Precision Calculations of Hadron Physics

Qian Wang (王倩)

qianwang@m.scnu.edu.cn

味物理讲座第一一七期, 15th October, 2026

Exotic hadrons?

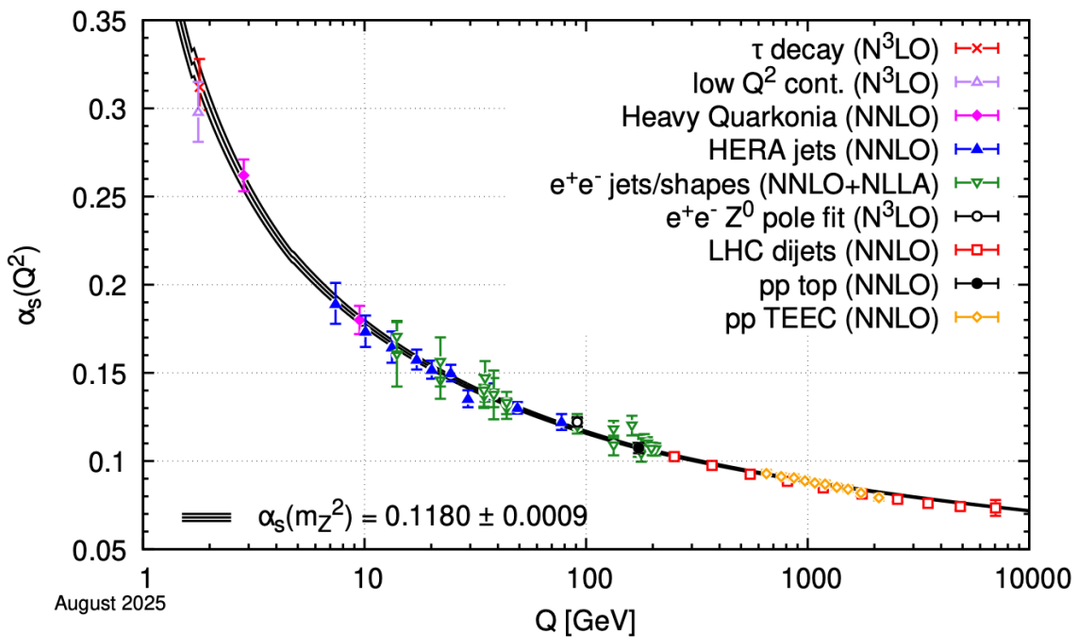
- Color Confinement

△ Allow all the color neutral objects

① Conventional mesons and baryons in QM

② Hadrons beyond QM, e.g. exotic hadrons

Running Coupling



- Quantum numbers in QM

△ For mesons $q\bar{q}$

① $P = (-1)^{L+1}, \quad C = (-1)^{L+S}$

② $0^{-+}, 0^{++}, 1^{--}, 1^{+-}, 1^{++}, 2^{++}, 2^{-+}, 2^{--}, 2^{+-}, \cdots$

△ For baryons qqq

① Allow for all half integer spin

② Allow for both positive and negative parities

- Exotic hadrons

△ With exotic quantum numbers

① $0^{--}, 0^{+-}, 1^{-+}, 2^{+-}, 3^{-+}, \cdots$

② e.g. $\pi_1(1600)$, $J^{PC} = 1^{-+}$

△ With exotic components

① $q\bar{q}g,\ gg,\ ggg, \cdots$

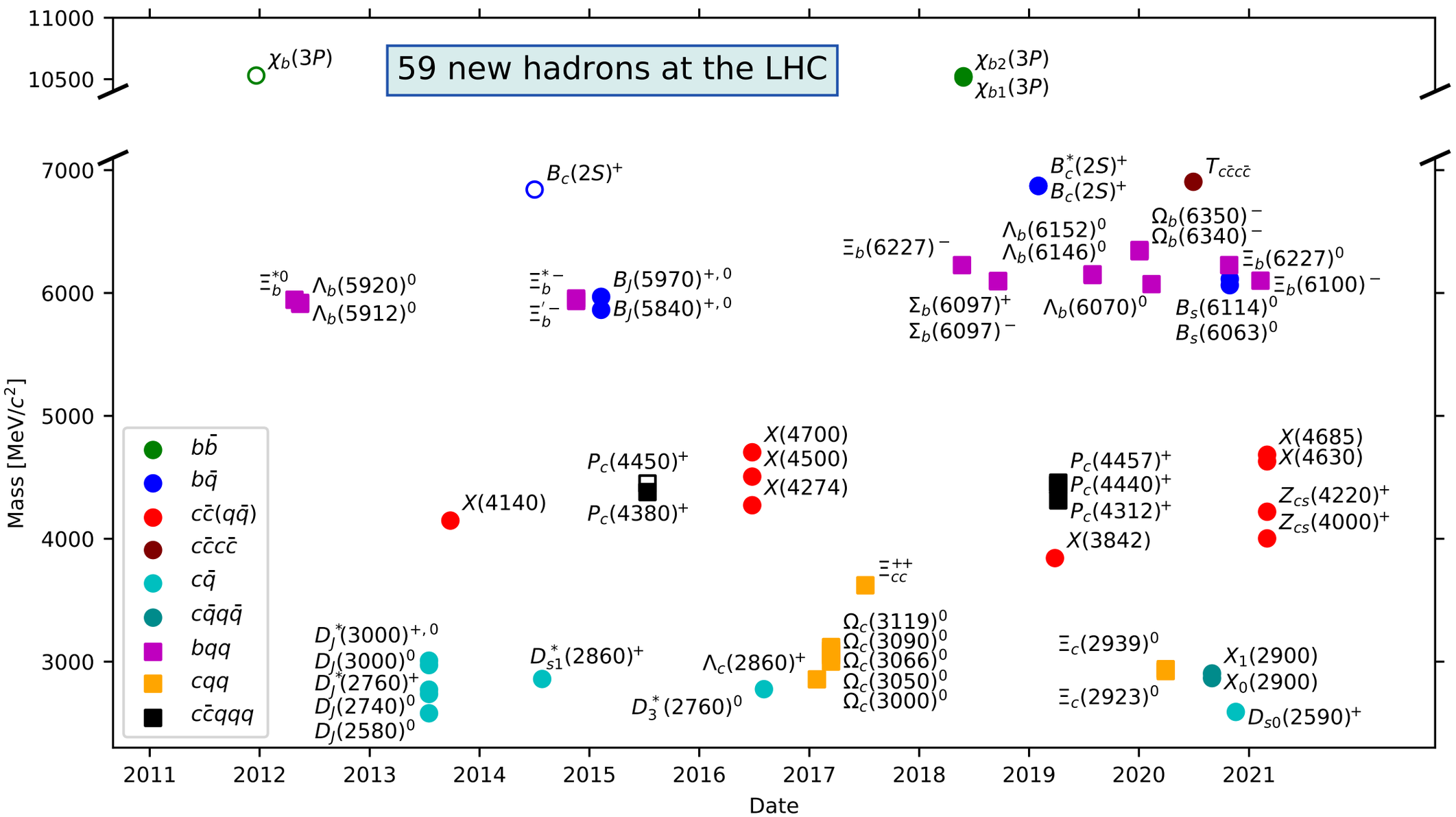
② $qq\bar{q}q, qqqq\bar{q}, qqqqq, \cdots$

0^{--} $\bar{D}_s DK$ three-body bound state, Wu et al., PRL135(2025)031902

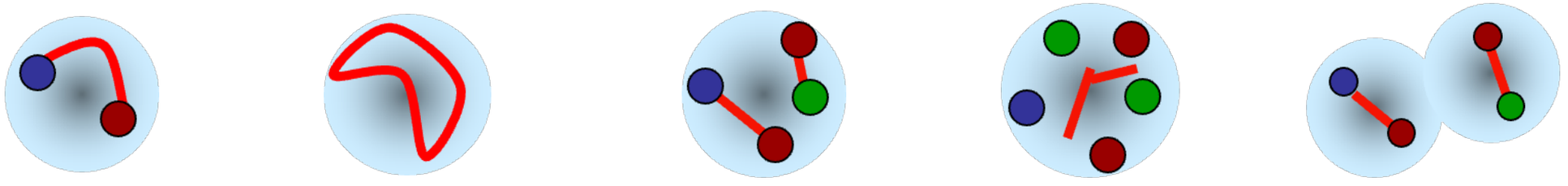
Prediction of 0^{--} state as $\psi(4260)$ HQSS partner, Ji et al., PRL129(2022)102002

Experimental data

- Tens of exotic candidates are observed
- Taking those observed by LHCb collaboration as an example



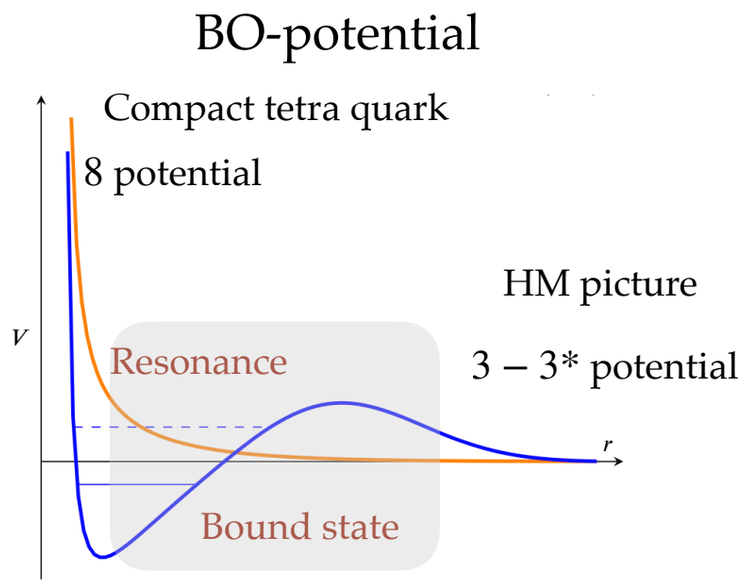
Interpretations



Hybrid Glueball Tetraquark Pentaquark Hadronic molecule

Chen:2016qju,Hosaka:2016pey,Richard:2016eis,Lebed:2016hpi,Esposito:2016noz,Guo:2017jvc,Ali:2017jda,Olsen:2017bmm,Liu:2019zoy,Brambilla:2019esw,Yamaguchi:2019vea,Guo:2019twa,Yang:2020atz,Chen:2022asf,Mai:2022eur,Meng:2022ozq,Liu:2024uxn,Huang:2023jec,Montana:2023sft,Chen:2024eaq,Mai:2025wjb,Lu:2025syk

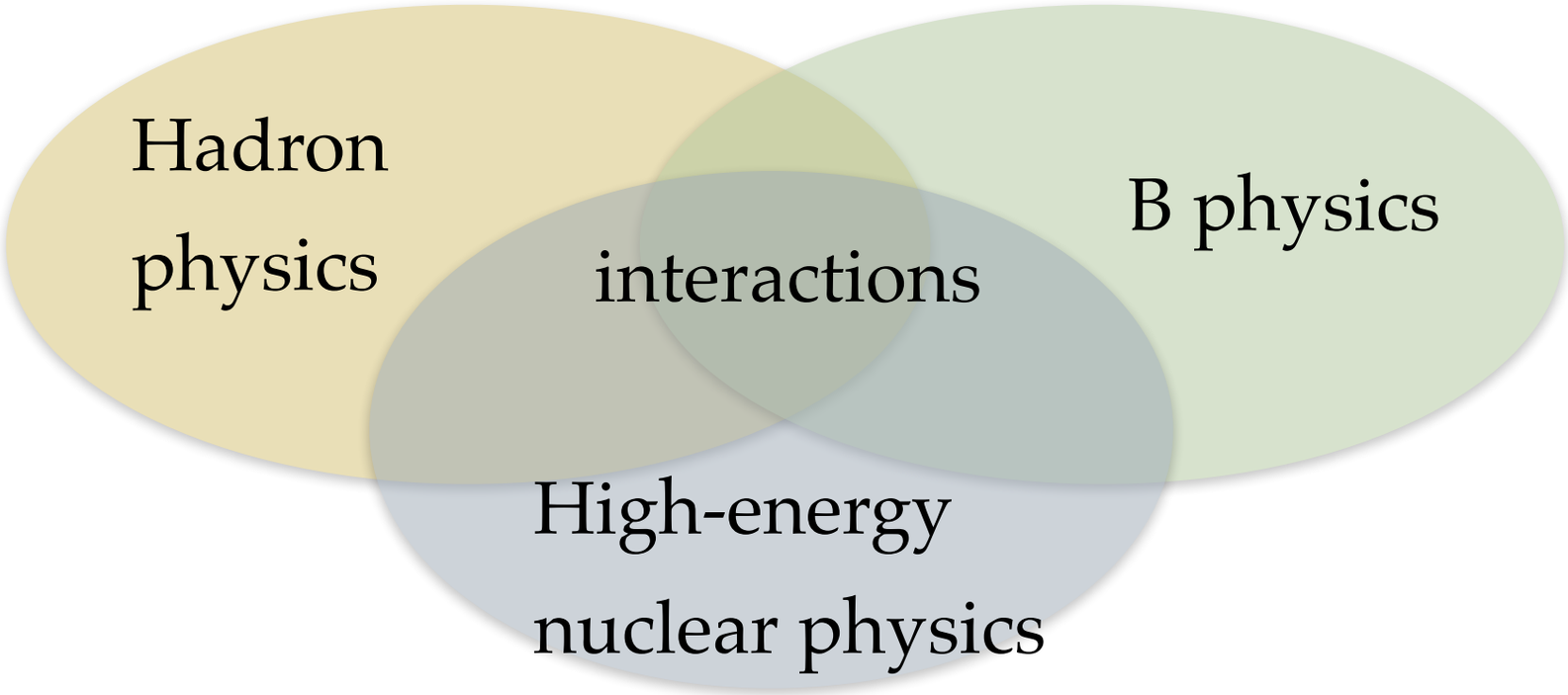
- With explicit gluon d.o.f., e.g. the famous $X(2370)$
- Multiquark spectra (diquark-antidiquark picture, GEM,…)
- Hadronic molecule, analogous to deuteron (hadron-hadron interaction)
- A unified picture



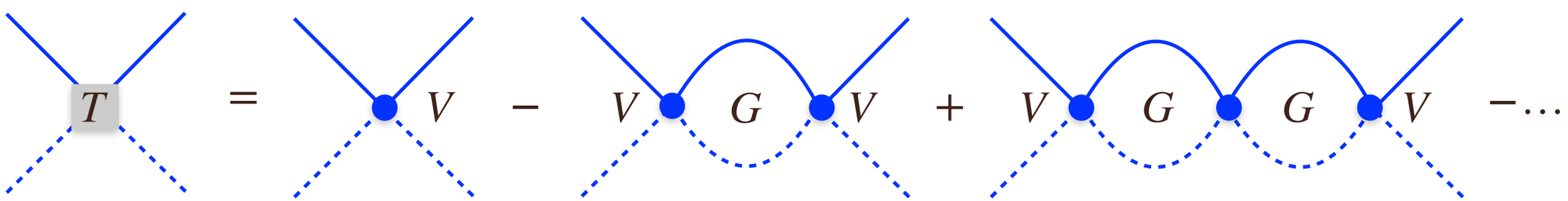
Braaten et al., PLB863(2025)139386

Hadron-hadron interaction

- The importance of hadron–hadron interactions



- Described by LSE or BSE



Examples

- $N\Omega$ interactions

  Near-threshold scattering of proton and Omega baryon and possible bound states,

  Feng, QW, Zhao, arXiv:2606.28319[hep-ph], accepted by PRD

- S-wave $D^{(*)}\bar{D}^{(*)}$ interaction

  Why is the $Z_c(3900)$ absent in the $h_c\pi$ final state?

  Ye, Zhang, Niu, QW, PRD113(2026)114017

- S-wave $D\bar{D}$ interaction

  Resolving the CP Asymmetry Puzzle in B decays with Unitized Final-State Interactions,

  Hu, Niu, QW, Wang, arXiv:2606.13278[hep-ph]

$N\Omega$ interaction

- Motivation
  - Exp./LQCD side

- $E_b \sim 27$ MeV for $^5S_2$ state from $p\Omega$ correlation function — STAR, PLB790(2019)490
- $E_b = 1.54$ MeV for HALQCD group — HAL QCD, PLB792(2019)284
- The depletion in the correlation function that is visible in the calculations around $k = 150$ MeV, owing to the presence of a $p\Omega-$ bound state, is not observed in the measured correlation. — ALICE, Nature 588(2020)232
- $E_b = 1.6^{+1.4}_{-0.5}$ MeV for $^5S_2$ state from $p\Omega$ correlation function — STAR, arXiv:2512.09452[nucl-ex]

$N\Omega$ interaction

- Motivation
  - Theoretical side

Both $^3S_1$ and $^5S_2$ $p\Omega$ are bound from quark model calculation Goldman et al., PRL59(1987)627

$E_b = 3.5 \sim 49.5$ MeV for $p\Omega$ $^5S_2$ bound state from various potentials Li et al.,EPJA8(2000)417

$E_b = 62$ MeV for $p\Omega$ $^5S_2$ bound state from quark model calculations Pang et al., PRC69(2004)065207

Exist $p\Omega$ $^5S_2$ bound state from quark model calculations Dai et al., CPL24(2007)389

Unbound for QDCSM, 53 MeV binding energy for ChQM Chen et al.,PRC83(2011)015202

Both $^3S_1$ and $^5S_2$ $p\Omega$ are bound with tens of MeV binding energy Huang et al., PRC92(2015)065202

No clear conclusions from χPT, because the results are cutoff dependent Haidenbauer et al., EPJC77(2017)760

$E_b = 21$ MeV for $p\Omega$ bound state $^5S_2$ from QCD sum rule, $^3S_1$ does not bound Chen et al., PRD103(2021)094011

$N\Omega$ interaction

- Motivation
  - Theoretical side

$^5S_2$ $p\Omega$ bound state with $E_b = 0.1$ MeV, short-range contact potential, $\Lambda\Xi$, $\Sigma\Xi$ channels are important

Sekihara et al., PRC98(2018)015205

Exist $p\Omega$ $^5S_2$ bound state, revisit short-range potential from quark model

Sekihara et al., PRC108(2023)065202

Coulomb interaction cannot be neglected for charged particle correlation, which could be one of the reason for the depletion of ALICE data

Yan et al., Sci. China Phys. Mech. Astron. 68(2025)232012

Potentials from LQCD makes this kind of study more precise.

$N\Omega$ interaction

- Debate in two-body correlation function

- Two-body correlation function $C(\mathbf{k}) = \int d\mathbf{r} S_{12}(\mathbf{r}) \left| \Psi(\mathbf{r}, \mathbf{k}) \right|^2$

  $S_{12}(\mathbf{r})$ source function, usually parameterized as Gaussian form $\dfrac{e^{-r^2/(4r_0^2)}}{(4\pi r_0^2)^{3/2}}$

  $\Psi(\mathbf{r}, \mathbf{k})$ relative wave function of out-going two-body system

- Whether the source function is universal or not?

The source function should not be universal. Epelbaum et al., PRL136(2026)21

A physical observable does not depend on the choice of the Hilbert space

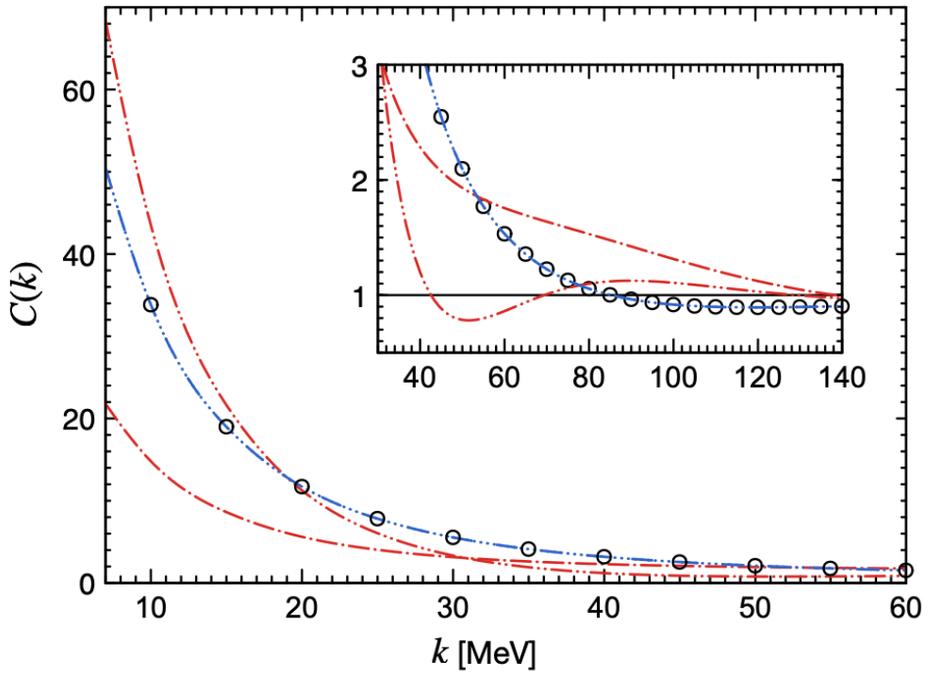
$$C(\mathbf{k}) = \left\langle \Psi^{(+)}_{-\mathbf{k}} \right| \hat{S}_{12} \left| \Psi^{(+)}_{-\mathbf{k}} \right\rangle \qquad C(\mathbf{k}) = \left(\left\langle \Psi^{(+)}_{-\mathbf{k}} \right| \hat{U}^\dagger \right) \left(\hat{U} \hat{S}_{12} \hat{U}^\dagger \right) \left(\hat{U} \left| \Psi^{(+)}_{-\mathbf{k}} \right\rangle \right)$$

$$\equiv \left\langle \Psi'^{(+)}_{-\mathbf{k}} \right| \hat{S}'_{12} \left| \Psi'^{(+)}_{-\mathbf{k}} \right\rangle .$$

Assuming $\hat{S}'_{12} = \hat{S}_{12}$ makes a model-dependent correlation function, which stems from the off-shell interaction.

$N\Omega$ interaction

- Debate in two-body correlation function
  - Whether the source function is universal or not?



Epelbaum et al., PRL136(2026)21

① An example to illustrate model-dependence

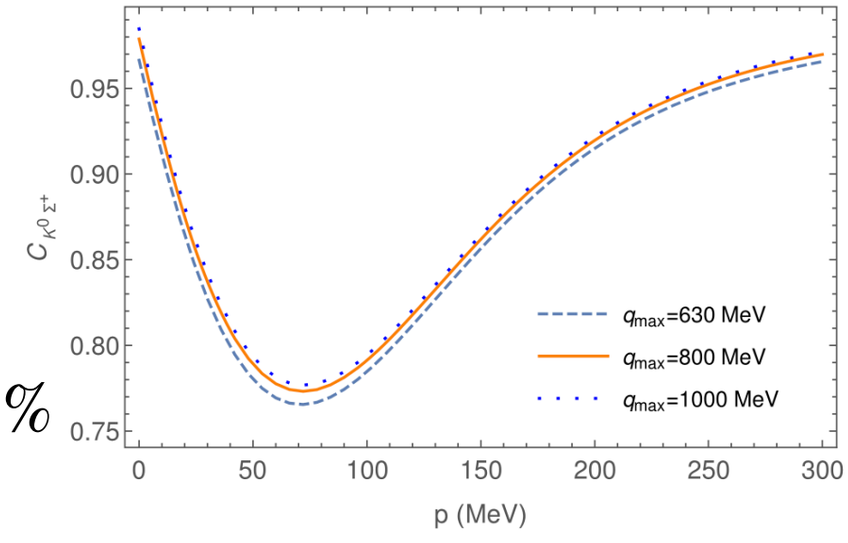
② Blue lines and red lines do not coincide

③ When $r_0 > r_{\text{range}}$, it is not a serious problem, since the large-distance behavior of the wave function $\Psi(\mathbf{r}, \mathbf{k})$ is unambiguously determined by the S-matrix

① $K\Lambda, K\Sigma, \eta N$ coupled channels for $N^*(1535)$

② Realistic interactions based on χ dynamics

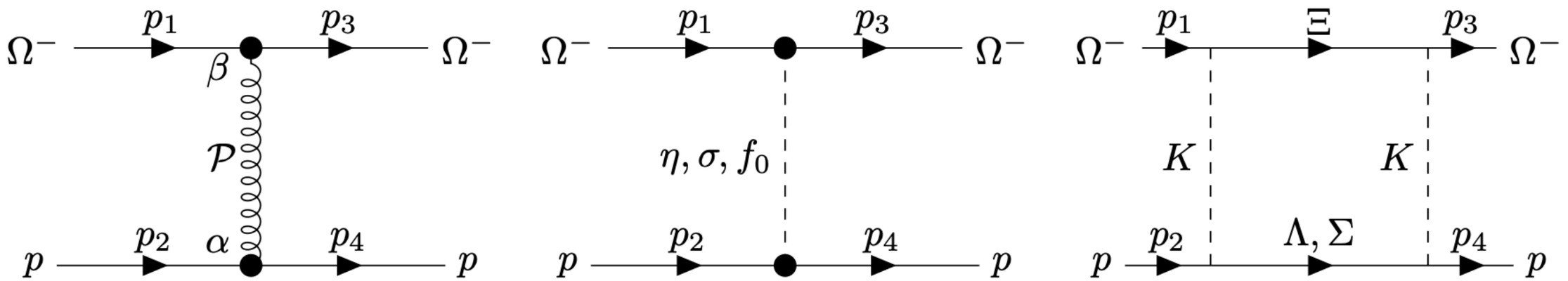
③ The uncertainty from off-shell ambiguity is $2 \sim 3$ %



Molina et al., PRD112(2025)096006

$N\Omega$ interaction

- Framework
  - Potentials



(a) Pomeron exchange (b) One meson exchange (c) Box-loop diagram

① Empirical parameters from other works

② Pomeron exchanged

$$\mathrm{T}(\mathrm{s},\mathrm{t}) = \left[\beta_{\mathrm{qA}} n_{\mathrm{A}} F_{\mathrm{A}}(t)\right]\left[\beta_{\mathrm{qB}} n_{\mathrm{B}} F_{\mathrm{B}}(t)\right] \times \frac{e^{-i\frac{\pi}{2}\left(\alpha_{\mathrm{p}}(t)-1\right)}}{2\sin\left(\frac{\pi}{2}\alpha_{\mathrm{p}}(t)\right)}\left(\alpha_{1,\mathrm{p}} s\right)^{\alpha_{\mathrm{p}}(t)},$$

Regge trajectory of Pomeron

$$\alpha_{\mathrm{p}}(t) = \alpha_{0,p} + \alpha_{1,\mathrm{p}} t = 1.08 + 0.25t$$

Feng, QW, Zhao, arXiv:2606.28319[hep-ph], accepted by PRD

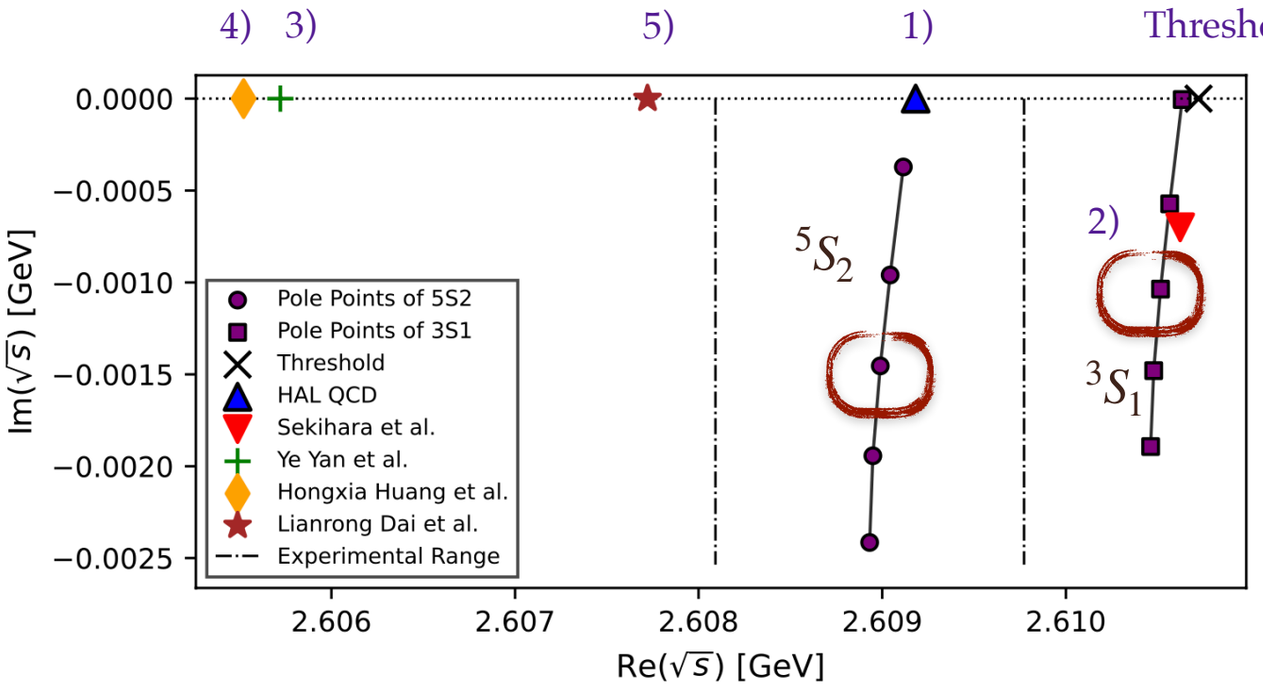
$N\Omega$ interaction

$$M(^5S_2) = 2608.98 - 1.45i \text{ MeV}$$

$$M(^3S_1) = 2610.52 - 1.04i \text{ MeV}$$

$$@\,\Lambda_p = 0.15 \text{ GeV}$$

- Results
  - Pole positions in a comparison with other works



$\Lambda_p = 0$ GeV

$\Lambda_p = 0.1$ GeV

$\Lambda_p = 0.15$ GeV

$\Lambda_p = 0.2$ GeV

$\Lambda_p = 0.25$ GeV

- Without Pomeron, pole is consistent with that from HALQCD
- However, scattering length is significant larger than the expected value of HALQCD

1) HAL QCD, PLB792(2019)284

2) Sekihara et al., PRC98(2018)015205

3) Yan et al., Sci. China Phys. Mech. Astron. 68(2025)232012

4) Huang et al., PRC92(2015)065202

5) Dai et al., CPL24(2007)389

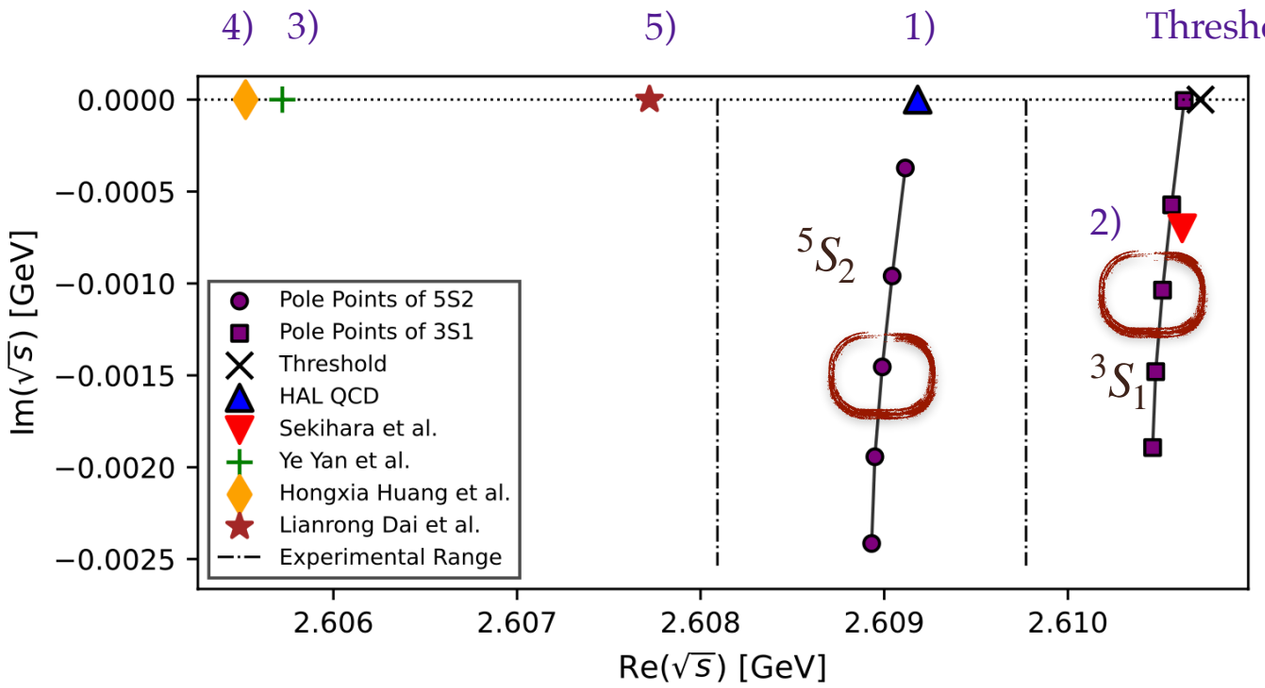
$N\Omega$ interaction

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- Results
  - Pole positions in a comparison with other works



$\Lambda_p = 0$ GeV

$\Lambda_p = 0.1$ GeV

$\Lambda_p = 0.15$ GeV

$\Lambda_p = 0.2$ GeV

$\Lambda_p = 0.25$ GeV

(GeV)	$\Lambda_p = 0$	$\Lambda_p = 0.1$	$\Lambda_p = 0.15$	$\Lambda_p = 0.2$
$^5S_2$	$6.99 - 1.06i$	$5.76 - 2.39i$	$4.87 - 2.49i$	$4.11 - 2.47i$
$^3S_1$	109 − 6.09i	$6.18 - 15.86i$	$3.79 - 9.05i$	$2.90 - 6.41i$

	$\Lambda_p = 0.25$	Spin ave	Quintet	HALQCD
$^5S_2$	$2.11 - 5.19i$	$4.89^{+0.66}_{-0.54}$	$4.27^{+0.68}_{-0.43}$	$5.30^{+0.16}_{-0.01}$
$^3S_1$	$2.38 - 5.05i$	none	none	none

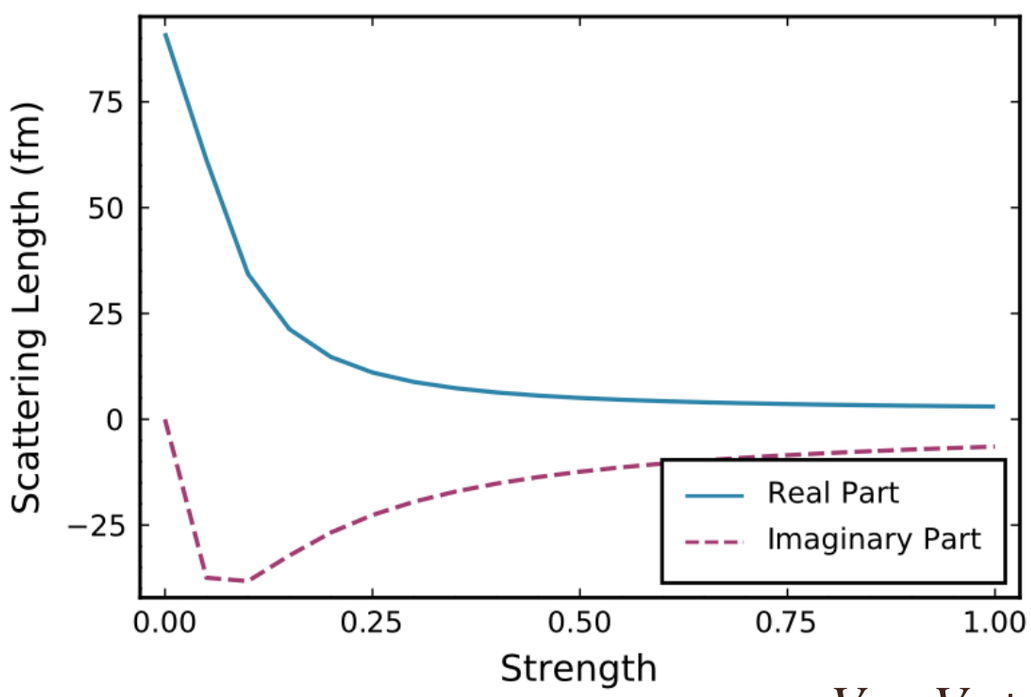
$N\Omega$ interaction

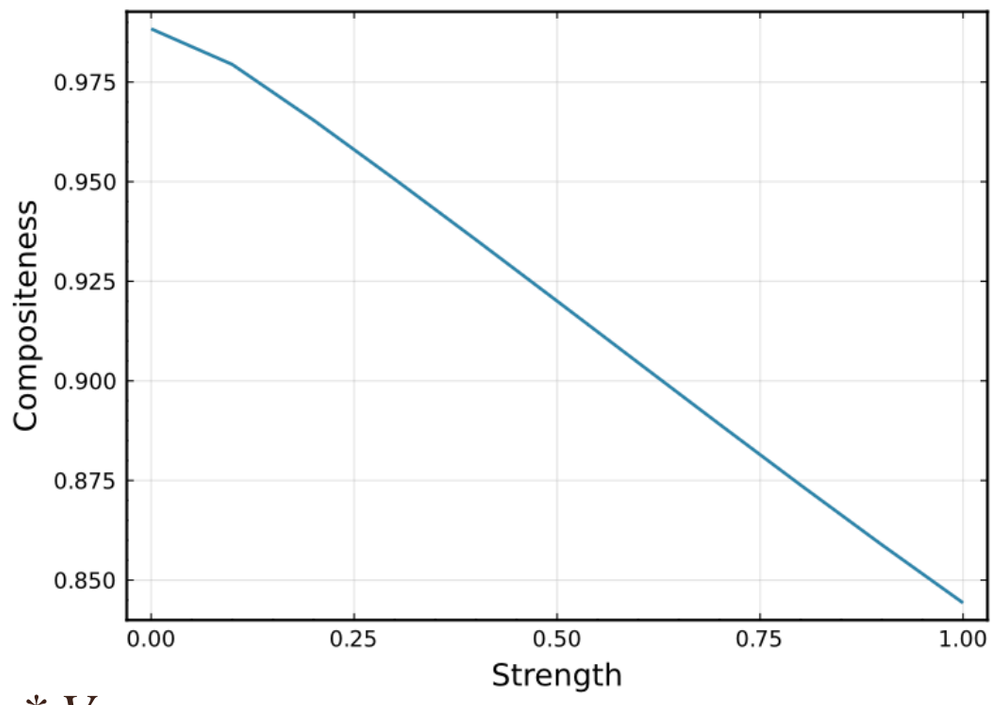
- Results
  - The compositeness $\bar{X} = \sqrt{\dfrac{1}{1 + |2r/a|}}$

The non relativistic scattering amplitude $f_s(\mathrm{k}) = -\dfrac{\mu}{2\pi}\, \mathrm{T}(\mathrm{E}; \mathrm{k}, \mathrm{k})$

Effective-range-Expansion $f_s(\mathrm{k})^{-1} = -\dfrac{1}{a} - i\mathrm{k} + \dfrac{1}{2} r_{\mathrm{eff}} \mathrm{k}^2 + \mathcal{O}\left(\mathrm{k}^4\right)$

Scattering length $a = -f_s(\mathrm{k} = 0)$ Effective range $r_{\mathrm{eff}} = 2\left[\dfrac{\mathrm{d}f_s^{-1}(\mathrm{k})}{\mathrm{dk}^2}\right]_{\mathrm{k}=0}$





$V = V_s + n * V_P$

Pomeron make it more compact!

What do we expect from experimental side

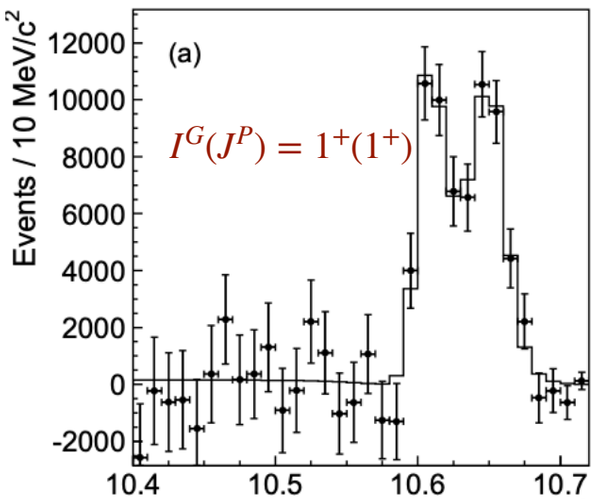
- More data about $p\Omega$, or dibaryon interaction from the processes other than correlation functions

S-wave $D^{(*)}\bar{D}^{(*)}$ interaction

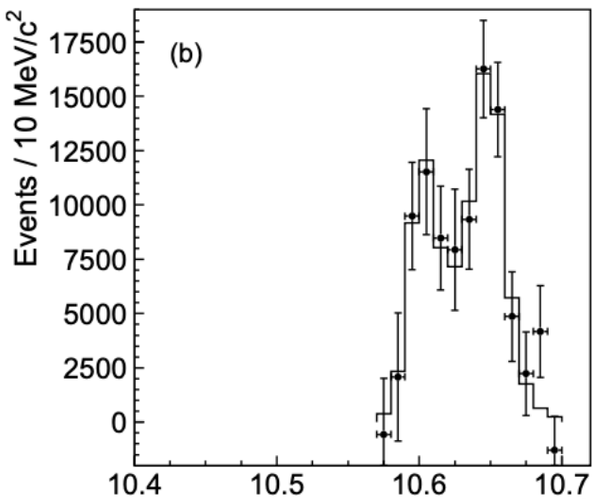
- Motivation

Belle, PRL108(2012)122001, PRD91(2015)072003

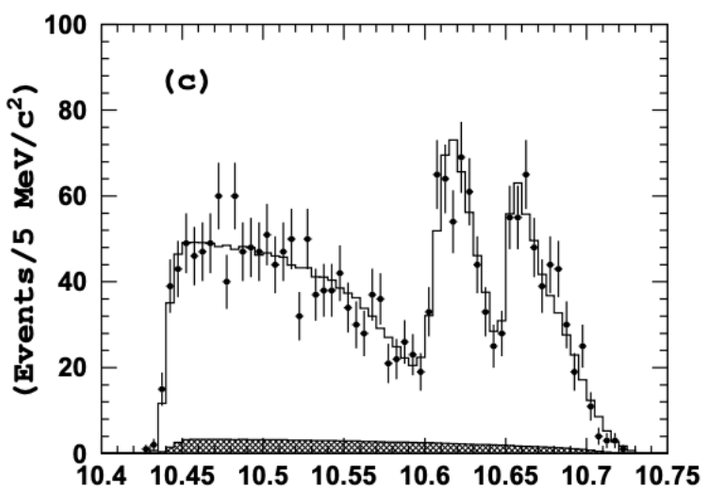
- The observation of two narrow structures in $\Upsilon(nS)\pi, h_b(mP)\pi$ channels



$e^+e^- \to h_b(1P)\pi^+\pi^-$



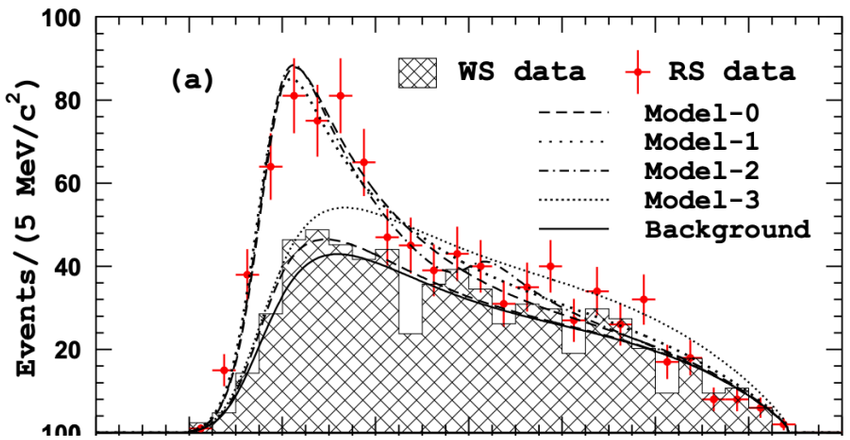
$e^+e^- \to h_b(2P)\pi^+\pi^-$



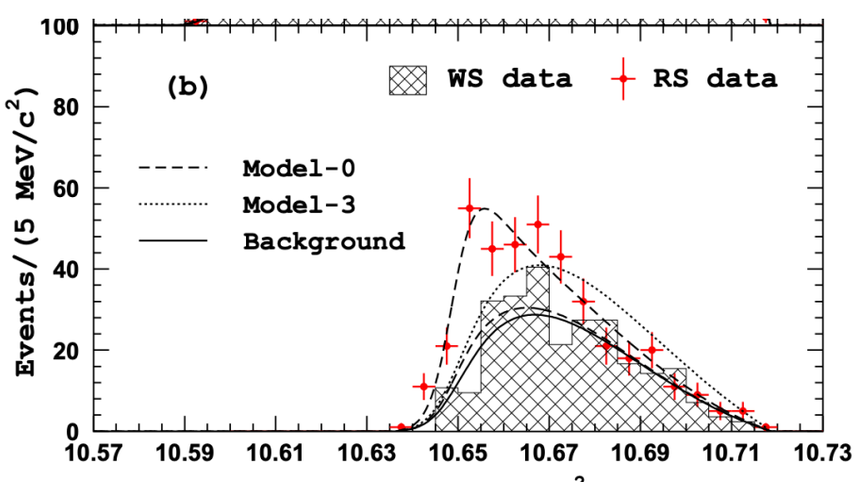
$e^+e^- \to \Upsilon(2S)\pi^+\pi^-$

- Confirm these two structures in $B^{(*)}\bar{B}^{(*)}$ channels

Belle, PRL116(2016)1212001



$e^+e^- \to B\bar{B}^*\pi$



$e^+e^- \to B^*\bar{B}^*\pi$

S-wave $D^{(*)}\bar{D}^{(*)}$ interaction

- Motivation
  - $B^*\bar{B}^{(*)}$ hadronic molecule v.s. compact tetra quark

Observation of two charged bottomonium-like resonances in Y(5S) decays #1

Belle Collaboration • A. Bondar (Novosibirsk, IYF) et al. (Oct, 2011)

Published in: *Phys.Rev.Lett.* 108 (2012) 122001 • e-Print: 1110.2251 [hep-ex]

pdf links DOI cite claim reference search 786 citations

Hadronic molecule

Guo, et al., RMP90(2018)015004,

Chen et al., Phys.Rept.639(2016)1,

Barambilla et al., Phys. Rept. 873(2020)1

Compact tetra quark

Esposito et al., Phys.Rept.668(2017)1,

Lebed et al., PPNP93(2017)143,

Ali et al., PPNP97(2017)123

The top three most cited review articles

Heavy quark spin structure in Z_b resonances #22

A.E. Bondar (Novosibirsk, IYF and Novosibirsk State U.), A. Garmash (Novosibirsk, IYF and Novosibirsk State U.), A.I. Milstein (Novosibirsk, IYF and Novosibirsk State U.), R. Mizuk (Moscow, ITEP), M.B. Voloshin (Moscow, ITEP and Minnesota U., Theor. Phys. Inst.) (May, 2011)

Published in: *Phys.Rev.D* 84 (2011) 054010 • e-Print: 1105.4473 [hep-ph]

pdf DOI cite claim reference search 294 citations

S-wave $D^{(*)}\bar{D}^{(*)}$ interaction

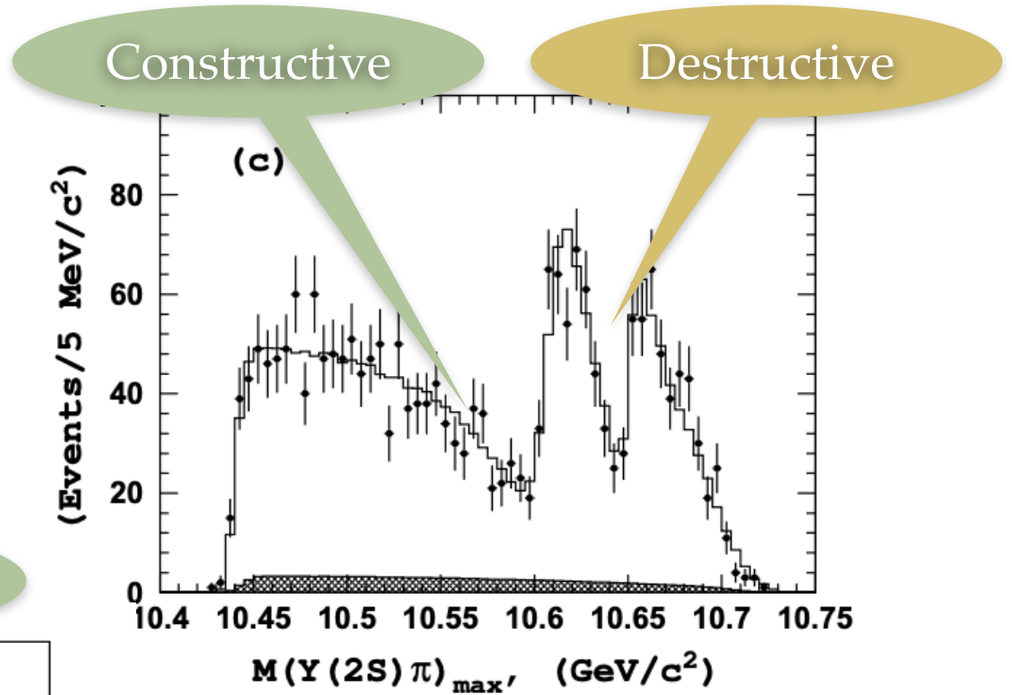
- Motivation
  - The heavy-light decomposition

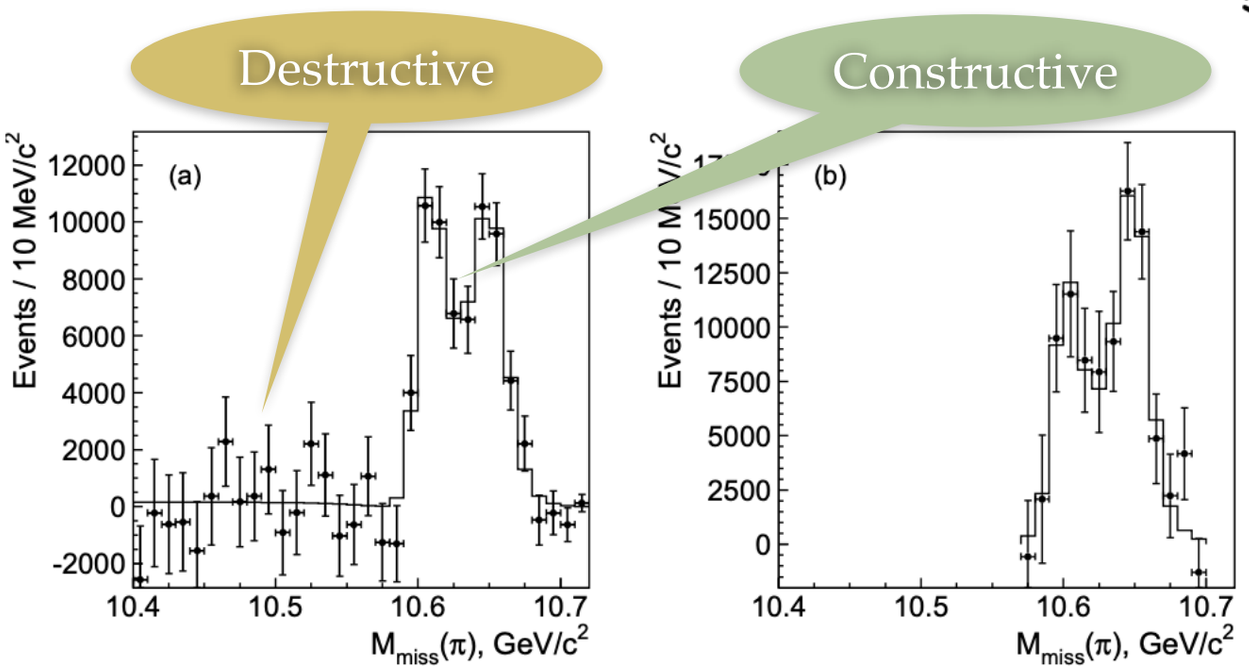
$h_b(mP)\pi$ $\quad$ $\Upsilon(nS)\pi$

$$\left|Z_b'\right\rangle = \frac{1}{\sqrt{2}}\left(0^-_{b\bar{b}} \otimes 1^-_{\bar{Q}_q} - 1^-_{b\bar{b}} \otimes 0^-_{\bar{Q}_q}\right),$$

$$\left|Z_b\right\rangle = \frac{1}{\sqrt{2}}\left(0^-_{b\bar{b}} \otimes 1^-_{\bar{Q}_q} + 1^-_{b\bar{b}} \otimes 0^-_{\bar{Q}_q}\right).$$

Bondar et al., PRD84(2011)054010





- The expected analog in charm sector

$b \to c$

$Z_b(10610) \to Z_c(3900)$

$B\bar{B}^* \to D\bar{D}^*$

$Z_b(10650) \to Z_c(4020)$

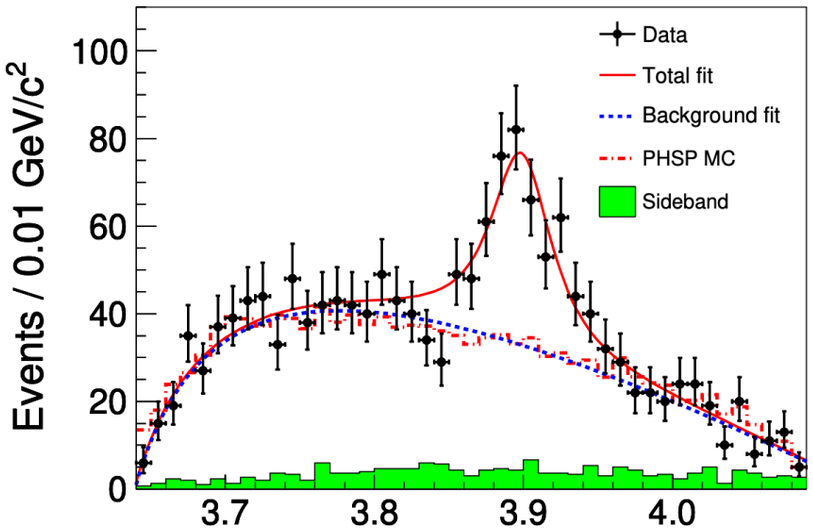
$B^*\bar{B}^* \to D^*\bar{D}^*$

Guo et al., PRD88(2013)054007

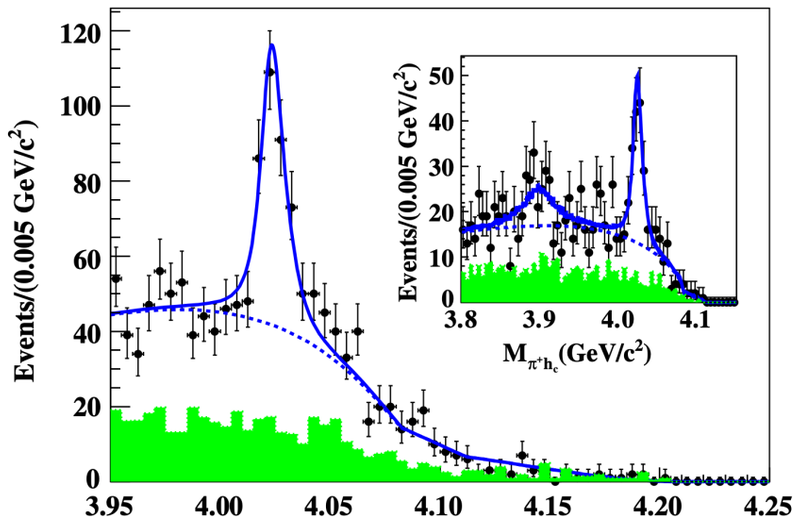
S-wave $D^{(*)}\bar{D}^{(*)}$ interaction

- Motivation

  - The observation of $Z_c(3900)$ and $Z_c(4020)$



$e^+e^- \to J/\psi\pi^+\pi^-$



$e^+e^- \to h_c\pi^+\pi^-$

BESIII, PRL110(2013)252001, Belle, PRL110(2013)019901, Xiao et al., PLB727(2013)366,BESIII, PRL110(2013)242001

  - Indicate significantly HQSS breaking effect

  - The HQFS is largely broken

S-wave $D^{(*)}\bar{D}^{(*)}$ interaction

- Potentials
  - Channels

    Bottom sector: $B\bar{B}^*, B^*\bar{B}^*, \Upsilon(1S)\pi, \Upsilon(2S)\pi, h_b(1P)\pi, h_b(2P)\pi$

    Charm sector: $D\bar{D}^*, D^*\bar{D}^*, J/\psi\pi, h_c\pi$

  - Potentials among elastic channels

$$V_{\rm e}^{\rm c} = \begin{pmatrix} \frac{1}{2}\left(\mathscr{C}_0+\mathscr{C}_1\right)+\mathscr{B} & \frac{1}{2}\left(\mathscr{C}_1-\mathscr{C}_0\right) \\ \frac{1}{2}\left(\mathscr{C}_1-\mathscr{C}_0\right) & \frac{1}{2}\left(\mathscr{C}_0+\mathscr{C}_1\right)-\mathscr{B} \end{pmatrix}$$

HQS parameters

$$\mathscr{C}_0 \equiv \langle 1\otimes 0|\,\hat{\mathscr{H}}\,|1\otimes 0\rangle,$$
$$\mathscr{C}_1 \equiv \langle 0\otimes 1|\,\hat{\mathscr{H}}\,|0\otimes 1\rangle,$$

HQSV parameters $\quad \mathscr{B} \equiv \langle 1\otimes 0|\,\hat{\mathscr{H}}'\,|0\otimes 1\rangle = \langle 0\otimes 1|\,\hat{\mathscr{H}}'\,|1\otimes 0\rangle$

  - Couplings between elastic and inelastic channels

HQS parameters

$$g_{1S}^{\psi} \equiv \langle \psi(1S)\pi|\,\hat{\mathscr{H}}_{\rm in}\,|1\otimes 0\rangle,$$
$$g_{1P}^{h_{\rm c}} \equiv \left\langle h_c(1P)\pi\right|\,\hat{\mathscr{H}}_{\rm in}\,|0\otimes 1\rangle,$$

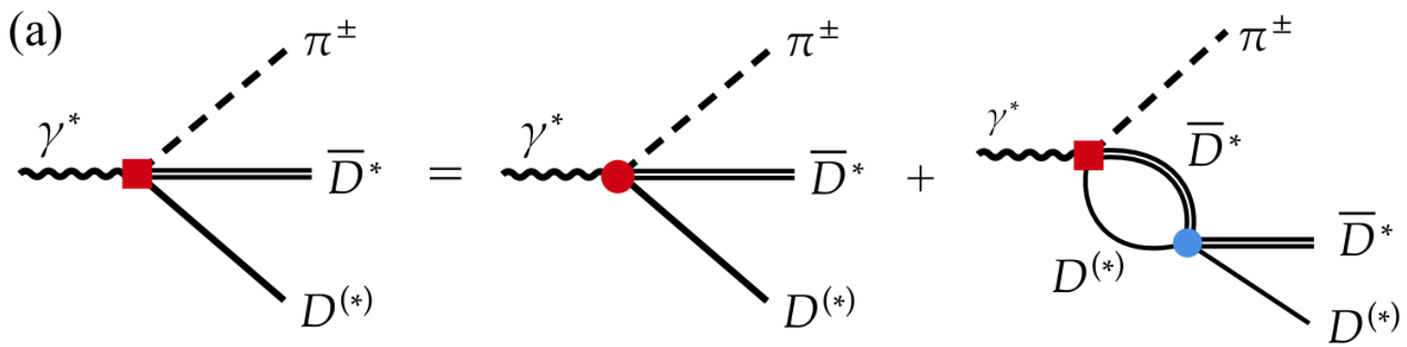
HQSV parameters

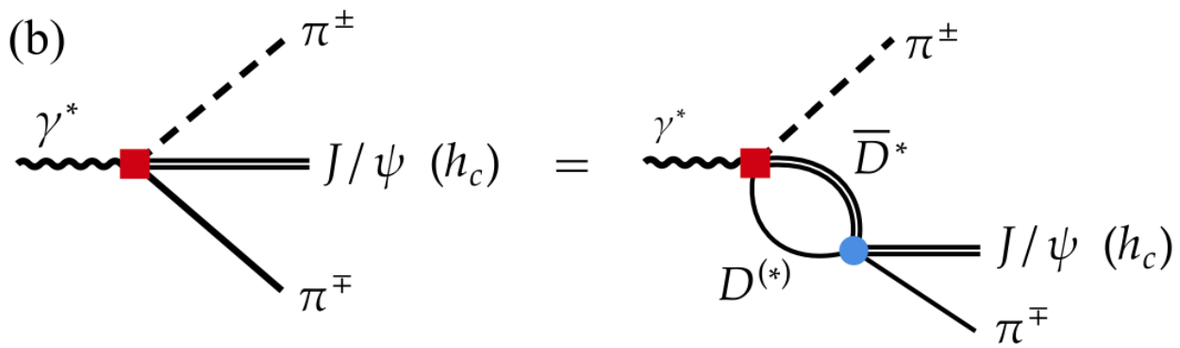
$$v_{1S}^{\psi} \equiv \langle \psi(1S)\pi|\,\hat{\mathscr{H}}'_{i\rm in}\,|0\otimes 1\rangle,$$
$$v_{1P}^{h_c} \equiv \left\langle h_c(1P)\pi\right|\,\hat{\mathscr{H}}'_{i\rm in}\,|1\otimes 0\rangle,$$

Ye et al., PRD113(2026)114017

S-wave $D^{(*)}\bar{D}^{(*)}$ interaction

- Production amplitudes





- Consider both S-wave and D-wave $D^{(*)}\bar{D}^{(*)}$ channels
- Bare production amplitudes vary at different $E_{\rm cm}$
- Do not consider three-body FSI

Chen et al., arXiv:2604.25607[hep-ph]

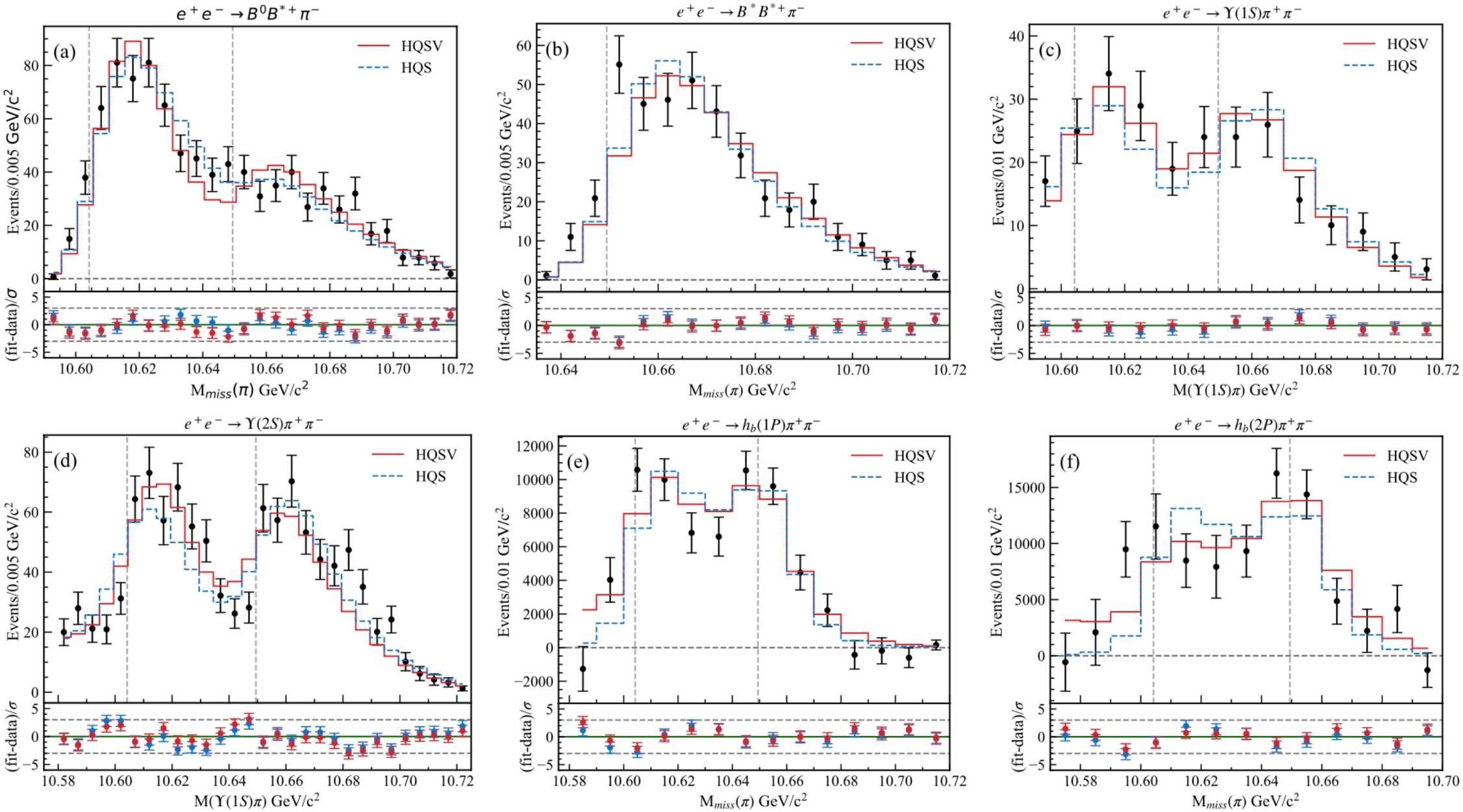
- Do not consider triangle diagrams

Chen et al., arXiv:2604.25607[hep-ph], Du et al., PRD105(2022)074018

Ye et al., PRD113(2026)114017

S-wave $D^{(*)}\bar{D}^{(*)}$ interaction

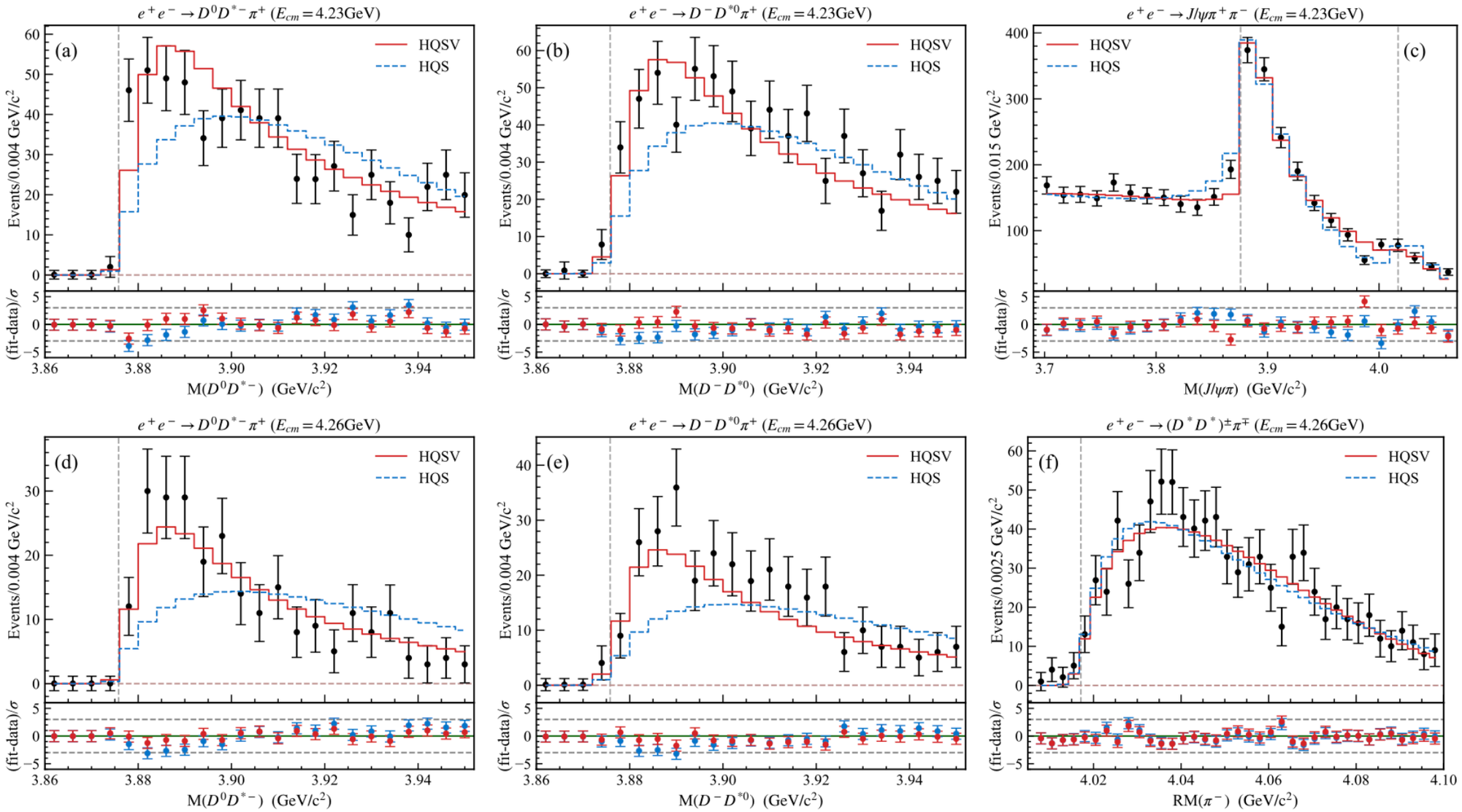
- Results



Ye et al., PRD113(2026)114017

S-wave $D^{(*)}\bar{D}^{(*)}$ interaction

- Results



Ye et al., PRD113(2026)114017

S-wave $D^{(*)}\bar{D}^{(*)}$ interaction

- Results
  - Fit parameters

Large HQSV

Respect light-quark symmetry

Parameters	HQSV (Z_c)	HQS (Z_c)	HQSV (Z_b)	HQS (Z_b)
$\mathcal{C}_0$ [GeV^{-2}]	-3.88 ± 0.37	-6.75 ± 0.13	-2.21 ± 0.06	-2.64 ± 0.12
$\mathcal{C}_1$ [GeV^{-2}]	9.41 ± 1.00	-3.32 ± 0.29	-2.67 ± 0.07	-2.88 ± 0.11
$\mathcal{B}$ [GeV^{-2}]	-14.65 ± 0.46	...	-0.25 ± 0.04	...
g_{1S}^{ψ} [GeV$^{-3/2}$]	-5.34 ± 0.09	-2.15 ± 0.07	...	...
$g_{1P}^{h_c}$ [GeV^{-2}]	0.98 ± 0.06	-0.31 ± 0.04	...	...
v_{1S}^{ψ} [GeV$^{-3/2}$]	1.99 ± 0.12	...	...	...
v_{1P}^{ψ} [GeV^{-2}]	0.27 ± 0.02	...	...	...
g_{1S}^{Υ} [GeV$^{-3/2}$]	...	...	-1.42 ± 0.05	-0.47 ± 0.12
g_{2S}^{Υ} [GeV$^{-3/2}$]	...	...	-0.13 ± 0.02	-1.99 ± 0.08
$g_{1P}^{h_b}$ [GeV^{-2}]	...	...	0.15 ± 0.04	0.24 ± 0.06
$g_{2P}^{h_b}$ [GeV^{-2}]	...	...	-4.30 ± 0.51	-4.95 ± 0.64
v_{1S}^{Υ} [GeV$^{-3/2}$]	...	...	-0.26 ± 0.07	...
v_{2S}^{Υ} [GeV$^{-3/2}$]	...	...	-0.03 ± 0.01	...
$v_{1P}^{h_b}$ [GeV^{-2}]	...	...	0.03 ± 0.01	...
$v_{2P}^{h_b}$ [GeV^{-2}]	...	...	-0.50 ± 0.28	...
Λ [GeV]	1.00	1.80	1.80	1.40
χ^2/d.o.f.	1.12	1.92	1.76	1.81

Improve the fit quantity

Ye et al., PRD113(2026)114017

S-wave $D^{(*)}\bar{D}^{(*)}$ interaction

Ye et al., PRD113(2026)114017

- Results
  - Poles

① Two poles in b sector

② Two poles for HQS in c sector

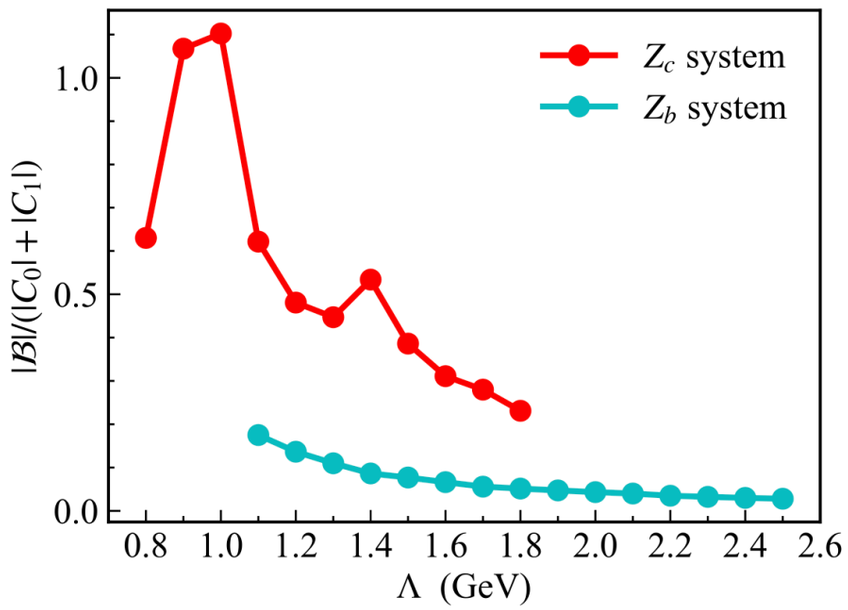
③ One pole for HQSV in c sector

④ In HQS,

$$g_{Z_b B\bar{B}^*} \simeq g_{Z_b' B^*\bar{B}^*}$$

$$g_{Z_b' B\bar{B}^*} \simeq g_{Z_b B^*\bar{B}^*}$$

  - Quantify HQSV



RS (Z_c)	HQSV (Z_c)	HQS (Z_c)
$(-,+)$	$3.875-0.011i$	$3.815-0.052i$
$g_{D\bar{D}^*}$	0.85	1.22
$g_{D^*\bar{D}^*}$	0.54	0.64
$(-,-)$	···	$3.988-0.028i$
$g_{D\bar{D}^*}$	···	0.57
$g_{D^*\bar{D}^*}$	···	1.02

RS (Z_b)	HQSV (Z_b)	HQS (Z_b)
$(-,+)$	$10.610-0.010i$	$10.600-0.013i$
$g_{B\bar{B}^*}$	0.42	0.39
$g_{B^*\bar{B}^*}$	0.11	0.13
$(-,-)$	$10.640-0.021i$	$10.649-0.016i$
$g_{B\bar{B}^*}$	0.25	0.13
$g_{B^*\bar{B}^*}$	0.42	0.40

$\lvert C/V\rvert$	Z_c system	Z_b system
$\lvert\mathcal{B}\rvert/(\lvert\mathcal{C}_0\rvert+\lvert\mathcal{C}_1\rvert)$	1.10	0.05
$\lvert v_{1S}^{\psi}/g_{1S}^{\psi}\rvert$	0.37	···
$\lvert v_{1P}^{\psi}/g_{1P}^{\psi}\rvert$	0.28	···
$\lvert v_{1S}^{\Upsilon}/g_{1S}^{\Upsilon}\rvert$	···	0.18
$\lvert v_{2S}^{\Upsilon}/g_{2S}^{\Upsilon}\rvert$	···	0.23
$\lvert v_{1P}^{h_b}/g_{1P}^{h_b}\rvert$	···	0.20
$\lvert v_{2P}^{h_b}/g_{2P}^{h_b}\rvert$	···	0.12

What do we learn?

- Provide a way to estimate HQSV effect quantitatively

S-wave $D\bar{D}$ interaction

- Motivation
- CP violation, a prerequisite for the observed matter-antimatter asymmetry of the universe A.D. Sakharov, Pisma Zh. Esp. Theos. Fiz. 5(1967)32
- CKM matrix elements ——fundamental parameters in SM
  - leptonic B decays ⇒ CKM matrix elements
  - Hadronic decays of B meson ⇒ Weak angle

    The best channels for $\gamma \Leftarrow \quad B \to DK, \;\; B \to D\pi$ J. Brod, J. Zupan, JHEP01(2014)051
  - We propose other channels to measure weak angle γ
  - e.g. $\bar{B}^0 \to D\bar{D}, \bar{B}_s^0 \to D\bar{D}$ with FSI
- FSI
  - Whether FSI is important or not? ⇐ deBroglie wave lengths of the final particles

FSI is not important in B decays H.Y. Cheng, C.K. Chuan, A. Soni, PRD71(2005)014030

S-wave $D\bar{D}$ interaction

- Motivation
- FSI

- For D mesons ($m_B - m_D \sim 4$ GeV), FSI is important

⇒ Long-distance rescattering effects are expected to influence CP asymmetry in $D \to PP$ decays

A. Pich, et.al, PRD108(2023)036026

⇒ FSI is important for the CP violation in both B and D decays

I. Bediaga et.al, PPNP114(2020)103808

⇒ FSI enhance CPV in $D^0 \to \pi^+\pi^-$ and $D^0 \to K^+K^-$ processes

I. Bediaga, et.al, PRL131(2023)051802

⇒ The one-loop approximation of FSI in B decays

H.Y. Cheng, et.al, PRD72(2005)014006, C. Smith EPJC33(2004)523,…

- FSI in B decays

⇒ $m_P \to m_D$, phase space is sufficient small

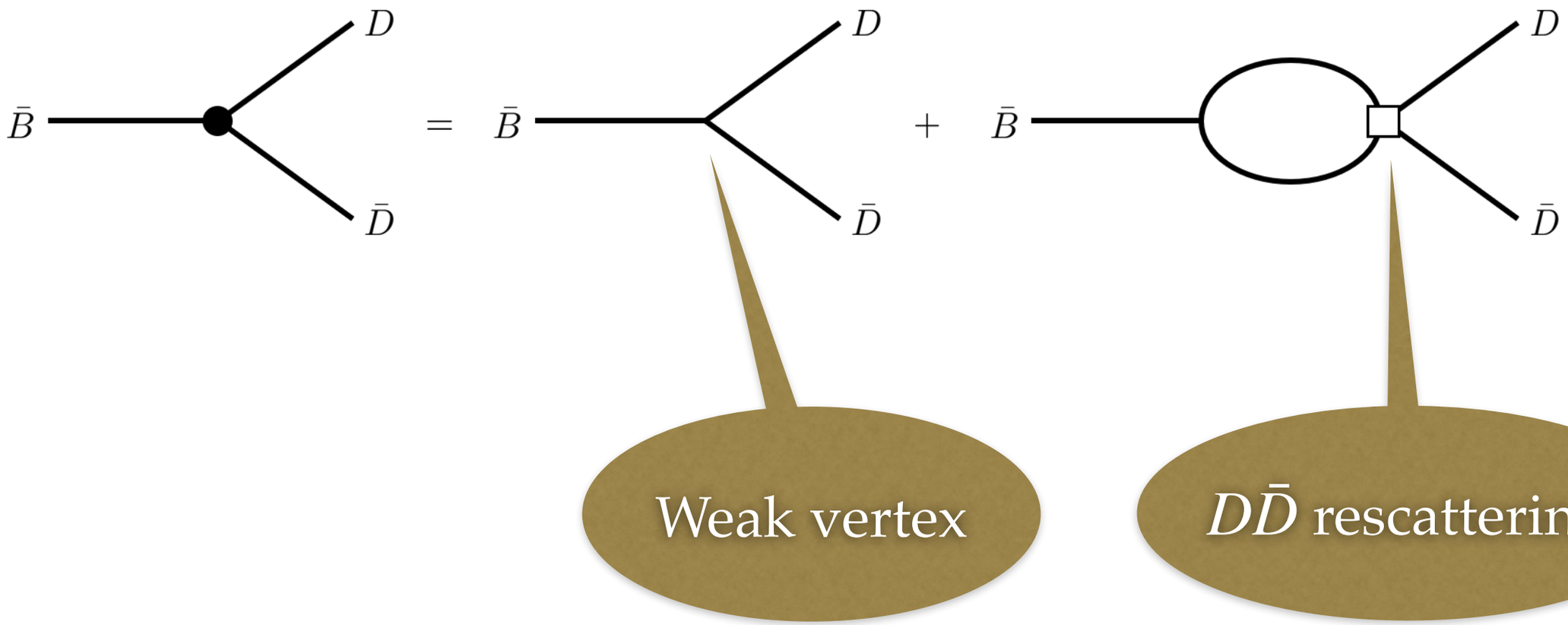
$$\mathcal{A}_{\rm CP}^{\rm dir}\left(\bar{B}^0 \to D_s^- D^+\right) = 0.0009 \pm 0.0053 \pm 0.0040$$
$$\mathcal{A}_{\rm CP}^{\rm dir}\left(\bar{B}_s^0 \to D_s^+ D^-\right) = 0.103 \pm 0.053 \pm 0.010$$

⇒ Take advantage of the recent experimental measurements

LHCb, arXiv:2603.28132

S-wave $D\bar{D}$ interaction

- Motivation



S-wave $D\bar{D}$ interaction

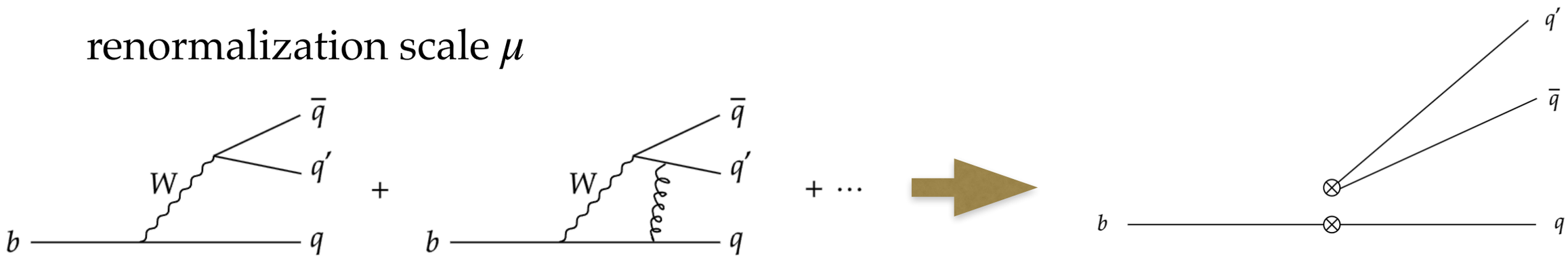
- Weak Effective Hamiltonian

$$H_{\text{eff}} = \frac{G_E}{\sqrt{2}} V_{qb}^* V_{qq'} \left(C_1(\mu)\mathcal{O}_1 + C_2(\mu)\mathcal{O}_2 \right)$$

$$\mathcal{O}_1 = \left(\bar{q}'_\alpha q_\beta\right)_{V-A} \left(\bar{q}_\beta b_\alpha\right) \quad q = (u, c)$$

$$\mathcal{O}_2 = \left(\bar{q}'_\alpha q_\alpha\right)_{V-A} \left(\bar{q}_\beta b_\beta\right) \quad q' = (d, s)$$

- $C_1(\mu)$ and $C_2(\mu)$ are Wilson coefficients encoded physics larger than the renormalization scale μ



- The coefficients of other eight penguin operators are much smaller (neglected)

- Decay Amplitude

A.J. Buras, et.al, NPB569(2000)3, M. Ciuchini, et.al, NPB501(1997)271

$$\mathcal{M}(B \to F) = \frac{G}{\sqrt{2}} V_{qb}^* V_{qq'} \sum_{n=1,2} C_n(\mu) \langle B \,|\, \mathcal{O}_n(\mu) \,|\, F \rangle$$

- $\langle B \,|\, \mathcal{O}_n(\mu) \,|\, F \rangle$ hadronic matrix elements

S-wave $D\bar{D}$ interaction

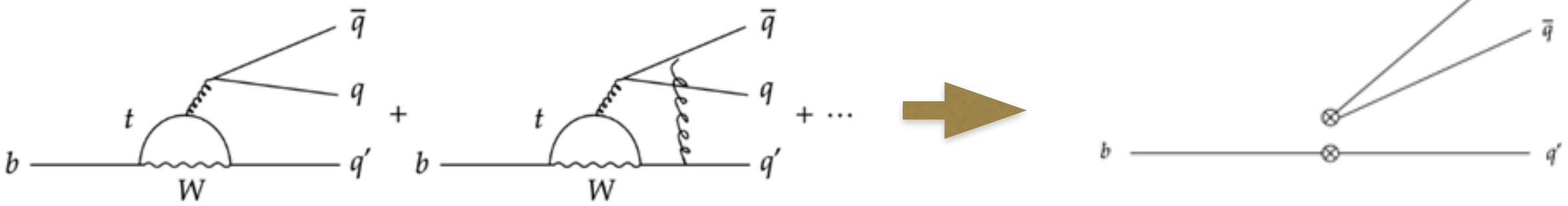
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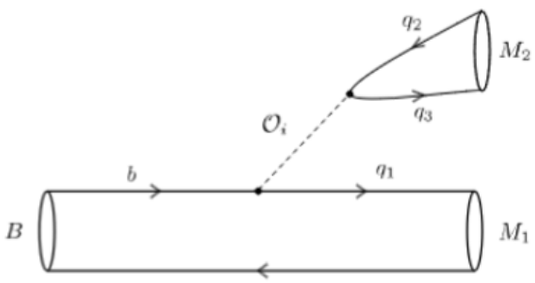
A.J. Buras, et.al, NPB569(2000)3, M. Ciuchini, et.al, NPB501(1997)271

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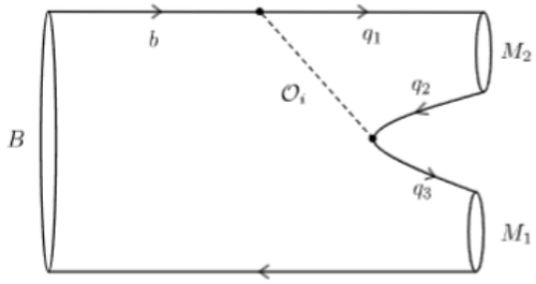
- $\langle B \,|\, \mathcal{O}_n(\mu) \,|\, F \rangle$ hadronic matrix elements

S-wave $D\bar{D}$ interaction

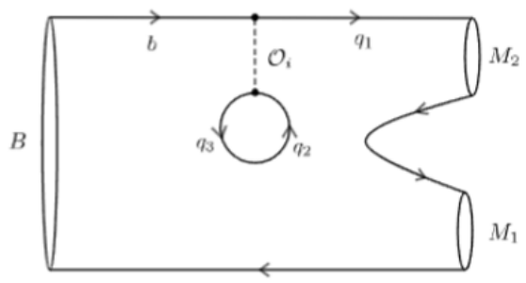
- Different topologies of the matrix element



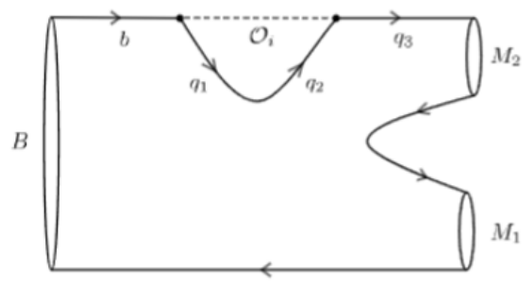
(a) DE: Disconnected Emission



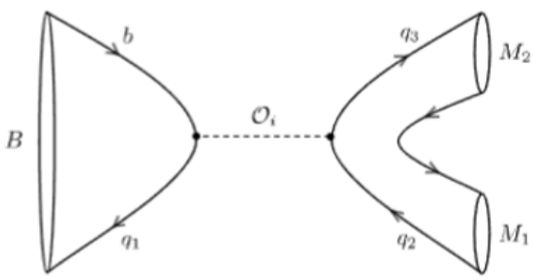
(b) CE: Connected Emission



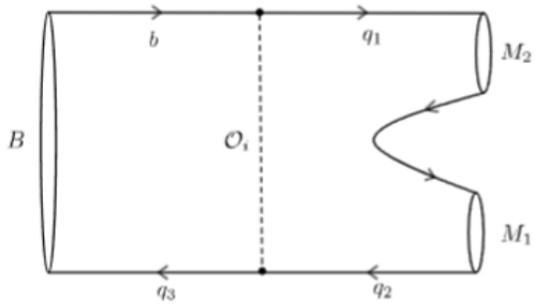
(e) DP: Disconnected Penguin



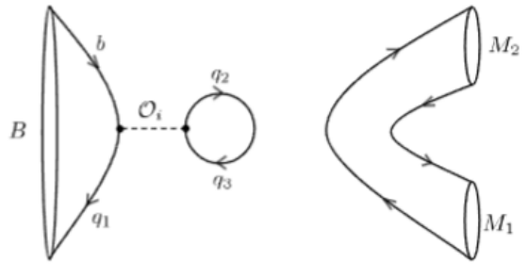
(f) CP: Connected Penguin



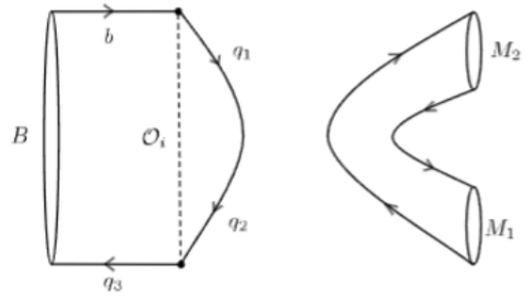
(c) DA: Disconnected Annihilation



(d) CA: Connected Annihilation



(g) DPA: Disconnected Penguin-Annihilation



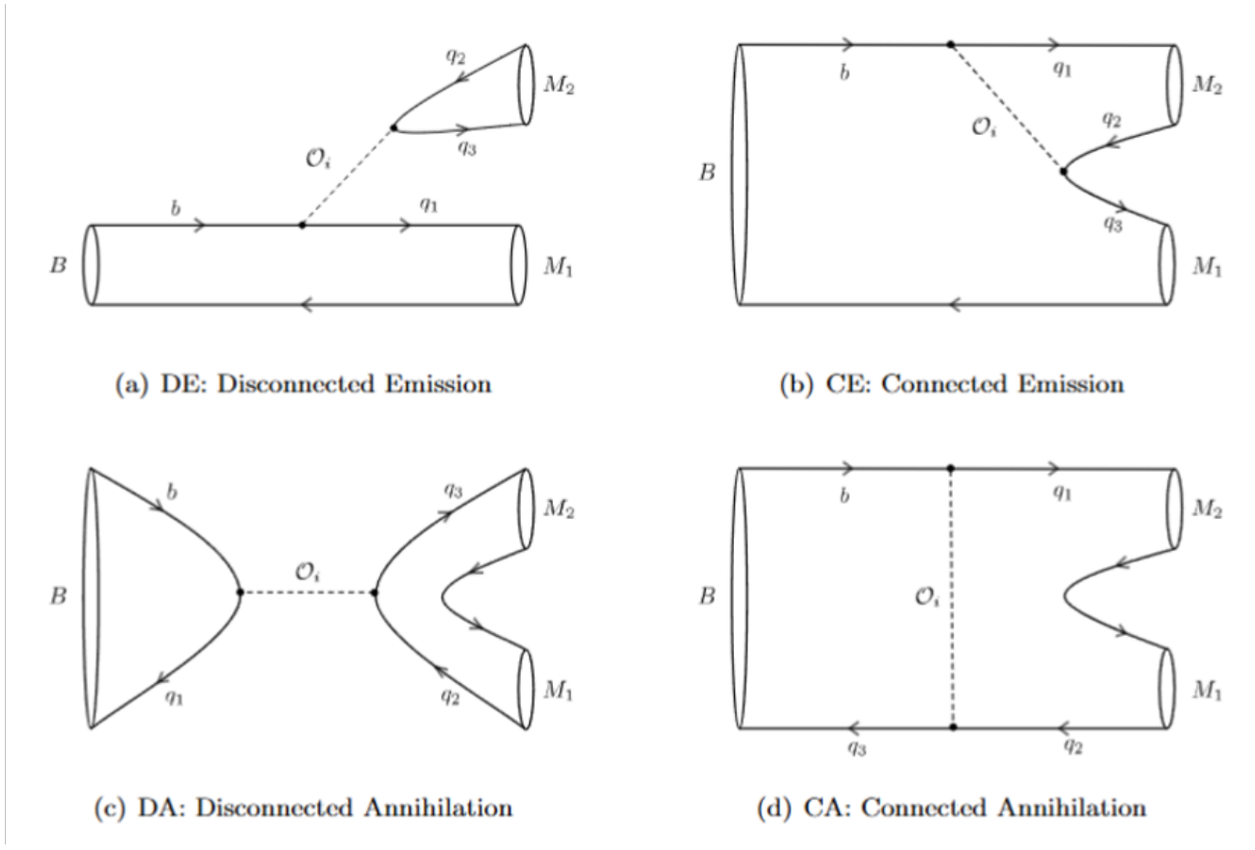
(h) CPA: Connected Penguin-Annihilation

- Eight topologies contribute to the $B_{(s)}$ to a pair of charmed mesons

Andrzej J. Buras and Luca Silvestrini, NPB 569 (2000) 3-52

S-wave $D\bar{D}$ interaction

- Different topologies of the matrix element



(a) DE: Disconnected Emission

(b) CE: Connected Emission

(c) DA: Disconnected Annihilation

(d) CA: Connected Annihilation

$$E^c = C_1(\mu)\left\langle O_1(\mu)\right\rangle^c_{\mathrm{CE}} + C_2(\mu)\left\langle O_2(\mu)\right\rangle^c_{\mathrm{DE}}$$

$$A^c = C_1(\mu)\left\langle \mathcal{O}_1(\mu)\right\rangle^c_{\mathrm{DA}} + C_2(\mu)\left\langle \mathcal{O}_2(\mu)\right\rangle^c_{\mathrm{CA}}$$

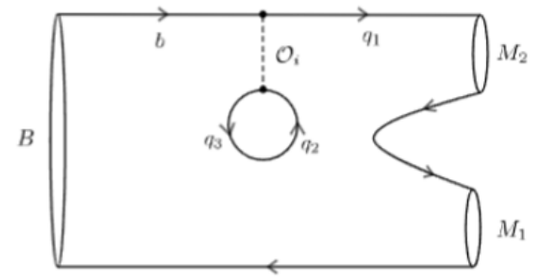
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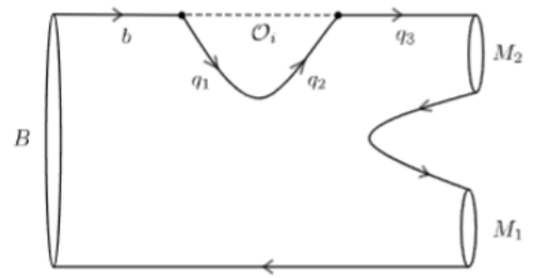
S-wave $D\bar{D}$ interaction

- Different topologies of the matrix element

$$P^q = C_1(\mu)\left\langle \mathcal{O}_1(\mu)\right\rangle^q_{\mathrm{DP}} + C_2(\mu)\left\langle \mathcal{O}_2(\mu)\right\rangle^q_{\mathrm{CP}}$$

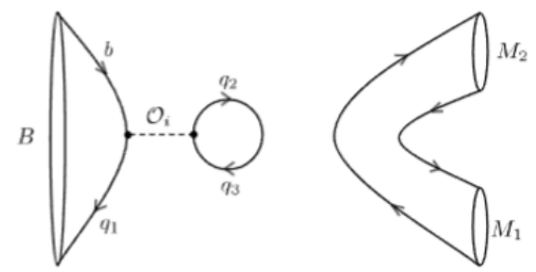


(e) DP: Disconnected Penguin

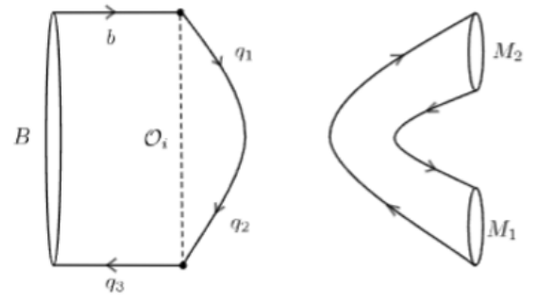


(f) CP: Connected Penguin

$$PA^q = C_1(\mu)\left\langle \mathcal{O}_1(\mu)\right\rangle^q_{\mathrm{DPA}} + C_2(\mu)\left\langle \mathcal{O}_2(\mu)\right\rangle^q_{\mathrm{CPA}}$$



(g) DPA: Disconnected Penguin-Annihilation



(h) CPA: Connected Penguin-Annihilation

- Eight topologies contribute to the $B_{(s)}$ to a pair of charmed mesons
- Different operators contribute to different topology diagrams
- For $B_{(s)} \to D\bar{D}$ process, only three combinations of those parameters appear

Andrzej J. Buras and Luca Silvestrini, NPB 569 (2000) 3-52

S-wave $D\bar{D}$ interaction

- Bare decay amplitudes

$$T \equiv E^c + P^c, \quad A \equiv A^c + PA^c, \quad P \equiv P^u$$

Channel	Amplitude	Channel	Amplitude
$B^- \to D^0 D^-$	$Y_{cd}T + Y_{ud}P$	$\bar{B}^0 \to D_s^+ D_s^-$	$Y_{cd}A$
$B^- \to D^0 D_s^-$	$Y_{cs}T + Y_{us}P$	$\bar{B}_s^0 \to D^+ D^-$	$Y_{cs}A$
$\bar{B}^0 \to D^+ D^-$	$Y_{cd}(T + A) + Y_{ud}P$	$\bar{B}_s^0 \to D_s^+ D_s^-$	$Y_{cs}(T + A) + Y_{us}P$
$\bar{B}^0 \to D^+ D_s^-$	$Y_{cs}T + Y_{us}P$	$\bar{B}_s^0 \to D^0 \bar{D}^0$	$Y_{cs}A$
$\bar{B}^0 \to D^0 \bar{D}^0$	$Y_{cd}A$	$\bar{B}_s^0 \to D_s^+ D^-$	$Y_{cd}T + Y_{ud}P$

- Under CPT symmetry, the direct

$$A_{\rm CP}(\bar{B} \to F) \equiv \frac{\Gamma(\bar{B} \to F) - \Gamma(B \to F)}{\Gamma(\bar{B} \to F) + \Gamma(B \to F)}$$

$$A_{CP}\left(\bar{B}^0 \to D^0 \bar{D}^0\right) = 0$$

$$A_{CP}\left(\bar{B}^0 \to D^+ D^-\right) = \frac{2R\delta_p \sin\gamma}{1 + R^2 - 2R\cos\gamma} + \cdots$$

$$A_{CP}\left(\bar{B}^0 \to D_s^+ D_s^-\right) = 0$$

$$Y_{qq'} \equiv V_{qb}^* V_{qq'}$$

$$r \equiv |A|/|T|, R \equiv |Y_{ud}/Y_{cd}|,$$

$$\gamma \equiv \arg(-Y_{ud}/Y_{cd}), |T| \simeq |P|$$

S-wave $D\bar{D}$ interaction

- LO $D\bar{D}$ interactions with respecting HQSS

$$\mathscr{L}_{4H} = \frac{1}{4}\mathrm{Tr}\left[\bar{H}^{(Q)a}H_b^{(Q)}\gamma_\mu\right]\mathrm{Tr}\left[H^{(\bar{Q})c}\bar{H}_d^{(\bar{Q})}\gamma^\mu\right]\left(F_A\delta_a^b\delta_c^d + F_A^\lambda\vec{\lambda}_a^b\cdot\vec{\lambda}_c^d\right)$$

$$+\frac{1}{4}\mathrm{Tr}\left[\bar{H}^{(Q)a}H_b^{(Q)}\gamma_\mu\gamma_5\right]\mathrm{Tr}\left[H^{(\bar{Q})c}\bar{H}_d^{(\bar{Q})}\gamma^\mu\gamma_5\right]\left(F_B\delta_a^b\delta_c^d + F_B^\lambda\vec{\lambda}_a^b\cdot\vec{\lambda}_c^d\right)$$

$$= v^2 P^{(Q)a\dagger}P_b^{(Q)}P^{(\bar{Q})c}P_d^{(\bar{Q})\dagger}(F_A\delta_a^b\delta_c^d + F_A^\lambda\vec{\lambda}_a^b\cdot\vec{\lambda}_c^d) + \cdots$$

M.B. Wise, PRD45(1992)R2188, M.T. AlFiky, et.al, PLB640(2006)238, T. Ji, et.al, PRD106(2022)094002

- The super field

$$H_a^{(Q)} = \frac{1+\not{v}}{2}(V_{a\mu}^{(Q)}\gamma^\mu - P_a^{(Q)}\gamma_5), \quad v\cdot V_a^{(Q)} = 0,$$

$$H_a^{(\bar{Q})} = (V_{a\mu}^{(\bar{Q})}\gamma^\mu - P_a^{(\bar{Q})}\gamma_5)\frac{1-\not{v}}{2}, \quad v\cdot V_a^{(\bar{Q})} = 0,$$

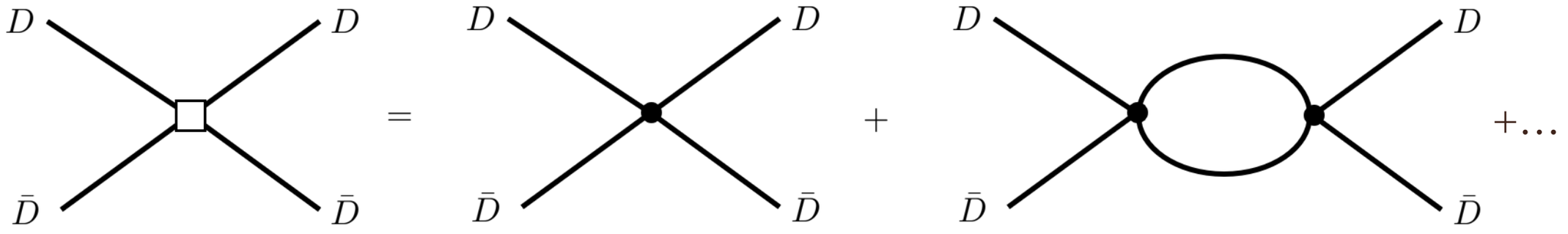
- S-wave $D\bar{D}$ contact potential

$$V_{\mathrm{CT}} = \begin{pmatrix} F_A+\frac{4}{3}F_A^\lambda & 2F_A^\lambda & 2F_A^\lambda \\ 2F_A^\lambda & F_A+\frac{4}{3}F_A^\lambda & 2F_A^\lambda \\ 2F_A^\lambda & 2F_A^\lambda & F_A+\frac{4}{3}F_A^\lambda \end{pmatrix}$$

- F_A and F_A^λ low-energy constants to be fitted from the $D\bar{D}$ scattering

S-wave $D\bar{D}$ interaction

- LSE of $D\bar{D}$ scattering



- $\gamma\gamma \to D\bar{D}$ bare production amplitude $\quad \mathscr{C}_i = (r\mathrm{U}, \mathrm{U}, \mathrm{U})$

$D^0\bar{D}^0, D^+D^-, D_s^+D_s^-$

- The $D\bar{D}$ two-body propagator

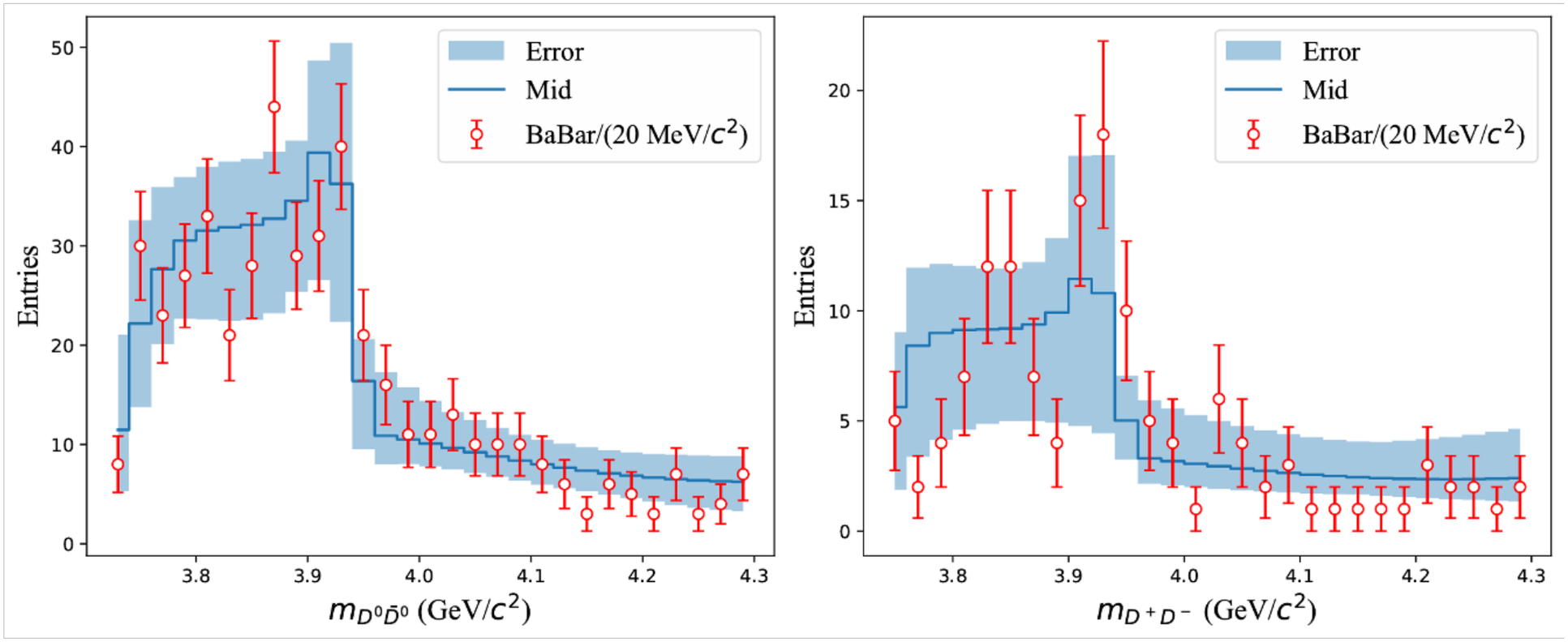
$$G_i^{\Lambda}(s) = -i\int \frac{d^4q}{(2\pi)^4} \frac{f^{\Lambda}(\mathbf{q}^2)}{(q^2 - m_i^2 + i\epsilon)[(P-q)^2 - m_i^2 + i\epsilon]}$$

$$= \frac{1}{4m_i^2}\left[\frac{\mu_i\Lambda}{(2\pi)^{3/2}} - \frac{\mu_i k_i}{2\pi} e^{-\frac{2k_i^2}{\Lambda^2}}\left(\mathrm{erfi}\left(\frac{\sqrt{2}k_i}{\Lambda}\right) - i\right)\right]$$

- The physical production amplitude $\quad \mathscr{M}\left(\gamma\gamma \to D^i\bar{D}^i\right) = \mathscr{C}_j G_j^{\Lambda}(s) T_{ji}(s) + \mathscr{C}_i$

S-wave $D\bar{D}$ interaction

- The fitted $D\bar{D}$ invariant mass



BaBar, PRD81(2010)092003

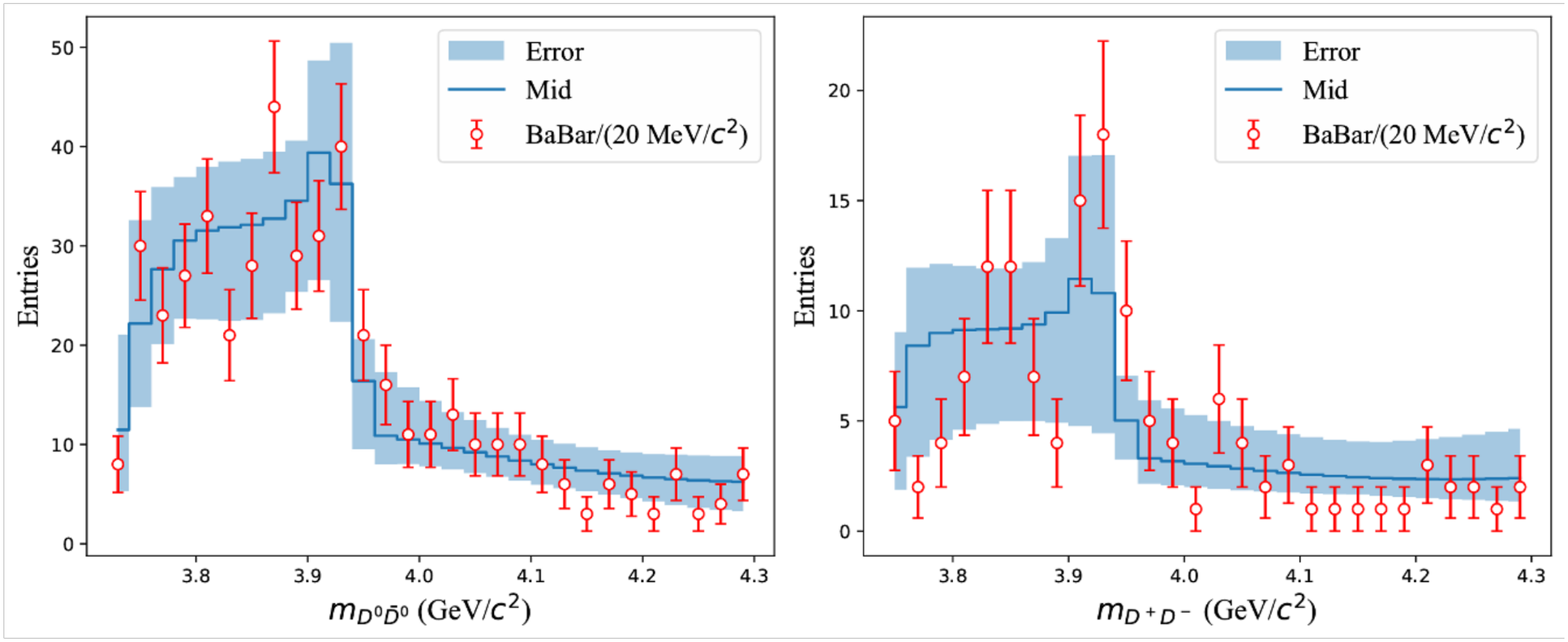
- The fitted parameters

Parameters	F_A (GeV^{-2})	F_A^λ (GeV^{-2})	Λ (GeV)	U (GeV^{-2})	r (GeV^0)	a (GeV^{-1})
Value	45.143 ± 39.667	-12.232 ± 7.646	1.942 ± 0.251	307.547 ± 33.112	0.823 ± 0.150	0.284 ± 0.036

- The errors are obtained by bootstrap
- $r < 1 \Rightarrow$ The magnetic force is typically much weaker than the electric force

S-wave $D\bar{D}$ interaction

- The fitted $D\bar{D}$ invariant mass



BaBar, PRD81(2010)092003

- The poles

RSs	$+++$	$-++$	$--+$	$---$
E	3.65	$3.76 \pm 0.003i$	3.73 $3.94 \pm 0.003i$	3.73

- The $3.94 \pm 0.003i$ pole has physical impact. Ji, Sci.Bull.68(2023)688

S-wave $D\bar{D}$ interaction

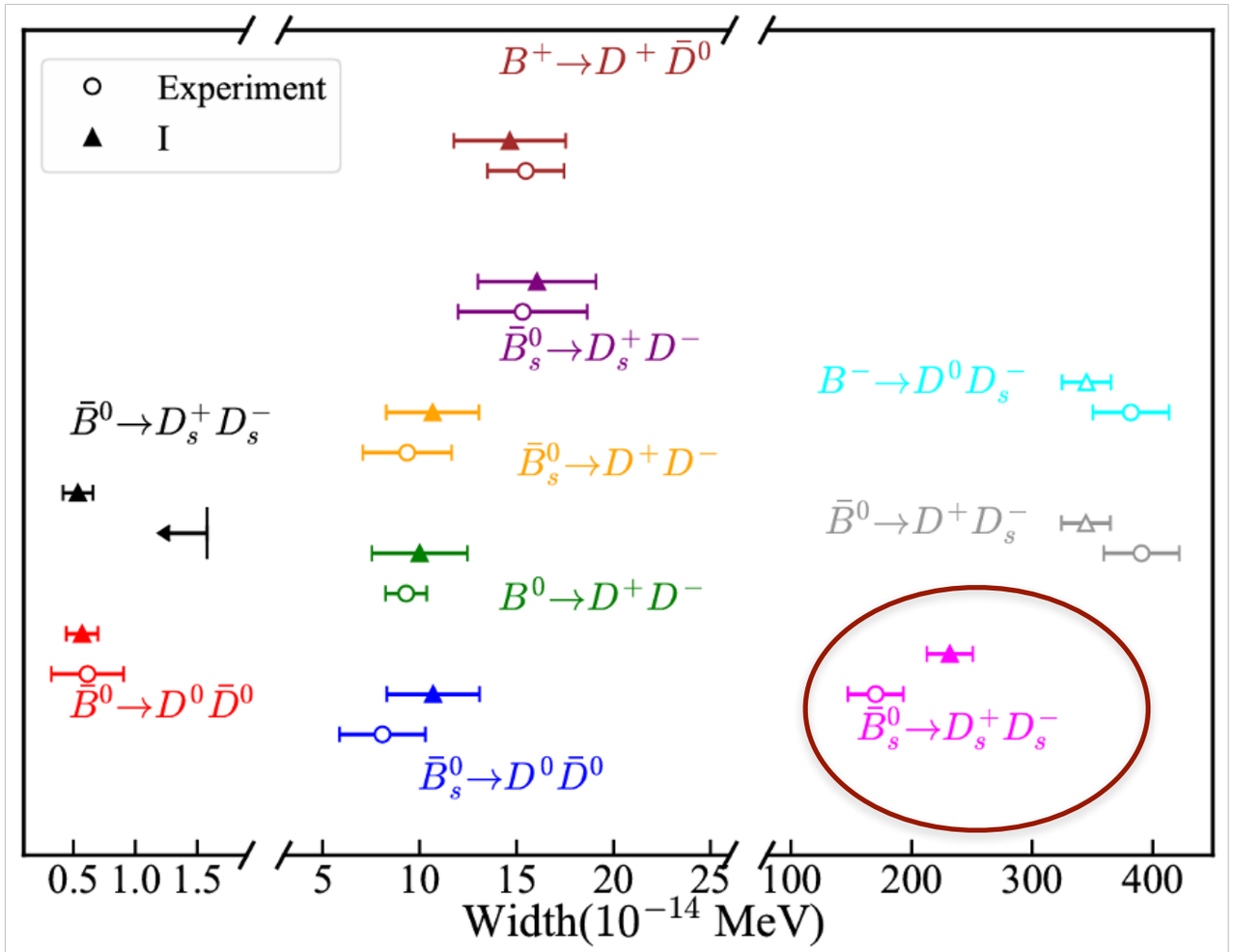
- Fit schemes

  - Scheme I: without FSI, fit to widths
  - Scheme II: with FSI, fit to widths
  - Scheme III, with FSI, fit to widths

  and A_{CP} of the channels

$$B^- \to D^- D^0 \quad B^- \to D^0 D_s^-$$

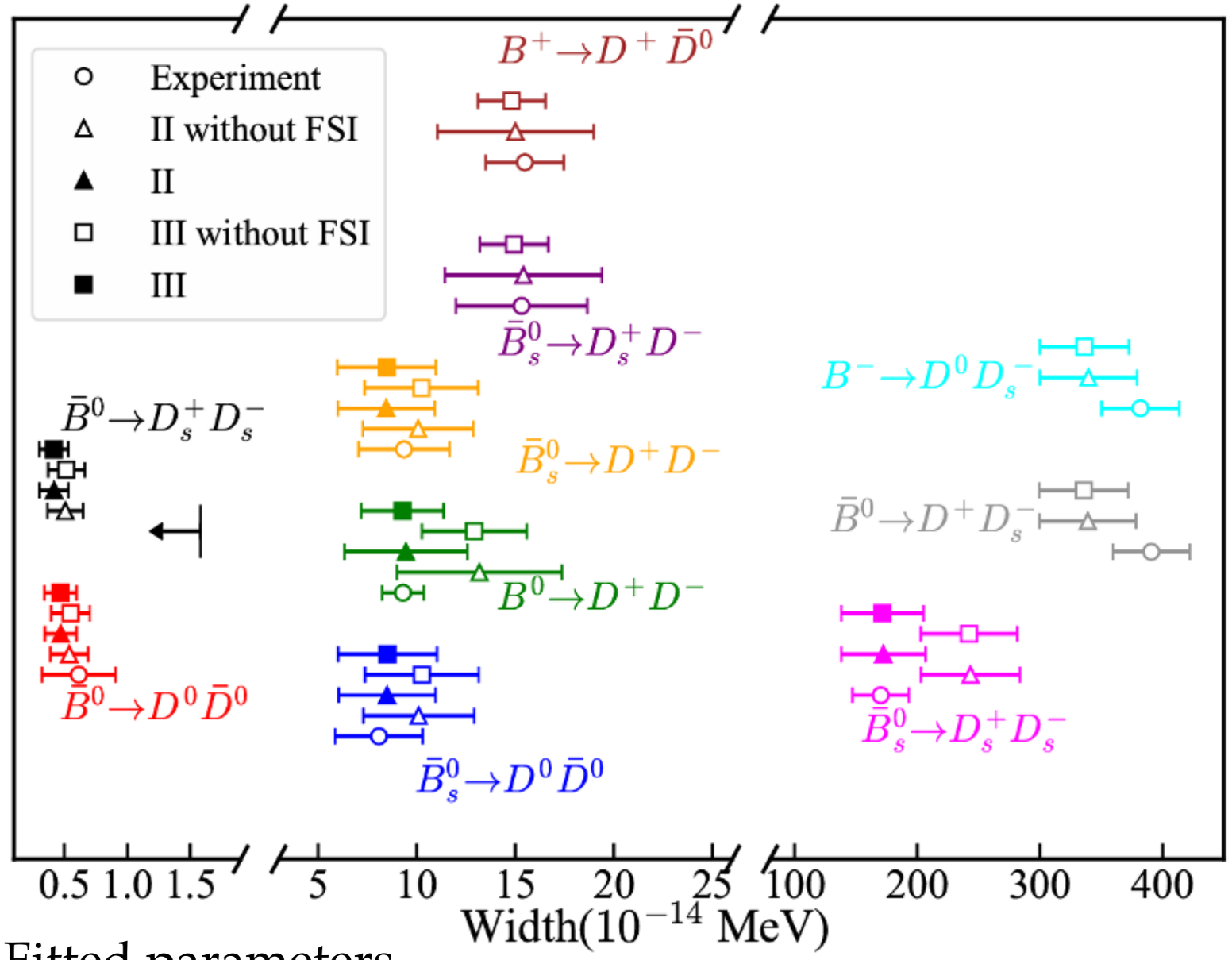
$$\bar{B}^0 \to D^+ D_s^- \quad \bar{B}_s^0 \to D_s^+ D^-$$



- Scheme I: quark-level calculation cannot describe the $\bar{B}_s^0 \to D_s^+ D_s^-$ channel

S-wave $D\bar{D}$ interaction

- widths



- Errors in Scheme III is smaller than those of Scheme II
- The reduced chi-square becomes smaller
- FSI increase widths

- Fitted parameters

Schemes	$\|T\|$ [MeV]	$\|A\|$ [MeV]	$\|P\|$ [MeV]	δ_a[rad]	δ_p[rad]	χ^2/d. o. f
I	$(2.78 \pm 0.07) \times 10^{-2}$	$(4.88 \pm 0.54) \times 10^{-3}$	$(3.86 \pm 1.05) \times 10^{-2}$	3.25 ± 0.08	$(0.52 \pm 1.54) \times 10^{-1}$	2.30
II	$(2.76 \pm 0.16) \times 10^{-2}$	$(4.74 \pm 0.66) \times 10^{-3}$	$(4.06 \pm 0.99) \times 10^{-2}$	2.59 ± 0.37	$(0.23 \pm 1.91) \times 10^{-1}$	1.15
III	$(2.74 \pm 0.14) \times 10^{-2}$	$(4.78 \pm 0.67) \times 10^{-3}$	$(3.69 \pm 0.99) \times 10^{-2}$	2.55 ± 0.38	$(1.60 \pm 2.27) \times 10^{-2}$	1.00

S-wave $D\bar{D}$ interaction

- The reasonability of the parameters

Schemes	$\lvert T\rvert$ [MeV]	$\lvert A\rvert$ [MeV]	$\lvert P\rvert$ [MeV]	δ_a[rad]	δ_p[rad]	χ^2/d. o. f
I	$(2.78 \pm 0.07) \times 10^{-2}$	$(4.88 \pm 0.54) \times 10^{-3}$	$(3.86 \pm 1.05) \times 10^{-2}$	3.25 ± 0.08	$(0.52 \pm 1.54) \times 10^{-1}$	2.30
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- The emission diagram is parameterized as

$$E \sim \frac{G_F}{\sqrt{2}} a_2 F^{B\to D}(m_D^2) f_D (m_B^2 - m_D^2) \sim 0.03 \text{ MeV} \quad \text{with} \quad F^{B\to D}(m_D^2) = \frac{F^{B\to D}(0)}{1 - \alpha_1 \frac{m_D^2}{m_{\text{pole}}^2} + \alpha_2 \frac{m_D^4}{m_{\text{pole}}^4}}$$

$$\alpha_1 = 1.71, \ \ \alpha_2 = 0.52, \ \ m_{\text{pole}} = m_B \qquad a_{1,2} = C_{1,2} + \frac{C_{2,1}}{N_c} = 0.090, 1.036$$

S.H. Zhou, et.al, PRD92(2015)094016, S.H. Zhou, et.al, PRD110(2024)056001

- The annihilation diagram cannot be calculated in quark-level, but estimated as

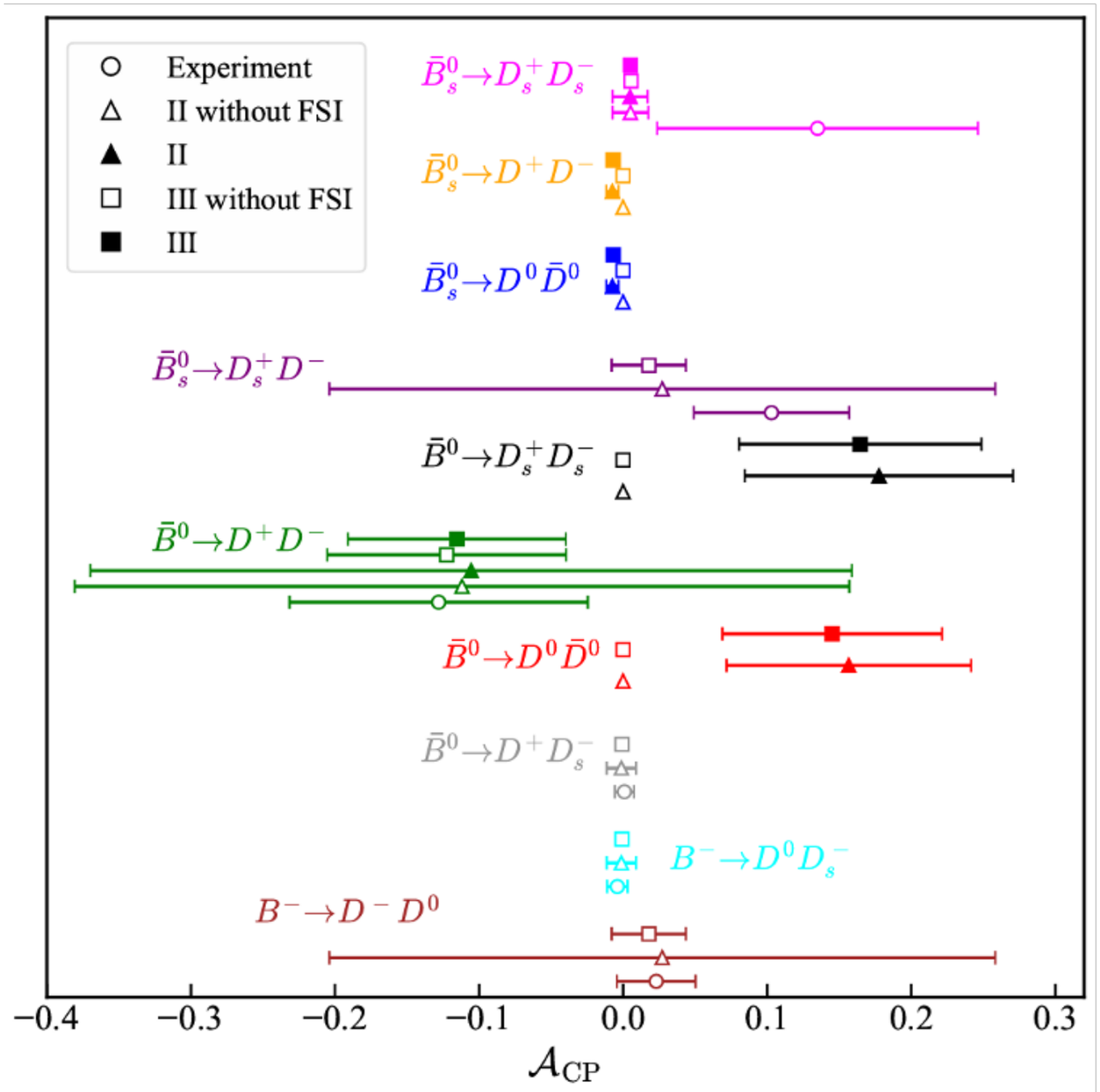
$$A \sim \frac{G_F}{\sqrt{2}} a_1 f_B f_{\chi_{c0}} g_{\chi_{c0} D\bar{D}} \frac{m_B^2 m_{\chi_{c0}}}{m_B^2 - m_{\chi_{c0}}^2} \sim 0.002 \text{ MeV}$$

Y.F. Sheng, et.al, PRD84(2011)074019, P. Colangelo, et.al, PRD69(2004)054023, PLB542(2002)71,

J. Ou-Yang, et.al, PRD112(2025)056005

S-wave $D\bar{D}$ interaction

- A_{CP}



- A_{CP} without FSI

$$A_{CP}\left(\bar{B}^0 \to D^0\bar{D}^0\right) = 0$$

$$A_{CP}\left(\bar{B}^0 \to D^+D^-\right) = \frac{2R\delta_p \sin\gamma}{1+R^2-2R\cos\gamma} +$$

$$A_{CP}\left(\bar{B}^0 \to D_s^+D_s^-\right) = 0$$

- A_{CP} with FSI

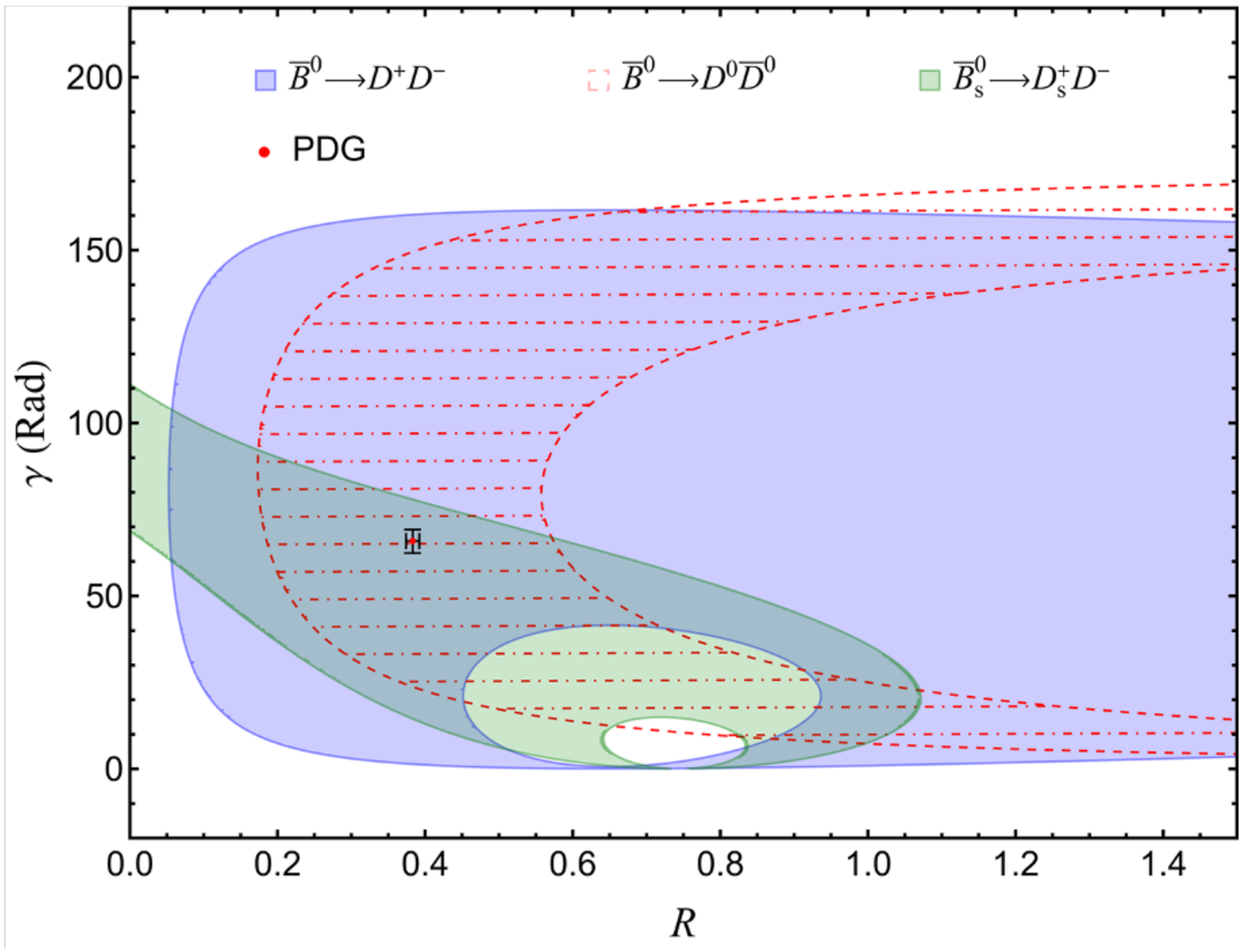
$$A_{\rm CP}(\bar{B}^0 \to D^0\bar{D}^0) \simeq f(\delta_p + \Delta^1_{12} + \Delta^1_{32} - \Delta^1_{22}),$$
$$A_{\rm CP}(\bar{B}^0 \to D^+D^-) \simeq f(\delta_p + \Delta^2_{12} + \Delta^2_{32}),$$
$$A_{\rm CP}(\bar{B}^0 \to D_s^+D_s^-) \simeq f(\delta_p + \Delta^3_{12} + \Delta^3_{32} - \Delta^3_{22}),$$

$$f \equiv \frac{2R\sin\gamma}{1+R^2-2R\cos\gamma}$$

- FSI significantly enhance the A_{CP} of the channels with only annihilation diagrams
- Predict A_{CP} of other channels
- Errors in III are smaller than that in II

S-wave $D\bar{D}$ interaction

- Precise measurements of A_{CP} can be used to determine the parameters in SM



- Filled band: Exp.
- Dashed band: Theory
- FSI makes channels with only annihilation diagrams can also be used for determine the parameters in SM

S-wave $D\bar{D}$ interaction

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Annihilation, Rescattering, and *CP* Asymmetries in *B* Meson Decays

Boris Blok and Michael Gronau
Department of Physics, Technion-Israel Institute of Technology, Haifa 32000, Israel

Jonathan L. Rosner
Enrico Fermi Institute and Department of Physics, University of Chicago, Chicago, Illinois 60637
(Received 28 January 1997)

TABLE I. $B \to PP$ amplitudes involving only suppressed graphs.

CKM factor	Order in λ	Process	Suppressed amplitude
$V_{cb}^* V_{ud}$	λ^2	$B^0 \to K^+ D_s^-$	$\overline{E}$
$V_{cb}^* V_{cs}$	λ^2	$B_s \to D^+ D^-$	$\hat{E}$
		$B_s \to D^0 \overline{D}^0$	$-\hat{E}$
$V_{cb}^* V_{us}$	λ^3	$B_s \to \pi^+ D^-$	$\lambda\overline{E}$
		$B_s \to \pi^0 \overline{D}^0$	$-\lambda\overline{E}/\sqrt{2}$
$V_{cb}^* V_{cd}$	λ^3	$B^0 \to D^0 \overline{D}^0$	$-\hat{E}'$
		$B^0 \to D_s^+ D_s^-$	$\hat{E}'$
$V_{ub}^* V_{ud}$	λ^3	$B^0 \to K^+ K^-$	$-E$
$V_{ub}^* V_{cs}$	λ^3	$B^+ \to K^0 D^+$	$\tilde{A}$
		$B_s \to \pi^- D^+$	$-\tilde{E}$
		$B_s \to \pi^0 D^0$	$\tilde{E}/\sqrt{2}$
$V_{ub}^* V_{us}$	λ^{4a}	$B_s \to \pi^+ \pi^-$	$-E'$
		$B_s \to \pi^0 \pi^0$	$E'/\sqrt{2}$
$V_{ub}^* V_{cd}$	λ^4	$B^+ \to \overline{K}^0 D_s^+$	$-\lambda\tilde{A}$
		$B^0 \to K^- D_s^+$	$\lambda\tilde{E}$

[a]Penguin annihilation (also of this order) ignored.

Observation of a *CP*-violating asymmetry based on the anticipated large final-state phases is more demanding, but within the capabilities of planned high-intensity sources of *B* mesons. The decays $B^0 \to D^0\overline{D}^0$ and $B^0 \to D_s^+ D_s^-$ are expected to be fed by rescattering from both tree and penguin contributions in $B^0 \to D^+ D^-$, with the branching ratio for the latter process about λ^2 that for the observed process $B^0 \to D^- D_s^+$, or $(1/20)\,(0.7\%)$. Rescattering may cost another factor of λ^2 in rate, leading to a branching ratio $B(B^0 \to D^0\overline{D}^0) \simeq 2 \times 10^{-5}$. As we have estimated, a rate asymmetry of order 10% could arise between this process and its charge conjugate. The situation in $B^0 \to K^+ K^-$, where an asymmetry at a similar level is expected, is less favorable. This process is fed by rescattering from $B^0 \to \pi^+ \pi^-$, whose rate is likely to be about 10^{-5} [22], so $B(B^0 \to K^+ K^-)$ should be below 10^{-6}. Finally rates at a similar level are expected for $B_s \to \pi^+ \pi^-$ and $B_s \to \pi^0 \pi^0$, which are fed by rescattering from $B_s \to K^+ K^- (K^0 \overline{K}^0)$ whose branching ratios are probably about 10^{-5}, similar to that of $B^0 \to K^+ \pi^-$ [22]. As we noted, the asymmetry in $B_s \to \pi\pi$ may be very large.

S-wave $D\bar{D}$ interaction

PHYSICAL REVIEW D **81,** 034006 (2010)

Decays of *B* meson to two charmed mesons

Run-Hui Li,[1,2] Cai-Dian Lü,[1,4] A. I. Sanda,[3] and Xiao-Xia Wang[1]

[1]*Institute of High Energy Physics, P.O. Box 918(4), Beijing 100049, People's Republic of China*
[2]*School of Physics, Shandong University, Jinan 250100, People's Republic of China*
[3]*Faculty of Technology, Kanagawa University, Yokohama, Kanagawa 221, Japan*
[4]*Theoretical Physics Center for Science Facilities, Beijing 100049, People's Republic of China*
(Received 12 October 2009; published 4 February 2010)

The factorization theorem in decays of $B_{(s)}$ mesons to two charmed mesons (both pseudoscalar and vector) can be proved in the leading order in m_D/m_B and $\Lambda_{\rm QCD}/m_D$ expansion. Working in the perturbative QCD approach, we find that the factorizable emission diagrams are dominant. Most of branching ratios we compute agree with the experimental data well, which means that the factorization theorem seems to be reliable in predicting branching ratios for these decays. In the decays of a B meson to two vector charmed mesons, the transverse polarization states contribute 40%–50% both in the processes with an external W emission and in the pure annihilation decays. This is in agreement with the present experimental data. We also calculate the CP asymmetry parameters. The results show that the direct CP asymmetries are very small. Thus observation of any large direct CP asymmetry will be a signal for new physics. The mixing induced CP asymmetry in the neutral modes is large. This is also in agreement with the current experimental measurements. They can give a cross-check of the $\sin 2\beta$ measurement from other channels.

10^{-3}

S-wave $D\bar{D}$ interaction

TABLE III. CP averaging branching ratios (unit: 10^{-4}) and the CP asymmetries for $B \to VP$ (characterized by $B \to V$ form factor) decays.

	Channels	$\mathcal{BR}$ Experiments	$\mathcal{BR}$ This work	$A_{CP}^{\rm dir}(\%)$ This work
1	$B^- \to D^{*0}D^-$	$6.3 \pm 1.4 \pm 1.0$ [*BABAR*] [23]	$4.8^{+3.4+1.1+0.1}_{-2.3-1.4-0.3}$	$-0.5^{+0.1+0.0}_{-0.2-0.3}$
2	$B^- \to D^{*0}D_s^-$	84 ± 17	$119^{+94+27+2}_{-56-34-7}$	$\sim -10^{-3}$
3	$\bar{B}^0 \to D^{*+}D^-$	8.8 ± 1.6	$4.6^{+3.5+0.9+0.1}_{-2.1-1.1-0.3}$	$-0.6^{+0.0+0.1}_{-0.1-0.2}$
4	$\bar{B}^0 \to D^{*+}D_s^-$	83 ± 11	$112^{+86+26+2}_{-53-32-6}$	...
5	$\bar{B}_s^0 \to D_s^{*+}D^-$		$2.7^{+1.7+0.9+0.1}_{-1.1-0.9-0.1}$	$-0.4^{+0.0+0.1}_{-0.0-0.1}$
6	$\bar{B}_s^0 \to D_s^{*+}D_s^-$		$70^{+44+19+1}_{-31-21-4}$	...
7	$\bar{B}^0 \to D^{*0}\bar{D}^0$		$0.21^{+0.06+0.03+0.00}_{-0.08-0.05-0.01}$	$-1.2^{+0.3+0.7+0.1}_{-1.2-1.4-0.1}$
8	$\bar{B}^0 \to D_s^{*+}D_s^-$	<1.3 [*BABAR*] [24]	$0.25^{+0.08+0.06+0.01}_{-0.08-0.08-0.01}$	$\sim -10^{-3}$
9	$\bar{B}_s^0 \to D^{*0}\bar{D}^0$		$4.3^{+1.3+0.8+0.1}_{-1.3-1.1-0.2}$	$0.2^{+0.0+0.0}_{-0.1-0.1}$
10	$10\bar{B}_s^0 \to D^{*+}D^-$		$4.4^{+1.4+0.9+0.1}_{-1.3-1.2-0.2}$	...

10^{-3}

Take-home messages

- Hadron-hadron interaction is not only useful for the property of exotic hadrons, but also useful for ACP in B physics

- What do we expect from BelleII and LHCb?

$A_{\mathrm{CP}}(\bar{B}^0 \to D^0\bar{D}^0), A_{\mathrm{CP}}(\bar{B}^0 \to D_s^+ D_s^-)$ $A_{\mathrm{CP}}(\bar{B}_s^0 \to D^0\bar{D}^0), A_{\mathrm{CP}}(\bar{B}_s^0 \to D^+ D^-)$

Summary and Outlook

- Summary

  - Pomeron exchanged potential makes ${}^5S_2$ $p\Omega$ bound state more compact
  - Provide a way to estimate HQSV effect for $Z_c^{(\prime)}$ and $Z_b^{(\prime)}$
  - Hadron-hadron interaction is important for both exotic hadrons and B physics

Thank you very much for your attention!