

Fermionic Hidden Zeros and New Factorizations

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Zeros can be the beginning, not the end

Scattering amplitudes often become interesting only **after identifying what must vanish**.

Low multiplicity

At tree level, the first few multiplicities are highly constrained,

$$A_2^{\text{tree}} = 0,$$

while three-point amplitudes exist only on special complex or split-signature kinematics.

Modern amplitude methods then use three-point amplitudes as basic building blocks for higher-point and even higher-loop amplitudes, through factorization and unitarity cuts.

Helicity sectors

For four-dimensional Yang–Mills,

$$A_n(+, +, \dots, +) = 0,$$

$$A_n(-, +, \dots, +) = 0,$$

while the first rich nonvanishing sector is

MHV.

The MHV amplitude became one of the early “smoking guns” of the modern amplitudes program, leading to many new structures and computational methods.

Zeros often mark the boundary between forbidden and nontrivial amplitude structures.

A third kind of zero

We have already seen two familiar ways in which zeros have played an important role in scattering amplitudes:

low multiplicity and special helicity sectors.

Here is a third possibility.

We do not start by selecting a vanishing helicity sector. Instead, we impose a rectangular condition on the **momenta**, together with bridge conditions on the **external wavefunctions**.

rectangular momentum locus
+ external-wavefunction bridges $\implies A_n = 0$

These zero loci lie

away from ordinary physical factorization poles

of the color-ordered amplitude.

We will study this third kind of zero for amplitudes containing fermions.

Hidden zeros and splitting structures

Over the past few years, several related structures have emerged on special kinematic subspaces.

- **Smooth splitting:** scalar amplitudes reorganize into products of semi-local currents.
- **Hidden zeros:** scalar, pion, gluon and string amplitudes vanish on special rectangular kinematics.
- **2-split and related structures:** special kinematics reorganize amplitudes into lower-point objects.
- **New factorization:** for Yang–Mills, opening the polarization data around a hidden zero reveals a nontrivial factorization without an ordinary physical pole.

The zero appears to be a special limit of a broader amplitude structure on the same kinematic locus.

Cachazo–Early–Giménez Umbert, 2112.14191; Arkani-Hamed et al., 2312.16282; Cao et al., 2403.08855, 2406.03838; Rodina, 2406.04234; Jones–Paranjape, 2505.02520; Guevara–Zhang, 2406.08969; Zhang, 2412.15198.

Hidden zeros already at four points

The simplest examples are visible directly at four points.

Cubic scalar

For the color ordering 1234,

$$A_4^{\phi^3} = \frac{1}{s_{12}} + \frac{1}{s_{23}}.$$

Since

$$s_{12} + s_{13} + s_{23} = 0,$$

setting

$$s_{13} = k_1 \cdot k_3 = 0$$

gives

$$A_4^{\phi^3} = 0.$$

Yang-Mills

The same kinematic condition appears,

$$s_{13} = 0,$$

now supplemented by polarization data,

$$\epsilon_1 \cdot \epsilon_3 = 0,$$

$$\epsilon_1 \cdot k_3 = 0,$$

$$k_1 \cdot \epsilon_3 = 0.$$

Then

$$A_4^{\text{YM}} = 0.$$

Gravity

For factorized graviton polarizations,

$$\varepsilon_{\mu\nu} = \epsilon_\mu \tilde{\epsilon}_\nu,$$

impose the gauge-theory conditions in both copies,

$$\epsilon_1 \cdot \epsilon_3 = 0, \quad \tilde{\epsilon}_1 \cdot \tilde{\epsilon}_3 = 0,$$

$$\epsilon_1 \cdot k_3 = 0, \quad \tilde{\epsilon}_1 \cdot k_3 = 0,$$

$$k_1 \cdot \epsilon_3 = 0, \quad k_1 \cdot \tilde{\epsilon}_3 = 0.$$

Then

$$\mathcal{M}_4 = 0.$$

The same special kinematics can produce zeros across scalar, gauge and gravitational amplitudes.

Opening the zero: a four-point new factorization

For Yang–Mills, first impose only $s_{13} = 0$,
but do **not** impose the polarization conditions.

Then the four-point amplitude does not vanish. Instead it reorganizes into

$$A_4(1, 2, 3, 4)|_{s_{13}=0} = -\frac{1}{s_{12}} \sum_{\lambda} A_3(1, 3, -\hat{3}^{\lambda}) A_3(2, \hat{3}^{\lambda}, 4)$$

with $\hat{k}_3 = k_1 + k_3$, $\hat{k}_3^2 = 0$.

special kinematics $\implies$ factorization without an ordinary factorization pole

If the extra hidden-zero polarization conditions are imposed,

$$A_3(1, 3, -\hat{3}^{\lambda}) = 0 \quad \text{for all } \lambda,$$

and the factorization collapses back to

$$A_4 = 0.$$

The hidden zero is the vanishing limit of a larger structure.

What changes when fermions are introduced?

We consider Yang–Mills theory coupled to massless adjoint real fermions,

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^a F^{a\mu\nu} + \frac{i}{2}\bar{\chi}^a \gamma^\mu D_\mu \chi^a, \quad D_\mu \chi^a = \partial_\mu \chi^a + g f^{abc} A_\mu^b \chi^c.$$

The rectangular geometry itself does not know about particle species.

Once fermions are included, each active pair can have four possible particle assignments,

$ff,$	$fg,$	$gf,$	$gg.$
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Global complexity

$$2f \longrightarrow 4f \longrightarrow 6f \longrightarrow 8f \longrightarrow \dots$$

Spinor contractions, fermion routings, and Clifford structures rapidly proliferate.

Local question

What data live on one cross pair?

Does increasing fermion multiplicity require an increasingly complicated set of local conditions?

Can complicated fermionic amplitudes be controlled by finite local data?

Zhang–Zhu, 2609.06552; Chang–Chen–Huang, 2506.15299.

Four-point example I: a mixed fg bridge

Consider $A_4(1_f, 2_f, 3_g, 4_g)$.

Choose the special kinematics $s_{13} = 0$,

so the active pair is

$$(1_f, 3_g).$$

We use

$$\not{k}_i^{\mu\nu} = k_i^\mu \epsilon_i^\nu - k_i^\nu \epsilon_i^\mu, \quad \not{f}_i \equiv \frac{1}{2} f_{i,\mu\nu} \gamma^{\mu\nu}.$$

The two planar diagrams reorganize into

$$A_4(1_f, 2_f, 3_g, 4_g)|_{s_{13}=0} = \frac{1}{s_{12}} [\chi_1 \not{f}_3 \not{\epsilon}_4 \chi_2 - (k_1 \cdot \epsilon_3) \chi_1 \not{\epsilon}_4 \chi_2].$$

There are therefore two independent structures associated with the active $f_1 g_3$ pair:

$$k_1 \cdot \epsilon_3, \quad \chi_1 \not{f}_3.$$

Hence

$$k_1 \cdot \epsilon_3 = 0, \quad \chi_1 \not{f}_3 = 0 \quad \implies \quad A_4 = 0.$$

Four-point example II: the ff bridge

Now consider the alternating ordering

$$A_4(1_f, 2_g, 3_f, 4_g).$$

Choose

$$\boxed{s_{13} = 0},$$

so that the active pair is

$$(1_f, 3_f).$$

The amplitude becomes

$$A_4|_{s_{13}=0} = \frac{1}{s_{12}} J_{13}^\mu B_\mu,$$

where $\boxed{J_{13}^\mu = \chi_1 \gamma^\mu \chi_3}$

and explicitly $B_\mu = 2(k_1 + k_3) \cdot \epsilon_2 \epsilon_{4\mu} - 2(k_1 + k_3) \cdot \epsilon_4 \epsilon_{2\mu} - (\epsilon_2 \cdot \epsilon_4)(k_2 - k_4)_\mu$.

Therefore

$$\boxed{J_{13}^\mu = 0} \quad \implies \quad A_4 = 0.$$

Four-point example III: the same amplitude also has a gg zero

Keep the same physical amplitude, $A_4(1_f, 2_g, 3_f, 4_g)$,
but choose the other four-point rectangular kinematics,

$$s_{24} = 0.$$

Now the active pair is $(2_g, 4_g)$, rather than $(1_f, 3_f)$.
The corresponding hidden-zero conditions are

$$\epsilon_2 \cdot \epsilon_4 = 0, \quad \epsilon_2 \cdot k_4 = 0, \quad k_2 \cdot \epsilon_4 = 0.$$

Together with $s_{24} = 0$, they imply

$$A_4(1_f, 2_g, 3_f, 4_g) = 0.$$

$$\begin{array}{ll} s_{13} = 0 & \Rightarrow \text{ff bridge,} \\ s_{24} = 0 & \Rightarrow \text{gg bridge.} \end{array}$$

Four-point example IV: four fermions and Fierz routing

Finally consider

$$A_4(1_f, 2_f, 3_f, 4_f).$$

The planar amplitude is

$$A_4 = \frac{J_{12} \cdot J_{34}}{s_{12}} - \frac{J_{23} \cdot J_{41}}{s_{23}}, \quad J_{ij}^\mu = \chi_i \gamma^\mu \chi_j.$$

On

$$s_{13} = 0, \quad s_{23} = -s_{12},$$

this becomes

$$A_4|_{s_{13}=0} = \frac{1}{s_{12}} (J_{12} \cdot J_{34} + J_{23} \cdot J_{41}).$$

Now use the $D = 4, 6, 10$ Fierz identity $J_{12} \cdot J_{34} + J_{13} \cdot J_{42} + J_{14} \cdot J_{23} = 0$.

Therefore $A_4|_{s_{13}=0} = -\frac{1}{s_{12}} J_{13} \cdot J_{24}.$

Hence

$$J_{13}^\mu = 0 \implies A_4 = 0.$$

All four bridges in one rectangle

Consider

$$A_6(1_f, 2_g, 3_g, 4_f, 5_g, 6_g),$$

with

$$L = \{1_f, 2_g\}, \quad R = \{4_f, 5_g\}.$$

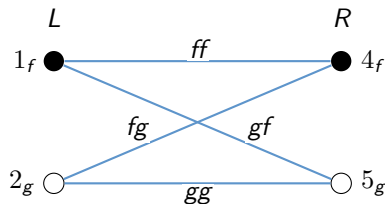
The rectangular kinematics is

$$\begin{array}{l} s_{14} = s_{15} = 0 \\ s_{24} = s_{25} = 0 \end{array}$$

The four cross pairs are

$$\begin{array}{ll} (1_f, 4_f) & : \quad ff, \\ (1_f, 5_g) & : \quad fg, \\ (2_g, 4_f) & : \quad gf, \\ (2_g, 5_g) & : \quad gg. \end{array}$$

Thus a single zero already contains all four local bridge types.



Different particle species simply decorate the same rectangle.

The general rectangular geometry

The four- and six-point examples suggest a common pattern.

Choose two nonadjacent distinguished legs a, b in the cyclic ordering, $\mathbb{I}_n = (a, L, b, R)$, which divide the remaining legs into two ordered sets, $L = L(a, b)$, $R = R(b, a)$.

For every cross pair

$$(i, j) \in L \times R,$$

impose the kinematic condition

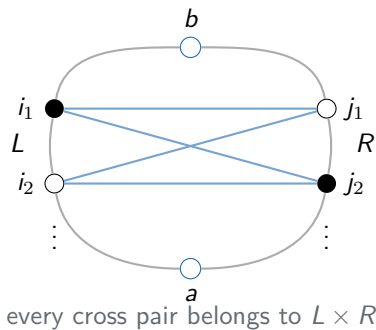
$$s_{ij} = k_i \cdot k_j = 0.$$

The cross pairs therefore form a rectangular set,

$$L \times R.$$

The particle species attached to the vertices may be arbitrary:

$$f, \quad g.$$



The rectangular geometry is universal; particle species only decorate its cross pairs.

Fermionic hidden zeros at arbitrary multiplicity

For each cross pair $(i, j) \in L \times R$, supplement $s_{ij} = 0$ with the bridge determined only by the species of that pair:

$$\begin{array}{l|l} f_i f_j & \chi_i \gamma^\mu \chi_j \\ f_i g_j & k_i \cdot \epsilon_j, \quad \chi_i \not{f}_j \\ g_i f_j & \epsilon_i \cdot k_j, \quad \not{f}_i \chi_j \\ g_i g_j & \epsilon_i \cdot \epsilon_j, \quad \epsilon_i \cdot k_j, \quad k_i \cdot \epsilon_j. \end{array}$$

Collect the corresponding bridge conditions in $x_{ij} = 0$, and define

$$H_{L|R} = \{s_{ij} = 0, x_{ij} = 0 \mid (i, j) \in L \times R\}.$$

Then

$$A_n(\mathbb{I}_n)|_{H_{L|R}} = 0, \quad D = 4, 6, 10.$$

This holds for

arbitrary n , arbitrary even fermion number, arbitrary f/g ordering.

No new primitive bridge appears at higher fermion multiplicity.

Six fermions: a nontrivial field-theory check

The theorem is simple, but the component amplitude is not.

$$A_6(1_f, 2_f, 3_f, 4_f, 5_f, 6_f) \quad \text{contains} \quad \boxed{14 \text{ planar tree graphs}}.$$

The calculation contains

fermion exchange graphs

and two genuine “snow graph” diagrams.

Individual graphs do not vanish on the rectangular locus.

After summing all diagrams,

$$A_6|_{h_m} = \sum_{(i,j) \in L \times R} J_{ij}^\mu B_\mu^{(ij)}.$$

Every term lies in the ideal generated by the vector-current bridges.

Six fermions already require nontrivial Fierz routing, but no new bridge.

Berends–Giele, 1988; Mafra–Schlotterer, 1510.08846; Zhang–Zhu, 2609.06552.

Eight fermions: the local data still do not grow

At eight fermions the raw amplitude becomes much more complicated,

$$A_8(1_f, \dots, 8_f) \quad \text{contains} \quad \boxed{134 \text{ planar contributions}}.$$

Before simplification, one encounters higher Clifford structures,

$$\chi_i \gamma^\mu \chi_j, \quad \chi_i \gamma^{\mu\nu\rho} \chi_j, \quad \chi_i \gamma^{\mu_1 \cdots \mu_5} \chi_j.$$

Nevertheless the full amplitude satisfies

$$\boxed{A_8|_{H_{L|R}} = 0.}$$

Higher-rank Clifford structures appear in intermediate representations of the amplitude, but they do not define new primitive local bridge data.

Global fermionic complexity grows; the primitive local fusion algebra stays finite.

Why does the structure remain finite?

At higher fermion multiplicity, one might have expected new local spinorial data,

$$\chi_i \gamma^\mu \chi_j, \quad \chi_i \gamma^{\mu\nu\rho} \chi_j, \quad \chi_i \gamma^{\mu_1 \cdots \mu_5} \chi_j, \quad \dots$$

but the hidden-zero conditions require only the finite bridge set already seen at four points.

The natural question is therefore not

“How complicated is the full amplitude?”

but rather

“What happens locally to one active cross pair?”

The answer becomes transparent on the worldsheet.

Worksheet view: rectangular kinematics becomes local collisions

The amplitudes of interest admit a scattering-equation representation,

$$A_n = \int d\mu_n \text{PT}(\mathbb{I}_n) I_n.$$

On the regulated rectangular locus,

$$s_{ij} = \tau \hat{s}_{ij}, \quad \tau \rightarrow 0,$$

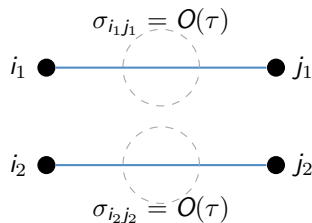
the singular solutions organize into cross-pair pinches,

$$\sigma_{i_\alpha j_\alpha} = O(\tau).$$

For a p -pinch sector,

$$\mathfrak{M}_p = \{(i_\alpha, j_\alpha)\}_{\alpha=1}^p,$$

the active pairs are row- and column-disjoint.



The universal Jacobian behaves as

$$(\det' \Phi)^{-1} = O(\tau^p).$$

The all-multiplicity problem reduces to local collisions of active pairs.

Local fusion: $ff \rightarrow g$

For two Ramond vertex operators,

$$V_{f,i}^{(-1/2)}(z) = \chi_i^\alpha S_\alpha(z) e^{-\phi/2}(z) e^{ik_i \cdot X(z)}.$$

The leading spin-field and superghost OPEs give

$$S_\alpha(z) S_\beta(0) \sim z^{-3/4} (C\gamma^\mu)_{\alpha\beta} \psi_\mu(0),$$

$$e^{-\phi/2}(z) e^{-\phi/2}(0) \sim z^{-1/4} e^{-\phi}(0).$$

Therefore,

$$V_{f,i}^{(-1/2)}(z) V_{f,j}^{(-1/2)}(0) = \frac{1}{2z} (\chi_i \gamma^\mu \chi_j) V_{g,\mu}^{(-1)}(0) + O(1).$$

Hence the bridge

$$\chi_i \gamma^\mu \chi_j = 0$$

removes the complete fermion–fermion principal part.

The ff bridge is exactly the coefficient of the physical fusion $ff \rightarrow g$.

Cohn–Friedan–Qiu–Shenker, 1986; Haertl–Schlotterer, 1011.1249; Belopolsky, hep-th/9706033; Stieberger–Taylor, 0708.0574.

Local fusion: $fg/gf \rightarrow f$

Use a $-\frac{1}{2}$ -picture Ramond vertex and a zero-picture gluon vertex,

$$V_{g,j}^{(0)} = \left[\epsilon_j \cdot P + \frac{1}{2} f_{j,\mu\nu} \psi^\mu \psi^\nu \right] e^{ik_j \cdot X}.$$

For the active pair (i_f, j_g) , the simple pole has two sources:

$$\epsilon_j \cdot P \Rightarrow -(k_i \cdot \epsilon_j) \chi_i.$$

$$\frac{1}{2} f_{j,\mu\nu} \psi^\mu \psi^\nu \Rightarrow \chi_i \not{f}_j.$$

$$\chi_i \not{f}_j - (k_i \cdot \epsilon_j) \chi_i = \sum_l A_3(i_f, j_g, -\hat{j}_f^l) \chi_j^l$$

Thus

$$V_{f,i}^{(-1/2)}(z) V_{g,j}^{(0)}(0) \Big|_{\text{principal}} = \frac{1}{z} \sum_l A_3(i_f, j_g, -\hat{j}_f^l) V_{\hat{j},f}^{(-1/2),l}(0).$$

The two mixed bridge conditions are precisely the components of one physical $fg \rightarrow f$ state.

Local fusion: $gg \rightarrow g$

The zero-picture gluon–gluon OPE contains both simple and double poles,

$$V_{g,i}^{(0)}(z) V_{g,j}^{(0)}(0) \sim \frac{1}{z^2}(\cdots) + \frac{1}{z}(\cdots) + O(1).$$

The double-pole part is

$$\left. V_{g,i}^{(0)}(z) V_{g,j}^{(0)}(0) \right|_{z^{-2}} = -\frac{s_{ij}(\epsilon_i \cdot \epsilon_j)}{z^2} e^{i(k_i+k_j) \cdot X}.$$

It cannot be discarded before taking the correlated limit, because

$$s_{ij} = O(\tau), \quad z = \sigma_{ij} = O(\tau), \quad \frac{s_{ij}}{z^2} = O(\tau^{-1}).$$

Using the local scattering equation,

$$\mathring{E}_i + \frac{s_{ij}}{z} = 0, \quad \frac{s_{ij}}{z^2} \longrightarrow -\frac{\mathring{E}_i}{z},$$

the double pole supplies the missing term in the fused-gluon current

$$\mathcal{E}_{ij}^\mu = (\epsilon_i \cdot \epsilon_j) k_i^\mu + (\epsilon_i \cdot k_j) \epsilon_j^\mu - (k_i \cdot \epsilon_j) \epsilon_i^\mu.$$

The gluon–gluon OPE reconstructs a physical gluon

After combining the genuine simple pole with the regulated double pole,

$$V_{g,i}^{(0)}(z) V_{g,j}^{(0)}(0) \Big|_{\text{principal}} = \frac{1}{z} [\mathcal{E}_{ij} \cdot P_{\text{reg}} + (K \cdot \psi)(\mathcal{E}_{ij} \cdot \psi)] e^{iK \cdot X}.$$

Here

$$K = k_i + k_j, \quad K^2 = 0, \quad K \cdot \mathcal{E}_{ij} = 0$$

on the rectangular locus.

Resolving $\mathcal{E}_{ij}$ on physical gluon polarizations gives

$$\mathcal{E}_{ij} \cdot \epsilon_j^\lambda = A_3(i_g, j_g, -\hat{j}_g^\lambda),$$

and therefore

$$V_{g,i}^{(0)}(z) V_{g,j}^{(0)}(0) \Big|_{\text{principal}} = \frac{1}{z} \sum_{\lambda} A_3(i_g, j_g, -\hat{j}_g^\lambda) V_{\hat{j},g}^{(0),\lambda}(0).$$

The only subtle channel is still an ordinary physical three-gluon state sum.

The local fusion lemma

The three active-pair OPEs all reduce to the same local structure.

$$\mathcal{I}_{2rf,n}^{\text{tree}} \Big|_{(i,j)} = \frac{1}{\sigma_{ij}} \sum_{\eta} A_3(i,j, -\hat{j}^{\eta}) \mathcal{I}_{\text{red},n-1}^{\text{tree}}(\hat{j}^{\eta}, \dots) + O(1).$$

The reduced sector is

$$\mathcal{I}_{\text{red},n-1}^{\text{tree}} = \begin{cases} \mathcal{I}_{(2r-2)f,n-1}^{\text{tree}}, & ff, \\ \mathcal{I}_{2rf,n-1}^{\text{tree}}, & fg/gf, \quad gg. \end{cases}$$

Equivalently,

$$ff \rightarrow g, \quad fg/gf \rightarrow f, \quad gg \rightarrow g.$$

All remaining fermions, gluons and picture-changing operators enter only through the spectator correlator after fusion.

Increasing fermion multiplicity changes the spectators, not the primitive local data.

Completing the proof: the all-multiplicity zero

Consider a p -pinch sector $\mathfrak{M}_p = \{(i_\alpha, j_\alpha)\}_{\alpha=1}^p$, $\sigma_{i_\alpha j_\alpha} = O(\tau)$.

Because the active collision regions are mutually disjoint, the local fusion analysis applies independently to every pair.

After imposing the corresponding bridge conditions,

$$I_n = O(1)$$

throughout the joint degeneration.

At the same time,

$$\text{PT}(\mathbb{I}_n) = O(1), \quad (\det' \Phi)^{-1} = O(\tau^p).$$

Therefore each singular branch contributes

$$A_n^{(\mathfrak{M}_p)} = O(\tau^p) \longrightarrow 0.$$

Every singular branch is suppressed by the same finite local fusion algebra.

Local bridge cancellation plus universal CHY suppression gives the general hidden zero.

Hidden zeros with gravitons and gravitinos

The same local mechanism extends to gravity through the factorized CHY representation,

$$\mathcal{M}_n = \int d\mu_n I_n \tilde{I}_n.$$

For factorized ten-dimensional states, take

$$h: \quad \varepsilon_{\mu\nu} = \epsilon_\mu \tilde{\epsilon}_\nu, \quad \psi_{3/2}: \quad \Psi_{\mu\alpha} = \epsilon_\mu \tilde{\chi}_\alpha, \quad \not\epsilon \tilde{\chi} = 0.$$

Each physical cross pair is seen independently by the two copies:

physical pair	I_n	$\tilde{I}_n$
hh	gg	gg
ψh	gg	fg
$h\psi$	gg	gf
$\psi\psi$	gg	ff

Thus for every cross pair, $s_{ij} = 0, \quad x_{ij} = 0, \quad \tilde{x}_{ij} = 0,$
and the two half-integrands remain regular on every pinch.

$$\mathcal{M}_n(h^{n-2r}, \psi_{3/2}^{2r}) = 0.$$

Single-trace EYM with gluinos and gravitons

A closely related extension is the single-trace EYM sector $A_n^{\text{EYM}}(\ell^r; h^{n-2r})$,

with CHY structure $A_n^{\text{EYM}} = \int d\mu_n \text{PT}(l_c) I_n \tilde{l}_{\text{grav}}(H).$

The two half-integrands see different state content: $f: (\chi, \emptyset), \quad h: (\epsilon, \tilde{\epsilon}).$
Therefore

$$ff, fh, hf, hh$$

in the physical amplitude inherit respectively

$$ff, fg, gf, gg$$

bridges in the fermionic half-integrand I_n .

The second copy only carries the graviton polarization data, so its bridge is the corresponding restricted bosonic one.

The only additional requirement comes from the color trace: $\text{PT}(l_c) = O(1)$
on the rectangular degeneration.

Equivalently, no Parke–Taylor edge should directly connect the two sides of the rectangle.

$$s_{ij} = 0, \quad x_{ij} = 0, \quad \tilde{x}_{ij} = 0 \quad \implies \quad A_n^{\text{EYM}} = 0.$$

The same hidden zero at finite string tension

The mechanism also survives in conventional string theory at finite α' .

For a rectangular partition $L \bowtie R$ a, $k_i \cdot k_j = 0$, $(i, j) \in L \times R$,

removes all cross Koba–Nielsen interactions, while the state-dependent bridge conditions

remove the singular cross OPEs, $\boxed{\text{PP}_{z_i=z_j} [V_i(z_i) V_j(z_j)]_{\text{phys}} = 0.}$

The two blocks are then coupled only through a relative dilation,

$$u_j = \lambda x_j,$$

and the worldsheet form becomes

$$\boxed{\Omega_n|_{\text{hz}} = \frac{d\lambda}{\lambda} \sum_{\rho} \lambda^{\Delta_{\rho}} \hat{\Omega}_{\rho}.}$$

Thus the remaining λ -integrals are scaleless Mellin sectors,

$$\int_0^{\infty} \frac{d\lambda}{\lambda} \lambda^{\Delta_{\rho}} \longrightarrow 0.$$

Hidden zeros arise from removing all direct cross-worldsheet interactions and exposing a relative-scale symmetry.

What if we do not kill the local fusion?

For the hidden zero, we impose

$$x_{ij} = 0$$

and remove every active local fusion channel.

Instead, keep the rectangular Mandelstam conditions

$$s_{ij} = 0, \quad (i, j) \in L \times R,$$

but leave the physical fusion data open.

This gives a new factorization.

Bosonic new factorization

For pure Yang–Mills, on the rectangular Mandelstam locus h_m ,

$$A_n^{\text{YM}}(\mathbb{I}_n)|_{h_m} = \sum_{\substack{i \in L_m \\ j \in R_m}} \mathcal{F}_{m,n}^{\text{YM}}(i,j).$$

Each completed pair contribution takes the form

$$\begin{aligned} \mathcal{F}_{m,n}^{\text{YM}}(i,j) &= \frac{(-1)^{n+1}}{s_{12\dots m+1}} \sum_{\lambda} A_3(i_g, j_g, -\hat{j}_g^{\lambda}) \\ &\quad \times \sum_{\rho \in S_{m-1}} X_m^{(i)}(s, \rho) A_{n-1}^{\text{YM}}(\rho; \hat{j}_g^{\lambda}). \end{aligned}$$

Here $A_{n-1}^{\text{YM}}(\rho; \hat{j}^{\lambda})$ denotes the lower-point amplitude with the ordering specified by ρ .

The kernel $X_m^{(i)}(s, \rho)$ depends on the rectangular kinematics and on the distinguished leg i , but is independent of the partner j and the total multiplicity n .

Guevara–Zhang, 2406.08969; Zhang, 2412.15198.

Fermionic new factorization

Now consider an amplitude with an arbitrary even number $2r$ of fermions, $\mathcal{A}_n^{(2rf)}(\mathbb{I}_n)$.
On the same rectangular Mandelstam locus h_m ,

$$\mathcal{A}_n^{(2rf)}(\mathbb{I}_n) \Big|_{h_m} = \sum_{\substack{i \in L_m \\ j \in R_m}} \mathcal{F}_{m,n}(i,j).$$

The global completion uses the same kernel $X_m^{(i)}$. What changes is only the local physical state produced by the active pair:

$$ff \rightarrow g, \quad fg/gf \rightarrow f, \quad gg \rightarrow g.$$

Accordingly, the lower fermion sector is

active pair	fused state	lower amplitude
ff	g	$(2r-2)f$
fg/gf	f	$2rf$
gg	g	$2rf$

same $X_m^{(i)}$ + species-dependent local fusion

What is inside one completed pair contribution?

For an active pair (i, j) ,

$$\mathcal{F}_{m,n}(i, j) = \frac{(-1)^{n+1}}{s_{12 \dots m+1}} \sum_{\eta} \mathcal{A}_3(i, j, -\hat{j}^{\eta}) \underbrace{\sum_{\rho \in S_{m-1}} X_m^{(i)}(s, \rho) \mathcal{A}_{n-1}^{\text{red}}(\rho; \hat{j}^{\eta})}_{\mathbf{X}_m^{(i)} \cdot \mathbf{A}_{n-1}^{(j, \eta)}}.$$

Thus the permutation sum is naturally a contraction $\boxed{\mathbf{X}_m^{(i)} \cdot \mathbf{A}_{n-1}^{(j, \eta)}}$ between a universal kernel vector and a vector of lower-point amplitudes.

Local fusion

$$\mathcal{A}_3(i, j, -\hat{j}^{\eta})$$

contains all species dependence,
 $ff \rightarrow g, fg/gf \rightarrow f, gg \rightarrow g.$

Universal kernel

$$\mathbf{X}_m^{(i)} = (X_m^{(i)}(s, \rho))_{\rho}$$

depends on m, i and the rectangular kinematics,
 $\boxed{\text{not on } j \text{ or } n}.$

Lower-amplitude vector

$$\mathbf{A}_{n-1}^{(j, \eta)} = (\mathcal{A}_{n-1}^{\text{red}}(\rho; \hat{j}^{\eta}))_{\rho}$$

carries the ordering, remaining particle content, and compatible further pinches.

Completed pair = local physical fusion $\times$ kernel–amplitude contraction.

The four-point amplitude revisited

Return to $A_4(1_f, 2_g, 3_f, 4_g)$, $s_{13} = 0$.

Earlier we wrote

$$A_4|_{s_{13}=0} = \frac{1}{s_{12}} J_{13}^\mu B_\mu.$$

Now the meaning of J_{13}^μ is clear:

$$\frac{1}{2} \chi_1 \gamma^\mu \chi_3 = \sum_\lambda A_3(1_f, 3_f, -\hat{3}_g^\lambda) \epsilon_{\hat{3}}^{\lambda, \mu}.$$

Therefore

$$A_4(1_f, 2_g, 3_f, 4_g)|_{s_{13}=0} = -\frac{1}{s_{12}} \sum_\lambda A_3(1_f, 3_f, -\hat{3}_g^\lambda) A_3(2_g, \hat{3}_g^\lambda, 4_g).$$

The fermion–fermion bridge was not just a condition for a zero.

It was the physical $f\bar{f} \rightarrow g$ state sum hiding inside the amplitude.

All four fusion channels in one factorization

Consider $A_6(1_f, 2_g, 3_g, 4_f, 5_g, 6_g)$ on $s_{14} = s_{15} = s_{24} = s_{25} = 0$.

The four active cross pairs contribute

$$A_6|_{h_2} = \mathcal{F}_{2,6}(1, 4) + \mathcal{F}_{2,6}(1, 5) + \mathcal{F}_{2,6}(2, 4) + \mathcal{F}_{2,6}(2, 5).$$

For example, the ff contribution has the form

$$\mathcal{F}_{2,6}(1, 4) = -\frac{s_{12} + s_{23}}{s_{12} s_{123}} \sum_{\lambda} \mathcal{A}_3(1_f, 4_f, -\hat{4}_g^{\lambda}) \mathcal{A}_5^{\text{YM}}(2_g, 3_g, \hat{4}_g^{\lambda}, 5_g, 6_g)$$

The four channels are

pair	fusion	lower sector
$(1_f, 4_f)$	$ff \rightarrow g$	$0f$
$(1_f, 5_g)$	$fg \rightarrow f$	$2f$
$(2_g, 4_f)$	$gf \rightarrow f$	$2f$
$(2_g, 5_g)$	$gg \rightarrow g$	$2f$

Zhang–Zhu, in preparation.

The factorization holds at arbitrary multiplicity

The same structure extends to arbitrary multiplicity and arbitrary fermion–gluon arrangements.

For every active pair (i, j) ,

$$\mathcal{F}_{m,n}(i, j) = \text{local physical fusion} \times \text{universal rectangular completion}.$$

The complete proof follows the same multi-pinch organization as in pure Yang–Mills.

The fermionic state-dependent factors match across all compatible multi-pinch sectors, while the remaining coefficients obey the same completion relations as in the bosonic theory.

No new fermionic completion identity is required.

All genuinely new fermionic information is local; the global completion is inherited from Yang–Mills.

Zhang, 2412.15198; Zhang–Zhu, in preparation.

The hidden zero is a special limit

The new factorization exists already on the rectangular Mandelstam locus,

$$s_{ij} = 0, \quad (i, j) \in L \times R.$$

There,

$$\mathcal{A}_n^{(2rf)} \Big|_{h_m} = \sum_{i,j} \mathcal{F}_{m,n}(i, j).$$

Now impose the fermionic hidden-zero bridges,

$$x_{ij} = 0 \quad \forall (i, j) \in L \times R.$$

Each local physical state sum vanishes,

$$A_3(i, j, -\hat{j}^\eta) \longrightarrow 0,$$

and hence

$$\boxed{\mathcal{A}_n^{(2rf)} \Big|_{H_{L|R}} = 0.}$$

What have we learned?

The fermionic problem separates into two remarkably simple pieces.

universal rectangular geometry + finite local physical state data

The local data close on

$$ff \rightarrow g, \quad fg/gf \rightarrow f, \quad gg \rightarrow g.$$

This finite algebra controls both

fermionic hidden zeros

and

fermionic new factorizations.

The complicated global amplitude is organized by a remarkably small local structure.

A broader perspective: start from the zero

The logic is reminiscent of familiar amplitude structures.

vanishing helicity sectors $\longrightarrow$ first nontrivial helicity amplitudes
fermionic hidden zeros $\longrightarrow$ fermionic new factorizations

The zero tells us which structures disappear.

The factorization tells us what was present before we switched them off.

find the zero $\longrightarrow$ open the zero $\longrightarrow$ constrain the full amplitude?

Zeros may provide useful boundary data for reconstructing more of the amplitude.

Toward QCD: a single quark line

The same local fusion mechanism survives for a single fundamental quark line.
For an open-compatible ordering,

$$\mathcal{A}_n^{\text{prim}}(1_q, \alpha, n_{\bar{q}}) = \mathcal{A}_n^{\text{adj}}(1_f, \alpha, n_{\bar{f}})$$

up to the common normalization conventions.

At four points, consider

$$\mathcal{A}_4^{\text{prim}}(1_q, 2_g, 3_g, 4_{\bar{q}}).$$

On $s_{13} = 0$,

the active pair $(1_q, 3_g)$ gives

$$\mathcal{A}_4^{\text{prim}}(1_q, 2_g, 3_g, 4_{\bar{q}}) \Big|_{s_{13}=0} = -\frac{1}{s_{12}} \sum_l \mathcal{A}_3(1_q, 3_g, -\hat{3}_q^l) \mathcal{A}_3(2_g, \hat{3}_q^l, 4_{\bar{q}})$$

Thus the local fusion channels become

$$qg \rightarrow q, \quad g\bar{q} \rightarrow \bar{q}, \quad q\bar{q} \rightarrow g, \quad gg \rightarrow g.$$

The local fusion is already the familiar quark–gluon interaction structure.

Massive quarks: the fusion survives

The same structure also survives for simple massive one-line descendants.

Consider $\mathcal{A}_4(1_q^m, 2_g, 3_g, 4_{\bar{q}}^m)$.

For the mixed active pair $(1_q^m, 3_g)$, $s_{13} = 0$,

and the factorization keeps the same form,

$$\mathcal{A}_4(1_q^m, 2_g, 3_g, 4_{\bar{q}}^m)|_{s_{13}=0} = -\frac{1}{s_{12}} \sum_l \mathcal{A}_3(1_q^m, 3_g, -\hat{3}_q^{m,l}) \mathcal{A}_3(2_g, \hat{3}_q^{m,l}, 4_{\bar{q}}^m).$$

Thus $q^m g \rightarrow q^m$ has the same local fusion structure as in the massless case.

The quark–antiquark channel is slightly different:

$$s_{q\bar{q}} = 0 \quad \longrightarrow \quad s_{q\bar{q}} = -m^2.$$

On the shifted locus,

$$q^m \bar{q}^m \rightarrow g$$

again produces a massless gluon.

Mass changes the active-pair kinematics, not the primitive local fusion.

Several quark lines: factorization becomes relation-valued

With more than one fermion line, the local fusion is still simple, but the global organization changes.

A real-adjoint four-fermion amplitude can be resolved into charge-assigned Dirac sectors,

$$4\mathcal{A}_M = \mathcal{A}_4(f, f, \bar{f}, \bar{f}) + \mathcal{A}_4(f, \bar{f}, f, \bar{f}) + \mathcal{A}_4(f, \bar{f}, \bar{f}, f) \\ + \mathcal{A}_4(\bar{f}, f, f, \bar{f}) + \mathcal{A}_4(\bar{f}, f, \bar{f}, f) + \mathcal{A}_4(\bar{f}, \bar{f}, f, f).$$

On the rectangular locus $s_{13} = 0$, the real-adjoint amplitude factorizes as

$$\mathcal{A}_M|_{s_{13}=0} = -\frac{1}{s_{12}} \sum_{\lambda} \mathcal{A}_3(\chi_1, \chi_3, -\hat{3}_g^{\lambda}) \mathcal{A}_3(\chi_2, \hat{3}_g^{\lambda}, \chi_4).$$

Resolving the external fermions into Dirac states turns this into

a linear relation among charge-assigned fermion amplitudes.

Equivalently,

single quark line: individual factorization

while

multiple quark lines: factorization relation.

Outlook

Several directions now become concrete.

Field theory and bootstrap

- Can the completed-pair structure be derived directly from Berends–Giele currents and Ward identities?
- Can locality, factorization and hidden-zero data constrain or determine the full amplitude?
- How does the structure interact with BCJ relations and color–kinematics duality?

Beyond the present setup

- Gravity and gauge–gravity new factorizations
- Finite- α' string amplitudes
- Multi-quark QCD-like sectors
- Loop-level and cosmological extensions

The next question is whether the finite local fusion algebra can organize amplitudes far beyond the present tree-level setting.

Arkani-Hamed et al., 2401.00041, 2408.11891; Cao–Dong–He–Zhu, 2504.21676; Mafra–Schlotterer–Stieberger, 1106.2645, 1106.2646; He–Teng–Zhang, 1812.03369, 1907.06041; Backus–Rodina, 2503.03805.

Fermionic Hidden Zeros and New Factorizations

$$ff \longrightarrow g, \quad fg/gf \longrightarrow f, \quad gg \longrightarrow g$$



local physical fusion + universal rectangular geometry



fermionic new factorization

with fermionic hidden zeros as its vanishing limit

Thank you!

Questions and comments are welcome.