2026年Belle II中国组冬季学校会议@东南大学

An overview of charmonium/like states

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2026年11月14日

Outline

- Charmonium(like) states and strong interactions
- Theoretical studies on charmonium(like) states
- Symmetry, Three-body system and Femtoscopy
- Summary

Discovery of charm quark in the vector charmonium state

➢ GIM mechanism

Experiment showed that the decay $K_L{}^0 \to \mu^+\mu^-$ is extraordingrily suppressed, lying orders of magnitude below tree-level weak-interaction predictions

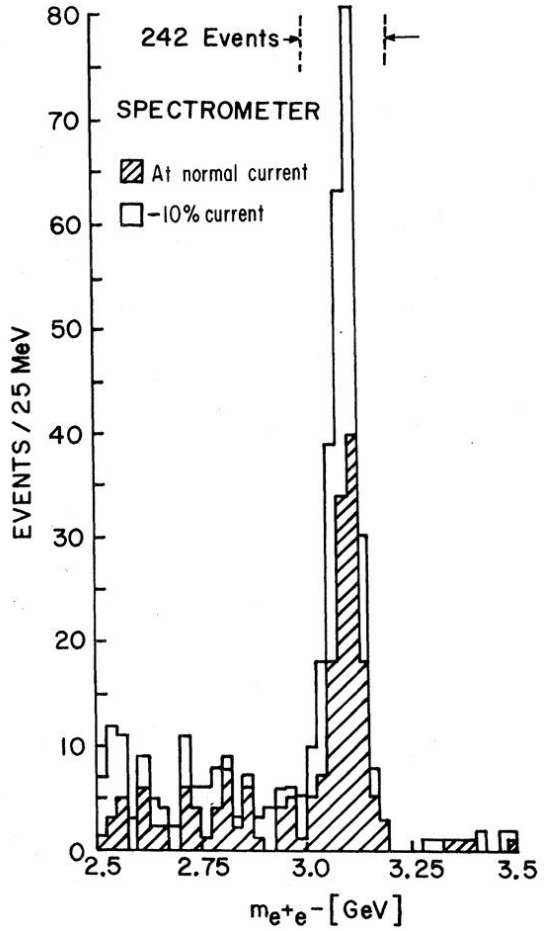
$$\begin{pmatrix} u \\ d' \end{pmatrix} = \begin{pmatrix} u \\ d\cos\theta_c + s\sin\theta_c \end{pmatrix}$$

$$\begin{pmatrix} c \\ s' \end{pmatrix} = \begin{pmatrix} c \\ s\cos\theta_c - d\sin\theta_c \end{pmatrix}$$

S.L. Glashow, J. Iliopoulos, L. Maiani,
Phys.Rev.D 2 (1970) 1285-1292

GIM mechanism predicted the existence of charm quark

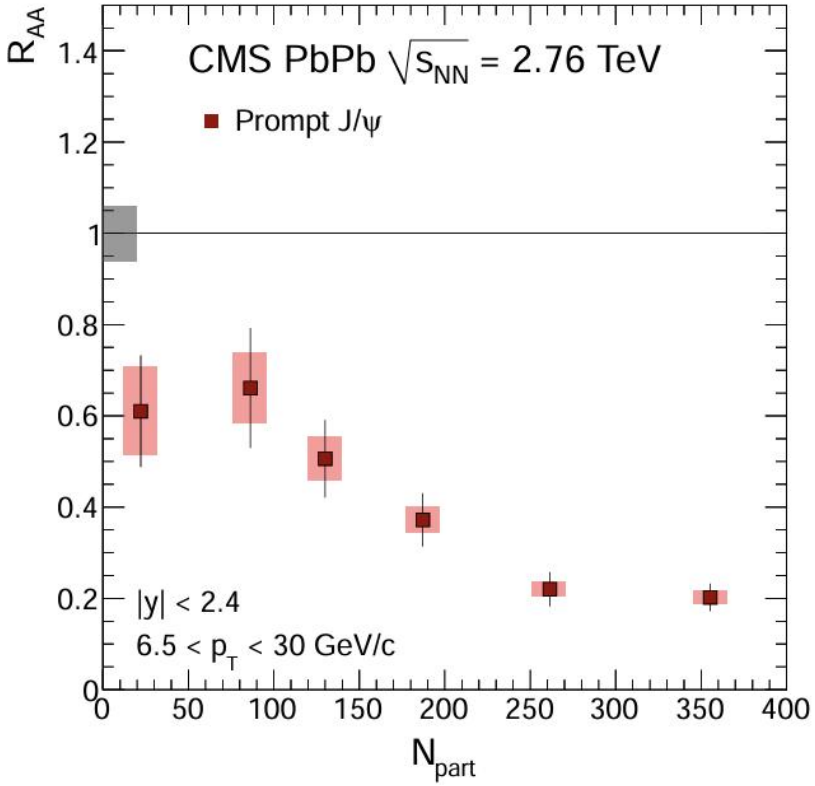
➢ Discovery of charm quark in J/psi



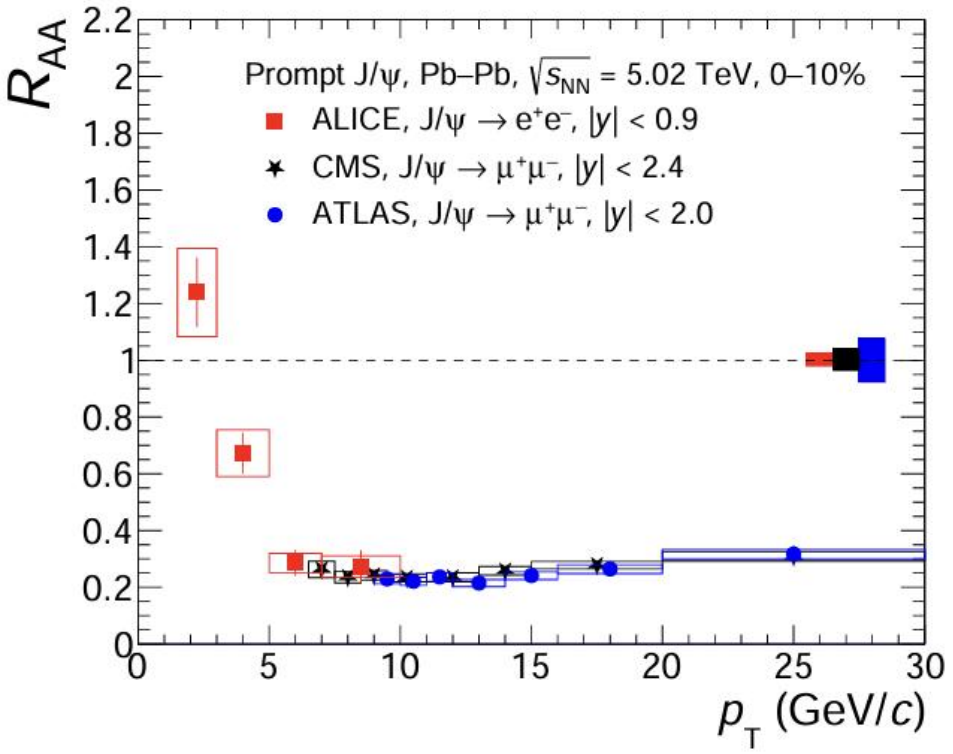
J.J. Aubert et al., Phys.Rev.Lett. 33 (1974) 1404-1406
J.E. Augustin et al., Phys.Rev.Lett. 33 (1974) 1406-1408
C. Bacci et al., Phys.Rev.Lett. 33 (1974) 1408

Charmonium states in high temperature

- **The QGP in heavy ion collision suppresses the formation of charmonium states**



CMS Collaboration, JHEP 05 (2012) 063



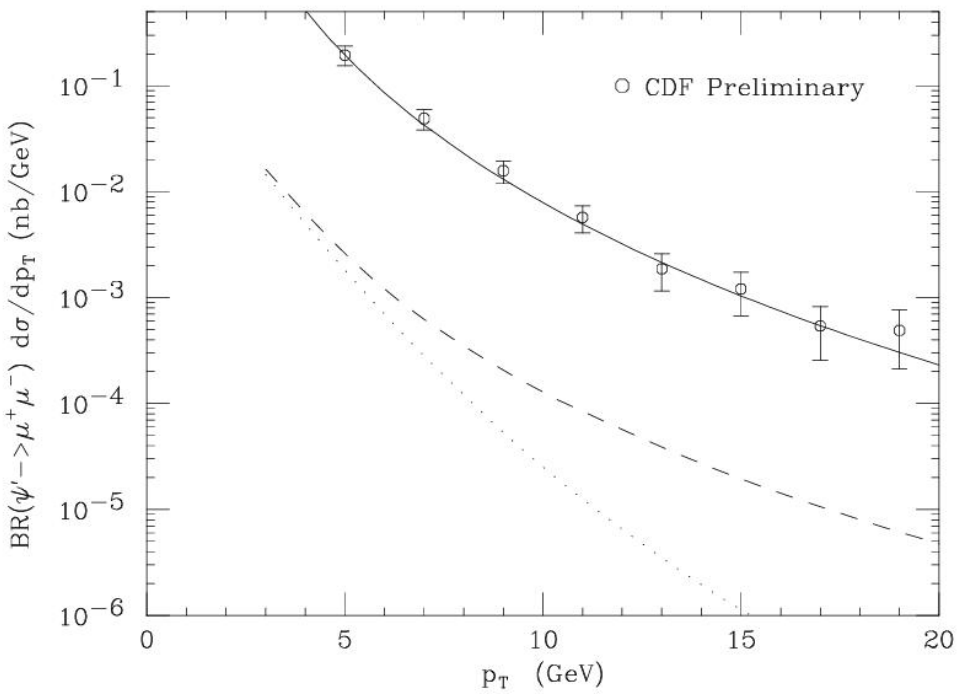
Alice Collaboration, Phys.Lett.B 849 (2024) 138451

The suppression of J/psi meson in the heavy ion collision implies the existence of QGP signal

T. Matsui and H. Satz, Phys.Lett.B 178 (1986) 416-422

Charmonium states in studying strong interaction QCD

➢ The prompt production of J/psi



Eric Braaten et al., Phys.Rev.Lett. 74 (1995) 3327-3330

➢ The leptonic decay of J/psi

$$\Gamma(\psi \to l^+ l^-) = \frac{4\pi e_Q^2 \alpha_{em}^2 f_\psi^2}{3M_\psi}$$

$$\langle \psi(p)|\bar{Q}\gamma_\mu Q|0\rangle = (-i) f_\psi M_\psi \, \epsilon_\mu^*(p)$$

$$\langle \psi(p)|\bar{Q}\gamma^\mu Q|0\rangle = \Lambda^{\mu i}(p)\left[C_0\left(\alpha_s, \frac{m_Q}{\mu}\right)\langle \psi|\psi^\dagger \sigma_i \chi|0\rangle(\mu) + \frac{C_1\left(\alpha_s, \frac{m_Q}{\mu}\right)}{6m_Q^2}\langle \psi|\psi^\dagger \vec{D}^2 \sigma_i \chi|0\rangle(\mu) + \mathcal{O}(v^4)\right]$$

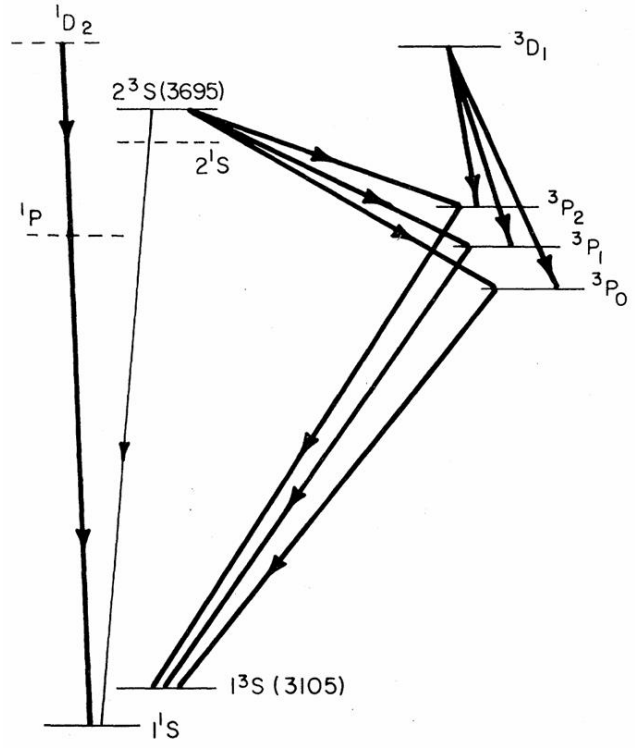
M. Beneke et al., Phys.Rev.Lett. 80 (1998) 2535-2538

J/ψ is an ideal probe to test perturbative QCD and nonperturbative NRQCD

Charmonium states in studying strong interaction QCD

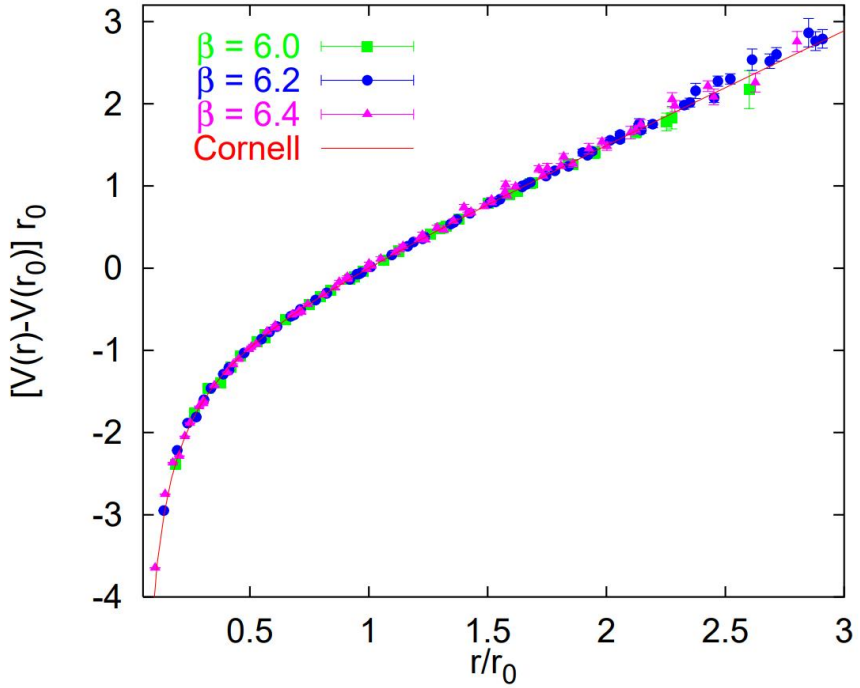
➢ Spectrum of charmonium states

Cornell model $V_0(r) = -\frac{4\alpha_s}{3r} + \sigma r$



E. Eichten et al., Phys.Rev.Lett. 34 (1975) 369-372

Lattice QCD

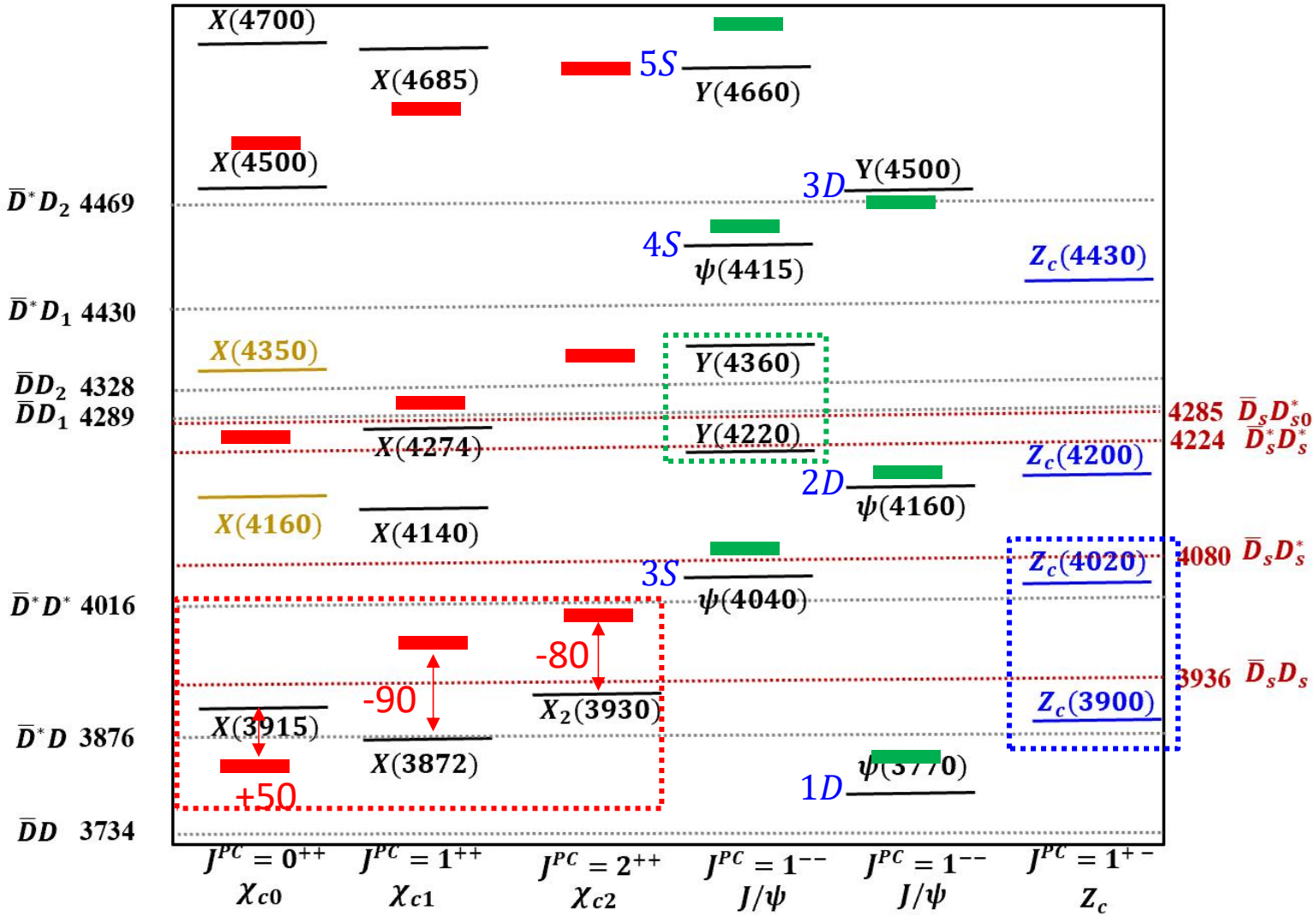


Gunnar S. Bali, Phys. Rept. 343 (2001) 1-136

From the spectrum of charmonium states, the long-range and short-range properities are verified

The states beyond conventional charmonium states

➢ Spectrum of charmonium states



- $Y(4230)$ and $Y(4360)$ states are not arranged into the vector charmonium spectrum
- $\chi_{cJ}(2P)$ states deviate from the experimental data, $\chi_{cJ}(3P)$ states are not well experimentally established
- Z_c states beyond conventional quark configuration $\bar{q}q$

These states are beyond conventional quark model !

The states beyond conventional charmonium states

➢ The decays of charmonium states

- **Isospin breaking**

$$\frac{\mathcal{B}(X(3872)\to\psi\pi^+\pi^-\pi^0)}{\mathcal{B}(X(3872)\to\psi\pi^+\pi^-)}=1.0\pm0.4\pm0.3$$

Belle Collaboration, arXiv:hep-ex/0505037

$$\frac{\mathcal{B}(X(3872)\to\psi\omega)}{\mathcal{B}(X(3872)\to\psi\pi^+\pi^-)}=0.7\pm0.3$$

BaBar Collaboration, Phys.Rev.D 82 (2010) 011101

$$\frac{\mathcal{B}(X(3872)\to\psi\omega)}{\mathcal{B}(X(3872)\to\psi\pi^+\pi^-)}=1.6^{+0.4}_{-0.3}\pm0.2$$

BESIII Collaboration, Phys.Rev.Lett. 122 (2019) 232002

$$\frac{g(X(3872)\to\psi\rho^0)}{g(X(3872)\to\psi\omega)}=0.29\pm0.04$$

LHCb Collaboration, Phys.Rev.D 108 (2023) L011103

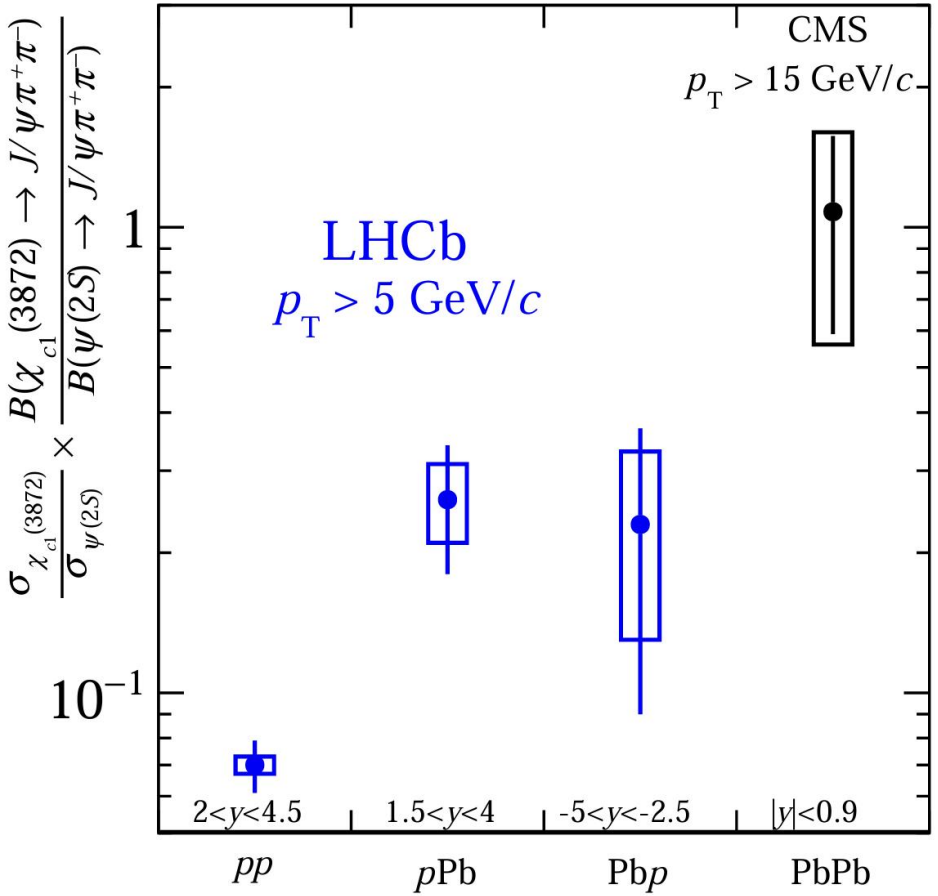
- **Exotic decay behavior**

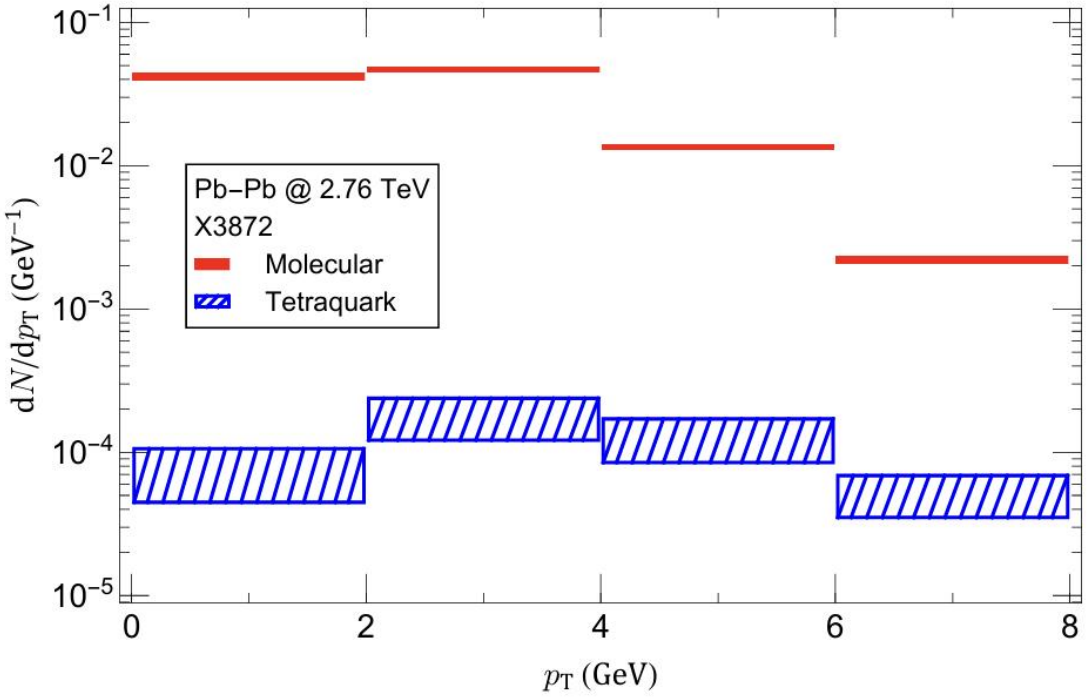
$Y(4260)\to J/\psi\pi^+\pi^-,\ J/\psi K^+K^-$

$Y(4260)$ dominantly decay into hidden charm modes

The states beyond conventional charmonium states

➢ The productions of X(3872) at heavy ion collisions





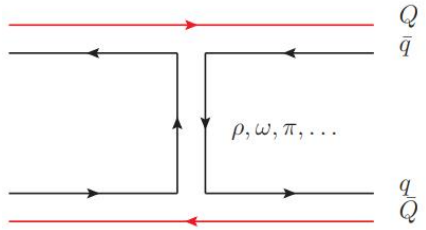
Zhang hui et al., Phys.Rev.Lett. 126 (2021) 012301

pPb LHCb Collaboration., Phys.Rev.Lett. 132 (2024) 242301

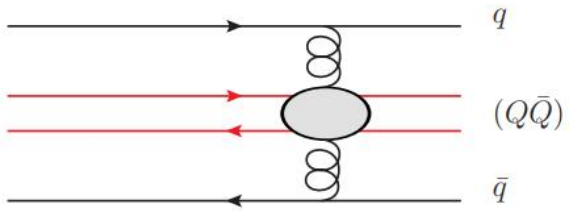
PbPb CMS Collaboration., Phys.Rev.Lett. 128 (2022) 032001

The challenge of XYZ states

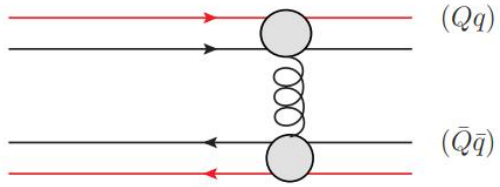
➢ The Genuine states



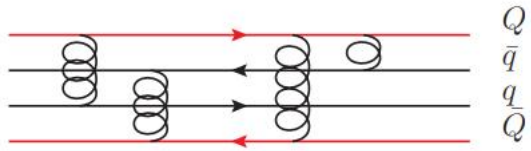
Molecule



Hadro-quarkonium



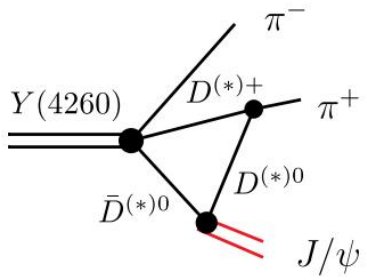
Diquarkonium



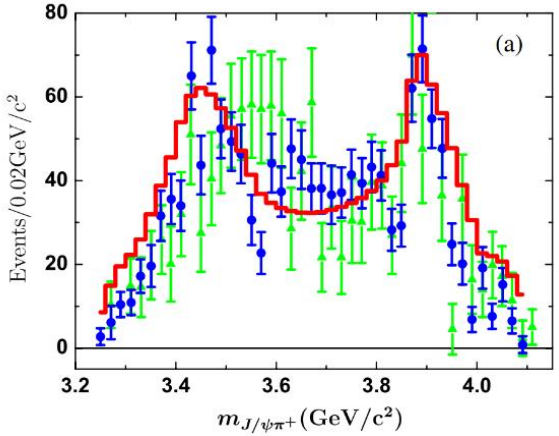
Mess

➢ Fake states

Cusp effect



Chen et al., Phys.Rev.D 88 (2013) 3, 036008



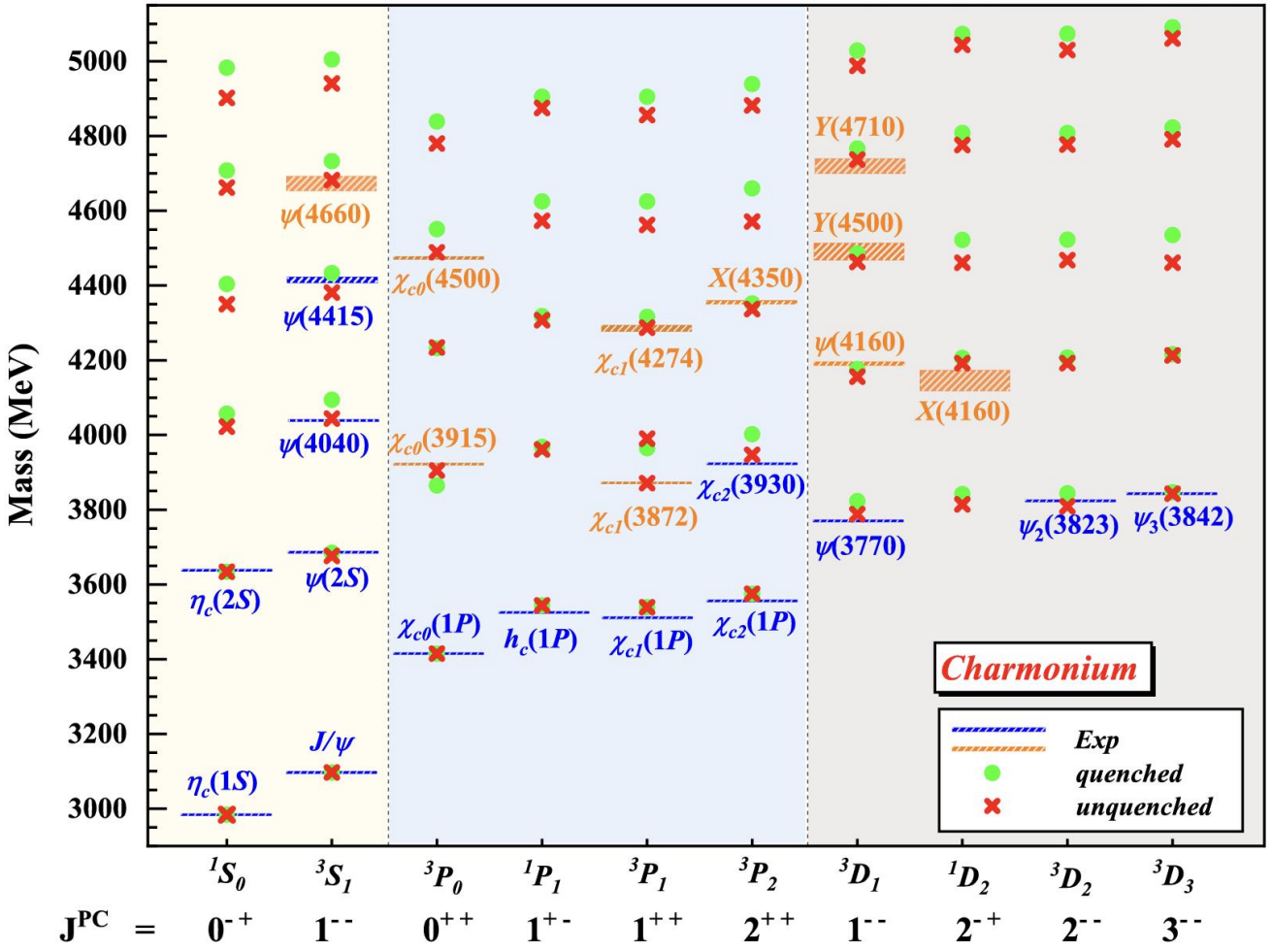
How the strong interaction bind the quarks and glous into the XYZ states?

Outline

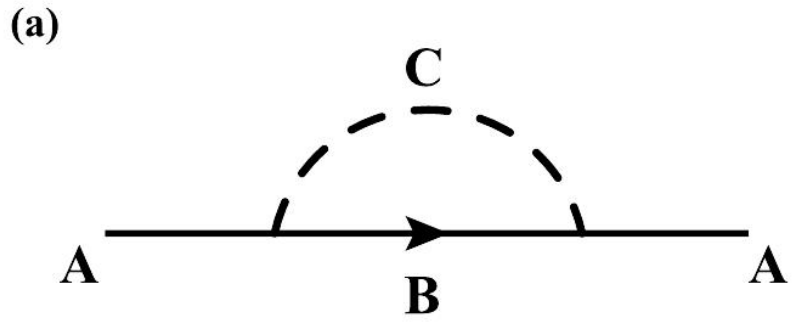
- Charmonium(like) states and strong interactions
- Theoretical studies on charmonium(like) states
- Symmetry, Three-body system and Femtoscopy
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Haron-hadron interactions for XYZ states

➢ Spectrum in the unquenched quark model



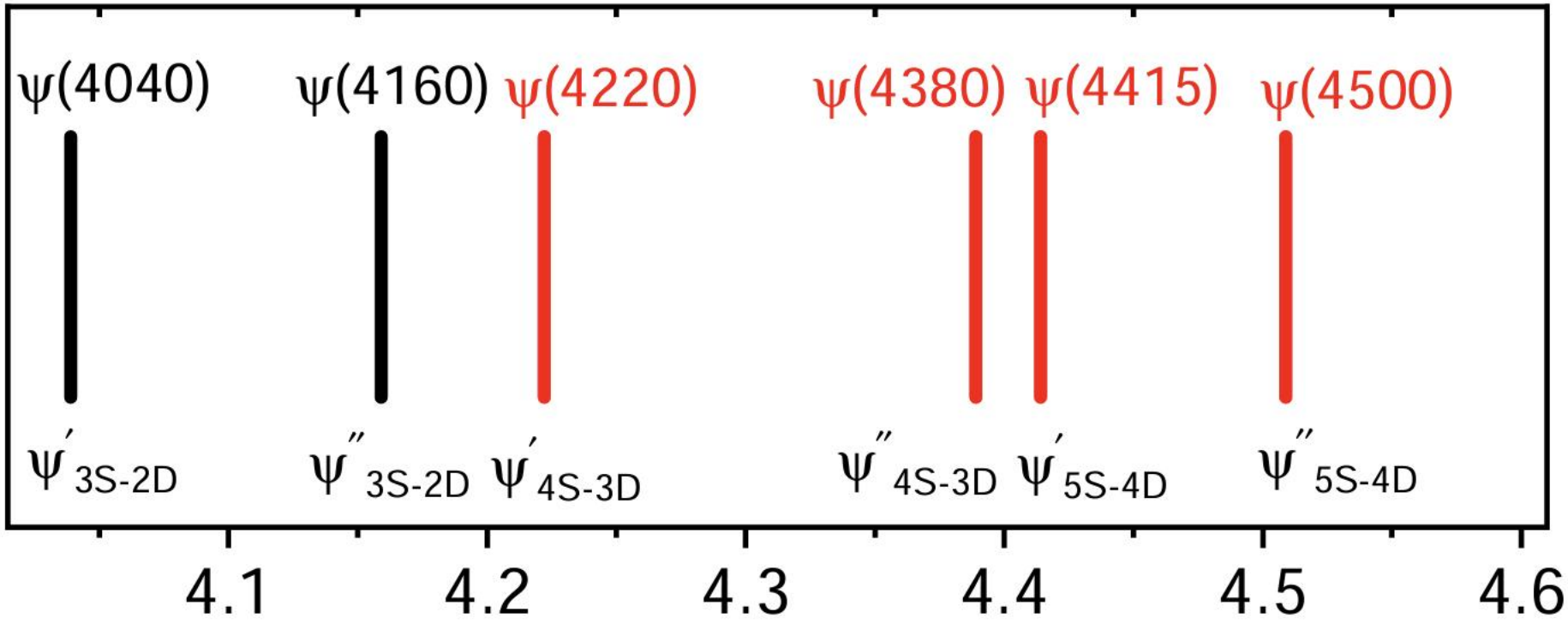
Deng Qian et al., Phys.Rev.D 110 (2024) 056034



The effect of hadron loop implies that the hadron-hadron interaction plays dominant role in explaining the XYZ states

S–D mixing provides a dynamical mechanism for XYZ states

➢ **Spectrum in the unquenched quark model**



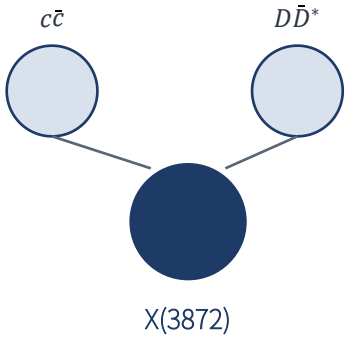
Jun-Zhang Wang et al., Phys.Lett.B 849 (2024) 138456

The orbit-angular momentum mixing can arrange the vector charmonium(like) states

Mixing hadronic molecule and bare state

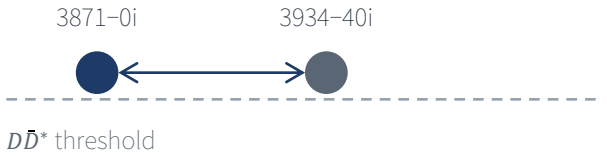
➢ Probability of hadronic molecule and bare state for X(3872)

	Ortega 0907. 3997	Yamaguchi 1908. 08790	Li (Lattice QCD) 2402. 14541	Deng 2312. 10296	BESIII 2309. 01502	Esposito 2108. 11413
$\bar{c}c$	7%	6%	4%	10%	18%	5%-14%
$\bar{D}^*D$	93%	94%	96%	90%	82%	95%-86%



$n^{2s+1}L_J$	M_{expt}	Γ_{expt}	M_{BW}	Γ_{BW}	pole	GI
2^3P_2	3927.2 ± 2.6	24 ± 6	3920	10	3920-4i	3979
2^3P_1	3942 ± 9	37^{+27}_{-17}			3934-40i	3953
	3871.69 ± 0.17	< 1.2	3871	0	3871-0i	
2^3P_0	3862^{+66}_{-45}	201^{+242}_{-149}	3878	11	3878-5i	3917
2^1P_1			3895	37	3902-27i	3956

The physical state X(3872) is originated from the hadronic moleucle rather than the bare state



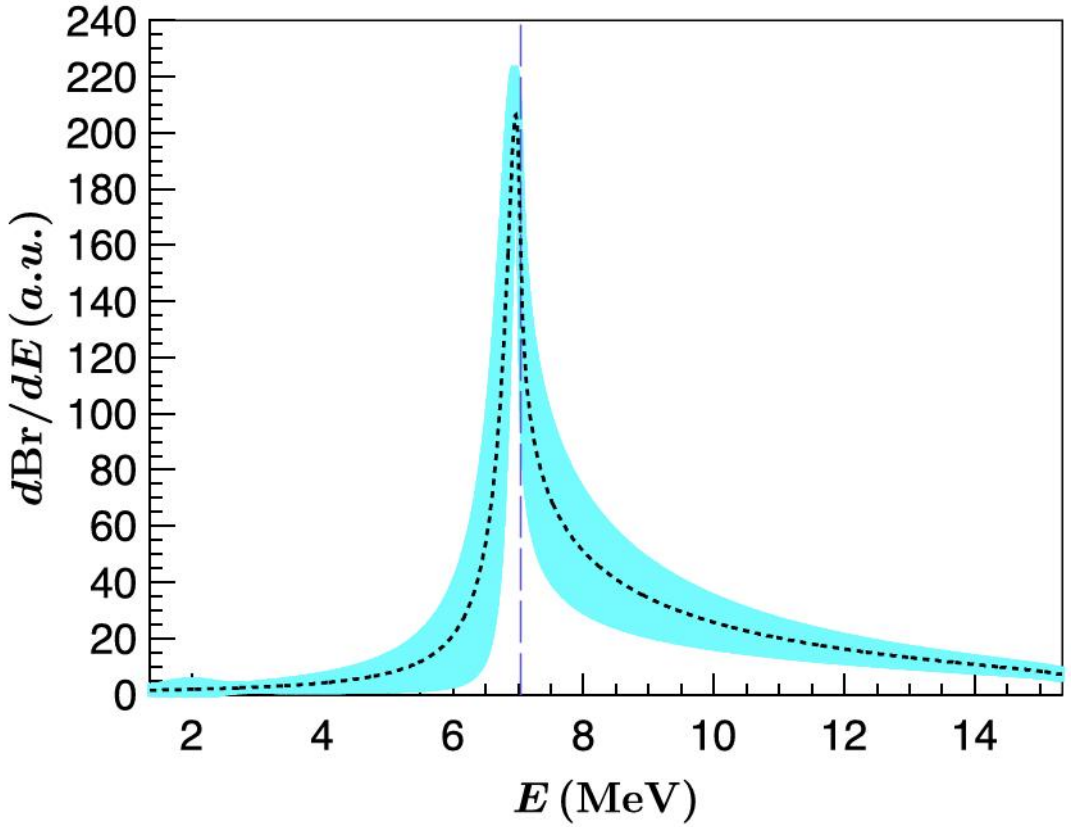
Zhi-Yong Zhou et al., Phys.Rev.D 96 (2017) 054031
Pan-Pan Shi et al., Phys.Rev.D 111 (2025) 074043



Jun-Zhang Wang et al., Phys.Rev.D 111 (2025) L111502

Invariant mass distributions for X(3872)

➢ Spectrum and decay widths (Invariant mass distributions)



$X(3872) \to J/\psi\pi^+\pi^-$ $X(3872) \to \bar{D}^0 D^0 \pi^0$

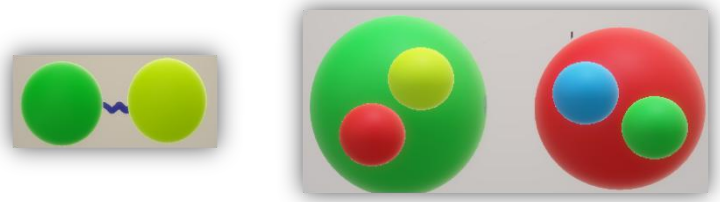
BESIII Collaboration., Phys.Rev.Lett. 132 (2024) 151903

- Pole position

$$M_X = (3871.63 \pm 0.13^{+0.06}_{-0.05})\ \text{MeV}$$

- Scattering length and effective range

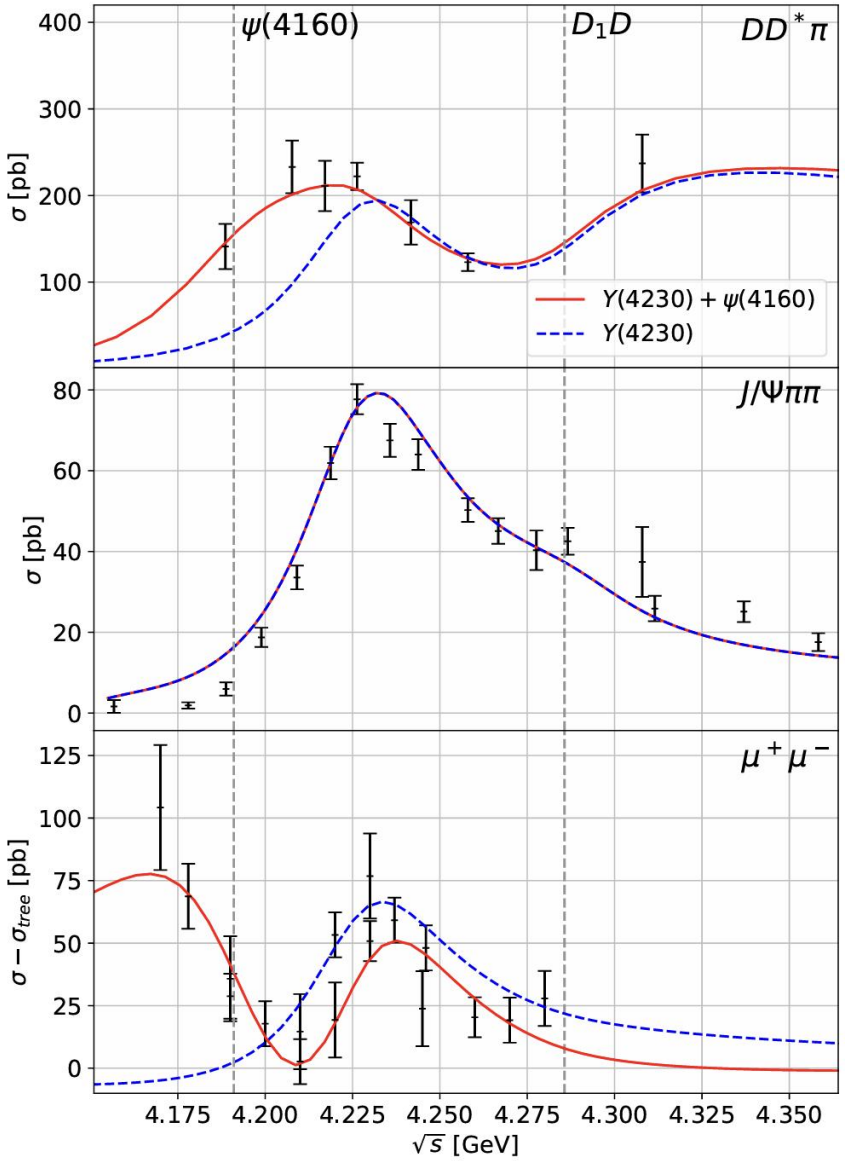
a =− 16.5 fm r =− 4.1 fm

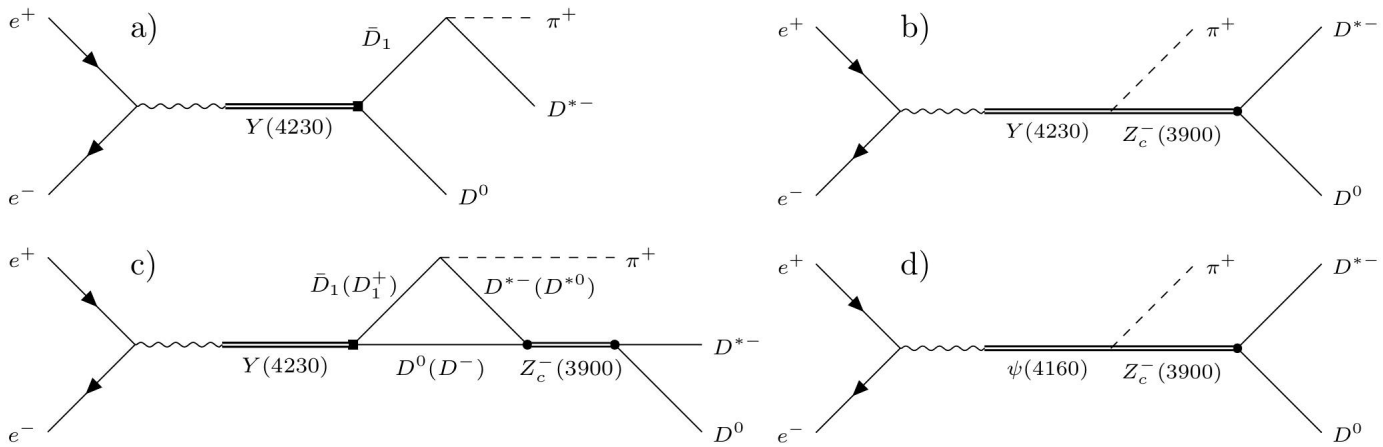


Molecule and bare state mixing

Invariant mass distributions for Y(4260)

➢ Spectrum and decay widths (Invariant mass distributions)





✓ $Y(4230)$ is treated as a $\bar{D}_1 D$ hadronic molecule

✓ The large interference between $Y(4230)$ and $\psi(4160)$

Leon von Detten et al., Phys.Rev.D 109 (2024) 116002

Invariant mass distributions for Y(4260)

➢ Spectrum and decay widths (Invariant mass distributions)

TABLE III. Vector charmonium poles and $\psi(4660)$ BW parameters. See the text for the notations.

This work		PDG(ψ) [4], BESIII [16, 38, 88]		
M (MeV)	Γ (MeV)	M (MeV)	Γ (MeV)	
3764.2 ± 2.0	47.3 ± 2.6	3751.9 ± 3.8	32.8 ± 5.8	$^{\mathrm{r}}D\bar{D}$
3780.2 ± 1.2	29.9 ± 2.3	3778.1 ± 0.7	27.5 ± 0.9	$\psi(3770)$
3898.4 ± 0.9	127.5 ± 6.7	3872.5 ± 14.2	179.7 ± 14.1	$^{\mathrm{r}}D^*\bar{D}$
3956.1 ± 1.0	96.8 ± 10.4	–	–	$^{\mathrm{r}}D_s\bar{D}_s$
4029.2 ± 0.4	26.3 ± 1.0	4039 ± 1	80 ± 10	$\psi(4040)$
4052.4 ± 0.4	49.0 ± 0.3	–	–	$^{\mathrm{v}}D_s^*\bar{D}_s$
4192.2 ± 2.2	129.3 ± 4.2	4191 ± 5	70 ± 10	$\psi(4160)$
4216.2 ± 0.5	40.3 ± 1.0	–	–	$^{\mathrm{v}}D_s^*\bar{D}_s^*$
4229.9 ± 0.9	46.4 ± 2.6	4222.5 ± 2.4	48 ± 8	$\psi(4230)$
4308.1 ± 2.2	138.2 ± 4.4	4298 ± 12	127 ± 17	$Y(4320)$
4346.2 ± 3.8	122.8 ± 6.7	4374 ± 7	118 ± 12	$\psi(4360)$
4390.1 ± 2.0	106.5 ± 4.1	4421 ± 4	62 ± 20	$\psi(4415)$
4496.3 ± 3.1	16.4 ± 2.1	–	–	$^{\mathrm{b}}D_{s1}\bar{D}_s$
4579.6 ± 1.7	-5.2 ± 7.6	–	–	$^{\mathrm{b}}\Lambda_c\bar{\Lambda}_c$
4655.9 ± 3.0	134.9 ± 5.9	4630 ± 6	72^{+14}_{-12}	$\psi(4660)$

- ✓ The width of $\psi(4160)$ becomes large
- ✓ $Y(4230)$ is treated as a mixing state of $\bar{D}_1 D$ hadronic molecule and bare states
- ✓ There exists an additional state between $\psi(4160)$ and $Y(4230)$

S.X. Nakamura et al., Phys.Rev.D 112 (2025) 054027

Radiative Decay for X(3872)

➢ Radiative Decays

- **Experimental measurements**

$$\frac{\mathcal{B}(X(3872)\to\psi(2S)\gamma)}{\mathcal{B}(X(3872)\to\psi\gamma)} = 3.4\pm1.4$$

BaBar Collaboration, Phys.Rev.Lett. 102 (2009) 132001

$$\frac{\mathcal{B}(X(3872)\to\psi(2S)\gamma)}{\mathcal{B}(X(3872)\to\psi\gamma)} < 2.1$$

Belle Collaboration, Phys.Rev.Lett. 107 (2011) 091803

$$\frac{\mathcal{B}(X(3872)\to\psi(2S)\gamma)}{\mathcal{B}(X(3872)\to\psi\gamma)} = 1.67\pm0.21\pm0.12\pm0.04$$

LHCb Collaboration, JHEP 11 (2024) 121

$$\frac{\mathcal{B}(X(3872)\to\psi(2S)\gamma)}{\mathcal{B}(X(3872)\to\psi\gamma)} < 0.59$$

BESIII Collaboration, Phys.Rev.Lett. 124 (2020) 242001

- **Pure charmonium state**

$$\frac{\mathcal{B}(X(3872)\to\psi(2S)\gamma)}{\mathcal{B}(X(3872)\to\psi\gamma)} > 1$$

T. Barnes et al., Phys.Rev.D 72 (2005) 054026

Bai-Qing Li et al., Phys.Rev.D 79 (2009) 094004

A.M. Badalian et al., Phys.Rev.D 85 (2012) 114002

- **Compact states**

$$\frac{\mathcal{B}(X(3872)\to\psi(2S)\gamma)}{\mathcal{B}(X(3872)\to\psi\gamma)} \geq 1$$

B. Grinstein et al., Phys.Rev.D 109 (2024) 074009

- **Molecules**

$$\frac{\mathcal{B}(X(3872)\to\psi(2S)\gamma)}{\mathcal{B}(X(3872)\to\psi\gamma)} < 1$$

Eric S. Swanson, Phys.Lett.B 598 (2004) 197-202

Yubing Dong et al., J.Phys.G 38 (2011) 015001

Feng-Kun Guo et al., Particles 9 (2026) 46

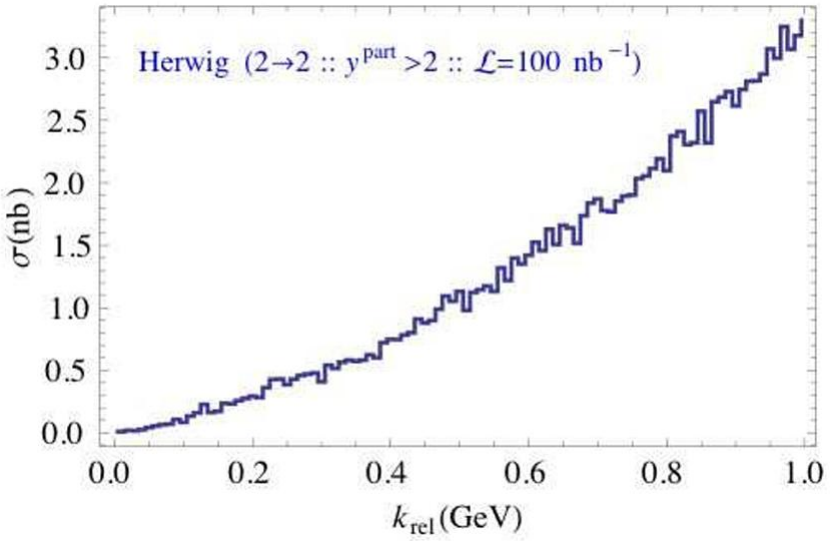
Prompt production of X(3872) at the Tevatron

➢ Production of X(3872) at the Tevatron

$$\frac{\sigma(p\bar{p} \to X(3872) + \text{All})_{\text{prompt}} \times \mathcal{B}(X(3872) \to J/\psi\pi^+\pi^-))}{\sigma(p\bar{p} \to \psi(2S) + \text{All})} = 4.7 \pm 0.8\%$$

$$\sigma(p\bar{p} \to X(3872) + \text{All})^{\text{min}} > \sigma(p\bar{p} \to X(3872) + \text{All}) \times \mathcal{B}(X(3872) \to J/\psi\pi^+\pi^-) = 3.1 \pm 0.7 \text{ nb}$$

σ = 0.08 nb



C. Bignamini et al., Phys.Rev.Lett. 103 (2009) 162001

$$\boldsymbol{\sigma_{\text{prompt}}(X(3872)) \approx 33 \sim 72 \text{ nb}}$$

$$\sigma[X(3872)] \approx \sigma_{\text{naive}}[D^{*0}\bar{D}^0(k < \Lambda)] \times \frac{6\pi\sqrt{2\mu E_X}}{\Lambda}$$

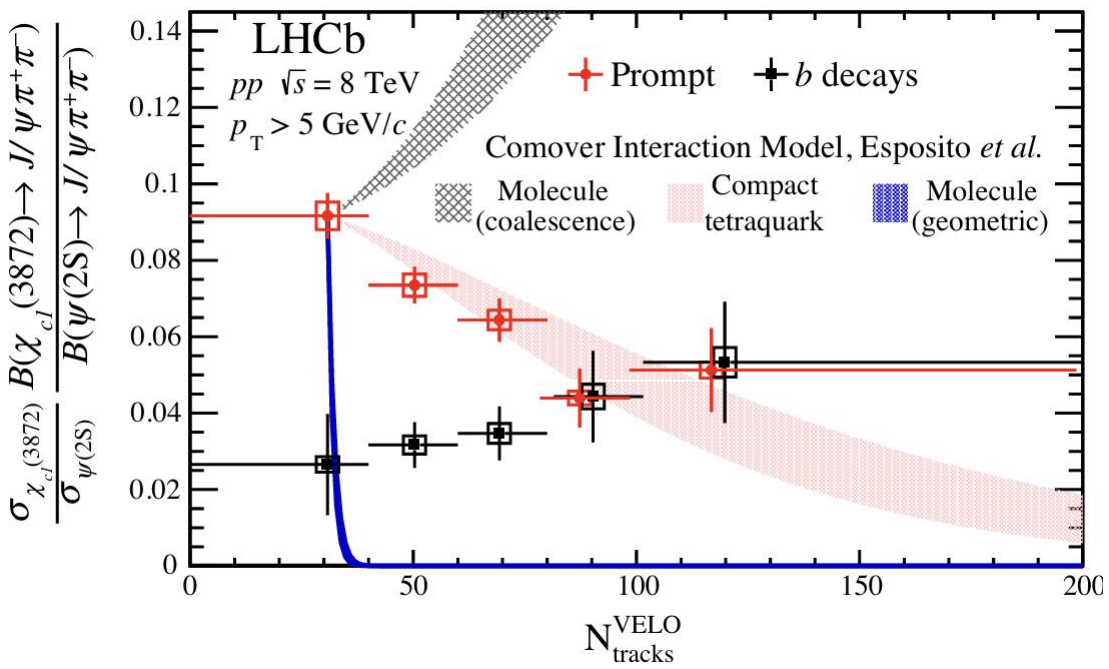
σ = 1.5~23 nb

Eric Braaten et al., Phys.Rev.D 81 (2010) 114018

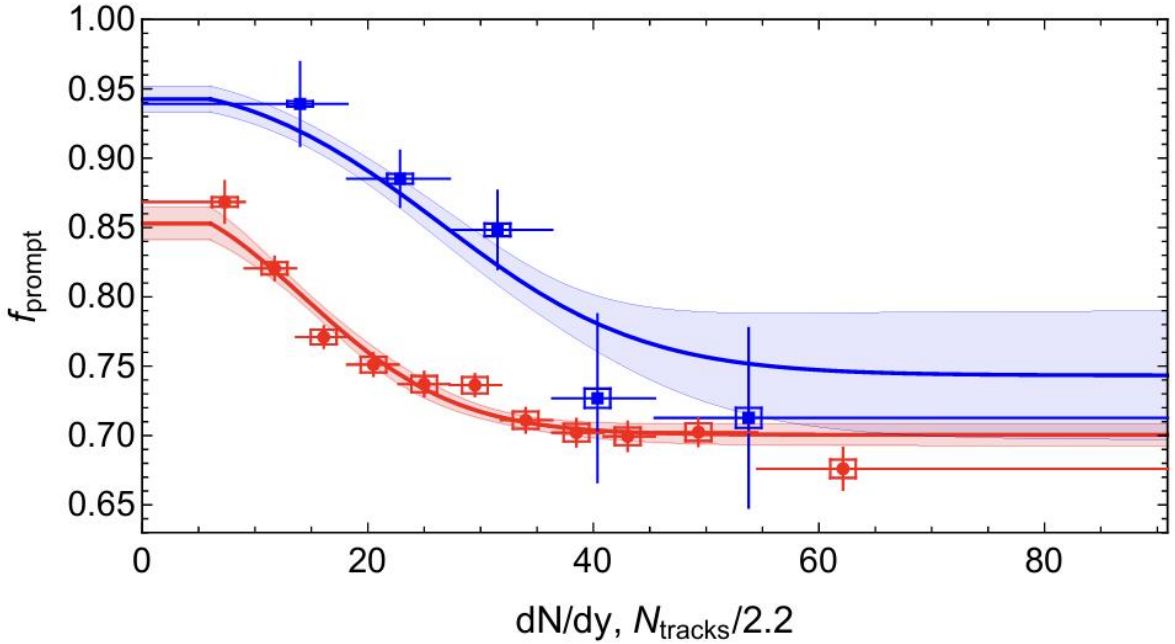
Favor the molecular interpretation

Prompt production of X(3872) at pp collision

➢ Production of X(3872) at the pp collision



LHCb Collaboration., Phys.Rev.Lett. 126 (2021) 9, 092001



Eric Braaten et al., Phys.Rev.D 103 (2021) L071901

- Prompt production probe the internal structure of X(3872)

Productions of exotic states in b-flavored hadron decays

➢ **A lot of new hadron states were discovered in b-flavored hadron decays**

Decay modes	New states	Decay modes	New states
$\Xi_b \to J/\psi\Lambda K$	$P_{cs}(4459)$	$\Lambda_b \to J/\psi pK$	$P_c(4312)/P_c(4440)/P_c(4457)$
$\Lambda_b \to Dp\pi$	$\Lambda_c(2940)$	$B^+ \to D_s{}^+\pi^0\overline{D}^0$	$D_{s0}(2317)$
$B_s \to J/\psi p\bar{p}$	$P_c(4337)$	$B^+ \to D_s{}^{*+}\pi^0\overline{D}^0$	$D_{s1}(2460)$
$B \to J/\psi\Lambda\bar{p}$	$P_{cs}(4338)$	$B^+ \to D^-K^+D^+$	$X_0(2900)/X_1(2900)$
$B^+ \to D_s{}^+\pi^+D^-$	$T_{c\bar{s}0}(2900)^{++}$	$B^+ \to J/\psi\phi K^+$	$X(4140)/X(4274)$
$B^0 \to D_s{}^+\pi^-\overline{D}^0$	$T_{c\bar{s}0}(2900)^0$	$B^+ \to J/\psi\phi K^+$	$X(4500)/X(4700)$
$B^+ \to D^0\overline{D}^{*0}K^+$	$X(3872)$	$B^+ \to J/\psi K^+\phi$	$Z_{cs}(4000)/Z_{cs}(4220)$
$B^+ \to D_s{}^+D_s{}^-K^+$	$X(3960)$	$B^0 \to \psi'\pi^\mp K^\pm$	$Z_c(4430)$
$B^+ \to J/\psi\omega K^+$	$X(3915)$	$B^0 \to \chi_{c1}\pi^\mp K^\pm$	$Z_c(4051)/Z_c(4248)$
$B^+ \to K^+K^-K^+$	$f_0(980)$	$B^0 \to J/\psi\pi^\mp K^\pm$	$Z_c(4200)$
$B^+ \to \pi^0\pi^0K^+$	$f_0(980)$		

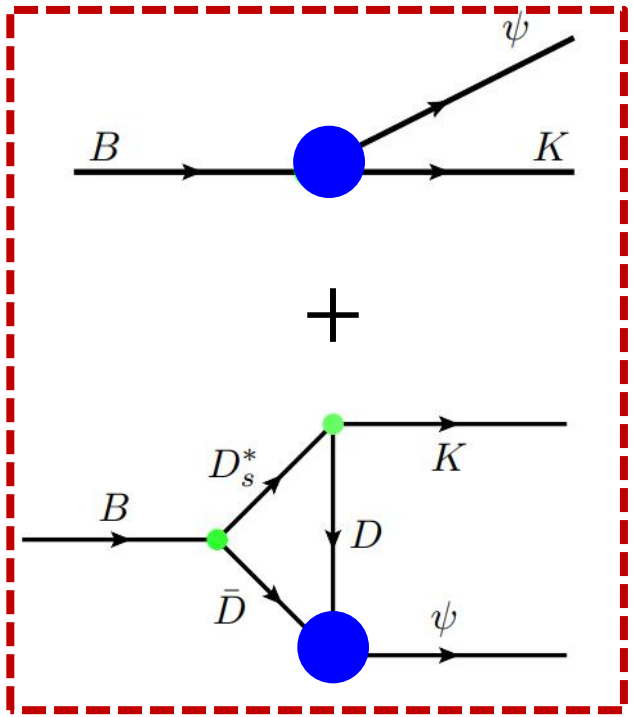
➢ **Providing rich physical observables to study the exotic states**

- **Branching fraction**
- **Invariant mass distribution**
- **CP violation**
- **Angular distributions**

Productions of charmonium(like) states in B decays

➢ A unified descriptions of the productions of vector charmonium(like) in B decays

	$\psi(1S)$	$\psi(2S)$	$\psi(4040)$	$\psi(3770)$	$\psi(4160)$
$B \to K$	$(1.02 \mp 0.19) \times 10^{-3}$	$(6.24 \mp 0.21) \times 10^{-4}$	$(1.\ 6 \mp 0.5) \times 10^{-3}$	$(4.3 \mp 1.1) \times 10^{-4}$	$(5.1 \mp 2.7) \times 10^{-4}$



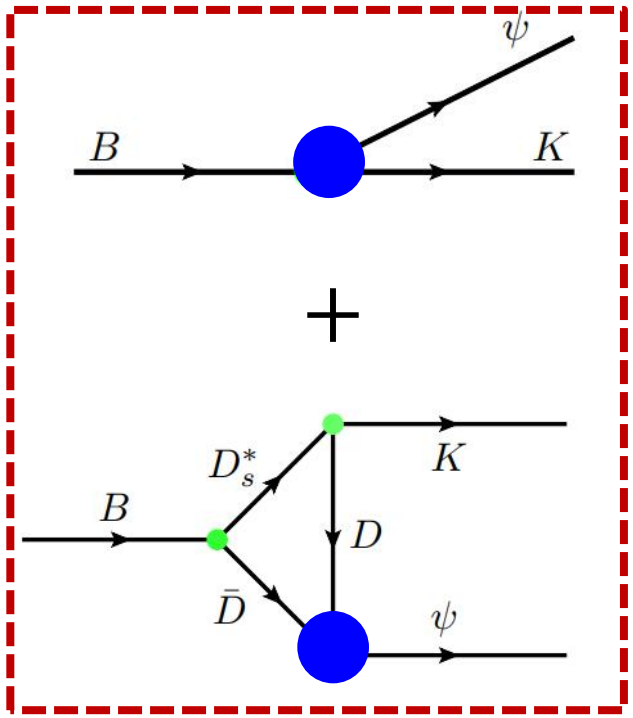
Decay modes	Short-range	Long range	Total	Experiments
$\to J/\psi$	1.551	$2.818 \sim 89.519$	$8.402 \sim 112.177$	10.20 ± 0.19
$\to \psi(2S)$	0.640	$2.299 \sim 59.555$	$5.351 \sim 71.948$	6.24 ± 0.21
$\to \psi(4040)$	0.193	$0.331 \sim 11.847$ $1.132 \sim 29.862$	$1.006 \sim 15.019$ $2.255 \sim 34.434$	16.0 ± 5.0
$\to \psi(3770)$	0.070	$0.269 \sim 5.954$	$0.508 \sim 6.048$	4.3 ± 1.1
$\to \psi(4160)$	0.090	$0.010 \sim 0.809$ $0.035 \sim 2.216$	$0.161 \sim 1.437$ $0.247 \sim 3.228$	5.1 ± 2.7

Qi-Wei Yuan et al., Phys.Rev.D 111 (2025) 114035

Productions of charmonium(like) states in B decays

➢ A unified descriptions of the productions of vector charmonium(like) in B decays

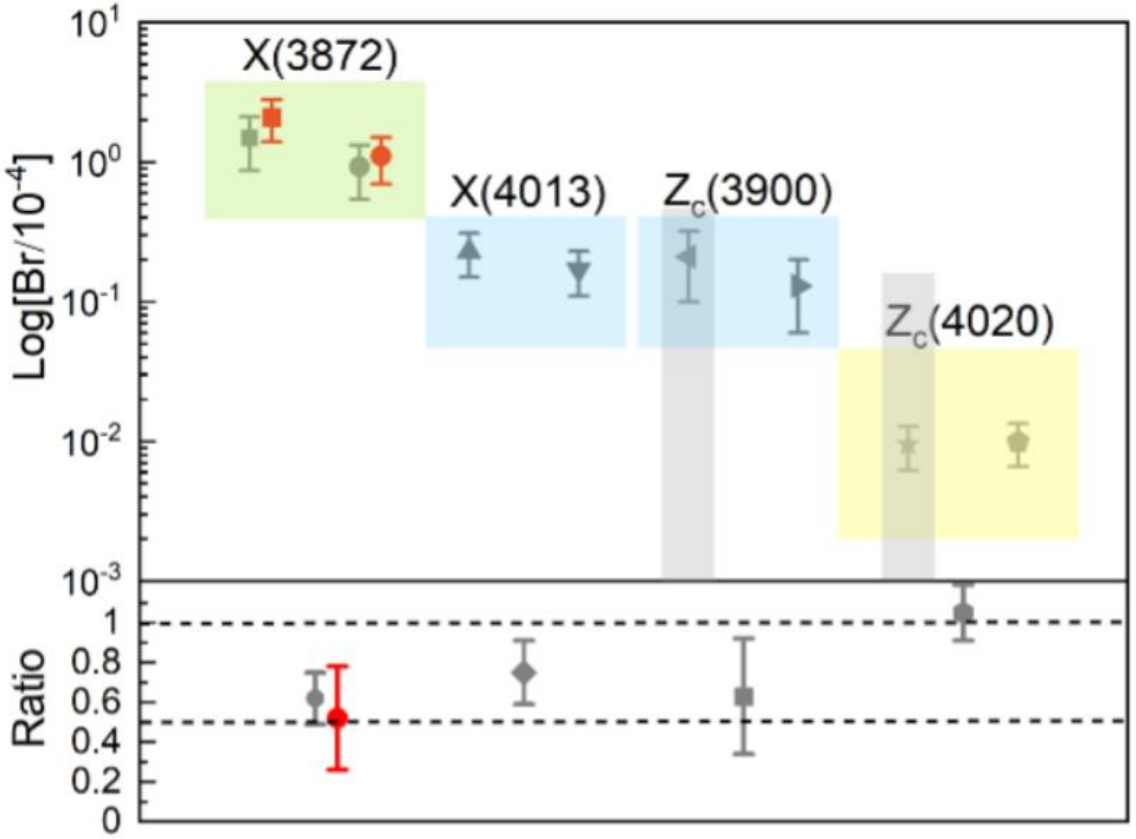
	$\psi(1S)$	$\psi(2S)$	$\psi(4040)$	$\psi(3770)$	$\psi(4160)$
$B \to K$	$(1.02 \mp 0.19) \times 10^{-3}$	$(6.24 \mp 0.21) \times 10^{-4}$	$(1.\ 6 \mp 0.5) \times 10^{-3}$	$(4.3 \mp 1.1) \times 10^{-4}$	$(5.1 \mp 2.7) \times 10^{-4}$



Decay modes	Short-range	Long range	Total	Experiments	
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$\to \psi(4160)$	0.090	$0.010 \sim 0.809$ $0.035 \sim 2.216$	$0.161 \sim 1.437$ $0.247 \sim 3.228$	5.1 ± 2.7	2%

- Long range contributions play the dominant role in the productions of charmonium states in B decays

Productions of charmoniumlike states in B decays



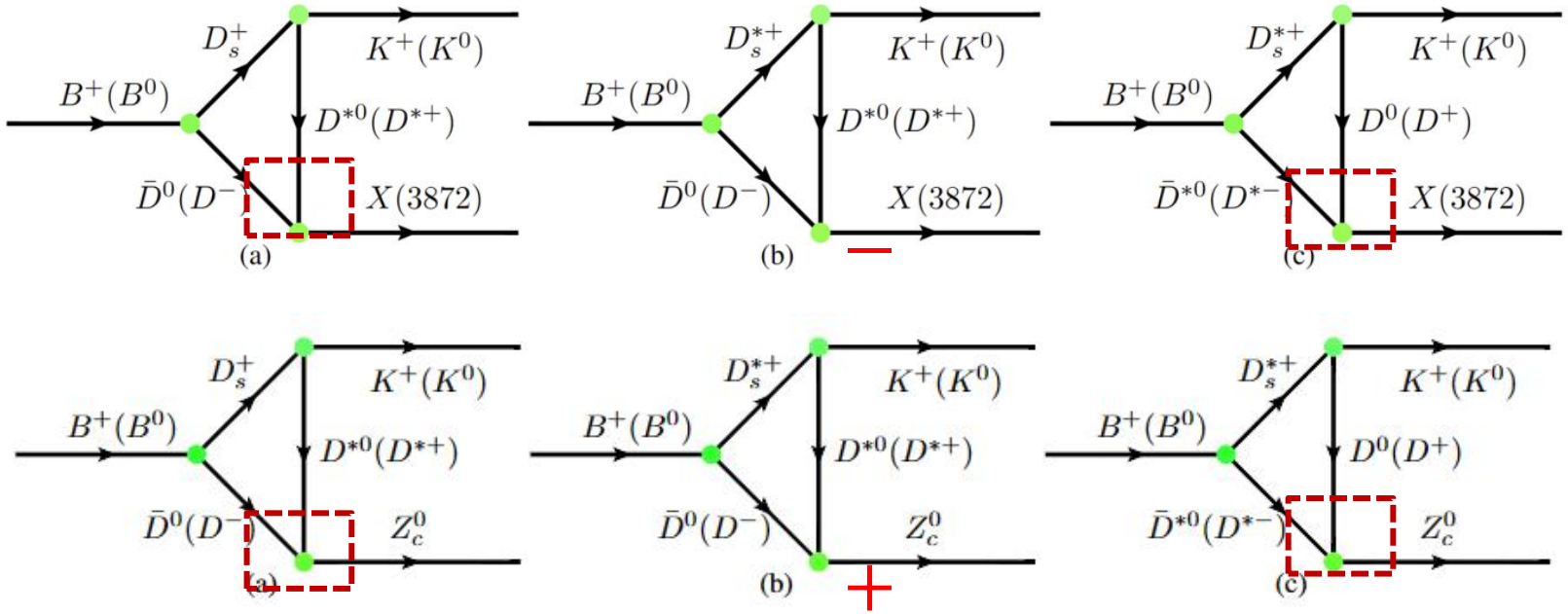
- Production rates of X(3872) in B meson decays are larger than those of X(4013) by one order of magnitude
- Production rates of Zc(3900) in B meson decays are larger than those of Zc(4020) by one order of magnitude
- Production rates of Zc(3900) in B meson decays are smaller than those of X(3872) by one order of magnitude

A hierarchy for the branching fractions of $\bar{D}^{(*)}D^{(*)}$ molecules in B decays

Qi Wu et al., Eur.Phys.J.C 84 (2024) 147

Productions of charmoniumlike states in B decays

- **Why branching fractions of isoscalar molecules are larger than those of isovector molecules**



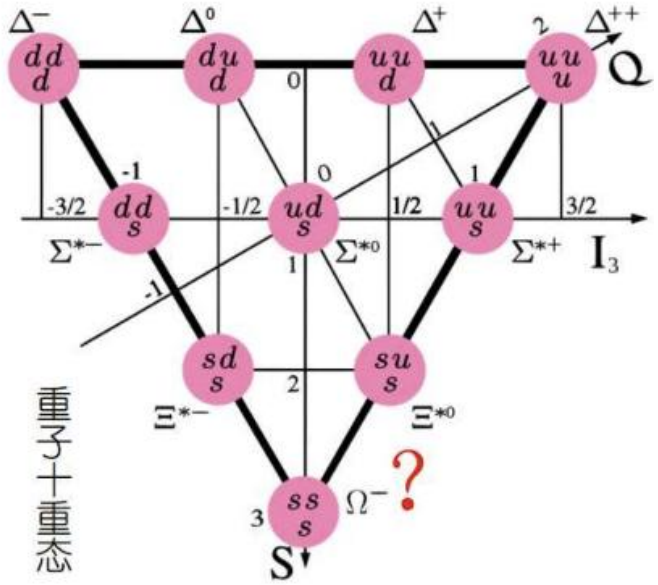
Molecules	$D^{*+}D^-$	D^+D^{*-}	$D^{*0}\bar{D}^0$	$D^0\bar{D}^{*0}$
$X(3872)$	$1/2$	$-1/2$	$1/2$	$-1/2$
$Z_c(3900)$	$1/2$	$1/2$	$-1/2$	$-1/2$

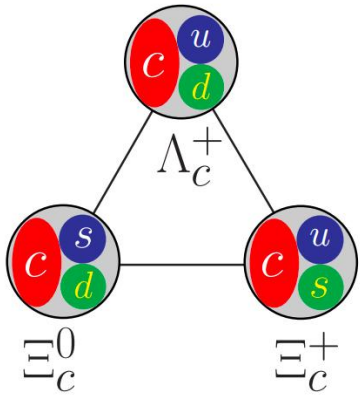
Amp a and c for isoscalar molecules add constructively, but those for isovector molecules add destructively

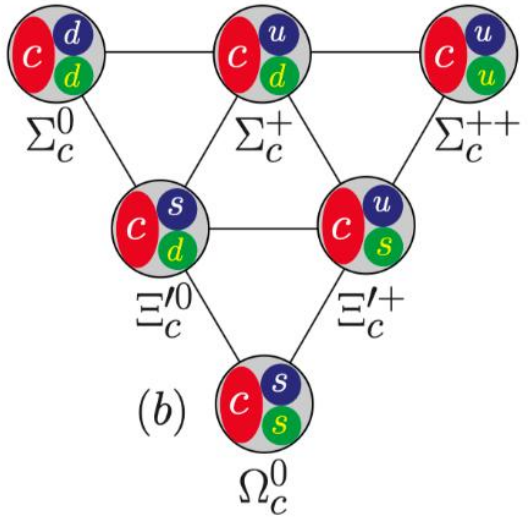
Outline

- Charmonium(like) states and strong interactions
- Theoretical studies on charmonium(like) states
- Symmetry, Three-body system and Femtoscopy
- Summary

Symmetry

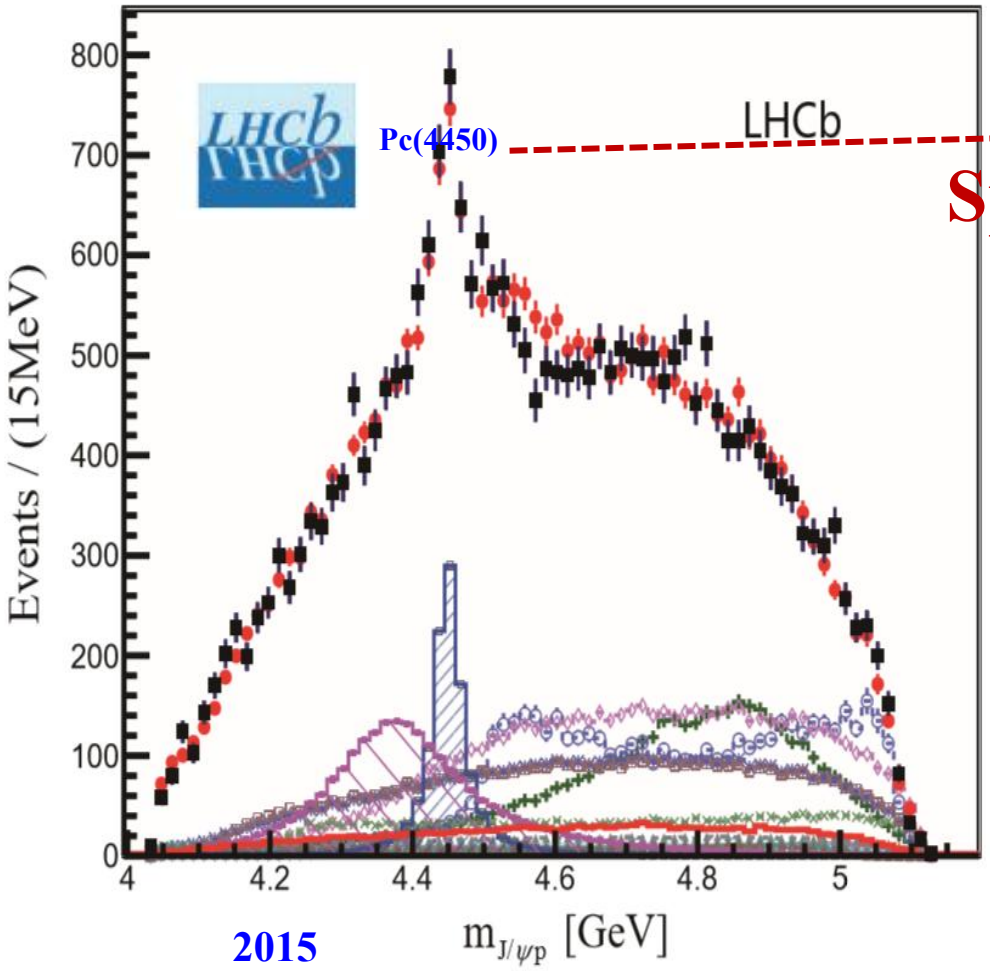

重子十重态
Q
I_3
S
Ω^-
?


Λ_c^+
Ξ_c^0
Ξ_c^+


Σ_c^0
Σ_c^+
Σ_c^{++}
$\Xi_c'^0$
$\Xi_c'^+$
Ω_c^0
(b)

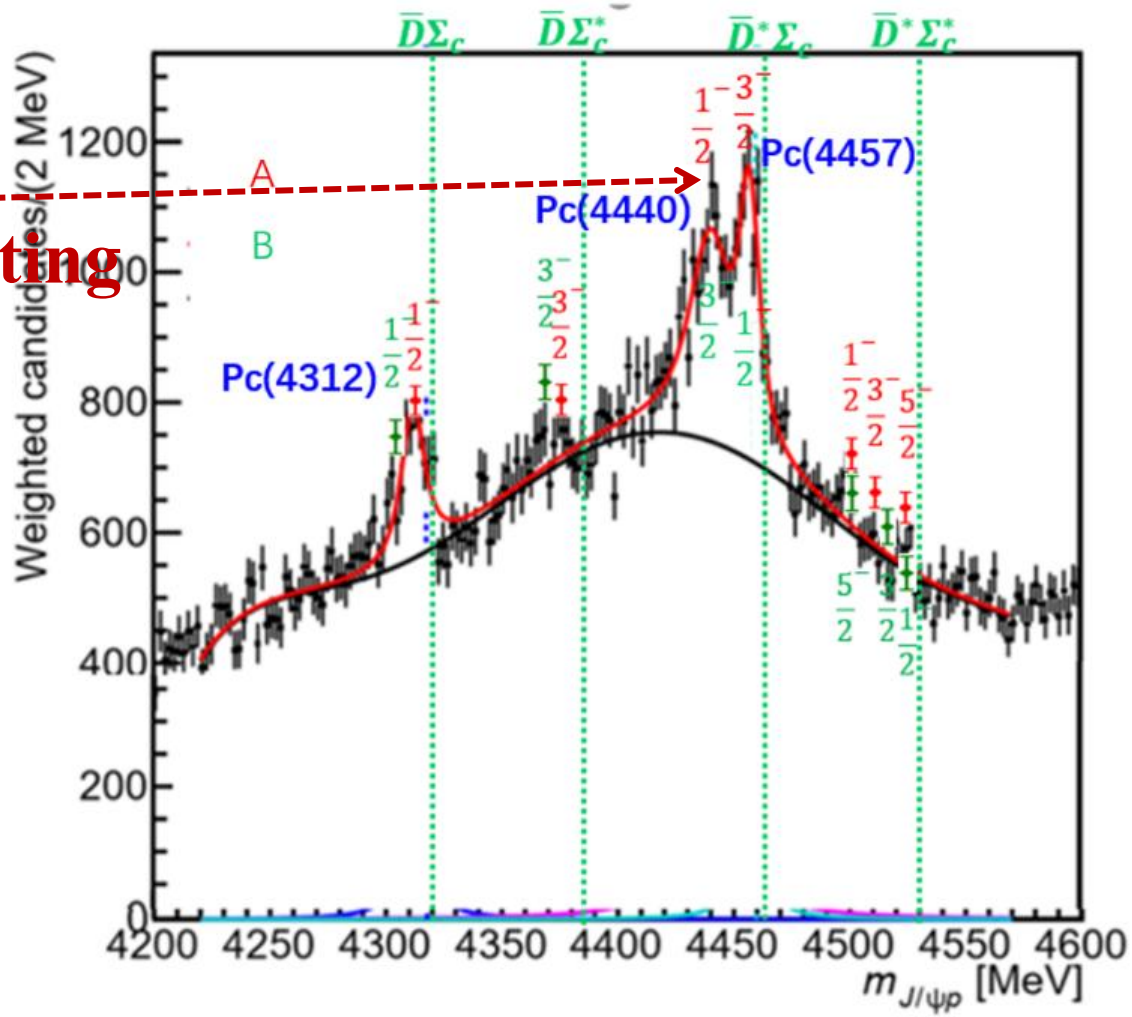
The experimental signal for fine structures of exotic states

➢ Hidden charm pentaquark states



2015

Splitting

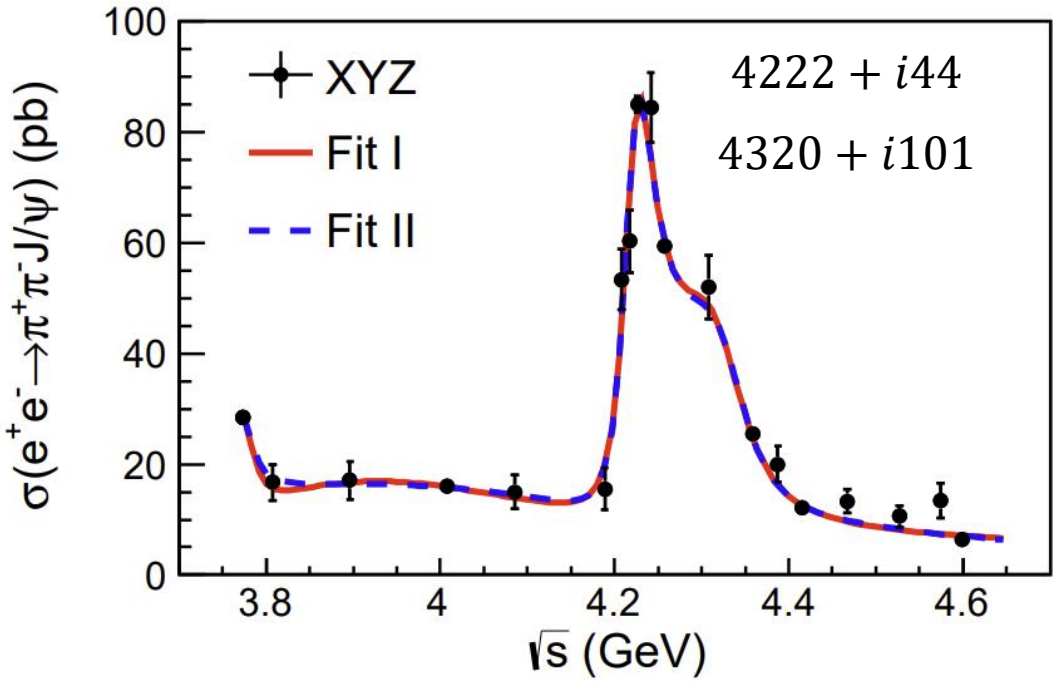


2019

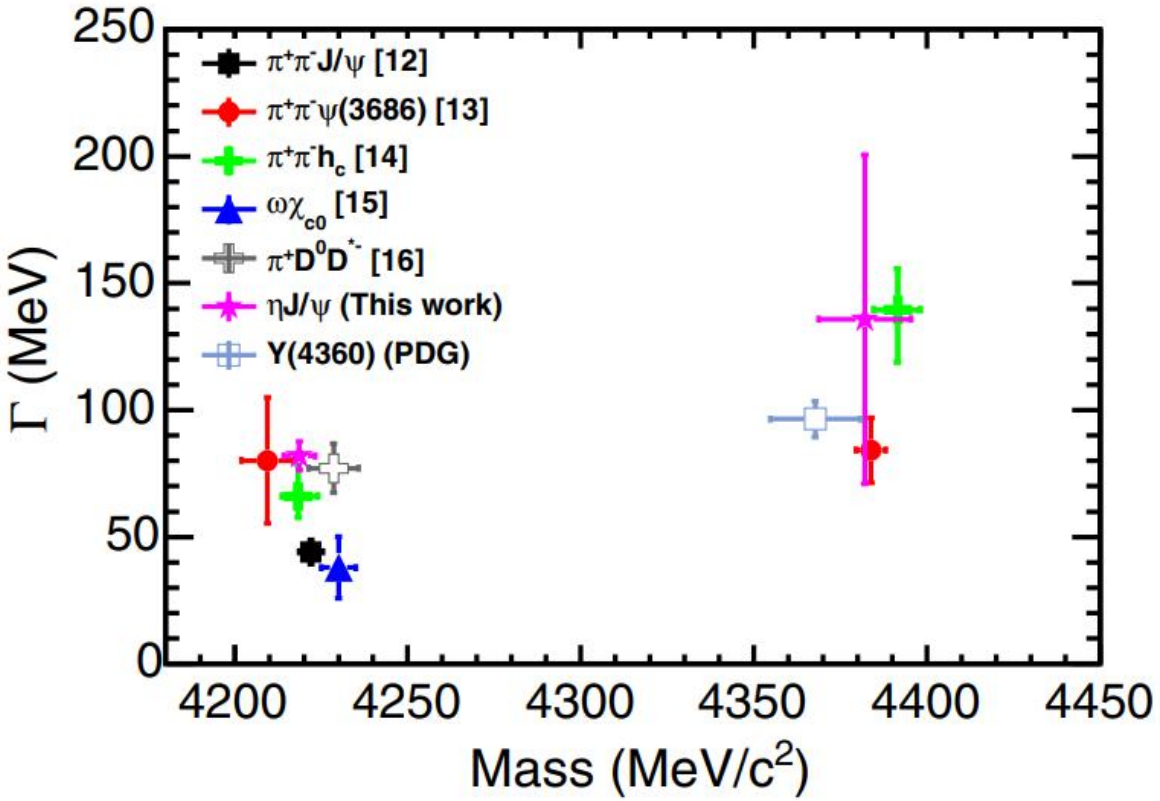
Fine structure!

The experimental signal for fine structures of exotic states

➢ Vector charmonium-like states



BESIII Collaboration., Phys.Rev.Lett. 118 (2017) 9, 092001



BESIII Collaboration, Phys.Rev.D 102 (2020) 3, 031101

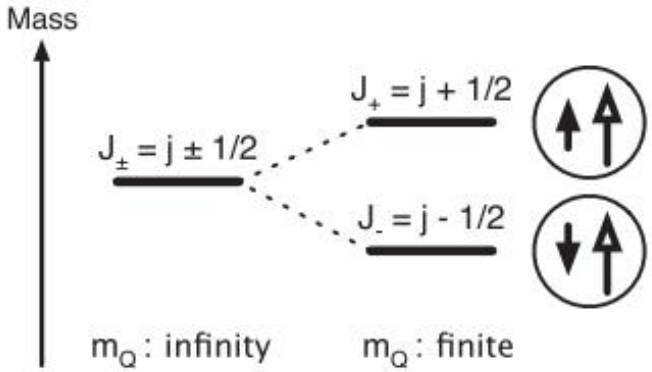
$$Y(4260) \longrightarrow \begin{cases} Y(4230) \\ Y(4360) \end{cases}$$

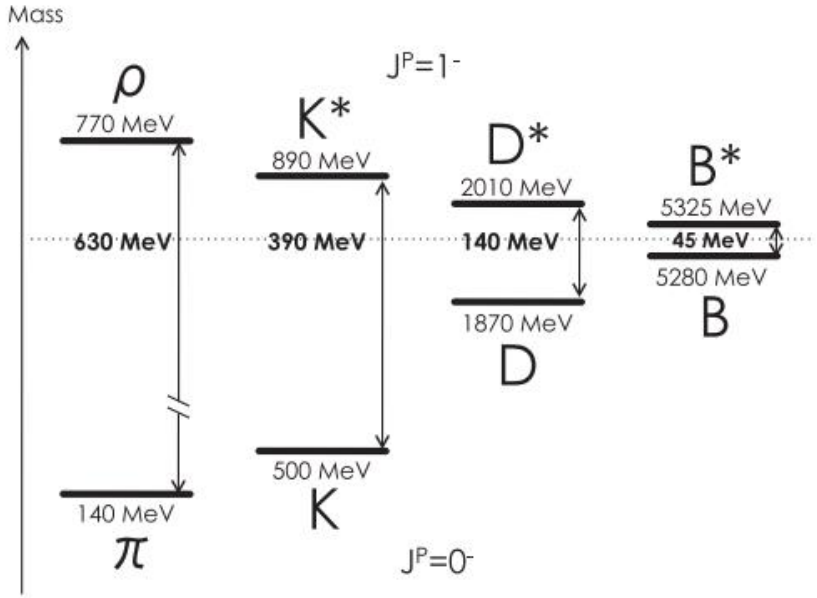
Fine structure!

Heavy quark spin symmetry(HQSS)

QCD interaction can not flip the spin of heavy quark

- **Spin doublet**





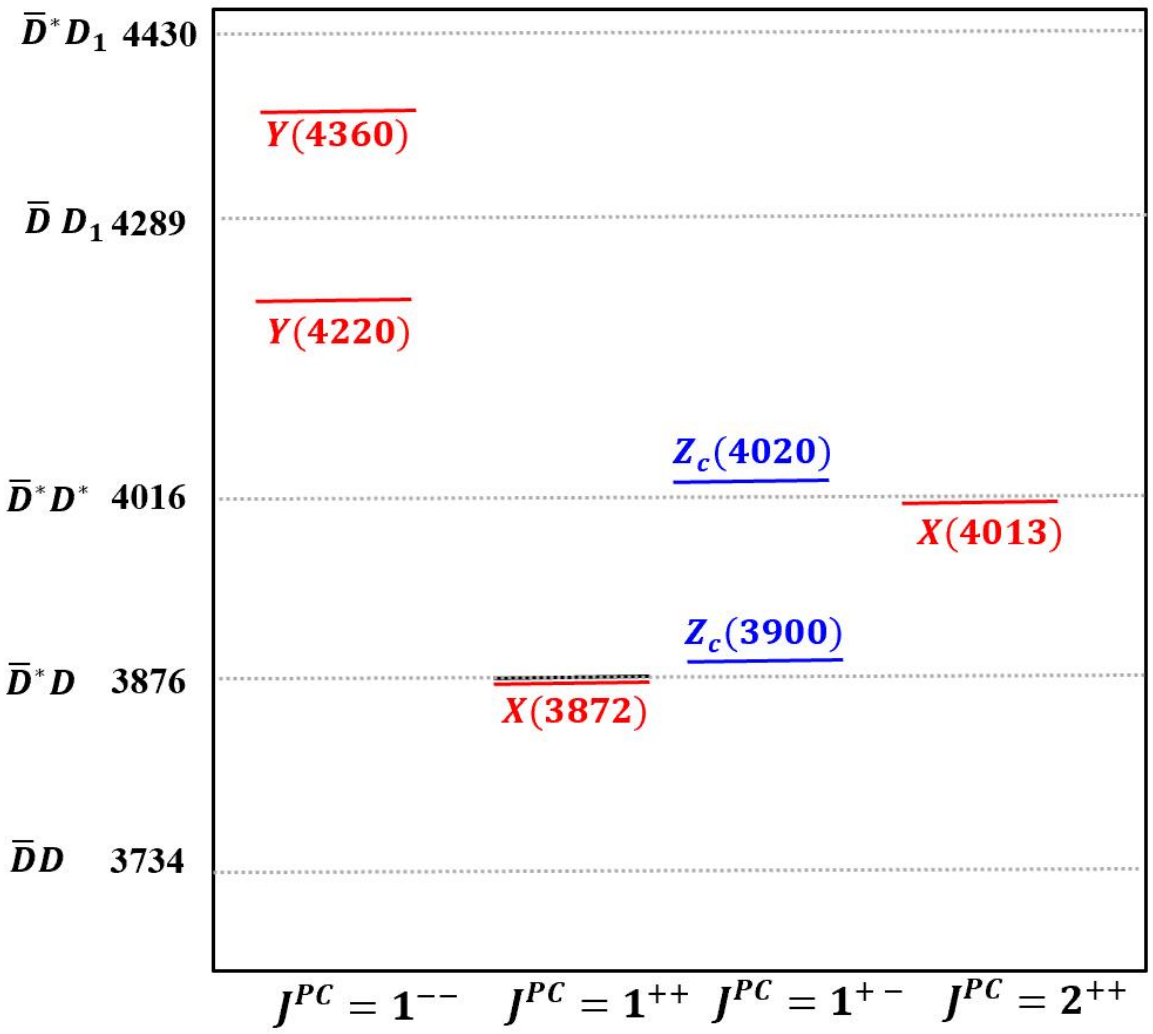
Superfield $H_c = \frac{1}{\sqrt{2}}(D + \vec{D}^*\vec{\sigma})$ $S_c = \frac{1}{\sqrt{3}}(\Sigma_c\vec{\sigma} + \vec{\Sigma}^*{}_c)$

Lagrangian $L = C_a Tr[H_c^\dagger H_c]Tr[H_c'^\dagger H_c'] + C_b Tr[H_c^\dagger \sigma H_c]Tr[H_c'^\dagger \sigma H_c']$

The low energy constants reduced from 4 to 2 within HQSS

The experimental signal for fine structures of exotic states

➢ HQSS multiplet hadronic molecules



- Contact-range potential of EFT

$$V(J^{PC}=1^{++}\ \bar{D}^*D) = C_a + C_b,$$
$$V(J^{PC}=2^{++}\ \bar{D}^*D^*) = C_a + C_b,$$
$$V(J^{PC}=1^{+-}\ \bar{D}^*D) = C_a - C_b,$$
$$V(J^{PC}=1^{+-}\ \bar{D}^*D^*) = C_a - C_b,$$
$$V(J^{PC}=1^{--}\ \bar{D}D_1) = C_a - \frac{2}{3}D_b,$$
$$V(J^{PC}=1^{--}\ \bar{D}^*D_1) = C_a - \frac{5}{4}C_b - \frac{1}{6}D_b - \frac{5}{4}D_c.$$

- Lippmann-Schwinger Equation

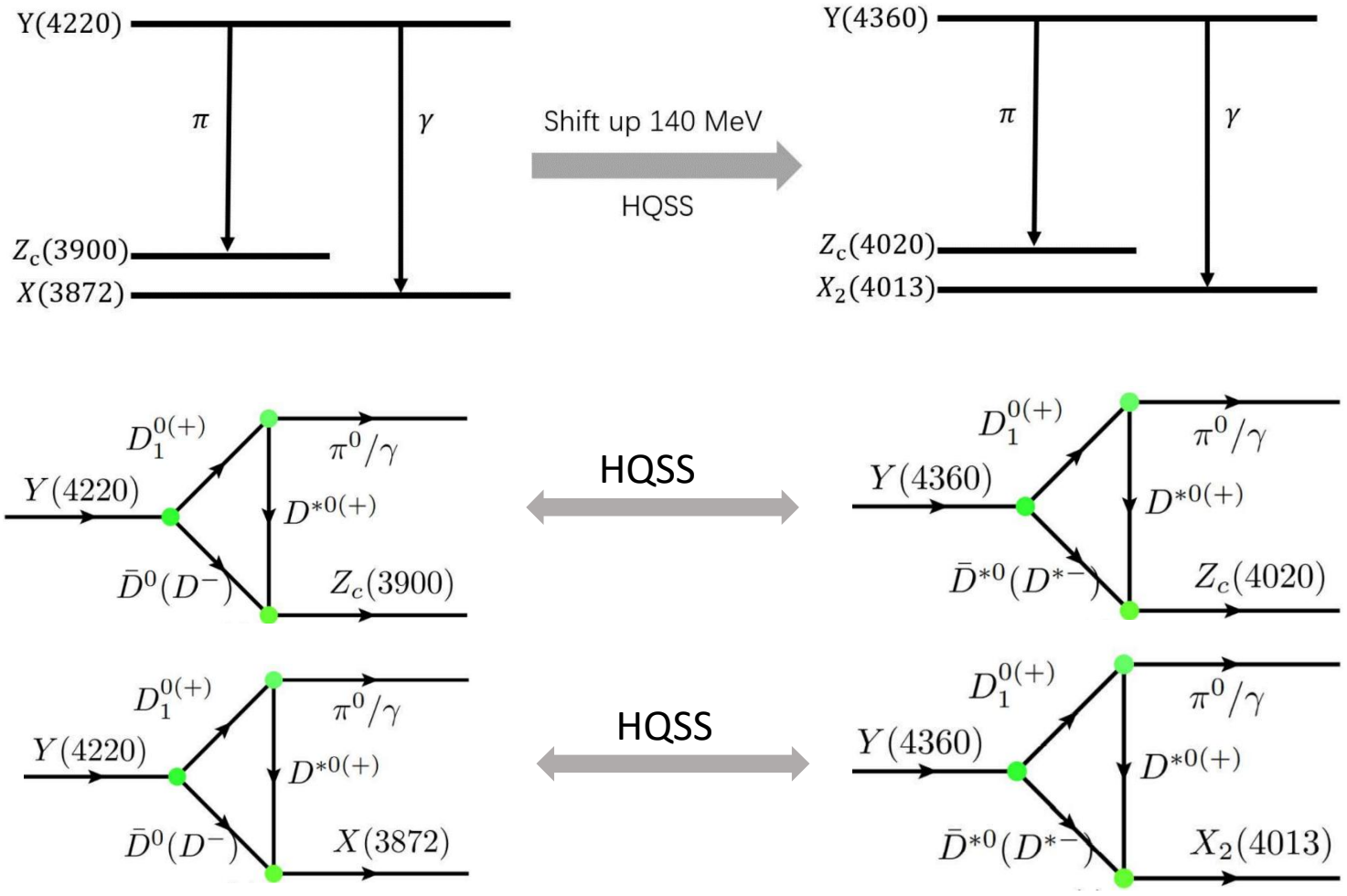
$$\langle \vec{k}'|T|\vec{k}\rangle = \langle \vec{k}'|T|\vec{k}\rangle + \int \frac{d^3\vec{q}}{(2\pi)^3}\langle \vec{k}'|V|\vec{q}\rangle\, G(s)\langle \vec{q}|T|\vec{k}\rangle$$

For multiplet hadronic molecules see

Mng-Zhu Liu, et al., Phys.Rept. 1108 (2025) 1-108

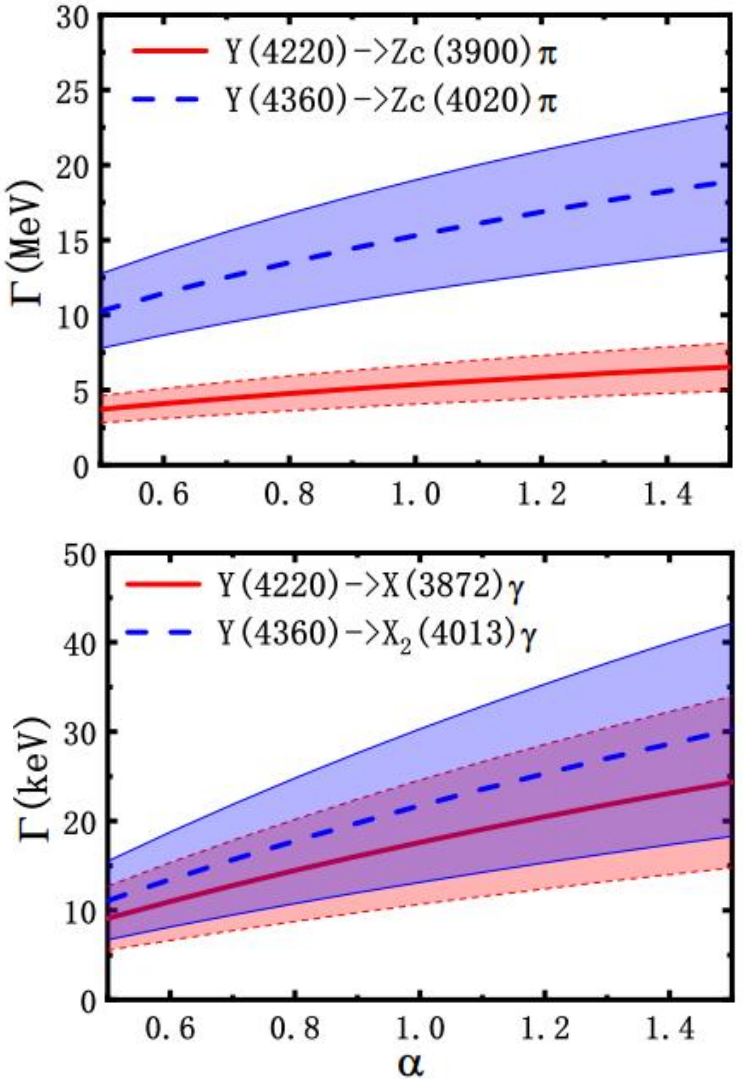
HQSS doublet hadronic molecules as a tool

➢ Studying other physical observables



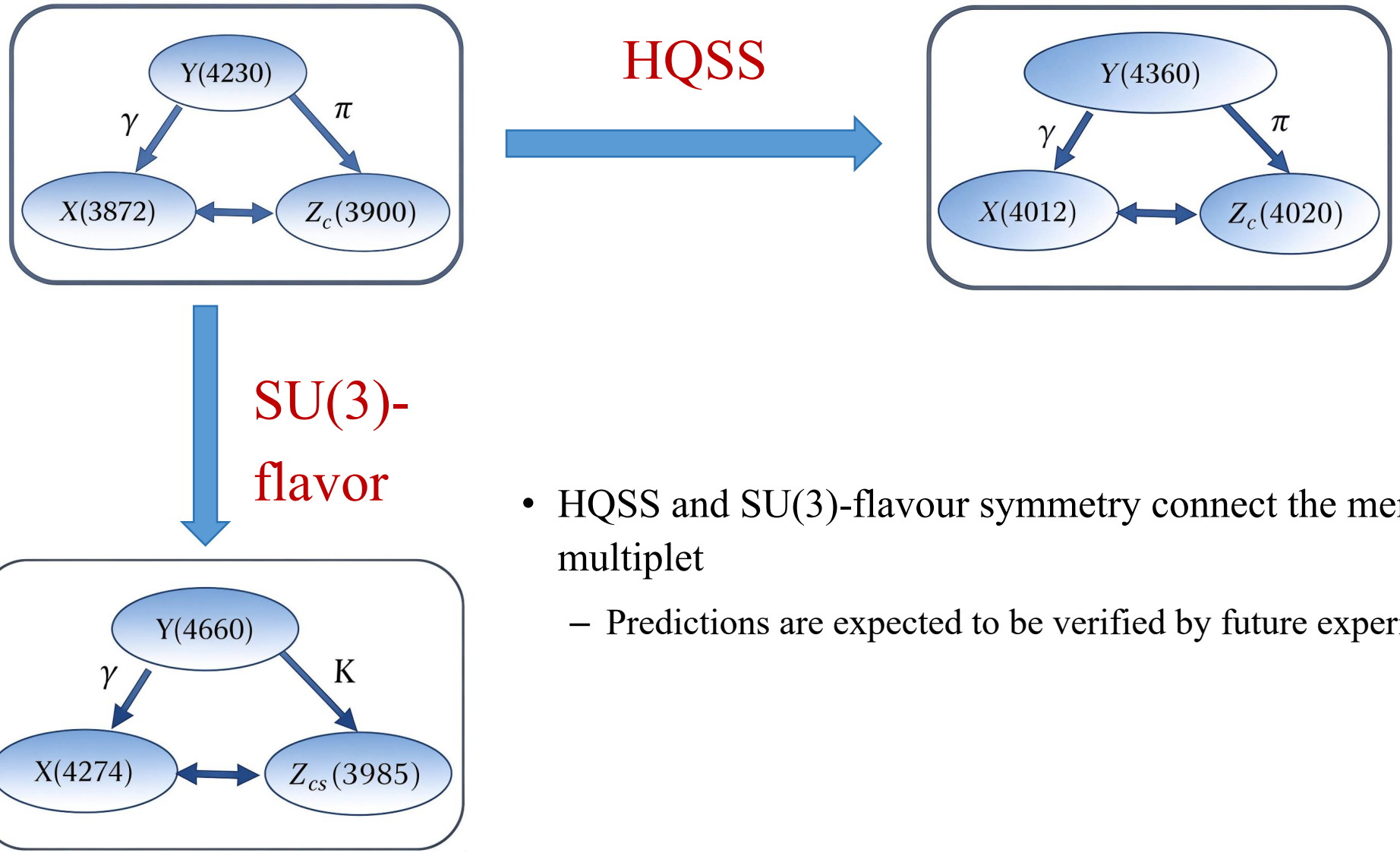
Mng-Zhu Liu, et al., Phys.Rev.D 110 (2024) 054035

➢ Results



The relationship between XYZ states

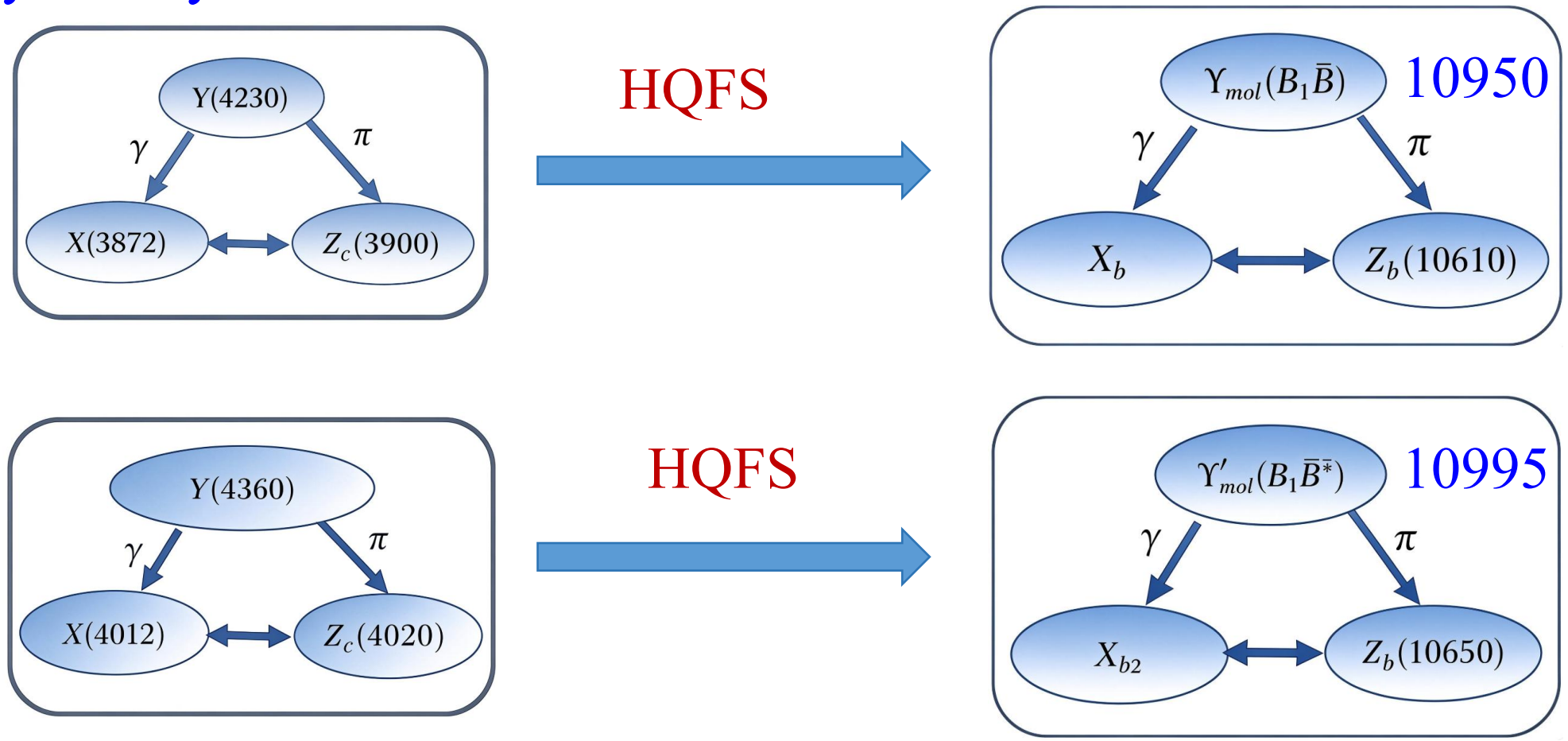
➢ Symmetry



- HQSS and SU(3)-flavour symmetry connect the members of a multiplet
  - Predictions are expected to be verified by future experiments

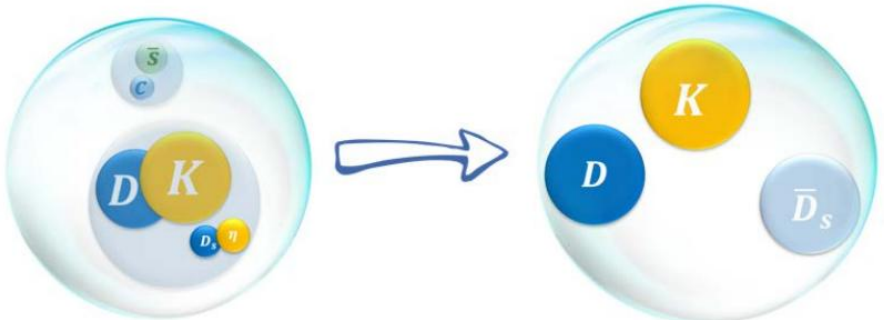
The relationship between XYZ states

➢ Symmetry



- Heavy-quark flavour symmetry (HQFS) relates the charm and bottom sectors
  - Predicted partner masses: 10950 MeV and 10995 MeV

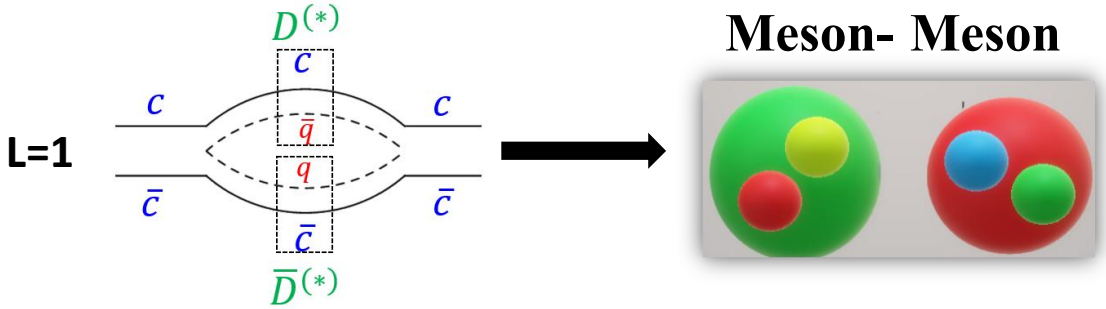
From two-body to three-body systems



$D_{s0}^*(2317)$ $\bar{D}_s DK$

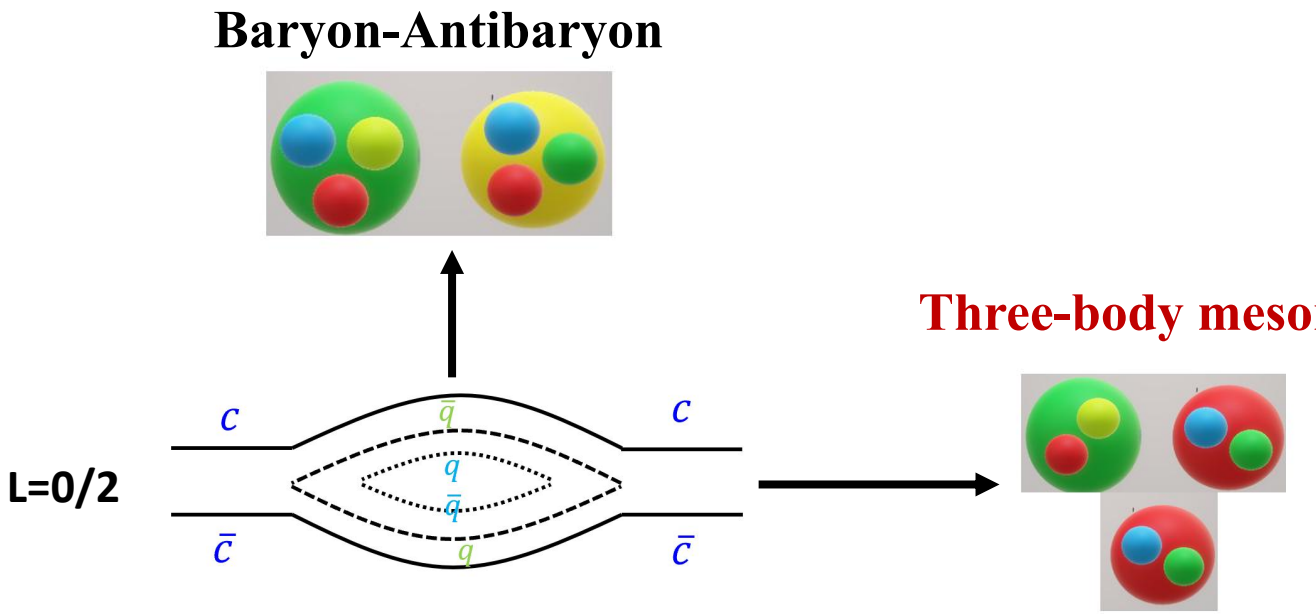
Hadronic molecule with new configurations

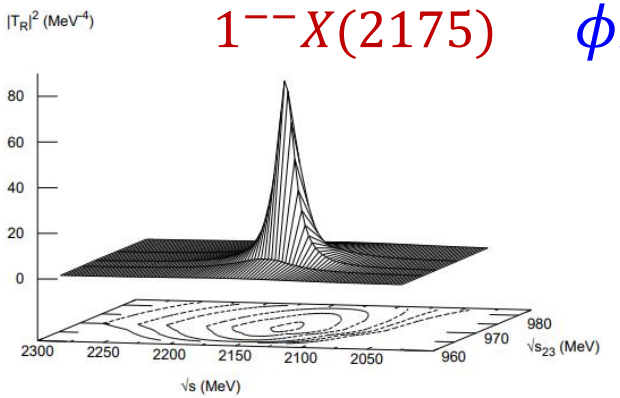
➢ Two-body hadronic molecules



Bing-Song Zou, Nucl. Phys. A 914 (2013) 454-460

➢ Three-body hadronic molecules





$1^{--}X(2175)$ $\phi K\bar{K}$

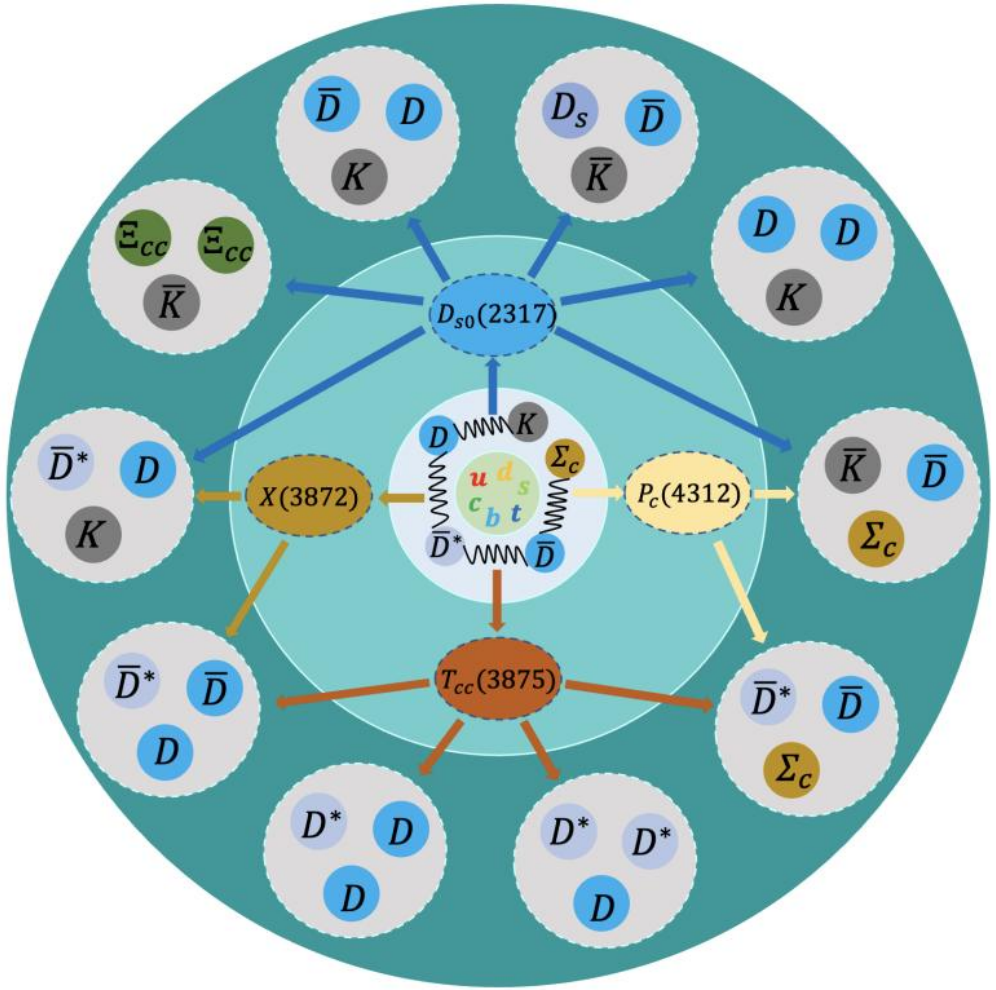
A. Martinez Torres et al., Phys.Rev.D 78 (2008) 074031

$1^{--}Y(4260)$ $J/\psi K\bar{K}$

A. Martinez Torres et al., Phys.Rev.D 80 (2009) 094012

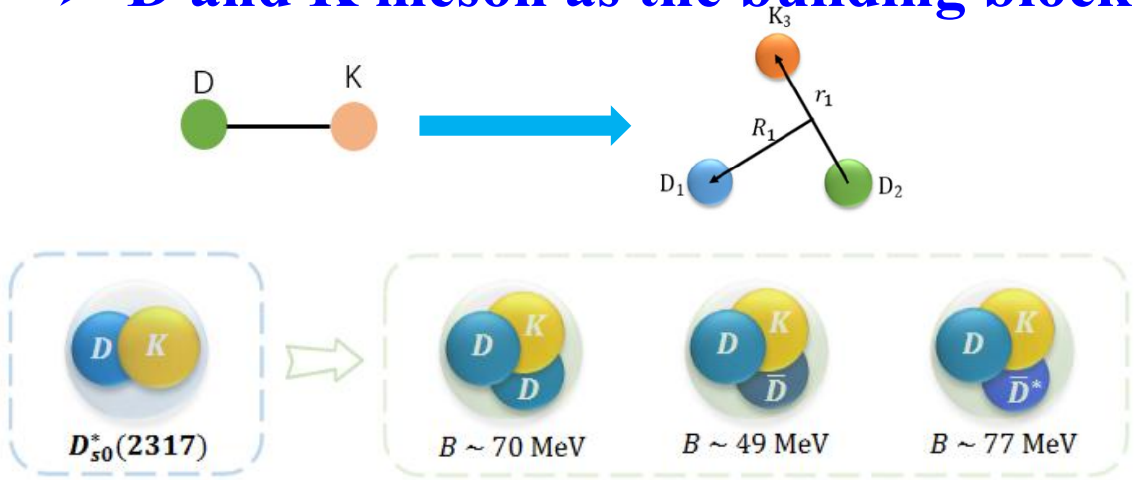
Three-body molecules can verify hadron-hadron interaction

➢ Existence of three-body hadronic molecules



Wu et al., Sci.Bull. 67 (2022) 1735-1738

➢ D and K meson as the building block



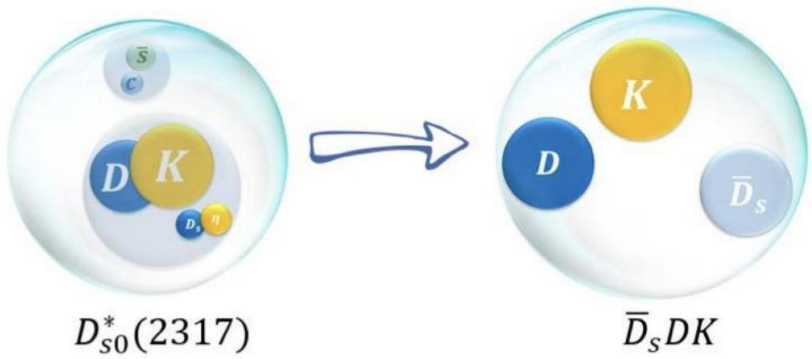
Liu et al., Phys. Rept. 1108 (2025) 1-108

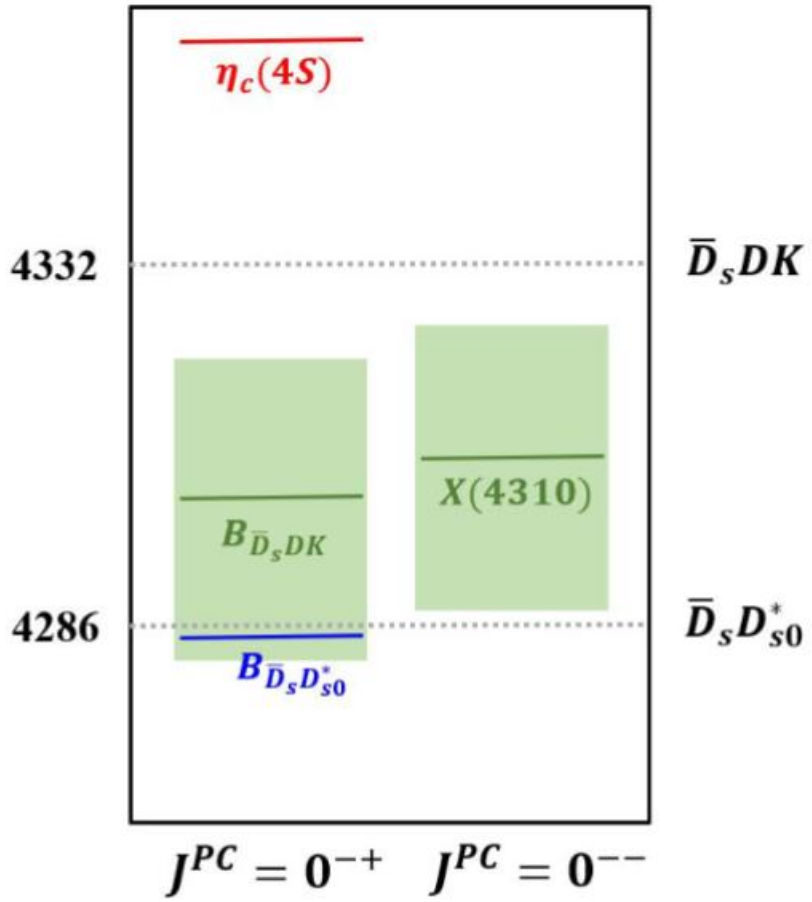
- ✓ **New configuration states generated by hadron-hadron interactions**
- ✓ **The existence of three-body molecules can verify hadron-hadron interaction**

Experimental search for the three-body molecule

➢ $J^{PC} = 0^{--}$ $\bar{D}_s DK$ molecule

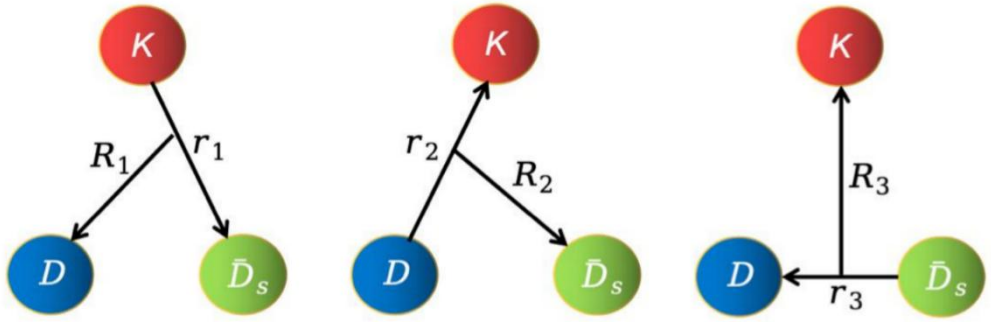
- ✓ **Contain charge parity**
  - This system with neutral charge
- ✓ **Impact of other non-perturbative effect is small**
  - No mixture of conventional $c\bar{c}$
  - Two-body subsystem $J^{PC} = 0^{--}$ $\bar{D}_s D_{s0}^*$ can not bind
- ✓ **Can be detected by experiment**
  - Can be produced at e^+e^- and pp collisions





A good candidate of three-body molecule!

Spectrum and decay of the $DK\bar{D}_s$ molecule



➢ The mass of $DK\bar{D}_s$ molecule

Scenarios	B.E.(0^{--})	$P_{\bar{D}_s K-D}$	$P_{DK-\bar{D}_s}$	$P_{\bar{D}_s D-K}$
$\alpha=1$	22^{+23}_{-14}	11^{+1}_{-1} %	78^{-1}_{+2} %	11^{+0}_{-1} %
$\alpha=2$	20^{+22}_{-13}	10^{+1}_{-1}%	80^{-1}_{+2} %	10^{+0}_{-1} %
Scenarios	$r_{\bar{D}_s K}$	r_{DK}	$r_{\bar{D}_s D}$	$\langle T\rangle$
$\alpha=1$	$1.6^{-0.4}_{+0.9}$	$1.2^{-0.3}_{+0.5}$	$1.4^{-0.3}_{+0.8}$	177^{+81}_{-74}
$\alpha=2$	$1.7^{-0.4}_{+1.4}$	$1.2^{-0.3}_{+0.7}$	$1.6^{-0.5}_{+1.3}$	169^{+82}_{-78}
Scenarios	$\langle V_{D_s\bar{K}}\rangle$	$\langle V_{DK}\rangle$	$\langle V_{D_s\bar{D}}\rangle$	$\langle V^{C=-}_{D_s\bar{D}^*_{s0}}\rangle$
$\alpha=1$	-40^{-25}_{+20}	-147^{-73}_{+62}	-14^{-7}_{+6}	2^{+0}_{-1}
$\alpha=2$	-37^{-25}_{+22}	-143^{-73}_{+64}	-13^{-6}_{+7}	3^{+2}_{-1}

➢ The decay of $DK\bar{D}_s$ molecule

Center	Lower	Upper
$\Gamma(X\to\bar{D}^*D)/\Gamma(X\to\bar{D}^*{}_sD_s)$		
9.3~12.6	9.9~13.6	8.7~11.8
$\Gamma(X\to\bar{D}^*{}_sD_s)/\Gamma(X\to J/\psi\eta)$		
5.1~5.7	5.2~6.1	4.9~5.4

$$\Gamma(X\to\bar{D}^*D):\Gamma(X\to\bar{D}^*{}_sD_s):\Gamma(X\to J/\psi\eta)$$

$$50 \;:\; 5 \;:\; 1$$

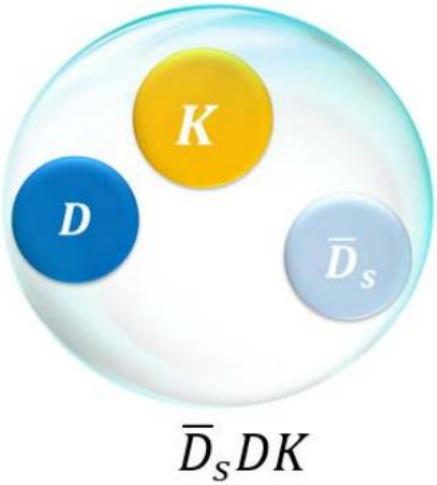
The mass of $DK\bar{D}_s$ molecule is around 4310 MeV, and its dominant decay mode is $\bar{D}^*D$

$$X(4310)\to\bar{D}^*D$$

Tian-Wei Wu et al., Phys. Rev. Lett. 135 (2025) 031902

Event number of the $DK\bar{D}_s$ molecule at LHCb experiment

➢ **Estimate the event number** $\mathcal{B}(B \to X(4310)K) \approx 2 \times 10^{-6}$



$\bar{D}_s DK$

✓ $\boldsymbol{B \to D^{(*)}\bar{D}K}$

$\mathcal{B}\left(X(4310) \to D^{*\pm}D^{\mp}\right) \approx 0.25$

$\mathcal{B}\left(B^+ \to \left(X(4310) \to D^{*\pm}D^{\mp}\right)K^+\right) \approx 5 \times 10^{-7}$

$\mathcal{B}\left(B^+ \to D^{*\pm}D^{\mp}K^+\right) \approx 6 \times 10^{-4}$

$\mathcal{B}\left(B^+ \to \left(X(4310) \to D^{*\pm}D^{\mp}\right)K^+\right)/\mathcal{B}\left(B^+ \to D^{*\pm}D^{\mp}K^+\right) \approx 10^{-3}$

LHCb Collaboration $9\ \text{fb}^{-1}$ $\mathrm{N}\left(B^+ \to D^{*\pm}D^{\mp}K^+\right) \approx 2 \times 10^3$

$\mathrm{N}\left(B^+ \to \left(X(4310) \to D^{*\pm}D^{\mp}\right)K^+\right) \approx 2$

LHCb Collaboration $50\ \text{fb}^{-1}$ $\mathrm{N}\left(B^+ \to \left(X(4310) \to D^{*\pm}D^{\mp}\right)K^+\right) \approx 10$

LHCb Collaboration $350\ \text{fb}^{-1}$ $\mathrm{N}\left(B^+ \to \left(X(4310) \to D^{*\pm}D^{\mp}\right)K^+\right) \approx 100$

Why do we need three-body forces?

➢ Binding energy of light nuclei

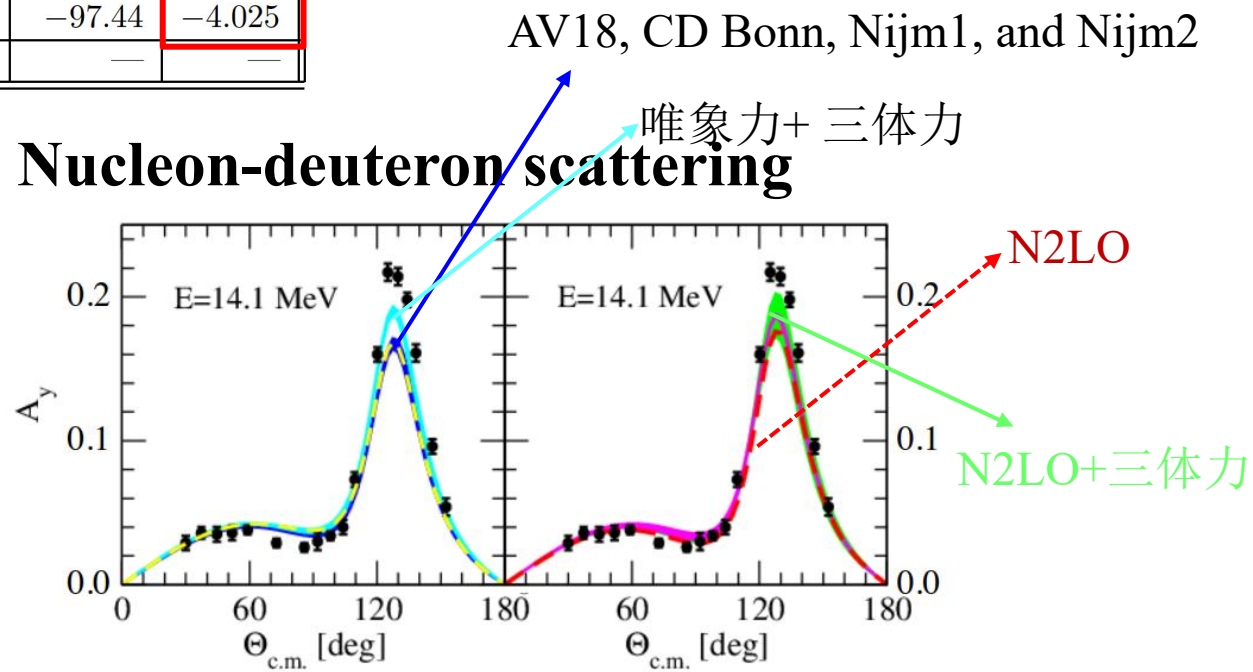
		^{3}H				^{4}He			
	Λ	E	T	V_{NN}	V_{3N}	E	T	V_{NN}	V_{3N}
NLO	500	−8.54	30.76	−39.30	—	−29.57	61.42	−91.00	—
	600	−7.53	39.24	−46.77	—	−23.87	77.61	−101.47	—
NNLO	500	−8.68	31.07	−39.43	−0.318	−29.51	61.83	−89.59	−1.753
	600	−8.68	34.44	−42.41	−0.712	−29.98	71.49	−97.44	−4.025
"Expt"	—	−8.68	—	—	—	-29.8 ± 0.1	—	—	—

E. Epelbaum et al., Phys. Rev. C 66 (2002) 064001

	B (MeV)	T (MeV)	$P_{S'}$ (%)	P_D (%)	P_P (%)	^2a (fm)	^4a (fm)
				^{3}H			
AV18	7.61	46.66	1.30	8.50	0.07	1.27	6.33
AV18+UR	8.49	51.34	1.05	9.31	0.13	0.63	6.33
				^{3}He			
AV18	6.91	45.62	1.53	8.46	0.06	1.17	13.6
AV18+UR	7.75	50.27	1.24	9.26	0.13	−0.02	13.7

A. Kievsky et al., Phys.Rev.C 52 (1995) R15-R19

➢ Nucleon-deuteron scattering

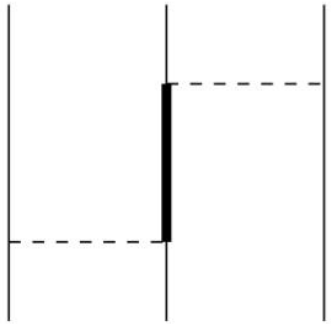


E. Epelbaum et al., Phys.Rev.C 99 (2019) 024313

Ay puzzle

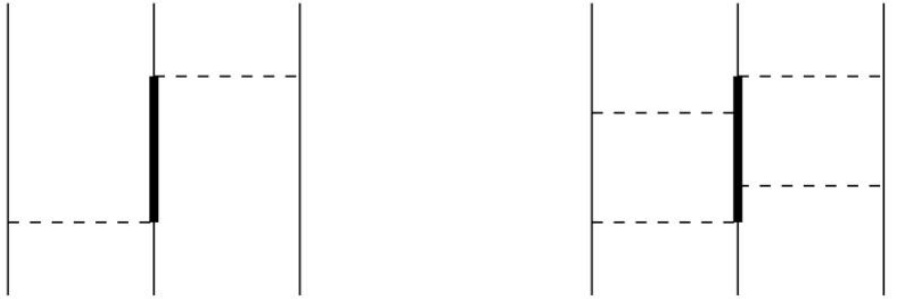
Three-body forces: phenomenological models

- **Fujita-Miyazawa ——Two-pion exchange**



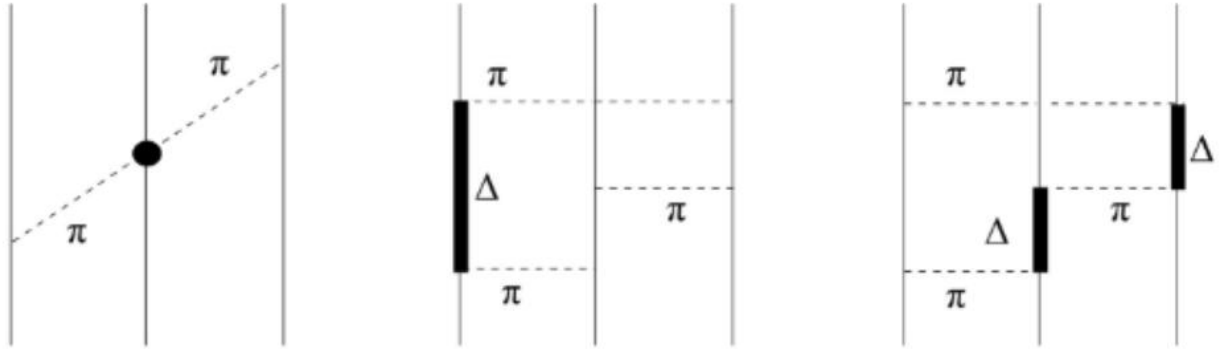
Fujita and Miyazawa, Prog.Theor.Phys. 17 (1957) 360-365

- **Urbana force**



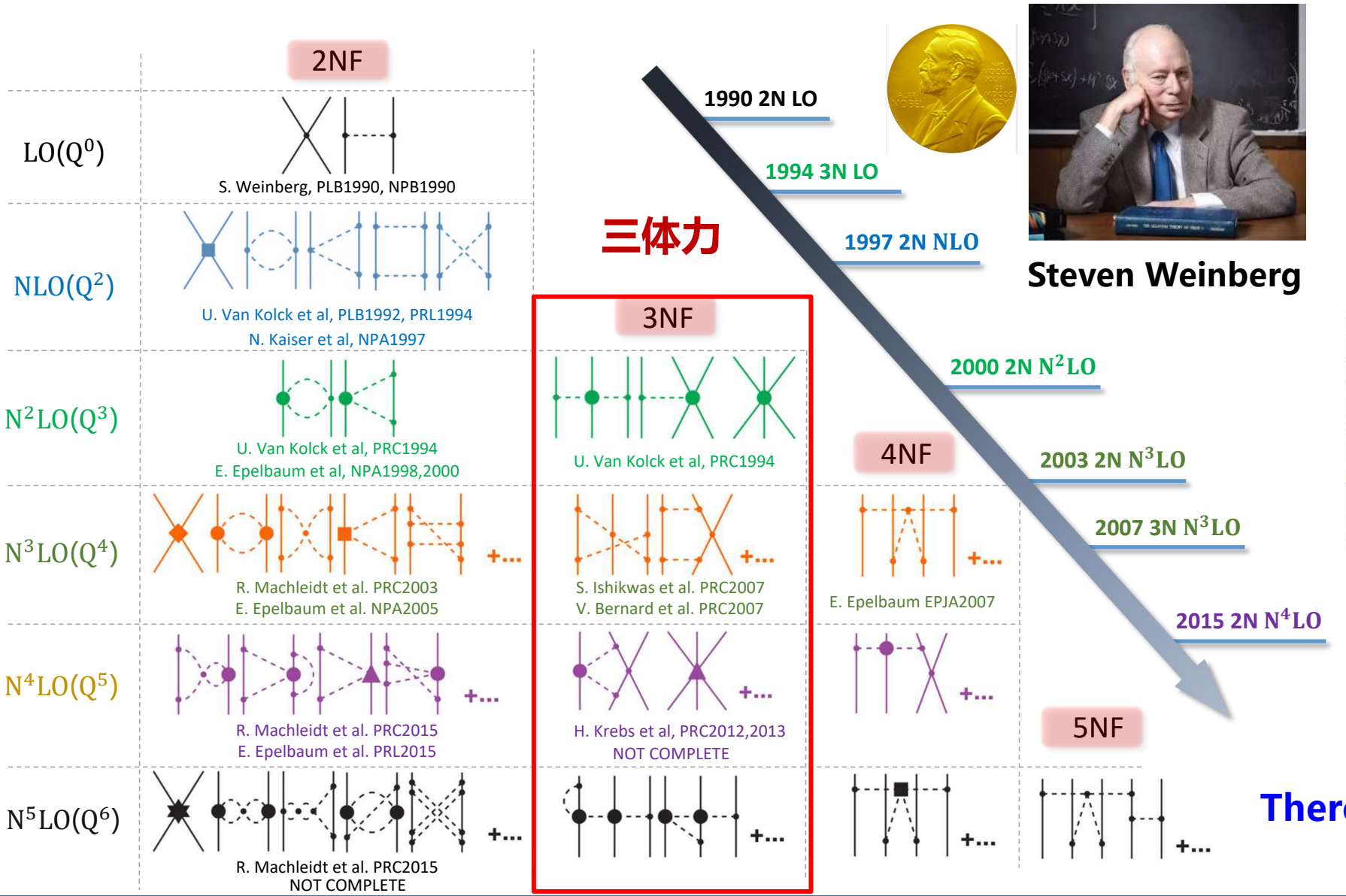
Carlson, Pandharipande,and Wiringa, Nucl.Phys.A 401 (1983) 59-85

- **Illinois force**



Steven C. Pieper, V. R. Pandharipande, R. B. Wiringa, J. Carlson, Phys.Rev.C 64 (2001) 014001

Three-body force: Chiral effective field theory



Steven Weinberg



van Kolck



Kaiser



Epelbaum



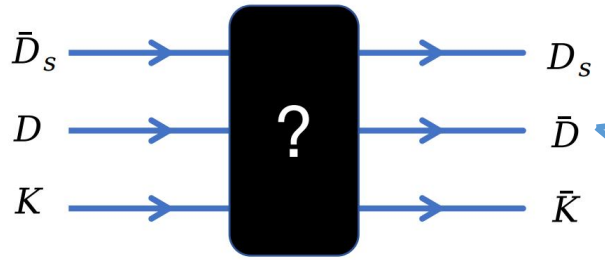
Machleidt

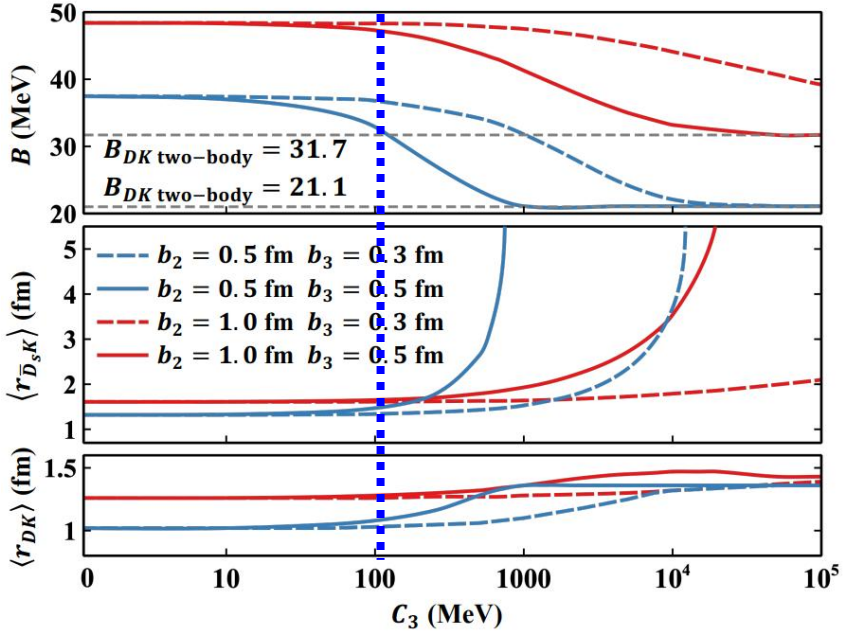
Theree-body force naturly exists

Three-body force in three hadron systems

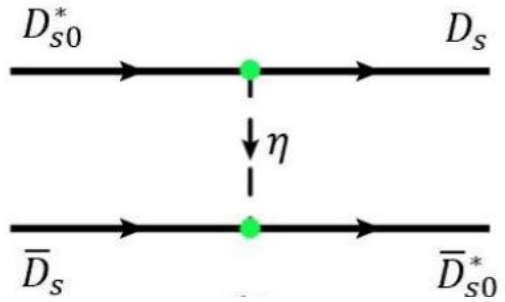
➢ Three-body system with C-charge parity

$$H = T + T' + V + V' + V^C$$



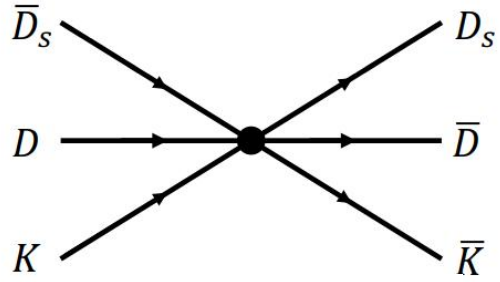


$$\Psi^C = \frac{1}{\sqrt{2}}(\Psi_{\bar{D}_s DK} + C\Psi'_{D_s \bar{D}\bar{K}})$$



$$V^{C=\pm} = \mp\frac{2}{3}\frac{k^2}{f_\pi^2}q_0^2\left(\frac{e^{-mr} - e^{-\Lambda r}}{4\pi r} - \frac{\Lambda^2 - m^2}{8\pi\Lambda}e^{-\Lambda r}\right)$$

Marek Karliner et al., Nucl.Phys.A 954 (2016) 365-370

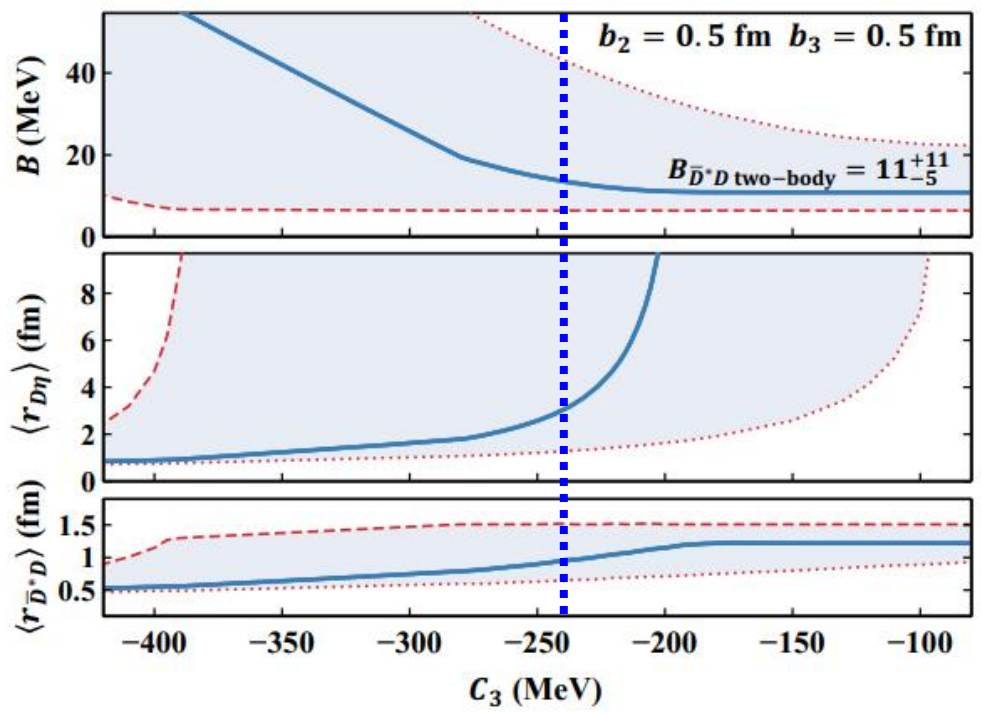


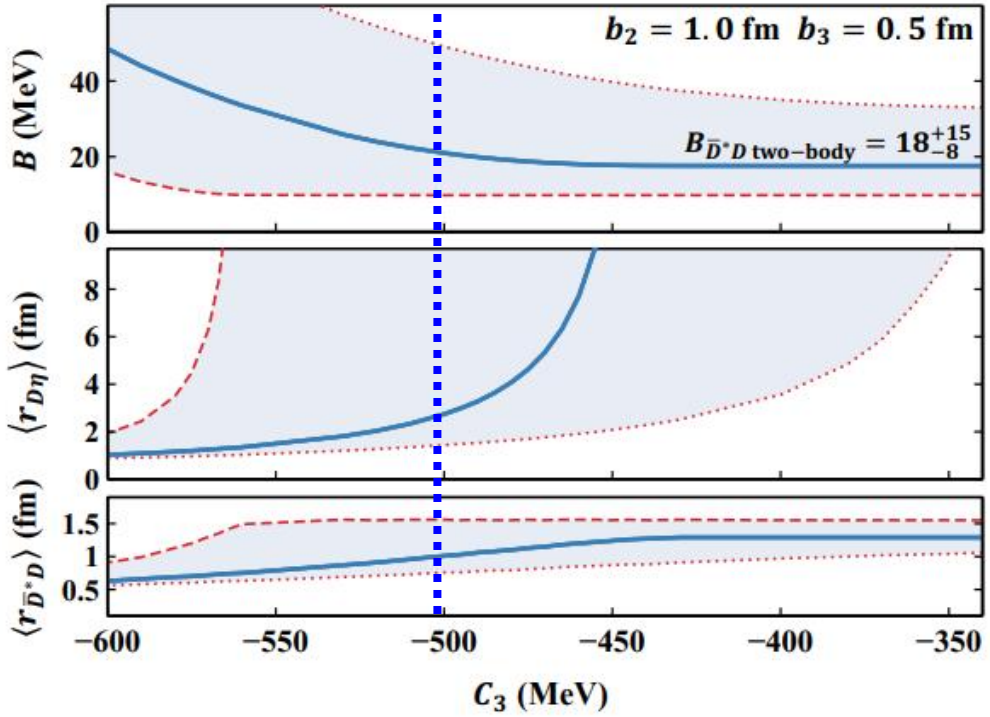
$$V^C = C_3 e^{-r^2/b_3^2} e^{-R^2/b_3^2}$$

$$\frac{|\langle V_3 \rangle|}{|\langle V_3 \rangle| + \sum_{1=i<j}^{3} |\langle V(r_{ij}) \rangle|}$$ **4%**

Three-body system $I(J^{PC}) = 0(1^{-+})$ $\bar{D}^* D\eta$

- **Binding energy and radius dependence on the three-body force**

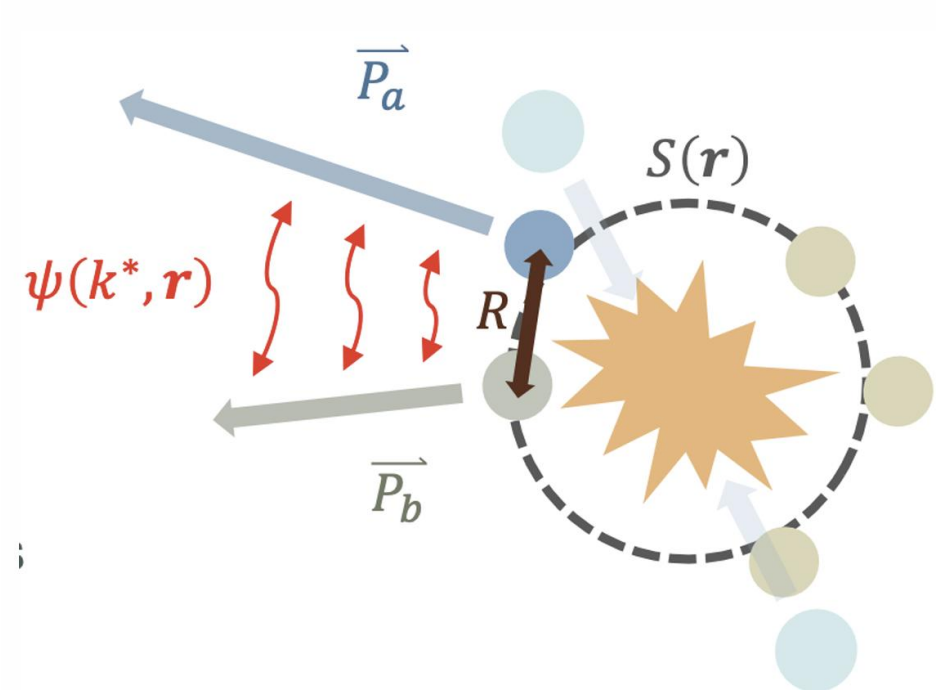




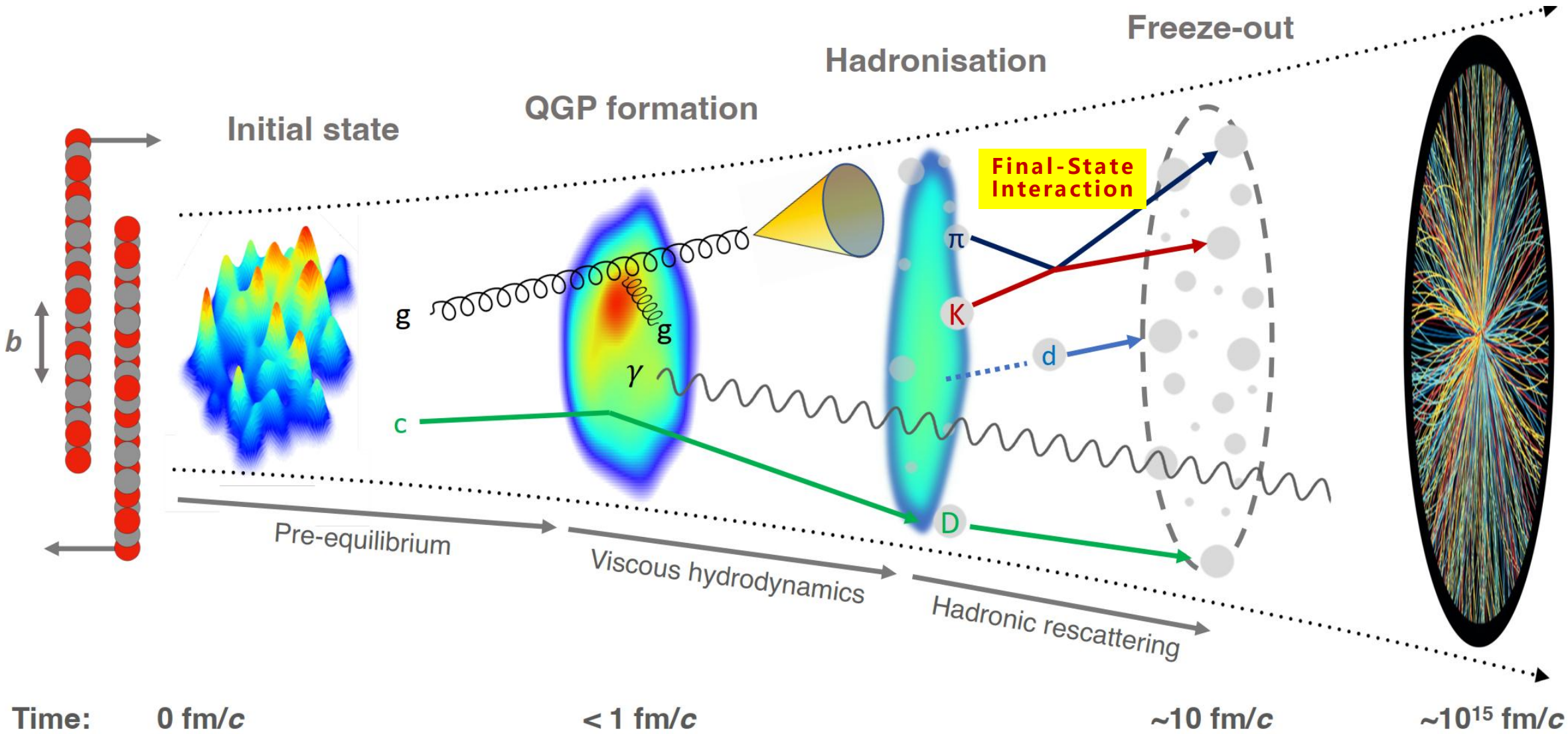
- ✓ **Three-body force contribution can account for20%**

This system is an ideal laboratory for studying three-body forces

Femtoscopy technique

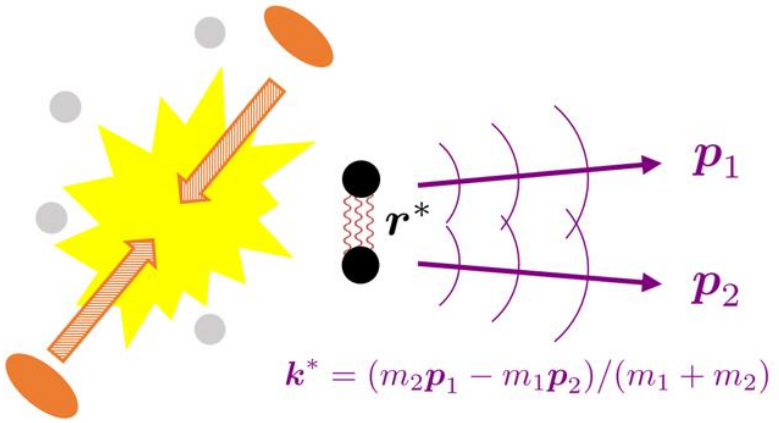


Dynamic evolution of heavy-ion collisions



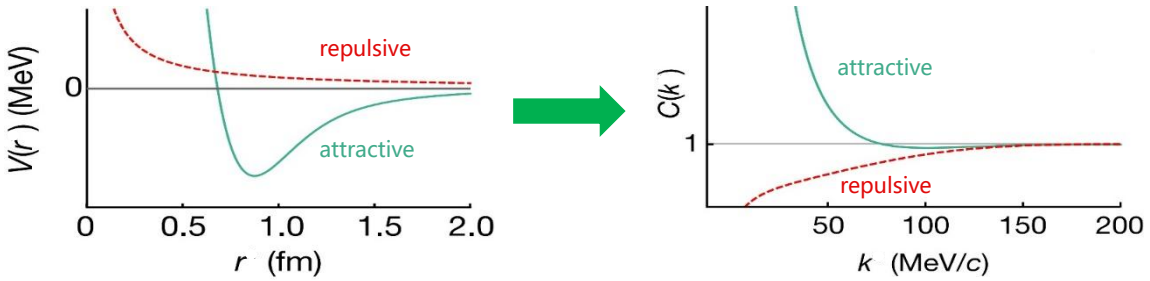
Femtoscopy

➢ Momentum correlation function



Emission source $S_{12}(r^*)$ Two-particle wavefunction $\psi(k^*, r^*)$

Interacting potential → **Correlation function**



Schrödinger equation

Two-particle wavefunction $\psi(k\ ,\ r\ )$

Theo. description

Koonin–Pratt formula

$$C(k) = \int S_{12}(\boldsymbol{r})\ |\psi(\boldsymbol{k}, \boldsymbol{r})|^2\ \mathrm{d}\boldsymbol{r}$$

$S_{12}(\boldsymbol{r})$: spacial structure

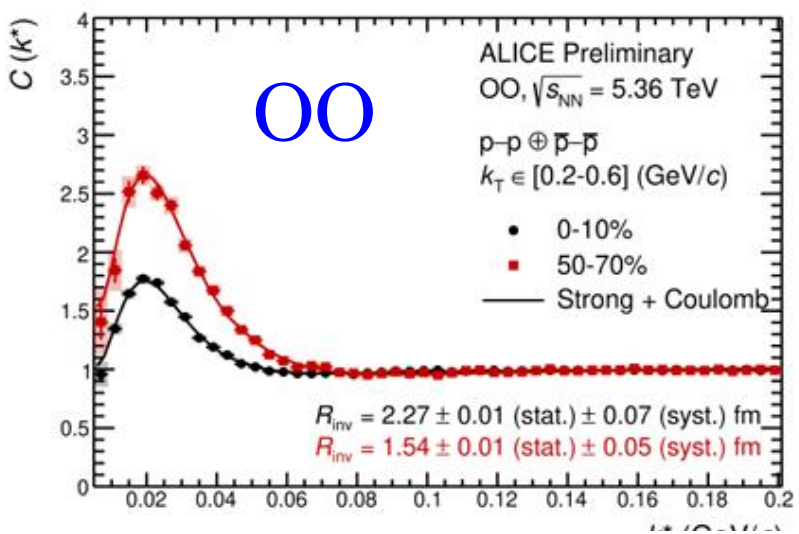
$|\psi(\boldsymbol{k}, \boldsymbol{r})|^2$: final-state interactions; quantum statistics effects

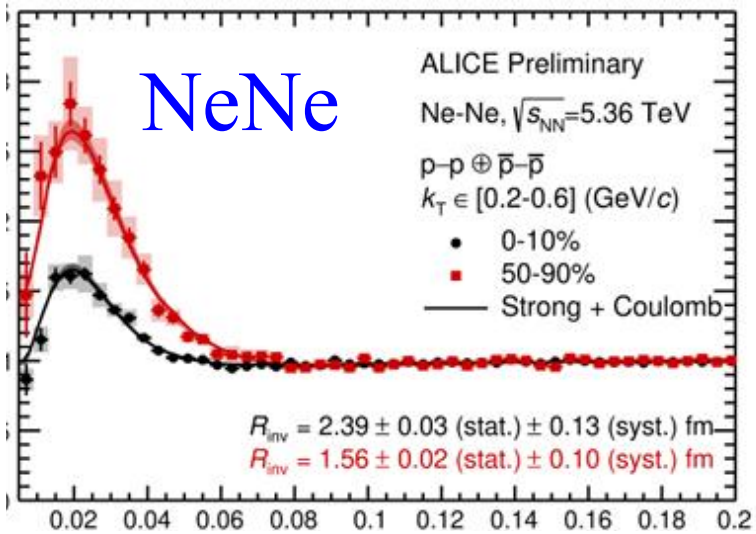
Basic Properties

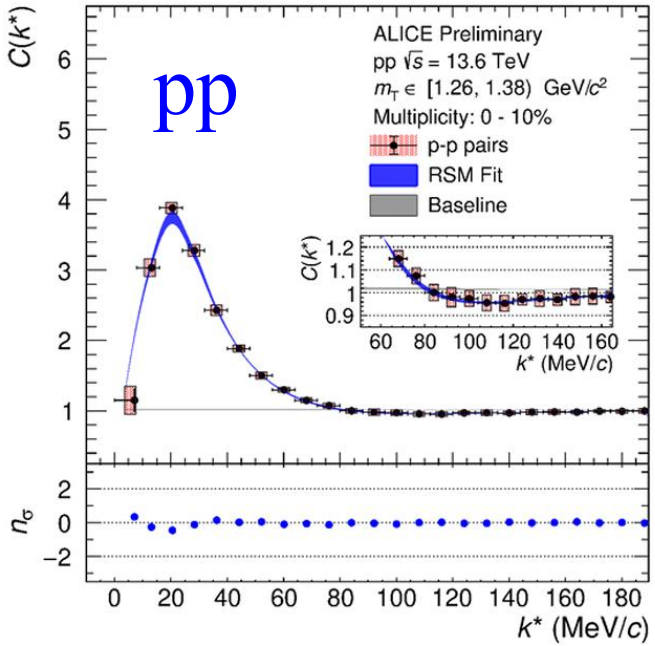
$C(k)$

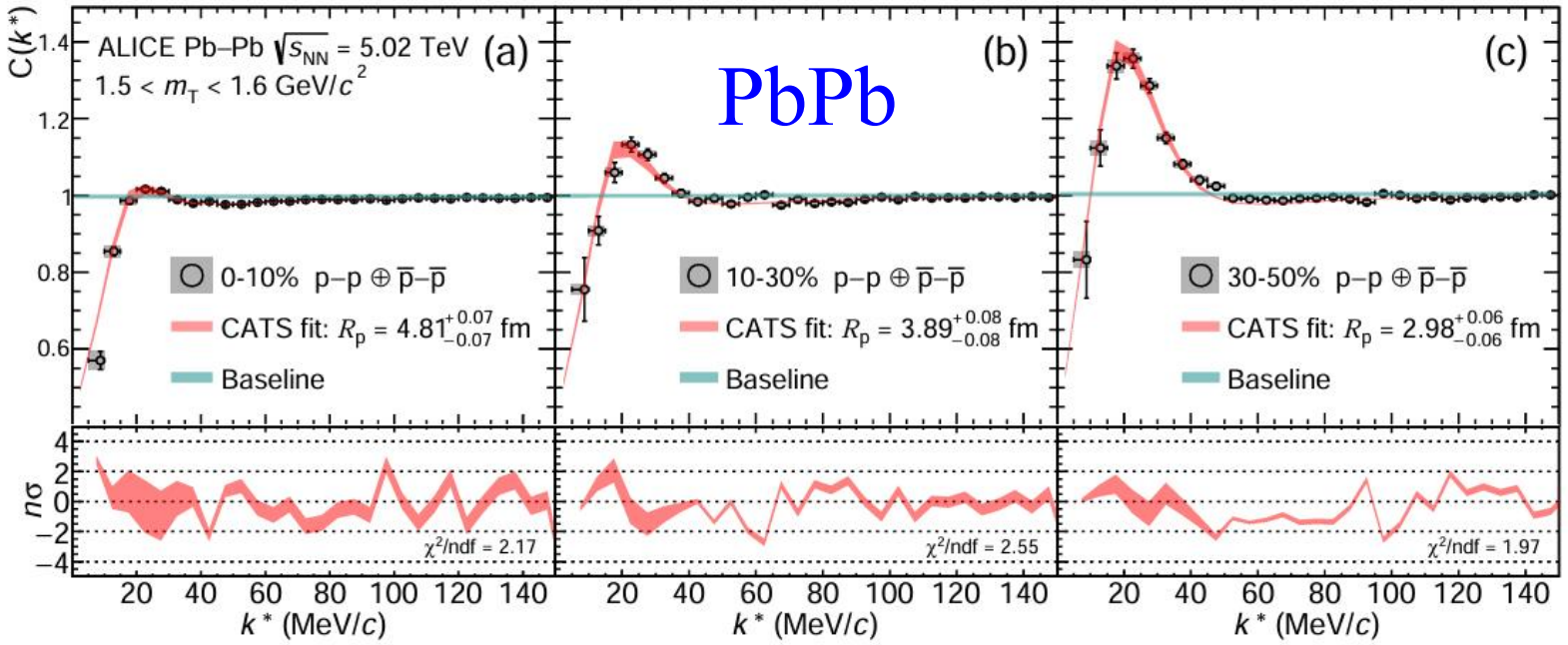
- >1 if the interaction is attractive
- =1 if there is no interaction
- <1 if the interaction is repulsive

Femtoscopy for proton-proton correlations



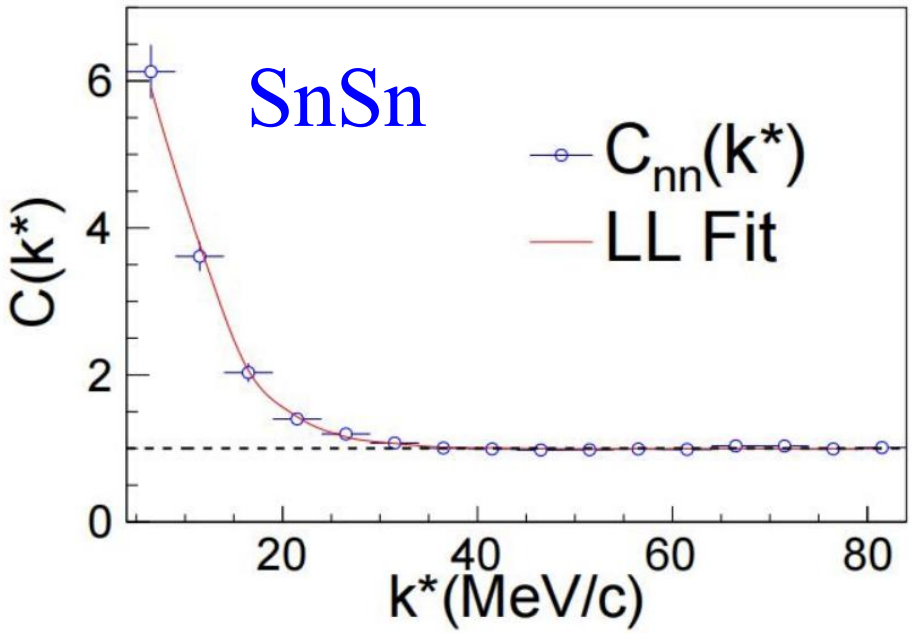




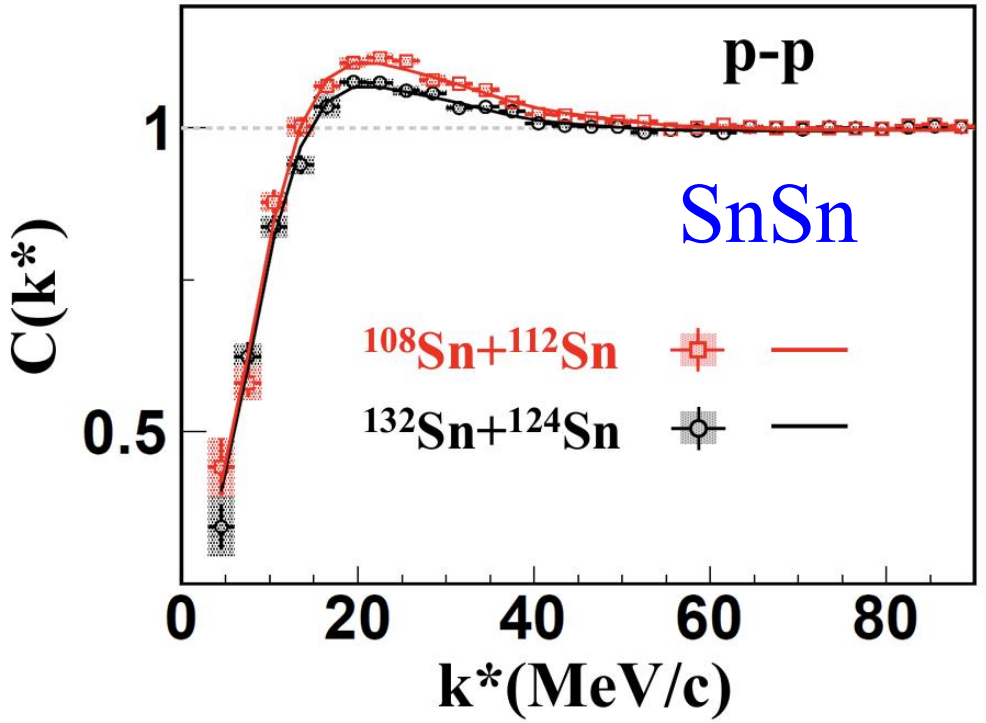


Femtoscopy for proton-proton correlations

CSHINE
Compact Spectrometer for Heavy IoN Experiments



Si Dawei, et al., Phys. Rev. Lett 134(2025) 222301

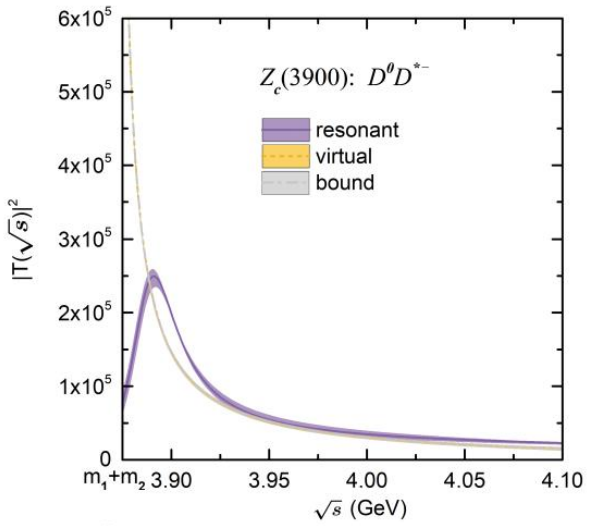


Wang yijie, et al., ArXiv 2604.25107

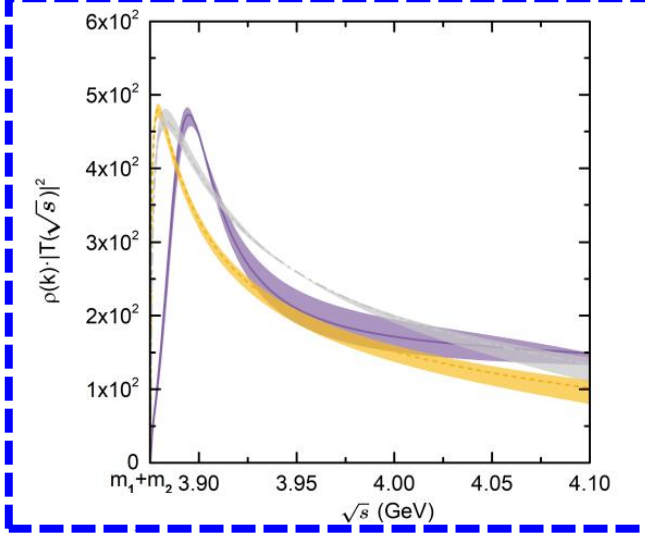
Systematic measurements in **SnSn** collisions constrain the p-p interaction

Correlation functions determine pole positions

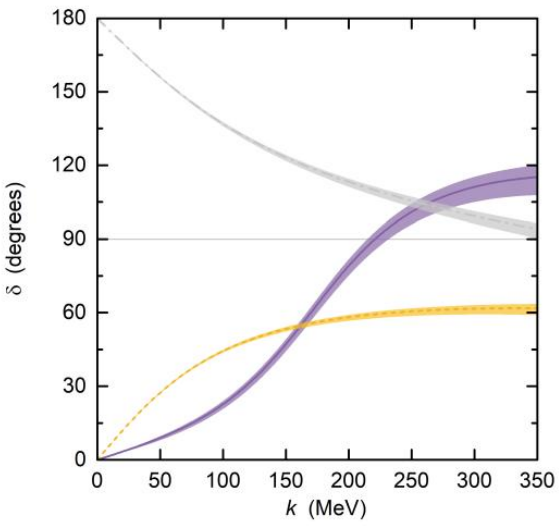
Amplitude square



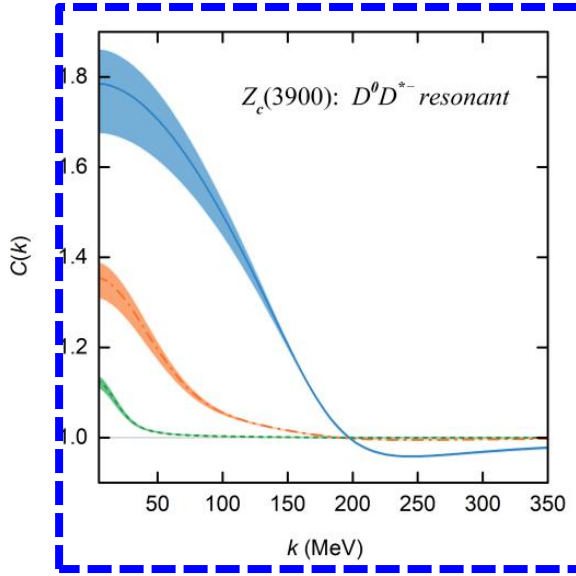
Invariant mass distribution

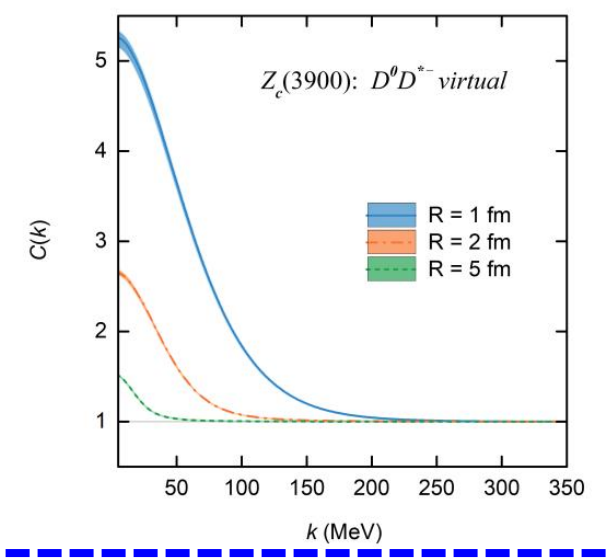


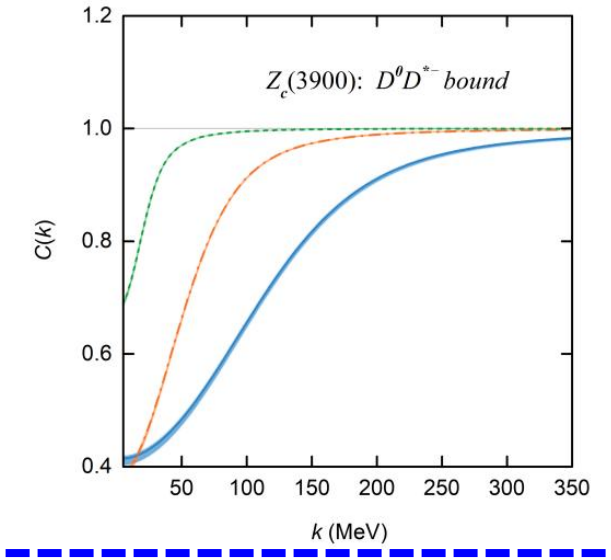
Phase shift



Correlation functions



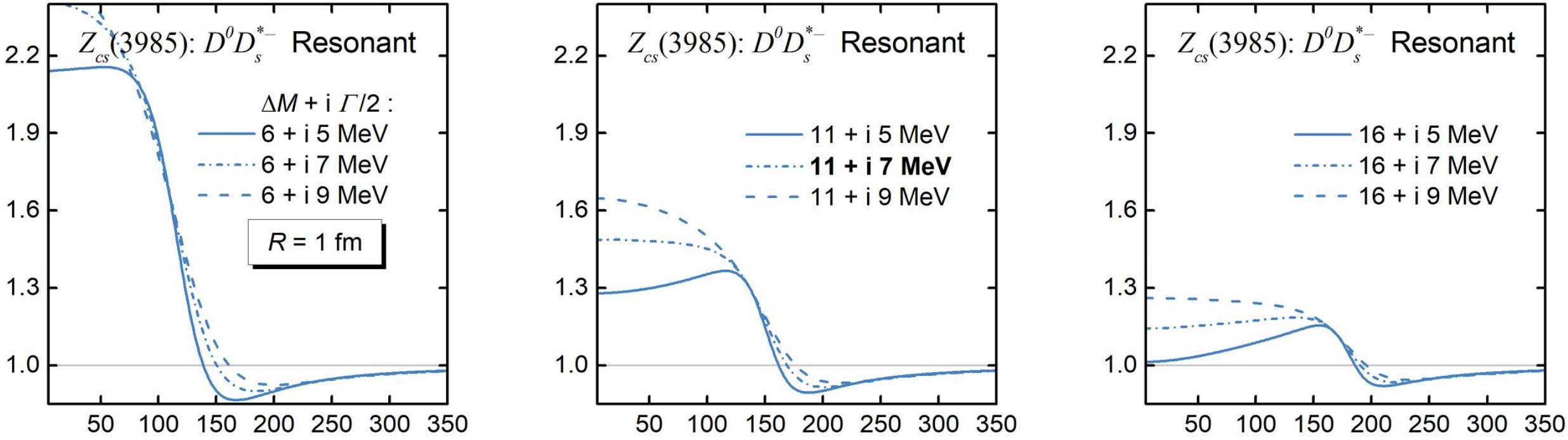




Zhi-Wei Liu, et al., Sci.Bull. 70 (2025) 3515-3521

Correlation functions determine pole positions

➢ Correlation functions for resonant states

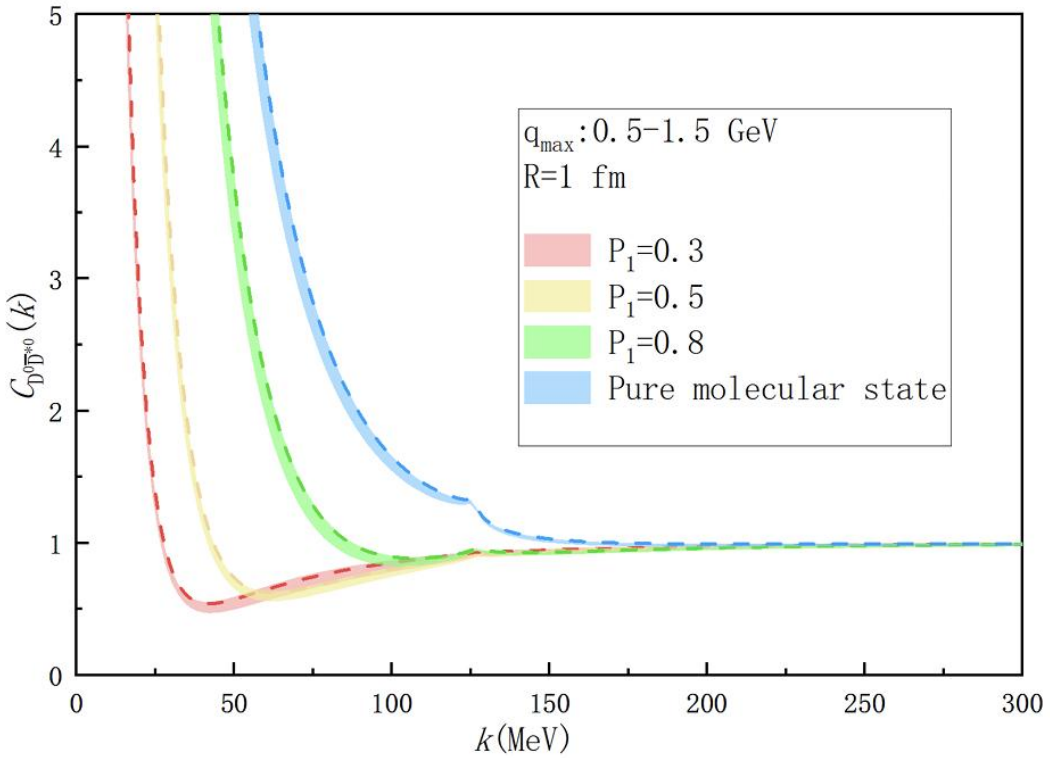


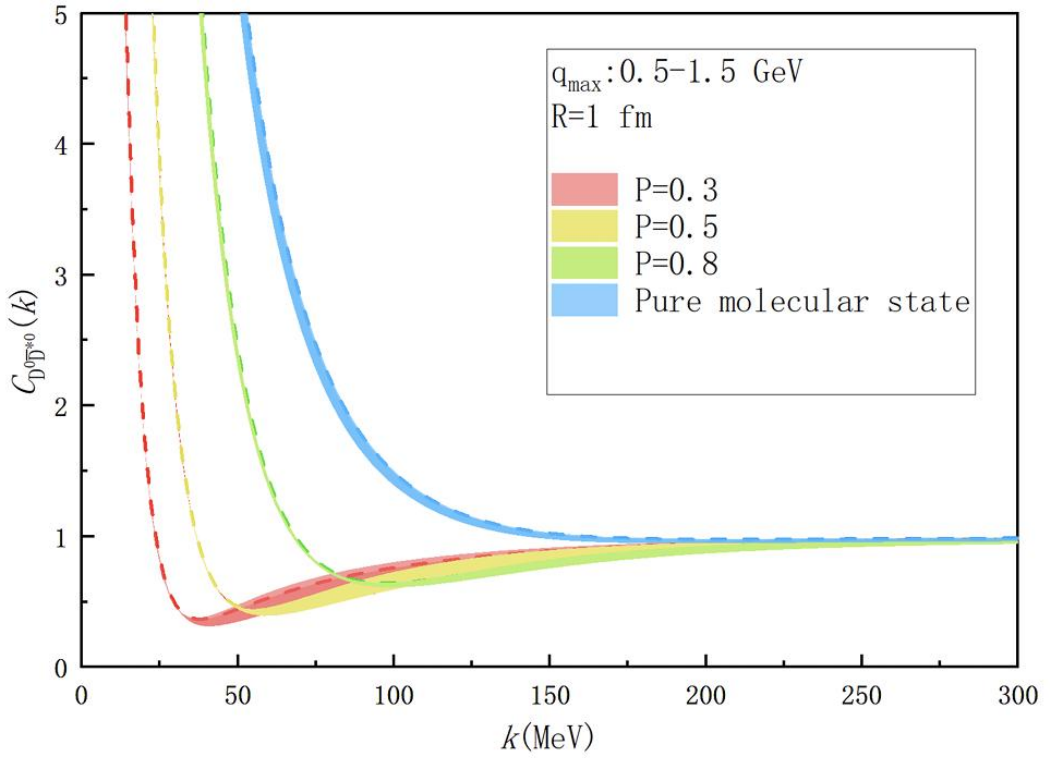
Zhi-Wei Liu, et al., Sci.Bull. 70 (2025) 3515-3521

The pole position of resonant state affect the lineshape of correlation functions

Correlation function for X(3872)

➢ Couple channel with a bare state





Yi-Bo Shen, et al., Phys.Lett.B 878 (2026) 140535

The dressing effect of the bare state significantly modifies the lineshape of the $D^0\overline{D}^{*0}$ correlation function

Outline

- Charmonium(like) states and strong interactions
- Theoretical studies on charmonium(like) states
- Symmetry, Three-body system and Femtoscopy
- Summary

Summary

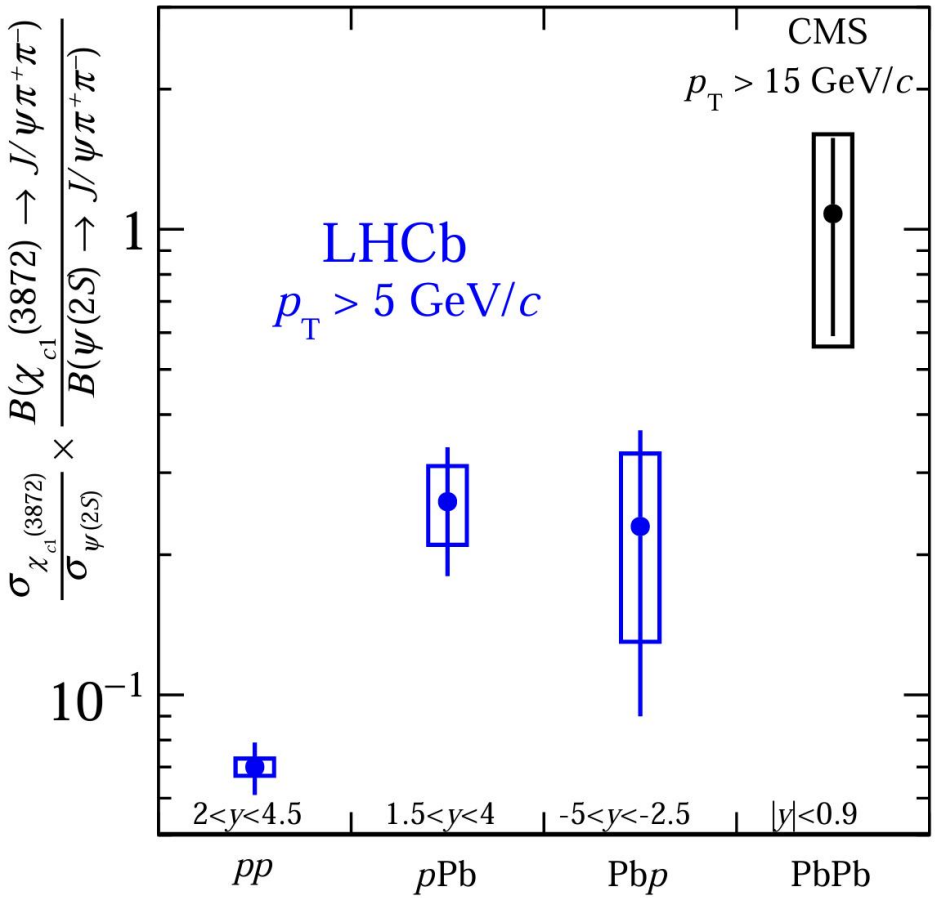
- From charmonium states to charmoniumlike states, the strong interaction is deepenly understood

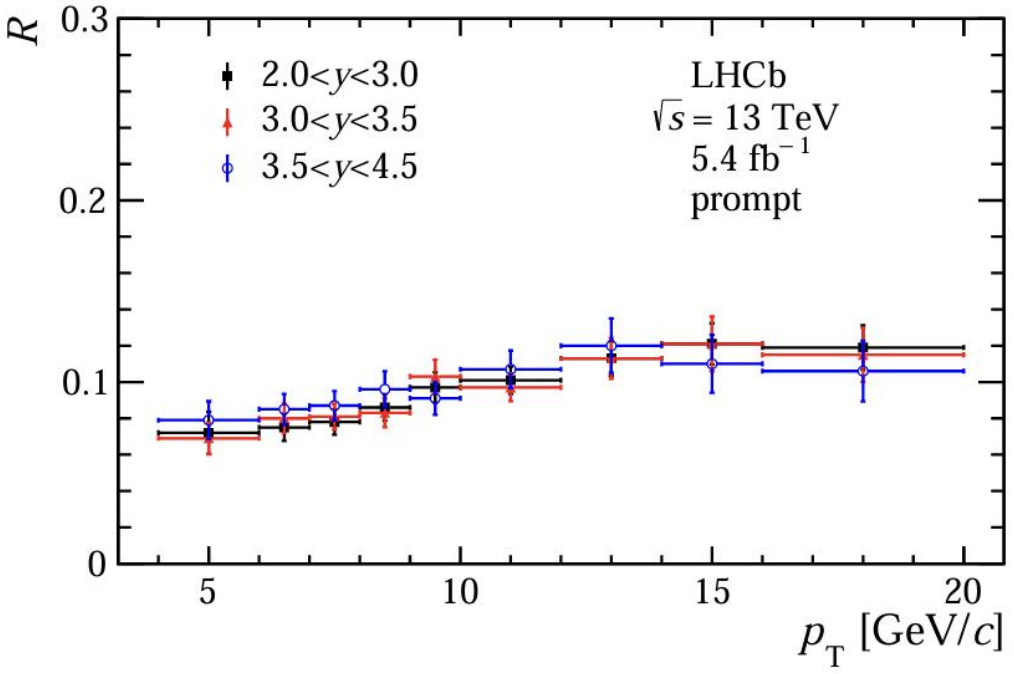
- The nature of charmoniumlike states is expected to be revealed by future experiments. Beyond ever more precise measurements, key inputs will come from theoretical predictions: the heavy-quark-flavour partner Xb, the heavy-quark-spin symmetry partner X2(4013), and their production in heavy-ion collisions.

Thanks for your attention!

The states beyond conventional charmonium states

➢ The productions of X(3872) at heavy ion collisions





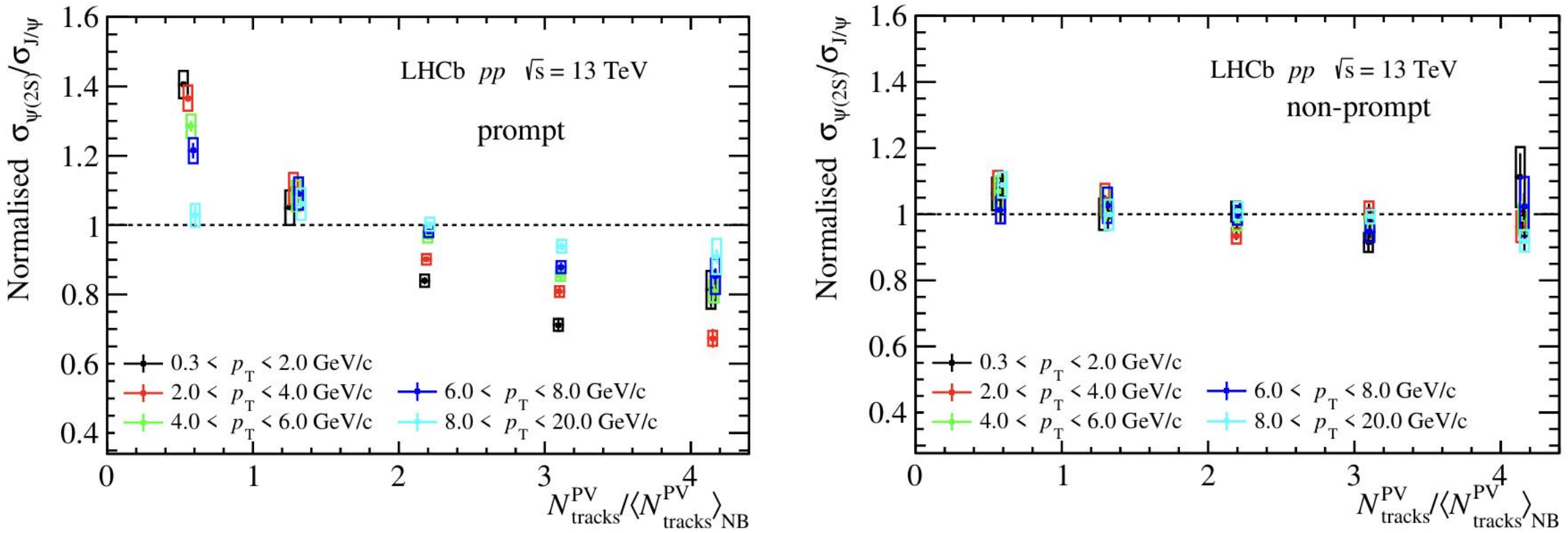
LHCb Collaboration., JHEP 01 (2022) 131

pPb LHCb Collaboration., Phys.Rev.Lett. 132 (2024) 242301

PbPb CMS Collaboration., Phys.Rev.Lett. 128 (2022) 032001

Resolve the challenge of XYZ states

➢ Production of X(3872) at the pp collision

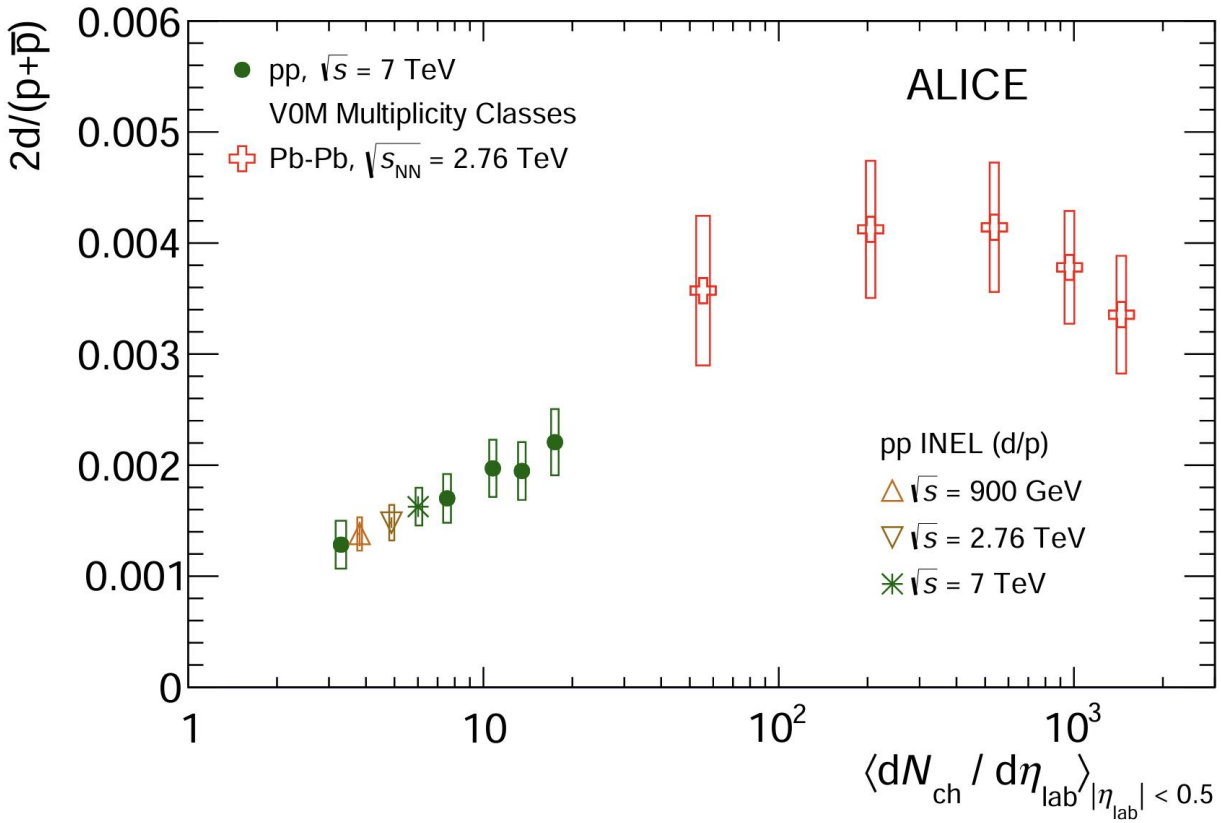


LHCb Collaboration., JHEP 05 (2024) 243

The prompt productions of charmonium is more senstive to the covermer effects, while it's insenstive to in the non-prompt productions

Resolve the challenge of XYZ states

➢ Production of deuteron



Alice Collaboration., Phys.Lett.B 794 (2019) 50-63

The increasing d/p ratio with multiplicity in pp collisions is caused by the alleviation of finite-source suppression, where the effect of rising nucleon density overwhelms the decrease of the coalescence parameter

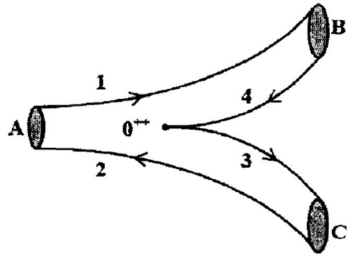
Backup

$$H|\Psi\rangle = E|\Psi\rangle \qquad H = \begin{pmatrix} T^q & 0 \\ 0 & T^h + \Delta \end{pmatrix} + \begin{pmatrix} V^q & V^t \\ V^t & V^h \end{pmatrix} \qquad |\Psi\rangle = \begin{pmatrix} |q\rangle \\ |h\rangle \end{pmatrix}$$

➢ Unquenched quark model $H^q_{\rm eff}(E) = T^q + V^q_{\rm eff}(E)$

$$m^2_{\rm bare} - s + \Pi(s)$$

$$\mathrm{Im}\Pi_n(m^2_{\rm phy}) = -2\pi P E_B E_C |M^{LS}(P)|^2$$



3P0 model

➢ Effective hadron-hadron potentials $H^h_{\rm eff}(E) = T^h + \Delta + V^h_{\rm eff}(E)$

Form factor $\langle D^0\bar{D}^{*0}(\boldsymbol{q})|U|c\bar{c}\rangle = \dfrac{g_{c\bar{c}}}{\sqrt{\Lambda_q}} \dfrac{\Lambda_q^2}{q^2+\Lambda_q^2}$

Yasuhiro Yamaguchi et al., J.Phys.G 47 (2020) 5, 053001

CDD pole $t(E) = \left(\dfrac{\lambda}{E - M_{\rm CDD}} + \beta - ik(E) \right)^{-1}$

Xian-Wei Kang et al., Eur.Phys.J.C 77 (2017) 6, 399

Compositeness rule

$$Z = 1 - \sum_i P_i \begin{cases} T = (1 - VG)^{-1} V \\ P_i = -g_i^2 \dfrac{\partial G_{ii}(\sqrt{s})}{\partial \sqrt{s}} \end{cases}$$

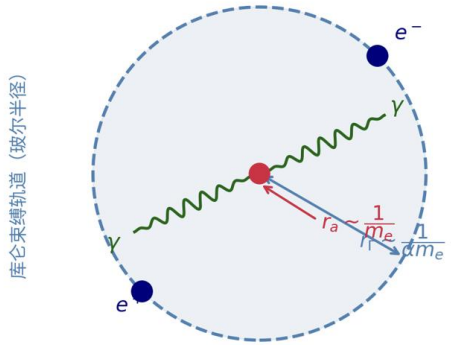
Hadronic molecules as a tag

Backup

两种尺度层级对比：情形 A（$r_\Gamma \gg r_a$）与情形 B（$r_\Gamma \ll r_a$）

情形 A：正电子素极限（如 e^+e^- 偶素）

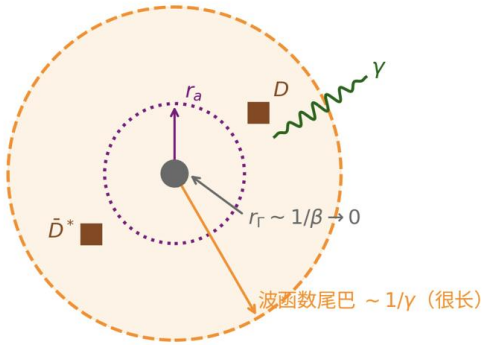
束缚由无质量光子传递，力程 = 轨道半径



湮灭只发生在“原点” → 宽度 ∝ $|\psi(0)|^2$

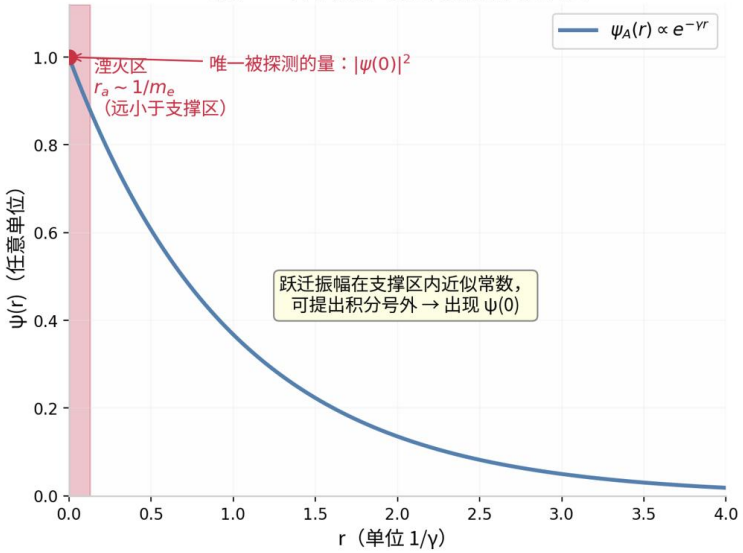
情形 B：点状 / 零程极限（如 $D\bar{D}^*$ 分子）

力程极短，波函数由普适长程尾巴主导



光子发射 / 末态形成遍及整个尾巴 → $|\psi(0)|^2$ 公式失效

情形 A：ψ(0) 有限，宽度由原点波函数决定



情形 B：宽度由尾巴上的圈积分决定，须完整计算

