Probing early thermalization in heavy-ion collisions

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Collectivity in Heavy-Ion Collisions

Pre-reaction **Detection**

Lorentz con

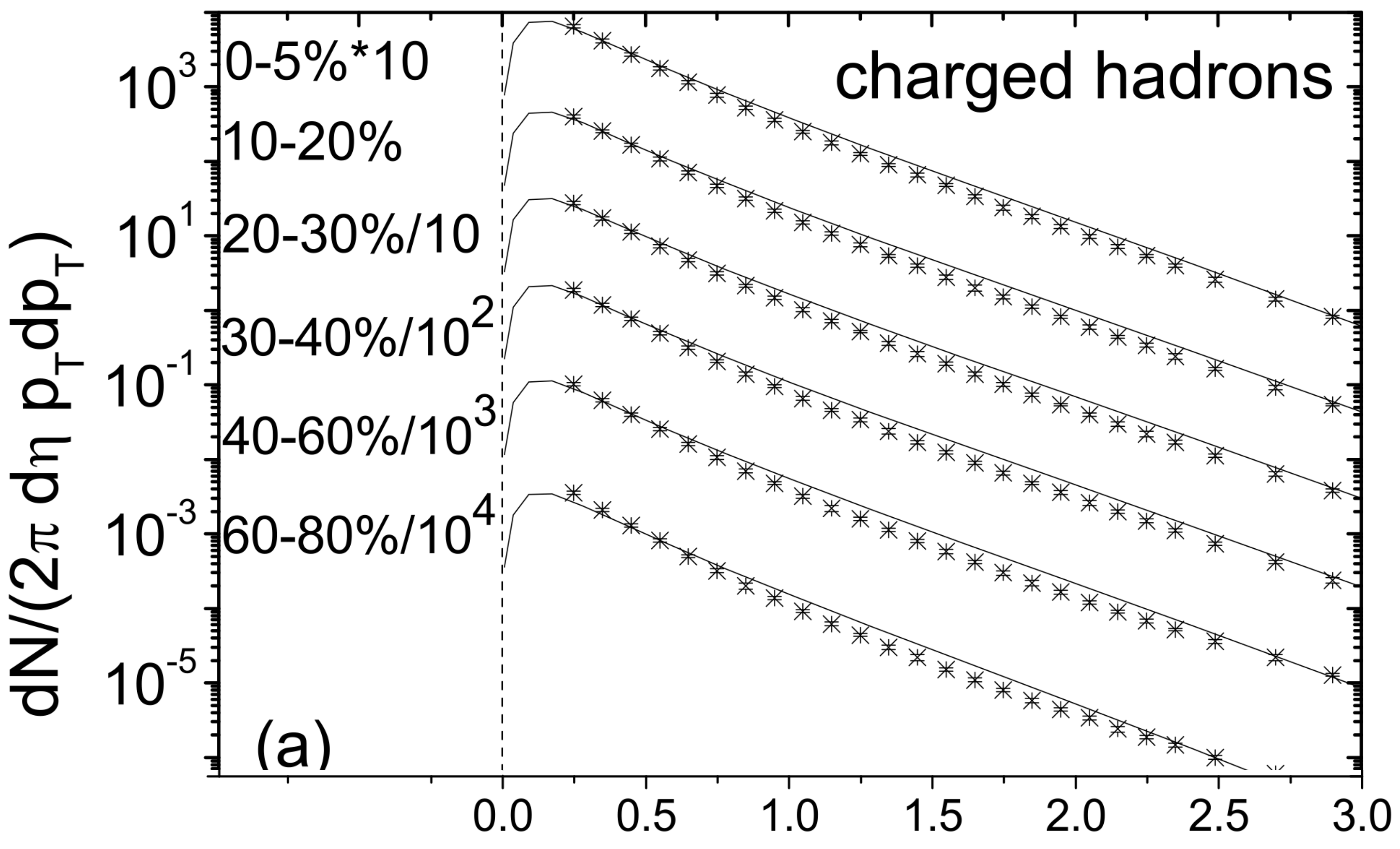
$\gamma \sim 100 - 2000$

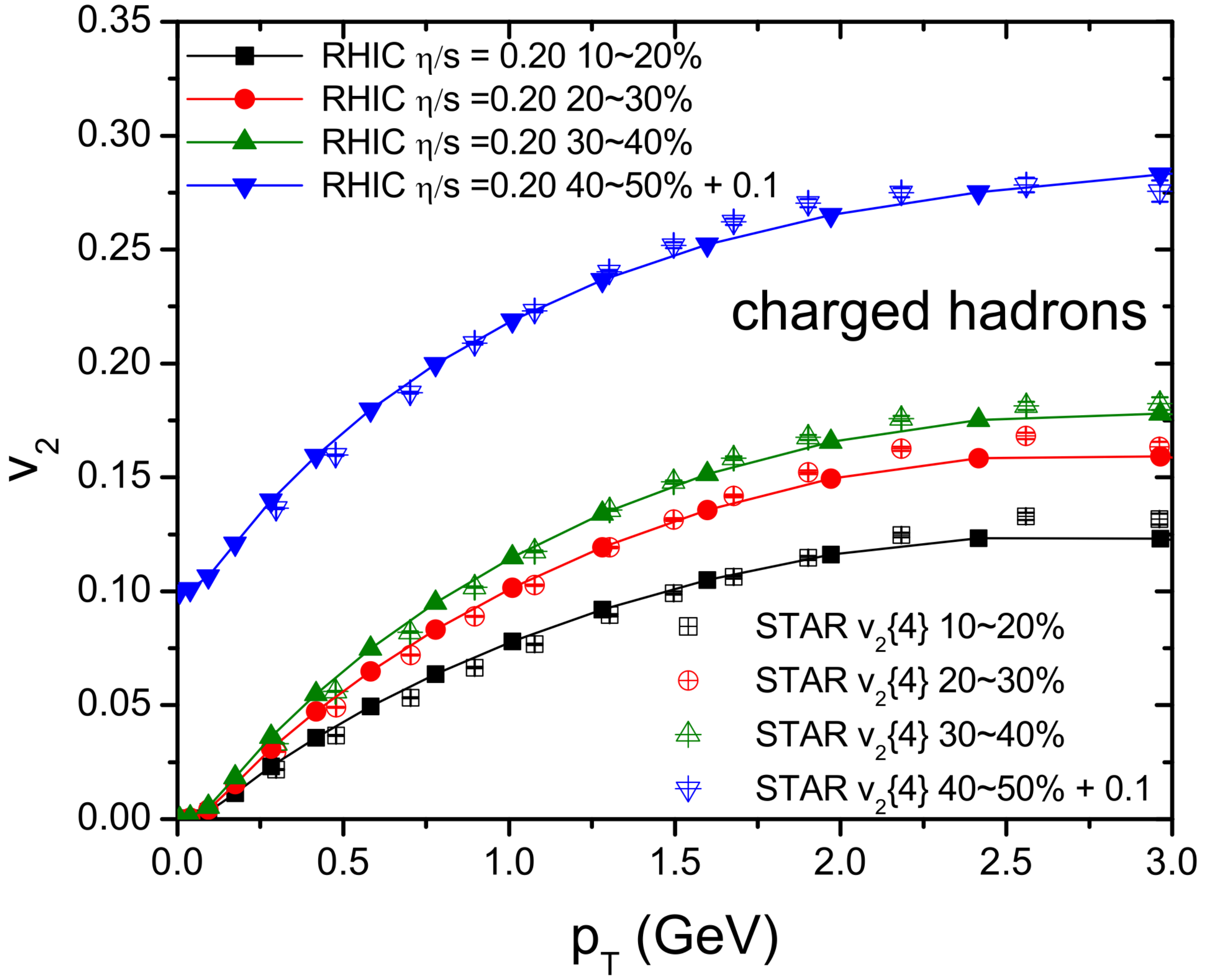
articles

$N \sim 10^{3-4}$

Collectivity in Heavy-Ion Collisions

***Collective Property** of final state particles*:





symbols: data

curves: hydrodynamics theory

(thermal w/ anisotropic blue-shift)

*+ many **independent probes** of the hot medium*

onset of hydrodynamics in Heavy-Ion Collisions

$$\partial_\mu T^{\mu\nu} = 0\,,$$

$$T^{\mu\nu} = (\varepsilon + P + \Pi)u^\mu u^\nu - (P + \Pi)g^{\mu\nu} + \pi^{\mu\nu}\,.$$

thermodynamic + kinetic quantities

viscous (non-equilibrium) corrections

(EOS: QCD physics validated)

Hydrodynamics $\Leftarrow$ near local-equilibrium (many body)

for ~ 10^{3-4} particles?

✓ mean free path (~0.5 fm) $\ll$ system size (~10 fm)

? rapid equilibration? (t ~ 0.5-1 fm/c)

? small system?

onset of hydrodynamics in Heavy-Ion Collisions

Hydrodynamic attractors

- focus on properties of *hydrodynamic equations*
- *common behavior* developed very rapidly

Heller, Spalinski, Phys.Rev.Lett. 115 (2015) 072501

⋮

S. Chen and SS, PhysRevC.111.L021902+PhysRevD.113.L071501

Hydrodynamization in strong-coupling quantum systems

- thermalization of quantum distribution function
- emergence of Bjorken flow

S. Chen, SS, and L. Yan, 2412.00662

H. Shao, S. Chen, and SS, 2509.10835 + 2509.10855

Understanding thermalization and hydrodynamization?

Bogolyubov-Born-Green-Kirkwood-Yvon Equations

$$\left(\frac{\partial}{\partial t}+\frac{\boldsymbol{p}}{E}\cdot\nabla\right)P_1(t,\boldsymbol{x},\boldsymbol{p})=\underline{\mathscr{C}[P_1,P_2]}\,,$$

$$\left(\frac{\partial}{\partial t}+\frac{\boldsymbol{p}_i}{E_i}\cdot\nabla_i\right)P_2(t,\{\boldsymbol{x},\boldsymbol{p}\}_2)=\underline{\mathscr{C}[P_2,P_3]},$$

$$\vdots$$

collision term

$$\left(\frac{\partial}{\partial t}+\frac{\boldsymbol{p}_i}{E_i}\cdot\nabla_i\right)P_n(t,\{\boldsymbol{x},\boldsymbol{p}\}_n)=\underline{\mathscr{C}[P_n,P_{n+1}]},$$

n-body joint distribution

$$\vdots$$

microscopic physics
classical: Newtonian equations
quantum: Schroedinger equation

If neglecting multi-body correlation,
$P_2(t,\boldsymbol{x}_1,\boldsymbol{p}_1,\boldsymbol{x}_2,\boldsymbol{p}_2)=P_1(t,\boldsymbol{x}_1,\boldsymbol{p}_1)P_1(t,\boldsymbol{x}_2,\boldsymbol{p}_2)$,
return to Boltzmann equation:

$$\left(\frac{\partial}{\partial t}+\frac{\boldsymbol{p}}{E}\cdot\nabla\right)f_1(t,\boldsymbol{x},\boldsymbol{p})=\mathscr{C}[f_1]\,.$$

linearized Boltzmann equation:

$$f_1=f_{\text{local eq.}}+f_\delta$$

$$\mathscr{C}[f_1]=\mathscr{L}[f_\delta]+\cancel{\mathcal{O}(f_\delta^2)}$$

$$\Big\downarrow \text{+ other truncations}$$

hydrodynamics

spectral Bogolyubov-Born-Green-Kirkwood-Yvon

$$\left(\frac{\partial}{\partial t}+\frac{\boldsymbol{p}}{E}\cdot\nabla\right)f(t,\boldsymbol{x},\boldsymbol{p})$$
$$=\int\frac{\mathrm{d}^3\boldsymbol{k}\,\mathrm{d}^3\boldsymbol{p}'\,\mathrm{d}^3\boldsymbol{k}'}{(2\pi)^9E_p\,E_k\,E_{p'}\,E_{k'}}\,W_{p,k\leftrightarrow p',k'}\left(f(t,\boldsymbol{x},\boldsymbol{p}')f(t,\boldsymbol{x},\boldsymbol{k}')-f(t,\boldsymbol{x},\boldsymbol{p})f(t,\boldsymbol{x},\boldsymbol{k})\right).$$

space-time dependent coefficients

$$P_n(t,\{\boldsymbol{x},\boldsymbol{p}\}_n)=\sum_{i_1,\cdots,i_n}f_{i_1,\cdots,i_n}(t,\{\boldsymbol{x}\}_n)\times\left(\underline{\mathrm{P}_{i_1}(\boldsymbol{p}_1)}\cdots\mathrm{P}_{i_n}(\boldsymbol{p}_n)\right)$$

momentum dependent basis

spectral Boltzmann

$$\left(\frac{\partial}{\partial t} + \frac{\boldsymbol{p}}{E_p} \cdot \nabla\right) f(t, \boldsymbol{x}, \boldsymbol{p})$$
$$= \int_{\boldsymbol{k}, \boldsymbol{k}', \boldsymbol{p}'} \frac{W_{p,k \leftrightarrow p',k'}}{E_p} \Big(f(t, \boldsymbol{x}, \boldsymbol{p}') f(t, \boldsymbol{x}, \boldsymbol{k}') - f(t, \boldsymbol{x}, \boldsymbol{p}) f(t, \boldsymbol{x}, \boldsymbol{k}) \Big) \,.$$

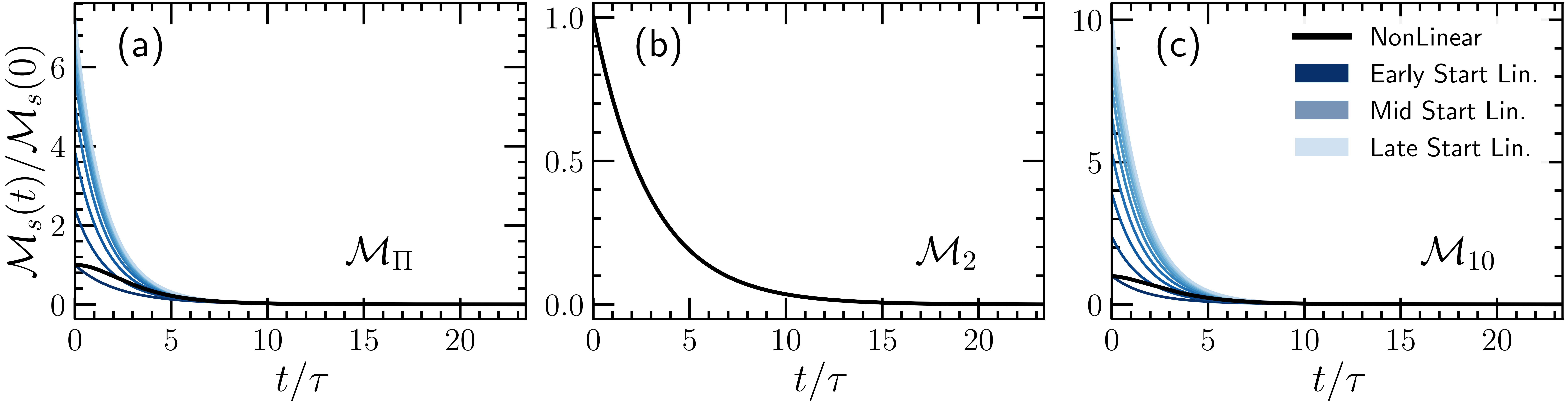
$$f(t, \boldsymbol{x}, \boldsymbol{p}) = \sum_i f_i(t, \boldsymbol{x})\, \mathrm{P}_i(\boldsymbol{p})$$

$$\frac{\partial}{\partial t} f_i + \boldsymbol{B}_j \cdot \nabla f_j(t, \boldsymbol{x}) = C_{ijk}\, f_j(t, \boldsymbol{x}) f_k(t, \boldsymbol{x}) \,.$$

$$f_i(t, \boldsymbol{x}) = \int_{\boldsymbol{p}} f(t, \boldsymbol{x}, \boldsymbol{p})\, \mathrm{P}_i(\boldsymbol{p}) w(\boldsymbol{p})$$

Xingjian Lu and SS, PhysRevC.113.034917

linearization



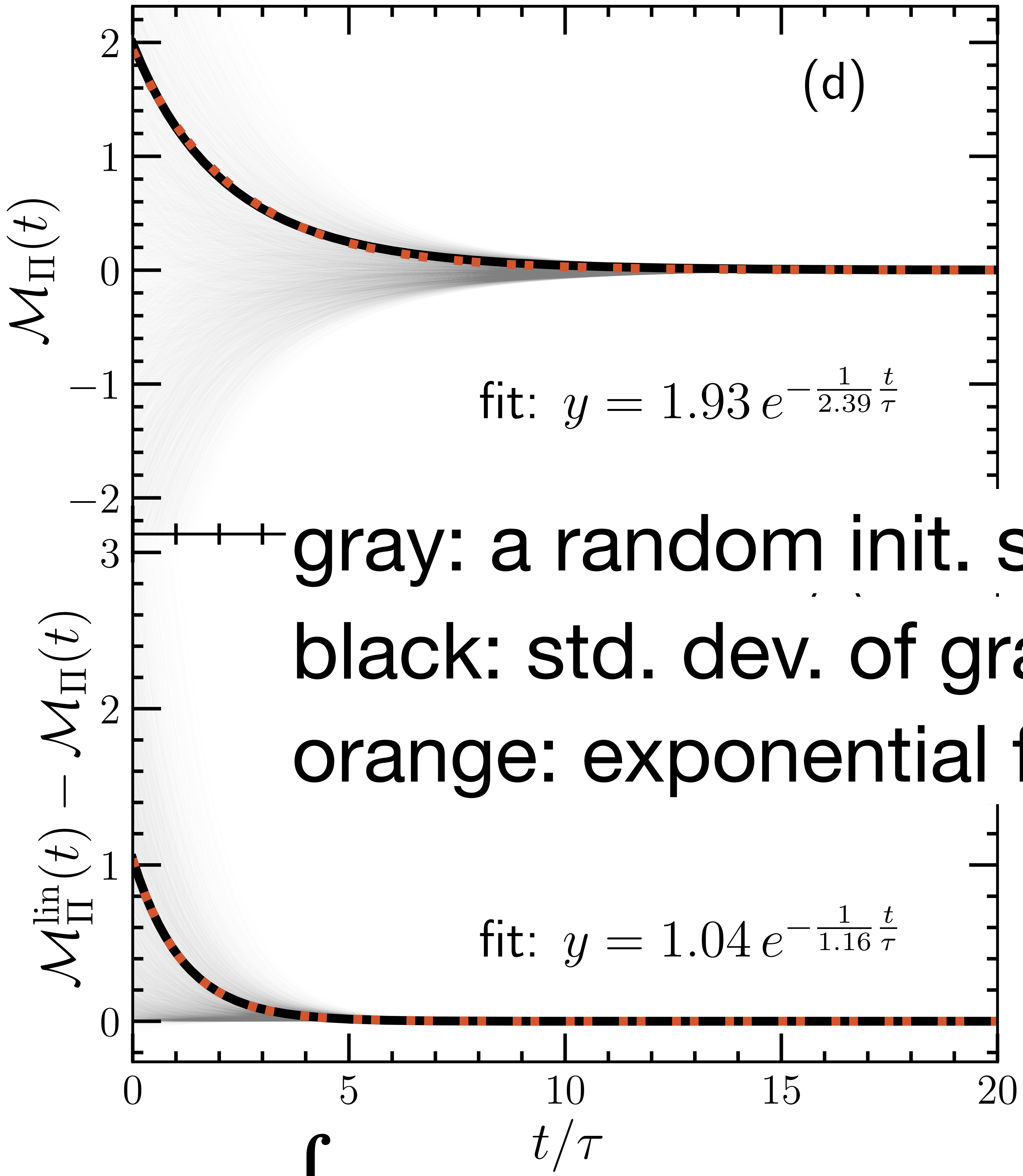
black: full evolution

blue: linear evolution (but match the full state at a fixed time slot)

$$\mathscr{M}_k \equiv \int_{\boldsymbol{p}} (E_p)^{k+1}\Big(f(\boldsymbol{p}) - f_{\rm eq}(\boldsymbol{p})\Big)$$

Xingjian Lu and SS, 2509.23978

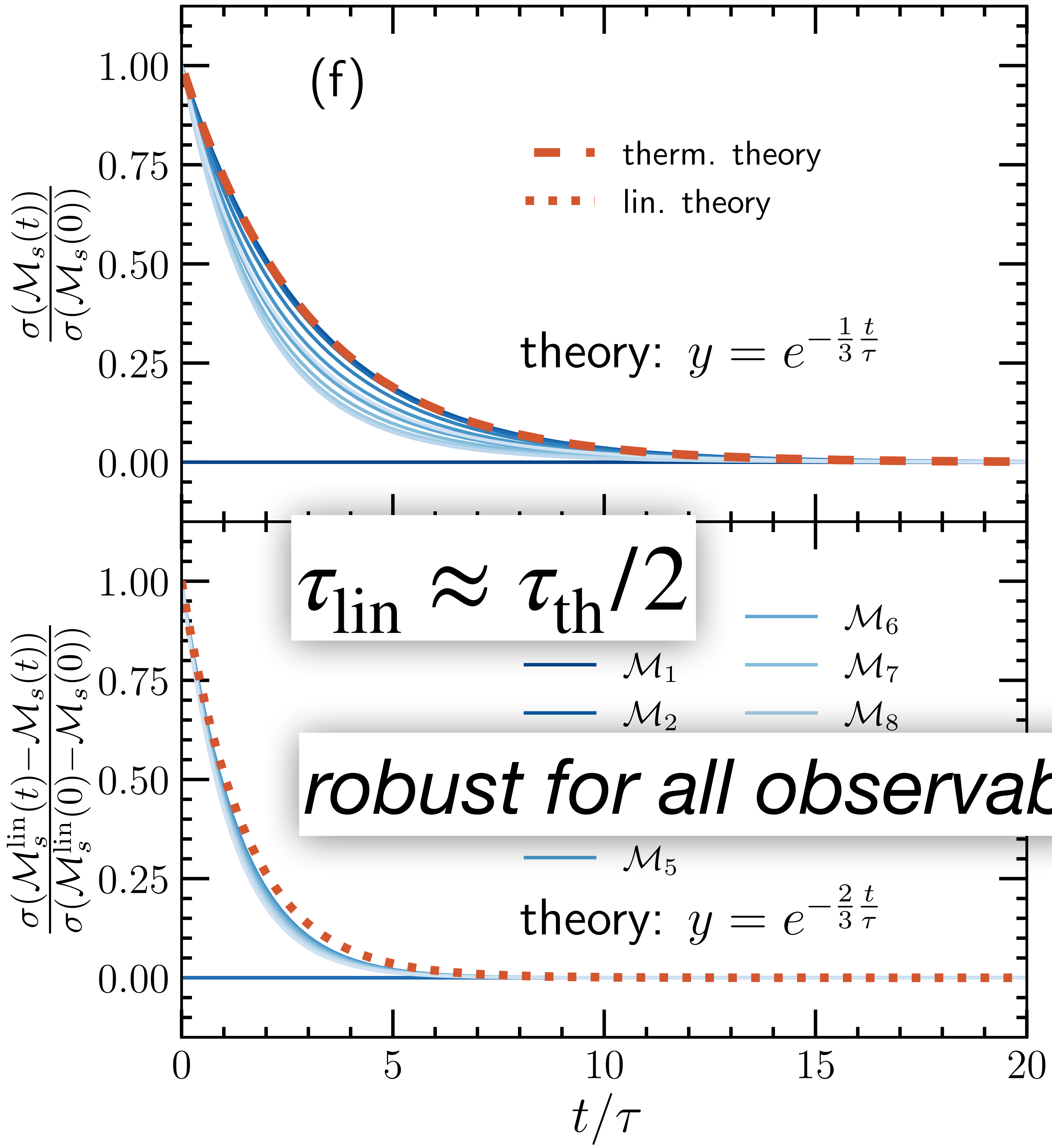
linearization versus thermalization



gray: a random init. state

black: std. dev. of gray

orange: exponential fit



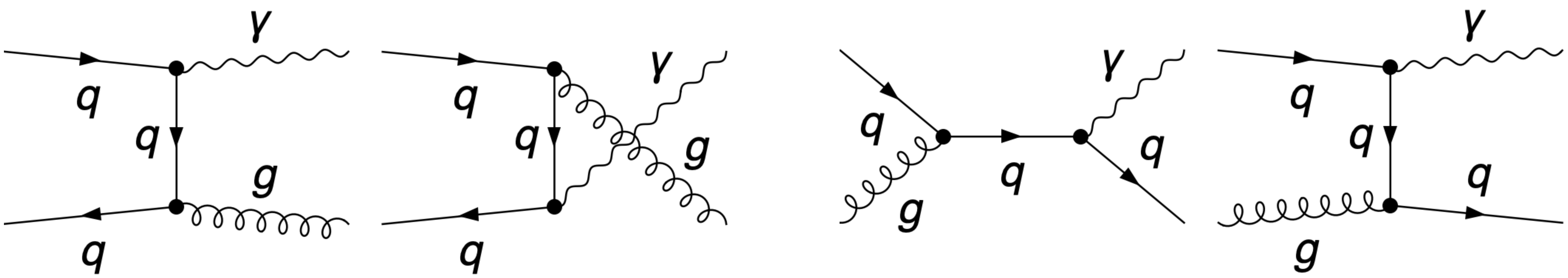
$\tau_{\mathrm{lin}} \approx \tau_{\mathrm{th}}/2$

robust for all observables

$$\mathcal{M}_k \equiv \int_{\boldsymbol{p}} (E_p)^{k+1}\big(f(\boldsymbol{p}) - f_{\mathrm{eq}}(\boldsymbol{p})\big)$$

Xingjian Lu and SS, 2509.23978

how to probe correlations?



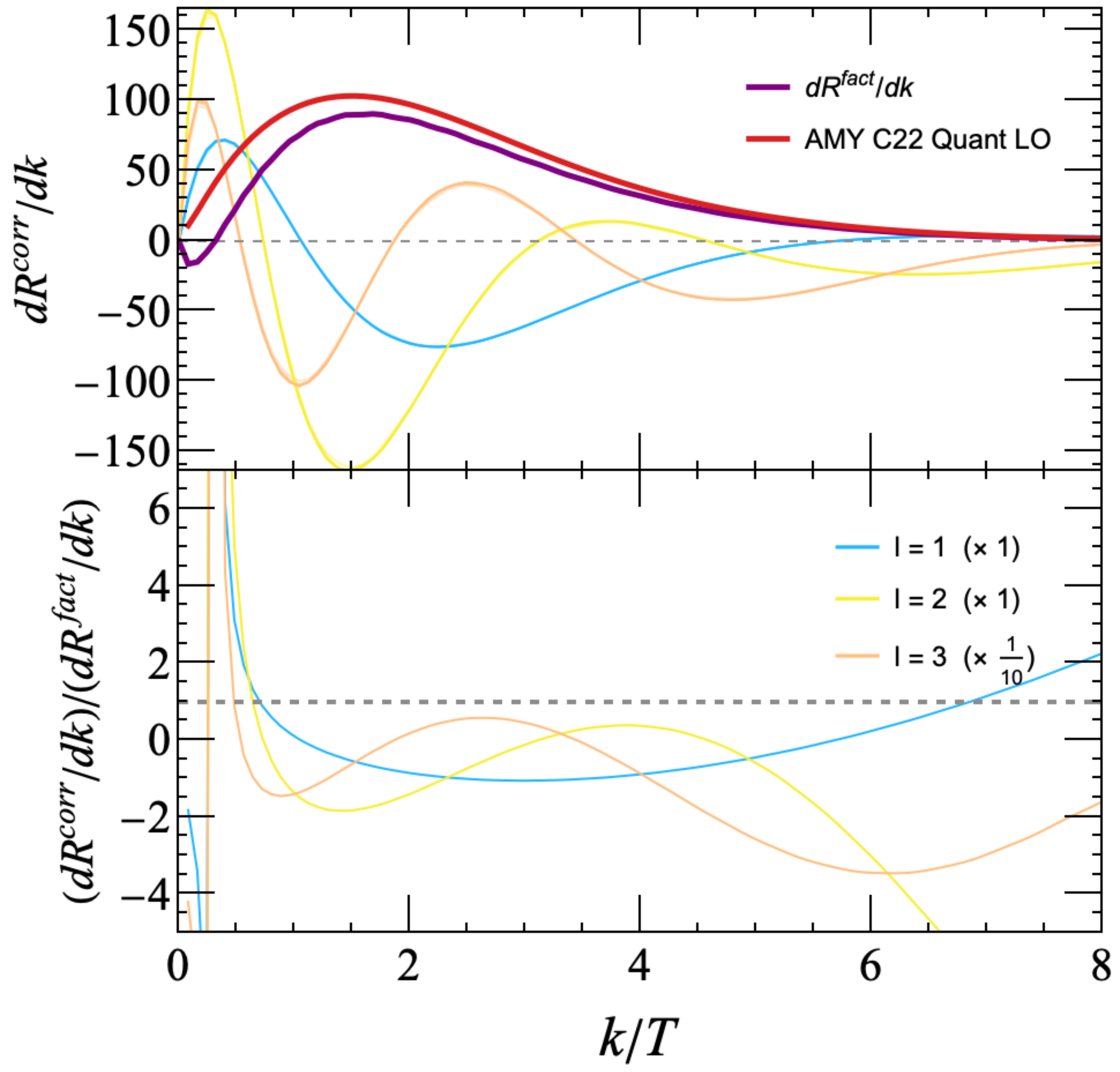
a, b: incoming particles

$$E_k \frac{dR(X)}{d^3k} = \sum_{ab\to c\gamma} \pi \int d\Pi_a\, d\Pi_b\, d\Pi_c\, \delta^{(4)}(p_a + p_b - p_c - k)\mathcal{K}_{ab\to c\gamma}\, f(p_a)f(p_b)$$

$$\to \quad \left(f(p_a)f(p_b) + g(p_a, p_b)\right)$$

Xingjian Lu and SS, 2606.02414

how to probe correlations?



purple: w/o correlation

thin lines: correlation contribution, separated by modes

Correlation can contribute sizably to photon production rate.

Xingjian Lu and SS, 2606.02414

summary

- Established numerical framework for correlation: spectral BBGKY
- Hydrodynamization occurs earlier, but of the same order, than thermalization
- Correlation could be observed in photon production