High Energy Quantum Teleportation and Quantum Information Tools for Particle Physics



Yu Shi

yu_shi@ustc.edu.cn

Wilczek Quantum Center, Shanghai Institute for Advanced Studies, USTC

Institute of High Energy Physics, 4/11/2026

70 Years of Parity Violation

PHYSICAL REVIEW VOLUME 104, NUMBER 1 OCTOBER 1, 1956

Question of Parity Conservation in Weak Interactions*

T. D. LEE, *Columbia University, New York, New York*

AND

C. N. YANG,† *Brookhaven National Laboratory, Upton, New York*

(Received June 22, 1956)

The question of parity conservation in β decays and in hyperon and meson decays is examined. Possible experiments are suggested which might test parity conservation in these interactions.

70 Years of Parity Violation

The Nobel Prize in Physics 1957



Photo from the Nobel Foundation archive.

Chen Ning Yang

Prize share: 1/2



Photo from the Nobel Foundation archive.

Tsung-Dao (T.D.) Lee

Prize share: 1/2

The Nobel Prize in Physics 1957 was awarded jointly to Chen Ning Yang and Tsung-Dao (T.D.) Lee "for their penetrating investigation of the so-called parity laws which has led to important discoveries regarding the elementary particles."

70 Years of Parity Violation

Chien-Shiung Wu

Wolf Prize Laureate in Physics 1978

Chien-Shiung Wu

Affiliation at the time of the award:

Columbia University, USA

Award citation:

"for her persistent and successful exploration of the weak interaction which helped establish the precise form and the non conservation of parity for this new natural force."



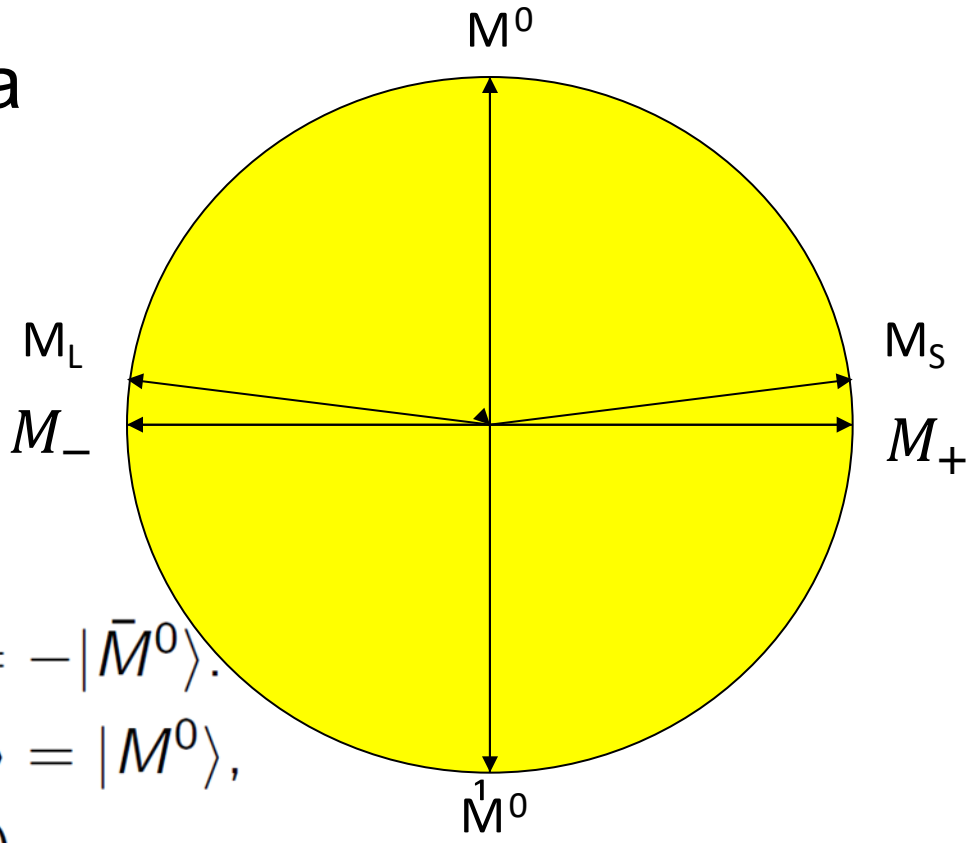
Wu, Yang, Lee: Also Pioneers of Quantum Entanglement in Particle Physics, not using the word “entanglement”

- 1949: Wu-Shaknov experiment on photon pairs from electron-position annihilation.
- 1961: Lee-Yang discussion on correlation between kaon pairs, unpublished, but Lee gave a talk on it.
- 1975: Kasday-Ullman-Wu test of Bell Inequality.
- My historic talks/articles:
  - C S Wu as the experimental pioneer in quantum entanglement: a 2022 note, Mod. Phys. Lett. A 40, 253000 (2025), arXiv:2502.06458
  - Scientific Spirit of C S Wu: From Quantum Entanglement to Parity Nonconservation, Int. Sym. Comm. 110th Birth Ann. of C S Wu, 2022.5.31; transcript:2023.6.2; English: Int J of Mod Phys A, **40**, 2546001 (2025). arXiv:2504.16978(吴健雄的科学精神：从量子纠缠到宇称)
  - C S Wu: From Quantum Entanglement to Parity Nonconservation, Emerging Science and Technology(新兴科学与技术趋势), **3**(3), 225 (2024).
  - Historical origins of quantum entanglement in particle physics, Chinese: Progress in Physics, 2023.3.17; English: Int J Mod Phys A **40**, 2546002 (2025), arXiv:2507.13582
  - The road of quantum entanglement: from Einstein to 2022 Nobel Prize in Physics, Chinese: 2022; English: Int J Mod Phys, arXiv:2602.14601.

Meson in a two dimensional Hilbert space

- Particle and antiparticle form a 2D Hilbert space.
- similar to qubit.

M=K,B,D.



- $\mathcal{F}|M^0\rangle = |M^0\rangle,\ \mathcal{F}|\bar{M}^0\rangle = -|\bar{M}^0\rangle.$
- $CP|M^0\rangle = |\bar{M}^0\rangle,\ CP|\bar{M}^0\rangle = |M^0\rangle,$
- $|M_\pm\rangle = \frac{1}{\sqrt{2}}(|M^0\rangle \pm |\bar{M}^0\rangle).$
- $CP|M_\pm\rangle = \pm|M_\pm\rangle.$

Lee-Yang 1961 unpublished work on entangled neutral kaons

- In a meeting of the ZGS Users Group at Argonne National Lab on 28 May 1960, Lee discussed the possibility of correlated kaons similar to EPR question, resulted from proton-antiproton annihilation.
- A paper by Inglis 1961 includes a chapter on the unpublished work of Lee and Yang, which gave the entangled state of neutral kaons similar to the spin singlet state.
- Lee-Yang noted that it is impossible for the two neutral kaons to be observed as both K0 or both K0-bar at a same time.
- They also calculated the probability that the two particles are observed to be both K0-bar at different moments.

1. Proposal for a High Enegy Quantum Information Process: quantum teleportation in terms of entangled mesons

- YS, Phys. Lett. B 641, 75 (06); 641,492 (2006).
- YS and Y. L. Wu, EPJC 55, 477 (2008).

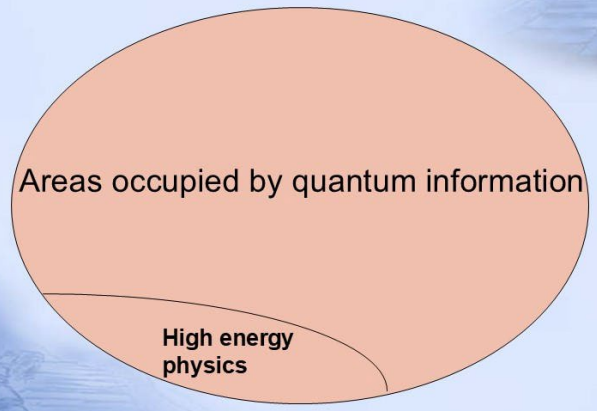
Towards high energy quantum information: quantum teleportation using neutral kaons

Yu Shi
(Fudan University)

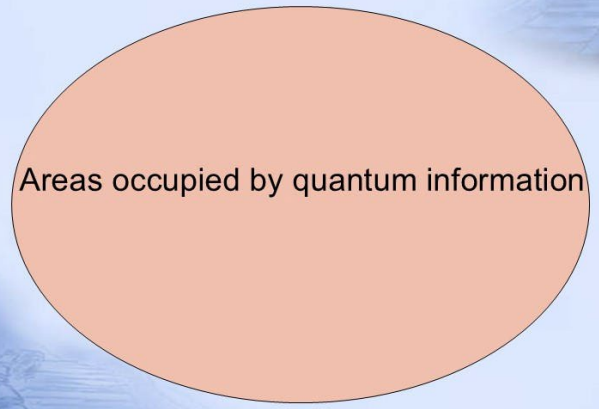
Ref: quant-ph/0605070; appearing in Phys. Lett. B

International Conference on Quantum Foundation and Technology: Frontier and Future, Hangzhou, 27/8/2006

Map of Quantum Physics



New Map of Quantum Physics



高能量子信息初探：中性K介子的超距传态
Towards high energy quantum information: quantum teleportation using neutral kaons

施郁
(复旦大学)

Ref: quant-ph/0605070; Phys. Lett. B 641, 75 (2006)

中国物理学会2006年秋季会议，北京，9.16.

Is quantum information relevant to particle physics?

Yu Shi (施郁)
Fudan University

Institute of Theoretical Physics, Aug. 2007

Is quantum information relevant to particle physics?

Yu Shi (施郁)
Fudan University

Conference in honor of CN Yang's 85th Birthday
Singapore, 2 Nov 2007

521

Is quantum information relevant to particle physics?

Yu Shi
Fudan University, Shanghai, China

Collaborator: Yue-Liang Wu (ITP, Beijing)

- YS, High energy quantum teleportation using neutral Kaons, Phys. Lett. B 641, 75-80 (2006). Erratum, Phys. Lett. B 641, 492 (2006).
- Yu Shi and Yueliang Wu, CP measurement in quantum teleportationof neutral mesons, European Physical Journal C 55, 477 (2008).

PROF. C. N. YANG AND QUANTUM ENTANGLEMENT IN PARTICLE PHYSICS

YU SHI

Department of Physics, Fudan University, Shanghai 200433, China

High Energy Quantum Teleportation and Quantum Information Tools for Particle Physics

QCD, radiation, entanglement and quantum decoherence, Nara, 30 Jan 2008

粒子物理中的量子纠缠与量子信息
Quantum entanglement and quantum information in particle physics

施郁
复旦大学物理系

第十届全国粒子物理会议，南京，2008.4.28.

粒子物理中的量子纠缠与量子信息
Quantum entanglement and quantum information in particle physics

施郁
复旦大学物理系

清华大学，2008.10.23

Conclusion

- Quantum information can be discussed in the setting of particle physics.
- Like that QI stimulates researches on quantum coherence in other areas, similar works could be done for particle physics.
- Such studies could reveal some new features unavailable in nonrelativistic regime, and deep connections between matter and information, provide new insights on particle physics!

Thank you for your attention!

Connecting quantum information to particle physics

施郁
复旦大学物理系

上海大学，2008.12.22

Generation of an EPR pair

- At t=0, a and b is created as

$$
\begin{aligned}
|\Psi_{ab}(0)\rangle &= \frac{1}{\sqrt{2}}(|K^0\rangle_a|\bar{K}^0\rangle_b - |\bar{K}^0\rangle_a|K^0\rangle_b) \\
&= \frac{1}{\sqrt{2}}(|K_2\rangle|K_1\rangle - |K_1\rangle|K_2\rangle) \\
&= \frac{r}{\sqrt{2}}(|K_L\rangle_a|K_S\rangle_b - |K_S\rangle_a|K_L\rangle_b),
\end{aligned}
$$

$$r = (|p|^2 + |q|^2)/2pq = (1 + |\epsilon|^2)/(1 - \epsilon^2)$$

- Decay under weak interaction:

$$|\Psi_{ab}(t)\rangle = M(t)|\Psi_-\rangle_{ab},$$

$$M(t) = \exp[-i(\lambda_S + \lambda_L)\Lambda_b t],\ \Lambda_b = 1/\sqrt{1 - v_b^2}$$

The third koan

- A third koan c is generated at t_z in an unknown state

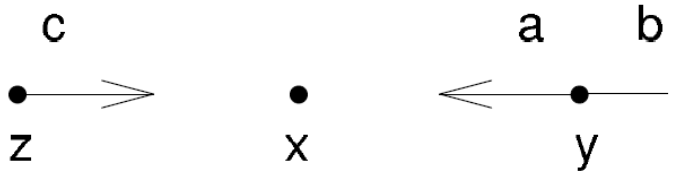
$$|\Psi_c(t_z)\rangle = \alpha|K^0\rangle_c + \beta|\bar{K}^0\rangle_c.$$

- Naturally decays

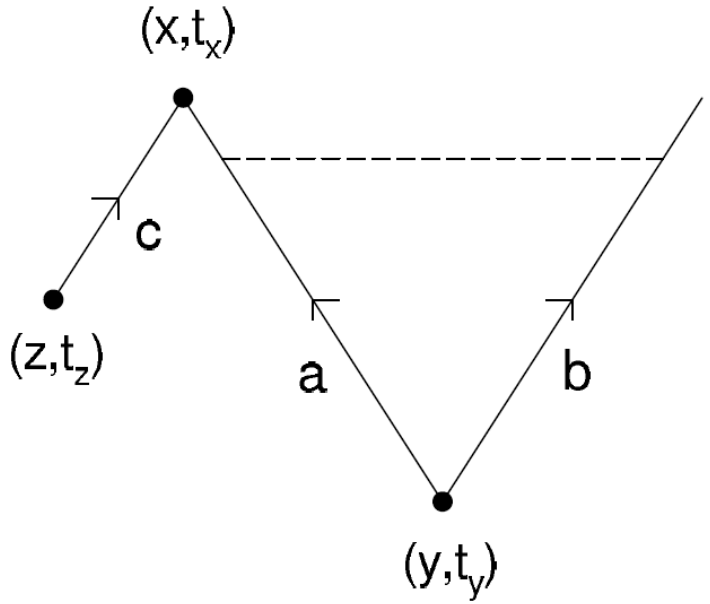
$$|\Psi_c(t)\rangle = F(t)|K^0\rangle_c + G(t)|\bar{K}^0\rangle_c,$$

$$F(t) = [(\alpha + \beta p/q)e^{-i\lambda_S \Lambda_c (t-t_z)} + (\alpha - \beta p/q)e^{-i\lambda_L \Lambda_c (t-t_z)}]/2,$$
$$G(t) = [(\alpha q/p + \beta)e^{-i\lambda_S \Lambda_c (t-t_z)} - (\alpha q/p - \beta)e^{-i\lambda_L \Lambda_c (t-t_z)}]/2.$$

Let c and a collide!



Spacetime diagram:



Eigenstates of P, S and I of c+a

- P, S, I are conserved by strong interaction, which governs c-a collision.

$$
\begin{aligned}
|\phi_1\rangle_{ca} &= |K^0 K^0\rangle \text{ with } P=1,\ S=2,\ I=1;\\
|\phi_2\rangle_{ca} &= |\bar{K}^0 \bar{K}^0\rangle \text{ with } P=1,\ S=-2,\ I=1;\\
|\phi_3\rangle_{ca} &= |\Psi_+\rangle = \tfrac{1}{\sqrt{2}}(|K^0\rangle|\bar{K}^0\rangle + |\bar{K}^0\rangle_c|K^0\rangle), \text{ with } P=1,\ S=0,\ I=1;\\
|\phi_4\rangle_{ca} &= |\Psi_-\rangle = \tfrac{1}{\sqrt{2}}(|K^0\rangle|\bar{K}^0\rangle - |\bar{K}^0\rangle_c|K^0\rangle \text{ with } P=-1,\ S=0,\ I=0.
\end{aligned}
$$

Three-kaon state before collision,decomposed in the strong interaction basis

$$
\begin{aligned}
|\Psi_{cab}(t)\rangle &= |\Psi_c(t)\rangle \otimes |\Psi_{ab}(t)\rangle \\
&= \frac{M(t)}{2}\{\sqrt{2}F(t)|\phi_1\rangle_{ca}|\bar{K}^0\rangle_b \\
&\quad -\sqrt{2}G(t)|\phi_2\rangle_{ca}|K^0\rangle_b \\
&\quad -|\phi_3\rangle_{ca}[F(t)|K^0\rangle_b - G(t)|\bar{K}^0\rangle_b] \\
&\quad -|\phi_4\rangle_{ca}[F(t)|K^0\rangle_b + G(t)|\bar{K}^0\rangle_b]\}.
\end{aligned}
$$

It is not a Bell basis. This basis is physical, while Bell basis is not.

Collision

- The collision effects a unitary transformation $\mathcal{S}$ in a negligible time interval

$$
\begin{aligned}
|\Psi_{cab}(t_x)\rangle \quad = \quad & \frac{M(t_x)}{2}\{\sqrt{2}F(t_x)\mathcal{S}|\phi_1\rangle_{ca}|\bar{K}^0\rangle_b \\
& -\sqrt{2}G(t_x)\mathcal{S}|\phi_2\rangle_{ca}|K^0\rangle_b \\
& -\mathcal{S}|\phi_3\rangle_{ca}[F(t_x)|K^0\rangle_b - G(t_x)|\bar{K}^0\rangle_b] \\
& -\mathcal{S}|\phi_4\rangle_{ca}[F(t_x)|K^0\rangle_b + G(t_x)|\bar{K}^0\rangle_b]\}.
\end{aligned}
$$

- As $\mathcal{S}$ is governed by strong interaction, $\mathcal{S}|\phi_i\rangle_{ca}$ $(i = 1, 2, 3, 4)$ is still an eigenstate of S, P and I, with the same eigenvalues as for $|\phi_i\rangle_{ca}$.

Detection of outgoing particles of c-a collision

- Using strong interaction with nuclear matter, the detection completes Alice's two-particle projection in the basis $\{\mathcal{S}|\phi_i\rangle_{ca}\}$.
- Probabilities:

$$|M(t_x)|^2|F(t_x)|^2/2,$$
$$|M(t_x)|^2|G(t_x)|^2/2,$$
$$|M(t_x)|^2[|F(t_x)|^2+|G(t_x)|^2]/4,$$
$$|M(t_x)|^2[|F(t_x)|^2+|G(t_x)|^2]/4.$$

- Corresponding projected state of b:

$$|\bar{K}^0\rangle_b,\ |K^0\rangle_b,\ [F(t_x)|K^0\rangle_b - G(t_x)|\bar{K}^0\rangle_b]/\sqrt{|F(t_x)|^2+|G(t_x)|^2},$$
$$[F(t_x)|K^0\rangle_b + G(t_x)|\bar{K}^0\rangle_b]/\sqrt{|F(t_x)|^2+|G(t_x)|^2}.$$

- Decay affects the probabilities.

Adopt a stochastic strategy

- This is because it is hard to implement subsequent unitary transformation on b.
- Bob chooses to retain or abandon b particle, based on the projection result of c-a.
- Teleportation of $F(t_x)|K^0\rangle_b + G(t_x)|\bar{K}^0\rangle_b$ is made if projection result of c-a is $\mathcal{S}|\Psi_-\rangle_{ca}$.

Verification scheme (1)

- Different projection results lead to different values of strangeness ratio ξ.
- For $|\Psi(t)\rangle_b = f(t)|K^0\rangle_b + g(t)|\bar{K}^0\rangle_b$, $\xi(t) = |f(t)|^2/g(t)|^2$.
- No projection/teleportation: $\xi(t) = 1.$
- Successful teleportation:

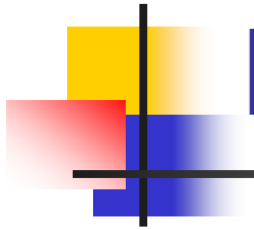
$$\xi(t) = \frac{|F(t_x)(e^{-\Gamma_S \tau/2} + e^{-\Gamma_L \tau/2}) + G(t_x)(e^{-\Gamma_S \tau/2} - e^{-\Gamma_L \tau/2})|^2}{|F(t_x)(e^{-\Gamma_S \tau/2} - e^{-\Gamma_L \tau/2}) + G(t_x)(e^{-\Gamma_S \tau/2} + e^{-\Gamma_L \tau/2})|^2},$$
$$\tau = \Lambda_b(t - t_x - 0).$$

very different from 1

Verification scheme (2)

- Different projection results lead to different values of CP ratio .
- For $|\Psi(t)\rangle_b = u_1(t)|K_1\rangle_b + u_2(t)|K_2\rangle_b$, $\zeta(t) = |u_1(t)|^2/u_2(t)|^2$.
- No projection/teleportation: $\zeta(t) = 1$.
- Successful teleportation: significantly different from 1.
- Advantage 1: valid no matter whether CP is violated.
- Advantage 2: easy experimental implementation, using non-leptonic decays (CP=1: decay to 2 pions; CP=-1: decay to 3 pions).

Another informational process: Entanglement swapping



- A and B are entangled, C and D are entangled.
- A and C are subject to a measurement (projection).
- Then B and D become entangled, though they never meet.
- Entangling partners are swapped.

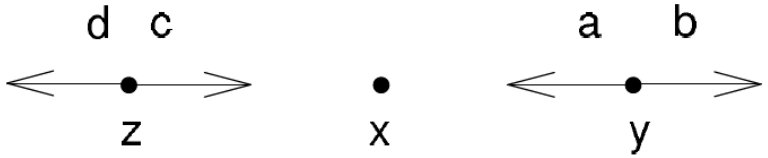
Preparation

In addition to $|\Psi_-\rangle_{ab}$ generated at $t = 0$,
another kaon pair d and c is generated as $|\Psi_-\rangle_{dc}$ at time t_z.
$|\Psi_{dc}(t)\rangle = M'(t - t_z)|\Psi_-\rangle_{dc}$,
$M'(t - t_z) = \exp[-i(\lambda_S + \lambda_L)\Lambda_d(t - t_z)]$.

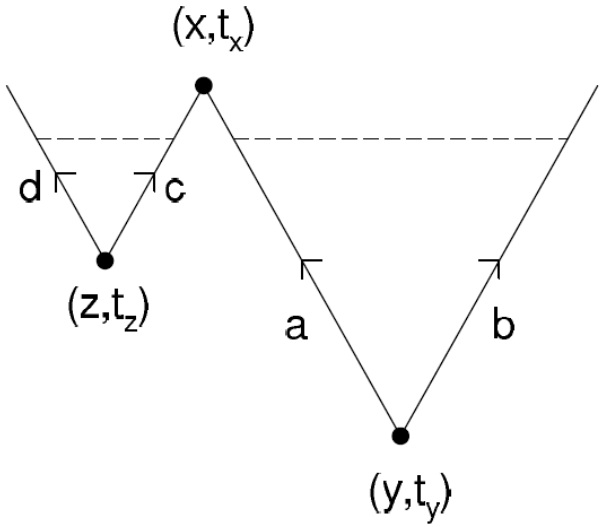
$$
\begin{aligned}
|\Psi_{dcab}(t)\rangle &= M'(t - t_z)M(t)|\Psi_-\rangle_{dc}|\Psi_-\rangle_{ab} \\
&= \frac{M'(t-t_z)M(t)}{2}(|\Psi_+\rangle_{ca}|\Psi_+\rangle_{db} - |\Psi_-\rangle_{ca}|\Psi_-\rangle_{db} \\
&\quad -|K^0K^0\rangle_{ca}|\bar{K}^0\bar{K}^0\rangle_{db} - |\bar{K}^0\bar{K}^0\rangle_{ca}|K^0K^0\rangle_{db}).
\end{aligned}
$$

Collision

Let c and a collide at t_x



Spacetime diagram:



Detection

Collision:

$$|\Psi_{dcab}(t_x+0)\rangle = \frac{M'(t_x-t_z)M(t_x)}{2}(\mathcal{S}|\Psi_+\rangle_{ca}|\Psi_+\rangle_{db} - \mathcal{S}|\Psi_-\rangle_{ca}|\Psi_-\rangle_{db} - \mathcal{S}|K^0K^0\rangle_{ca}|\bar{K}^0\bar{K}^0\rangle_{db} - \mathcal{S}|\bar{K}^0\bar{K}^0\rangle_{ca}|K^0K^0\rangle_{db}).$$

In detecting outgoing particles from the collision, c and a are projected to: $\mathcal{S}|\Psi_+\rangle_{ca}$, $\mathcal{S}|\Psi_-\rangle_{ca}$, $\mathcal{S}|K^0K^0\rangle_{ca}$ or $\mathcal{S}|\bar{K}^0\bar{K}^0\rangle_{ca}$. Correspondingly d and b are projected to: $|\Psi_+\rangle_{ca}$, $|\Psi_-\rangle_{ca}$, $|K^0K^0\rangle_{ca}$ and $|\bar{K}^0\bar{K}^0\rangle_{ca}$, respectively, each with probability $|M'(t_x-t_z)M(t_x)|^2/4$. .

The projection result is revealed by P, S and I of the outcomes of $c-a$ collision, according to which b and d are retained or abandoned.

Verification scheme (1)

- Measure the S asymmetry of b and d

$$A(t) = [p_{diff}(t) - p_{same}(t)]/[p_{diff}(t) + p_{same}(t)]$$

$p_{diff}(t)$ $(p_{same}(t))$:
probability to have different (same) strangeness values

- Many runs are needed.
- No entanglement swapping (all runs are considered): A(t)=0
- Entanglement swapping succeeds (consider those runs in which c-a project to $\mathcal{S}|\Psi_-\rangle_{ca}$: A(t)=1

Verification scheme (2)

- Measure the CP asymmetry of b and d

$$A(t) = [p_{diff}(t) - p_{same}(t)]/[p_{diff}(t) + p_{same}(t)]$$

$p_{diff}(t)$ $(p_{same}(t))$:
probability to have different (same) values of CP.

- Many runs (events) are needed.
- No entanglement swapping (all runs are considered): A(t)=-1
- Entanglement swapping succeeds (consider those runs in which c-a project to $\mathcal{S}|\Psi_-\rangle_{ca}$: A(t)=1

Question

- Can we do single-copy quantum systems in high energy experiments?
- Quantum weirdness mostly comes from such a circumstance.

2. Using Quantum Entanglement to Study CP and CPT Violations

- Z. Huang, YS, Euro. Phys. J. C 72, 1900 (2012).
- YS, Euro. Phys. J. C 73, 2506 (2013).
- Z. Huang, YS, Phys. Rev. D 89, 016018 (2014).

Time evolution and joint decay of $|\Psi_{\mp}\rangle$

- $$|\Psi_{\mp}(t_a, t_b)\rangle = \frac{1}{\sqrt{2}}(|M^0(t_a)\rangle_a|\bar{M}^0(t_b)\rangle_b \mp |\bar{M}^0(t_a)\rangle_a|M^0(t_b)\rangle_b)$$
- For state $|\Psi_{\mp}(t_a, t_b)\rangle$, the joint rate that Alice decays to $|\psi_a\rangle$ at t_a while Bob decays to $|\psi_b\rangle$ at t_b is

$$I(\psi_a, t_a; \psi_b, t_b) = |\langle\psi_a, \psi_b|\mathcal{H}_a\mathcal{H}_b|\Psi_{\mp}(t_a, t_b)\rangle|^2.$$

$$\begin{aligned}\langle\psi_a, \psi_b|\mathcal{H}_a\mathcal{H}_b|\Psi_{\mp}(t_a, t_b)\rangle & \\ = & \frac{1}{\sqrt{2}}(\langle\psi_a|\mathcal{H}_a|M^0(t_a)\rangle\langle\psi_b|\mathcal{H}_b|\bar{M}_0(t_b)\rangle \\ - & \langle\psi_a|\mathcal{H}_a|\bar{M}_0(t_a)\rangle\langle\psi_b|\mathcal{H}_b|M^0(t_b)\rangle).\end{aligned}$$

Integrated rates and asymmetries

Integrated rate

$$I'(\psi^1, \psi^2, \Delta t) = \int_0^\infty I(\psi^1, t_a; \psi^2, t_a + \Delta t) dt_a,$$

which is simply $I(\psi^1, t_a; \psi^2, t_a + \Delta t)$ with $e^{-(\Gamma_S + \Gamma_L)t_a}$ replaced as $1/(\Gamma_S + \Gamma_L)$.

$$A(\psi^1\psi^2, \psi^3\psi^4, \Delta t) \equiv \frac{I'(\psi^1, \psi^2, \Delta t) - I'(\psi^3, \psi^4, \Delta t)}{I'(\psi^1, \psi^2, \Delta t) + I'(\psi^3, \psi^4, \Delta t)}.$$

We use notation $A_\mp$ for $|\Psi_\mp\rangle$.

Theorem 1: For $\Delta t = 0$, any unequal-state asymmetry $A_\mp(\psi^a\psi^b, \psi^b\psi^a; \Delta t = 0)$ always vanishes no matter whether there is CP or CPT violation.

Could be used to confirm the entanglement of the pair.

Semileptonic decays of $|\Psi_-\rangle$ into flavor eigenstates $|l^{\pm}\rangle$

- Examples for $|l^+\rangle$:
  $M^- \bar{l}\nu$, $D^- D_s^+$, $D^- K^+$, $\pi^- D_s^+$, $\pi^- K^+$ from B^0;
  $D_s^- \pi^+$, $D_s^- D^+$, $K^- \pi^+$, $K^- D^+$ from B_S^0.
- Examples for $|l^-\rangle$:
  $M^+ l\bar{\nu}$, $D^+ D_s^-$, $D^+ K^-$, $\pi^+ D_s^-$, $\pi^+ K^-$ from $\bar{B}^0$;
  $D_s^+ \pi^-$, $D_s^+ D^-$, $K^+ \pi^-$, $K^+ D^-$ from $\bar{B}_s^0$.
- Define:
  $$R^+ \equiv \langle l^+|\mathcal{H}|M^0\rangle = a + b,$$
  $$R^- \equiv \langle l^-|\mathcal{H}|\bar{M}^0\rangle = a^* - b^*,$$
  $$S^+ \equiv \langle l^+|\mathcal{H}|\bar{M}_0\rangle = c^* - d^*,$$
  $$S^- \equiv \langle l^-|\mathcal{H}|M_0\rangle = c + d.$$
  a, b, c, d: quantities defined in literature.
  - Direct CP conservation $\Longrightarrow R^+ = R^-$ and $S^+ = S^-$.
  - Direct CPT conservation $\Longrightarrow (R^+)^* = R^-$ and $(S^+)^* = S^-$.
  - $\Delta\mathcal{F} = \Delta Q$ rule $\Longrightarrow S^{\pm} = 0$.

Equal-flavor asymmetry for $|\Psi_-\rangle$

$$A_-(++,--,\Delta t) = \frac{|x_S(S^+)^2 - x_L^{-1}(R^+)^2 + (1-\Omega)R^+S^+|^2 - |x_S(R^-)^2 - x_L^{-1}(S^-)^2 + (1-\Omega)R^-S^-|^2}{|x_S(S^+)^2 - x_L^{-1}(R^+)^2 + (1-\Omega)R^+S^+|^2 + |x_S(R^-)^2 - x_L^{-1}(S^-)^2 + (1-\Omega)R^-S^-|^2}.$$

- **Theorem 2:** $A_-(++,--,\Delta t)$ is always a constant independent of Δt.
- **Theorem 3:** *If $A_-(++,--,\Delta t) \neq 0$, then there exists one or two of the following violations: (1) CP is violated indirectly, (2) both CP and CPT are violated directly.*
- **Theorem 4:** *If $A_-(++,--,\Delta t) \neq 0$ while CPT is assumed to be conserved both directly ($(R^+)^* = R^-$, $(S^+)^* = S_-$) and indirectly ($x_S = x_L = q/p$, $\Omega = 1$), then in addition to indirect CP violation ($q/p \neq 1$), we can draw the following conclusions:*
  (1) $|q/p| \neq 1$, i.e. T must also be violated indirectly;
  (2) $|R^\pm| \neq |S^\pm|$.

Unequal-flavor asymmetry for $|\Psi_-\rangle$

$$A_-(+-,-+,\Delta t) = \frac{(|U|^2-|V|^2)(e^{-\Gamma_L\Delta t}-e^{-\Gamma_S\Delta t})+4\Im(U^*V)\sin(\Delta m\Delta t)}{(|U|^2+|V|^2)(e^{-\Gamma_L\Delta t}+e^{-\Gamma_S\Delta t})+4\Re(U^*V)\cos(\Delta m\Delta t)}.$$

- **Theorem 5** $A_-(+-,-+,\Delta t = 0) = 0$.
- **Theorem 6** *If* $A_-(+-,-+,\Delta t) \neq 0$, *then CP must be violated, directly or indirectly or both.*
- **Theorem 7** *If* $A_-(+-,-+,\Delta t) \neq 0$ *for* $\Delta t \neq 0$ *while CPT is assumed to be conserved both directly and indirectly, then we can draw the following conclusions:*
  (1) $|R^+| = |R^-| \neq |S^+| = |S^-|$;
  (2) $S^{\pm} \neq 0$, *i.e.* $\Delta\mathcal{F} = \Delta Q$ *rule must be violated.*

Hadronic decays of $|\Psi_-\rangle$ into CP eigenstates $|h^\pm\rangle$

- Examples of $|h^+\rangle$: $\pi^+\pi^-$, $\pi^0\pi^0$.
- Examples of $|h^-\rangle$: $\pi^0\pi^0\pi^0$.
- Define:
  $Q^\pm \equiv \langle h^\pm|\mathcal{H}|M_\pm\rangle$,
  $X^\pm \equiv \langle h^\pm|\mathcal{H}|M_\mp\rangle$,
  $\xi_\pm \equiv \frac{X^\pm}{Q^\pm}$.
- They are related to $\eta's$ usually defined:
  $$\eta_{h^+} \equiv \frac{\langle h^+|\mathcal{H}|M_L\rangle}{\langle h^+|\mathcal{H}|M_S\rangle} = \frac{\xi^+ + \epsilon_L}{1+\epsilon_S\xi^+},$$
  $$\eta_{h^-} \equiv \frac{\langle h^-|\mathcal{H}|M_S\rangle}{\langle h^-|\mathcal{H}|M_L\rangle} = \frac{\xi^- + \epsilon_S}{1+\epsilon_L\xi^-}.$$
  - If CP is conserved directly $\Rightarrow X^\pm = 0$.
  - If CPT is conserved directly $\Rightarrow X^\pm$ is purely imaginary, i.e. $X^\pm = -X^{\pm *}$.
- One can, of course, calculate using basis $|M^0\rangle$ and $|\bar{M}^0\rangle$, rather than $M_\pm\rangle$.

Equal-CP asymmetry for $|\Psi_-\rangle$

Asymmetry:

$$B_-(h_a^x h_b^y, h_a^y h_b^x, \Delta t) = \frac{I[h_a^x,t_a;h_b^y,t_a+\Delta t]-I[h_a^y,t_a;h_b^x,t_a+\Delta t]}{I[h_a^x,t_a;h_b^y,t_a+\Delta t]+I[h_a^y,t_a;h_b^x,t_a+\Delta t]} = \frac{I'[h_a^x,h_b^y,\Delta t]-I'[h_a^y,h_b^x,\Delta t]}{I'[h_a^x,h_b^y,\Delta t]+I'[h_a^y,h_b^x,\Delta t]}.$$

Equal-CP asymmetry: $B_-(++,--,\Delta t) = \frac{P(Q^+,X^+)-P(X^-,Q^-)}{P(Q^+,X^+)+P(X^-,Q^-)}$,

$P(\beta,\gamma) \equiv$

$|-(1-x_L^{-1}+x_S-\Omega)\beta^2+2(x_L^{-1}+x_S)\beta\gamma+(1+x_L^{-1}-x_S-\Omega)\gamma^2|^2.$

Theorem 9: *$B_-(++,--,\Delta t)$ is always a constant independent of Δt.*

Simple results concerning joint decays of $|\Psi_+\rangle$

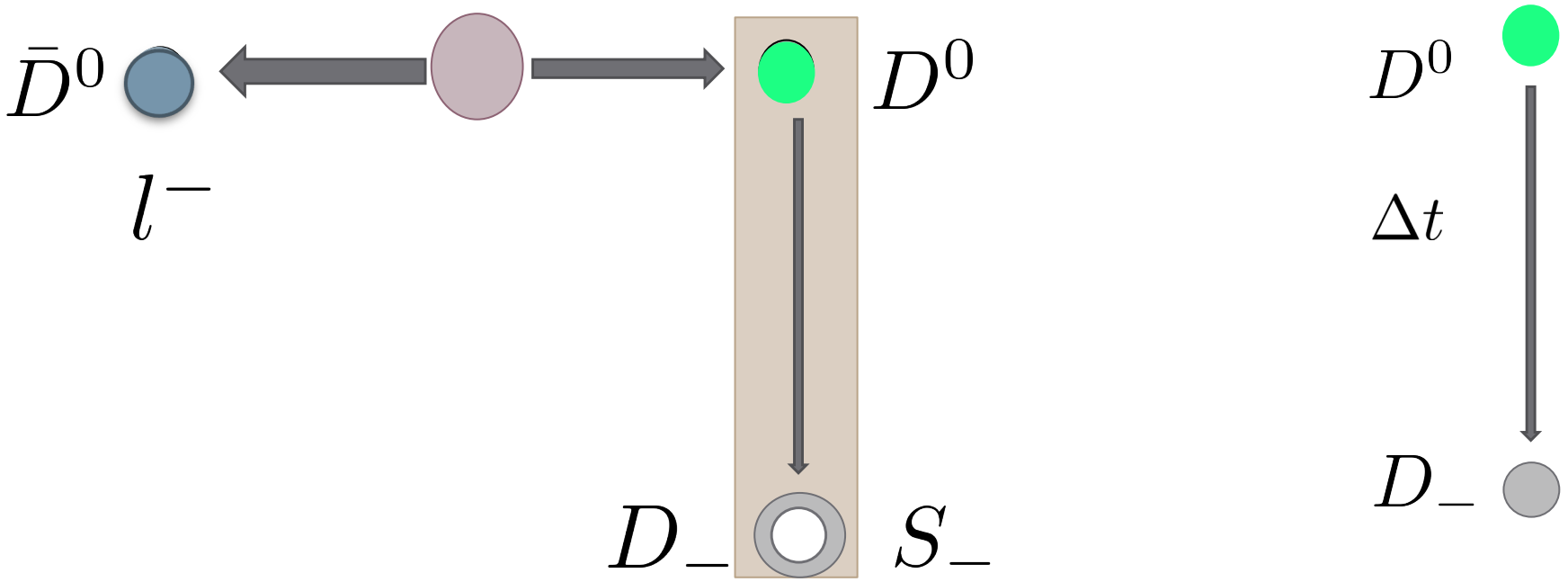
We have also calculated joint decays of $|\Psi_+\rangle$. Here are some simple results:

- **Theorem 10:** If $A_+(l^+l^-, l^-l^+; \Delta t)$ is nonzero, then CP must be violated indirectly.
- **Theorem 11:** If $A_+(l^+l^-, l^-l^+; \Delta t)$ depends on the first order of ϵ_M, then CP must be violated directly and the $\Delta\mathcal{F} = \Delta Q$ rule must be violated.
- **Theorem 12:** The deviation of $I_+(h^+, t_a; h^-, t_a)$ or $I_+(h^-, t_a; h^+, t_a)$ from zero implies CP violation, direct or indirect or both. [Immediately implied by $|\Psi_+\rangle = \frac{1}{\sqrt{2}}(|M_+\rangle|M_+\rangle - |M_-\rangle|M_-\rangle)$.]
- **Theorem 13:** If $I_+(h^+, t_a; h^-, t_a)$ or $I_+(h^-, t_a; h^+, t_a)$ or both are of the order of $O(\Delta_M)$ and $O(\epsilon_M)$, then CP is violated directly.

3. Genuine T Violation Signal in terms of entangled Dmesons

YS, JC Yang, Time reversal symmetry violation in entangled pseudoscalar neutral charmed mesons, Phys. Rev. D 98, 075079 (2018).

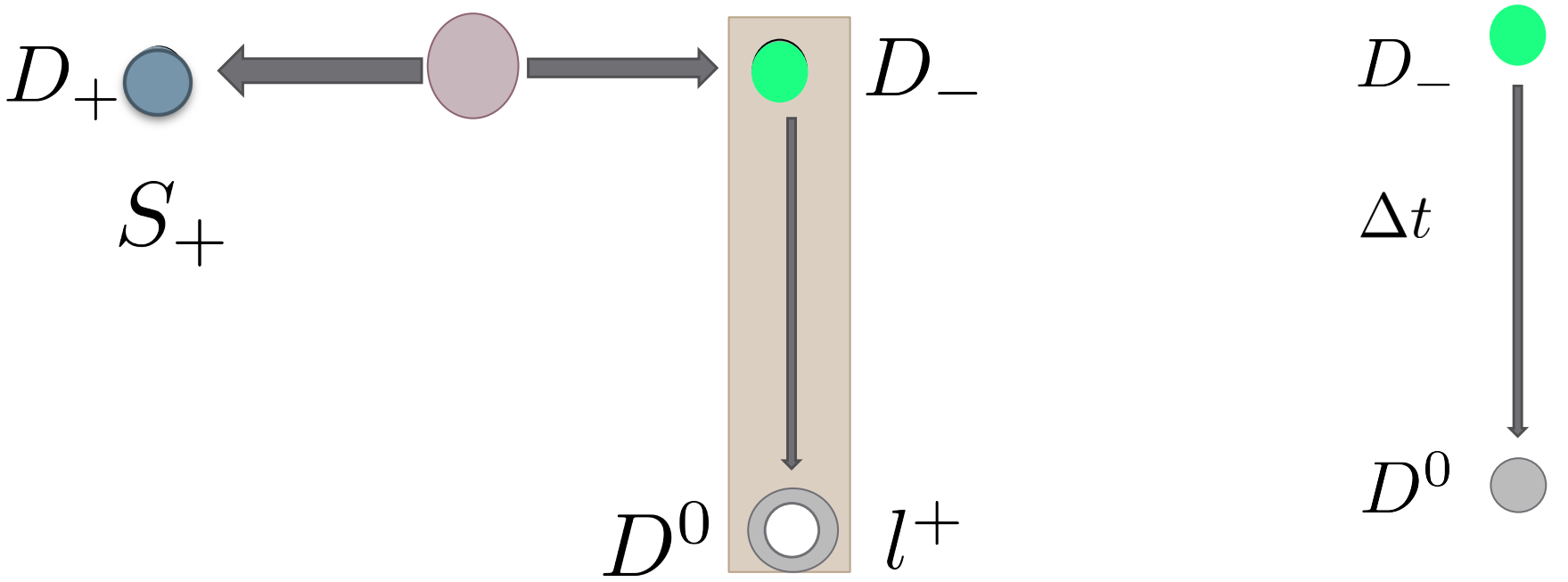
T-conjugate processes from C=-1 entangled state (1)



$$|\Psi_-\rangle = |M^0\rangle \otimes |\bar{M}^0\rangle - |\bar{M}^0\rangle \otimes |M^0\rangle$$

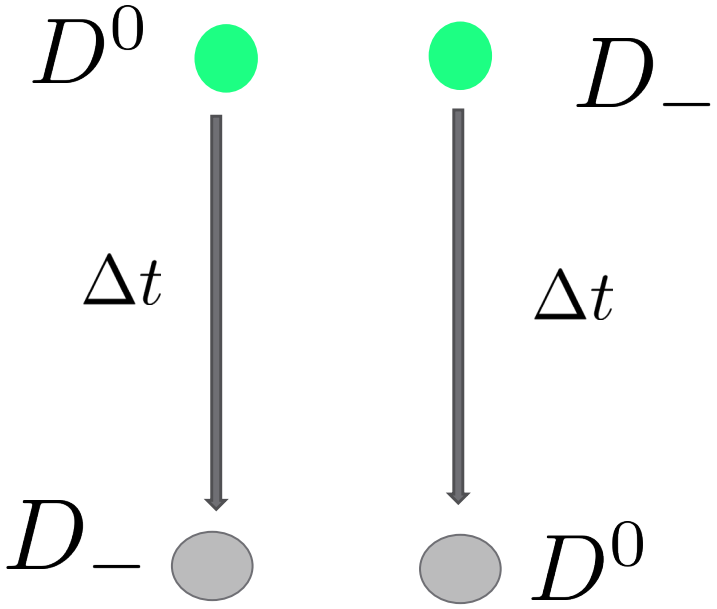
Yu Shi, Quantum entanglement in particle physics

T-conjugate processes from C=-1 entangled state (2)



$$|\Psi_-\rangle = |M_+\rangle \otimes |M_-\rangle - |M_-\rangle \otimes |M_-$$

T-conjugate processes from C=-1 entangled state



Transition rates can be calculated from joint decay rates of the entangled state.

transition of meson b	final state of meson a	final state of meson b	T-conjugate transition	final state of meson a	final state of meson b
$D^0 \to D_-$	l^-	S_-	$D_- \to D^0$	S_+	l^+
$D^0 \to D_+$	l^-	S_+	$D_+ \to D^0$	S_-	l^+
$\bar{D}^0 \to D_-$	l^+	S_-	$D_- \to \bar{D}^0$	S_+	l^-
$\bar{D}^0 \to D_+$	l^+	S_+	$D_+ \to \bar{D}^0$	S_-	l^-

T-violation signals

$$A_-^1(\Delta t) = \frac{R_-(l^-, S_-, \Delta t) - R_-(S_+, l^+, \Delta t)}{R_-(l^-, S_-, \Delta t) + R_-(S_+, l^+, \Delta t)},$$

$$A_-^2(\Delta t) = \frac{R_-(l^-, S_+, \Delta t) - R_-(S_-, l^+, \Delta t)}{R_-(l^-, S_+, \Delta t) + R_-(S_-, l^+, \Delta t)},$$

$$A_-^3(\Delta t) = \frac{R_-(l^+, S_-, \Delta t) - R_-(S_+, l^-, \Delta t)}{R_-(l^+, S_-, \Delta t) + R_-(S_+, l^-, \Delta t)},$$

$$A_-^4(\Delta t) = \frac{R_-(l^+, S_+, \Delta t) - R_-(S_-, l^-, \Delta t)}{R_-(l^+, S_+, \Delta t) + R_-(S_-, l^-, \Delta t)}.$$

$$\hat{A}_-^1 = \frac{R_-(l^-, S_-) - R_-(l^+, S_+)}{R_-(l^-, S_-) + R_-(l^+, S_+)}, \quad \hat{A}_-^2 = \frac{R_-(l^-, S_+) - R_-(l^+, S_-)}{R_-(l^-, S_+) + R_-(l^+, S_-)}.$$

Results within Standard model

- Calculations come down to a few parameters:

$$x \equiv \frac{\Delta m}{\Gamma} = 0.0037, \quad y \equiv \frac{\Delta \Gamma}{2\Gamma} = 0.0066,$$
$$|q/p| = 0.91, \quad arg(q/p)/2 = 0.91.$$

- The time-dependent T-asymmetries for $\Delta t = 1/\Gamma$, and time-independent T-asymmetries are found to be of the order of 10^{-5}

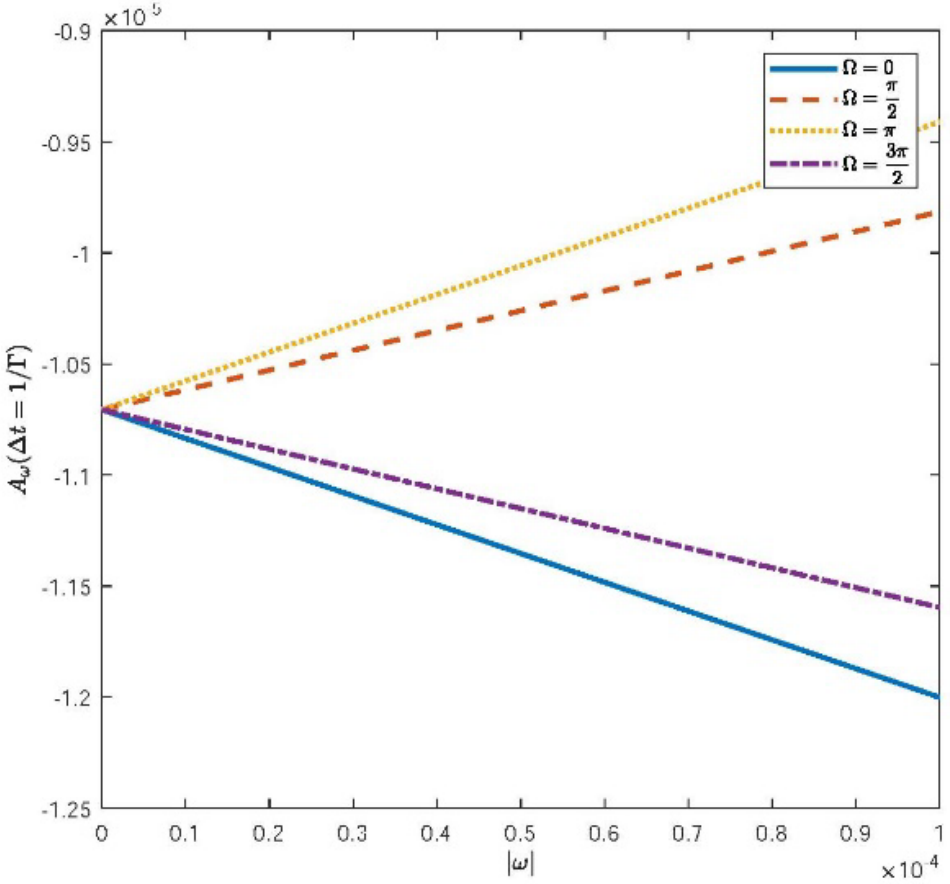
CPT violation

- Local quantum field theory (Lorentz invariance, local interaction,unitarity) ⟹ CPT.
- In some quantum gravity theories, because of objects unaccessible to low energy observers, CPT may be violated.

- Such CPTV leads to "omega effect"(Bernabeu, Ellis, Mavromatos, etc.)

$$|\Psi_\omega\rangle = |\Psi_-\rangle + \omega|\Psi_+\rangle$$

T-violating signals in presence of omega effect



$$\omega \equiv |\omega| \exp i\Omega.$$

For $|\omega| = 10^{-4}$, T-violation signals in D system are changed as large as 27%

4. General discussions on entanglement properties of quantum field theory

PHYSICAL REVIEW D, VOLUME 70, 105001

Entanglement in relativistic quantum field theory

Yu Shi

Center for Advanced Study, Tsinghua University, Beijing 100084, China
Department of Physics, University of Illinois at Urbana-Champaign, Urbana, Illinois 61891, USA
(Received 24 May 2004; published 2 November 2004)

I present some general ideas about quantum entanglement in relativistic quantum field theory, especially entanglement in the physical vacuum. Here, entanglement is defined between different single particle states (or modes), parametrized either by energy-momentum together with internal degrees of freedom, or by spacetime coordinate together with the component index in the case of a vector or spinor field. In this approach, the notion of entanglement between different spacetime points can be established. Some entanglement properties are obtained as constraints from symmetries, e.g., under Lorentz transformation, space inversion, time reverse, and charge conjugation.

Eur. Phys. J. Plus (2021) 136:907
https://doi.org/10.1140/epjp/s13360-021-01902-1

THE EUROPEAN PHYSICAL JOURNAL PLUS

Regular Article

Entanglement and quantum tomography with top quarks at the LHC

Yoav Afik[1,a], **Juan Ramón Muñoz de Nova**[2,b]

[1] Experimental Physics Department, CERN, 1211 Geneva, Switzerland
[2] Departamento de Física de Materiales, Universidad Complutense de Madrid, 28040 Madrid, Spain

The standard model (SM) of particle physics is a quantum field theory, based on special relativity and quantum mechanics. Therefore, it allows to test fundamental properties of quantum mechanics. For instance, entanglement has been studied in the context of particle physics [17–20] and is currently a hot topic of research in the field [21–25].

18. Yu. Shi, Entanglement in relativistic quantum field theory. Phys. Rev. D **70**, 105001 (2004)
21. Yu. Shi, J.-C. Yang, Entangled baryons: violation of inequalities based on local realism assuming dependence of decays on hidden variables. Eur. Phys. J. C **80**(2), 116 (2020)

5. Entangled mesons violate a generalized Leggett Inequality，in which decays also depend on hidden variables

- YS and JC Yang, Particle physics violating crypto-nonlocal realism, European Physical Journal C 80, 861 (2020).

Crypto-nonlocal realism and Leggett Inequalities

- BI Violation: Which should be abandoned, locality or realism?
- Leggett consider crypto-nonlocal realism and derived Leggett Inequality (violated by QM) :
  - The physical state is a statistical mixture of sub-ensembles of various definite polarization directions.
  - With the hidden variables, the measured quantity also depends on the polarizer axis on the other side (non-local).
  - Local HVT can be regarded as a special case of this kind of local HVT.
  - For each sub-ensemble, the measured quantity obeys Malus Law (which is local) for the average of the hidden parameters.

$$\bar{A}(\mathbf{u}) = \int \mathrm{d}\lambda \rho_{\mathbf{u},\mathbf{v}}(\lambda) A(\mathbf{a},\mathbf{b},\lambda) = \mathbf{u}\cdot\mathbf{a}$$

Yu Shi, Quantum entanglement in particle physics

Using high energy particles for Bell/Leggett tests

- Relativistic, massive, involving strong, weak and electromagnetic interactions.
- Decays are similar to measurements.
- However, the decay modes, products, times are possibly dependent on the hidden variables at the source, thus the decays of different particles may be correlated.
- It is a kind of loophole of measurement setup.

Our work

- Assume that the measurement setting possibly depends on hidden variables： decay (analogous to a rotating polaroid) is determined by the hidden variables at the particle source.
- Generalized crypto-nonlocal realism, Leggett inequalities, local realism, Bell inequalities.

Assume the measurement setup also depends on hidden variables

Average over distribution of nonlocal hidden variables:

$$\int d\lambda \rho'_{\mathbf{u},\mathbf{v},\mathbf{a},\mathbf{b}}(\lambda) A(\mathbf{u},\mathbf{v},\tilde{\mathbf{a}}(\lambda),\tilde{\mathbf{b}}(\lambda),\lambda) = \bar{A}(\mathbf{u},\mathbf{a}) = \mathbf{u}\cdot\mathbf{a},$$

$$\int d\lambda \rho'_{\mathbf{u},\mathbf{v},\mathbf{a},\mathbf{b}}(\lambda) B(\mathbf{v},\mathbf{u},\tilde{\mathbf{b}}(\lambda),\tilde{\mathbf{a}}(\lambda),\lambda) = \bar{B}(\mathbf{v},\mathbf{b}) = \mathbf{v}\cdot\mathbf{b}.$$

$$\overline{AB}(\mathbf{u},\mathbf{v},\mathbf{a},\mathbf{b}) = \int d\lambda \rho'_{\mathbf{u},\mathbf{v},\mathbf{a},\mathbf{b}}(\lambda) A(\mathbf{u},\mathbf{v},\tilde{\mathbf{a}}(\lambda),\tilde{\mathbf{b}}(\lambda),\lambda) B(\mathbf{v},\mathbf{u},\tilde{\mathbf{b}}(\lambda),\tilde{\mathbf{a}}(\lambda),\lambda),$$

where $\rho'_{\mathbf{u},\mathbf{v},\mathbf{a},\mathbf{b}}(\lambda) \equiv \rho_{\mathbf{u},\mathbf{v}}(\lambda)\delta(\tilde{\mathbf{a}}(\lambda)-\mathbf{a})\delta(\tilde{\mathbf{b}}(\lambda)-\mathbf{b})$ is a shorthand.

Statistical average over polarizations:

$$E(\mathbf{a},\mathbf{b}) = \int d\mathbf{u}d\mathbf{v} F(\mathbf{u},\mathbf{v}) \overline{AB}(\mathbf{u},\mathbf{v},\mathbf{a},\mathbf{b}),$$

Measuring after evolution, equivalent to measurement in a time-dependent basis, like a rotating polaroid

$$U(\theta_{\mathbf{a}}, \rho_{\mathbf{a}}) = \left(\cos \frac{\theta_{\mathbf{a}}}{2} - i \sin \frac{\theta_{\mathbf{a}}}{2} \left(\cos(\rho_{\mathbf{a}}) \sigma^x + \sin(\rho_{\mathbf{a}}) \sigma^y \right) \right). \tag{10}$$

Suppose that following the evolution $U(\theta_{\mathbf{a}}, \rho_{\mathbf{a}})$, a signal A is recorded as $A = +1$ if $|0\rangle$ is detected, while $A = -1$ if $|1\rangle$ is detected. The QM expectation value of A is

$$\bar{A}(\mathbf{u}) = \frac{|\langle 0|U|\mathbf{u}\rangle|^2 - |\langle 1|U|\mathbf{u}\rangle|^2}{|\langle 0|U|\mathbf{u}\rangle|^2 + |\langle 1|U|\mathbf{u}\rangle|^2} = \mathbf{u} \cdot \mathbf{a}, \tag{11}$$

Semileptonic decay on the flavor basis, satisfies Malus Law, thus the measurement "direction" can be determined.

For a B_d meson, the measurement in the flavor basis $\{|B^0\rangle, |\bar{B}^0\rangle\}$, corresponding to $A_l = \pm 1$, can be made in the semileptonic decay channel, as the direct CP violation or wrong sign decay is negligible [58],

$$\bar{A}_l(\mathbf{u}) = \frac{|\langle B^0|U(t)|\mathbf{u}\rangle|^2 - |\langle \bar{B}^0|U(t)|\mathbf{u}\rangle|^2}{|\langle B^0|U(t)|\mathbf{u}\rangle|^2 + |\langle \bar{B}^0|U(t)|\mathbf{u}\rangle|^2} = \mathbf{u}\cdot\mathbf{a}^l(t), \tag{12}$$

where $\mathbf{a}^l(t) = (\sin(2\beta)\sin(x\Gamma t), -\cos(2\beta)\sin(x\Gamma t), \cos(x\Gamma t))$.

Use B meson, as the decay width are close to each other.
Indirect CP violation effects in terms of phase β。
xΓ is the difference of mass-decay width.

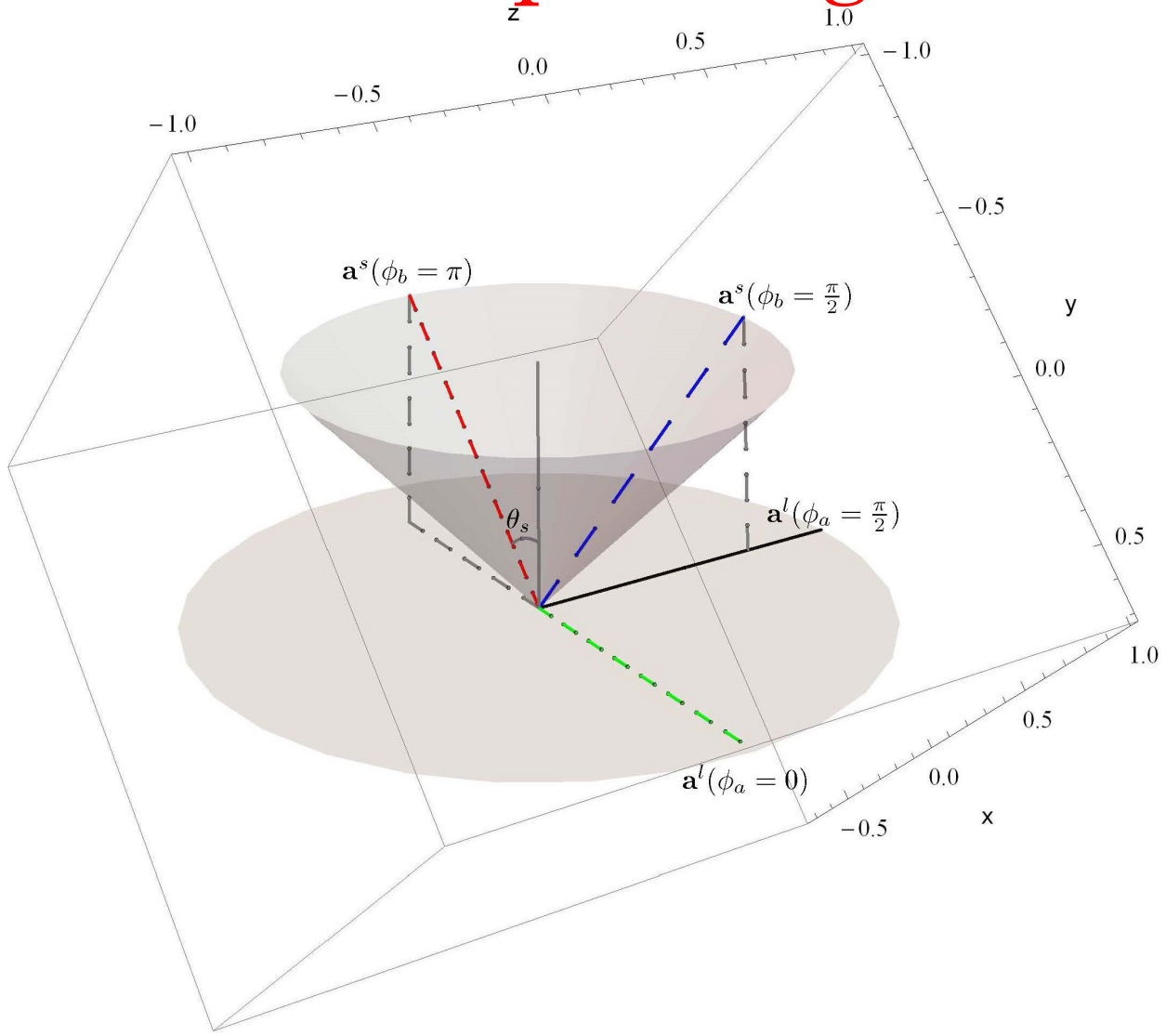
Decay on CP basis also satisfies Malus Law, from this the measuring "direction" can be determined

Likewise, observing the decay product to be CP eigenstates $S_\pm$ effectively measures the meson to be $|B_\pm\rangle \equiv \left(|B^0\rangle \pm |\bar{B}^0\rangle\right)/\sqrt{2}$, as the direct CP violation is negligible [58]. With $B_\pm$ corresponding to $A_s = \pm 1$,

$$\bar{A}_s(\mathbf{u}) = \frac{|\langle B_+|U(t)|\mathbf{u}\rangle|^2 - |\langle B_-|U(t)|\mathbf{u}\rangle|^2}{|\langle B_+|U(t)|\mathbf{u}\rangle|^2 + |\langle B_-|U(t)|\mathbf{u}\rangle|^2} = \mathbf{u}\cdot\mathbf{a}^s(t), \tag{13}$$

where $\mathbf{a}^s(t) = \left(\sin^2(2\beta)\cos(x\Gamma t) + \cos^2(2\beta), \sin(4\beta)\sin^2(x\Gamma t/2), -\sin(2\beta)\sin(x\Gamma t)\right)$.

The direction of “polaroid” corresponding to the decay



$\theta_s = 2\beta$

FIG. 1: The effective measuring directions $\mathbf{a}^l$ and $\mathbf{a}^s$. In a certain coordinate system, $\mathbf{a}^l(\phi_l)$ is on xy plane, $\mathbf{a}^s(\theta_s, \phi_s)$ is on a cone. For B_d mesons, $\phi_l = x\Gamma t$,

High Energy Quantum Teleportation and Quantum Information Tools for Particle Physics

Correlation function

We first consider correlation functions of various combinations of $\mathbf{a}^l$ and $\mathbf{a}^s$. Define $\hat{E}^{\pm}(\mathbf{a},\mathbf{b}) \equiv E(\mathbf{a},\mathbf{b}) + E(\mathbf{b},\pm\mathbf{b})$, and rewrite $\hat{E}^{\pm}(\mathbf{a}^s(\theta_s,\phi_a),\mathbf{a}^l(\phi_b))$ as $\hat{E}^{\pm}_{sl}(\theta_s,\xi,\varphi)$, where $\xi \equiv (\phi_a+\phi_b)/2$, $\varphi \equiv \phi_a - \phi_b$. $\hat{E}^{\pm}_{ll}(\theta_s,\xi,\varphi)$ and $\hat{E}^{+}_{ss}(\theta_s,\xi,\varphi)$ are similarly defined. Furthermore, we consider the averages over ξ, $\hat{E}^{-}_{sl}(\theta_s,\varphi) \equiv \int \frac{d\xi}{2\pi}\hat{E}^{-}_{sl}(\theta_s,\xi,\varphi)$ and so on.

Upper bound

$$\hat{E}^{-}_{sl}(\theta_s,\varphi_1)+\frac{\pi\cos(\theta_1(\theta_s,\varphi_1))L_1(\theta_s,\varphi_1)}{4\cos(\frac{\varphi_2}{2})}\hat{E}^{-}_{ll}(\theta_s,\varphi_2)$$
$$\leq 2\left(1+\frac{\pi\cos(\theta_1(\theta_s,\varphi_1))L_1(\theta_s,\varphi_1)}{4\cos(\frac{\varphi_2}{2})}\right)-\cos(\theta_1(\theta_s,\varphi_1))L_1(\theta_s,\varphi_1), \quad (26)$$

where $L_1(\theta_s,\varphi)\equiv|\mathbf{a}^s+\mathbf{a}^l|=\sqrt{2+2\cos(\varphi)\sin(\theta_s)}$, $\theta_1(\theta_s,\varphi)=\cos^{-1}\frac{\cos(\theta_s)}{\sqrt{2+2\cos(\varphi)\sin(\theta_s)}}$. With $0<\theta_s<\pi/2$, we have $\sin(\theta_1)>0$, $\cos(\theta_1)>0$.

Lower bound

We find two lower bounds. The first is given as

$$\hat{E}^{+}_{sl}(\theta_s,\varphi_1)+\frac{\pi\cos(\theta_2(\theta_s,\varphi_1))L_2(\theta_s,\varphi_1)}{4\left|\sin(\frac{\varphi_2}{2})\right|}\hat{E}^{+}_{ll}(\theta_s,\varphi_2)$$
$$\geq -2\left(1+\frac{\pi\cos(\theta_2(\theta_s,\varphi_1))L_2(\theta_s,\varphi_1)}{4\left|\sin(\frac{\varphi_2}{2})\right|}\right)+\cos(\theta_2(\theta_s,\varphi_1))L_2(\theta_s,\varphi_1).$$

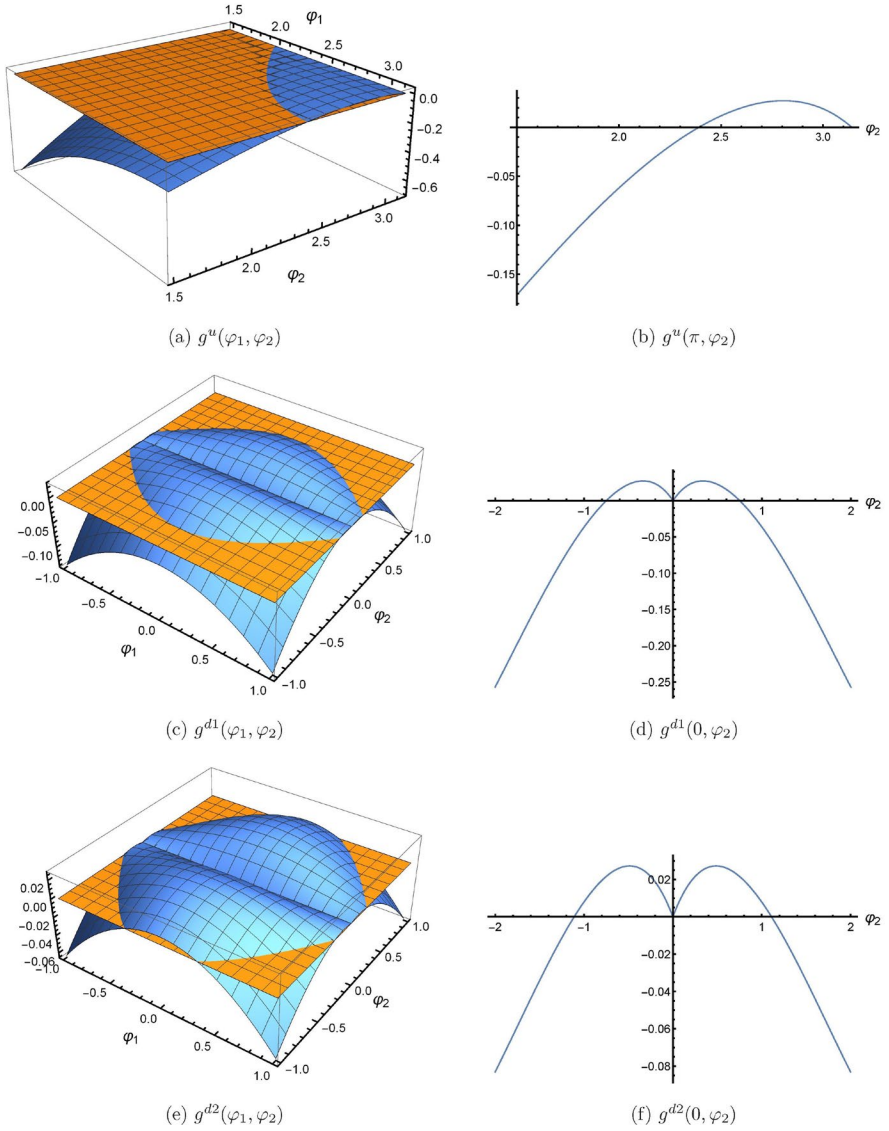
where $L_2(\theta_s,\varphi)=\sqrt{2-2\cos(\varphi)\sin(\theta_s)}$, $\theta_2(\theta_s,\varphi)=\cos^{-1}\frac{\cos(\theta_s)}{\sqrt{2-2\cos(\varphi)\sin(\theta_s)}}$.

The second lower bound is given as

$$\hat{E}^{+}_{sl}(\theta_s,\varphi_1)+\frac{\pi\cos(\theta_2(\theta_s,\varphi_1))L_2(\theta_s,\varphi_1)}{4\sin(\theta_s)\left|\sin(\frac{\varphi_2}{2})\right|}\hat{E}^{+}_{ss}(\varphi_2)$$
$$\geq -2\left(1+\frac{\pi\cos(\theta_2(\theta_s,\varphi_1))L_2(\theta_s,\varphi_1)}{4\sin(\theta_s)\left|\sin(\frac{\varphi_2}{2})\right|}\right)+\cos(\theta_2)L_2(\theta_s,\varphi_1).$$

Li is violated standard model of particle physics

- g=(LHS−RHS)/|LHS|。
- condition： θs≠0 (CP violation)
- Consistent with the real world (CP is indeed violated)!
- QM (LI violation) is consistent with CP violation.



6. Generalized Bell inequalities for hyperon entanglement (decay also depends on hidden variables)

YS and JC Yang, Entangled baryons: violation of inequalities based on local realism assuming dependence of decays on hidden variables, European Physical Journal C 80, 116 (2020).

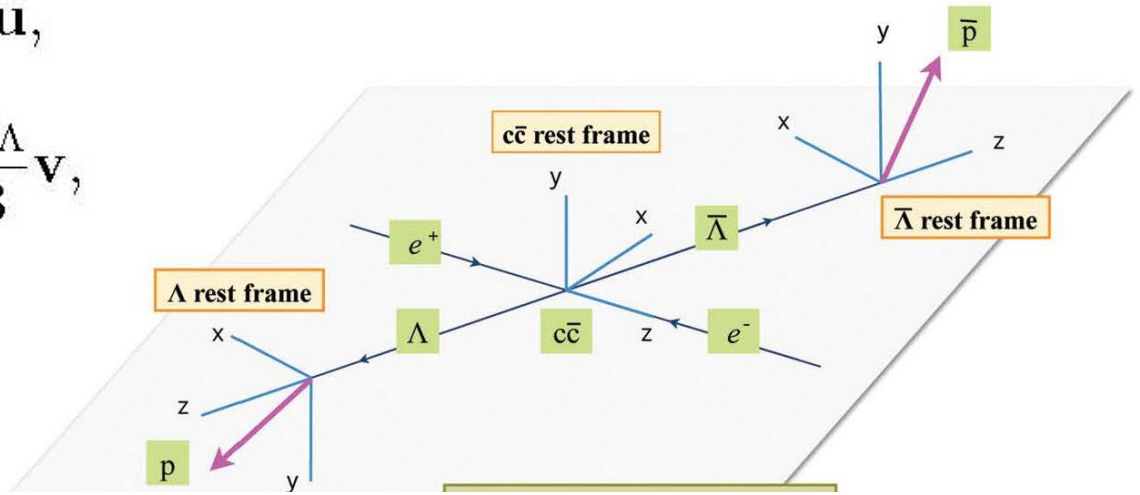
- Hyperon entanglement.
- Momentum direction of proton from the hyperon decay is correlated with hyperon spin, and plays a role similar to measurement agent. (scattering as “measurement”)
- The angular distribution plays a role similar to Malus Law.
- In the hidden variable theory, consider statistical mixture of polarizations.
- Study the correlation function of momenta directions of proton and antiproton.

$$\frac{d\sigma(\Lambda \to p\pi^-)}{d\Omega_p} = \frac{1}{4\pi}(1 + \alpha_\Lambda \mathbf{s}_\Lambda \cdot \mathbf{n}_p), \quad \frac{d\sigma(\Lambda \to \bar{p}\pi^+)}{d\Omega_{\bar{p}}} = \frac{1}{4\pi}(1 - \alpha_\Lambda \mathbf{s}_{\bar{\Lambda}} \cdot \mathbf{n}_{\bar{p}})$$

In HV theory, A and B are momentum direction of proton and antiproton, required to give the same averages as QM

$$\bar{\mathbf{A}} = \int d\lambda_A d\lambda_B \rho_A(\lambda_A)\rho_B(\lambda_B)\mathbf{A}(\lambda_A) = \frac{\alpha_\Lambda}{3}\mathbf{u},$$

$$\bar{\mathbf{B}} = \int d\lambda_A d\lambda_B \rho_A(\lambda_A)\rho_B(\lambda_B)\mathbf{B}(\lambda_B) = -\frac{\alpha_\Lambda}{3}\mathbf{v},$$



$$E(\mathbf{a},\mathbf{b}) \equiv -\overline{\langle \mathbf{A}\cdot\mathbf{a}\mathbf{B}\cdot\mathbf{b}\rangle} = -\sum_{ij} a_i b_j \overline{\langle A_i B_j\rangle}$$

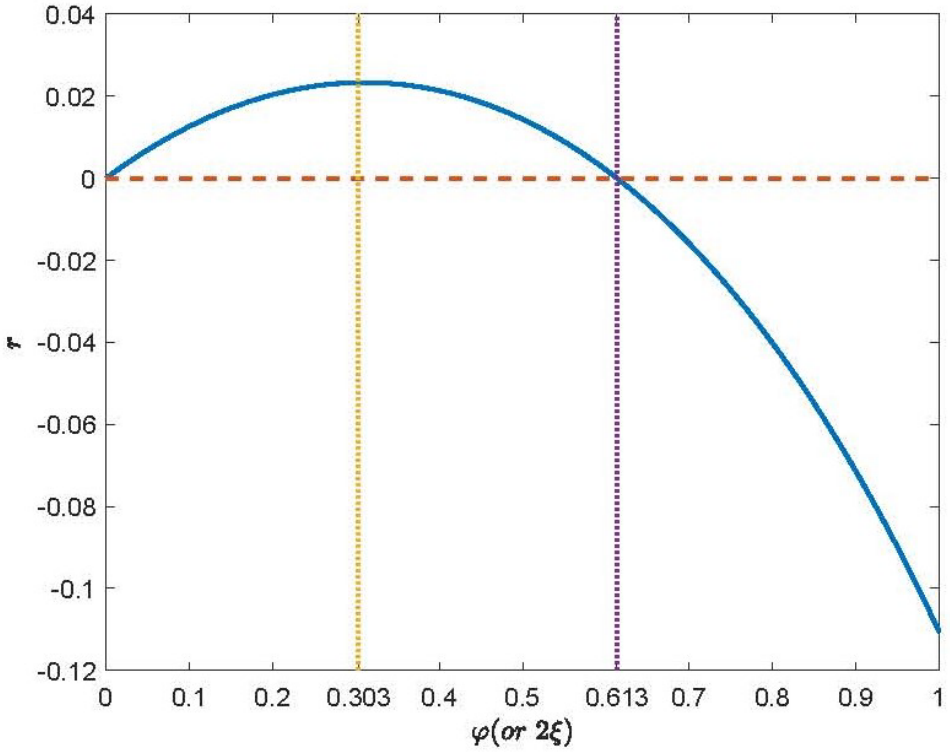
$$= -\int d\mathbf{u}d\mathbf{v}d\lambda_A d\lambda_B F(\mathbf{u},\mathbf{v})\rho_A(\lambda_A,\mathbf{u},\mathbf{v})\rho_B(\lambda_B,\mathbf{u},\mathbf{v})\mathbf{A}(\lambda_A,\mathbf{u},\mathbf{v})\cdot\mathbf{a}\mathbf{B}(\lambda_B,\mathbf{u},\mathbf{v})\cdot\mathbf{b}$$

$$= \frac{\alpha_\Lambda^2}{9}\int d\mathbf{u}d\mathbf{v}F(\mathbf{u},\mathbf{v})\mathbf{u}\cdot\mathbf{a}\mathbf{v}\cdot\mathbf{b},$$

$$\left|E_N^{\mathbf{ab}}(\varphi)+E_N^{\mathbf{ab}}(0)\right|+\left|E_N^{\mathbf{cd}}(\varphi)+E_N^{\mathbf{cd}}(0)\right|\leq\frac{\alpha_\Lambda^2}{9}\left(4-2u_N\left|\sin\frac{\varphi}{2}\right|\right)$$

$$\left|\hat{E}_N^{\mathbf{ab}}(\xi)+\hat{E}_N^{\mathbf{ab}}(0)\right|+\left|\hat{E}_N^{\mathbf{cd}}(\xi)+\hat{E}_N^{\mathbf{cd}}(0)\right|\leq\frac{\alpha_\Lambda^2}{9}\left(4-2u_N\left|\sin\xi\right|\right)\ u_N=\cot\left(\pi/2N\right)/N$$

- r=(L-R)/|L|, L is calculated from QM.



The violation ratio r for the first (second) inequality, as a function of φ (2ξ) for η_c (χ_{c0}).

7. Some more recent work: Quantum Fisher Information for neutrino oscillations and quantum correlations of quarks

Quantum Information Tools for Particle Physics

Neutrino Oscillations and Top-Quark Quantum Correlations

Yu Shi

Wilczek Quantum Center, Shanghai Institute for Advanced Studies
University of Science and Technology of China

Quantum Entanglement Studies in Nuclear and Particle Physics
Fudan University, October 25, 2026

Outline

1. Introduction
2. Quantum Fisher Information in Neutrino Oscillations
3. Multiparameter Estimation and Degeneracy Structure
4. Quantum Correlations of $t\bar{t}$ at the LHC
5. Non-Classical Correlations in the tW System
6. Summary

Quantum information for two subjects

- **Neutrino oscillations**: an evolving quantum state whose flavor content carries information on θ_{23}, δ_{CP}, Δm^2_{31}.
- **Top quarks at the LHC**: decay before hadronization ($\tau \sim 10^{-25}\,\mathrm{s}$) $\Rightarrow$ spin qubits at the TeV scale, accessible through decay angular distributions.
- Quantum estimation theory quantifies the ultimate precision allowed by quantum mechanics (quantum Cramér–Rao bound).
- Entanglement / discord / Bell-type observables probe non-classical correlations and new physics.

Two questions of this talk

1. How much information does the neutrino quantum state *really* carry about oscillation parameters?
2. How do quantum-correlation observables of top-quark systems respond to anomalous interactions?

Tool kit

Quantum parameter estimation

- Quantum Fisher information (matrix): F_{Q} (QFIM)
- Quantum Cramér–Rao bound: $\mathrm{Var}(\hat{\theta}_i) \geq [\mathcal{F}^{-1}]_{ii}/N$
- Classical Fisher information from flavor-probability measurements: F_{C} (CFIM)
- Quantum fidelity f between oscillation states

Quantum correlations

- Concurrence $\mathcal{C}$ (entanglement, two qubits)
- Negativity $\mathcal{C}^{\mathrm{PPT}}$ (PPT criterion, arbitrary bipartition)
- Geometric quantum discord D_g (beyond entanglement)
- Bell parameter $\mathcal{B}$ (CHSH)

Applied to: neutrinos ($\nu_\mu \to \nu_e$) · $t\bar{t}$ two-qubit spin state · tW qubit–qutrit state

QFIM and QCRB

- QFIM: $\mathcal{F}_{ij} = \mathrm{Re}[\langle\partial_i\psi|\partial_j\psi\rangle - \langle\partial_i\psi|\psi\rangle\langle\psi|\partial_j\psi\rangle], \quad (F_{\mathrm{Q}})_i = 4\mathcal{F}_{ii}$.
- Multiparameter QCRB: $\mathrm{Cov}(\hat{\boldsymbol{\theta}}) \succeq \mathcal{F}^{-1}/N$.
- CFIM from flavor measurement: $\mathcal{F}^{\mathrm{C}}_{ij} = \sum_\alpha \frac{\partial_i P_\alpha \, \partial_j P_\alpha}{P_\alpha}$.
- Fidelity between oscillation states: $f(1,2) = \left|\langle\psi(\theta^{(1)}_{23}, \delta^{(1)}_{\mathrm{CP}})|\psi(\theta^{(2)}_{23}, \delta^{(2)}_{\mathrm{CP}})\rangle\right|^2$.

Quantum correlation measures

- Concurrence (two qubits): $\mathcal{C}[\rho] = \max(0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4)$, λ_i = eigenvalues of $\sqrt{\sqrt{\rho}\,\tilde{\rho}\,\sqrt{\rho}}$, $\tilde{\rho} = \sigma_y \otimes \sigma_y\, \rho^*\, \sigma_y \otimes \sigma_y$.
- Negativity (PPT): $\mathcal{C}^{\mathrm{PPT}}[\rho] = \big(\|\rho^{T_B}\|_1 - 1\big)/2$.
- Discord: $D = I - J$, the gap between total correlation $I(\rho) = S(A) + S(B) - S(AB)$ and the best measurement-accessible correlation J — captures quantum correlations beyond entanglement: separable states can have $D > 0$. $D = 0$ iff classical–quantum, $\rho = \sum_i p_i \, |i\rangle\langle i| \otimes \rho_B^i$.
- Geometric discord: Hilbert–Schmidt distance to the zero-discord set Ω_0, $D_g[\rho] = \min_{\pi \in \Omega_0} \|\rho - \pi\|^2 = \frac{1}{2}\big(\|\vec{x}\|^2 + \|T\|_F^2 - k_{\max}\big)$ in Bloch form — replaces the hard measurement optimization by a distance; closed form, computable also for qubit–qutrit.
- CHSH Bell parameter $\mathcal{B}$; LHV bound $\mathcal{B} \leq 2$.

Section 2

Quantum Fisher Information in Neutrino Oscillations

Bhavna Yadav, Amir Subba, Yu Shi, *Quantum Fisher Information Revealing Parameter Sensitivity in Long-Baseline Neutrino Experiments*, arXiv:2602.05221.

Setup: neutrino state and QFI

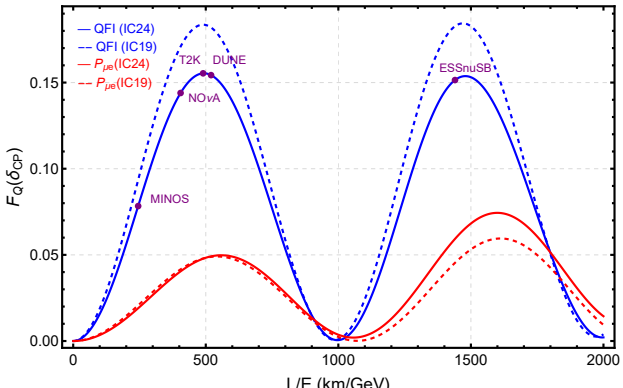
- Flavor state at baseline L: $|\nu_\mu(L)\rangle = \sum_i U^*_{\mu i}\, e^{-i m_i^2 L/2E}\, |\nu_i\rangle$ (pure state, PMNS mixing)
- Single-parameter QFI for parameter θ: $F_{\mathrm{Q}}(\theta) = 4\left[\langle\partial_\theta\psi|\partial_\theta\psi\rangle - |\langle\psi|\partial_\theta\psi\rangle|^2\right]$
- F_{Q} is measurement-independent: the ultimate sensitivity of the quantum state itself (via QCRB).
- Benchmark parameters from NuFit-6.0 (IC24 / IC19); focus on $\nu_\mu \to \nu_e$ appearance channel, vacuum vs. matter.

Programme

Map $F_{\mathrm{Q}}(\theta)$ as a function of L/E and compare with the classical Fisher information F_{C} of the flavor-probability measurement: $F_{\mathrm{C}} = \sum_\alpha P_\alpha^{-1}(\partial_\theta P_\alpha)^2$.

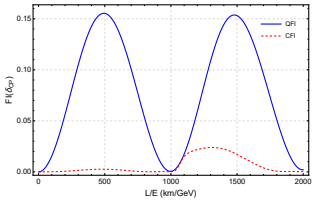
δ_{CP}: bimodal structure tracks oscillation maxima

- $F_{\mathrm{Q}}(\delta_{\mathrm{CP}})$ peaks at $L/E \sim 500$ and 1500 km/GeV — the first and second oscillation maxima.
- Maximal intrinsic sensitivity $F_{\mathrm{Q}} \sim 0.15$.
- Marked: working points of MINOS, T2K, NOνA, DUNE, ESSνSB.
- NuFit-6.0 IC24 vs. IC19 (no S–K atm.): robust conclusions, difference is caused by S–K atm.

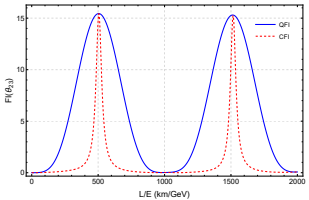


$F_{\mathrm{Q}}(\delta_{\mathrm{CP}})$ (blue) and $P(\nu_\mu \to \nu_e)$ (red) vs. L/E

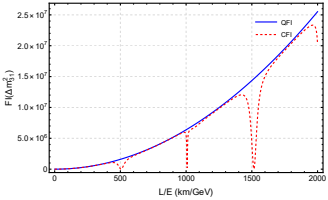
Strong hierarchy of intrinsic sensitivity



δ_{CP}



θ_{23}



Δm^2_{31}

- Peak values: $F_{\mathrm{Q}}(\delta_{\mathrm{CP}}) \sim 0.15$, $F_{\mathrm{Q}}(\theta_{23}) \sim 15$, $F_{\mathrm{Q}}(\Delta m^2_{31}) \sim 10^7$ — several orders of magnitude hierarchy.
- δ_{CP}, θ_{23}: bimodal, tied to oscillation maxima; Δm^2_{31}: grows rapidly with L/E.

Quantum vs. classical information: measurement matters

- For δ_{CP}: F_{C} stays well below F_{Q} over most of $L/E \Rightarrow$ flavor-probability counting does not extract all available information.
- For θ_{23}: $F_{\mathrm{C}} \approx F_{\mathrm{Q}}$ near the maxima — the standard measurement is nearly optimal there.
- For Δm^2_{31}: F_{C} captures most information, with localized losses near $L/E \sim 500$, 1000, 1500 km/GeV.
- Matter effects (DUNE baseline): enhance $F_{\mathrm{Q}}(\delta_{\mathrm{CP}})$ in the few-GeV region — sensitivity benefit at the oscillation maximum.

Message

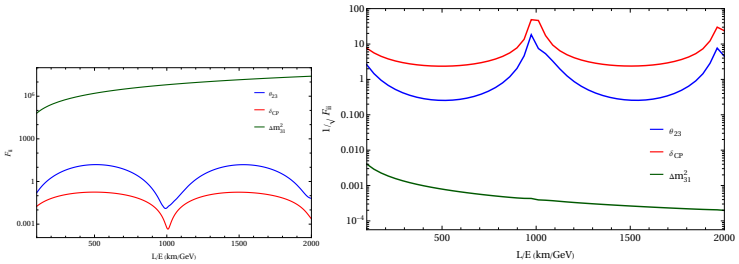
F_{Q} = measurement-independent quantum benchmark; $F_{\mathrm{Q}} - F_{\mathrm{C}}$ gap = headroom for optimized measurement strategies.

Section 3

Multiparameter Estimation and Degeneracy Structure

Bhavna Yadav, Amir Subba, Yu Shi, *Multiparameter Quantum Estimation and Degeneracy Structure in Three-Flavor Neutrino Oscillations*, arXiv:2606.30222.

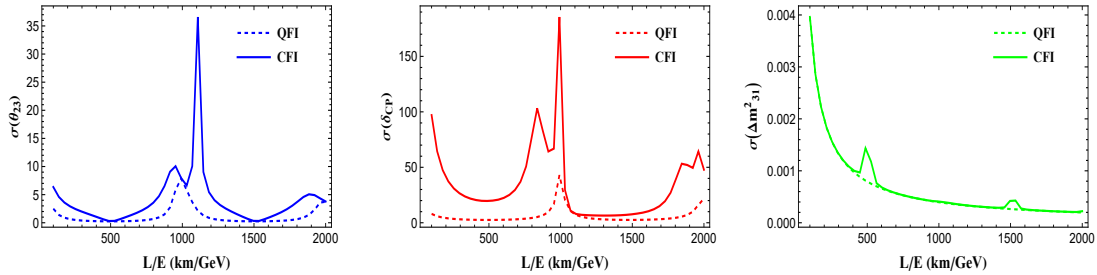
Full QFIM: six independent elements



Diagonal QFIM elements (left) and QCRB, $1/\sqrt{F_{ii}}$ (right) vs. L/E

- $F_{\Delta m^2_{31}\Delta m^2_{31}}$ dominates ($\sim 10^6$–10^7); QCRB 10^{-7}–10^{-5}.
- θ_{23}: QCRB 10^{-1}–10^{2}; $\delta_{\rm CP}$: can exceed 10^3.
- Off-diagonal elements: pronounced L/E dependence, sign changes $\Rightarrow$ non-trivial parameter correlations.

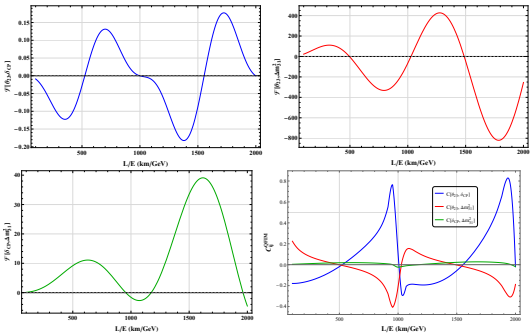
Diagonal vs. full QFIM; QFI vs. CFI uncertainties



Standard deviations from diagonal QFI (dashed) vs. CFI (solid) for θ_{23}, δ_{CP}, Δm^2_{31}

- θ_{23}: CFI uncertainty close to QFI limit $\Rightarrow$ efficient extraction.
- δ_{CP}: large Q–C gap, strongly L/E-dependent.
- Full vs. diagonal: noticeable for θ_{23} and δ_{CP} only near $L/E \simeq 1000$ and $\gtrsim 1700$ km/GeV; Δm^2_{31} essentially unaffected.
- Large raw off-diagonal element $\neq$ strong normalized correlation.

Off-diagonal structure: normalize before interpreting

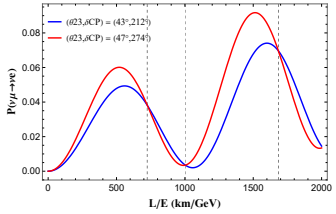


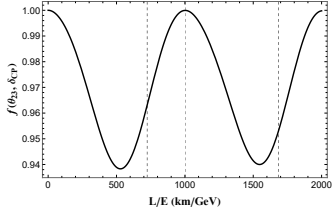
Raw off-diagonal QFIM elements (left, top to bottom) and normalized correlations C_{ij} (bottom right) vs. L/E

- Normalized correlation coefficients $C_{ij} = F_{ij}/\sqrt{F_{ii}F_{jj}}$: dimensionless, scale-free.
- Raw elements span wildly different scales (± 0.2 to ± 800) — large raw $\neq$ strong correlation.
- All $|C_{ij}| \lesssim 0.8$; sign flips track the correlation direction seen before.

Probability degeneracy $\neq$ state degeneracy

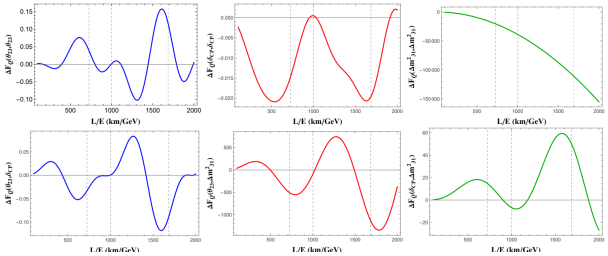
- Eightfold degeneracy: $(\theta_{23}, \delta_{\rm CP}) = (43^\circ, 212^\circ)$ vs. $(47^\circ, 274^\circ)$ give identical $P(\nu_\mu \to \nu_e)$ at $L/E \simeq$ 725, 1002, 1685 km/GeV.
- But the quantum states remain distinguishable: fidelity $f \simeq 0.953 \neq 1$.
- Probability equality does not imply state equality.





$P(\nu_\mu \to \nu_e)$ (top) and fidelity f (bottom)

Information geometry of degenerate solutions



Element-wise QFIM differences $\Delta\mathcal{F}_{ij} = \mathcal{F}_{ij}^{(1)} - \mathcal{F}_{ij}^{(2)}$ between the two degenerate parameter sets

- All six $\Delta\mathcal{F}_{ij} \neq 0$ at the degenerate points: the two solutions have distinct local quantum-information geometry.
- θ_{23} element: oscillates (-0.11 to $+0.16$).
- δ_{CP}: smaller (-0.021 to $+0.002$).
- Δm^2_{31}: grows to $\sim -1.6 \times 10^4$ with L/E.
- CFIM differences quantify what the flavor measurement can actually resolve.

Section 4

Quantum Correlations of $t\bar{t}$ at the LHC

Amir Subba, Yu Shi, *Quantumness of top quark pairs produced at LHC within SMEFT framework*, arXiv: 2605.12033.

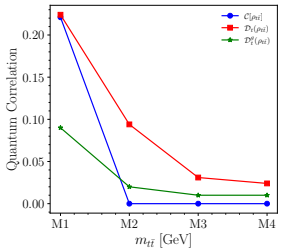
$t\bar{t}$ spin state as a two-qubit system

- Top decays before hadronization / spin decorrelation $\Rightarrow$ spin information transferred to decay products.
- Spin density matrix reconstructed from the joint angular distribution of the charged leptons in the *k*-*r*-*n* helicity basis:

$$\frac{1}{\sigma}\frac{\mathrm{d}\sigma}{\mathrm{d}\cos\theta_1 \mathrm{d}\cos\theta_2} \propto 1 + B_i^+ \hat{\ell}_i^+ + B_i^- \hat{\ell}_i^- - C_{ij}\, \hat{\ell}_i^+ \hat{\ell}_j^- .$$

- In the SM the Bloch vectors vanish ($B_i^{\pm} = 0$) by discrete symmetries; correlations encoded in C_{ij}.
- Observables: concurrence $\mathcal{C}$, geometric discord $D_g^{\pm}$, Bell parameter $\mathcal{B}$; four $m_{t\bar{t}}$ bins M1–M4, pp @ $\sqrt{s} = 13$ TeV, parton level.

SM quantum-correlation pattern



$\mathcal{C}$, D_t, D_g^t (left) and $\mathcal{B}$ (right) across $m_{t\bar{t}}$ bins

- Entanglement (concurrence ~ 0.2) only in the threshold region $m_{t\bar{t}} \lesssim 400$ GeV (spin-singlet dominance).
- Higher bins: separable, yet $D_g \neq 0$ everywhere — quantum correlations persist without entanglement.
- $\mathcal{B} < 2$ throughout: no Bell non-locality in the SM.
- Hierarchy confirmed: $\mathrm{QE} \Rightarrow \mathrm{QD}$, not conversely.

Section 5

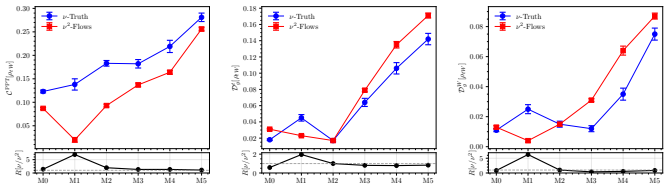
Non-Classical Correlations in the *tW* System

Amir Subba, Yu Shi, *Non-Classicality in tW system via negativity and geometric quantum discord* , arXiv: 2610.27404.

tW: a mixed-dimensional (qubit–qutrit) system

- $pp \to tW$ at $\sqrt{s} = 13$ TeV, leptonic final state $\ell^{\pm}\ell^{\mp}b\not{E}_T$: two missing neutrinos $\Rightarrow$ top spin (qubit) + W spin (qutrit): $\rho_{tW} \in \mathcal{H}^2 \otimes \mathcal{H}^3$.
- Neutrinos reconstructed with conditional normalizing flows (ν^2-Flows) trained on detector-level objects: RMS ~ 50 GeV ($p_{x,y}$), ~ 155 GeV (p_z), bias < 1 GeV.
- Spin density matrix from generalized Bloch representation of decay angular distributions.
- Backgrounds $t\bar{t}$ and WWj tagged with boosted decision trees.
- Correlations: PPT negativity $\mathcal{C}^{\mathrm{PPT}}$ and geometric discord $D_g^{t,W}$ at truth and reconstruction level.

Negativity and discord across m_{tW}



$\mathcal{C}^{\mathrm{PPT}}$, D_g^t, D_g^W in five m_{tW} regions, truth vs. reconstructed

- Monotonic increase with m_{tW}: $\mathcal{C}^{\mathrm{PPT}}$: $0.138 \to 0.281$ (truth), $0.020 \to 0.256$ (reco).
- Genuine qubit–qutrit entanglement reconstructed from realistic LHC final states.
- $t\bar{t}$, WWj contamination: negativity diluted up to $\sim 30\%$ (intermediate bins); discords by ~ 10–$25\% \Rightarrow$ discord-type observables more resilient.

Takeaway messages

1. **Neutrinos as quantum sensors:** the QFI reveals a strong intrinsic sensitivity hierarchy ($\Delta m^2_{31} \gg \theta_{23} \gg \delta_{\rm CP}$) and identifies where the standard flavor measurement fails to extract the available quantum information.
2. **Degeneracies are quantum-mechanically resolved:** probability-degenerate ($\theta_{23}, \delta_{\rm CP}$) solutions remain distinguishable at the state level (fidelity, QFIM geometry).
3. $t\bar{t}$ **as a two-qubit laboratory:** entanglement only near threshold, discord everywhere, no SM Bell violation — cleanly deformed by SMEFT dipole operators with CP-even/odd discrimination.
4. tW **as a qubit–qutrit system:** genuinely mixed-dimensional entanglement and discord are reconstructible at the LHC and give competitive sensitivity to anomalous top dipole moments, robust to backgrounds and systematics.

Quantum-information observables = complementary, qualitatively new probes of fundamental parameters and new physics.

Summary

1. Early entanglement study (not using this term) in particle physics.
2. Using Quantum Entanglement to Study CP and CPT Violations.
3. Genuine T Violation Signal in terms of entangled mesons.
4. A Proposal for a High Energy quantum teleportation.
5. Entangled mesons violate a generalized Leggett Inequality, in which decays also depend on hidden variables.
6. Entangled hyperons violated a generalized Bell inequality.
7. Neutrinos: the QFI is useful, e.g. reveals a strong intrinsic sensitivity hierarchy.
8. ttbar as a two-qubit laboratory
9. tW as a qubit–qutrit system
10. Quantum-information observables: complementary, qualitatively new probes of fundamental parameters and new physics.

Thank you!

Backup: why is the QFIM defined this way?

It is the Hessian of the fidelity

$$\left|\langle\psi(\boldsymbol{\theta})\,|\,\psi(\boldsymbol{\theta}+\mathrm{d}\boldsymbol{\theta})\rangle\right|^2 = 1 - \frac{1}{4}\,\mathcal{F}_{ij}\,\mathrm{d}\theta_i\,\mathrm{d}\theta_j + O(\mathrm{d}\theta^3) \qquad \text{(pure states)}$$

$\mathcal{F}_{ij}$ measures how fast states become distinguishable, thus limits parameter estimation.

Quantum geometric tensor: $\mathcal{Q}_{ij} = \langle\partial_i\psi|(1-|\psi\rangle\langle\psi|)|\partial_j\psi\rangle = \frac{1}{4}\mathcal{F}_{ij} - \frac{i}{2}\,\Omega_{ij}$

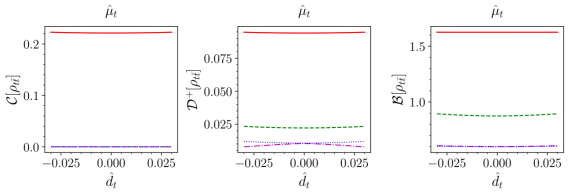
- Symmetric real part $\Rightarrow$ QFIM ($\div 4$: Fubini–Study metric).
- Antisymmetric imaginary part $\Rightarrow$ Berry curvature $\Omega_{ij} = \partial_i A_j - \partial_j A_i$, the field strength of $A_i = i\langle\psi|\partial_i\psi\rangle$ — *this* is the curvature; the QFIM itself is a metric.
- Metric $\neq$ curvature: $\mathcal{F}$ contains only first derivatives of $|\psi\rangle$. All the physics (bimodal $\mathcal{F}(\delta_{\mathrm{CP}})$, 10^7 hierarchy in Δm^2_{31}) lives in the metric.

Three faces of the same object: fidelity Hessian (geometry) = Bures-distance coefficient (geometry) = $\max_{\text{all POVMs}} F_{\mathrm{C}}$ (statistics) $\Rightarrow$ QCRB $\mathrm{Cov}(\hat{\boldsymbol{\theta}}) \geq (\nu\mathcal{F})^{-1}$, asymptotically attainable.

Backup: SMEFT dipole operators

- Chromo-dipole (CP-even $\hat{\mu}_t$, CP-odd $\hat{d}_t$): modify the $gt\bar{t}$ / tWg vertices.
- Weak dipoles: vector/axial $\Delta C_1^{A,V}$ and tensor $C_2^{A,V}$ modify the $t\bar{t}Z$ vertex and the $q\bar{q} \to t\bar{t}$ amplitude.
- All coefficients varied strictly within current experimental bounds — QI responses are deformations, not free fits.
- Consistent picture across basis choices (helicity vs. beam basis, four $m_{t\bar{t}}$ bins M1–M4).

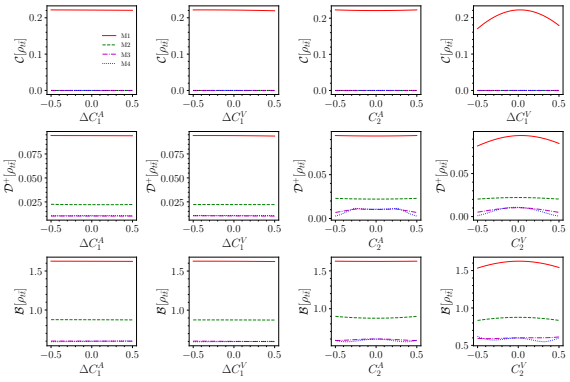
SMEFT dipole deformations



$\mathcal{C}$, D^+, $\mathcal{B}$ vs. chromo-dipole couplings $\hat{\mu}_t$ (top, M1) and $\hat{d}_t$ (bottom, M1–M4)

- Dimension-6 operators modifying $gt\bar{t}$ and $t\bar{t}Z$ vertices; all couplings varied within experimental bounds ($\hat{\mu}_t \in [-0.04, 0.005]$, $|\hat{d}_t| \leq 0.03$, CMS).
- CP-even $\hat{\mu}_t$: asymmetric shifts, mainly near threshold.
- CP-odd $\hat{d}_t$: mild symmetric response; no Bell violation.
- QI observables = complementary probe of top dipole structure.

Weak dipole moments: operator-class discrimination



Response to ΔC_1^A, ΔC_1^V, C_2^A, C_2^V (all $m_{t\bar{t}}$ bins)

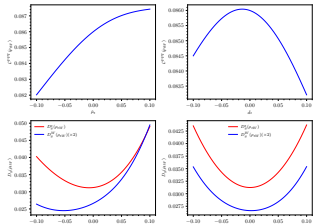
- CP-even tensor coupling C_2^V: largest deformation of all three QI observables.
- Vector/axial $\Delta C_1^{A,V}$: completely flat — no sensitivity at this level.
- Distinct sensitivity to CP-even vs. CP-odd operator structures $\Rightarrow$ QI observables as a discriminating handle on EFT operator classes.

Backup: ν^2-Flows reconstruction performance

	p_x^ν	p_y^ν	p_z^ν	$p_x^{\bar{\nu}}$	$p_y^{\bar{\nu}}$	$p_z^{\bar{\nu}}$
RMS [GeV]	49.5	50.3	153.1	50.0	51.5	155.1
Bias [GeV]	-0.24	-0.05	-0.02	-0.54	0.34	-0.95
Corr	0.57	0.56	0.44	0.57	0.55	0.41

- Correlation between predicted and true neutrino momenta 0.41–0.57; no systematic offsets (bias < 1 GeV).
- Longitudinal component hardest (no direct detector constraint along the beam).
- Residuals propagate into QI observables: quantified via bootstrap ($\mathcal{O}(10^3)$ repetitions, 40% event subsamples).

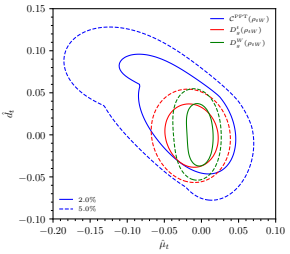
Response to anomalous chromo-dipole moments



Negativity (top) and $D_g^{t,W}$ (bottom, D_g^W rescaled $\times 2$) vs. $\hat{\mu}_t$, $\hat{d}_t$ at reconstruction level

- Negativity: concave, extremal at the SM point; discords: convex $\Rightarrow$ opposite quadratic dependence on $\hat{\mu}_t$, $\hat{d}_t$.
- D_g^W gives the dominant sensitivity to $\hat{\mu}_t$; for $\hat{d}_t$ the two discords are comparable.
- High-m_{tW} bin dominates the combined constraint.

Sensitivity: current LHC and HL-LHC



95% CL contours in the $(\hat{\mu}_t, \hat{d}_t)$ plane; luminosity (left) and systematic (right) dependence

- At $\mathcal{L} = 140$ fb^{-1}: D_g^W dominates the $\hat{\mu}_t$ reach; discords comparable for $\hat{d}_t$.
- HL-LHC (3000 fb^{-1}): contours shrink by ~ 2 (negativity) and 2.5–4 (discords).
- Systematics 2% $\to$ 5%: contours enlarge by about a factor 2 — still competitive.
- Geometric discord sensitivity comparable to vector-polarization and spin-correlation observables.