

Non-congruence of the nuclear liquid-gas and deconfinement phase transitions

Matthias Hempel Veronica Dexheimer Stefan Schramm Igor Iosilevskiy

arxiv:1302.2835v2

Xuefeng Ma

2026.09.24

Motivation Why the composition of coexisting phases matters.

Framework Gibbs equilibrium, conserved charges, and the phase fraction.

Symmetric matter A congruent reference case: LGS.

Asymmetric matter Non-congruent LGAS and its forced-congruent counterpart.

Main phenomena Separated endpoints, isospin distillation, and retrograde condensation.

Congruent vs. non-congruent coexistence

Congruent transition

The two coexisting phases retain the same conserved-charge ratio.

At fixed temperature, one coexistence pressure and one Gibbs free energy per baryon.

A phase-transition line in the T–P plane.

Non-congruent transition

The two phases can have different local charge fractions.

The composition of both phases changes during an isothermal transformation.

A finite coexistence region in the T–P plane.

Key control variable: the local versus global constraint on charge fraction.

Nuclear matter model and three scenarios

Model FSUgold relativistic mean-field EOS for uniform neutron–proton matter.

LGS Symmetric matter: global $Y_Q = 0.5$; congruent due to isospin symmetry.

LGAS Asymmetric matter: global $Y_Q = 0.3$; local phase compositions may differ.

LGAS fc Forced-congruent variant: $Y_Q(G) = Y_Q(L) = 0.3$.

Thermodynamic equilibrium for LGAS

Independent conserved charges: baryon number B and electric charge Q .

Particle chemical potentials: $\mu_n = \mu_B$ and $\mu_p = \mu_B + \mu_Q$.

Gibbs equilibrium: $T_G = T_L$, $P_G = P_L$, $\mu_B(G) = \mu_B(L)$, $\mu_Q(G) = \mu_Q(L)$.

Global composition: $Y_Q = Q / B = 0.3$ — not imposed separately on each phase.

At fixed T , pressure equality defines a coexistence curve in (μ_B, μ_Q) .

Two-phase mixture: the global charge constraint

Let α be the liquid volume fraction, $\alpha = V_L / (V_G + V_L)$.

$$\rho_B = (1 - \alpha) \rho_G + \alpha \rho_L$$

$$Y_Q \rho_B = (1 - \alpha) Y_Q(G) \rho_G + \alpha Y_Q(L) \rho_L$$

Saturation curve (SC): $\alpha = 0$; an infinitesimal liquid phase appears.

Boiling curve (BC): $\alpha = 1$; an infinitesimal gas phase appears.

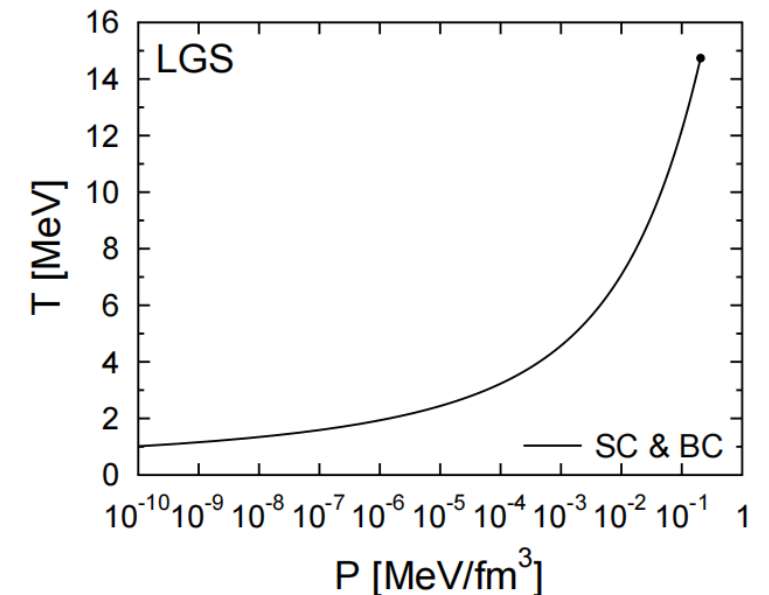
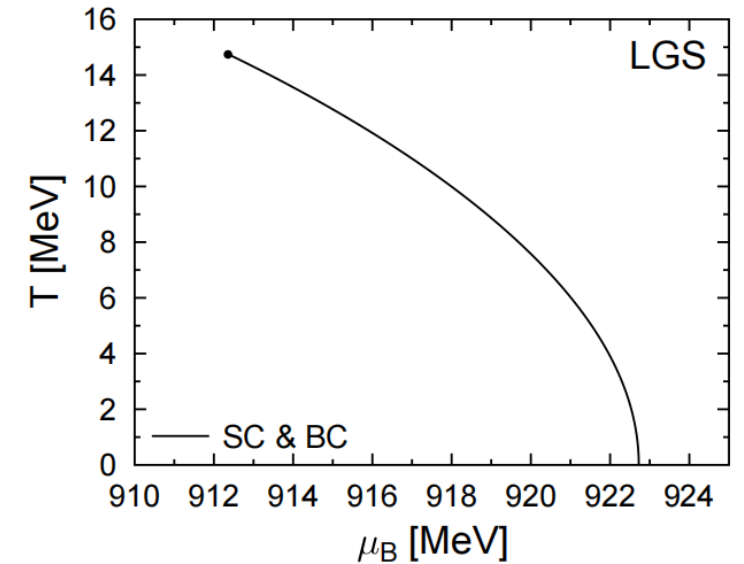
Symmetric nuclear matter: the congruent baseline

Isospin symmetry keeps both phases at $Y_Q = 0.5$.

The electric charge chemical potential vanishes: $\mu_Q = 0$.

SC and BC coincide in $T-\mu_B$ and $T-P$.

One coexistence μ_B and one coexistence pressure at fixed T .



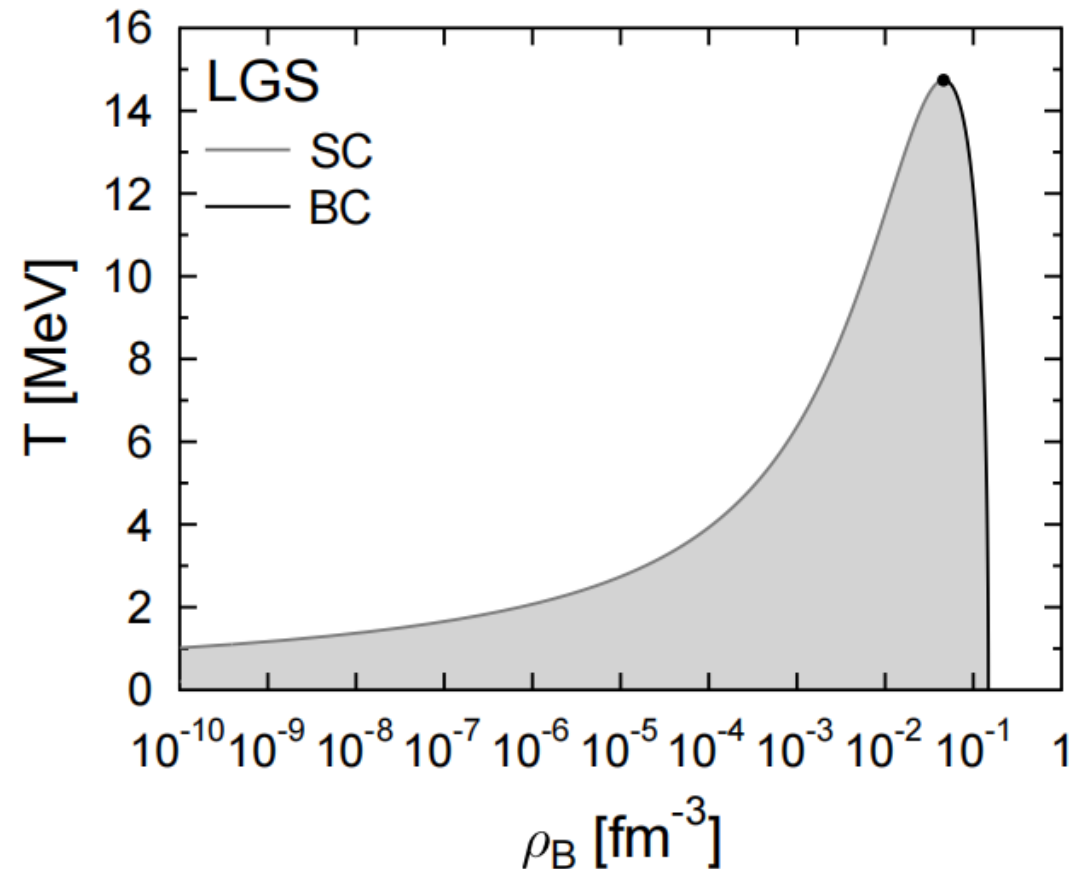
The congruent binodal in the T- ρ_B plane

Gas and liquid still have different baryon densities.

The binodal encloses a finite two-phase density interval.

SC and BC meet where the two phases become identical.

LGS critical temperature: $T(\text{CP}) \approx 14.75$ MeV.



Asymmetric matter: non-congruence in $T-\mu_B$

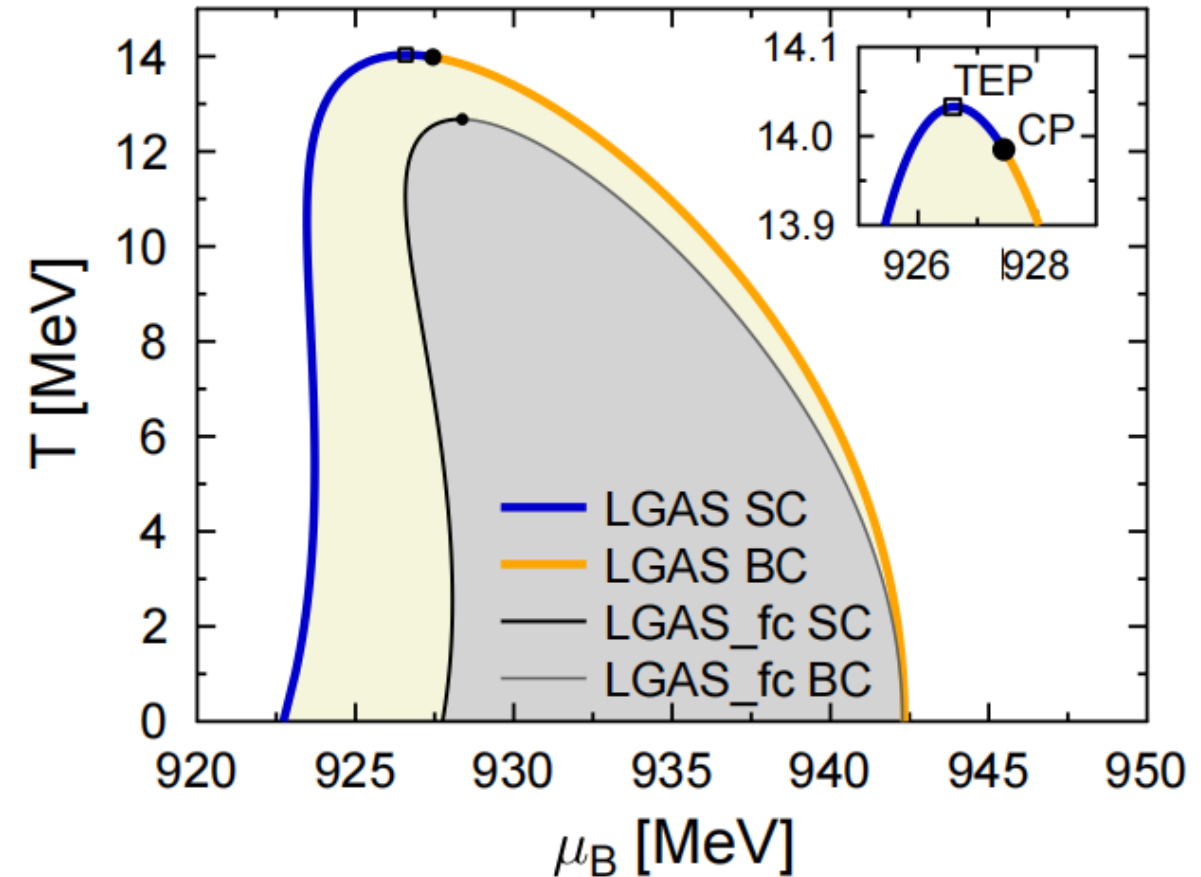
The total charge fraction is fixed at $Y_Q = 0.3$.

In LGAS, gas and liquid are free to exchange charge.

The coexistence boundaries split in $T-\mu_B$.

For LGAS fc, locally fixed Y_Q produces a different projected boundary structure.

A $T-\mu_B$ projection alone does not reveal whether a forced-congruent case is a line.



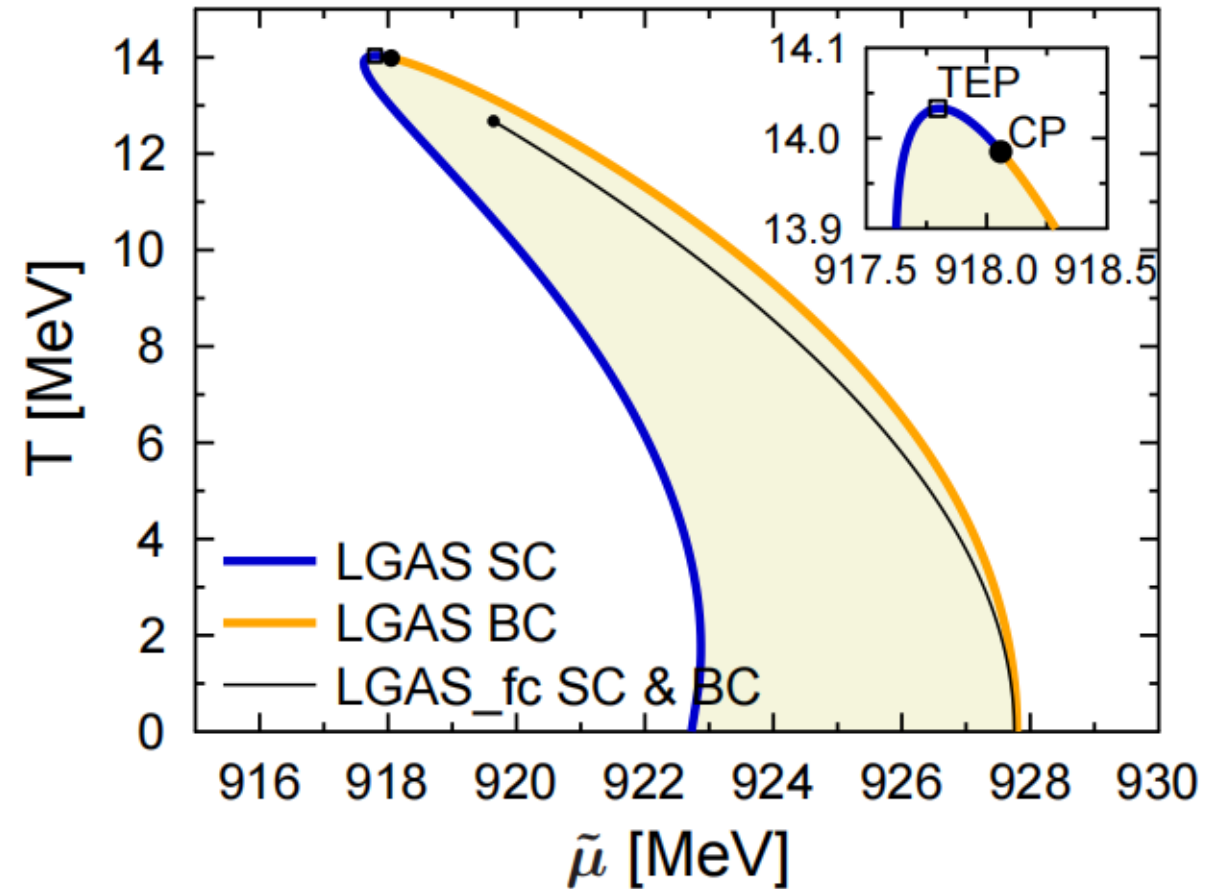
Why use the Gibbs free energy per baryon?

$$\tilde{\mu} = \mu_B + Y_Q \mu_Q = G / B$$

For LGAS, a finite “banana-shaped” coexistence region remains in $T-\tilde{\mu}$.

For LGAS fc, $\tilde{\mu}$ is equal in both phases.

The forced-congruent coexistence region collapses to a line in $T-\tilde{\mu}$.



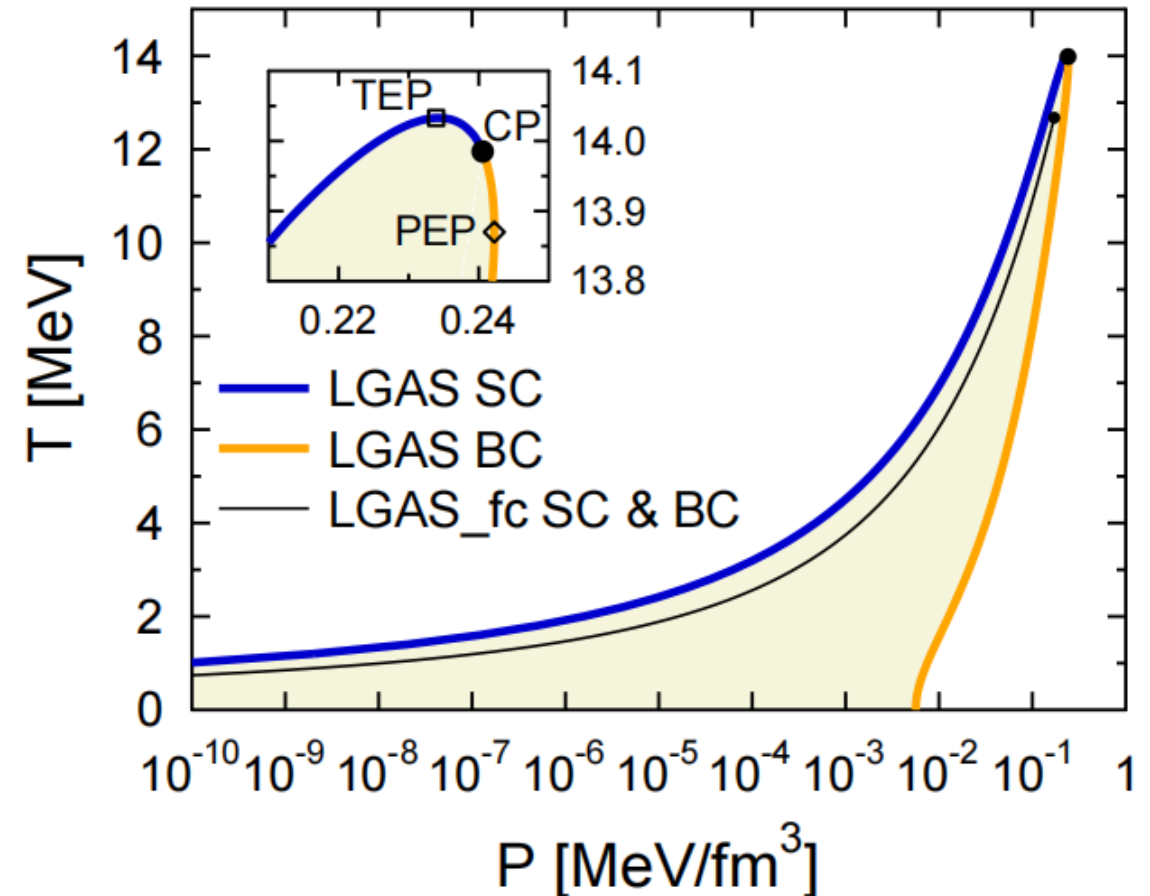
The T–P diagram: coexistence region and endpoints

At fixed T, LGAS allows a range of coexistence pressures.

Each state still satisfies $P_G = P_L$; the common pressure varies along the region.

The temperature endpoint (TEP) is the maximum-T point.

The pressure endpoint (PEP) lies on the boiling curve in this model.



The critical point is not the temperature endpoint

True critical point (CP) Gas and liquid become identical in density and charge fraction.

Temperature endpoint (TEP) The highest temperature attained by the coexistence region.

LGAS result $T(\text{CP}) \approx 13.99 \text{ MeV}$, whereas $T(\text{TEP}) \approx 14.03 \text{ MeV}$.

Location TEP lies on the saturation curve; CP occurs at a higher density.

Forced-congruent case Its pseudo-critical point and topological endpoints coincide.

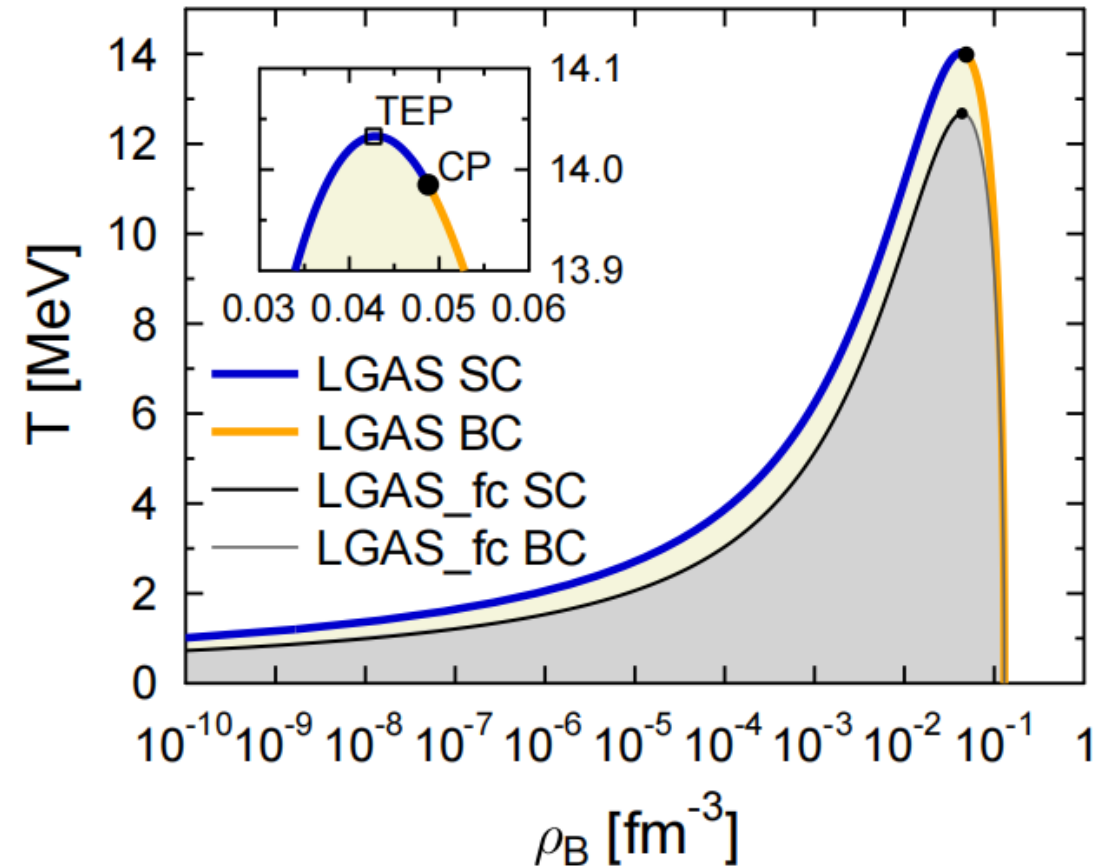
Density binodal and retrograde condensation

The SC and BC enclose the two-phase region in T - ρ_B .

For $T(\text{CP}) < T < T(\text{TEP})$, an isothermal compression can cross the SC twice.

The liquid fraction first increases, then decreases back to zero.

This is retrograde condensation: gas $\rightarrow$ gas + liquid $\rightarrow$ gas.



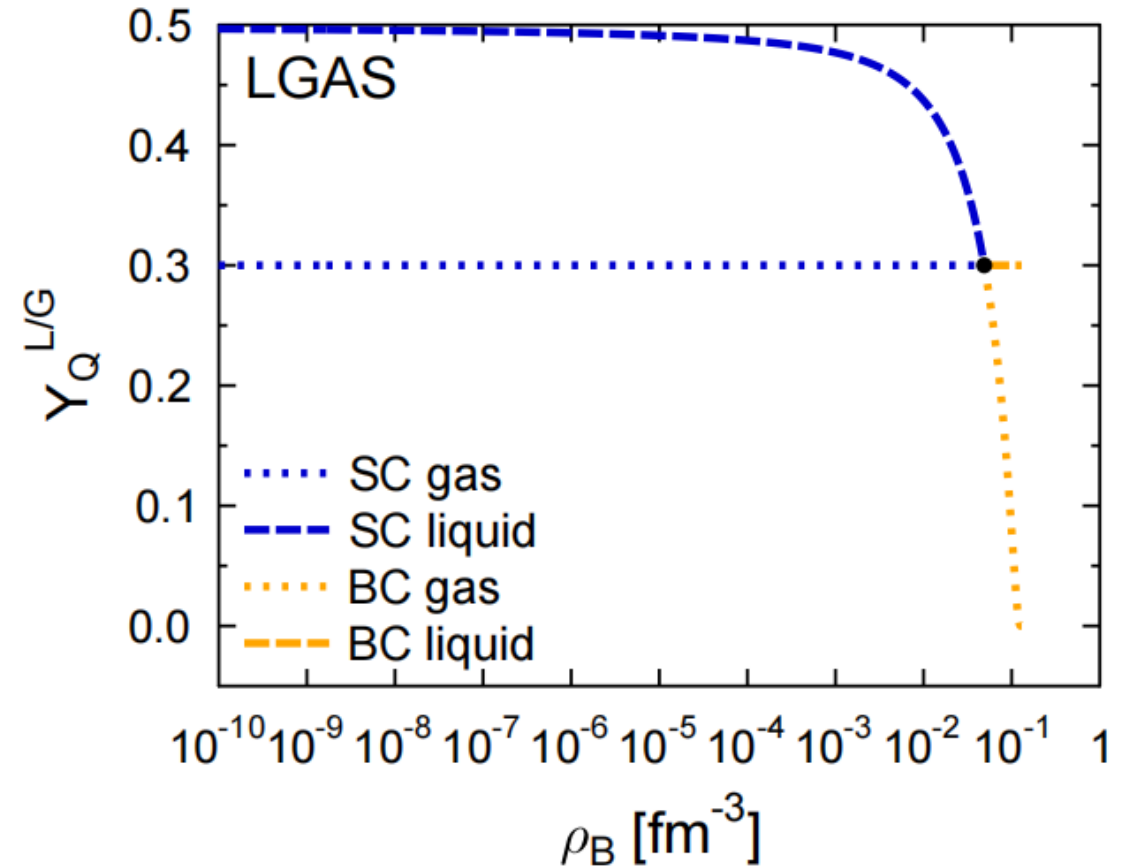
Isospin distillation: gas and liquid compositions

The gas is more neutron-rich than the liquid: $Y_Q(G) < Y_Q(L)$.

On SC: $Y_Q(G) = 0.3$; the incipient liquid has $Y_Q(L) > 0.3$.

On BC: $Y_Q(L) = 0.3$; the incipient gas has $Y_Q(G) < 0.3$.

Only at the true CP do the charge fractions become equal.



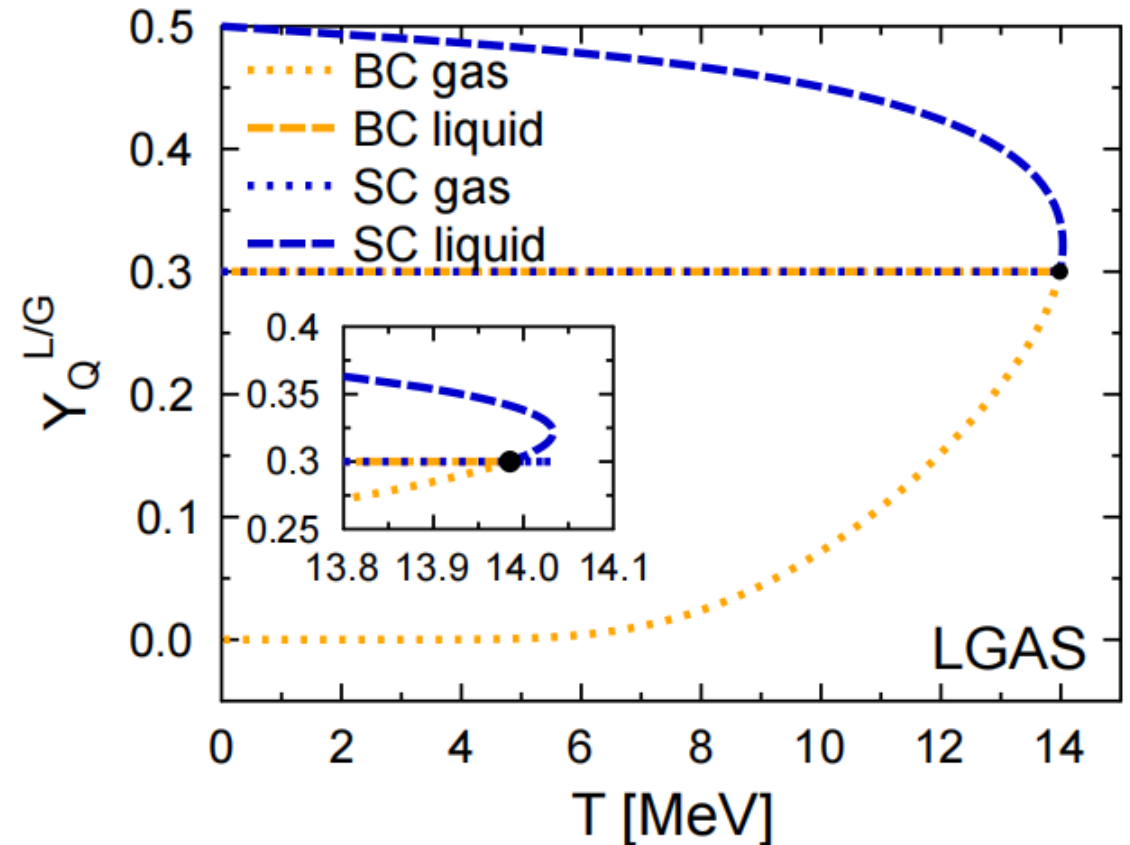
Charge fraction as a function of temperature

Below $T(\text{CP})$, SC and BC give two distinct coexistence states at each T .

As $T \rightarrow 0$, the boiling-side gas approaches pure neutron matter.

As $T \rightarrow 0$, the saturation-side liquid approaches $Y_Q(\text{L}) = 0.5$.

Above $T(\text{CP})$ but below $T(\text{TEP})$, both binodal branches are saturation branches.



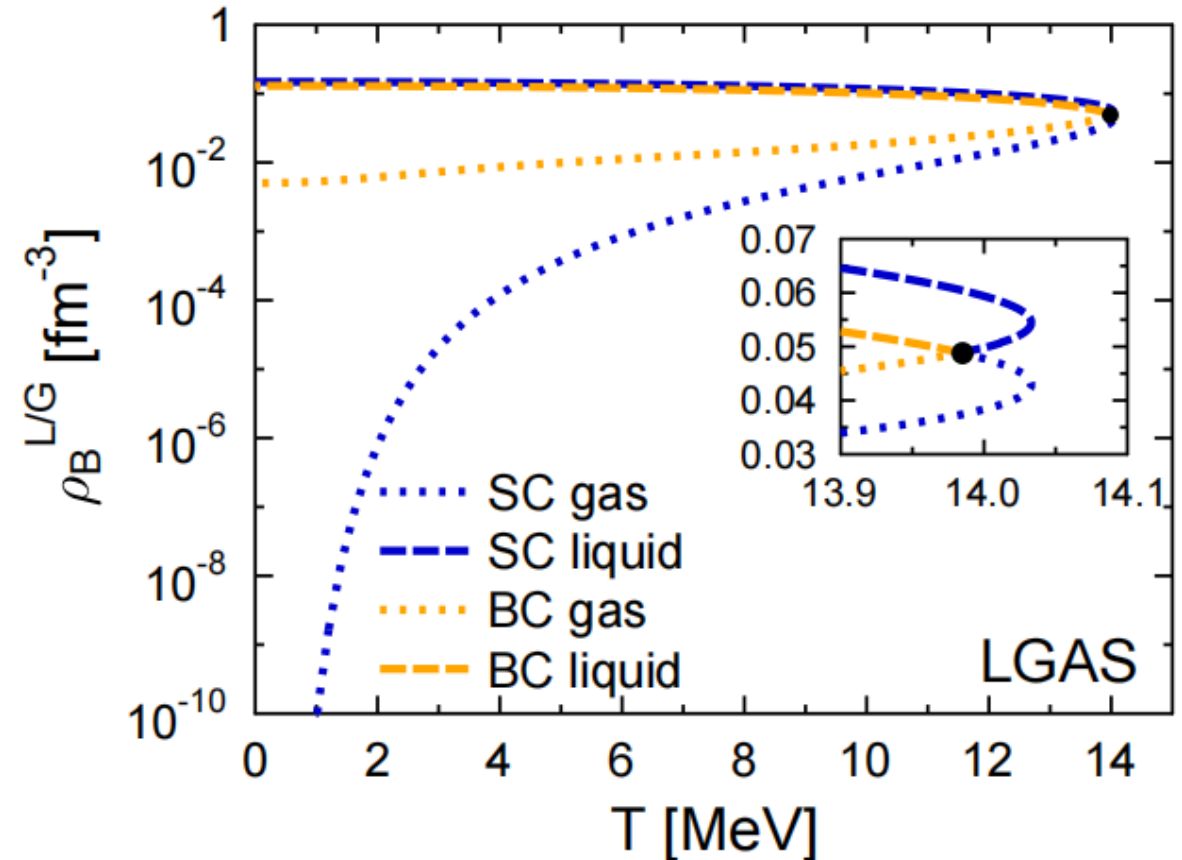
Density as an order parameter

Gas and liquid have distinct baryon densities along coexistence.

The liquid is the denser phase: $\rho_L > \rho_G$.

At the CP, both density and charge fraction differences vanish.

The phases cannot be distinguished thermodynamically at that point.



Representative model results

Case	T [MeV]	ρ_B [fm ⁻³]
LGS CP	14.75	0.046
LGAS CP	13.99	0.049
LGAS fc pseudo-CP	12.68	0.044

For LGAS, the temperature endpoint is $T(\text{TEP}) \approx 14.03$ MeV.

Main conclusions: nuclear liquid–gas transition

Symmetric nuclear matter is azeotropic and has a congruent liquid–gas transition.

Asymmetric nuclear matter is non-congruent when charge is conserved globally.

Non-congruence changes a coexistence line into a region in intensive-variable diagrams.

The critical point and the temperature/pressure endpoints need not coincide.

Charge fractionation and retrograde condensation are distinctive LGAS features.

Scope and discussion points

Bulk approximation The study neglects explicit Coulomb interactions and finite-size effects.

Low-density astrophysical matter Realistic applications may require nuclei and nuclear clusters.

Numerical implication Track the two phase compositions as well as their densities.

Endpoint identification Do not identify the highest-T point with the true CP without checking phase identity.