Millicharged Fermion Vacuum Polarization and CMB Elliptic Polarization

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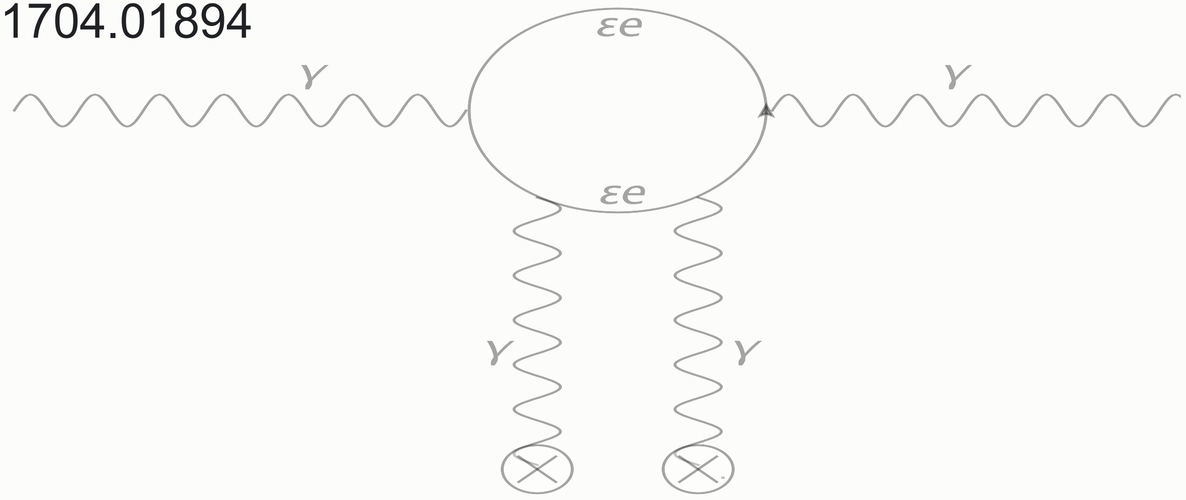


Motivation

1. Millicharged particles are theoretically allowed

$$q_\epsilon = \epsilon e, \qquad \epsilon < 1.$$

They arise naturally in several extensions or deformations of the Standard Model.

2. Magnetic fields turn vacuum polarization into an optical probe

$$\mathrm{Re}\,\Pi \to \text{birefringence},$$

$$\mathrm{Im}\,\Pi \to \text{dichroism}.$$

This is already the basis of laboratory polarization searches for weakly charged particles.

3. The Universe provides a natural long-baseline version of the experiment

$$\boxed{\text{CMB photons} + \text{cosmic magnetic fields}}$$

provide the conditions for magnetized-vacuum polarization over cosmological distances.

Motivation

4. Why MCP rather than ordinary electrons?

   MCP birefringence can be strongly enhanced in the low-χ regime:

$$\boxed{\Delta\widetilde{M} \propto \left(\frac{\epsilon}{m_\epsilon}\right)^4 B_\perp^2 \omega}$$

5. MCPs can also open absorptive channels unavailable to ordinary CMB photons

$\gamma \to \psi_\epsilon \bar{\psi}_\epsilon$ ➡ $\mathrm{Im}\,\Pi \neq 0$ ➡ dichroism

6. Can this propagation physics convert the existing CMB linear polarization into circular/elliptical polarization?

$$\boxed{P_C(T_0) = \frac{|V(T_0)|}{I(T_0)}\ ?}$$

Wave equation for the photon field in the FRW metric

$$i\partial_t \Psi(k,t) = \left(M(k, B_e, \Phi) - \frac{3}{2} iH\, \boldsymbol{I} \right) \Psi(k,t),$$

Definitions:

$\Psi(k,t) = (A_+, A_\times)^{\mathrm{T}}$ A_+: ⊥ to $B_\perp$ $A_\times$: ∥ to $B_\perp$

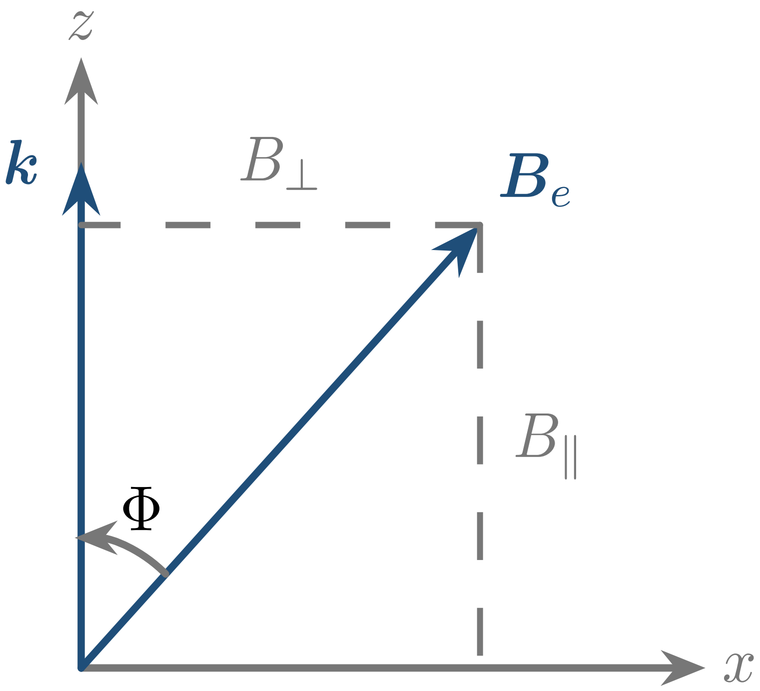
$$M = \begin{pmatrix} k - M_+ & -iM_F \\ iM_F & k - M_\times \end{pmatrix},$$

$M_F \rightarrow$ Faraday mixing

$$M_+ = -\Pi^{22}/(2\omega),\ M_\times = -\Pi^{11}/(2\omega),\ iM_F = -\Pi^{12}/(2\omega)$$

Igoring M_F,

$$\boxed{\begin{aligned} i\dot{A}_+ &= (k - M_+)A_+, \\ i\dot{A}_\times &= (k - M_\times)A_\times. \end{aligned} \rightarrow \begin{aligned} A_+(t) &\propto e^{-i(k-M_+)t}, \\ A_\times(t) &\propto e^{-i(k-M_\times)t}. \end{aligned} \rightarrow \Delta\phi(t) = \int dt\,(M_+ - M_\times).}$$

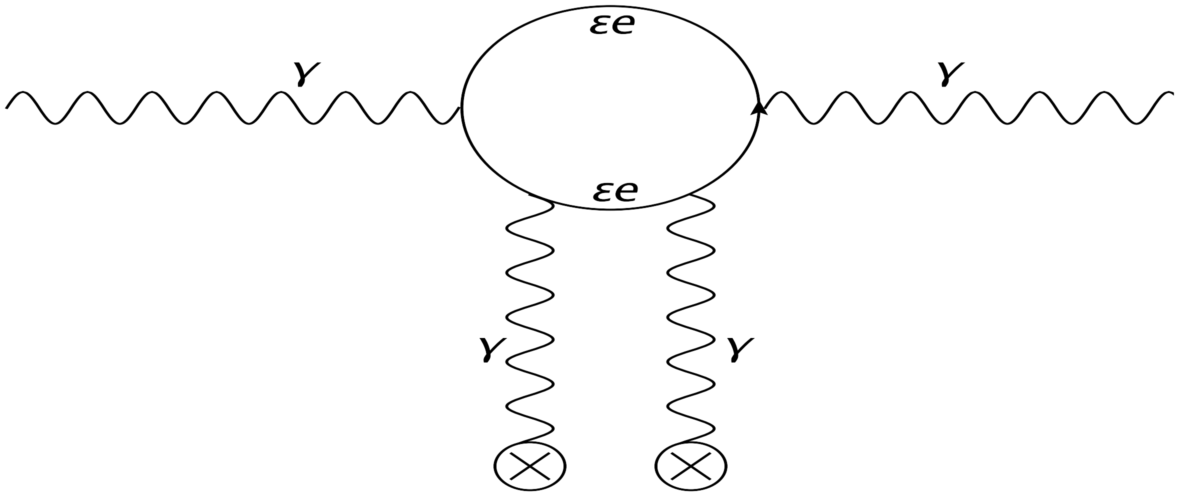


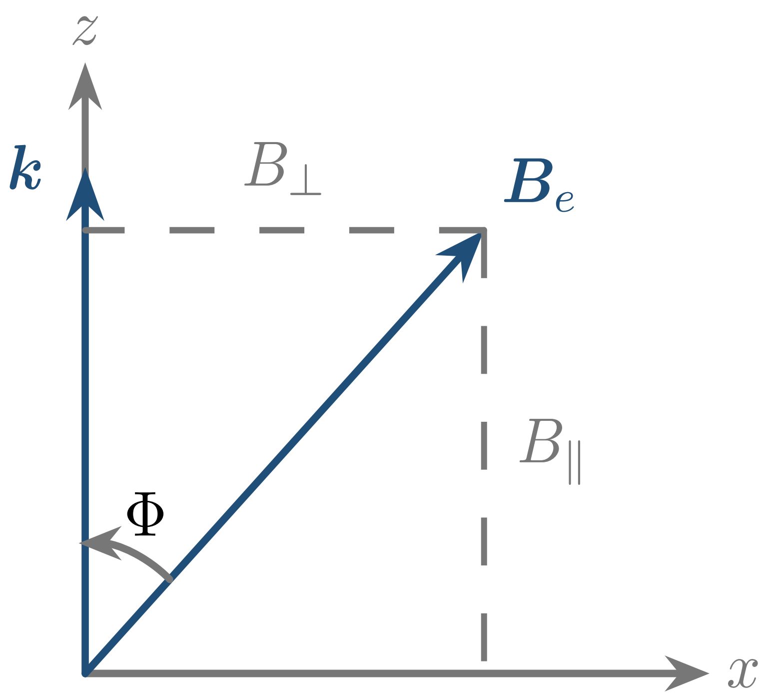
$\boldsymbol{k} = (0,0,k)$, $\boldsymbol{B}_e = (B_e \sin\Phi,\ 0,\ B_e \cos\Phi)$

A transverse cosmic field splits the photon eigenmodes

$$Q_\epsilon = \epsilon e, \qquad m_\epsilon$$

$$S^{(1)}_{\mathrm{MCP}} = -i\ln\det\left(i\gamma^\mu D_{\epsilon\mu} - m_\epsilon\right), \qquad D_{\epsilon\mu} = \partial_\mu + i\epsilon e A_\mu$$





$$\boldsymbol{k} = (0,0,k), \qquad \boldsymbol{B}_e = (B_e\sin\Phi,\ 0,\ B_e\cos\Phi)$$

$B_\perp$ **breaks transverse isotropy:** $\Pi^{11} \neq \Pi^{22}$**, hence** $n_+ \neq n_\times$.

Evolution of the density matrix in $(A_+, A_\times)$ basis

$$\boxed{\frac{\partial\rho}{\partial t} = -i[\tilde{M},\rho] - \{D,\rho\},}$$

Definitions:

$$\boxed{\tilde{M} = \begin{pmatrix} k - \tilde{M}_0^{(+)} & -iM_F \\ iM_F & k - \tilde{M}_0^{(\times)} \end{pmatrix}}$$

$$\rho_{ij} = \langle A_i A_j^* \rangle.$$

$$\boxed{D = \frac{3}{2}HI + \begin{pmatrix} M_1^{(+)} & 0 \\ 0 & M_1^{(\times)} \end{pmatrix}}$$

$$M = \begin{pmatrix} k - M_+ & -iM_F \\ iM_F & k - M_\times \end{pmatrix}$$

$$M_+ = \tilde{M}_0^{(+)} + iM_1^{(+)}, \qquad M_\times = \tilde{M}_0^{(\times)} + iM_1^{(\times)}$$

$$\Delta\tilde{M} \equiv \tilde{M}_0^{(+)} - \tilde{M}_0^{(\times)} = \frac{\mathrm{Re}\,\Pi^{11} - \mathrm{Re}\,\Pi^{22}}{2\omega} = \omega(n_+ - n_\times)$$

$$\Delta M_0 \equiv M_1^{(+)} - M_1^{(\times)}, \; \Delta M_1 \equiv M_1^{(+)} + M_1^{(\times)}$$

Three distinct effects

$\mathrm{Re}\Pi \rightarrow \Delta\tilde{M} \rightarrow$ **birefringent phase**

$\mathrm{Im}\Pi \rightarrow \Delta M_{0,1} \rightarrow$ **differential absorption**

$M_F \rightarrow$ **Faraday rotation**

Stokes parameters (I, Q, U, V)

$$\rho = \frac{1}{2}\left(I\sigma_0 + Q\sigma_3 + U\sigma_1 + V\sigma_2\right)$$

$$\longrightarrow \quad \rho = \frac{1}{2}\begin{pmatrix} I+Q & U-iV \\ U+iV & I-Q \end{pmatrix}_{(A_+, A_\times)}$$

$$I = \langle |E_x|^2 + |E_y|^2 \rangle,$$ $\longrightarrow$ Total intensity

$$Q = \langle |E_x|^2 - |E_y|^2 \rangle,$$ $\longrightarrow$ Horizontal vs vertical linear polarization

$$U = 2\,\mathrm{Re}\langle E_x E_y^* \rangle \sim \cos\delta,$$ $\longrightarrow$ Linear polarization along $+45^\circ$ vs -45°

$$V = 2\,\mathrm{Im}\langle E_x E_y^* \rangle \sim \sin\delta.$$ $\longrightarrow$ Circular polarization

Stokes evolution

$$\dot{I} = -\Delta M_1 I - \Delta M_0 Q - 3HI,$$
$$\dot{Q} = -\Delta M_0 I - \Delta M_1 Q - 2M_F U - 3HQ,$$
$$\dot{U} = 2M_F Q - \Delta M_1 U + \Delta \tilde{M} V - 3HU,$$
$$\dot{V} = -\Delta \tilde{M} U - \Delta M_1 V - 3HV,$$

$I \leftrightarrow Q$: dichroism

$$\dot{I} = -\Delta M_0 Q,$$
$$\dot{Q} = -\Delta M_0 I.$$

Unpolarized $\to$ linearly polarized

$$Q_{\text{ini}} = 0$$

$$\Gamma_+ \neq \Gamma_\times \quad \Rightarrow \quad I_+ \neq I_\times \quad \Rightarrow \quad Q \neq 0$$

Differential absorption generates linear polarization

Stokes evolution

$Q \leftrightarrow U$: Faraday rotation

$$Q = P_L I \cos 2\psi,$$
$$U = P_L I \sin 2\psi,$$

$$\dot{Q} = -2M_F U,$$
$$\dot{U} = 2M_F Q.$$

$$\begin{pmatrix} Q \\ U \end{pmatrix} \to \begin{pmatrix} \cos 2\psi & -\sin 2\psi \\ \sin 2\psi & \cos 2\psi \end{pmatrix} \begin{pmatrix} Q \\ U \end{pmatrix}.$$

$U \leftrightarrow V$: vacuum birefringence

$$\dot{U} = \Delta\widetilde{M} V,$$
$$\boxed{\dot{V} = -\Delta\widetilde{M} U.}$$

$$U_i \neq 0, \qquad V_i = 0.$$
$$\dot{V}(t_i) = \boxed{-\Delta\widetilde{M} U_i} \neq 0$$
$$V \neq 0.$$

$$\boxed{I \overset{\Delta M_0}{\longleftrightarrow} Q \overset{M_F}{\longleftrightarrow} U \overset{\Delta\widetilde{M}}{\longleftrightarrow} V}$$

$$\boxed{\Delta\tilde{M} = \frac{\mathrm{Re}\,\Pi^{11} - \mathrm{Re}\,\Pi^{22}}{2\omega}}$$

$\Delta M_0 =$ dichroism, $M_F =$ Faraday rotation, $\Delta\widetilde{M} =$ vacuum birefringence.

One-loop birefringence is controlled by the parameter χ

$$\Delta\widetilde{M} = -\left(\frac{\epsilon m_e}{m_\epsilon}\right)^4 \omega \left(\frac{\alpha}{4\pi}\right)\left(\frac{B_e}{B_c}\right)^2 \sin^2\Phi \; \Delta\mathcal{I}(\chi)$$

$$\Delta\widetilde{M} = \frac{\mathrm{Re}\Pi^{11} - \mathrm{Re}\Pi^{22}}{2\omega} = \omega(n_+ - n_\times), \qquad B_c = \frac{m_e^2}{e}$$

> **The real polarization tensor supplies a relative refractive phase.**

$$\chi = \frac{3}{2}\,\epsilon\left(\frac{\omega}{m_e}\right)\left(\frac{m_e}{m_\epsilon}\right)^3\left(\frac{B_e}{B_c}\right)\sin\Phi$$

χ increases with

- photon energy ω
- transverse field $B_e\sin\Phi$
- smaller m_ϵ

Small-χ gives the Euler–Heisenberg fourth-power scaling

$$\Delta\mathcal{J}(\chi) = \begin{cases} \dfrac{6}{45}, & \chi \ll 1, \\ -\dfrac{9}{7}\sqrt{\pi}\, 2^{1/3}\,\dfrac{\Gamma^2(2/3)}{\Gamma(1/6)}\,\chi^{-4/3}, & \chi \gg 1. \end{cases}$$

$\chi \ll 1$: the birefringence is a power law with strong m_ϵ^{-4} dependence.

$\chi \gg 1$: $\Delta\mathcal{J} \propto \chi^{-4/3}$

The allowed effect is practically tiny.

$$\longrightarrow \quad \Delta\widetilde{M} \simeq -\frac{2}{15}\alpha^2\epsilon^4\,\frac{\omega B_e^2 \sin^2\Phi}{m_\epsilon^4} \qquad (\chi \ll 1)$$

Dichroism is exponentially suppressed when $\chi \ll 1$

$$\Delta M_0 = \kappa_+ - \kappa_\times = \frac{1}{2}\epsilon^3 \left(\frac{m_e}{m_\epsilon}\right) \alpha \omega_c \sin\Phi \, \Delta\mathcal{T}_0(\chi),$$

$$\Delta M_1 = \kappa_+ + \kappa_\times = \frac{1}{2}\epsilon^3 \left(\frac{m_e}{m_\epsilon}\right) \alpha \omega_c \sin\Phi \, \Delta\mathcal{T}_1(\chi), \qquad \omega_c = \frac{eB_e}{m_e}.$$

$$\Delta\mathcal{T}_0(\chi) = -\frac{1}{4}\sqrt{\frac{3}{2}}\, e^{-4/\chi}, \qquad \Delta\mathcal{T}_1(\chi) = \frac{3}{4}\sqrt{\frac{3}{2}}\, e^{-4/\chi} \qquad (\chi \ll 1)$$

birefringence

power law in χ

dichroism

$\propto \exp(-4/\chi)$

For $k \perp B$, Faraday rotation vanishes and sectors decouple

$$\Phi = \frac{\pi}{2} \quad \Rightarrow \quad M_F \propto \cos\Phi = 0$$

(I, Q)

dichroism sector

(U, V)

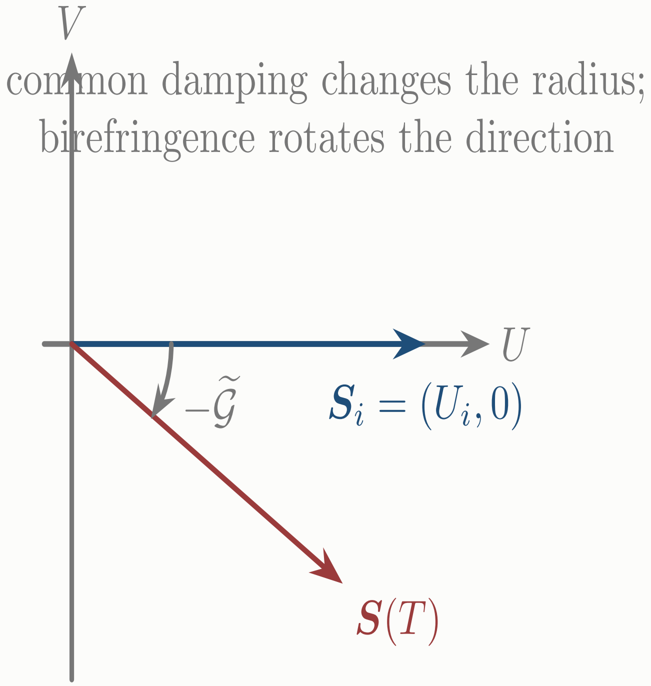
birefringence sector

$$\begin{aligned} \dot{U} &= -(\Delta M_1 + 3H)U + \Delta\widetilde{M}V, \\ \dot{V} &= -\Delta\widetilde{M}U - (\Delta M_1 + 3H)V. \end{aligned}$$

> $[B(T_1), B(T_2)] = 0 \Rightarrow$ **the solution is an exact matrix exponential.**

The perpendicular case is an exact U–V rotation

$$\begin{pmatrix} U(T) \\ V(T) \end{pmatrix} = \left(\frac{T}{T_i}\right)^2 e^{-\mathcal{G}_1(T)} \begin{pmatrix} \cos\tilde{\mathcal{G}} & \sin\tilde{\mathcal{G}} \\ -\sin\tilde{\mathcal{G}} & \cos\tilde{\mathcal{G}} \end{pmatrix} \begin{pmatrix} U_i \\ V_i \end{pmatrix}$$



$$\tilde{\mathcal{G}}(T) = \int_T^{T_i} \frac{\Delta\widetilde{M}(T')}{H(T')T'}\,\mathrm{d}T', \qquad \mathcal{G}_a(T) = \int_T^{T_i} \frac{\Delta M_a(T')}{H(T')T'}\,\mathrm{d}T' \quad (a = 0,1)$$

$$V(T) = -U_i \sin\left[\tilde{\mathcal{G}}(T)\right] \left(\frac{T}{T_i}\right)^2 e^{-\mathcal{G}_1(T)} \qquad (V_i = 0)$$

Cosmological redshifting weights the phase toward decoupling

$$\omega(T) = \omega_0 \frac{T}{T_0}, \qquad B_e(T) = B_{e0} \left(\frac{T}{T_0}\right)^2, \qquad H(T) \simeq H_0 \sqrt{\Omega_M} \left(\frac{T}{T_0}\right)^{3/2}$$

$$\tilde{\mathcal{G}}(T) = \mathcal{C} \int_T^{T_i} \mathrm{d}T' \, T'^{5/2} \, \Delta\mathcal{I}[\chi(T')] \qquad \partial_t = -HT\,\partial_T, \qquad \mathrm{d}t = -\frac{\mathrm{d}T}{HT}$$

$$\Delta\widetilde{M} \propto \omega B_e^2 \propto T^5, \qquad \frac{\Delta\widetilde{M}}{HT} \propto T^{5/2}, \qquad \tilde{\mathcal{G}} \propto T_i^{7/2} - T^{7/2}$$

$$\tilde{\mathcal{G}}(T) = \frac{12}{315}\,\mathcal{C}\left(T_i^{7/2} - T^{7/2}\right) \qquad (\chi \ll 1)$$

The low-χ signal inherits a fourth-power MCP lever arm

$$P_C(T_0) \simeq \left| -\sin\left[\tilde{\mathcal{G}}(T_0)\right] \frac{U_i}{I_i} \right| \qquad (V_i = 0,\ |\mathcal{G}_0| \ll 1)$$

$$P_C(T_0) \simeq \left| \sin\left[2.7 \times 10^{-3} \left(\frac{\epsilon m_e}{m_\epsilon}\right)^4 \left(\frac{\nu_0}{\mathrm{Hz}}\right) \left(\frac{B_{e0}}{\mathrm{G}}\right)^2 \right] \frac{U_i}{I_i} \right|$$

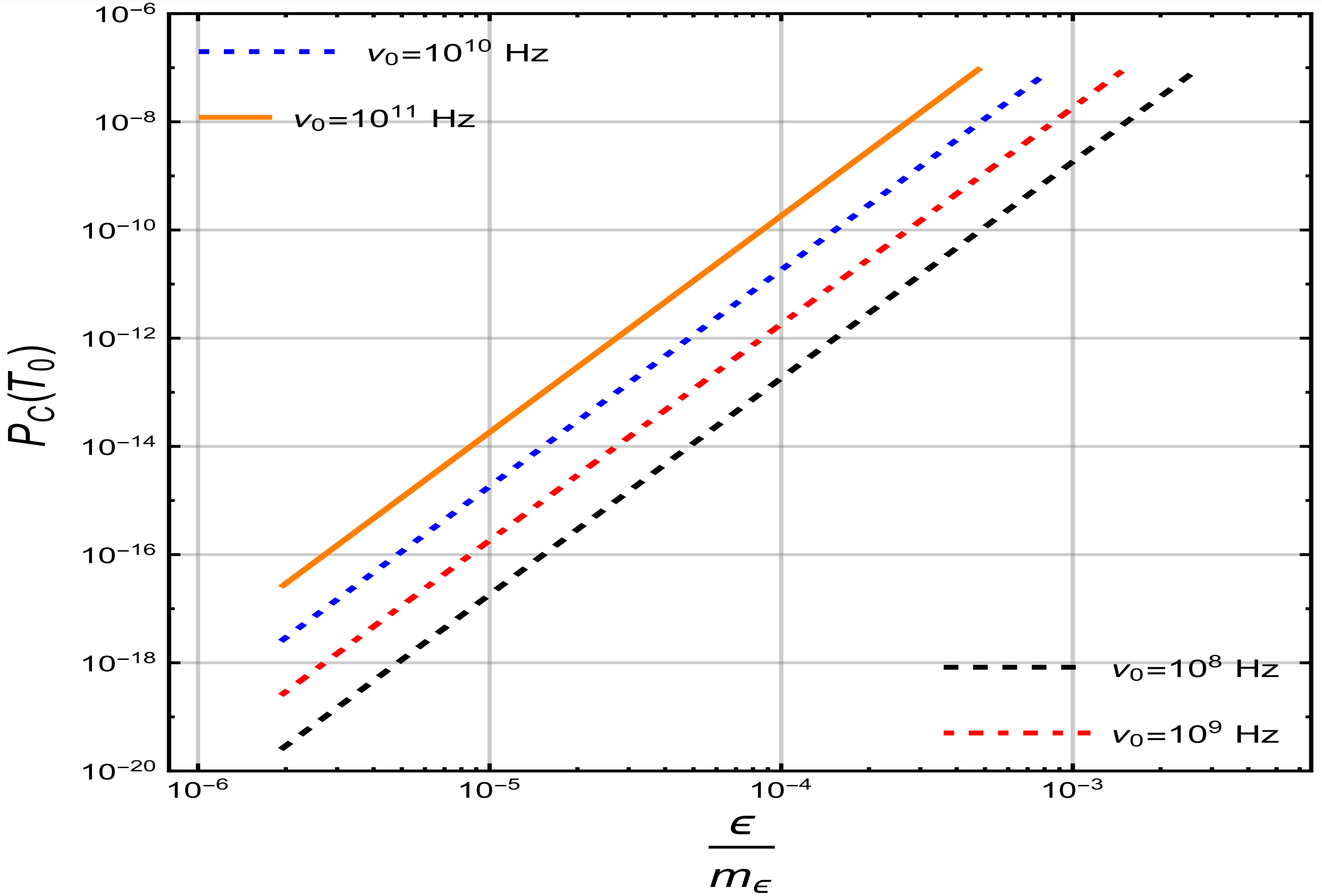
ν_0: photon frequency at present

$$P_C(T_0) \simeq 2.7 \times 10^{-3} \left(\frac{\epsilon m_e}{m_\epsilon}\right)^4 \left(\frac{\nu_0}{\mathrm{Hz}}\right) \left(\frac{B_{e0}}{\mathrm{G}}\right)^2 \left| \frac{U_i}{I_i} \right|$$

$$P_C \propto \left(\frac{\epsilon}{m_\epsilon}\right)^4 B_{e0}^2 \nu_0 \qquad (\chi \ll 1,\ |\tilde{\mathcal{G}}| \ll 1)$$

Assumptions: $V_i = 0$; negligible dichroism.

The quoted reach is optimistic and parameter-dependent



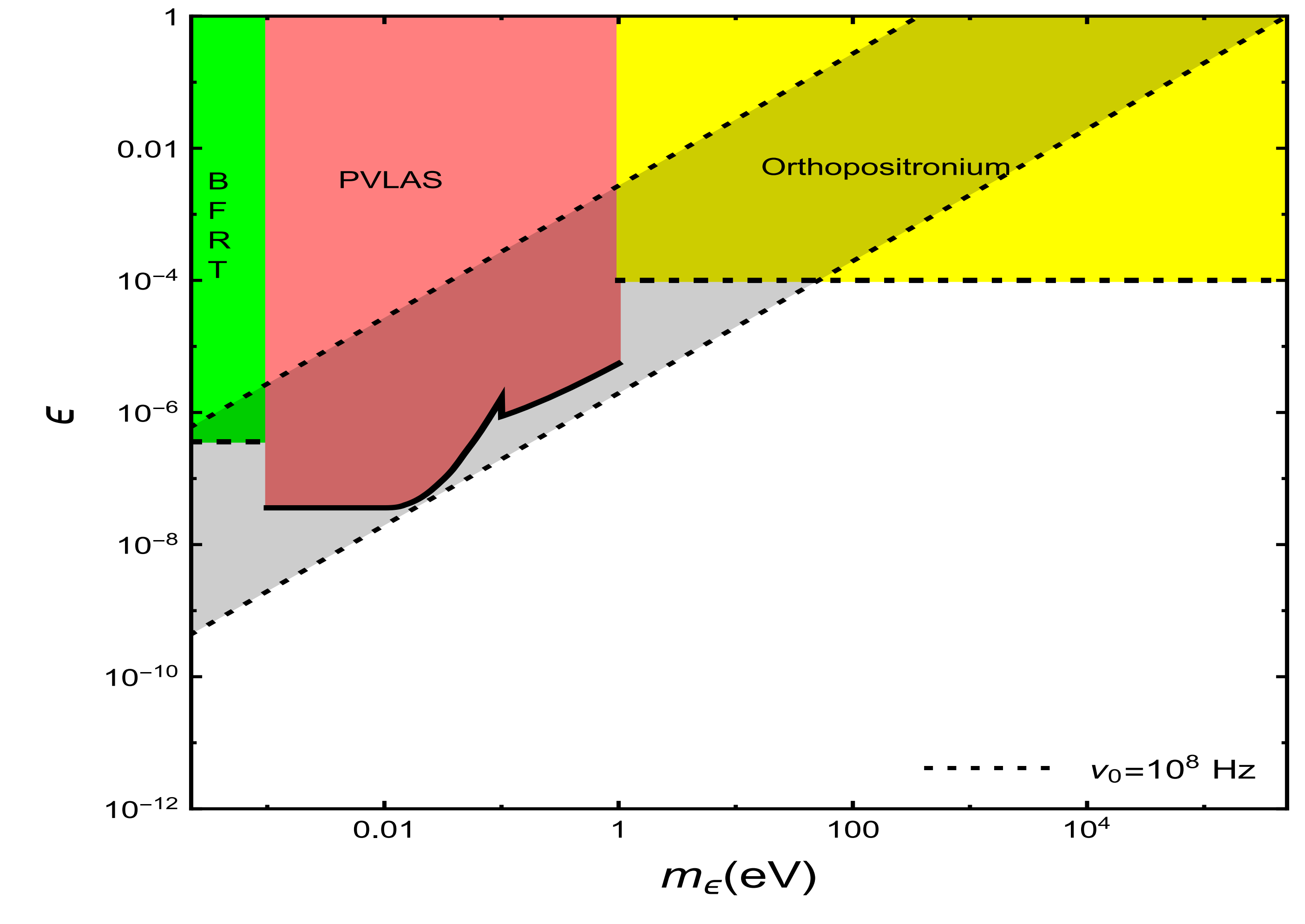
Paper Fig. 4(a): present field 1 nG, $|r| = 0.1$, and $\frac{Q_i}{I_i} = 10^{-6}$.

$$10^{-10}\text{–}10^{-6}$$

optimistic future-detection range quoted by the paper

increases with charge-to-mass ratio and observing frequency

Parameter constraints used in 2017



present field 1 nG; observing frequency 10^8 Hz .

How to read this slide

gray region: allowed by the joint conditions

colored regions: PVLAS, BFRT, and orthopositronium exclusions used in 2017

white does not mean excluded; the paper's joint conditions are simply not valid there

do not treat these contours as current limits

Summary

- MCP vacuum polarization in an external field produces birefringence.
- One Stokes system unifies dichroism, Faraday rotation, and birefringence.
- Existing linear CMB polarization can acquire a circular component.

$$I \xleftrightarrow{\text{dichroism } \Delta M_0} Q \xleftrightarrow{\text{Faraday } 2M_F} U \xleftrightarrow{\text{birefringence } \Delta\widetilde{M}} V$$