

Roper Resonance and 1^{-+} Meson

- Roper resonance: lattice calculations and its nature
- Is 1^{-+} meson a hybrid?

χ QCD Collaboration:

A. Alexandru, Y. Chen, S.J. Dong, T. Draper, M. Glatzmaier,
M. Gong, F.X. Lee, A. Li, H.W. Lin, K.F. Liu, N. Mathur, Y. Yang

Beijing, May 11, 2013

Many Facets of Roper Resonance

- Roper resonance (P_{11} in πN scattering, with $I = \frac{1}{2}$, $M = 1440$ MeV, $\Gamma \sim 300$ MeV) was discovered in pion-nucleon partial wave analysis in 1963.
- πN and γN has large contamination from Δ .
- Confirmation from (α, α') scattering and $J/\psi \rightarrow \bar{N}N\pi$ decay.
(review by L. Alvarez-Ruso)

M. Ablikim, et al., PRL 97, 062001 (2006)

PRL 97, 062001 (2006) PHYSICAL REVIEW LETTERS

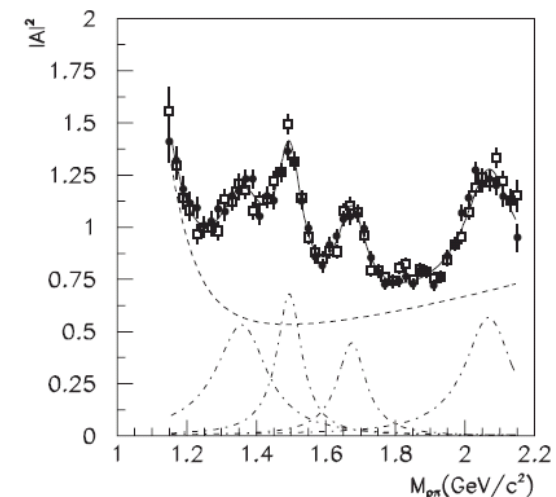
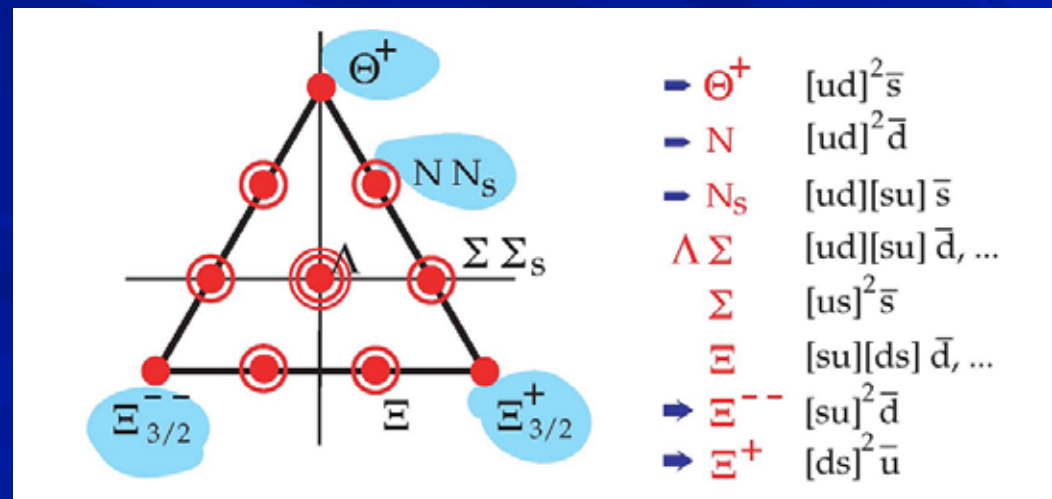


FIG. 6. Data divided by Monte Carlo phase space versus $p\pi$ invariant mass for $J/\psi \rightarrow p\pi^-\bar{n}$ (solid circles) and $J/\psi \rightarrow \bar{p}\pi^+n$ (open squares), compared with our fit (solid curve). The contributions of each resonance peak are shown by the dot-dashed lines in the same figure. The dashed line is the contribution of background terms including the nucleon pole term.

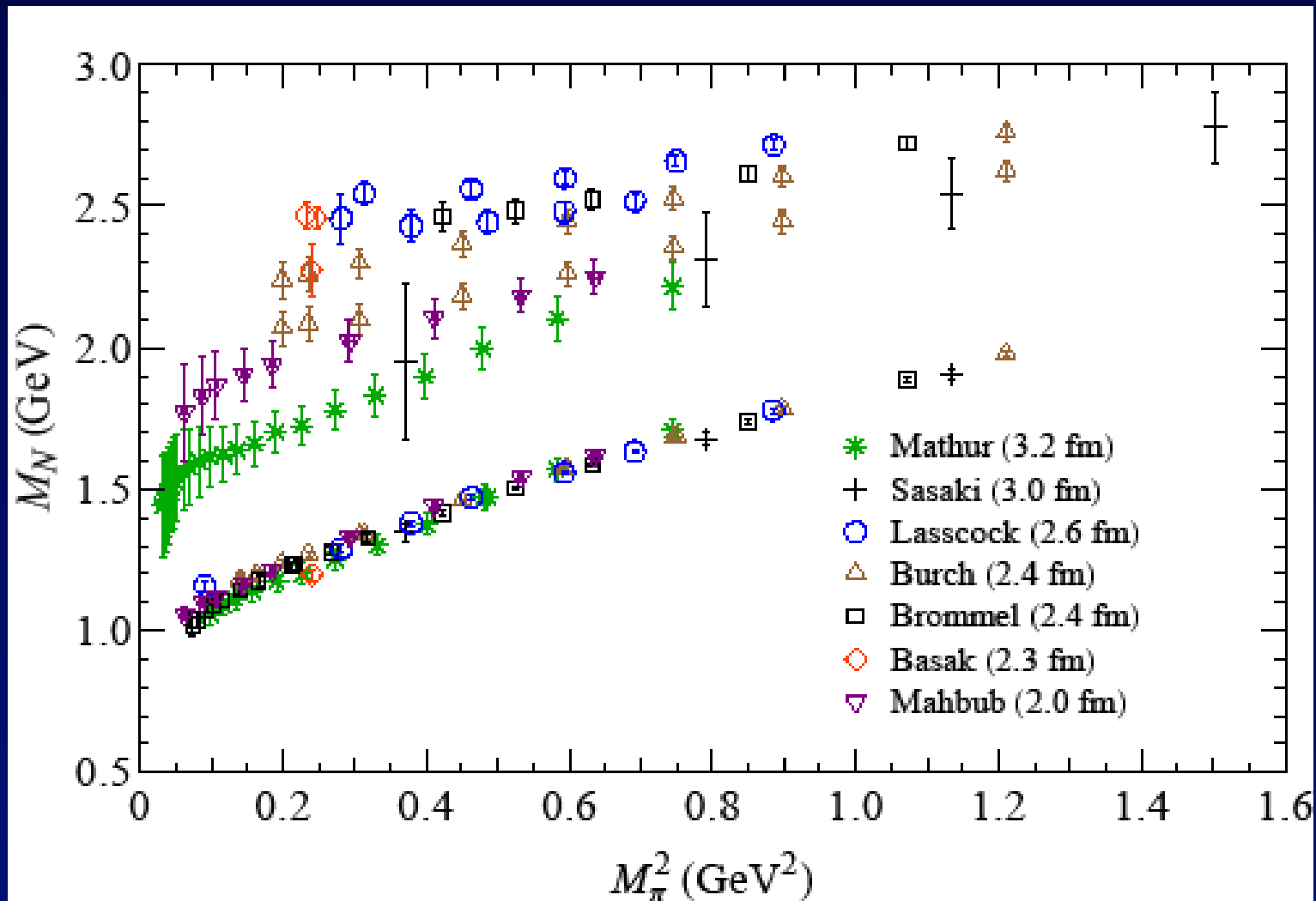
Many Facets of Roper Resonance

Theory:

- Quark potential model prediction is 100-200 MeV too high (Liu and Wong, 1983, Capstick and Isgur, 1986)
- Skyrmion can accommodate it as a radial excitation (J. Breit and C. Nappi, 1984, Liu, Zhang, Black, 1984; U. Kaulfuss and U. Meissner, 1985)
- Suggestion as a pentaquark (Krewald 2000); as a member of the antidecuplet (Jaffe, Wilczek, 2003)
- Perhaps a hybrid (Barnes, Close, etc. 1983)
- → Lattice calculations



Quenched Lattice Calculations of Roper



Roper on the lattice

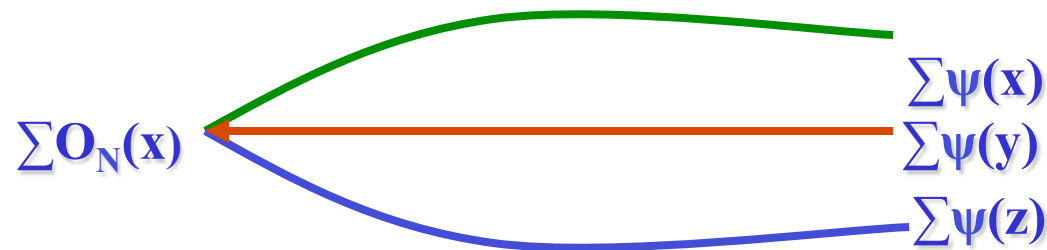
- 4 issues about lattice calculations:
 - Radial excitation or pentaquark state?
 - Dynamical fermions
 - Variation vs Bayesian fitting
 - Chiral dynamics

Roper

Radial excitation? q^4q State?

- Roper is seen on the lattice with **three-quark** interpolation field.
- Weight :

$$| \langle 0 | O_N | R \rangle |^2 > | \langle 0 | O_N | N \rangle |^2 > 0 \quad (\text{point source, point sink})$$

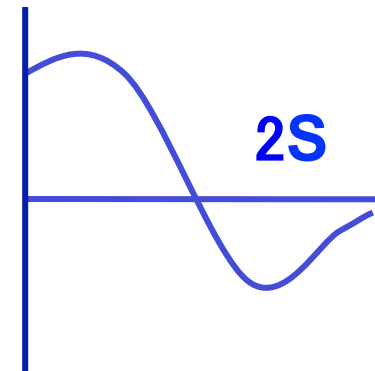
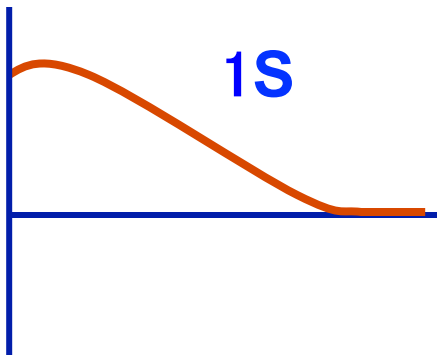


Point sink

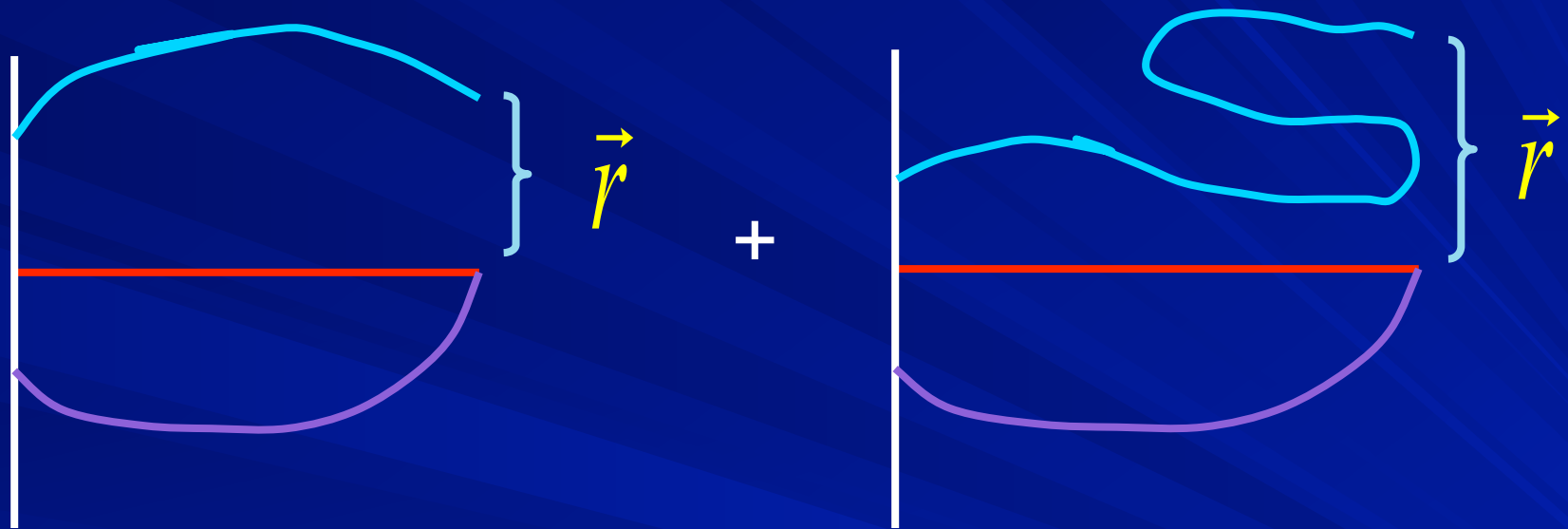
Wall source

$$\langle 0 | O_N(0) | N \rangle \langle N | \sum \psi(x) \sum \psi(y) \sum \psi(z) | 0 \rangle > 0$$

However, $\langle 0 | O_N(0) | R \rangle \langle R | \sum \psi(x) \sum \psi(y) | \sum \psi(z) | 0 \rangle < 0$



Bethe-Salpeter Wavefunction

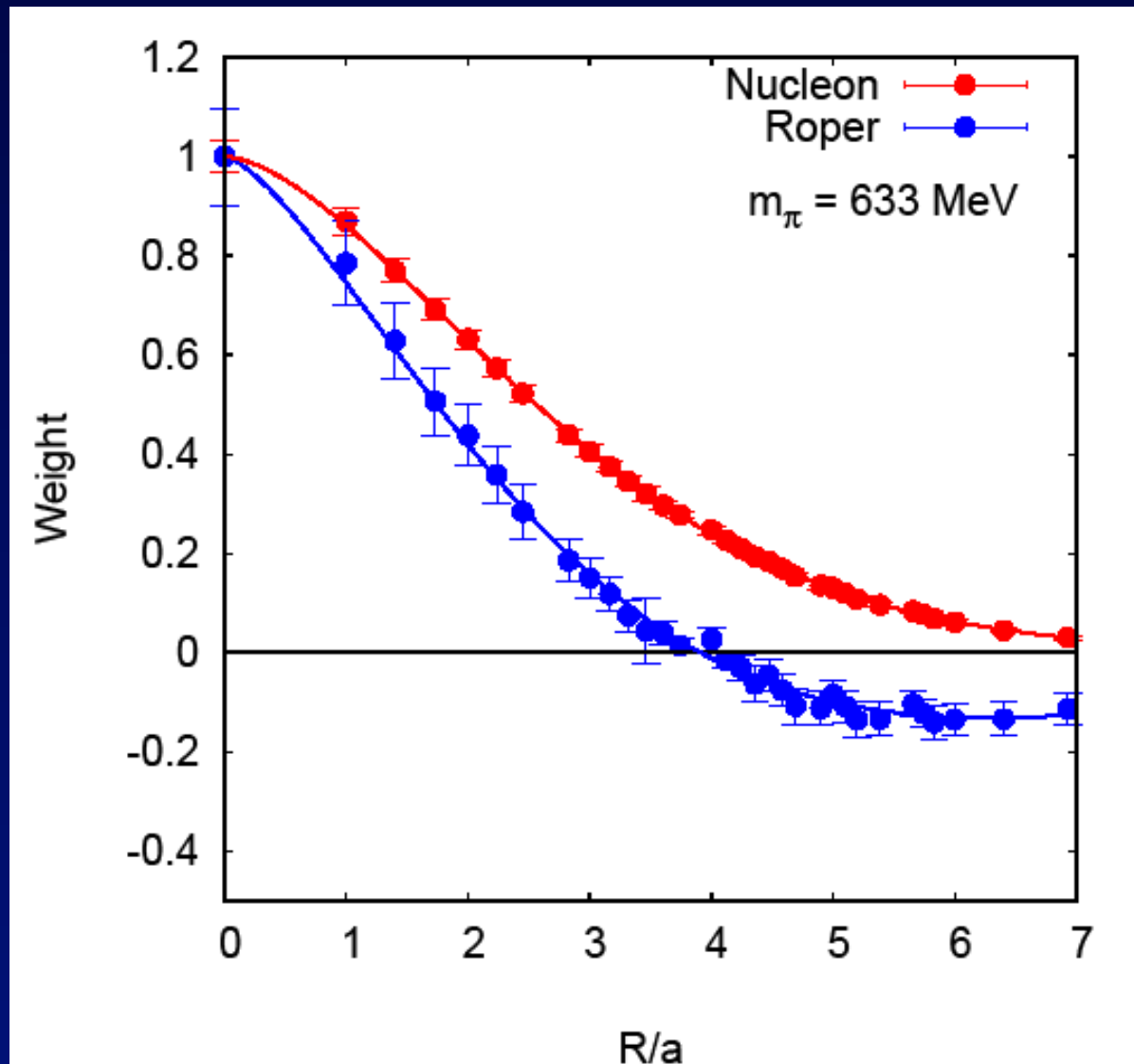


$$O_{RN} = \int dr \Psi_R^*(r) \Psi_N(r) = 0 \quad \text{at non-relativistic limit,}$$

$$O_{RN} = \int dr \Psi_R^*(r) \Psi_N(r) \quad \uparrow \quad \text{as } m_q \downarrow$$

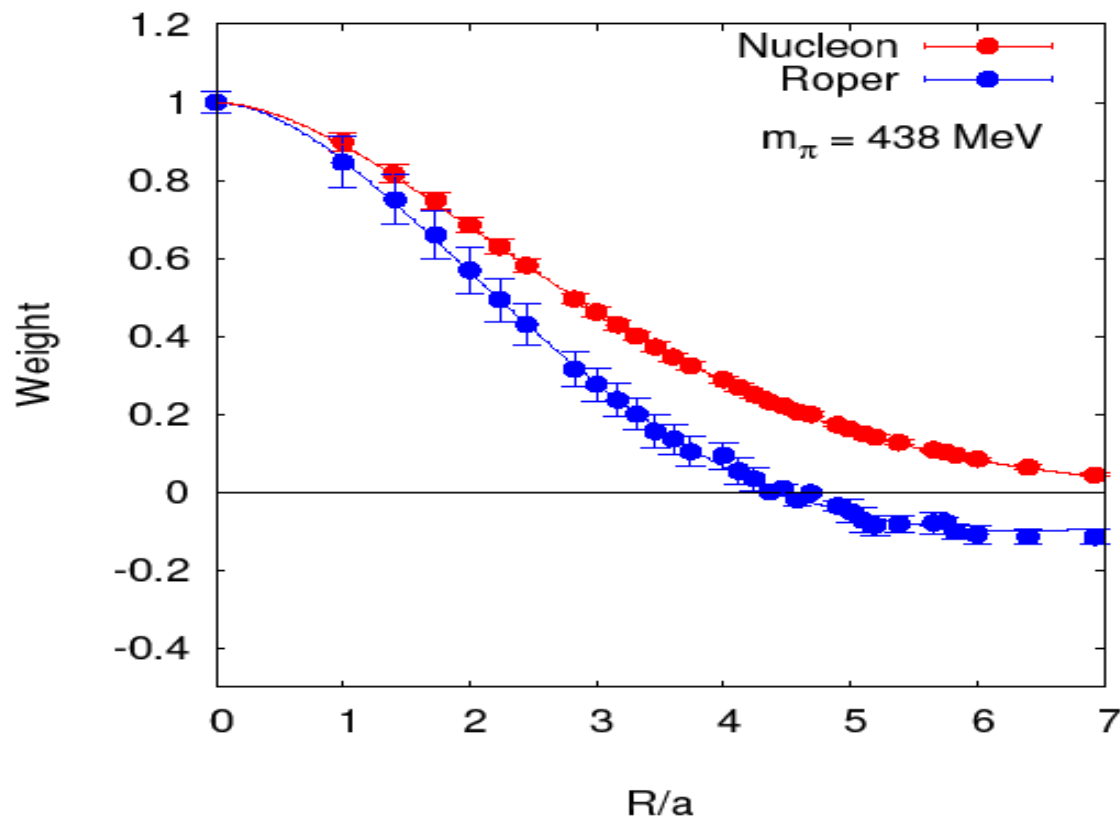
Nucleon and Roper wavefunctions for $m_\pi = 633$ MeV

$$O_{RN} = 0.30$$

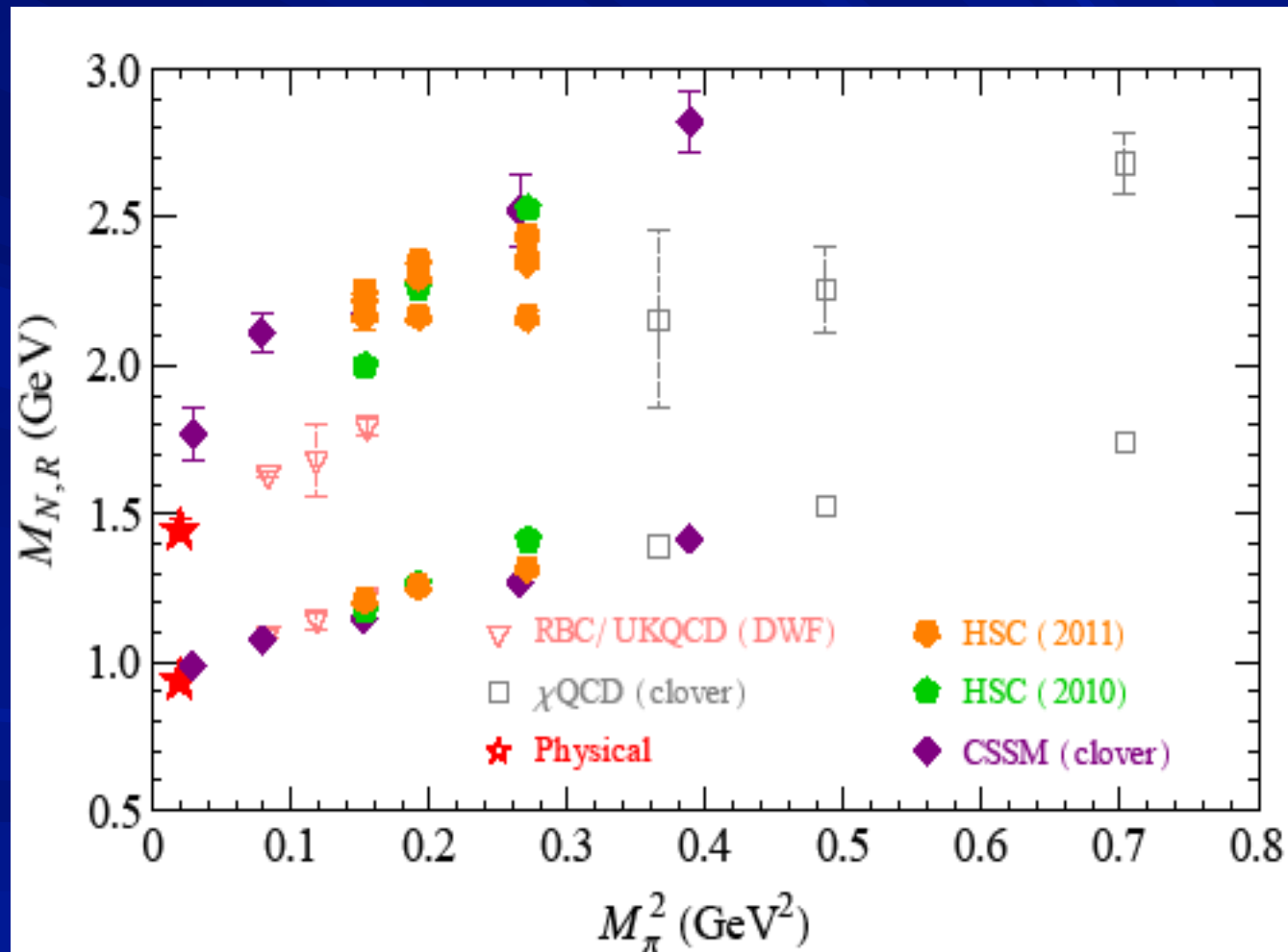


Roper and Nucleon Wavefunctions at $m_\pi = 438$ MeV

$$O_{RN} = 0.59$$

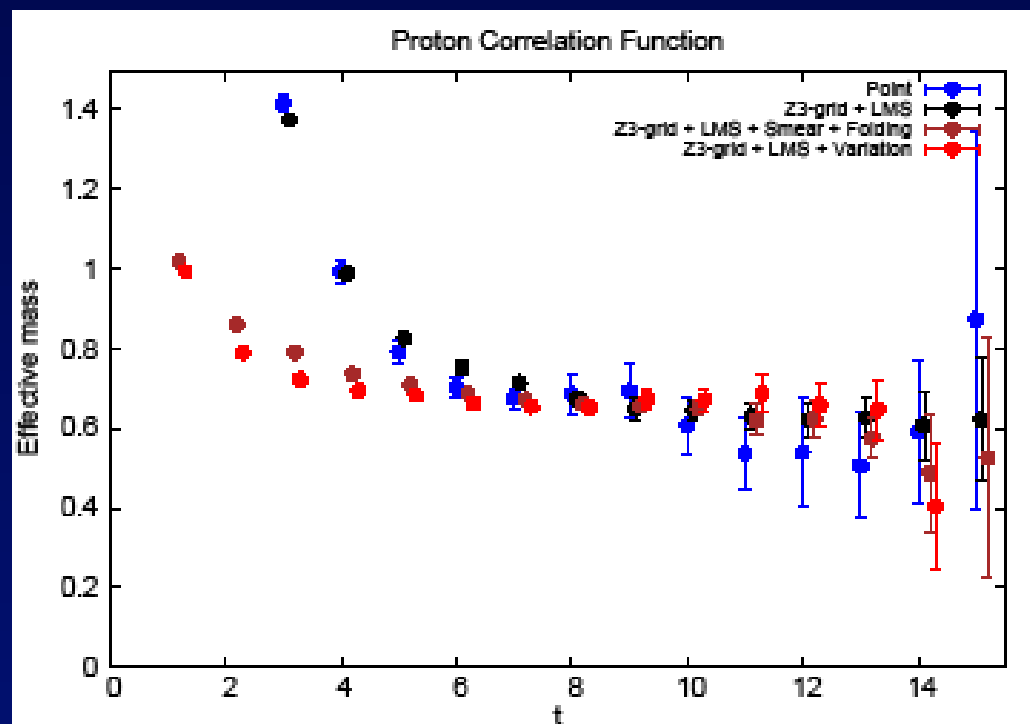


Dynamical Fermions

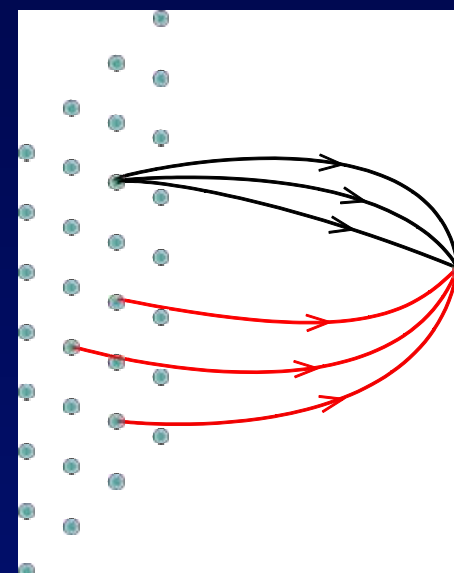


Dynamical Fermions (Overlap on DWF Configurations)

- Improvement of nucleon correlator with low-mode substitution

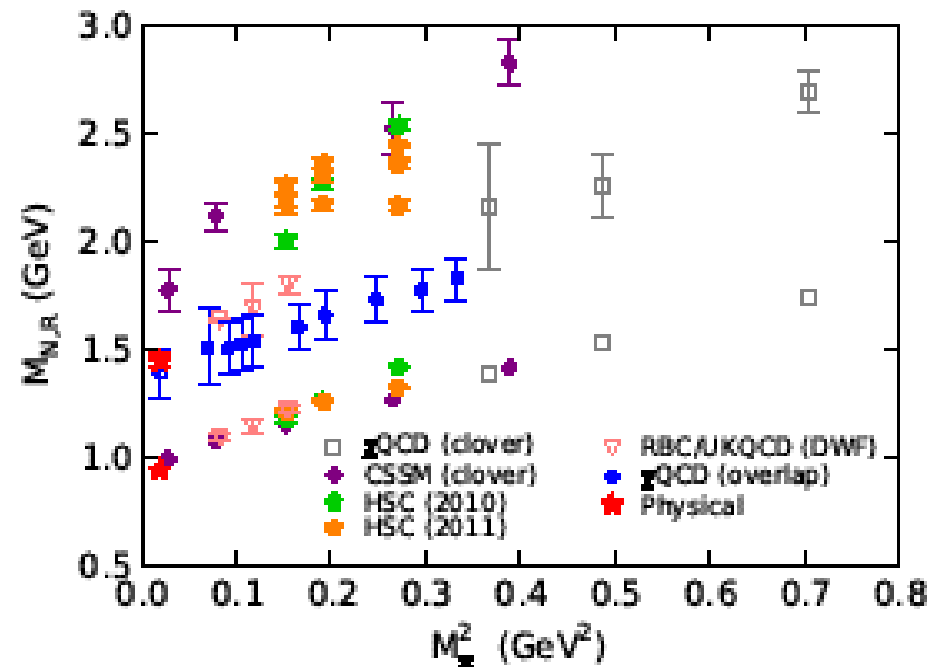
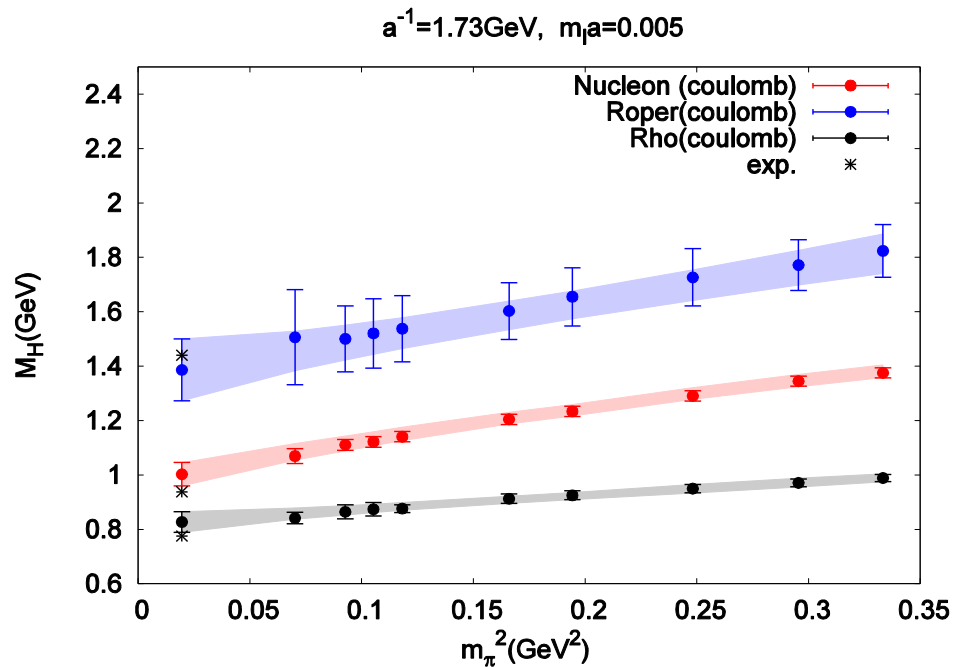


$24^3 \times 64$ lattice with $m_\pi = 331$ MeV, $a = 1.73$ GeV $^{-1}$
47 configurations



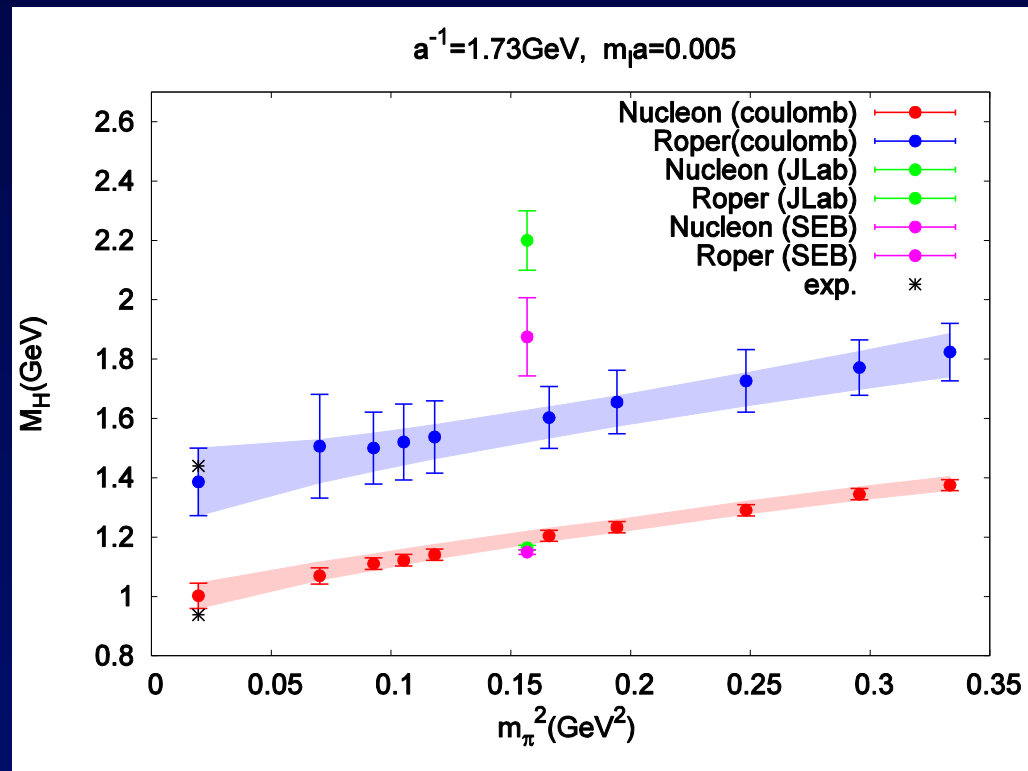
Point source: $m_N = 1.13(14)$ GeV;
 Z_3 grid source: $m_N = 1.08(5)$ GeV;
 Z_3 grid smeared source: $m_N = 1.14(2)$ GeV;
 Variation: $m_N = 1.16(1)$ GeV

- Roper resonance from Coulomb wall source



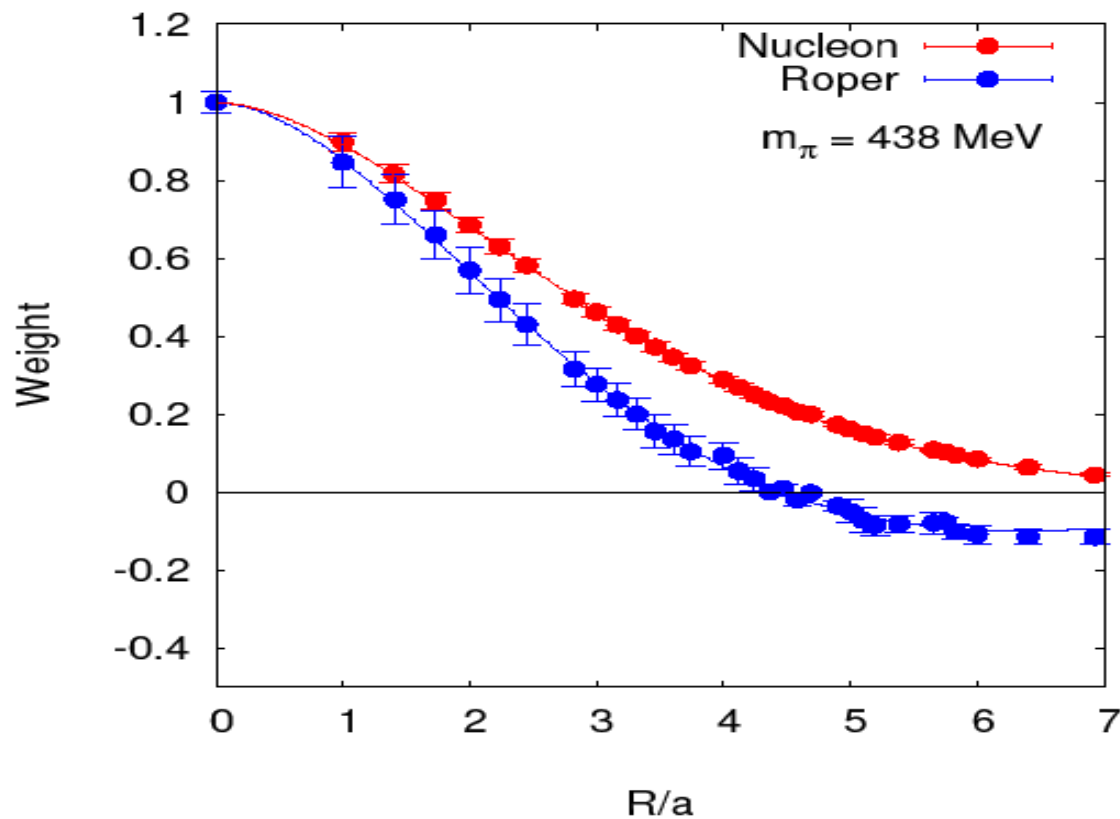
$$m_N = m_0 + c_1 m_\pi^2 + c_2 m_\pi^3 (\text{mixed}) + c_3 m_\pi^4 \ln(m_\pi^2 / \mu^2) + \dots$$

$24^3 \times 64$ lattice with $m_\pi = 331 \text{ MeV}(\text{sea})$, $a = 1.73 \text{ GeV}^{-1}$



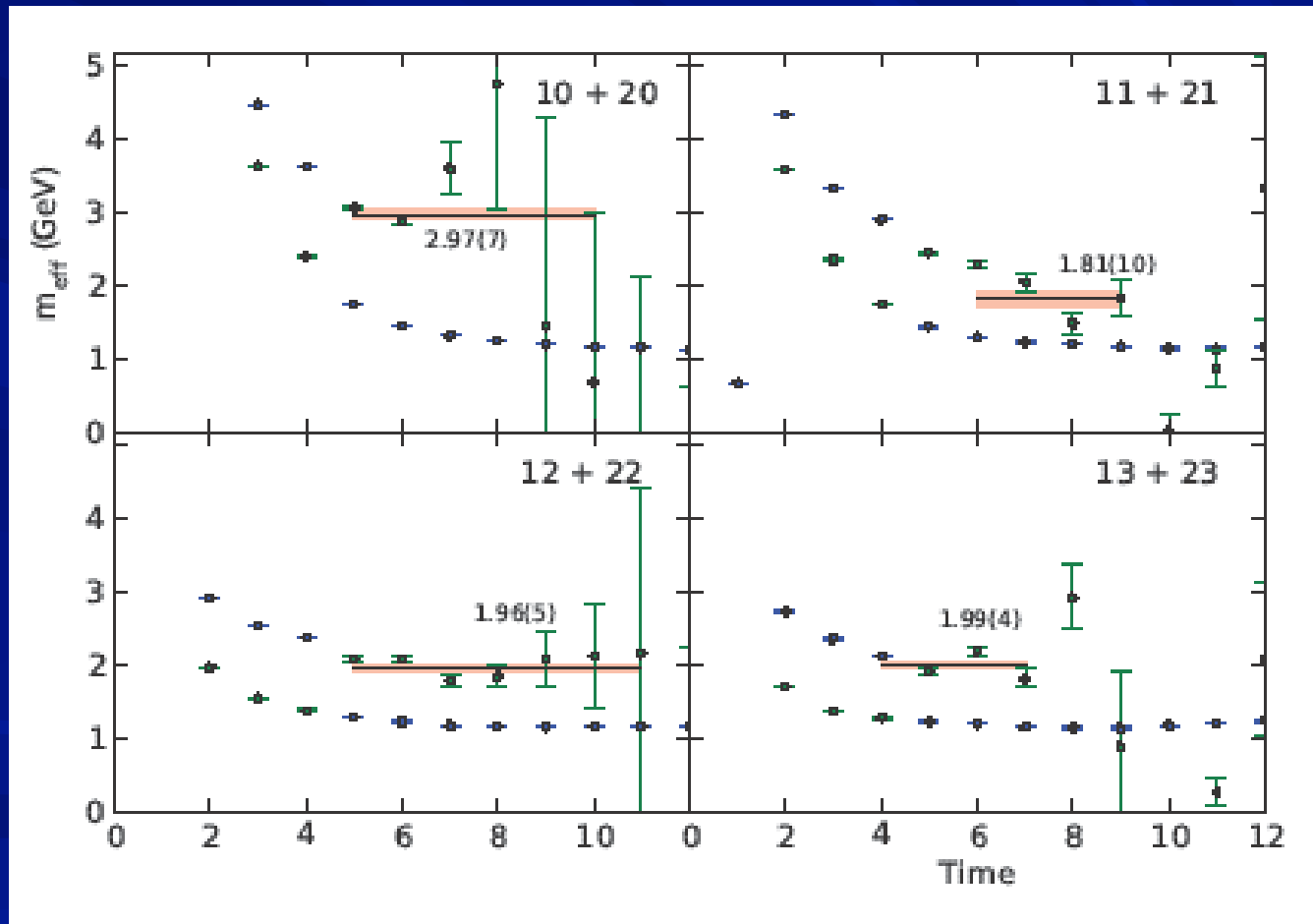
Roper and Nucleon Wavefunctions at $m_\pi = 438$ MeV

$$O_{RN} = 0.59$$



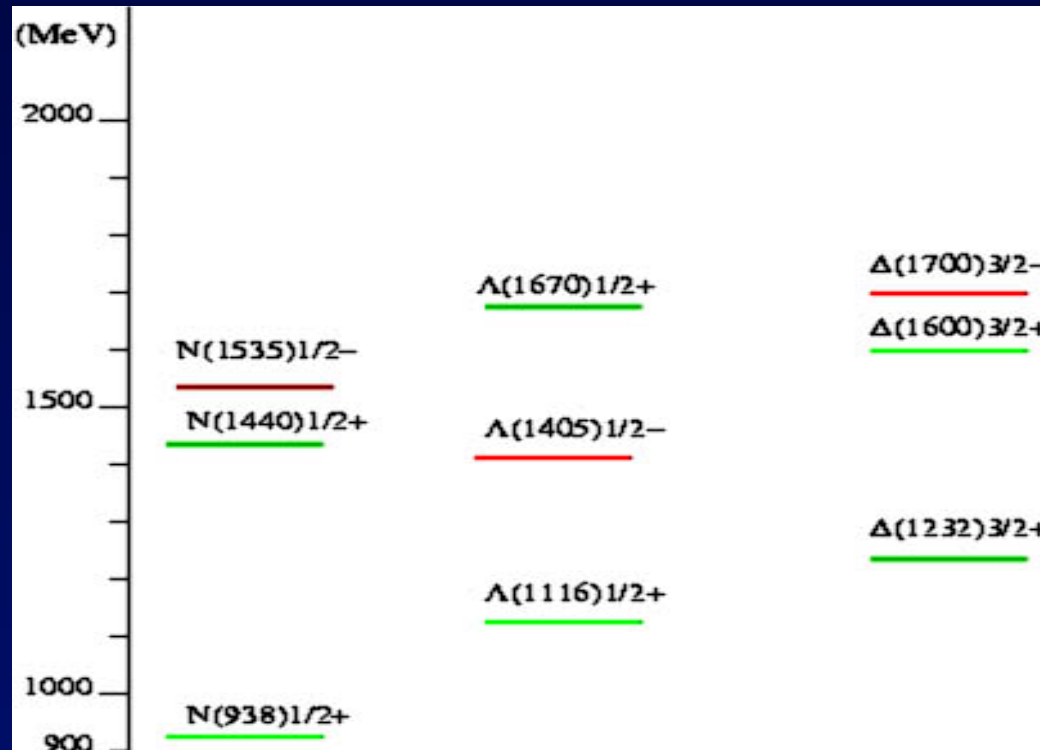
Variation with 2 operators

(10 – operator 1, no smearing, 23 – operator 2, 3 smearing)



Variation with wall and point sources?

Glozman and Riska Challenge Phys. Rep. 268, 263 (1996)



Hyperfine Interaction of Quarks in Baryons

- **Color-spin**

$$\lambda_1^C \cdot \lambda_2^C \vec{\sigma}_1 \cdot \vec{\sigma}_2$$

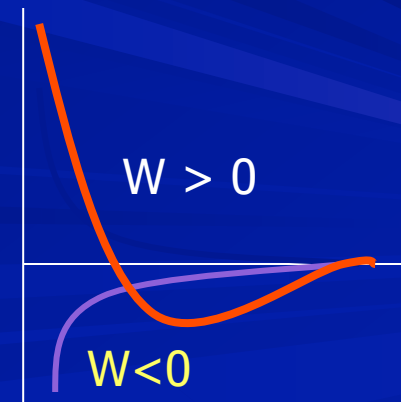
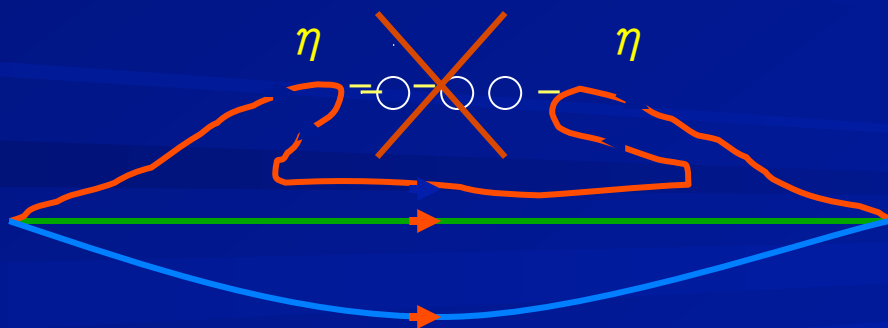
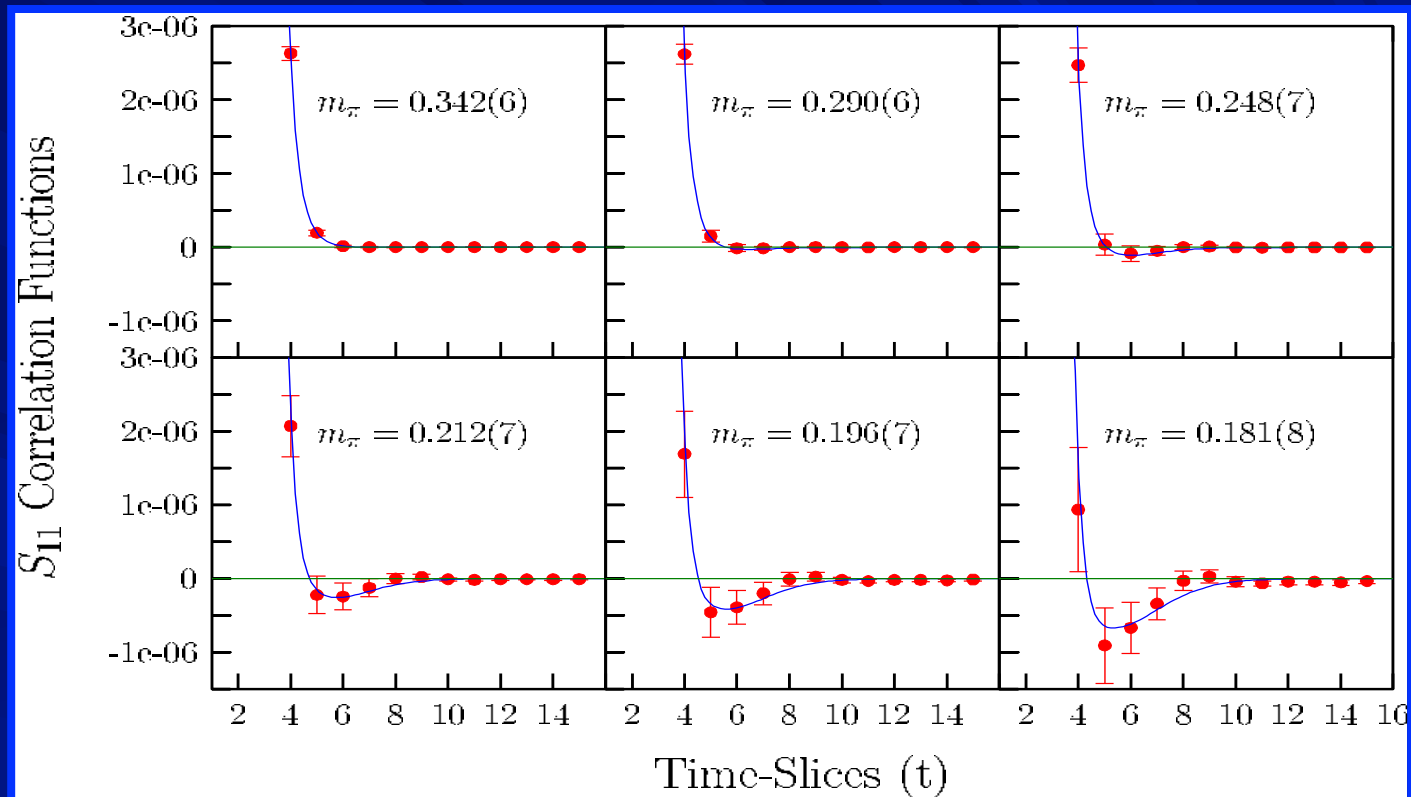
- One-gluon exchange

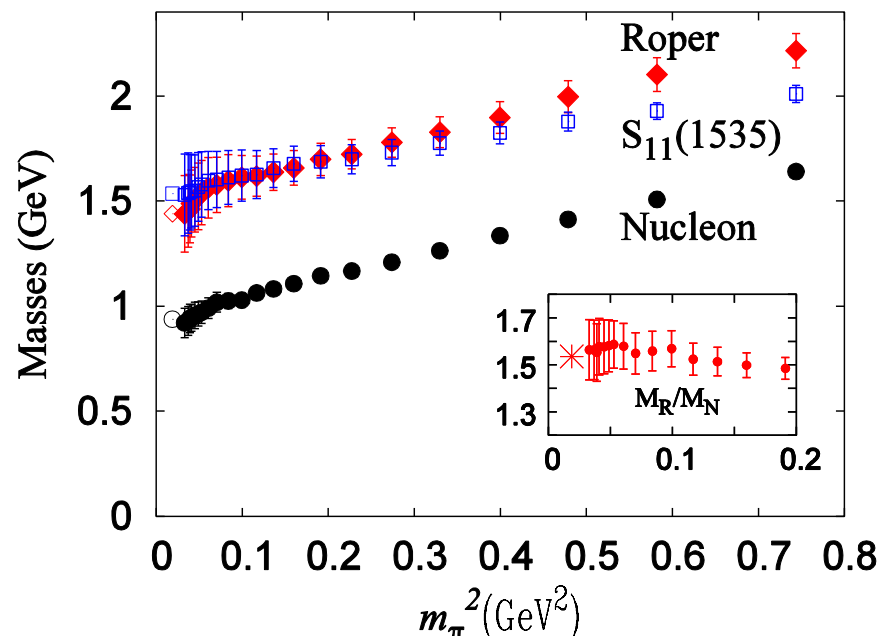
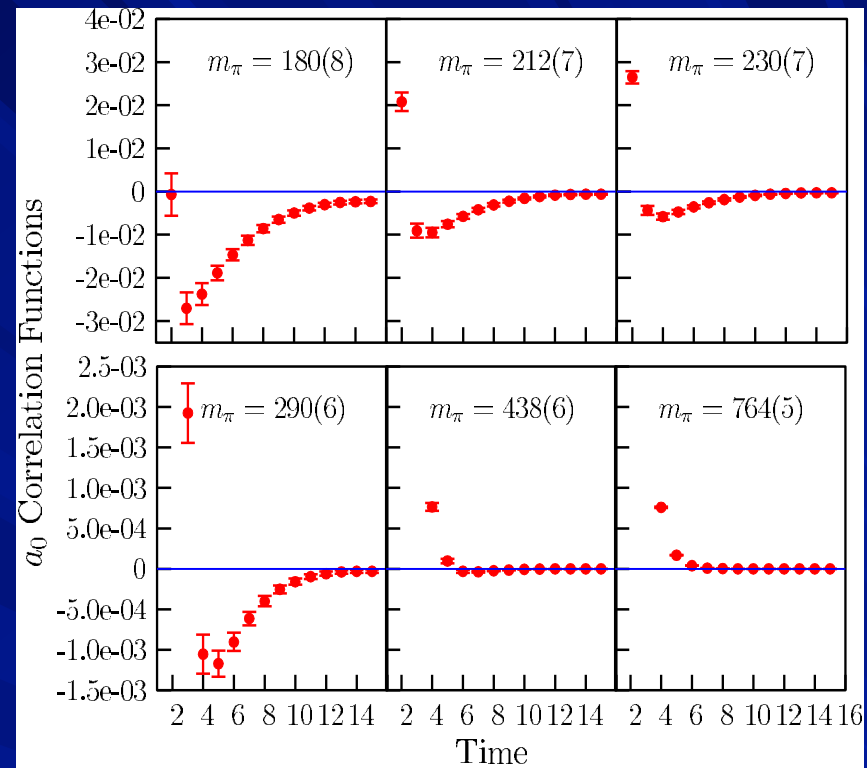
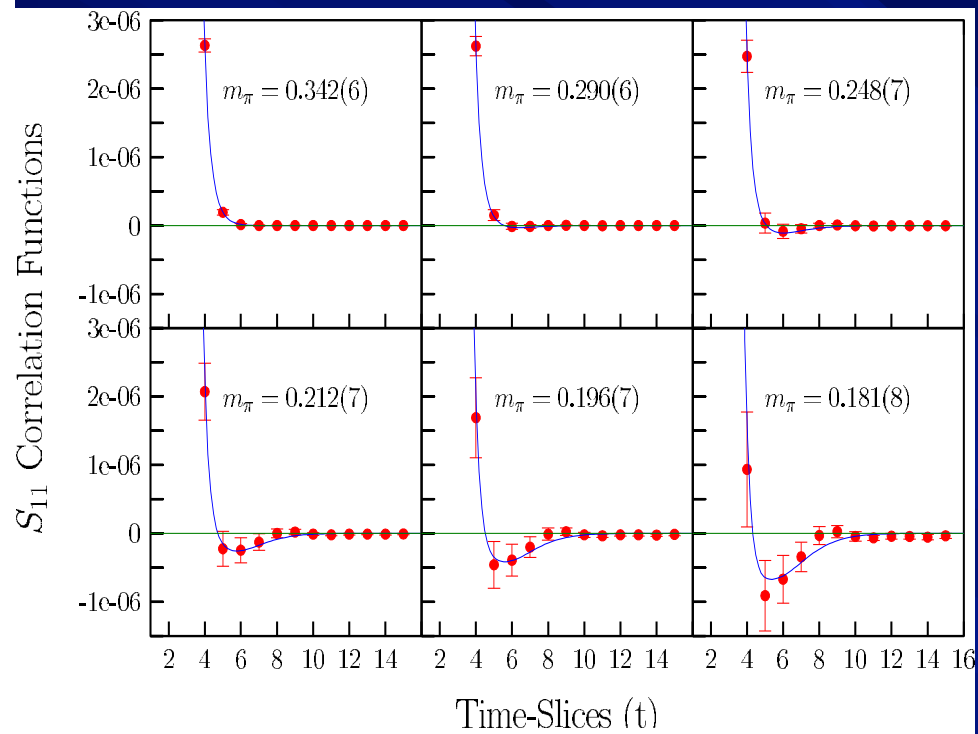
- **Flavor-spin**

$$\lambda_1^F \cdot \lambda_2^F \vec{\sigma}_1 \cdot \vec{\sigma}_2$$

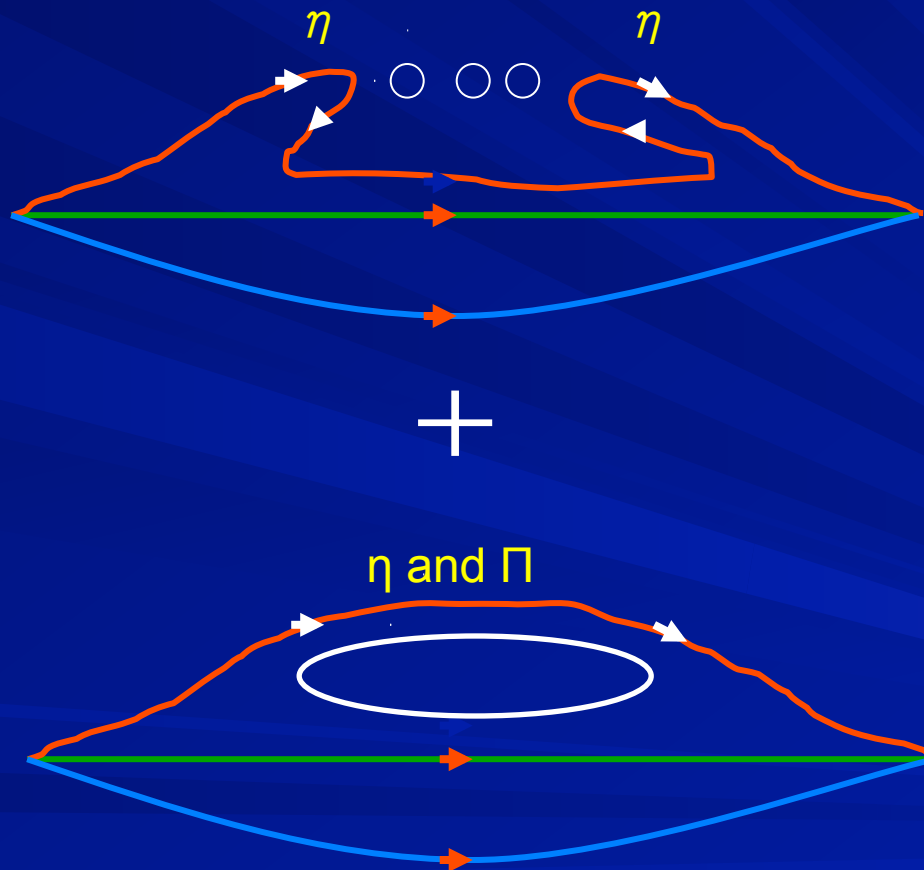
- Goldstone boson exchange

Evidence of η' N GHOST State in $S_{11}(1535)$ Channel

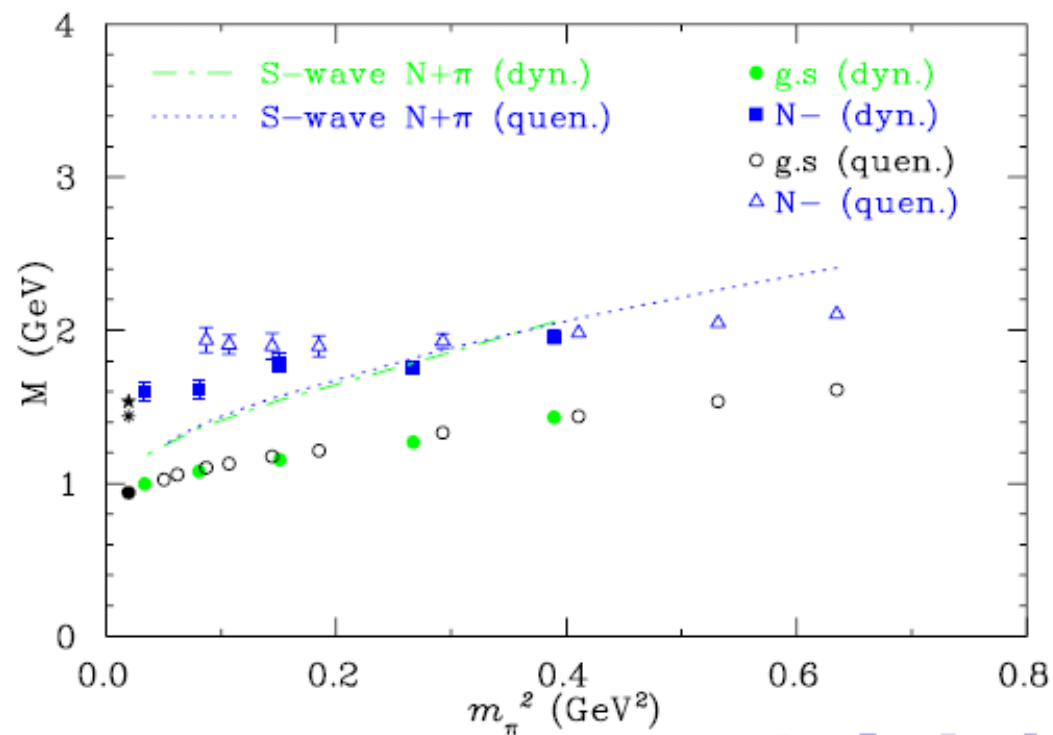




Dynamical Fermions

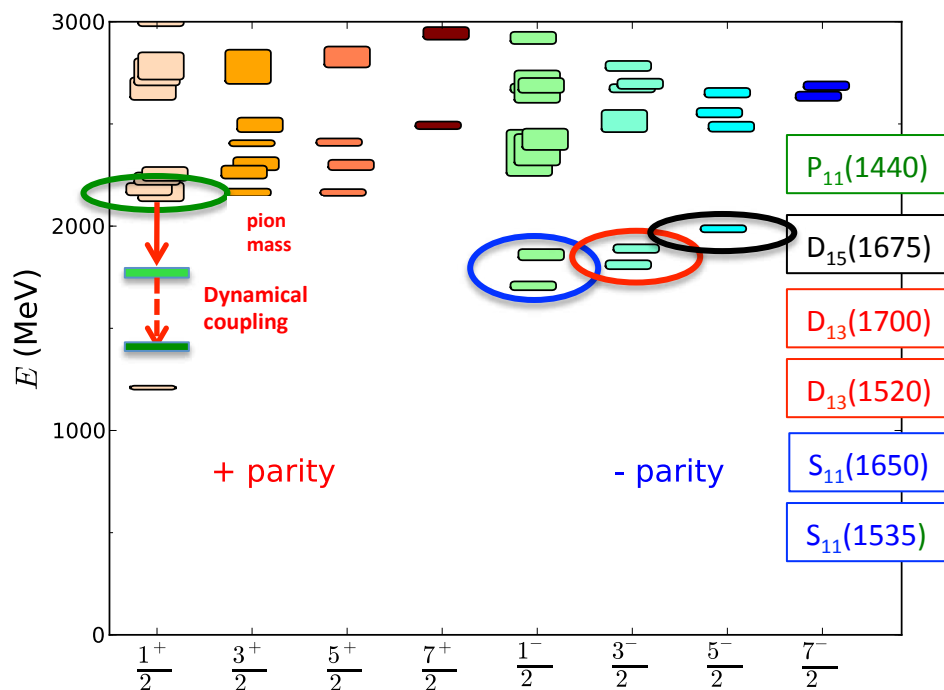


Quenched Vs Dynamical $N1/2^-$ (1535) (Sommer scale)



N* spectrum in LQCD & dynamical coupling

Lattice N* states ($m_\pi=396\text{MeV}$)

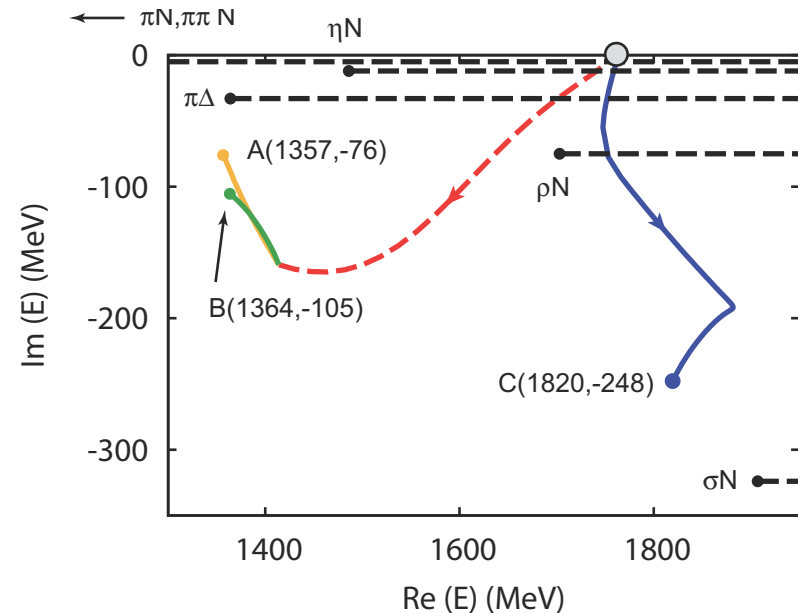


LQCD finds states as predicted in $SU(6) \times O(3)$

R. Edwards, J. Dudek, D. Richards,
S. Wallace, PRD84, 074508 (2011)

Dynamics of P_{11} -states:

The bare state at ~ 1750 MeV through coupling to inelastic channels generates 2 poles below 1400 MeV. They are identified with the “Roper” resonance.



N. Suzuki et al. (JLab/EBAC),
Phys.Rev.Lett.104:042302,2010

Nature of Roper Resonance

--- current understanding

- Roper is the radial excitation of nucleon with large couplings to $N\eta$ and $N\pi$. The real part of the $N\eta$ and $N\pi$ loops pushes down the pole of radial excitation and the imaginary part gives the width of Roper, much the same way the $N\pi$ coupling changes the Δ mass and gives rise of its width.
- Issues with lattice calculations:
 - Variation vs Bayesian fitting: the size of the operators.
 - Chiral dynamics of the fermion action.

Is 1^{-+} Meson a Hybrid?

Y. Yang, Y. Chen, KFL, PRD 86, 094511 (2012)
arXiv:1202.2205

Exotics:

- ❖ Glueballs
- ❖ Hybrids ($q\bar{q}g$)
- ❖ Tetraquark mesoniums ($q\bar{q}q\bar{q}$) ;
pentaquark baryons ($q^4\bar{q}$)

How to identify glueballs and hybrids in experiments and lattice calculations and distinguish them from ordinary $q\bar{q}$ mesons?

➡ exotic quantum numbers, e.g.

$$J^{PC} = 0^{+-}, 1^{-+}, 2^{-+}$$

Meson Interpolation Fields

- dim 3 $\bar{\Psi}\Gamma\Psi$
- dim 4 $\bar{\Psi}\Gamma\vec{D}\Psi$, GG
- dim 5 $\bar{\Psi}\Gamma\Psi G$, $\bar{\Psi}\Gamma DD\Psi$, $G DG$
- dim 6 $\bar{\Psi}\Gamma\Psi\bar{\Psi}\Gamma\Psi$, GGG , $GDDG$, etc

Dim 3 operators $\bar{\Psi}\Gamma\Psi$ do not generate exotic mesons with $J^{PC} = 1^{-+}, 0^{--}, 0^{+-}, 2^{+-}$ but they can be produced with dim 5 $\bar{\Psi}\Gamma\Psi G$ ops.
→ they are hybrids.

Experiments and Lattice Results

■ Expts on 1^{-+} :

- $\pi_1(1400)$, $M = 1376 \pm 17$ MeV, $\Gamma = 300 \pm 40$ MeV
- $\pi_1(1600)$, $M = 1653 \pm 17$ MeV, $\Gamma = 225 \pm 38$ MeV (?)

■ Lattice calculations (Quenched):

- UKQCD (1997): 2.0(2) GeV
- MILC (1997): 2.0(1) GeV, 2.1(1) GeV
- Lacock and Schilling (1998): 1.9(2) GeV
- MILC (2003): ~ 1.6 GeV
- Adelaide (2004): ~ 2.4 GeV; ~ 1.6 GeV
- HSC (2010)
- J. Dudek (2011)

$$\vec{J} = \vec{L} + \vec{S}, \quad P = (-)^{l+1}, \quad C = (-)^{l+S}$$

are non-relativistic definitions!

■ For example:

$$l = 1, S = 1, J = 0$$

Pauli Spinor

$$\bar{\psi} \vec{\sigma} \cdot \vec{\Delta} \psi$$

Dirac Spinor

$$\bar{\Psi} \Psi$$

$$l = 1, S = 1, J = 1$$

$$\bar{\psi} \vec{\sigma} \times \vec{\Delta} \psi$$

$$\bar{\Psi} \gamma_i \gamma_5 \Psi$$

$$l = 1, S = 1, J = 2$$

$$\bar{\psi} \vec{\sigma} \otimes \vec{\Delta} \psi$$

$$\bar{\Psi} \gamma \otimes \vec{D} \Psi$$

$$l = 1, S = 0, J = 1$$

$$\bar{\psi} \vec{\Delta} \psi$$

$$\bar{\Psi} \sigma_{ij} \Psi$$

■ Lower component of Dirac spinor has a different parity from the upper one.

$$\Psi_{free} \propto \begin{pmatrix} 1 \\ \frac{\vec{\sigma} \cdot \vec{p}}{E + m} \end{pmatrix} \chi e^{ip \cdot x}$$

1⁻⁺ Meson

■ Exotic:

- $\vec{J} = \vec{l} + \vec{S}$, $P = (-)^{l+1}$, $C = (-)^{l+S}$
- P $\rightarrow$ $l = \text{even}$
- J $\rightarrow$ $S = 1$
- C = -
- Cannot be $q\bar{q}$ meson

■ Yet dim 4 $\bar{\Psi}\gamma_4\vec{D}_i\Psi$ is 1⁻⁺ (B.A. Li, 1975)

- P: $\Psi(\vec{x}) \rightarrow \gamma_4\Psi(-\vec{x})$
- C: $\Psi \rightarrow C\tilde{\Psi}$
- Exotic $q\bar{q}$?
- Note: there is no dim 3 op. for 2⁺⁺

It is produced with $\bar{\Psi}\gamma_i\vec{D}_j\Psi$.

Content and Interpolation Field

- One cannot always judge the content of a hadronic state by its interpolation field. For example
 - the lowest state from $\bar{\Psi}\Psi$ is $\eta\pi$ and $\pi\pi$, not a_0 and f_0 .
 - the lowest P-wave state from χ_N is πN not S_{11} .
- The lowest pseudoscalars from $G\tilde{G}$ is η and η' not glueball.
$$\langle 0 | G\tilde{G} | \eta \rangle, \langle 0 | G\tilde{G} | \eta' \rangle \geq \langle 0 | G\tilde{G} | G \rangle$$
 - will need $\langle 0 | \bar{\Psi}\gamma_5\Psi | \eta \rangle, \langle 0 | \bar{\Psi}\gamma_5\Psi | \eta' \rangle \gg \langle 0 | \bar{\Psi}\gamma_5\Psi | G \rangle$ to help decide which one is a glueball.

(Cheng, Li and Liu, PRD 79, 014024 (2009))

Criteria for a meson to be a hybrid:

Compare the matrix elements of both the dim 4 and dim 5 operators of 1^{-+} against other ordinary mesons, particularly the 2^{++}

- Dim 4 m.e. $\langle 0 | \bar{\Psi} \gamma_4 \vec{D}_i \Psi | 1^{-+} \rangle \ll \langle 0 | O_4 | 0^{-+}, 1^{-+}, 0^{++}, 1^{++}, 2^{++} \rangle$
- Dim 5 m.e. $\langle 0 | \bar{\Psi} \varepsilon_{ijk} \gamma_j B_k \Psi | 1^{-+} \rangle \gg \langle 0 | O_5 | 0^{-+}, 1^{-+}, 0^{++}, 1^{++}, 2^{++} \rangle$

Dim 3, 4 (D-type) and dim 5 (B-type) operators

Table 1: Interpolation operators $\bar{\psi}\Gamma\psi$ (dimension 3, Γ -type), $\bar{\psi}\Gamma \times \overleftrightarrow{D}\psi$ (dimension 4, D-type), and $\bar{\psi}\Gamma \times B\psi$ (dimension 5, B-type). $\Sigma_i \equiv \frac{1}{2}\epsilon_{ijk}\sigma_{jk}$ and repeated indices are summed over.

	Γ	D	B
0^{-+}	γ_5	$\Sigma_i \overleftrightarrow{D}_i$	$\gamma_i B_i$
1^{--}	γ_i	$\overleftrightarrow{D}_i$	$\gamma_5 B_i$
0^{++}	$\mathbb{I}$	$\gamma_i \overleftrightarrow{D}_i$	$\Sigma_i B_i$
1^{++}	$\gamma_5 \gamma_i$	$\epsilon_{ijk} \gamma_j \overleftrightarrow{D}_k$	$\epsilon_{ijk} \Sigma_j B_k$
1^{+-}	Σ_i	$\gamma_5 \overleftrightarrow{D}_i$	B_i
2^{++}		$ \epsilon_{ijk} \gamma_j \overleftrightarrow{D}_k$	$ \epsilon_{ijk} \Sigma_j B_k$
1^{-+}		$\gamma_4 \overleftrightarrow{D}_i$ $\epsilon_{ijk} \Sigma_j \overleftrightarrow{D}_k$	$\epsilon_{ijk} \gamma_j B_k$

Non-relativistic Reduction

- why no such operators in quark model

Table 2: Non-relativistic form for the three kinds of operators (Γ , D and B) as shown in Table 1. Here we list the operators $\mathcal{O}$ in the interpolation field $\chi^\dagger \mathcal{O} \phi$. Repeated indices are summed over.

	Γ	D	B
0^{-+}	$\mathbb{I}$	$\frac{1}{2m_c} \overleftrightarrow{D}_i \overleftrightarrow{D}_i$	$i\sigma_i B_i$
1^{--}	σ_i	$\frac{1}{2m_c} \sigma_j \overleftrightarrow{D}_j \overleftrightarrow{D}_i$	B_i
0^{++}	$\frac{1}{2m_c} \overleftrightarrow{D}_i \sigma_i$	$\sigma_i \overleftrightarrow{D}_i$	$\frac{1}{2m_c} \overleftrightarrow{D}_i B_i$
1^{++}	$\frac{1}{2m_c} \varepsilon_{ijk} \overleftrightarrow{D}_j \sigma_k$	$\varepsilon_{ijk} \sigma_j \overleftrightarrow{D}_k$	$\frac{1}{2m_c} (\varepsilon_{ijk} \overleftrightarrow{D}_j B_k + i\partial_i (\sigma_j B_j))$
1^{+-}	$\frac{1}{2m_c} \overleftrightarrow{D}_i$	$\overleftrightarrow{D}_i$	$\frac{1}{2m_c} \sigma_j \overleftrightarrow{D}_j B_i$
2^{++}		$ \varepsilon_{ijk} \sigma_j \overleftrightarrow{D}_k$	$\frac{1}{2m_c} \varepsilon_{ijk} (\overleftrightarrow{D}_j B_k + i\varepsilon_{jmn} \sigma_m \partial_n (B_k))$
1^{-+}		$\frac{1}{2m_c} (\sigma \cdot \overleftrightarrow{D} \overleftrightarrow{D}_i + \overleftrightarrow{D}_i \sigma \cdot \overleftrightarrow{D})$ $\frac{1}{2m_c} (\overleftrightarrow{D}_i \sigma_j \overleftrightarrow{D}_j + \sigma_j \overleftrightarrow{D}_j \overleftrightarrow{D}_i)$	$\varepsilon_{ijk} \sigma_j B_k$

Note: $\vec{D} + \overleftarrow{D}$ is the $q\bar{q}$ center of mass momentum which is not a dynamical d.o.f. in the constituent quark model.

Charmoniums

Anisotropic $12^3 \times 96$ lattice with Wilson action, $\beta = 2.8$, $\zeta = 5$, $a_s = 0.138$ fm.

Table 3: Masses of charmonium states from Γ - and B -type sources and point sinks.

	$\Gamma_w \rightarrow \Gamma_p$	$B_w \rightarrow \Gamma_p$	$B_w \rightarrow B_p$	$B_w \rightarrow D_p$	PDG
0^{-+}	3000 ± 3	3000 ± 3	2999 ± 3	3000 ± 3	2980.3 ± 1.2
1^{--}	3096 ± 3	3095 ± 3	3093 ± 3	3094 ± 3	3096.916 ± 0.011
0^{++}	3458 ± 30	3485 ± 18	3485 ± 21	3476 ± 18	3414.75 ± 0.31
1^{++}	3497 ± 21	3491 ± 10	3492 ± 28	3492 ± 28	3510.66 ± 0.07
1^{+-}	3489 ± 30	3475 ± 21	3486 ± 12	3494 ± 6	3525.42 ± 0.29
2^{++}	—	—	3529 ± 40	3501 ± 13	3556.20 ± 0.09
1^{-+}	—	—	4205 ± 84	4234 ± 42	—

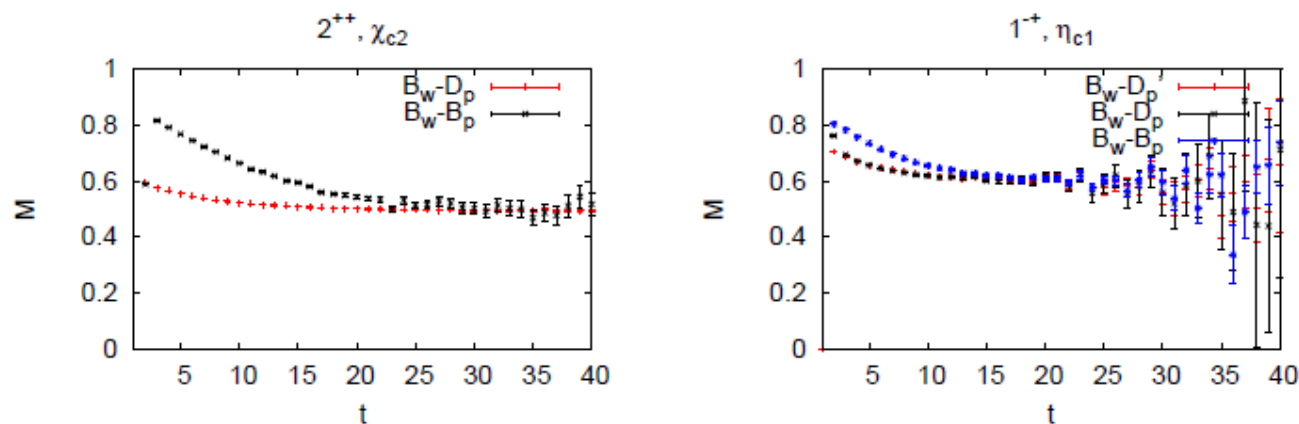


Figure 4: The same as Fig. 2 for $\chi_{c2}(2^{++})$ and $\eta_{c1}(1^{-+})$

Table 2: Non-relativistic form for the three kinds of operators (Γ , D and B) as shown in Table 1. Here we list the operators $\mathcal{O}$ in the interpolation field $\chi^\dagger \mathcal{O} \phi$. Repeated indices are summed over.

	Γ	D	B
0^{-+}	$\mathbb{I}$	$\frac{1}{2m_c} \overleftrightarrow{D}_i \overleftrightarrow{D}_i$	$i\sigma_i B_i$
1^{--}	σ_i	$\frac{1}{2m_c} \sigma_j \overleftrightarrow{D}_j \overleftrightarrow{D}_i$	B_i
0^{++}	$\frac{1}{2m_c} \overleftrightarrow{D}_i \sigma_i$	$\sigma_i \overleftrightarrow{D}_i$	$\frac{1}{2m_c} \overleftrightarrow{D}_i B_i$
1^{++}	$\frac{1}{2m_c} \varepsilon_{ijk} \overleftrightarrow{D}_j \sigma_k$	$\varepsilon_{ijk} \sigma_j \overleftrightarrow{D}_k$	$\frac{1}{2m_c} (\varepsilon_{ijk} \overleftrightarrow{D}_j B_k + i\partial_i (\sigma_j B_j))$
1^{+-}	$\frac{1}{2m_c} \overleftrightarrow{D}_i$	$\overleftrightarrow{D}_i$	$\frac{1}{2m_c} \sigma_j \overleftrightarrow{D}_j B_i$
2^{++}		$ \varepsilon_{ijk} \sigma_j \overleftrightarrow{D}_k$	$\frac{1}{2m_c} \varepsilon_{ijk} (\overleftrightarrow{D}_j B_k + i\varepsilon_{jmn} \sigma_m \partial_n (B_k))$
1^{-+}		$\frac{1}{2m_c} (\sigma \cdot \overleftrightarrow{D} \overleftrightarrow{D}_i + \overleftrightarrow{D}_i \sigma \cdot \overleftrightarrow{D})$ $\frac{1}{2m_c} (\overleftrightarrow{D}_i \sigma_j \overleftrightarrow{D}_j + \sigma_j \overleftrightarrow{D}_j \overleftrightarrow{D}_i)$	$\varepsilon_{ijk} \sigma_j B_k$

$$\int d^3x \bar{\psi} \gamma_4 \overleftrightarrow{D}_i \psi \xrightarrow{N.R.} \int d^3x \frac{-\varepsilon_{ijk}}{m} \chi^\dagger \sigma_j B_k \phi;$$

$$\int d^3x \varepsilon_{ijk} \bar{\psi} \Sigma_j \overleftrightarrow{D}_k \psi \xrightarrow{N.R.} \int d^3x \frac{\varepsilon_{ijk}}{m} \chi^\dagger \sigma_j B_k \phi$$

Table 4: Matrix elements $\langle 0 | \mathcal{O}_p | J^{PC} \rangle$ for charmoniums.

	Γ_p	D_p	B_p
0^{-+}	0.0697 ± 0.0014	0.0503 ± 0.0007	0.0251 ± 0.0006
1^{--}	0.0502 ± 0.0005	0.0149 ± 0.0001	0.0075 ± 0.0002
0^{++}	0.035 ± 0.005	0.075 ± 0.015	0.009 ± 0.003
1^{++}	0.020 ± 0.003	0.062 ± 0.005	0.0023 ± 0.0002
1^{+-}	0.014 ± 0.002	0.045 ± 0.005	0.0019 ± 0.0002
2^{++}		0.044 ± 0.003	0.00080 ± 0.00008
1^{-+}		0.0059 ± 0.0005	0.0082 ± 0.0006
		0.0054 ± 0.0004	

$$\times \frac{1}{v_{c\bar{c}}} = \begin{cases} 0.020(2) \\ 0.018(1) \end{cases}$$

$$\times \frac{1}{v_{rel}} \sim 0.027(3)$$

$$\times \frac{1}{ma} = 0.0058(4)$$

$$\frac{1}{ma} = \frac{v}{m_c a_t \zeta} \sim 0.7048;$$

$$\varepsilon = \frac{1}{2ma} \sim 0.352,$$

$$v_{c\bar{c}} = \frac{\vec{D} + \vec{\bar{D}}}{2m} \sim v_c \sim 0.3c;$$

$$v_{rev} = \frac{\vec{D} - \vec{\bar{D}}}{2m} \sim 0.3c$$

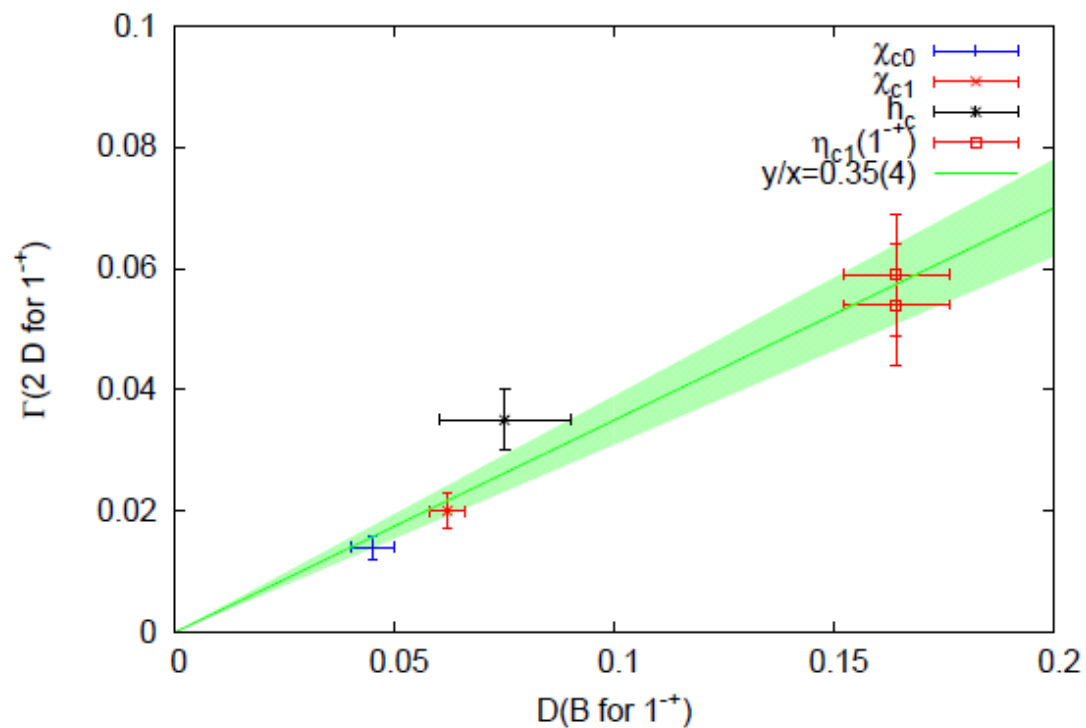


Figure 5: Global fit for the ratios of Γ for χ_{c0}, χ_{c1} and h_c ($2 D$ for 1^{-+}) m.e. to the corresponding m.e. of D (B for 1^{-+}).

$$0^{++}, 1^{++}, 1^{-+} : \Gamma \text{ m.e.} \approx \frac{1}{2m} D \text{ m.e.}$$

$$1^{-+} : D \text{ m.e.} \approx \frac{1}{m} B \text{ m.e.}$$

$$\varepsilon = \frac{1}{2m} \sim 0.352$$

- Leading NR reduction is reasonably good.
- Mixing of operators with different dimensions seems to be small.

Strangeness mesons

Table 6: The matrix elements $\langle 0|\mathcal{O}_p|J^{PC} \rangle$ for strange quarkoniums.

	Γ_p	D	B
0^{-+}	0.0247 ± 0.0002	0.021 ± 0.002	0.005 ± 0.0001
1^{--}	0.0141 ± 0.0002	0.0113 ± 0.0005	0.0025 ± 0.0001
0^{++}	0.043 ± 0.006	0.033 ± 0.005	0.017 ± 0.004
1^{++}	0.029 ± 0.004	0.034 ± 0.004	0.0018 ± 0.0002
1^{+-}	0.019 ± 0.006	0.029 ± 0.005	0.0019 ± 0.0004
2^{++}		0.010 ± 0.007	0.0003 ± 0.0001
1^{-+}		0.007 ± 0.001	0.004 ± 0.001
		0.006 ± 0.002	

- D and B m.e. of 1^{-+} are comparable in size to those of other ordinary mesons.
- No evidence for 1^{-+} in the charmonium and strangeonium regions to be 'hybrids'.

Exotic Quantum Numbers

- NR reduction shows that 1^{-+} involves a center of mass motion of the $q\bar{q}$ pair.
- MIT bag model (Jaffe and Johnson, 1976; DeGrand and Jaffe, 1976)

$$\Psi(2^{+\pm}) = \frac{1}{\sqrt{2}}(S_{1/2}\bar{P}_{3/2} \mp P_{3/2}\bar{S}_{1/2}) \rightarrow \text{cm motion for } 2^{+\pm}$$

$\Rightarrow$ Spectrum is doubled. Similarly (harmonic oscillator wf),

$$\Psi(1^{-\pm}) = \frac{1}{\sqrt{2}}(1S_{1/2}2\bar{S}_{1/2} \mp 2S_{1/2}1\bar{S}_{1/2})$$

- The 'exotic' q.n. can be accommodated by

$$C = (-)^{l+s}$$

$$\vec{J} = \vec{L} + \vec{l} + \vec{s}$$

$$P = (-)^{L+l+1}$$

$$1^{-+} : L=l=1, s=1$$

Conclusion

- By examining m.e. of dim 4 and dim 5 operators of 1^- against those of ordinary mesons, we find no evidence for it to be a 'hybrid' in the $c\bar{c}$ and $s\bar{s}$ regions.
- NR reduction shows that it involves a center of mass AM of the $q\bar{q}$ pair.
- These 'exotic' q.n. are accessible in chiral quark models, bag models, flux-tube models, and QCD.
- To accommodate these q.n., the parity and AM rules need to be modified to

$$C = (-)^{l+s}, \quad \vec{J} = \vec{L} + \vec{l} + \vec{s}, \quad P = L + l + 1$$

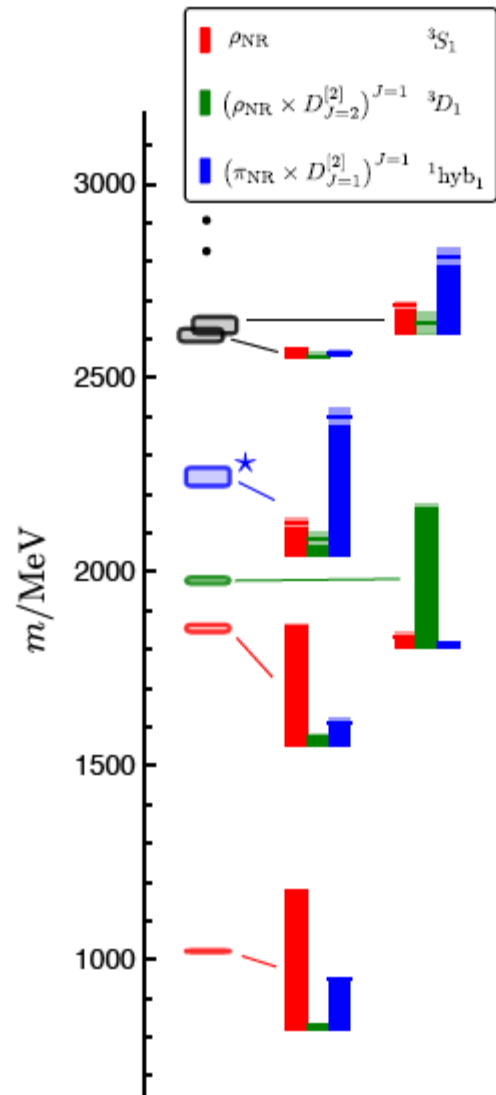
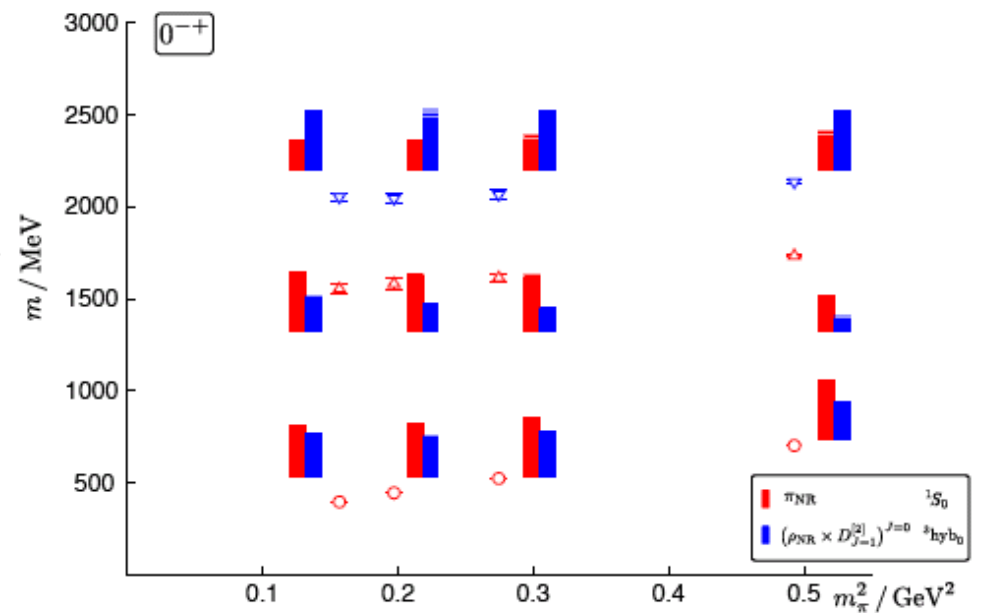


FIG. 2: Spectrum of 1^{--} mesons (as in Figure 1) along with relative size of overlaps onto operators, $\rho_{NR} \sim {}^3S_1$, $\rho_{NR} \times D_{J=2}^{[2]} \sim {}^3D_1$ and $\pi_{NR} \times D_{J=1}^{[2]} \sim {}^1\text{hyb}_1$. Histograms normalised such that the state with largest overlap across the entire excited 1^{--} spectrum has mean value of 1. The lighter



J. Dudek, PRD [arXiv:1106.5515]

Effective Theories

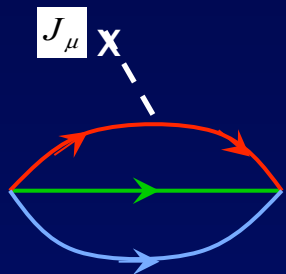
QCD

$$\Lambda_\chi \sim 4\pi f_\pi \sim 1 \text{ GeV}, \quad l_M \sim 0.2 \text{ fm}$$

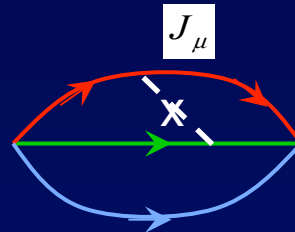
0.2 fm

$$\psi = \psi_L + \psi_S \quad A_\mu = A_{\mu L} + A_{\mu S}$$

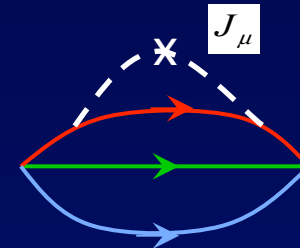
Chiral Quark Model $q_L, g_L(\alpha_s^R), \pi, \rho, \omega, a_1, K, \phi, \dots$



meson dominance
valence + connected sea



meson exchange
connected sea



meson loop
disconnected sea

1 fm

Chiral Perturbation Theory

$$\Lambda_B \sim 300 \text{ MeV}, \quad l_B \sim 0.6 \text{ fm}$$

