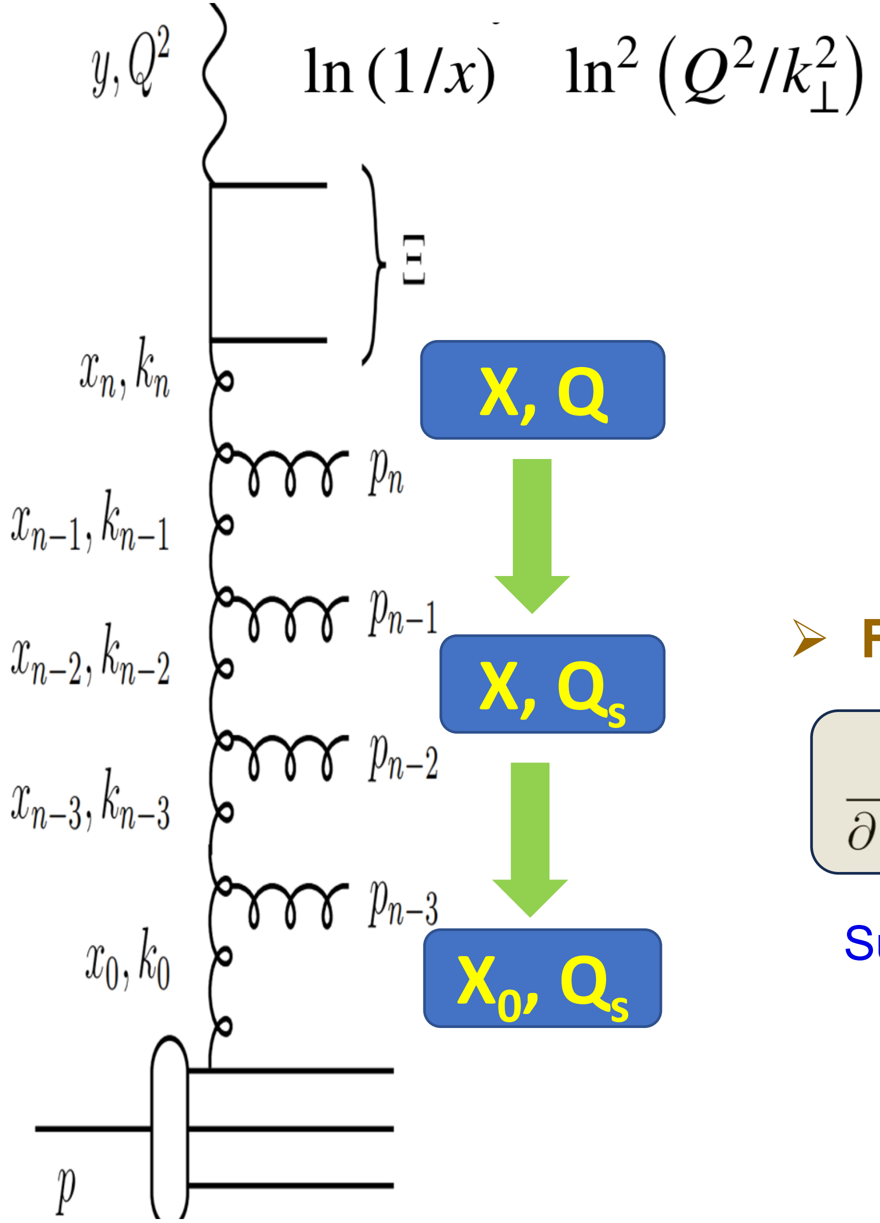
Compute UV part first; employ the Eikonal approximation next!



2019, ZJ

- Single log is not affected by saturation effect.

Monte Carlo implementation of the joint resummation



- Combing Collins-Soper equation

$$\frac{\partial N(\mu^2,\zeta^2,x,k_\perp)}{\partial \ln \zeta^2} = \frac{2\alpha_s N_c}{\pi^2}\int_0^{\zeta} \frac{d^2 l_\perp}{l_\perp^2}\left[N(\mu^2,\zeta^2,x,k_\perp+l_\perp) - N(\mu^2,\zeta^2,x,k_\perp)\right]$$

with renormalization group equation

$$\frac{\partial N(\mu^2,\zeta^2,x,k_\perp)}{\partial \ln \mu^2} = \frac{\alpha_s N_c}{\pi}\left[\frac{\beta_0}{6} - \ln\frac{\zeta^2}{\mu^2}\right] N(\mu^2,\zeta^2,x,k_\perp)$$

- **Folded CS+RG:** $N(Q^2,x,k_\perp) \equiv N(\mu^2 = Q^2, \zeta^2 = Q^2, x, k_\perp)$

$$\frac{\partial}{\partial \ln Q^2}\frac{N(Q^2,x,k_\perp)}{\Delta_s(Q^2,k_\perp)} = \frac{2\bar{\alpha}_s}{\pi}\int_{Q_0}^{Q}\frac{d^2 l_\perp}{l_\perp^2}\frac{N(Q^2,x,k_\perp+l_\perp)}{\Delta_s(Q^2,k_\perp)}$$

Shi-Wei-ZJ, 2023

Sudakov form factor

$$\Delta_s(Q^2,k_\perp) = \exp\left[-\bar{\alpha}_s\int_{Q_0^2}^{Q^2}\frac{dt}{t}\left(2\ln\frac{t}{Q_0^2} - \frac{\beta_0}{6}\right)\right]$$

Monte Carlo implementation of kt resummation

- The modified Sudakov factor in the backward evolution:

$$\frac{\Delta_s(Q_n^2,k_{\perp,n})N(Q_{n-1}^2,x_n,k_{\perp,n})}{\Delta_s(Q_{n-1}^2,k_{\perp,n})N(Q_n^2,x_n,k_{\perp,n})}=\exp\left[-\int_{Q_{n-1}^2}^{Q_n^2}\frac{dt}{t}\int_{Q_0}^{t}\frac{d^2l_\perp}{l_\perp^2}\frac{2\bar{\alpha}_s(l_\perp^2)}{\pi}\frac{N(t,x_n,k_{\perp,n}+l_\perp)}{N(t,x_n,k_{\perp,n})}\right]=\mathcal{R}$$

- Sample lt and reconstruct z

Shi-Wei-ZJ, 2023

$$\int\frac{d^2l_\perp}{l_\perp^2}\frac{\bar{\alpha}_s(l_\perp^2)}{\pi}N(Q_{n-1}^2,x_n,k_{\perp,n}+l_\perp)\qquad l_{\perp,n}^2\approx Q_{n-1}^2(1-z_n)$$

- Weight factor

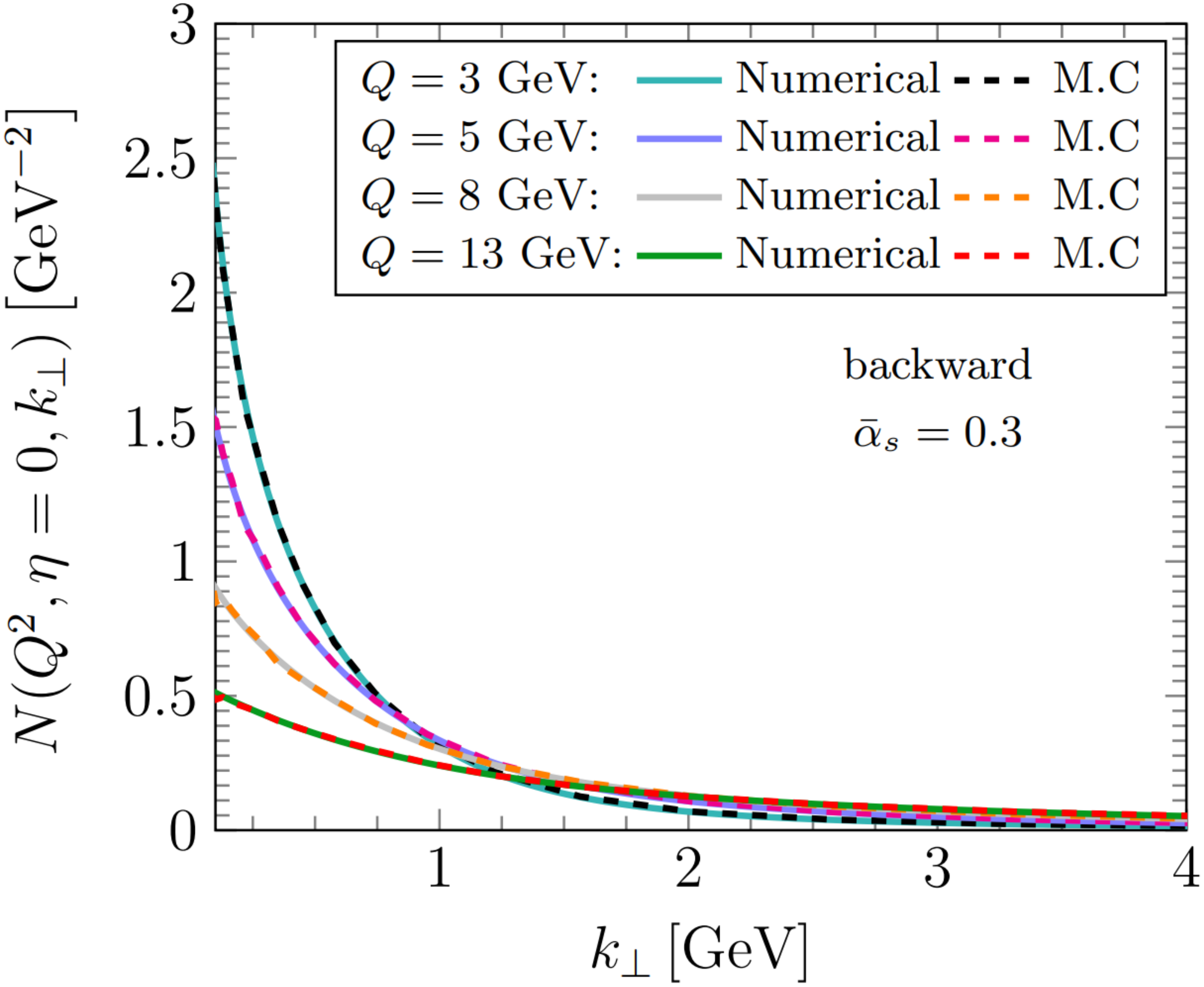
$$\mathcal{W}=\frac{\int_{Q_{n-1}^2}^{Q_n^2}\frac{dt}{t}\ln\frac{t^2}{Q_0^2}}{\int_{Q_{n-1}^2}^{Q_n^2}\frac{dt}{t}\left[\ln\frac{t^2}{Q_0^2}-\frac{\beta_0}{12}\right]}$$

- Unitary is preserved only within the double leading log approximation

- Terminate kt PS and turn on small x PS

Z_i>0.5 Q_i>Q_s

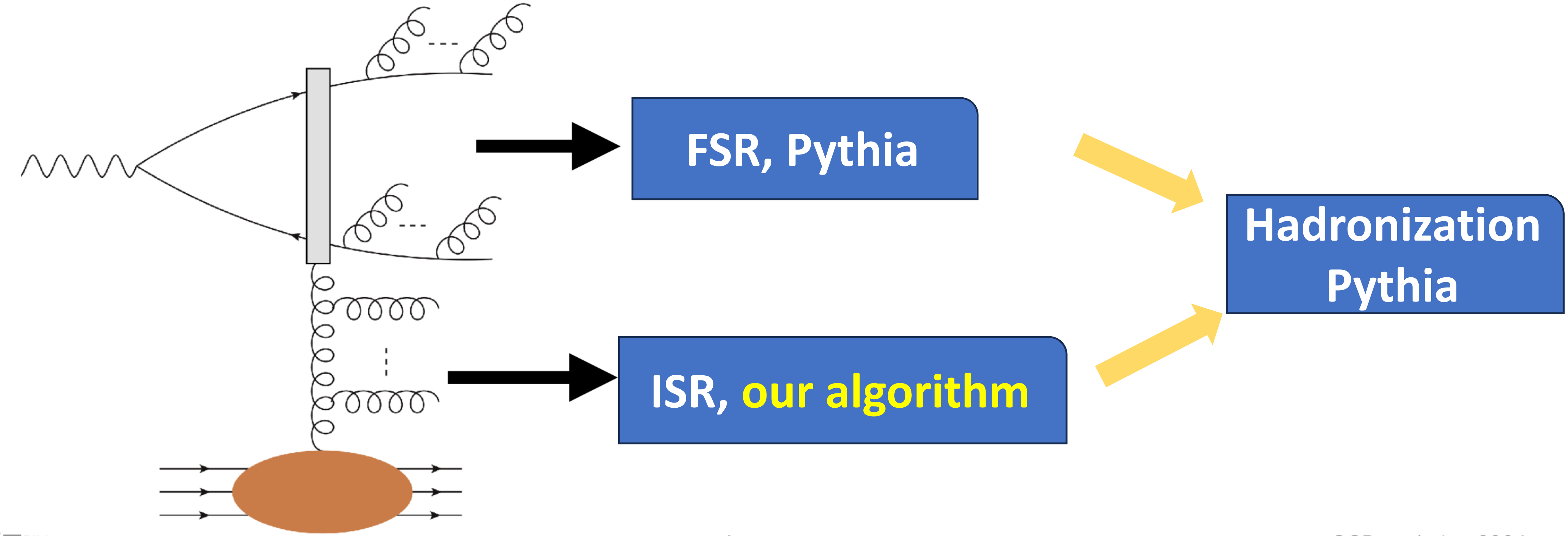
Numerical V.S. MC:



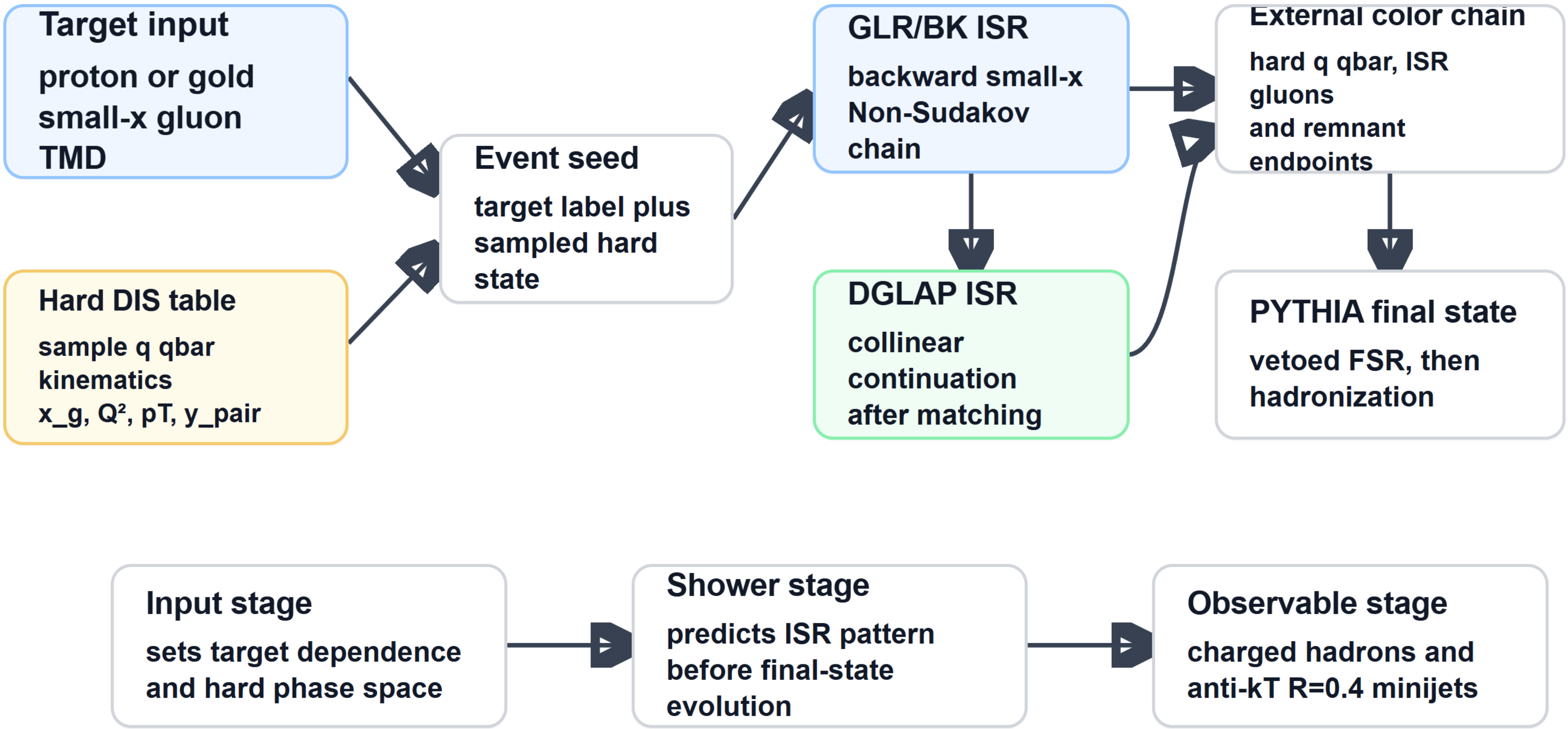
Initial condition:

$$N(Q_0^2 = 1\ \mathrm{GeV}^2, x = 0.01, k_\perp) = \int \frac{d^2 r_\perp}{2\pi} e^{ik_\perp \cdot r_\perp} \frac{1}{r_\perp^2} \left[1 - e^{-\frac{Q_s^2 r_\perp^2}{4} \log(\frac{1}{r_\perp \Lambda_{\mathrm{MV}}} + e)}\right]$$

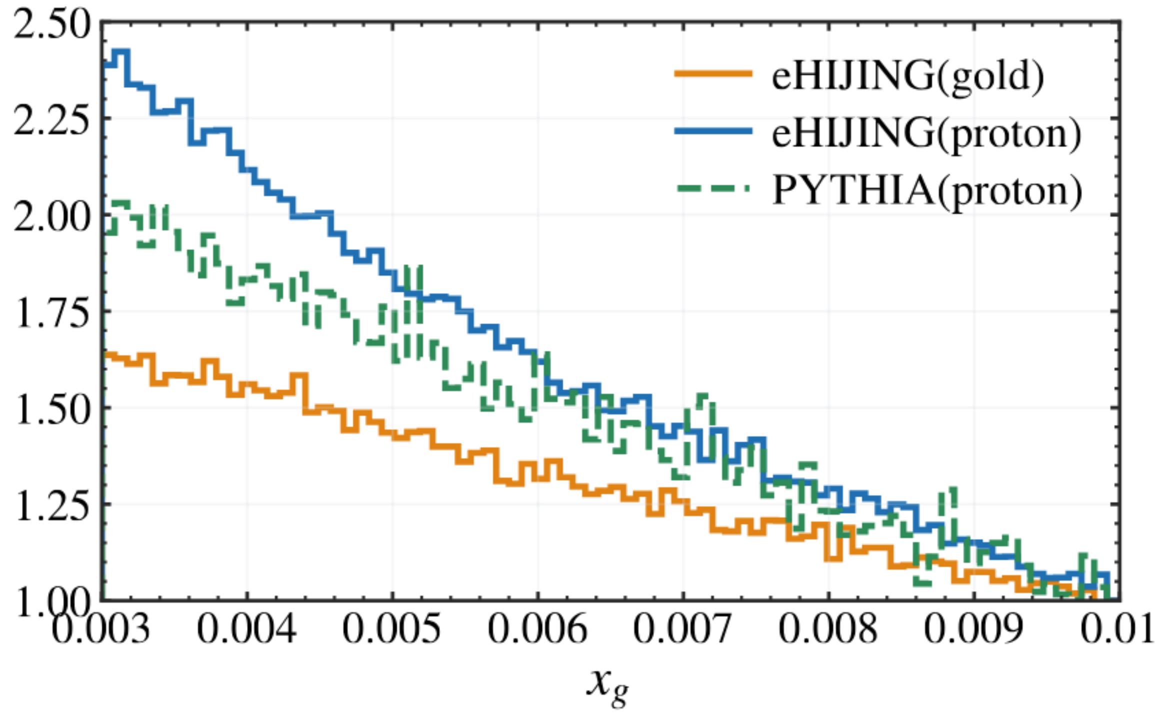
Hadronization and final state radiations



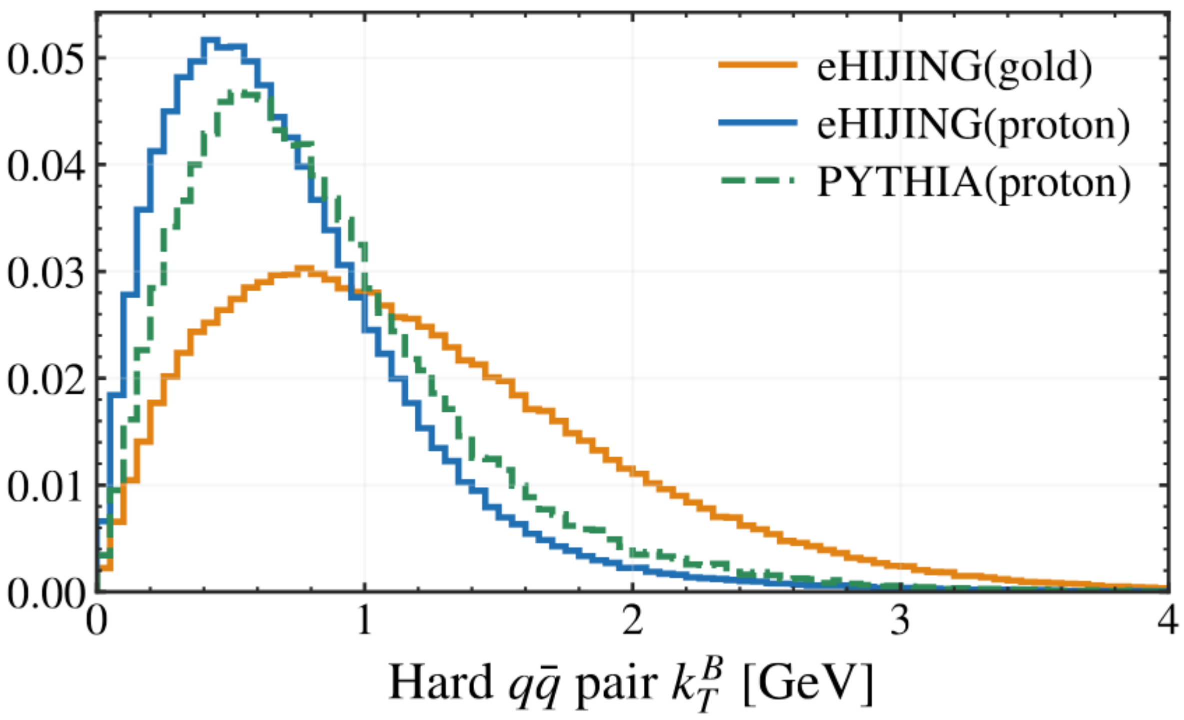
Architecture: target gluon field to hadronized event



Parton level baseline

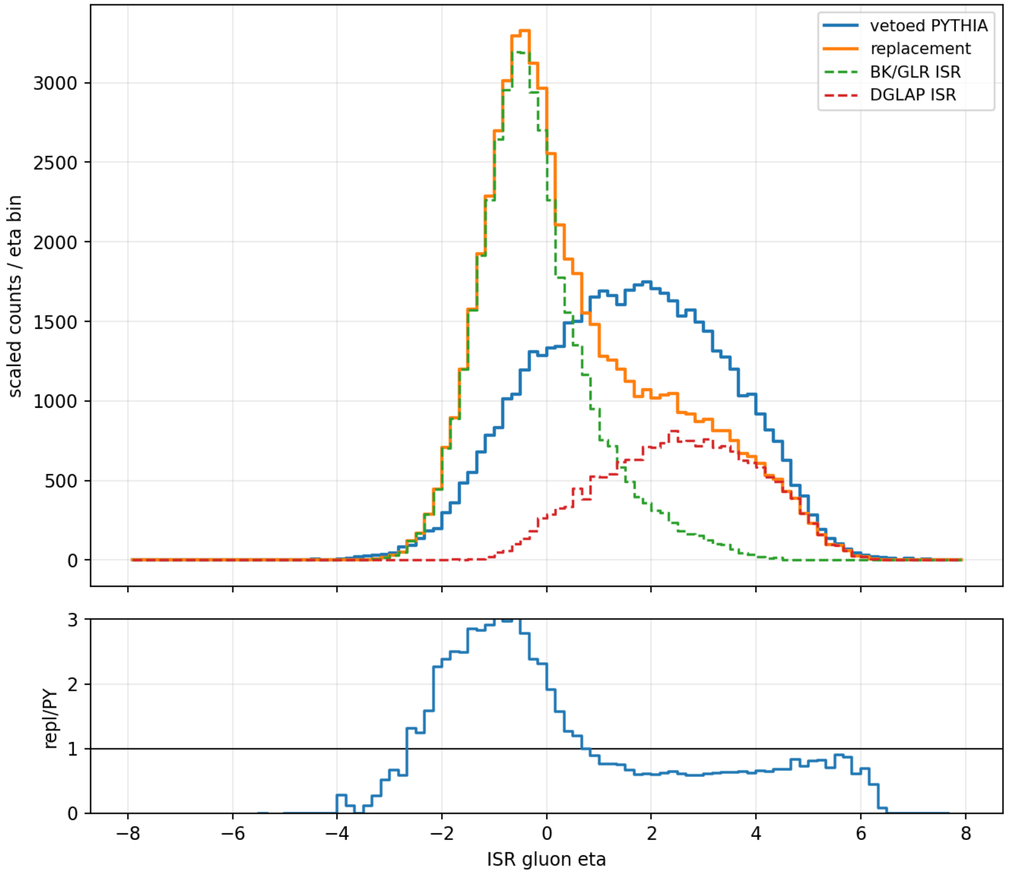


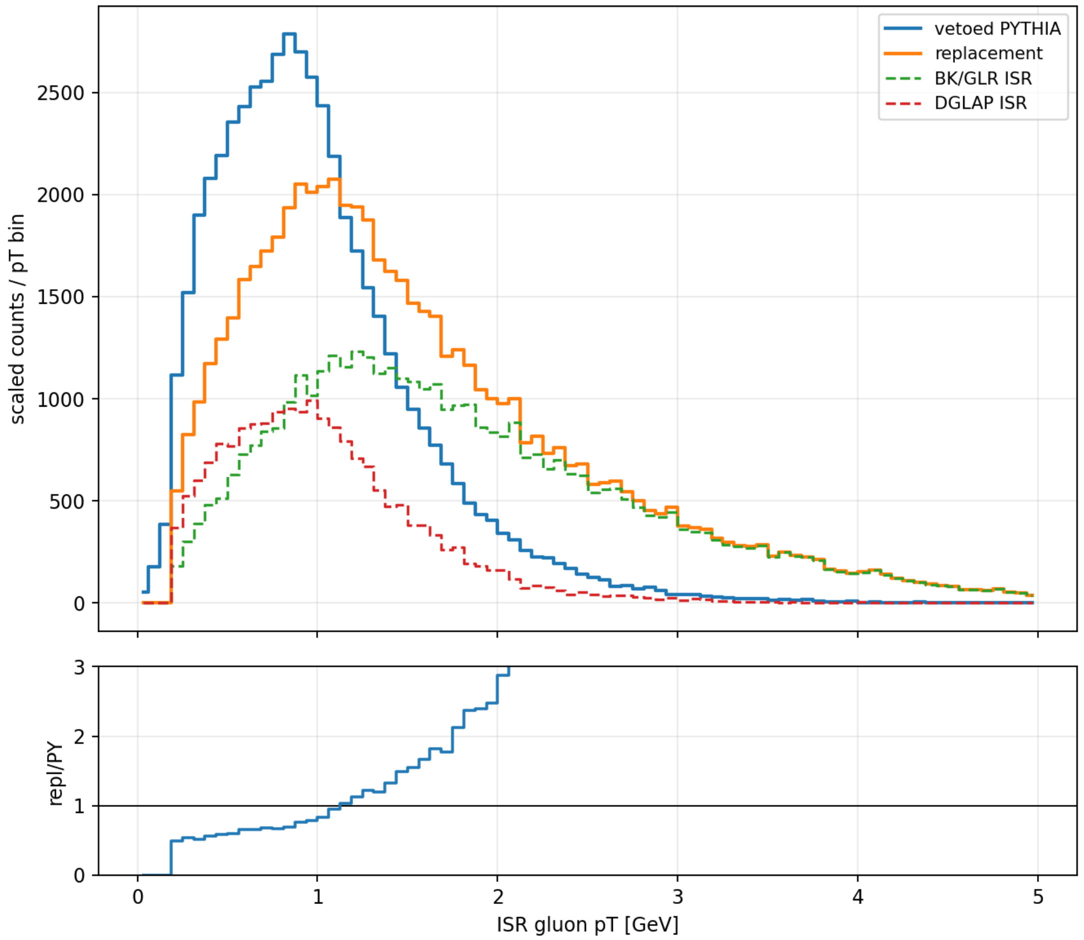
Hard events xg distribution



Hard quark-antiquark pair kt distribution

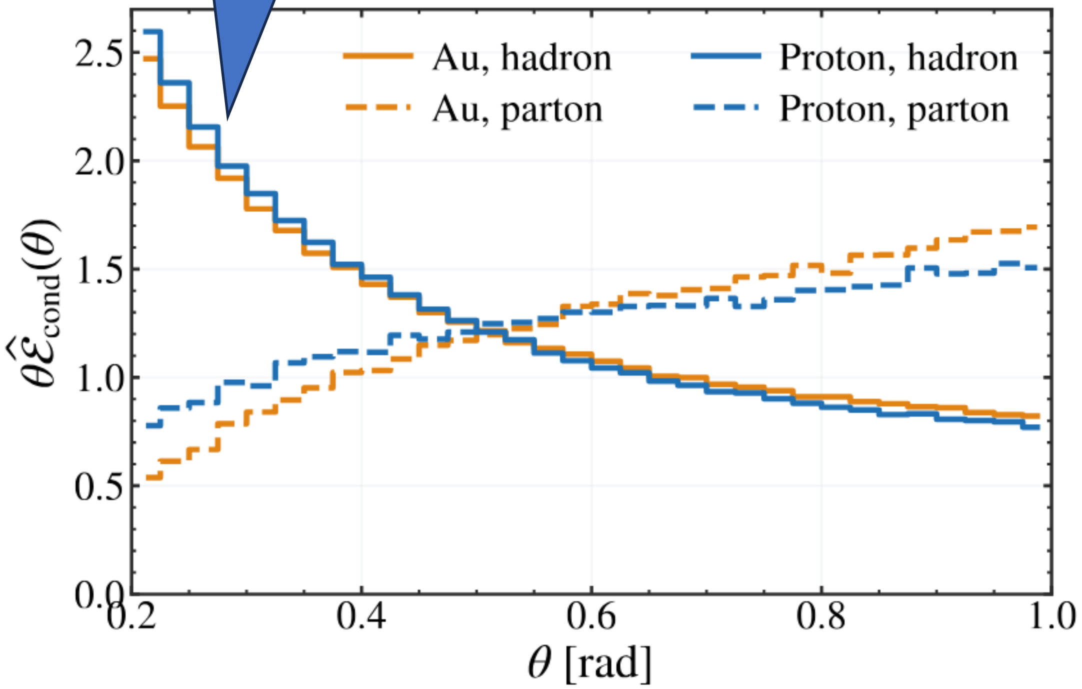
ISR对比图


vetoed PYTHIA
replacement
BK/GLR ISR
DGLAP ISR
scaled counts / eta bin
repl/PY
ISR gluon eta

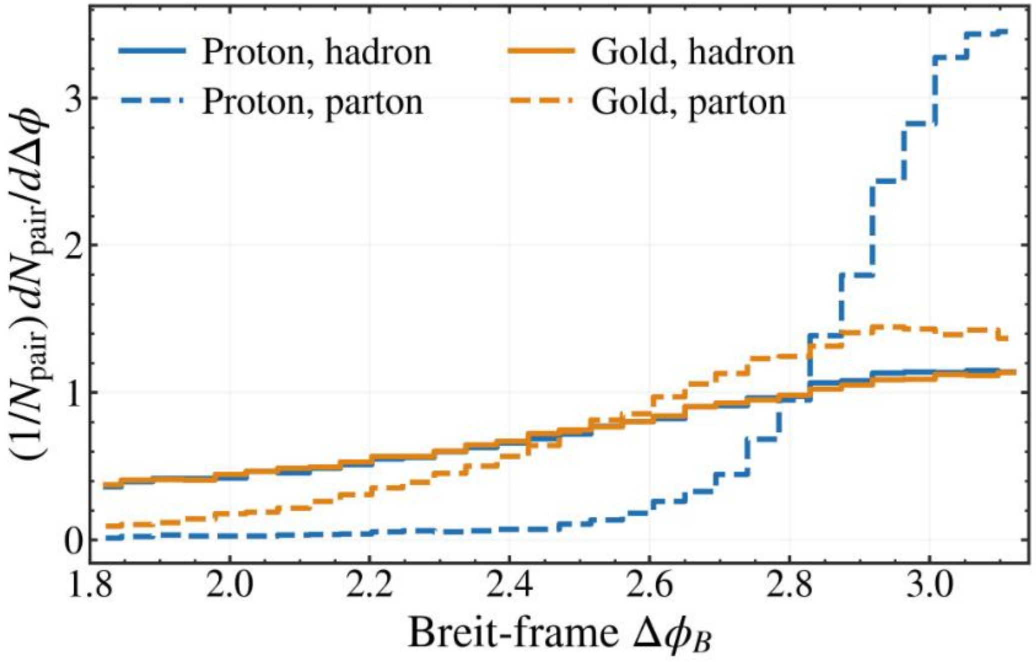

vetoed PYTHIA
replacement
BK/GLR ISR
DGLAP ISR
scaled counts / pT bin
repl/PY
ISR gluon pT [GeV]

NEC and acoplanarity at EIC

靶碎裂贡献



Nucleon energy correlation



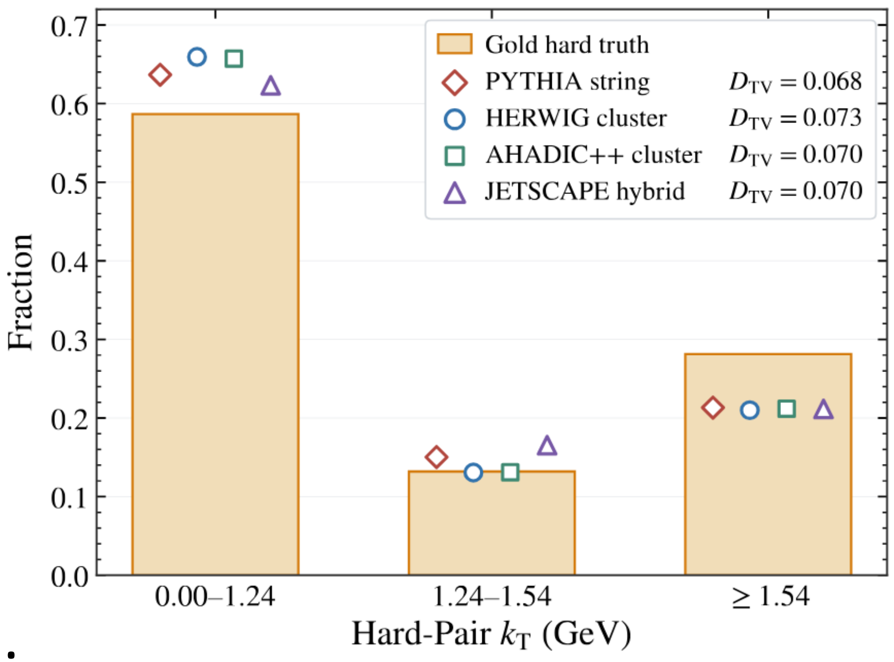
Di-hadron acoplanarity

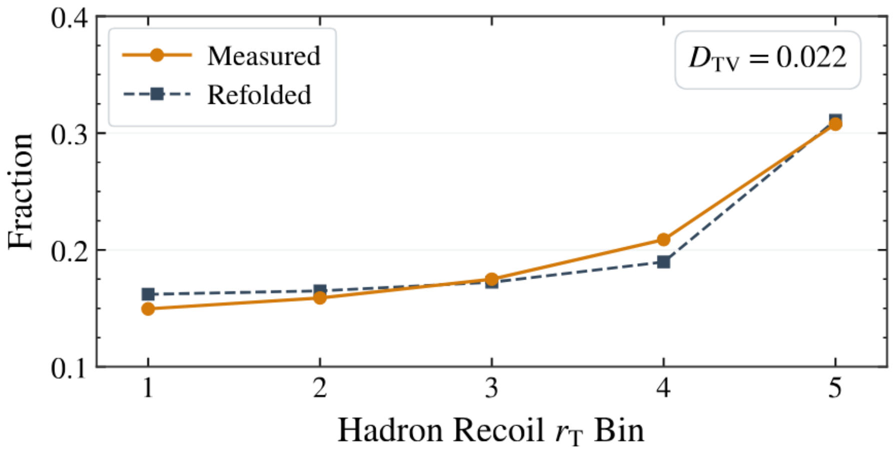
Hard-Pair Imbalance from Global Hadronic Recoil

- The hard-pair imbalance kt leaves a direct imprint on the global hadronicrecoil rt

- For a fixed ensemble of hard partonic events, final-state showering and hadronization map the kt distribution onto the rt distribution.

- Iterative Bayesian unfolding





Summary and Outlook

- The first event generator incorporating saturation effect
- The implementation of the joint resummation is sketched

- Recoiled effect; linear polarization effect; multiple gluon fusion effect
- Figure out the strongest saturation signal from PS data

Thank you for your attention!

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SHANDONG UNIVERSITY, QINGDAO