

# Six lectures on the Standard Theory of Elementary Particle Physics

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Beijing, 12 October 2026*



Lecture 1  
Gauge Symmetry

# The Plan of my Lectures

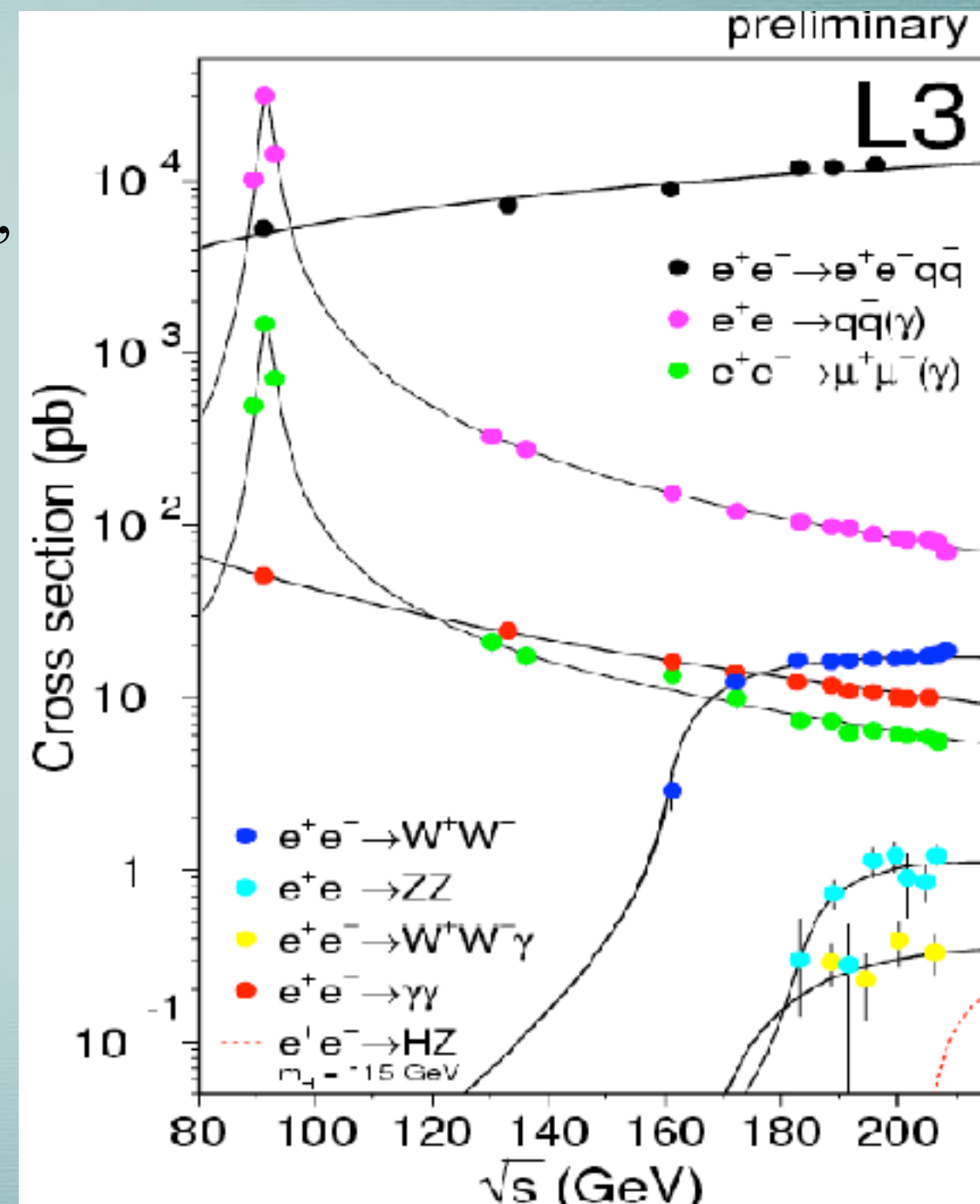
- 1. Gauge Symmetry: QED,  $g=2$ , Yang-Mills, Asymptotics.
- 2. Electroweak Unification 1; breaking the Symmetry; the Higgs-Brout-Englert boson 3.
- 3. Interactions and symmetries of subnuclear particles; Quarks: spectroscopy and fundamental interactions.
- 4. Quarks and Partons, Color and Gluons, Hadrons in the Constituent Quark model.
- 5. Electroweak Unification 2: GIM Mechanism, Charm, Beauty and CP violation.
- 6. The Higgs Boson search. The Standard Model: What's missing? What's next ?

# 1. Spinor Electro Dynamics

- Originally a theory of electrons and photons;
- $\mu$  and  $\tau$ -particles behave the same;
- $e$ ,  $\mu$  and  $\tau$  numbers separately conserved;
- $e^+ e^-$  annihilation gives access to muons,  $\tau$ , Intermediate Vector Bosons and the Higgs particle.
- Spinor QED is determined by few general principles:
  1. Lorentz invariance
  2. Gauge invariance
- matter particles are fermions with spin  $1/2$  (Dirac particles)
- is renormalizable
- The prototype of a fundamental field theory and an extraordinary success.

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# QED and local gauge invariance (abelian)

- Electric charge conservation follows from invariance of the lagrangian under *global phase transformations*:  $\psi(x) \rightarrow \psi'(x) = e^{i\phi} \psi(x)$
- We want to promote global to *local phase (gauge) transformations*

$$\psi(x) \rightarrow \psi'(x) = e^{i\phi(x)} \psi(x); \quad A_\mu(x) \rightarrow A'_\mu(x) = A_\mu(x) + \partial_\mu \phi(x)$$

- The minimal substitution produces a derivative field which transforms exactly like  $\psi$ :

$$\partial_\mu \psi \rightarrow D_\mu \psi = (\partial_\mu - iA_\mu) \psi$$

$$D'_\mu \psi'(x) = (\partial_\mu - iA'_\mu) \psi' = (\partial_\mu - iA_\mu - i\partial_\mu \phi) e^{i\phi} \psi = e^{i\phi} (\partial_\mu - iA_\mu) \psi = e^{i\phi} D_\mu \psi$$

- $A_\mu$  transforms in a complicated way, but the Maxwell's tensor is invariant:

$$F_{\mu\nu} = \partial_\nu A_\mu - \partial_\mu A_\nu; \quad F'_{\mu\nu} = F_{\mu\nu}$$

- Conclusion (W. Pauli): if  $L_0(\psi, \partial_\mu \psi)$  is invariant under global phase transformations, the Lagrangian:

$$L_{QED} = L_0(\psi, D_\mu \psi) - \frac{1}{4e^2} F_{\mu\nu} F^{\mu\nu}$$

- is invariant under local transformations;
- rescaling  $A \rightarrow -eA$ , we see that “e” gives the strength of the interaction;
- Symmetry determines the form of the photon-electron interaction (ex.  $g=2!!$ )
- No photon mass is allowed!

# QED histories

- Using the Dirac's Lagrangian, we get:

$$L_0(\psi, D_\mu \psi) = \bar{\psi}_e (i\partial_\mu \gamma^\mu - m) \psi_e - e \bar{\psi}_e A_\mu \gamma^\mu \psi_e + (\psi_e \rightarrow \psi_\mu) + (\psi_e \rightarrow \psi_\tau).$$

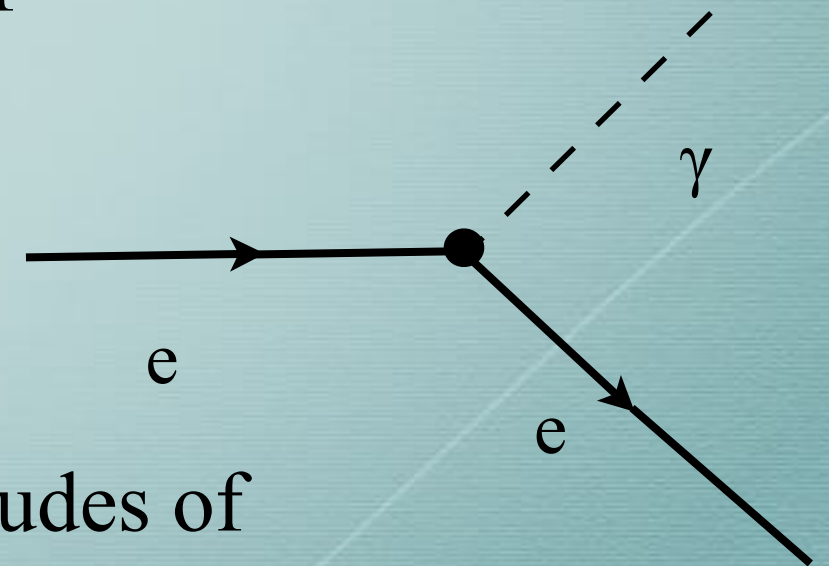
- that is:

$$L_I = -e A_\lambda [\bar{\psi}_e \gamma^\lambda \psi_e + \bar{\psi}_\mu \gamma^\lambda \psi_\mu + \bar{\psi}_\tau \gamma^\lambda \psi_\tau]$$

- QED histories:

- amplitudes representing the propagation of photons and electrons/mu/tau
- vertices
- lepton mass allowed by gauge invariance (because vector current!)

- The total amplitude is the sum of the amplitudes of the different stories with the same initial and final configurations.



# Gauge invariance and the photon propagator

- Gauge invariance means that not all components of  $A_\mu$  are dynamical variables
- Thus we have to require a supplementary condition (gauge condition) to be obeyed by the field configurations over which we do the path integral.
- We can enforce the gauge condition by adding to the Lagrangian a Lagrange multiplier, to get:

$$L = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{\lambda}{2}(\partial_\mu A^\mu)^2; \quad F^{\mu\nu} = \partial^\nu A^\mu - \partial^\mu A^\nu;$$
$$S = \int d^4x L = \int d^4x \frac{1}{2} (A^\mu K_{\mu\nu} A^\nu);$$
$$K_{\mu\nu} = g_{\mu\nu}(\partial)^2 - (1 - \lambda)\partial_\mu\partial_\nu$$

- after Fourier transforming and taking the inverse, we find:

$$K_{\mu\nu}^{-1}(q) = \frac{1}{q^2 + i\epsilon} \left( -g_{\mu\nu} + \left(1 - \frac{1}{\lambda}\right) \frac{q_\mu q_\nu}{q^2} \right)$$

# Gauge invariance and the photon propagator

$$K_{\mu\nu}^{-1}(q) = \frac{1}{q^2 + i\varepsilon} \left( -g_{\mu\nu} + \left(1 - \frac{1}{\lambda}\right) \frac{q_\mu q_\nu}{q^2} \right)$$

- the  $\lambda$  dependence of the propagator disappears when computing physical amplitudes, since photon propagator indices are always contracted with currents which are conserved:

$$q_\mu \langle J^\mu \rangle = 0$$

- the disappearance of the  $\lambda$  dependence happens only *after we add up all Feynman diagrams related to each other by gauge invariance* and it thus provides a powerful check of your calculation!
- useful gauges:
  - $\lambda = 1$ : Feynman gauge
  - $\lambda = \infty$ : Landau gauge (the propagator is explicitly transverse,

$$q^\mu K_{\mu\nu}^{-1}(q) = 0$$

# Non minimal couplings ?

- Using the Dirac's Lagrangian, we get:

$$L_0(\psi, D_\mu \psi) = \bar{\psi}(i\partial_\mu \gamma^\mu - m)\psi - eA_\mu \bar{\psi} \gamma^\mu \psi, \quad (\psi = \psi_e, \psi_\mu, \psi_\tau)$$

- From the Dirac Lagrangian one finds the electron magnetic moment

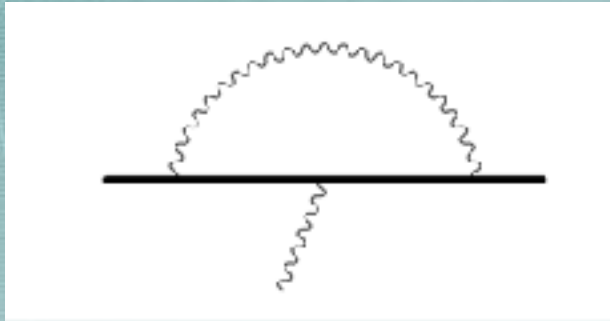
$$\mu = \frac{-e}{2m} \sigma = \frac{-e}{2m} \frac{\sigma}{2} 2 = \frac{-e}{2m} 2\mathbf{S} = \frac{-e}{2m} g\mathbf{S} \rightarrow g = 2$$

- we could add a non-minimal couplings formed with  $F_{\mu\nu}$  (Pauli):

$$\frac{\kappa}{4m} F^{\mu\nu} \bar{\psi} \sigma_{\mu\nu} \psi$$

- the magnetic moment would be anomalous:  $g = 2 + \kappa$ , with respect to electrons and muons, which have  $g-2$  very close to 0 (next slide).
- The Pauli term makes the high-energy behaviour of scattering amplitudes more singular, and it leads to a non-renormalizable theory;
- For particles which are ***not elementary***, e.g proton and neutron, non-minimal terms are indeed required by experiments.

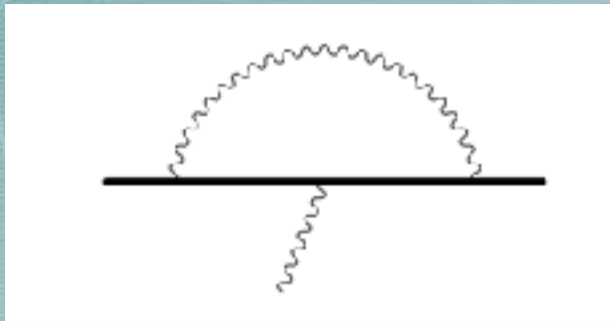
# Precision Frontier: the muon g-2



2<sup>nd</sup> order  
(J. Schwinger, 1947)  
$$g-2 = \frac{\alpha}{2\pi} = 0.00116\dots$$

Histories: photons and  
electrons ... (or muons..or  
hadrons)

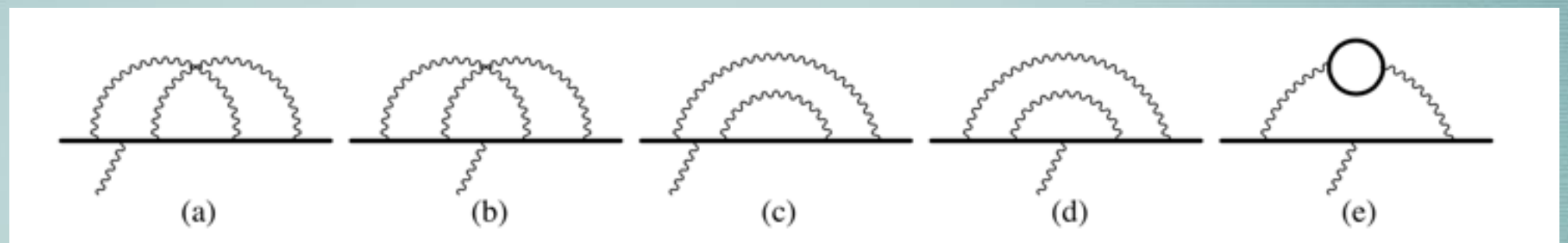
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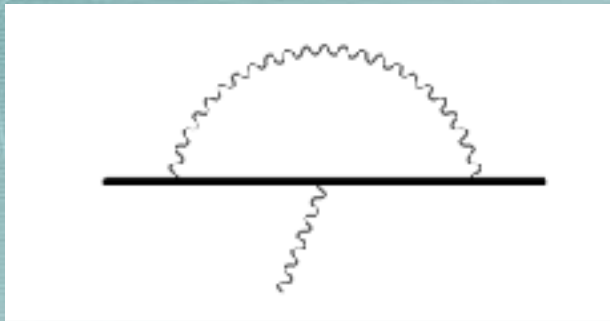
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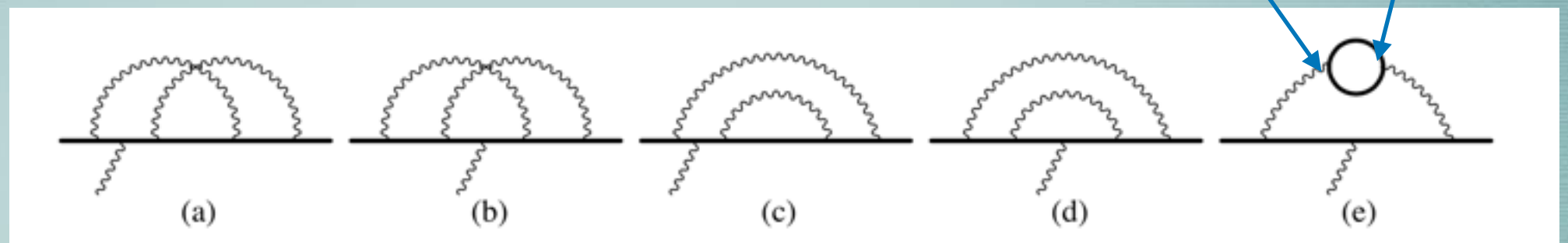
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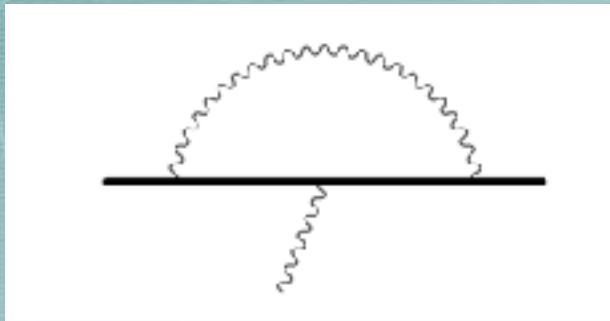
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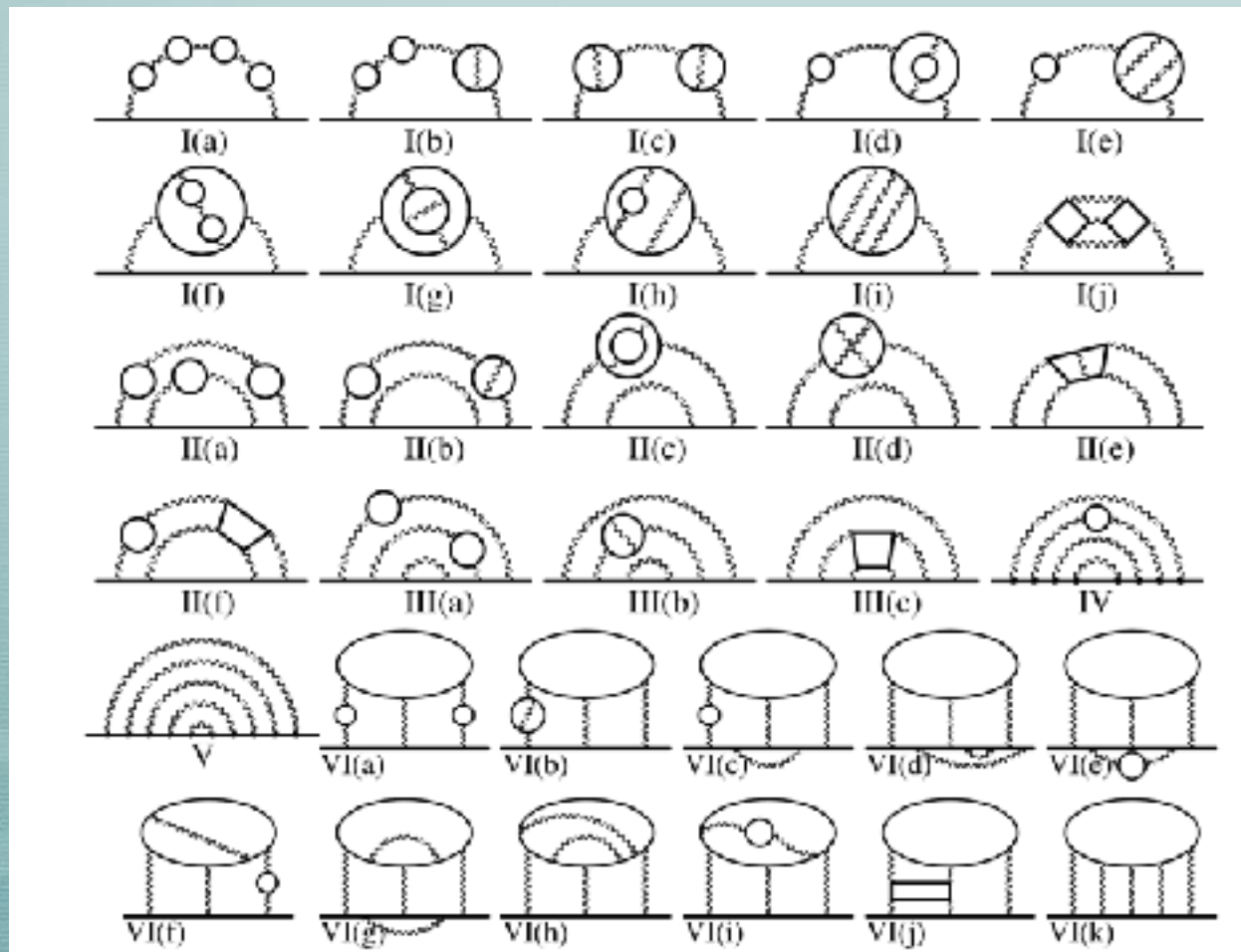
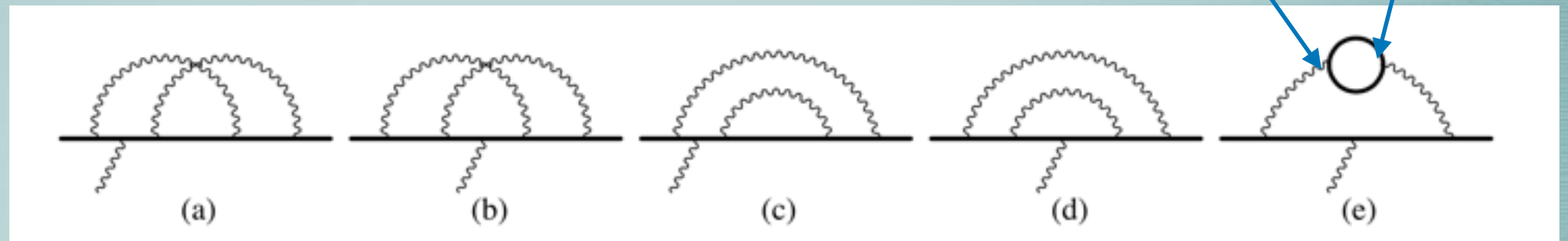
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... up to 10<sup>th</sup> order

# Precision Frontier: the muon g-2 anomaly (cont'd)

$$a_\mu = \frac{\alpha}{2\pi} = 0.00116, \text{ J. Schwinger, 1947}$$

- Fermilab and BNL most recent experimental result (2020):

- $g_\mu^{expt} - 2 = a_\mu^{expt} = 11659208.9 \cdot 10^{-10}$

- define:  $\Delta a_\mu^{expt} = (11659208.9 - 11659000.0) = 208.9 \pm 2.2$

- in the same convention, QED corrections are:  $\Delta a_\mu^{QED} = -528.2$

- ElectroWeak corrections:  $\Delta a_\mu^{EW} = 15.4$

ST predictions:

T. Aoyama *et al.* Phys. Rept. 887 (2020)

- Hadronic corrections (**HVP**, HLbL):

Hadron Vacuum Polarization- estimated from data on  $e^+e^- \rightarrow \text{hadrons}$   
Subleading: Hadron Light-by Light scattering- resonance insertion in loop

- $\Delta a_\mu^{HVP} = 684.5, \quad \Delta a_\mu^{HLbL} = 9.2$

HVP, from experim. data. T. Aoyama *et al.*

- In total:  $\Delta a_\mu^{ST} = 181 \pm 4.3: 5\sigma$  discrepancy

- **New evaluation of HVP** in a recent Lattice QCD contribution,

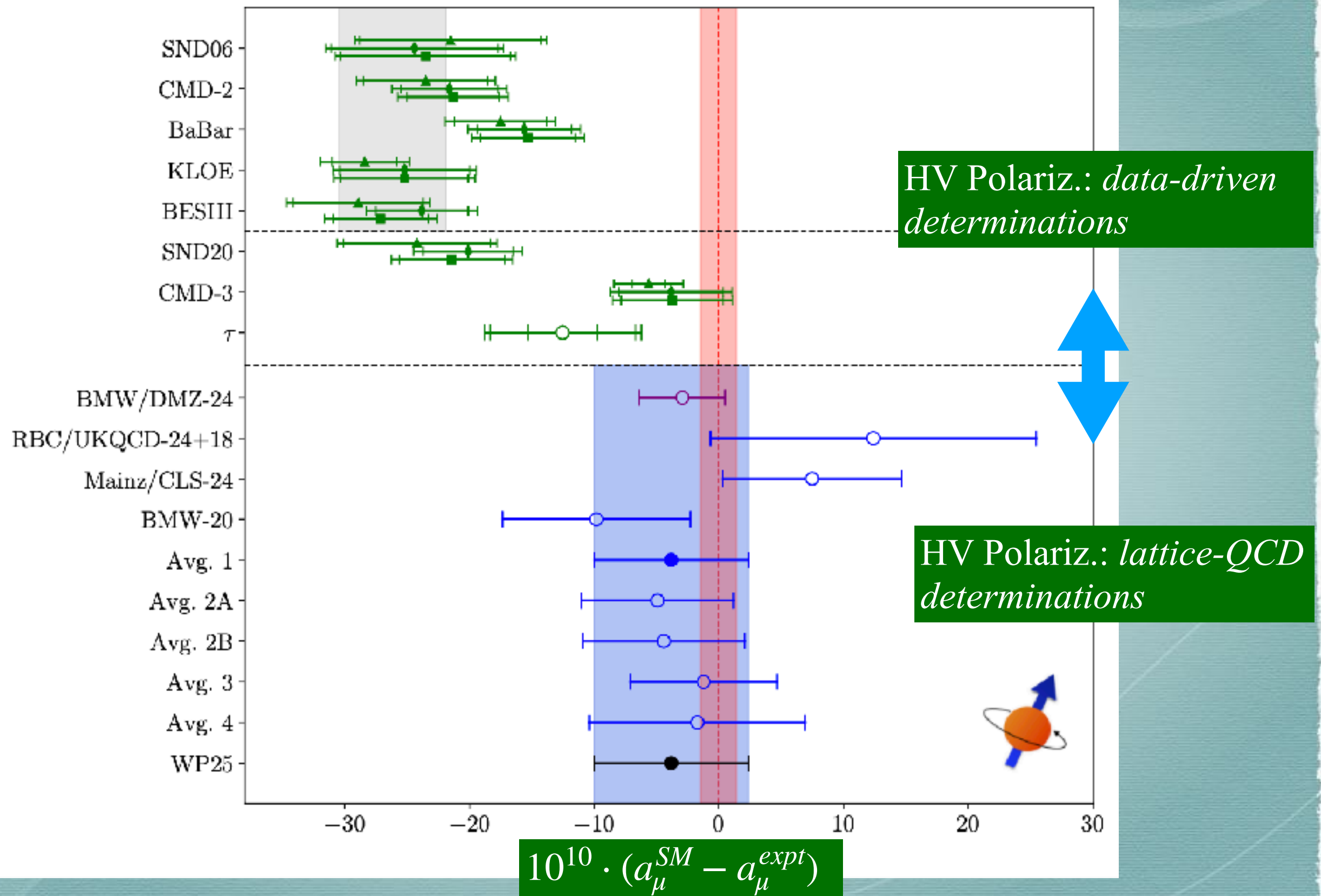
by the **BMW-DMZ Collaboration**:

A. Boccaletti *et al.*, arXiv:2407.10913

$$\Delta a_\mu^{\text{BMW-DMZ}} = 201.9 \pm 3.8 \text{ (QED + EW + HVP lattice + subleading hadronic)}$$

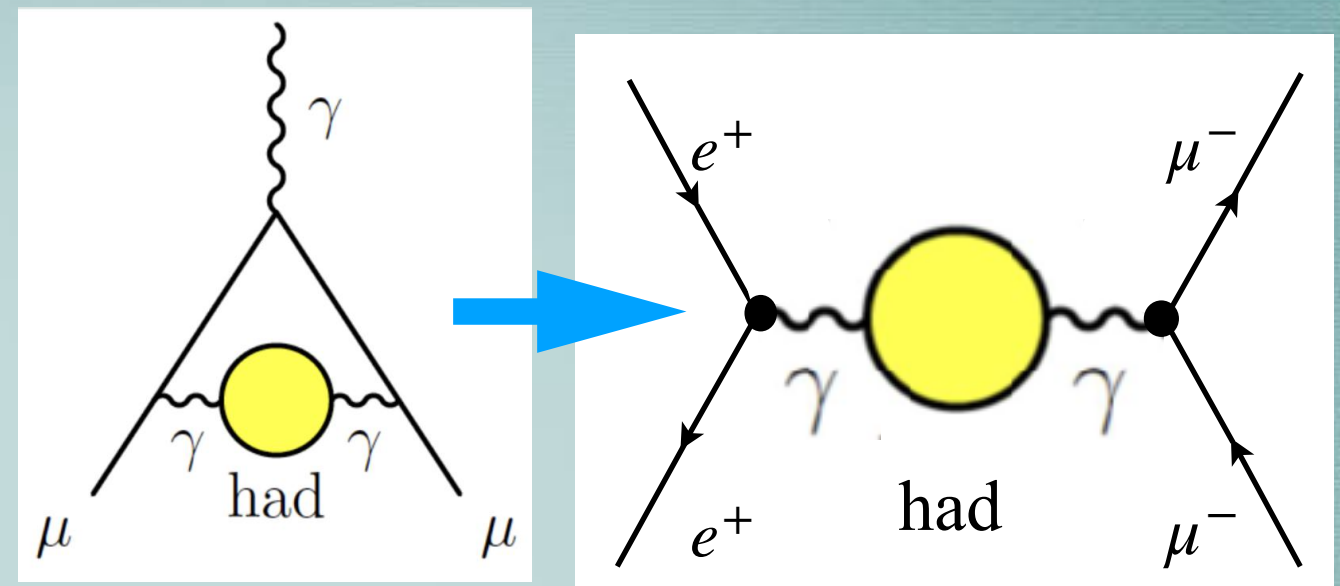
Last Update:

T. Aliberti *et al.*, arXiv:2505.21476v3 [hep-ph] 11 Sept. 2025



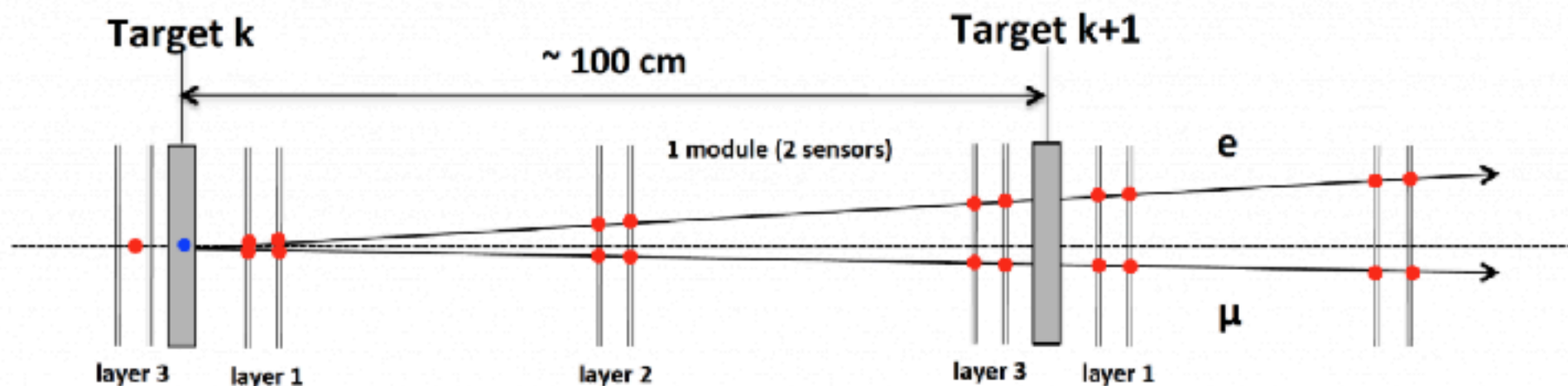
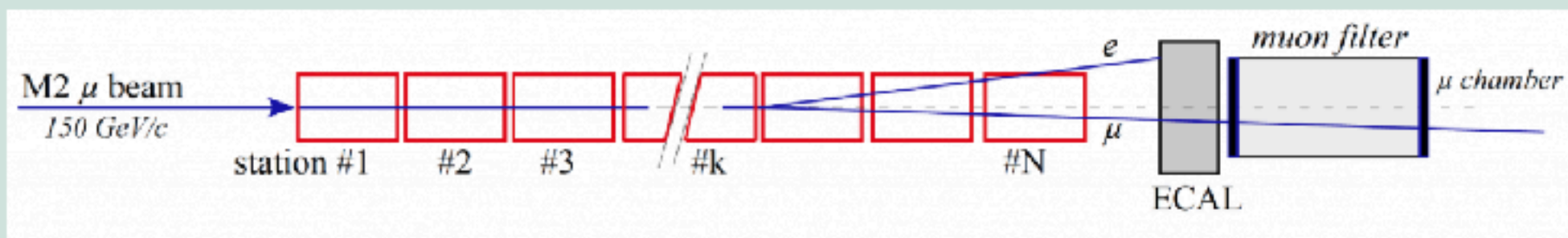
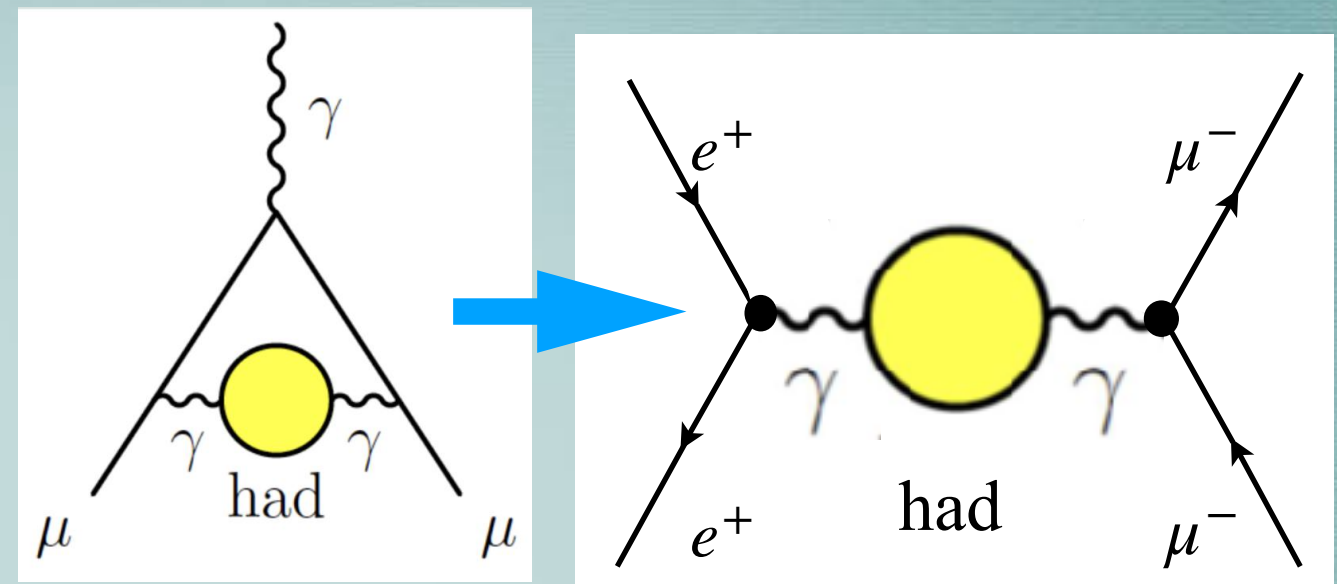
# An experimental way out??

- To *estimate* the Hadronic Vacuum Polarization contribution to  $g-2 \rightarrow$  *measure* the cross section of elastic muon-electron scattering !!



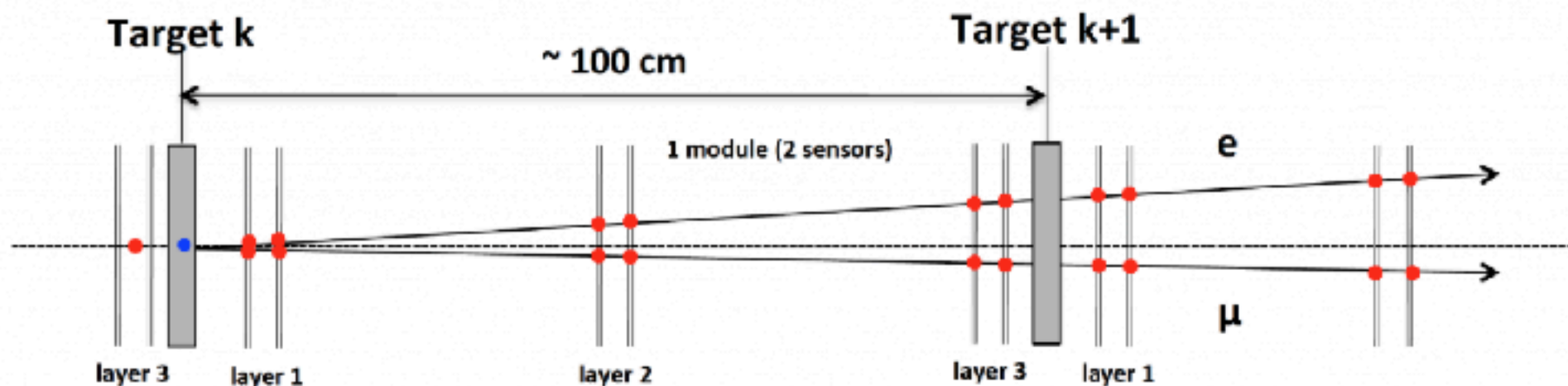
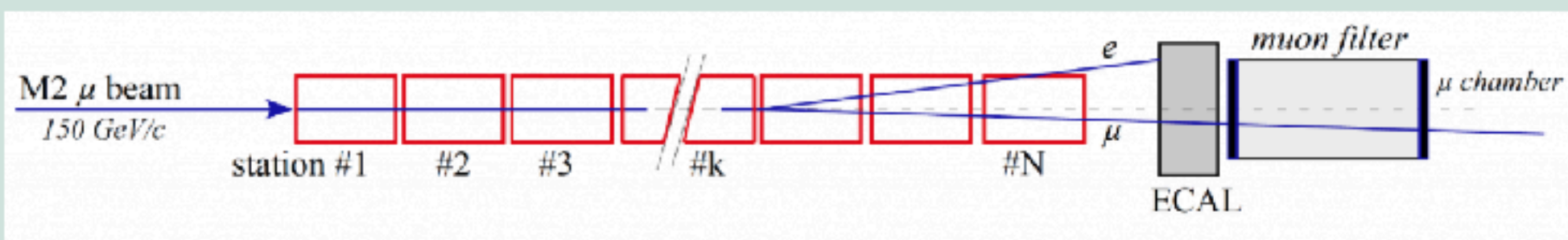
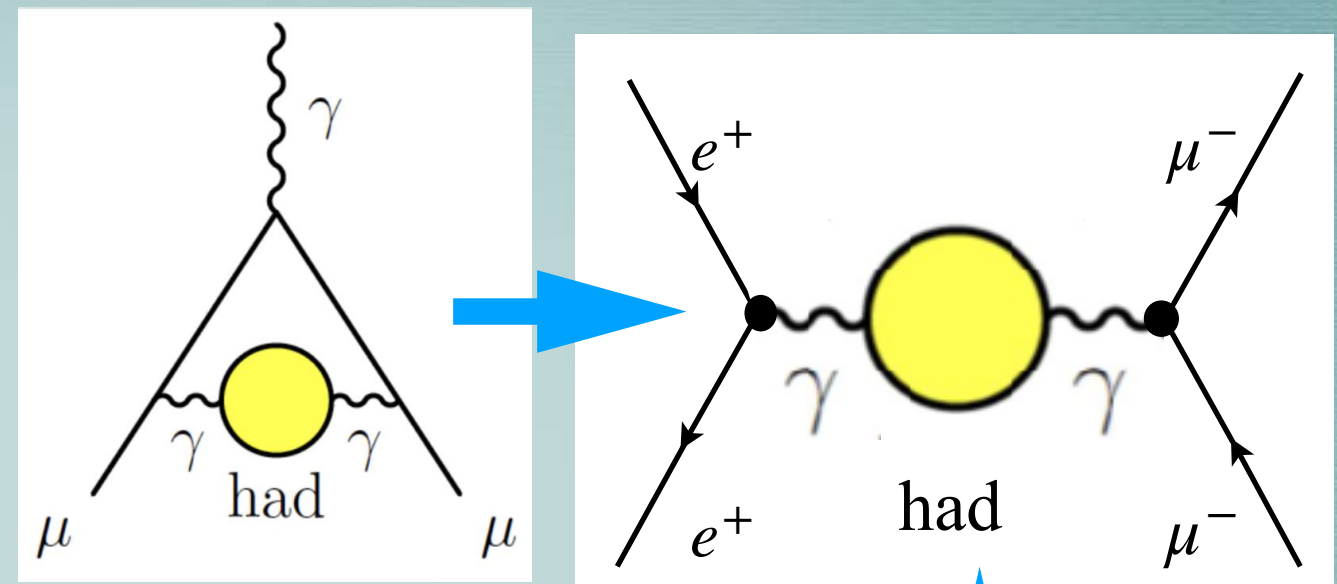
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**MUonE**  
*experiment* :  
 approved at  
 CERN,  
 Data analysis  
 ongoing

# 3. The Yang - Mills Theory

- Motivated by the global Isospin symmetry of the strong interactions:
- Can we have local (gauge) invariance under a non-abelian symmetry group?
- Consider local transformations of “matter fields” e.g. nucleons, under  $g \in G$ , compact, simple group. We take SU(2) ( $\tau^A$  = Pauli matrices,  $A=1, 2, 3$ ):

$$\psi(x) \rightarrow \psi'(x) = U(x)\psi(x); \quad U(x) = e^{i\alpha^A(x) \cdot \tau^A / 2}$$

- Introduce the gauge fields:

$$A_\mu(x) = \sum A_\mu^A(x) T^A; \quad T^A = \frac{\tau^A}{2}$$

$$A'_\mu(x) = U(x)A_\mu(x)U(x)^\dagger + iU(x)\partial_\mu U(x)^\dagger$$

- Covariant derivative

$$D_\mu \psi(x) = (\partial_\mu - iA_\mu)\psi$$

- and....

$$\begin{aligned} D'_\mu \psi'(x) &= \partial_\mu (U\psi) - i(UA_\mu U^\dagger + iU\partial_\mu U^\dagger)U\psi = \\ &= (\partial_\mu U)\psi + U\partial_\mu \psi - iUA_\mu \psi + U(\partial_\mu U^\dagger)U\psi = \\ &= U(\partial_\mu - iA_\mu)\psi = UD_\mu \psi \end{aligned}$$

- if  $L_0(\psi, \partial_\mu \psi)$  is invariant for global transformations (isospin),  $L_0(\psi, D_\mu \psi)$  is gauge invariant.
- We need the analog of the Maxwell term !!!????!!!!

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- and....

$$D'_\mu \psi'(x) = \partial_\mu (U\psi) - i(U A_\mu U^\dagger + iU \partial_\mu U^\dagger) U\psi =$$

$$= (\partial_\mu U) \psi + U \partial_\mu \psi - iU A_\mu \psi + U (\partial_\mu U^\dagger) U \psi =$$

$$= U(\partial_\mu - iA_\mu) \psi = U D_\mu \psi$$

$$U(\partial_\mu U^\dagger) U \psi = -(\partial_\mu U) U^\dagger U \psi = -(\partial_\mu U) \psi$$

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# Yang - Mills Lagrangian

- Define the Y-M tensor:  $G_{\mu\nu} = \partial_\nu A_\mu - \partial_\mu A_\nu - i[A_\nu, A_\mu]$
- One can prove (a good exercise) that G transforms like the regular representation:

$$G'_{\mu\nu} = \partial_\nu A'_\mu - \partial_\mu A'_\nu - i[A'_\nu, A'_\mu] = U G_{\mu\nu} U^\dagger$$

- Yang-Mills lagrangian: invariant, quadratic in the derivatives of A:

$$L_{Y-M} = \frac{1}{8g^2} \text{Tr}(G_{\mu\nu} G^{\mu\nu})$$

- Minimal substitution: given  $L_0(\psi, \partial_\mu \psi)$  invariant under global transformations, the new Lagrangian:

$$L_{tot} = L_0(\psi, D_\mu \psi) - \frac{1}{8g^2} \text{Tr}(G_{\mu\nu} G^{\mu\nu})$$

is invariant under local transformations

$$L_{tot} = L_0(\psi, D_\mu \psi) - \frac{1}{8g^2} \text{Tr}(G_{\mu\nu} G^{\mu\nu})$$

- Rescaling  $A \rightarrow gA$ , we see that “g” is the coupling constant (g=0, no interaction;
- There are as many g as there are simple components in G (es.  $SU(2)_W \otimes U(1)_Y$  has 2 constants)
- ***No mass term for the gauge bosons is allowed.***
- ***The form of the interaction matter-gauge fields is fixed by the symmetry***
- Gauge fields are interacting, since they carry a non vanishing charge: unlike QED, pure Yang-Mills is non-trivial

Y-M is a toy model for gravity

- Spin 1/2 matter fields:

$$L_{int} = g(\bar{\psi} \gamma^\lambda T^A \psi) A_\lambda^A = g A_\lambda^A (J^{A\lambda})$$

$J^A$  are the means by which matter interacts and the currents associated to the global symmetry.

- Scalar matter fields: besides the current mediated interaction  $(A^A)_\mu J^A \mu$  have a quadri-linear interaction (seagull):

$$L_{seagull} = \frac{g^2}{2} (\phi^\dagger g^{\mu\nu} \{T^A, T^B\} \phi) A_\mu^A A_\nu^B$$

- Write  $L_0$  for scalar fields.

## 4. Large $q^2$ behaviour in field theory: QED

- Asymptotic behaviour of deep inelastic processes, e.g.:

$$R = \frac{\sigma(e^+e^- \rightarrow \text{hadrons})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)}$$

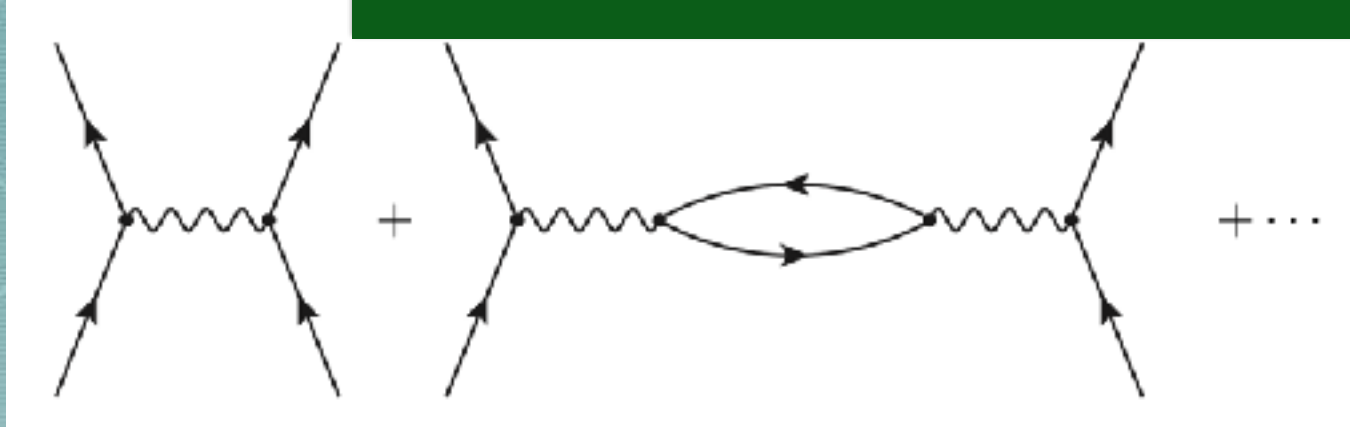
- The cross section for muon pairs scales with  $s$  like  $1/s$
- $R = \text{constant}$ , above low-energy resonances, says that  $1/s$  scaling is approximately true for hadron production.
- Naive scaling. How could it be different?
  - For large  $s$  we can set to 0 all particle masses
  - the dimension of the cross sections is determined by the only dimensional variable at our disposal,  $s$ :

$$[s] = [\text{lenght}^{-2}], [\sigma] = [\text{lenght}^2], R = \frac{\sigma(\text{hadrons})}{\sigma(\mu^+\mu^-)} = \text{constant}.$$

# Failure of naive scaling

- The argument is wrong!
  - field theory needs to be computed with an UltraViolet cut off,  $\Lambda$
  - renormalized physical quantities (e.g. charges) have to be defined at some value of  $q^2$ .
  - If we renormalize the physical quantities (e.g. charge) at  $q^2=0$ , we cannot send all masses to 0, because of mass singularities (i.e.  $\log(q^2/m^2)$ );
  - Alternatively, we can renormalize at a mass scale  $q^2 = \mu^2 < 0$ , and let masses  $\rightarrow 0$  to examine the limit  $q^2 \rightarrow \infty$  or  $s \rightarrow \infty$ ;
  - however, even in the massless theory, I can now have large logs of the type  $[\log(q^2/\mu^2)]^d$  or  $[\log(s/\mu^2)]^d$  *which spoil the naive scaling laws* !
  - Scaling Laws in the asymptotic behaviour are *non-trivial dynamical properties*: why do we see approximate naive scaling to hold???

# The $\beta$ function of QED



$$\mathcal{A} = \frac{e_0^2}{q^2} (1 + e_0^2 \Pi(q^2) + \dots)$$

$$\Pi(q^2) - \Pi(\mu^2) = \text{finite} = \Pi_c(q^2, \mu^2) = \frac{1}{12\pi^2} \ln \frac{q^2}{\mu^2}; \quad (-q^2, -\mu^2 \gg m^2)$$

- vertex and fermion propagator corrections carry a factor  $Z_2/Z_1=1$  (Ward identity). After subtraction

$$\mathbf{d} = q^2 A = e^2(\mu) [1 + e^2(\mu) [\Pi(q^2) - \Pi(\mu^2)]]$$

- $\mathbf{d}(q^2, e^2(\mu), \mu^2)$  is a physical quantity, which we call  $e(\mu)$ , and it cannot depend from  $\mu$ , which is arbitrary. A change of  $\mu$  must be compensated by a change in  $e(\mu)$ :

$$0 = \mu \frac{d\mathbf{d}}{d\mu} = \mu \frac{\partial \mathbf{d}}{\partial e^2} \frac{\partial e^2}{\partial \mu} + \mu \frac{\partial \mathbf{d}}{\partial \mu} \Big|_{(q^2=0)} \simeq \mu \frac{\partial e^2}{\partial \mu} - \frac{e^4}{6\pi^2} \rightarrow$$

$$\rightarrow \mu \frac{\partial e^2}{\partial \mu} = \beta(e) = + \frac{e^4}{6\pi^2}$$

# QED, the Gell-Mann and Low equation for the running coupling

- We found:  $\rightarrow \mu \frac{\partial e^2}{\partial \mu} = + \frac{e^4}{6\pi^2}$ . Defining:  $t = \ln(q^2/\mu^2)$ ,  $dt = \frac{2d\mu}{\mu}$ ,  $\alpha = \frac{e^2}{4\pi}$ ,

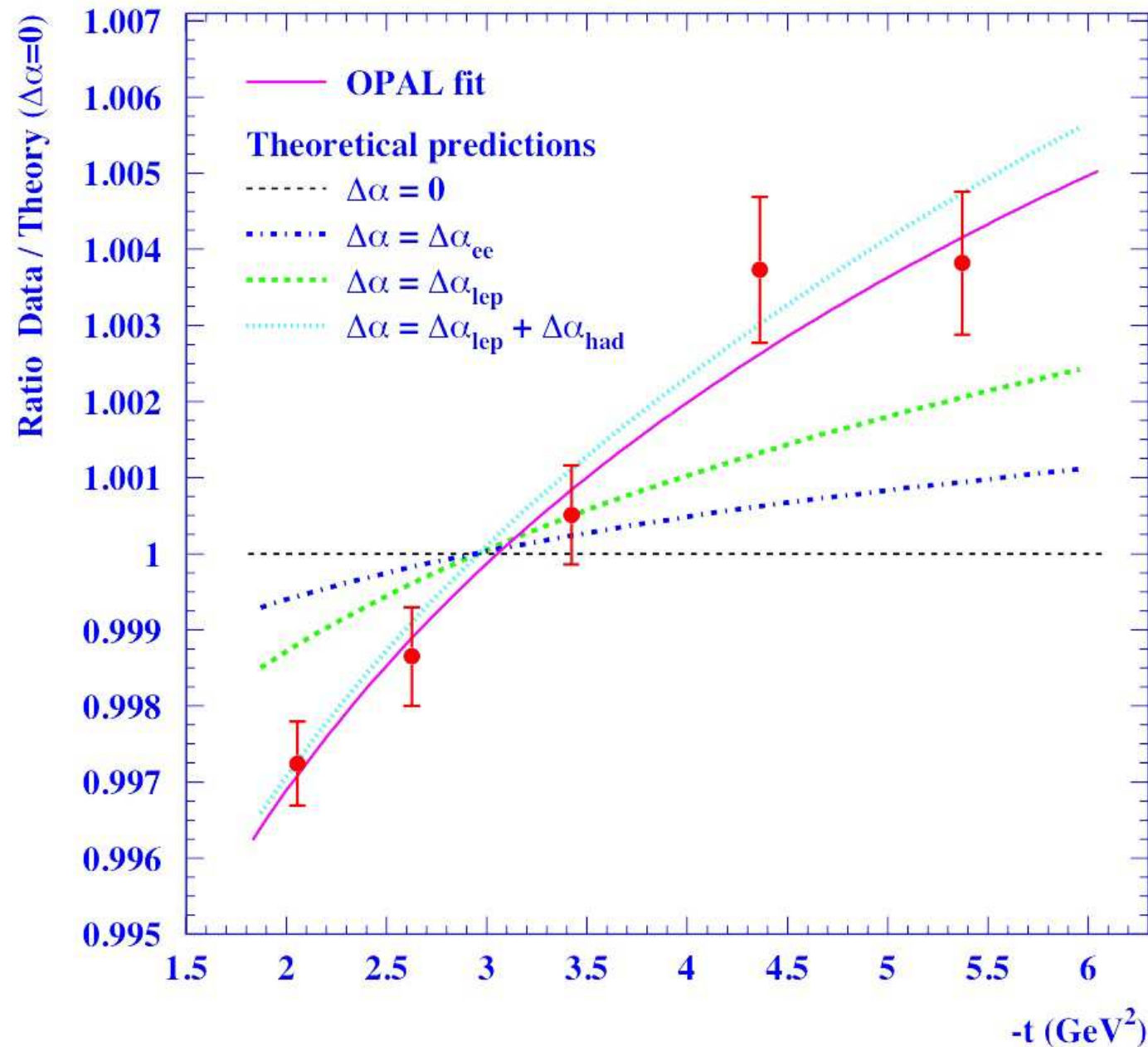
we obtain the Gell-Mann Low equation

$$\frac{d\alpha}{dt} = \mu \frac{d\alpha}{2d\mu} = \frac{\alpha^2}{3\pi}$$

- The solution with the initial condition  $\alpha(\mu)$  at  $t=0$  is:

$$\alpha(q) = \frac{\alpha(\mu)}{1 - \frac{\alpha(\mu)}{3\pi} \ln \frac{q^2}{\mu^2}}$$

- BAD and GOOD NEWS:
  - I have resummed all large logs: “improved perturbation theory”
  - all terms of order  $(\alpha(0) t)^n$  (leading logs) are included
  - neglected terms of order  $(\alpha(0) t)^{n-1}$  (next- to- leading)
  - in QED, the running coupling INCREASES !
  - QED at the Z scale ( $E=100$  GeV) is described by  $\alpha(100 \text{ GeV})=1/128$ .



QED: the “running” fine structure constant from Bhabha scattering.

At  $-q^2 \sim (M_Z)^2$ ,  $\alpha^{-1} \sim 128$

Dependence on  $-t = Q^2$  of the ratio between the elastic scattering cross section for  $e^+e^-$  (Bhabha scattering) measured at LEP and the results of theoretical calculations. The horizontal line corresponds to the case in which the value of  $\alpha$  is kept constant as  $Q^2$  varies

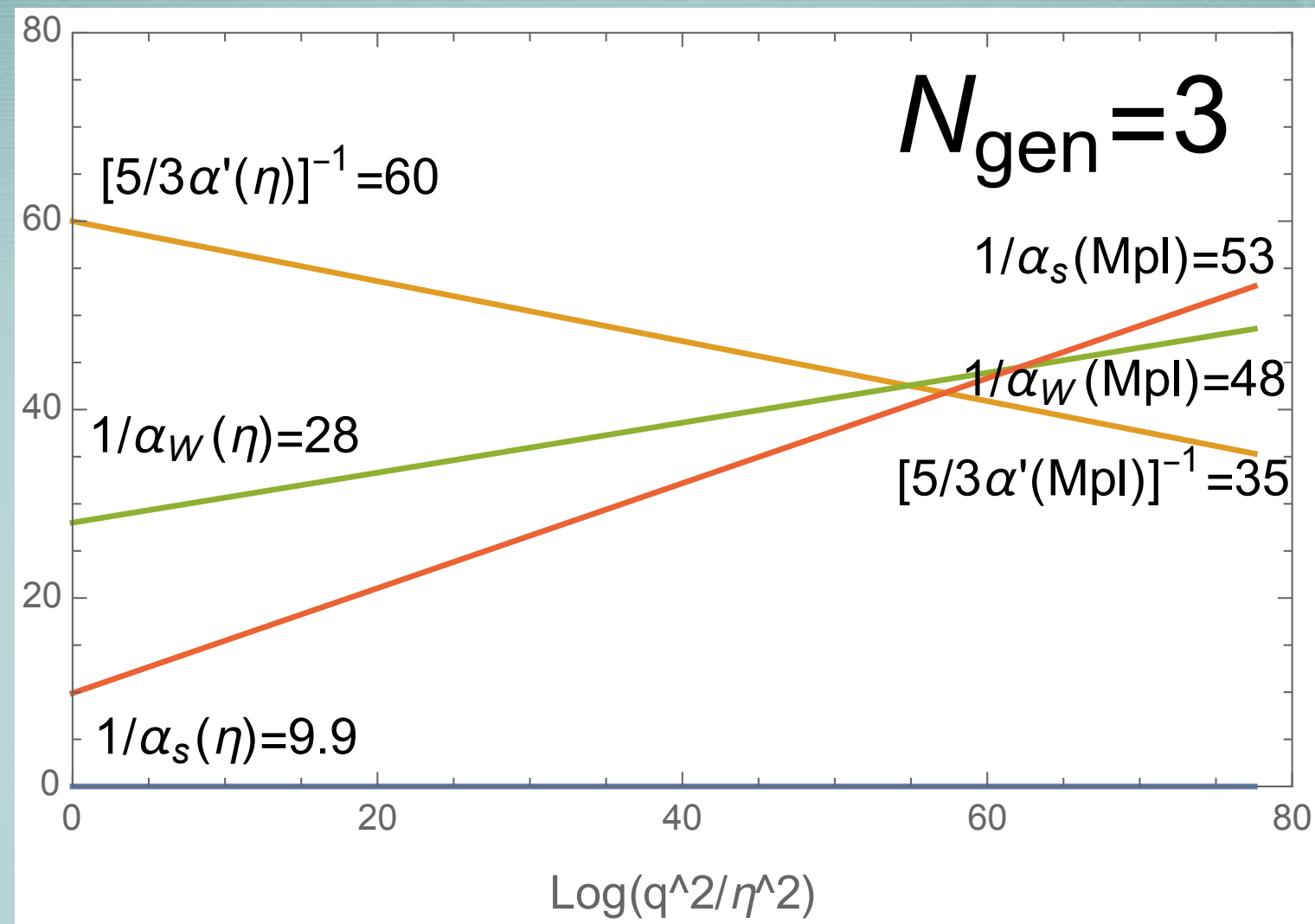
# Bare vs. running coupling

At the ultraviolet cut-off,  $\Lambda^2$ , I get the “bare coupling”

$$\alpha(\Lambda) = \frac{\alpha(\mu)}{1 - \frac{\alpha(\mu)}{3\pi} \ln\left(\frac{\Lambda^2}{\mu^2}\right)}$$

- BAD NEWS:
  - $\alpha(\Lambda)$  *becomes negative above the Landau pole*: for fixed physical constant  $\alpha(\mu)$ , we cannot send  $\Lambda \rightarrow \infty$ :
  - the continuum theory is recovered only for  $\alpha(\mu) = 0$
  - a similar phenomenon happens for the  $\phi^4$  theory (where we have much better control)
- If  $\beta(\alpha) < 0$ ,  $\alpha(\Lambda) \rightarrow 0$ : “asymptotic freedom”.
- A reasonable conjecture: the limit  $\Lambda \rightarrow \infty$  exists only for asymptotically free theories
- GOOD NEWS:
  - Yang-Mills theory with not too many fermions has  $\beta < 0$  and is asymptotically free (Gross, Wilczek, Politzer 1973). *Asymptotic freedom provides the basis of scaling in deep inelastic scattering* (see Lecture 4).

# Ultraviolet behaviour of the gauge couplings of the Electroweak gauge group, $SU(2)_W \otimes U(1)_Y$



Running of the Standard Model couplings for 3 generations.  $5/3 \alpha'$  is the coupling of the abelian component, and it does increase towards the Landau pole.

Around  $\text{Log}(q^2/\eta^2) = 56$ ,  $q \simeq 10^{14} \simeq M_{GUT}$ , the three lines cross. In  $SU(5)$  GUT, the symmetry group is non abelian and, from this point on, the couplings all would converge toward 0 (asymptotic freedom).