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Some Information from Energy Correlators

arXiv: 2506.09119 by Moult, Ian and Zhu, Hua Xing

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Energy Correlator

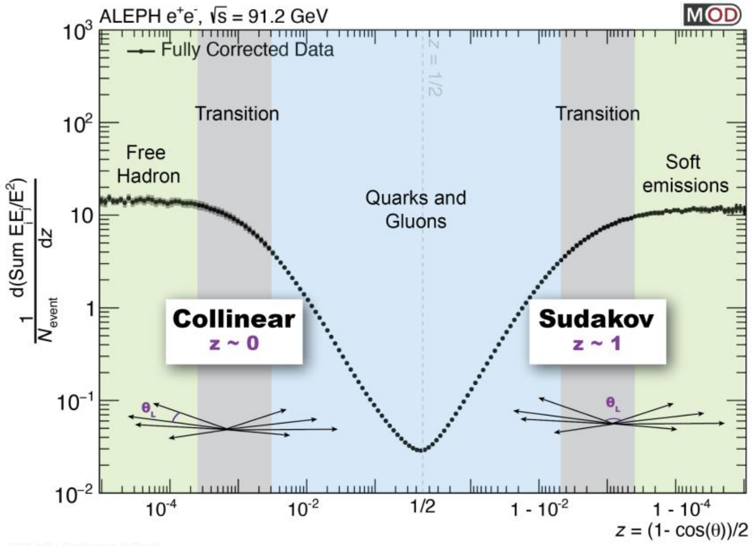
The energy correlators is defined as the joint probability of simultaneously discovering energy $\varepsilon(\theta_n)$ at the detector $(\theta_1, \theta_2, \ldots, \theta_n)$.

$$\langle \mathcal{E}(\theta_1) \cdots \mathcal{E}(\theta_n) \rangle \equiv \frac{\langle 0 | \mathcal{O}^\dagger \mathcal{E}(\theta_1) \cdots \mathcal{E}(\theta_n) \mathcal{O} | 0 \rangle}{\langle 0 | \mathcal{O}^\dagger \mathcal{O} | 0 \rangle}$$

where $\mathcal{O}$ makes the vacuum excited at $(t = 0, x^i = 0)$, such as the virtual photons in $e^- e^+ \to \gamma^* \to A$. Energy flow $\varepsilon(\theta_n)$ is defined as a light-ray operator:

$$\mathcal{E}(\theta) = \lim_{r \to \infty} r^2 \int_{-\infty}^{\infty} dt\, n^i T^0_i(t, r\vec{n}^i)$$

$$\mathcal{E}(\vec{n}) = \left(1 + |y^\perp|^2\right)^3 \int_{-\infty}^{\infty} dy^- T_{--}(y^+ = 0, y^-, y^\perp)$$



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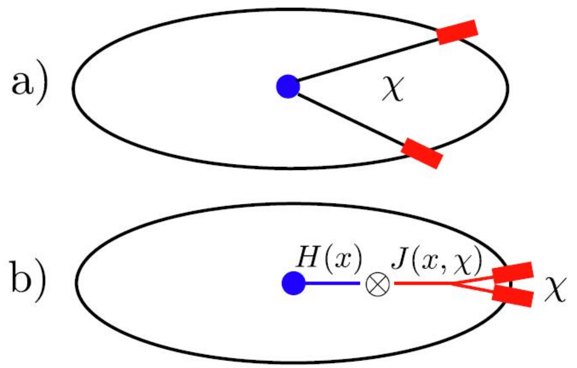
Collinear limit

In QCD Energy-Energy Correlator is given by:

$$\mathrm{EEC_{QCD}} = \sum_{ab,X'} \int d\sigma_{\gamma^* \to abX'} \frac{E_a E_b}{\sigma_{\mathrm{tot}} Q^2} \delta\left(z - \frac{1-\cos\chi_{ab}}{2}\right).$$

$$z = \frac{1-\cos\theta}{2}.$$



when the angle is small, then consider that $a \to b + c$

$$m_{bc}^2 = 2E_b E_c(1-\cos\theta) \simeq x(1-x)Q^2\theta^2.$$

so $m_{bc} \approx Q\theta$， we can use light-ray OPE solve it

$$O_1(x_1)O_2(x_2) = \sum_k C_{12k}(x_{12}) O_k(x_2)$$

$$C_{12k}(x) \sim \frac{1}{|x|^{\Delta_1+\Delta_2-\Delta_k}}$$

light-ray

$$\mathcal{E}(n_1)\mathcal{E}(n_2) = \int du_1 du_2 T_{u_1u_1}(u_1,n_1) T_{u_2u_2}(u_2,n_2)$$

$$T_{\mu\nu}(x_1)T_{\rho\sigma}(x_2) = \sum_k C^{(k)}_{\mu\nu\rho\sigma}(x_{12}) O_k(x_2) \qquad \mathbb{O}_k = \int du\, O_k(u,n)$$

$$\mathcal{E}(n_1)\mathcal{E}(n_2) = \sum_k C_k(\theta)\mathbb{O}_k(n_2) \qquad C_{\delta,j}(\theta) \sim \theta^{\delta-2}$$

$$EEC(\theta) \sim \theta^{\tau_k - 4} \qquad \tau = 2 + \gamma(J)$$

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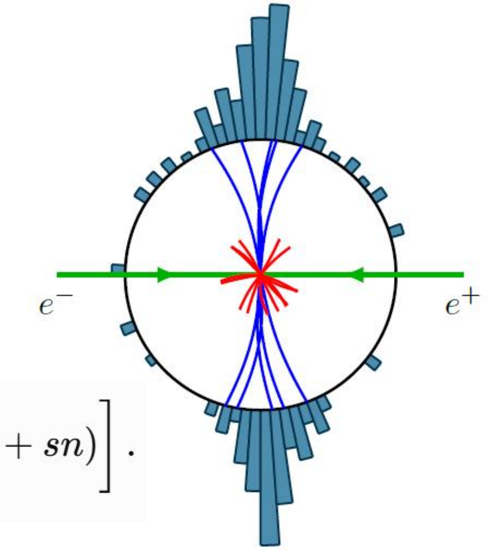
Back-to-Back limit

Transverse momentum between two jet is defined as $q_T \sim Q(\pi - \chi)$

When $\chi \to \pi$, we have $\pi - \chi \leftrightarrow \frac{q_T}{Q}$

a example: $e^+e^- \to q\bar{q}$ there is gluon radiation $q \to q + g$

Sum up the amplitudes emitted by any number of gluons (Wlison line)



$$gT^a \frac{p^\mu}{p \cdot k + i0} \simeq gT^a \frac{n^\mu}{n \cdot k + i0}. \qquad g^2 T^{a_1} T^{a_2} \frac{n^{\mu_1} n^{\mu_2}}{(n \cdot k_1)[n \cdot (k_1 + k_2)]} \qquad S_n(x) = P \exp\left[ig \int_0^\infty ds\, n \cdot A_s(x + sn)\right].$$

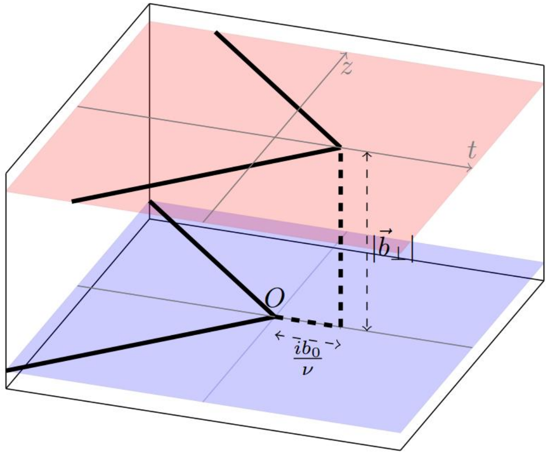
$$S_{\mathrm{EEC}}(b_\perp) = \frac{1}{N_c} \left\langle 0 \left| \mathrm{Tr}\left[T\left(S_n^\dagger S_{\bar{n}}\right)(b_\perp) \overline{T}\left(S_{\bar{n}}^\dagger S_n\right)(0)\right] \right| 0 \right\rangle.$$

The factorization structure of the EEC in the back-to-back limit can be written schematically as

$$\frac{d\sigma}{dz} \sim H(Q,\mu) \int_0^\infty db_T\, b_T\, J_0\left(b_T Q\sqrt{1-z}\right) J_n(b_T) J_{\bar{n}}(b_T) S_{\mathrm{EEC}}(b_T).$$

Here:

- $H(Q,\mu)$ describes the hard process that produces the two energetic jets;
- $J_n(b_T)$ and $J_{\bar{n}}(b_T)$ describe collinear radiation along the two jet directions;
- $S_{\mathrm{EEC}}(b_T)$ describes soft radiation correlated between the two jets;
- b_T is the impact parameter conjugate to the transverse recoil momentum q_T.



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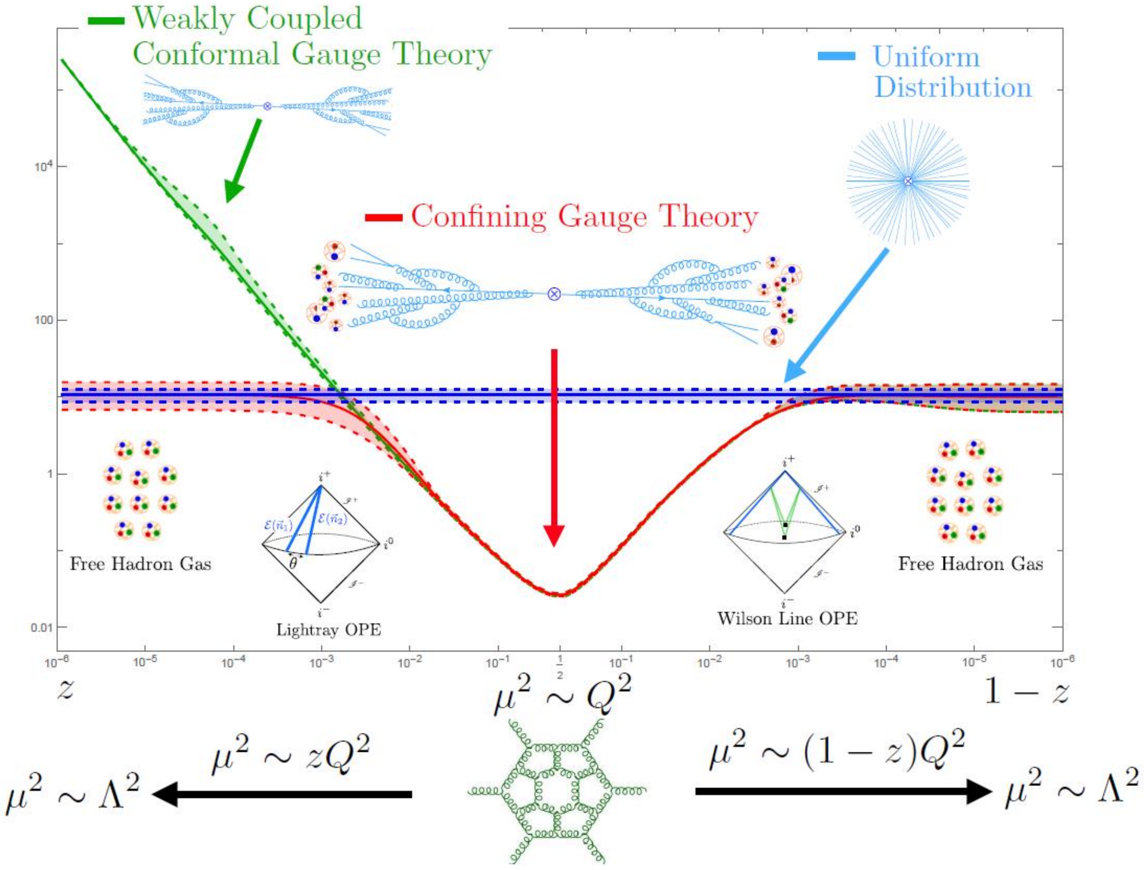
Energy Correlator in QCD phase structure

Two methods from UV to IR

Hard scattering, jet splitting, soft radiation, hadronization (QCD phase transition)



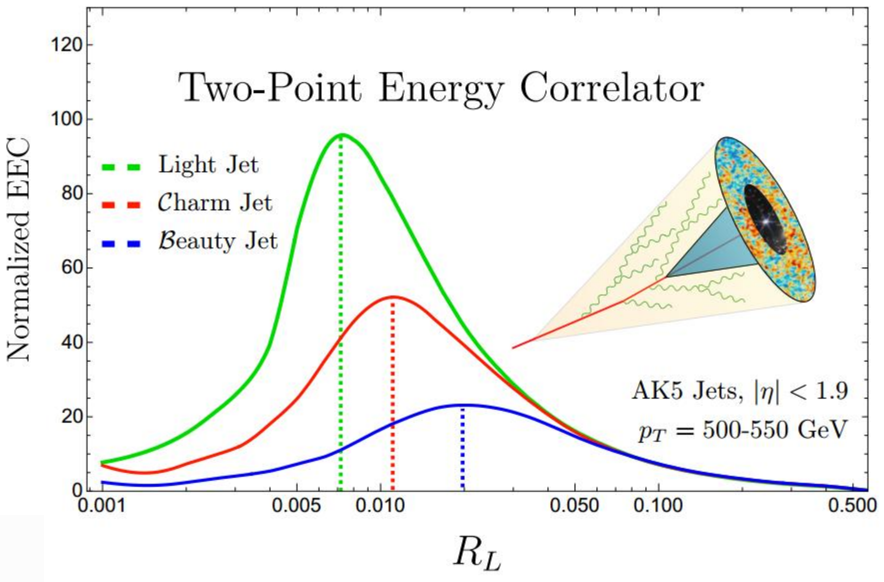
Massive quark

$$R_L = R_{12} = \sqrt{(\Delta\eta_{12})^2 + (\Delta\phi_{12})^2}$$

When the angle is small, the center of the jet can be approximated as θ_{12}

$$z \simeq \frac{\theta_{12}^2}{4} \simeq \frac{R_L^2}{4}$$

$$k_\perp \sim p_T R_L.$$



$$\begin{aligned}
p_T R_L \gg m_Q :& \quad \text{the quark-mass effect is negligible,} \\
p_T R_L \sim m_Q :& \quad \text{the quark mass begins to affect the radiation pattern,} \\
p_T R_L \ll m_Q :& \quad \text{the quark mass strongly suppresses collinear radiation.}
\end{aligned}$$

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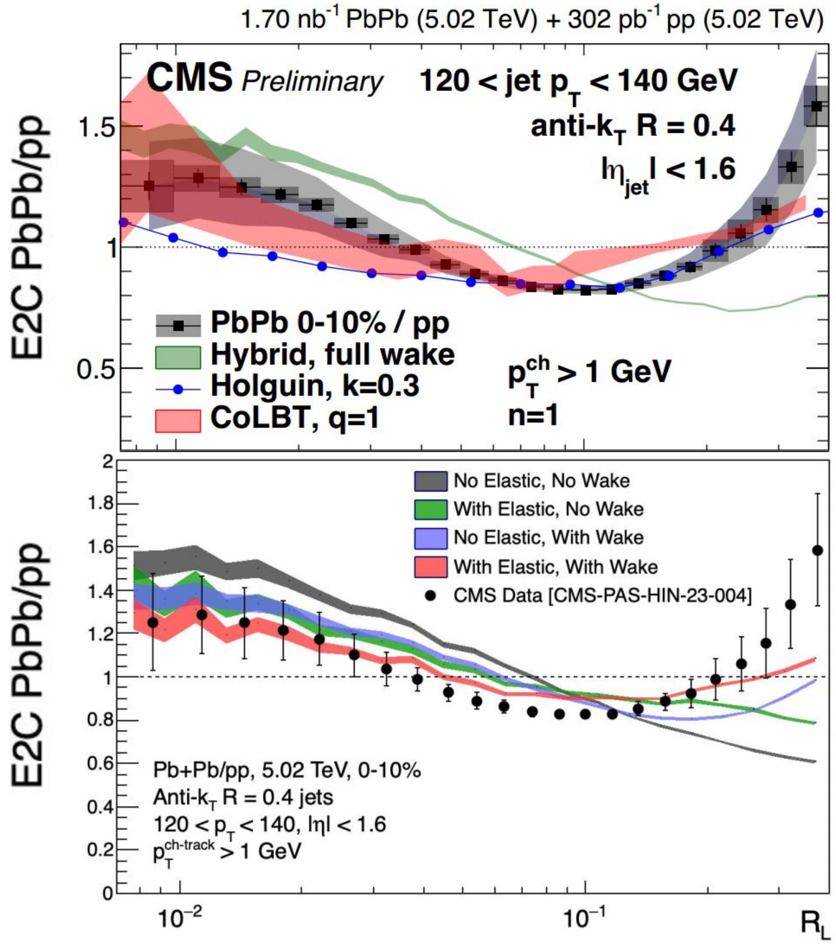
EEC in Heavy ion collision

$A + A \to QGP \to jet$

So the high-energy partons generated by hard scattering will pass through the medium, causing energy lossing

$$q \to q + g \qquad q + \text{medium} \to q + \text{medium}$$

The redistribution of energy in angle causes deformation of EEC



Energy Loss

Heavy quark`s energy loss in SYM. arXiv: 0605158

$$\boxed{\frac{dp}{dt} = -\frac{\pi}{2}\sqrt{\lambda}T^2\gamma v}$$

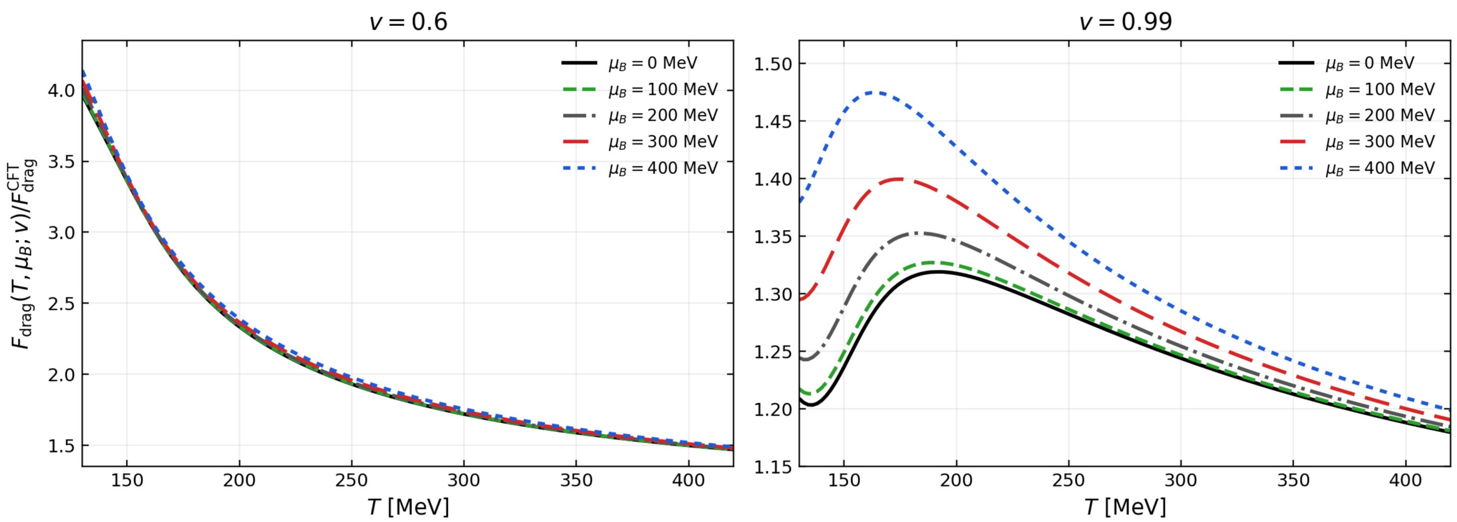
$$\boxed{\frac{dp}{dt} = -\mu p, \qquad \mu = \frac{\pi\sqrt{\lambda}T^2}{2m}}.$$

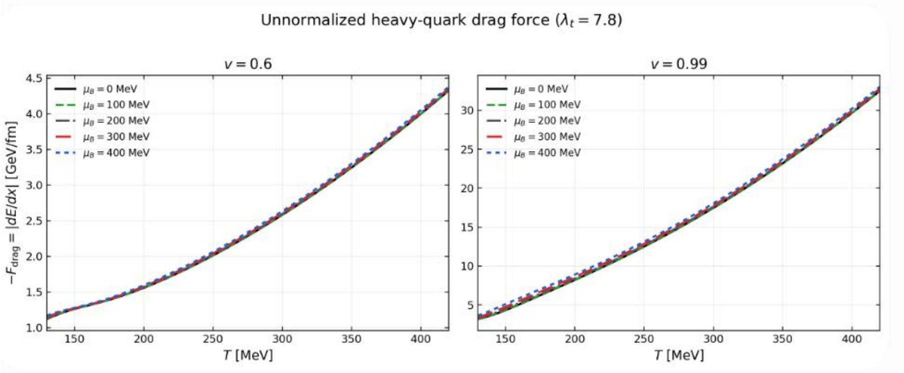
In non-conformal holography. arXiv:1507.06556

$$S = \frac{1}{16\pi G_5}\int d^5x\sqrt{-g}\left[R - \frac{1}{2}(\partial\phi)^2 - V(\phi) - \frac{f(\phi)}{4}F_{\mu\nu}F^{\mu\nu}\right]. \quad \beta_\phi \equiv \frac{d\phi}{d\ln Q} \simeq \frac{\phi'(r)}{A'(r)}.$$

$$F_{\text{drag}} = -\frac{e^{\sqrt{2/3}\phi(r_*)}\sqrt{-g_{tt}(r_*)g_{xx}(r_*)}}{2\pi\alpha'},$$

Heavy-quark drag force in the lattice-calibrated EMD plasma





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EEC in holography

When considering inserting ε into a $\mathrm{Ad}S_5$ spacetime with the boundary condition $f(x^{\perp}, z=0) = \delta^2$

$$ds^2 = e^{-2A(z)}\left(dx^+dx^- - |dx^{\perp}|^2 - dz^2\right) \qquad \delta ds^2 = \epsilon e^{-2A(z)}\delta(x^+)f(x^{\perp}, z)\left(dx^+\right)^2$$

The introduction of $\mathrm{A(z)}$ causes a scalar field to appear in the bulk

$$\int d^5x\sqrt{g}\left[\frac{1}{2}(\nabla\phi)^2 - V(\phi)\right] \xrightarrow{\text{E-L equation}} \begin{aligned}\left(\phi'(z)\right)^2 &= \frac{3}{\kappa^2}\left(A'(z)^2 + A''(z)\right),\\ V(\phi) &= -\frac{6}{\kappa^2}e^{2A(z)}A'(z)^2.\end{aligned}$$

Perturbation makes the Einstein tensor G and the stress-energy tensor T as

$$\delta G = \frac{\epsilon}{2}\delta(x^+)\left(dx^+\right)^2\left[-6\left(A'(z)^2 - A''(z)\right) - 3A'(z)\partial_z + \partial_z^2 + \partial_1^2 + \partial_2^2\right]f(x^{\perp}, z),$$
$$\delta T = \epsilon\delta(x^+)\left(dx^+\right)^2\left[e^{-2A(z)}V(\phi) + \phi'(z)^2\right].$$

Then,we have $\left(3A'(z)\partial_z - \partial_z^2 - \partial_1^2 - \partial_2^2\right)f(x^{\perp}, z) = 0.$

EEC in holography

The excitation operator $\mathcal{O}$ is also a scalar field in bulk

$$\left(3A'(z)\partial_z - \partial_z^2 + \partial^\mu\partial_\mu\right)\chi(x^\mu, z) = 0.$$

we can set the field as

$$\chi = e^{iqt} q^{3/2} e^{3A(z)/2} \psi(z).$$

Then

$$\left[-\partial_z^2 + V(z)\right]\psi(z) = 0,$$

$$\begin{aligned} V(z) &= \frac{9}{4}A'(z)^2 - \frac{3}{2}A''(z) - q^2 \\ &= \frac{15}{4z^2} - q^2 + \frac{9w'(z/z_{\mathrm{IR}})}{2zz_{\mathrm{IR}}} + \frac{9w'(z/z_{\mathrm{IR}})^2}{4{z_{\mathrm{IR}}}^2} - \frac{3w''(z/z_{\mathrm{IR}})}{2{z_{\mathrm{IR}}}^2}. \end{aligned}$$

$$\chi_{\text{after transformation}}(x^+ = 0, x^-, x^\perp, z) \sim \delta(z-1)\delta^2(x^\perp)e^{iqx^-/2},$$

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EEC in holography

we have

$$\left\langle e^{\epsilon_1 \mathcal{E}(y_1^\perp)} e^{\epsilon_2 \mathcal{E}(y_2^\perp)} \right\rangle \sim \int \frac{dz}{z^3} d^2x^\perp dx^- i\chi^* \left(e^{-\epsilon_1 \tilde{f}(y_1^\perp)\partial_-} e^{-\epsilon_2 \tilde{f}(y_2^\perp)\partial_-}\right) \partial_-\chi$$

$$\langle \mathcal{E}(0)\mathcal{E}(y^\perp)\rangle \sim \left(1+(y^\perp)^2\right)^3 f(y^\perp, z=1).$$

When $exp(2A(z)) = 1/r^2$,$\langle \varepsilon\varepsilon \rangle = \frac{q}{4\pi}$, where q is the total energry


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