Six lectures on the Standard Theory of Elementary Particle Physics

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Beijing, 26 October2026

Lecture 4
Quarks & Partons

The Plan of my Lectures

- 1. Gauge Symmetry: QED, g-2, Yang-Mills, Asymptotics.

- 2. Electroweak Unification 1; breaking the Symmetry; the Higgs-Brout-Englert boson.

- 3. Interactions and symmetries of subnuclear particles; Quarks: spectroscopy and fundamental interactions.

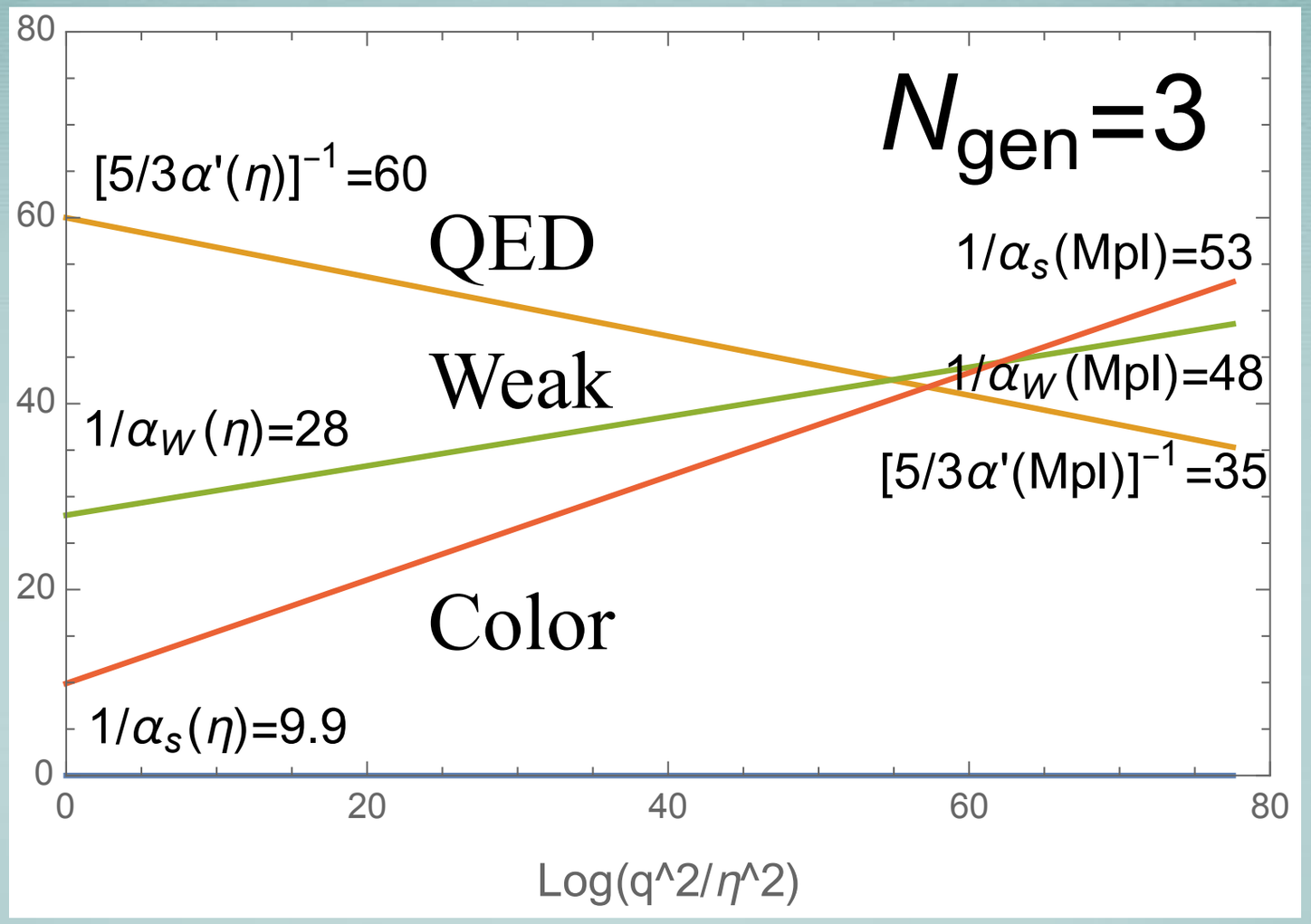
- 4.***Quarks and Partons, Color and Gluons, Hadrons in the Constituent Quark model.***

- 5. Electroweak Unification 2: GIM Mechanism, Charm, Beauty and CP violation.

- 6. The Higgs Boson search. The Standard Model: What's missing? What's next ?

Ultraviolet behaviour of the (gauge couplings)$^{-1}$ of QED, Weak and Color gauge groups (Lecture 1)

(Coupling Constants)$^{-1}$



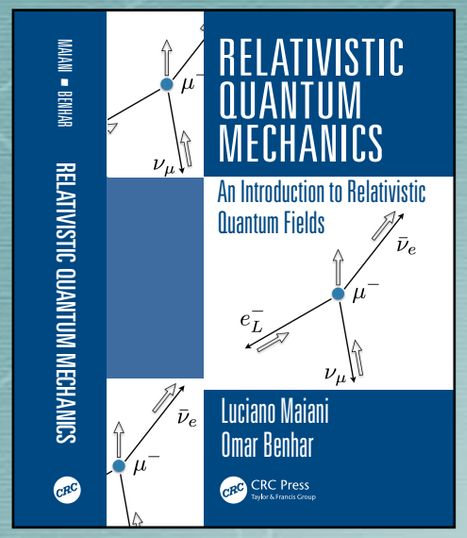
Asymptotic freedom?…..

To the Landau pole? Grand Unification?…..

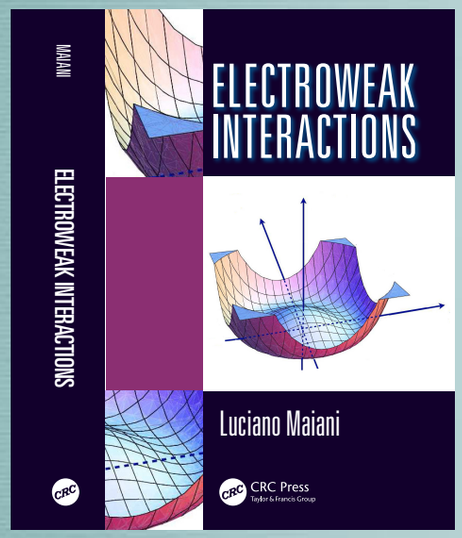
Running of the Standard Model couplings for 3 generations. 5/3 α' is the coupling of the abelian component, and it does increase towards the Landau pole.

Around $Log(q^2/\eta^2) = 56$, $q \simeq 10^{14} \simeq M_{GUT}$, the three lines cross. In SU(5) GUT, the symmetry group is non abelian and, from this point on, the couplings all would converge toward 0 (asymptotic freedom).

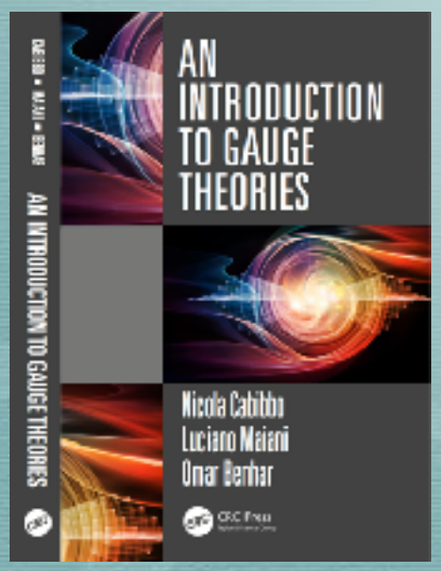
References of my Talks



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Taylor & Francis, 2024,
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Electroweak Interactions,
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N. Cabibbo, L. Maiani and O. Benhar,
An Introduction to Gauge Theories,
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The Beginnings

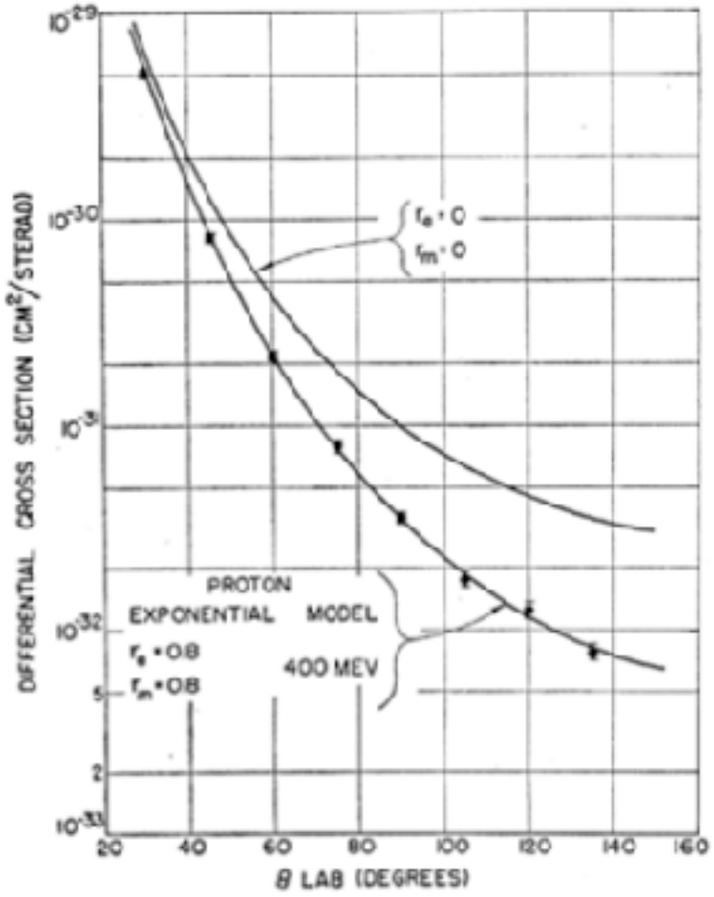


FIG. 26. Typical angular distribution for elastic scattering of 400-Mev electrons against protons. The solid line is a theoretical curve for a proton of finite extent. The model providing the theoretical curve is an exponential with rms radii $=0.80\times10^{-13}$ cm.

R. Hofstadter, Rev. Mod. Phys. 56 (1956) 214

Robert Hofstadter
Nobel prize 1961



ep-elastic
Finite size of the proton

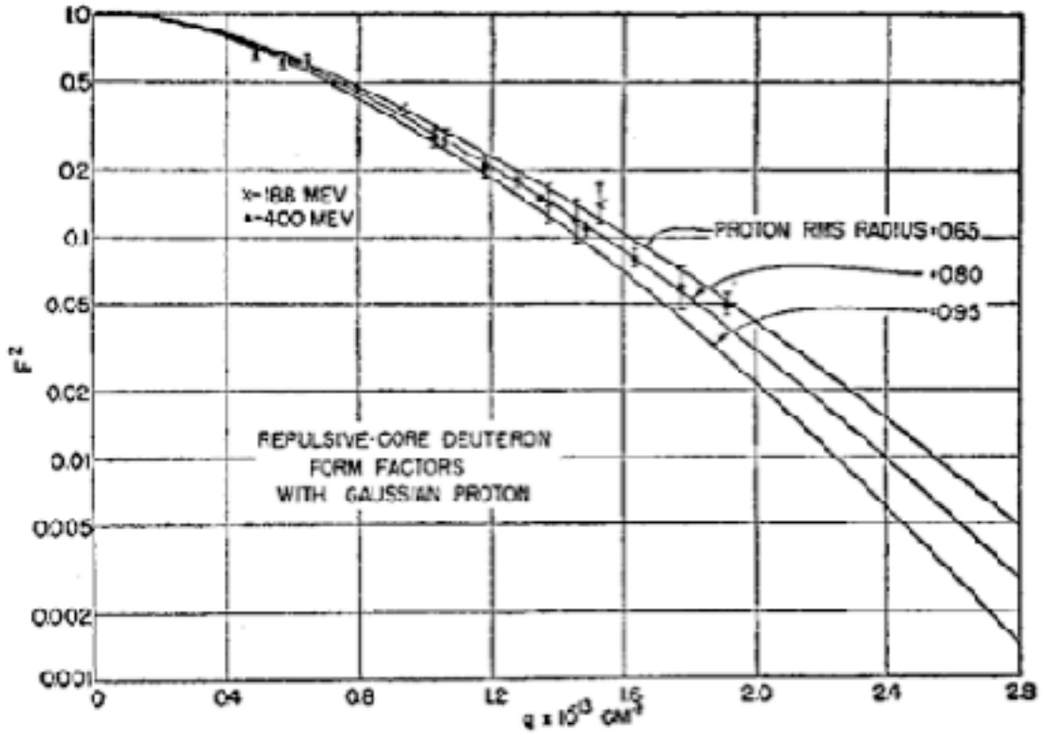


FIG. 31. Introduction of a finite proton core allows the experimental data to be fitted with conventional form factors (McIntyre).

ed-elastic
Finite size + nuclear structure

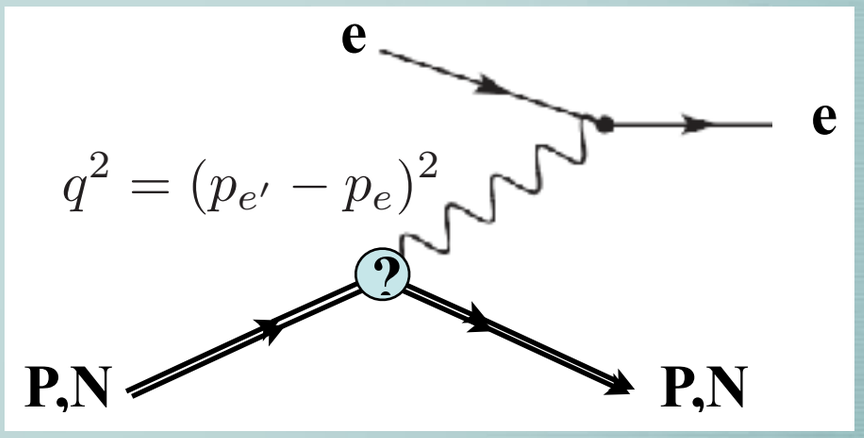
1. The Proton's Form Factor

Rutherford's discovery of a massive, pointlike nucleous inside the atom, 1911

... It was quite the most incredible event that has ever happened to me in my life. It was almost incredible as if you fired a 15-inch shell at a piece of tissue paper and it came back and hit you.
E. Rutherford, 1936

... things did not go this way with elastic electron scattering on protons

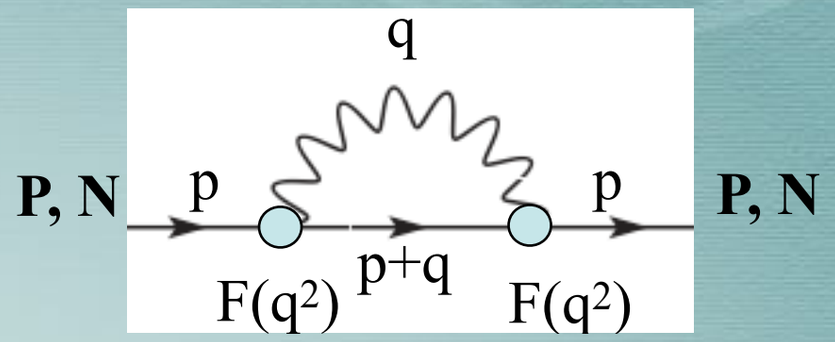
- Send pointlike electrons to scatter off protons and (neutrons, inside deuteron)
  - measure the probability of elastic scattering at different q^2;
  - compare with what expected for pointlike proton
  - the ratio measures the electric and magnetic "form factor" of proton and neutron.



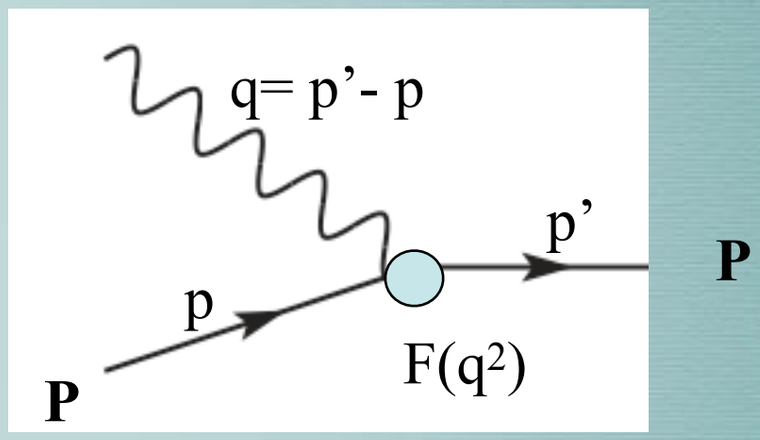
- First determinations made by R. Hosfstadter at Stanford University, California, gave the radius of a uniformely distributed, spherical charge density of the proton
- $R \simeq 0.8\ 10^{-13}$ cm = 0.8 fermi, with $F(q^2) \to 0$ decreasing rapidly for $q^2 \to -\infty$
- proton and neutron are not pointlike: 1961 Nobel Prize for Physics to Robert Hofstadter.

Subnuclear matter: is it *SOFT* ?

- Due to the form factor, the interaction between proton or neutron and the photon decreases rapidly with the increase of momentum transfer
- virtual effects may be finite and calculable. Is the neutron-proton mass difference (=1.4 MeV) calculable?



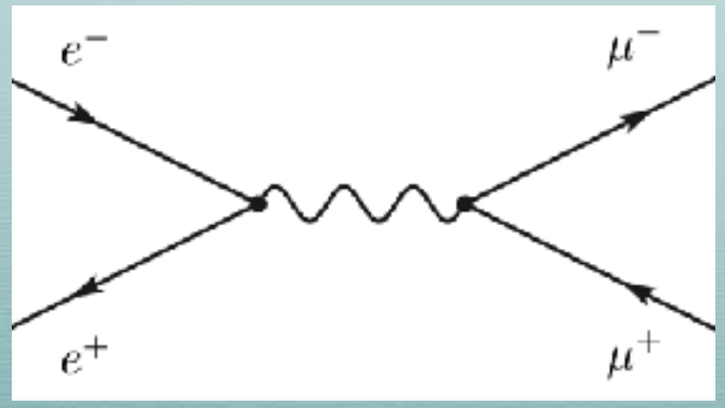
- In the 60's, many believed that probing hadron matter with electron scattering at large q^2 , one would find less and less interactions, unlike what happens with scattering off the pointlike particles, electrons and muons;

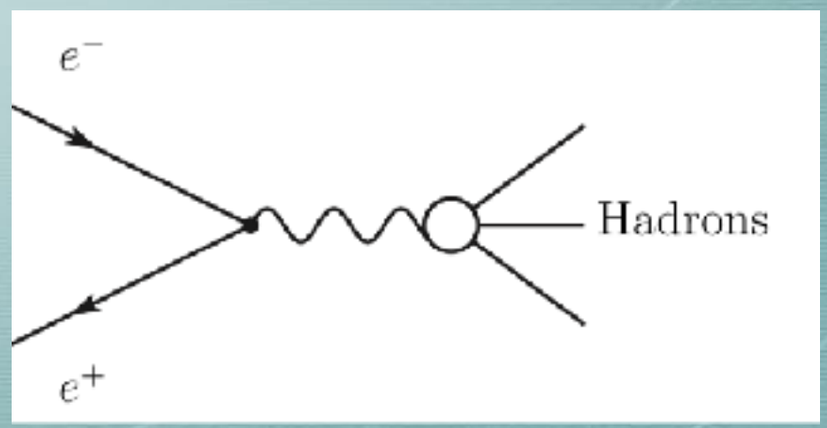


Electron proton's elastic scattering $q^2<0$

- ***Electron-Positron annihilation into μ+μ- and hadrons, $q^2>0$***
- Prob ($e^+e^- \to \mu^+\mu^-$) ≈ $1/|\,q^2\,|$ while Prob ($e^+e^- \to$ hadrons) would disappear soon after ~1 GeV (corresponding to sizes ~ 0.2 fermi): the ***hadron desert***, as was called by J. J. Sakurai.

$$R = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

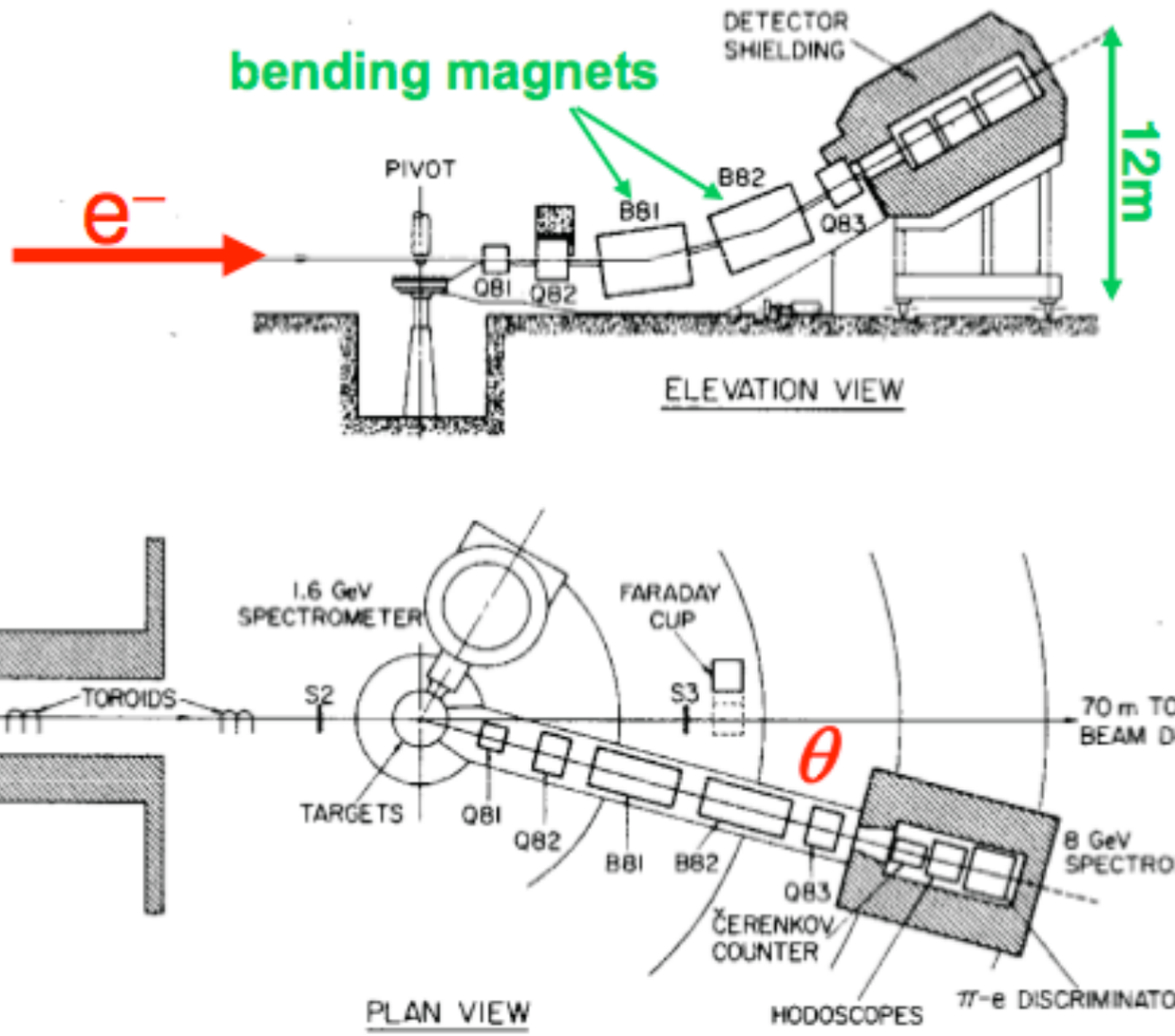




Higher Energy Electron-Proton Scattering

★ **Use electron beam from SLAC LINAC:** $5 < E_{beam} < 20$ GeV

- **Detect scattered electrons using the "8 GeV Spectrometer"**



P.N.Kirk et al., Phys Rev D8 (1973) 63

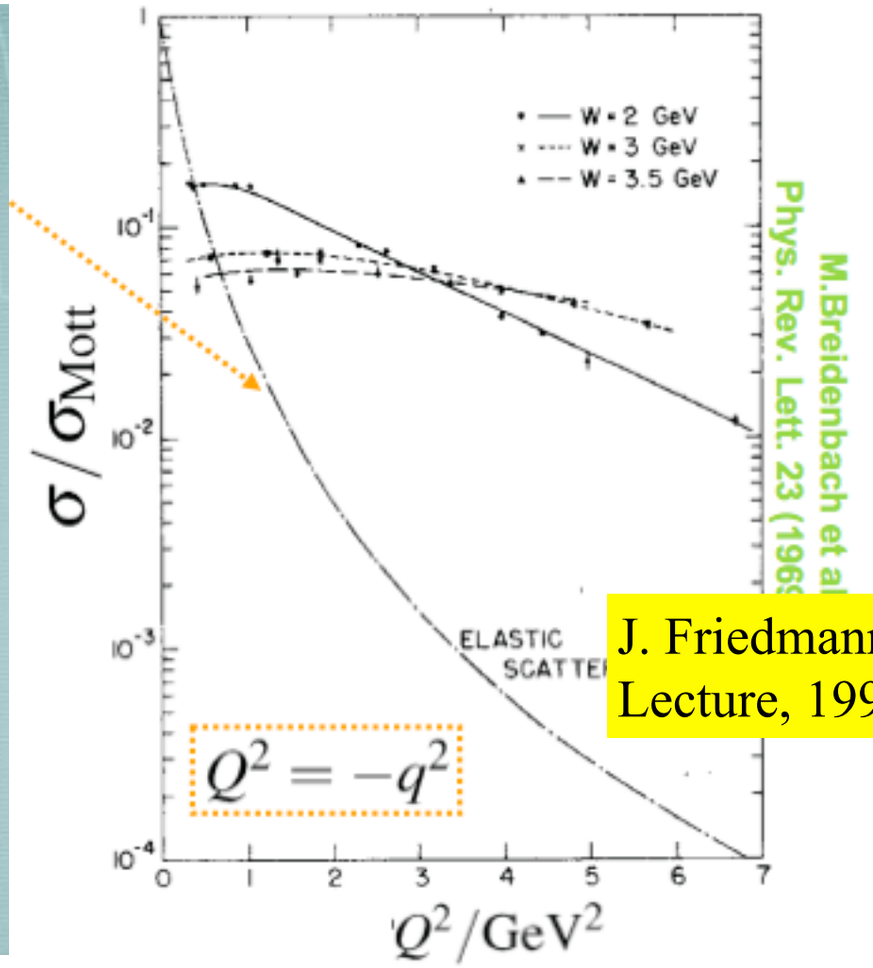


First results in 1968

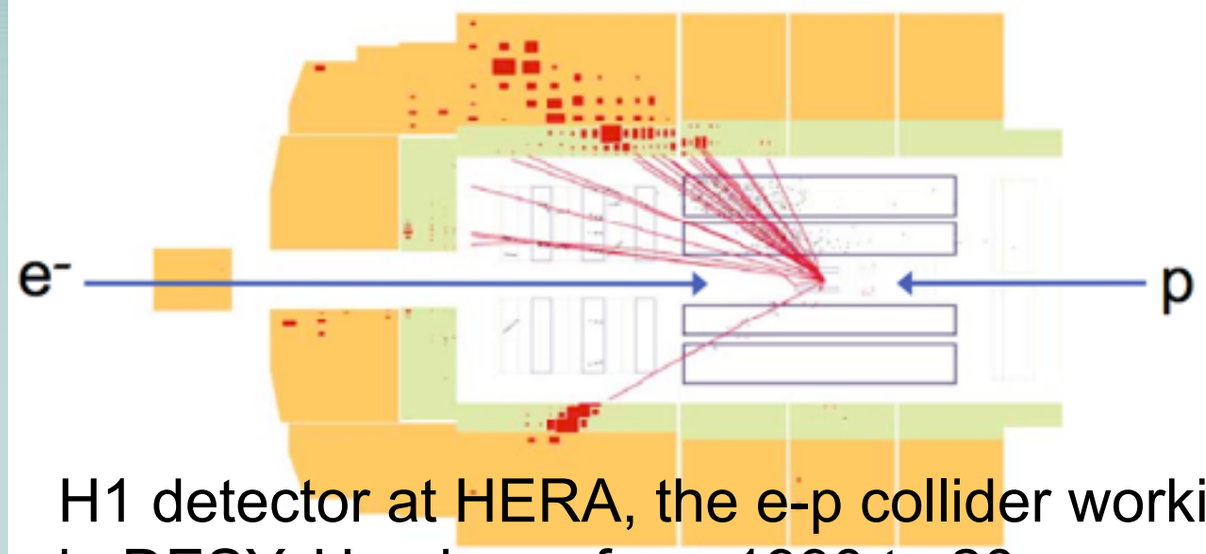
Nobel Prize 1990
J.Friedmann, H. Kendall, R. Taylor

2. Deep inelastic electron proton scattering reveals elementary constituents in the proton

Process: $e + p \rightarrow e +$ anything



J. Friedmann, Nobel Lecture, 1990

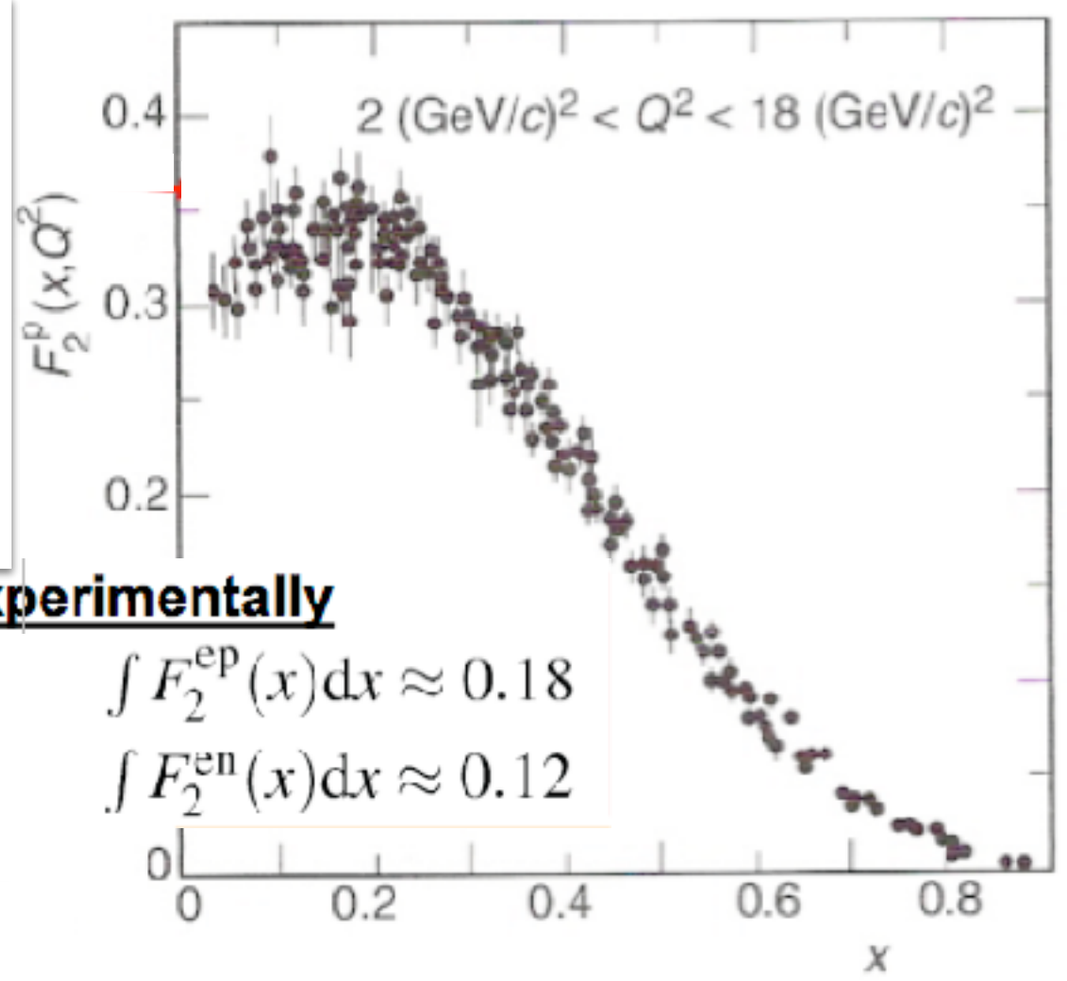


H1 detector at HERA, the e-p collider worki
in DESY, Hamburg, from 1990 to 20

$$\frac{\sigma(e + p \rightarrow e + \text{anything})}{\sigma_{Mott}} = F_2(Q^2, x),$$

2 variables : $Q^2 = -q^2 > 0$, $\nu =$ electron energy loss

$$\nu = (p \cdot q)/M, \; x = \frac{Q^2}{2M\nu} = \text{Bjorken variable}$$



Experimentally

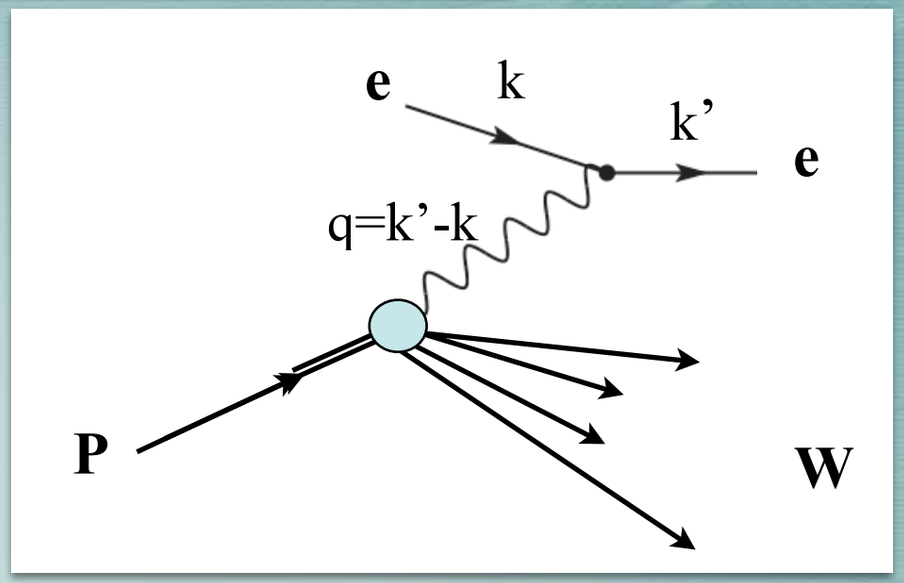
$$\int F_2^{ep}(x)dx \approx 0.18$$

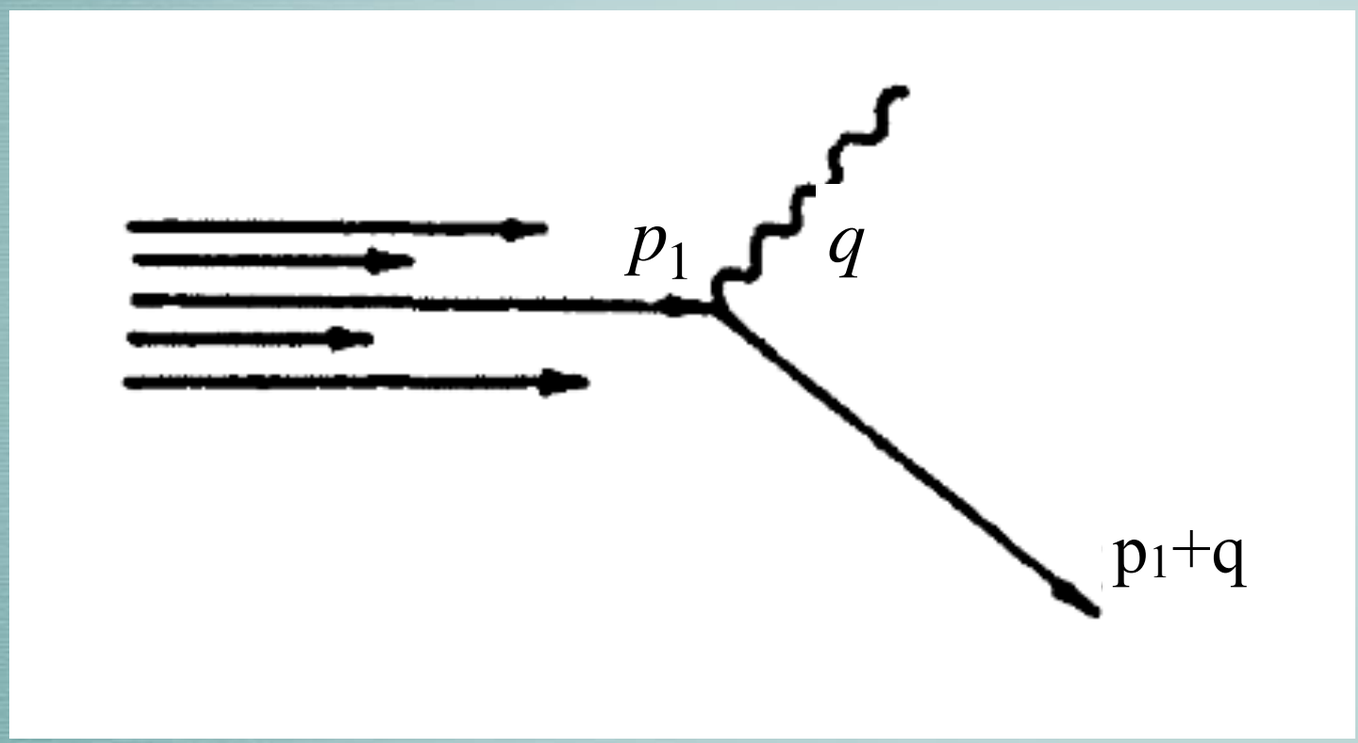
$$\int F_2^{en}(x)dx \approx 0.12$$

Bjorken scaling: F2 is only function of x

Feynman's "partons" (1970)

seen by the virtual photon, a high energy proton looks like a beam of elementary constituents, "partons", massless and with a limited transverse momentum, each with a fraction x of the total proton's momentum, P





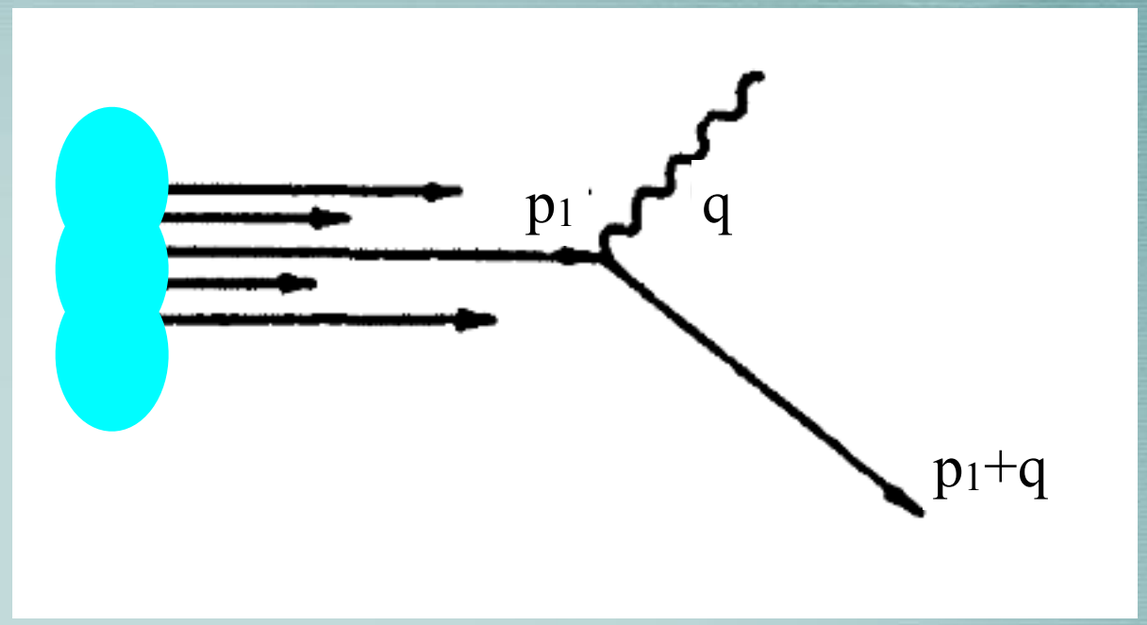
$$m_{parton} \simeq 0,\ P_{transverse} \simeq 0,$$
$$p_1 = xP$$
$$x = \frac{-q^2}{2M\nu}$$

$$d\sigma \propto \delta((xP+q)^2 - m^2_{part}) \sim \delta(q^2 + 2M\nu)$$

Bjorken's variable is the fraction of momentum carried by the struck parton; scaling is implied by the fact that the final parton is on mass shell: the electron scatters elastically off pointlike particles

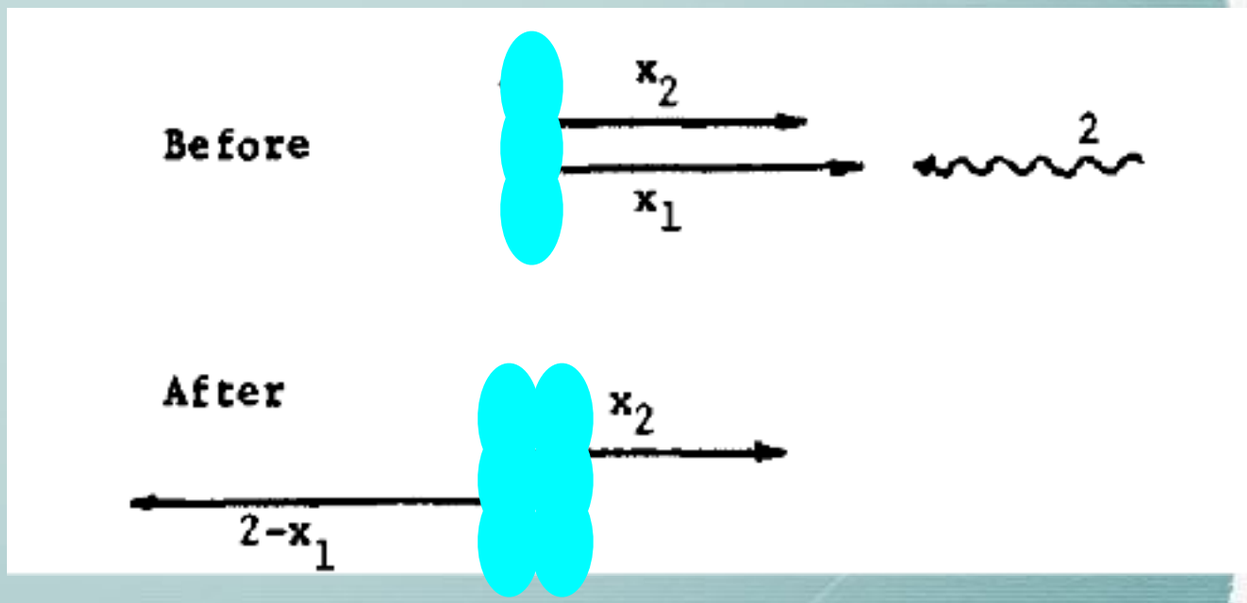
Wee partons (Feynman)

- The 'hard' partons (x∿ 0.2-0.8) are accompanied by a cloud of very soft partons, the ***wee partons***
- wee partons can move ***forward and backward directions!!***



- when the collision sends the hard parton"out of the chorus", hard and soft partons make up a jet of normal hadrons

No isolated quarks seen, only hadron jets



Is this the reason why we do not see quarks among the products of the collisions (not because they have a large mass) ???

see later

Momentum crisis: neutral partons ?

$P(x) =$ Probability to find a parton with momentum fraction x

$$F_2(x) = \sum_i Q_i^2 \; x_i P(x_i)$$

Experimentally

$$\int F_2^{ep}(x)dx \approx 0.18$$
$$\int F_2^{en}(x)dx \approx 0.12$$

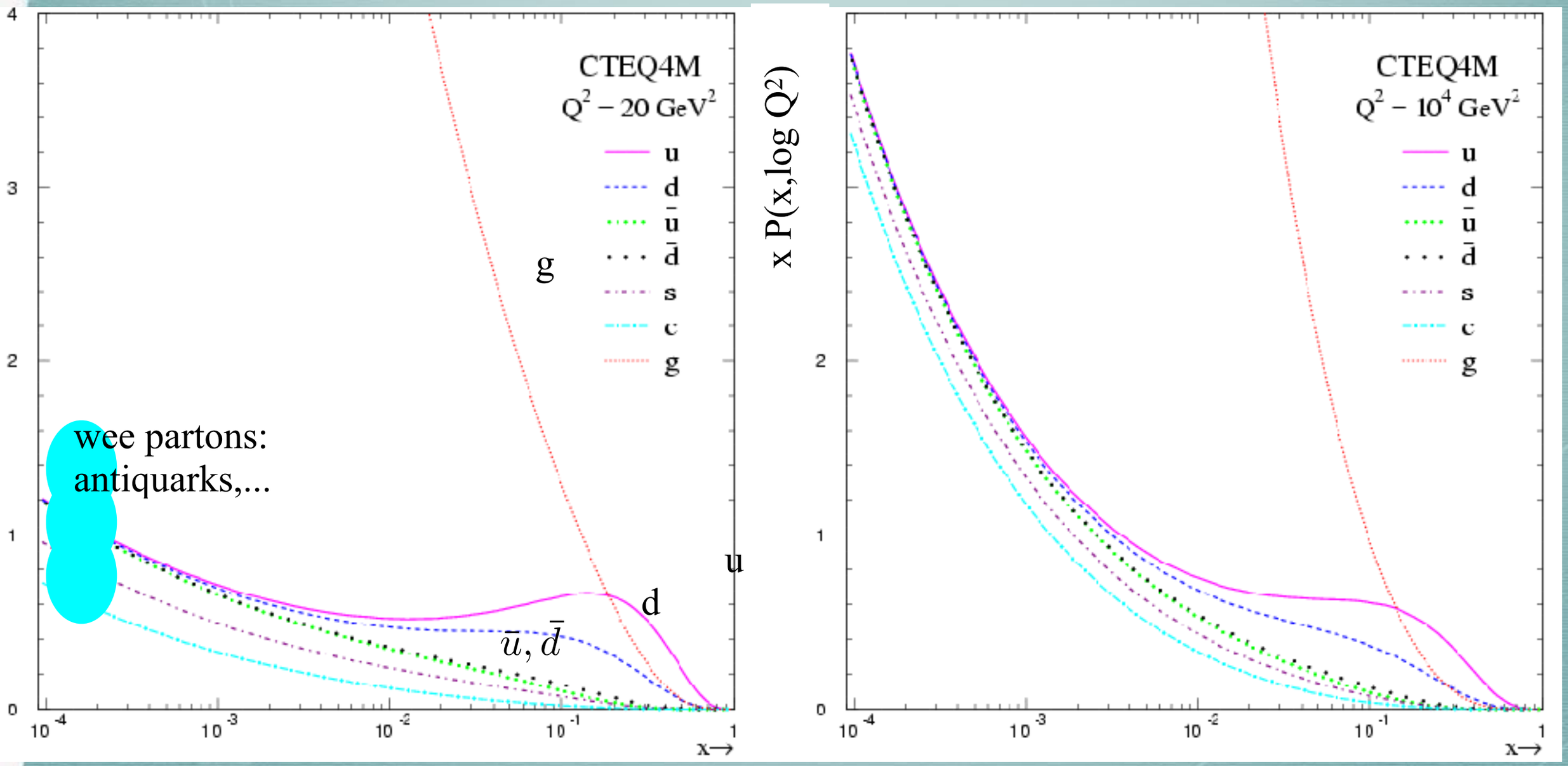
- the integral of F_2 (x) gives the average momentum of each parton, weighted with the electric charge squared
- assume partons= quarks, proton =uud, neutron=udd
- assume that all quarks carry a total momentum fraction $\mathbf{x_q}$
- u should carry 2/3 of total quark momentum, that is: $(F_{2u}^p)_{quark} = \frac{4}{9} \times \frac{2}{3} = \frac{8}{27}$
- d should carry 1/3 $(F_{2d}^p)_{quark} = \frac{1}{9} \times \frac{1}{3} = \frac{1}{27}$
- total: $(F_2^p) = 9/27\ x_q = 3/9\ x_q$ $(F_2^p)_{quark} = \frac{9}{27} = \frac{3}{9}$
- similarly: $(F_2^n) = (4/9*1/3+1/9*2/3)\ x_q = 2/9\ x_q$ $(F_2^n)_{quark} = \frac{1}{9} \times \frac{2}{3} + \frac{4}{9} \times \frac{1}{3} = \frac{2}{9}$
- $(F_2^p/F_2^n) = 3/2$, very good!

if proton was made of quarks only, $x_q = 1$ and $(F_2^p) = 3/9 = 0.33$
while observed $F_2^p = 0.18$, that is: $x_q = 54\%$

46% of proton's momentum is taken by electrically neutral partons ?!!!?

Partons in the proton, seen at different momenta

Parton composition of High Energy protons at
SLAC (Q2=20) and LHC (Q2=10.000)



Quarks, scaling and color

- ***Three successful but problematic concepts***
  - ***Quarks*** $\Delta^{++} = u\uparrow(x_1)u\uparrow(x_2)u\uparrow(x_3)$
    - ground state must be symmetric in the coordinates, howewer ***what about Fermi Statistics?***
      - ***Where are the quarks ?*** The lightest quark should be absolutely stable due to charge conservation
      - Why not produced in High Energy collisions? Large Mass? but chiral SU(2)xSU(2) indicates up/down quark masses are O(MeV), m_s=150 MeV!!!
      - ***Abundance in the Universe ? Energetic cosmic rays should produce them?***
  - ***Scaling***
    - how to reproduce free partons (i.e. scaling) in a non trivial field theory?
    - Revival of renormalization group techniques. The beta function "must" send the interaction to vanish for $q^2 \to -\infty$, unlike in QED.
  - ***Color***
    - Han & Nambu introduced the idea that a quark of given flavor (e.g. ***u*** quark) comes in ***3 unseen different quantum states*** (colors): Baryons: ***qqq in color singlet obey Fermi statistics***
      - quark interactions = Yang Mills theory based on unbroken color symmetry, gauge vectors=gluons
      - ***for massless gluons, self-energy gap between colored and uncolored bound states may be ∞: quark confinement?***

QCD: it all happened in few years

The years 1970-1973 brought decisive discoveries.

- 1970. The ***quark charm*** made possible electroweak unification for quarks without flavour changing neutral currents (***GIM mechanism***, next Lecture)
- In 1971, 't-Hooft and Veltman showed the ***renormalisability of the Weinberg-Salam theory***.
- In 1972, Bouchiat, Iliopoulos and Meyer proved the ***cancellation of Adler anomalies in an Electroweak theory*** with quark-lepton symmetry and fractionally charged quarks in three colors.
- 1973, the idea of color interaction of quarks was put forward by Fritzsch, Gell-Mann and Leutwyler: ***Quantum Chromo Dynamics (QCD)***:
- Fractionally charged quarks in 3 colors, color group commuting with the electroweak group, 8 electrically neutral gluons.
- 1973. Discovery of ***neutral current events*** by Gargamelle at CERN

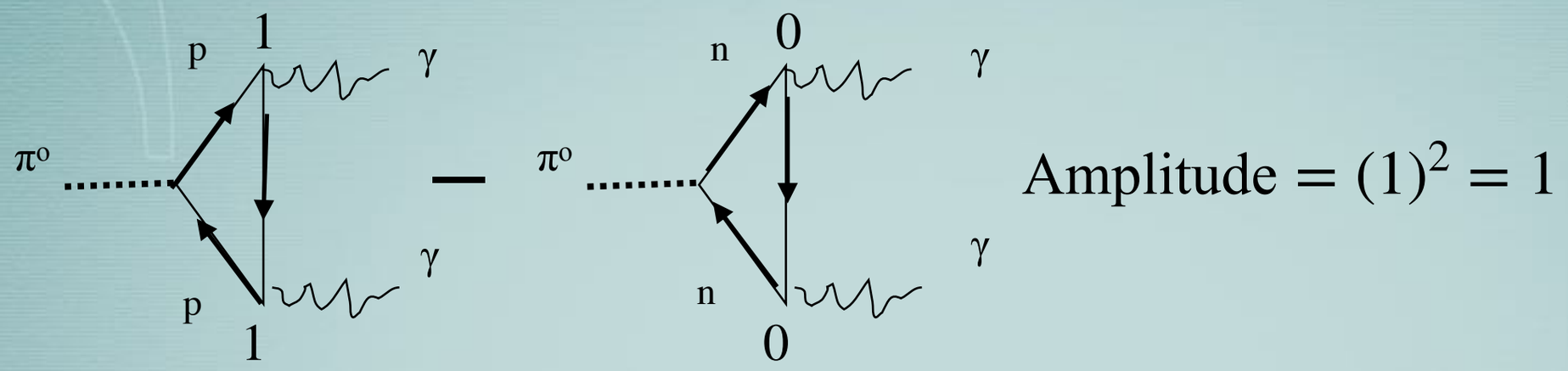
$$\nu_\mu + e \to \nu_\mu + e$$
$$\nu_\mu + N \to \nu_\mu + \text{hadrons}$$

- 1973…and in the same year came the discovery of ***asymptotic freedom of the Yang-Mills theor***y by Gross and Wilczeck and by Politzer.
- 1973, Kobayashi and Maskawa: a ***fourth quark generation to explain CP Violation*** ?

$\pi_0 \to \gamma + \gamma$ and quark colour

- $\pi^0 \to \gamma\gamma$: a long history:
  - Computed by J. Steinberger in Fermi-Yang theory, theory agrees with the observed rate:

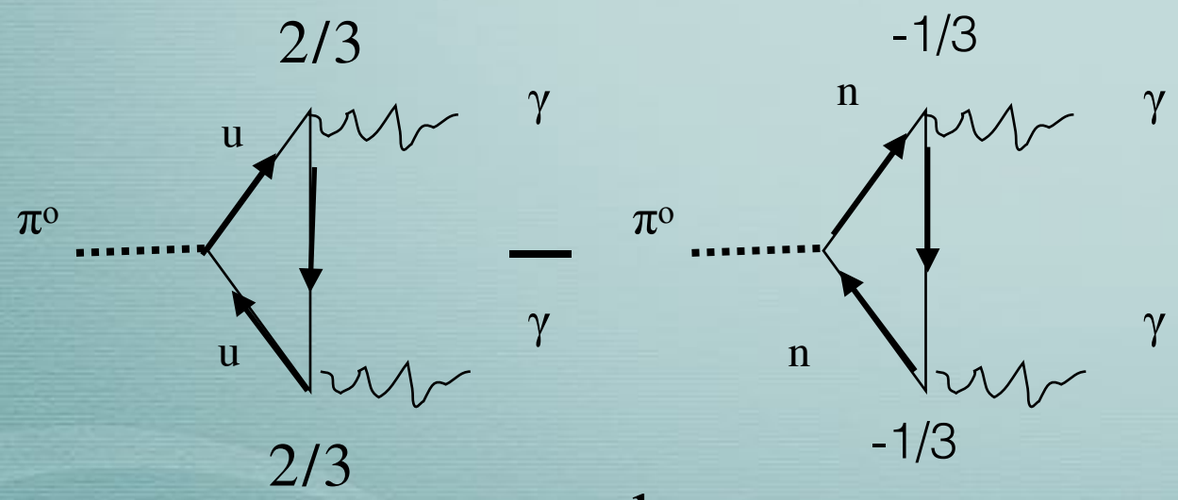
$$\pi_0(\bar{N}\tau_3 N) = \pi_0(\bar{p}p - \bar{n}n)$$



Amplitude $= (1)^2 = 1$

- With quarks:

$$\pi_0(\bar{q}\tau_3 q) = \pi_0(\bar{u}u - \bar{d}d)$$



no color

$A = (2/3)^2 - (-1/3)^2 = 1/3$

3 colors

$A = 1/3 \times 3 = 1$

Deep Inelastic Annihilation: SLAC, Adone



Adone (Frascati Laboratories, Italy)

- e^+e^-Colliding Ring 1969-1993
- C.o.M energy: up to 3.1 GeV
-deep inelastic annihilation at high energy:

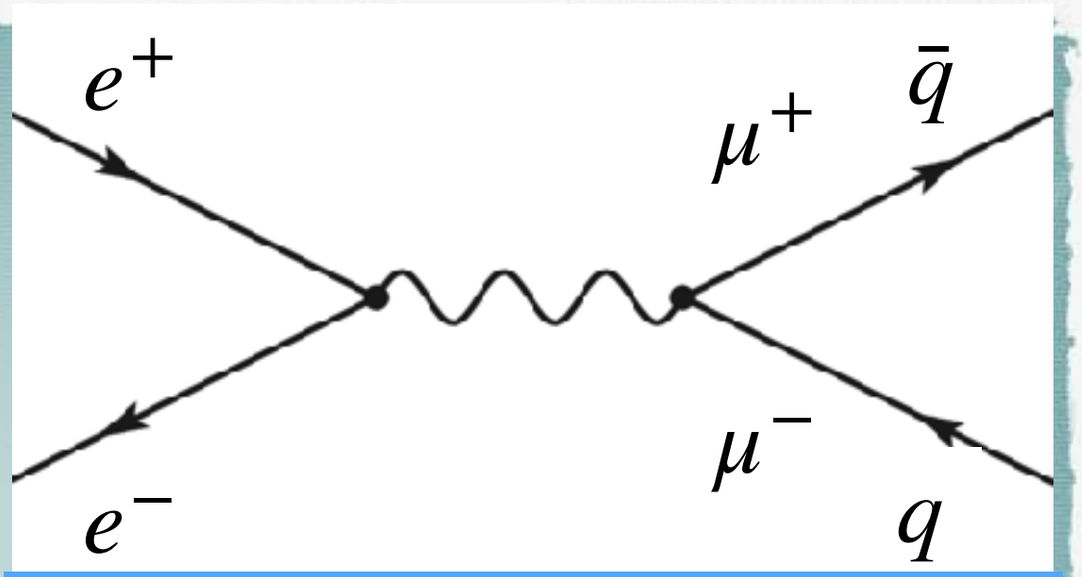
$$\frac{e^+e^- \to \text{hadrons}}{e^+e^- \to \mu^+\mu^-}$$

- confirmation of J/Ψ (1974)

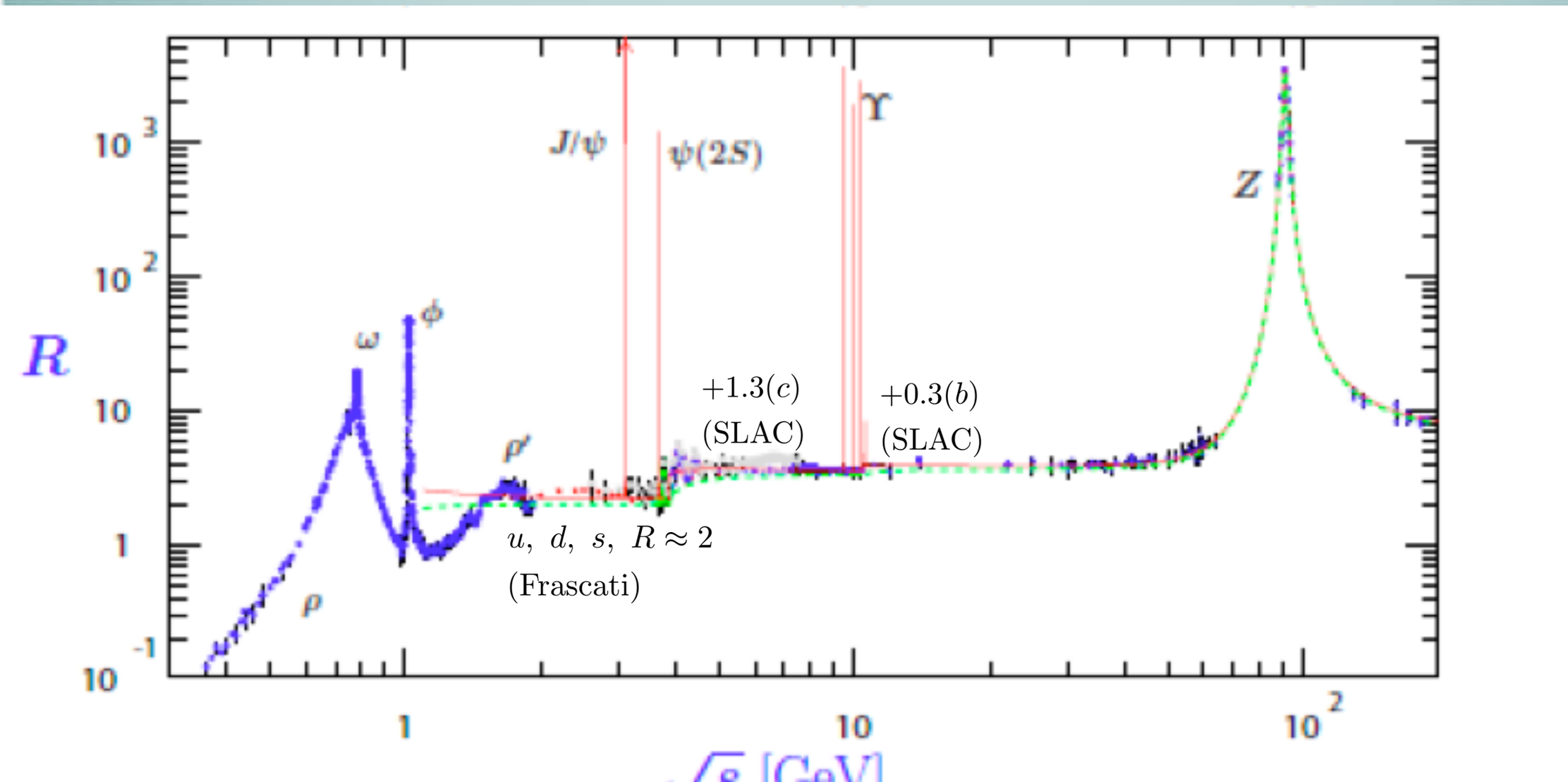
Confirmations of three colors (2)

Electron-Positron annihilation into μ+μ- and hadrons, $q^2>0$

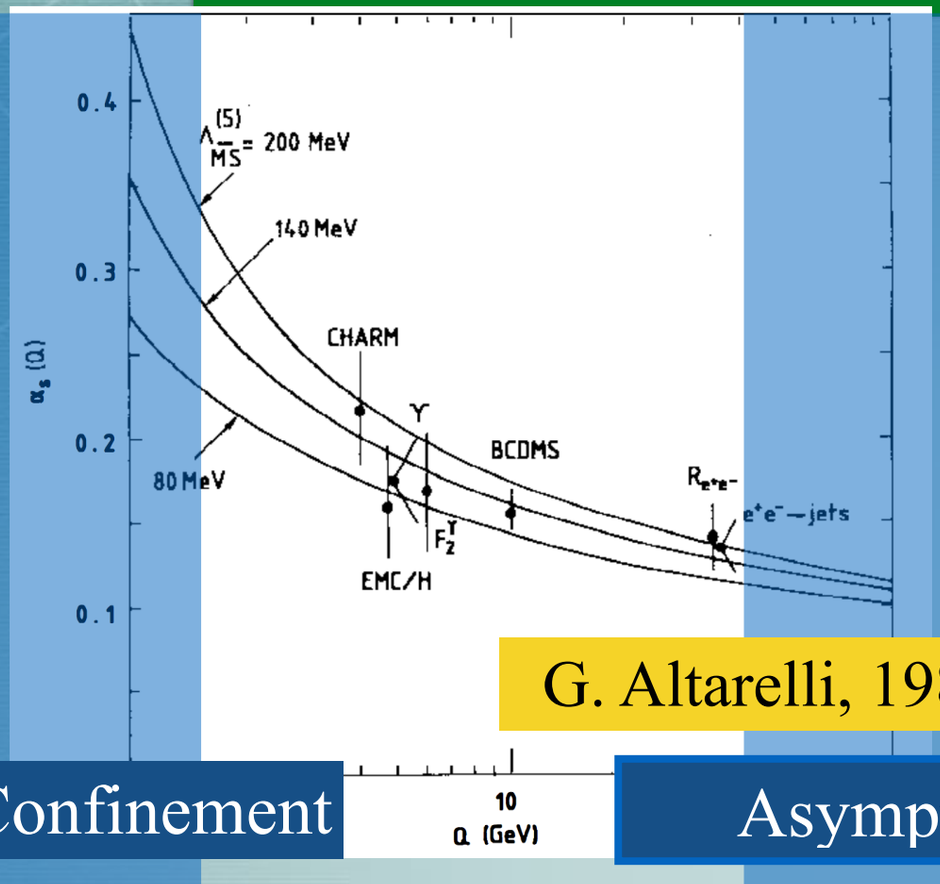
- Above resonances, hadron process is described by a simple diagram
- At ADONE, Frascati, expt: R~ 2; three colors predict: R = 3 (4/9+1/9+1/9)=2
  - above the charm threshold R increases by about 1.3 units, that is: 4/9x3=4/3 !!



$$R = \frac{\sigma(e^+e^- \to \mu^+\mu^-)}{\sigma(e^+e^- \to \text{hadrons})} = N_c(\Sigma_i\, Q_i)$$



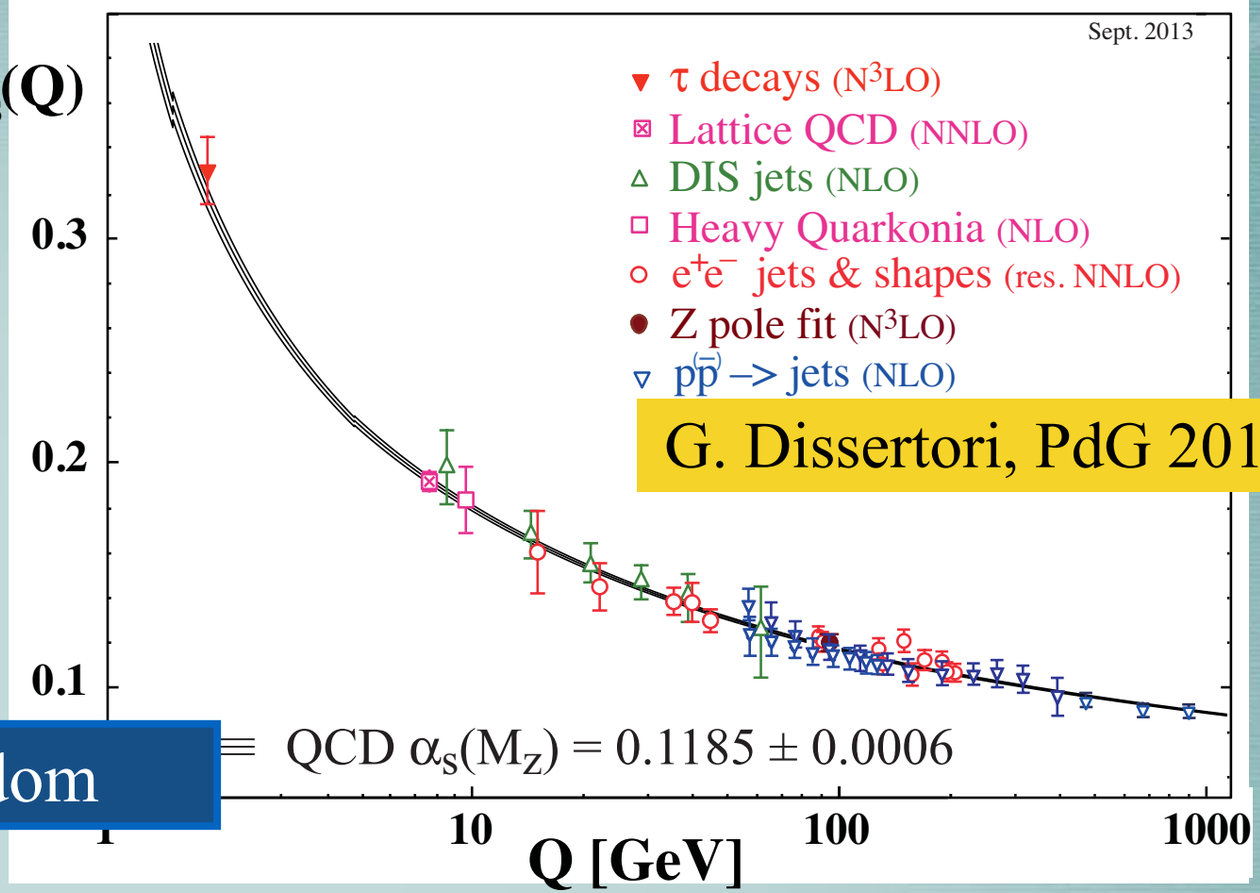
3. QCD is the answer to (almost) any question



G. Altarelli, 1989

Confinement

Asympt. freedom



G. Dissertori, PdG 2013

- QCD is asymptotically free
- quarks carry color and are confined inside color singlet hadrons
- only one way to make a color singlet with tree quarks: Fermi statistics is obeyed $\Delta^{++} = u^{\uparrow}_{\alpha} u^{\uparrow}_{\beta} u^{\uparrow}_{\gamma} \epsilon^{\alpha\beta\gamma}$
- proton at rest: only 3 quarks, dressed by strong QCD

Constituent Quarks

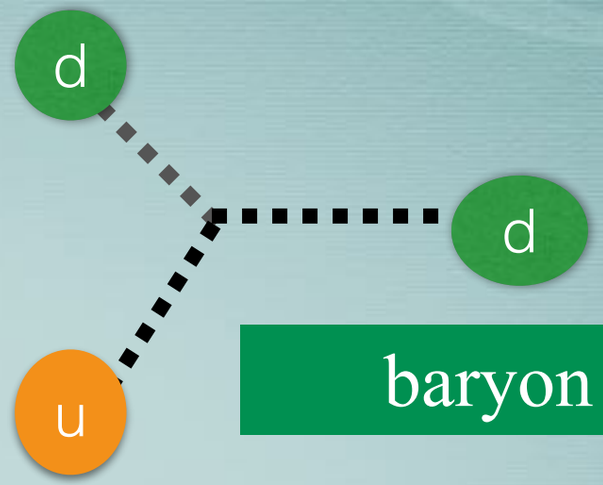
- increasing q^2, quarks radiate gluons (the Altarelli-Parisi picture of scaling violations)
- at large q^2, we see quarks and neutral gluons as almost free partons.

QCD Partons

QCD strings and confinement

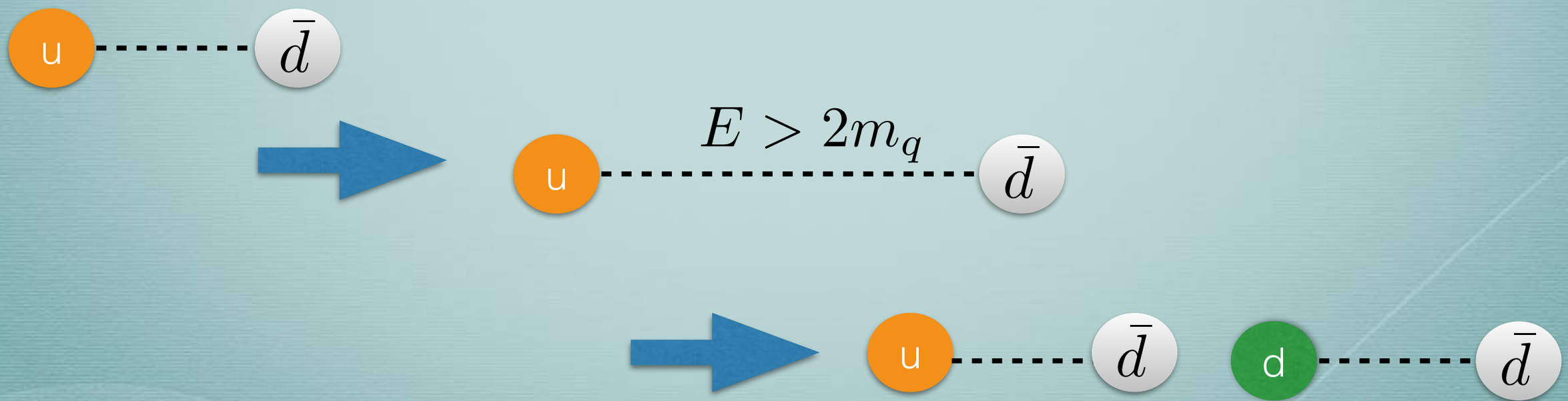


meson



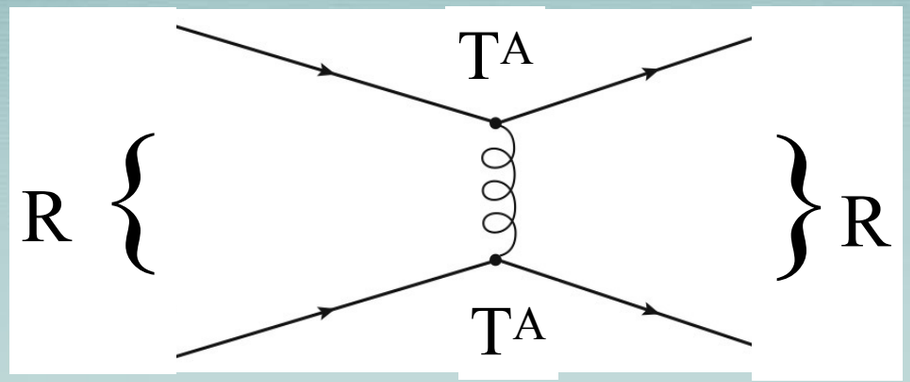
baryon

energy on the string is proportional to distance...



quarks do not come out

Forces between colored objects (Han-Nambu)



- Interaction proportional to: $\frac{g^2}{q^2} \times 2\langle R|\mathbf{T}\cdot\mathbf{T}|R\rangle = \frac{g^2}{q^2}\left[C_2(R) - C_2(A) - C_2(B)\right]$
- quark-antiquark
  - singlet: attractive (-8/3)
  - octet: repulsive (+1/3)
- quark-quark
  - three bar: attractive (-4/3)
  - six: repulsive (+2/3)

$$C_2(R) = \mathbf{T}_R \cdot \mathbf{T}_R$$

R	$C_2(R)$
1	0
8	3
$\mathbf{3}, \bar{\mathbf{3}}$	4/3
6	10/3

- quark-antiquark pairs bind in color singlet mesons;

- diquarks bind:

to another quark, to make a color singlet baryon

to an antidiquark, to make a color singlet tetraquark.

4. Constituent quark model

Old days: Sakharov&Zeldovich

In QCD: De Rujula, Georgi and Glashow, PRL 38 (1977) 317

Revisited & applied to tetraquarks: L.M., A. Polosa, V. Riquer, F, Piccinini, PRD 71 (2005) 014028

- color string forces produce an overall spin-independent potential that confines quarks inside a definite volume (bag), with some wave functions $\psi(x_1,x_2,x_3)$
- residual quark-quark or quark-antiquark interactions are color-magnetic, spin- spin, forces of the form

$$H_{ij} = \frac{g^2}{m_1 m_2}(T_1 \cdot T_2)\ \mathbf{s}_i \cdot \mathbf{s}_j\ \delta^{(3)}(x_1 - x_2)$$

- T are the color charges, g the color coupling, the form is derived from the non relativistic limit of QCD
- if i,j are in a color representation R , the formula simplifies to (CF is a color factor similar to the one introduced for color interactions)

$$H = \sum_i m_i + \sum_{i<j} 2\kappa_{ij}\ \mathbf{s}_j \cdot \mathbf{s}_j \qquad \kappa_{ij} = \text{Color Factor} \times \frac{g^2}{m_i m_j} |\Psi(0)|^2$$

$$H = \sum_i m_i + \sum_{i<j} 2\kappa_{ij}\ \mathbf{s}_j \cdot \mathbf{s}_j \qquad \kappa_{ij} = \text{Color Factor} \times \frac{g^2}{m_i m_j} |\Psi(0)|^2$$

- the Hamiltonian can be developed to first order in the small mass difference, m_s-$m_{u,d}$, the origin of flavour symmetry breaking
- there is also a first order contribution from the spin-spin interaction, which is very crucial for baryons
- one usually assumes that the wave function overlap is the same in all mesons and baryons, but this is dubious in the case of hadrons with heavy quarks, c or b.

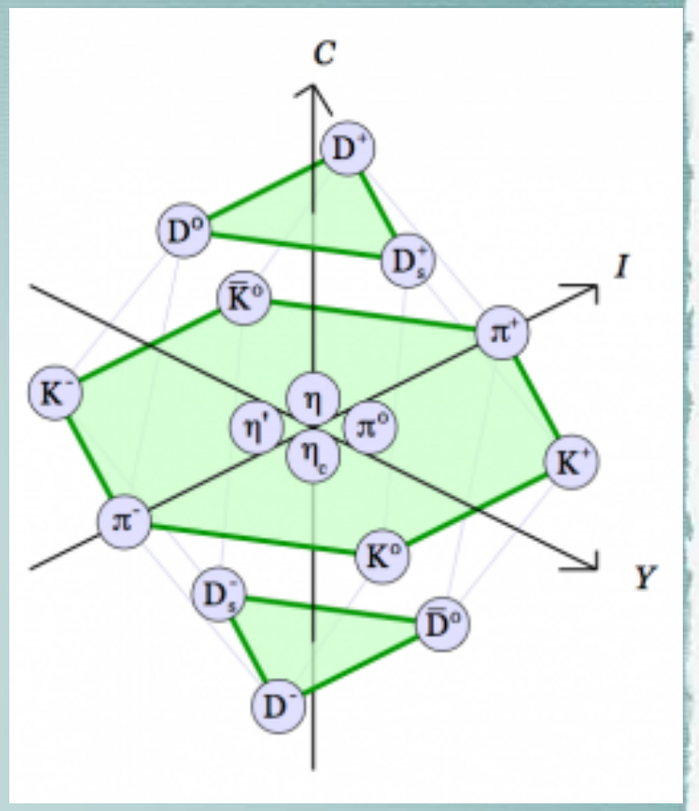
- Works well for mesons and baryons with charm=0 (too well?), still not bad for c=1 hadrons
- Few parameters: m_u, m_d, m_s, m_c…., κ_{ij}
- reproduce masses of different hadrons (within ±30 MeV)
- κ_{ij} scale approx. like $1/m_i m_j$ and not far from scaling with color factors

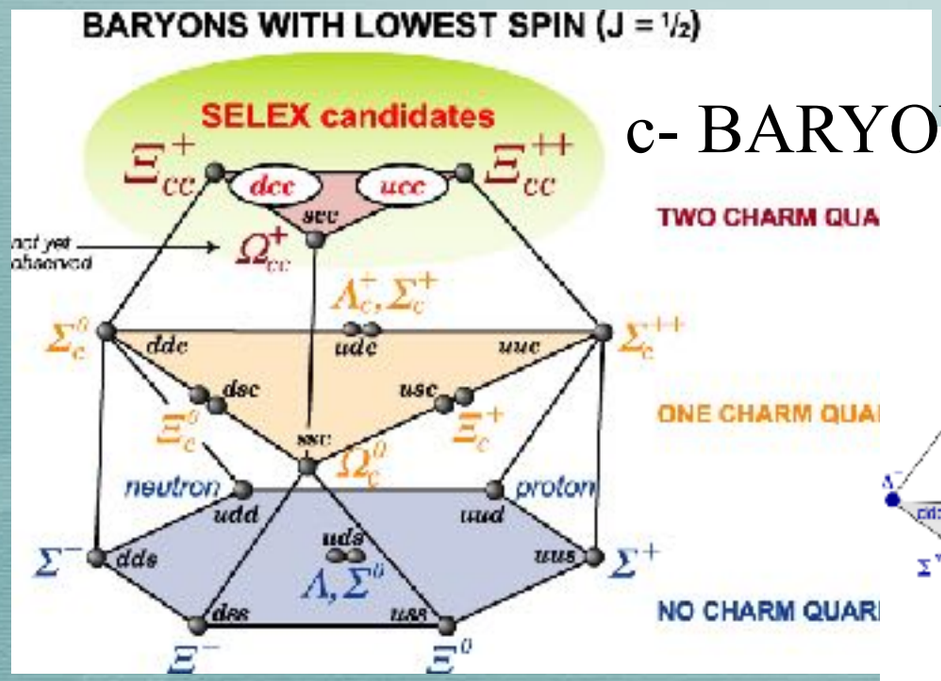
Remember. With $\mathbf{s_1} + \mathbf{s_2} = \mathbf{J}$:

$$2(\mathbf{s_1} \cdot \mathbf{s_2}) = J(J+1) - s_1(s_1+1) - s_2(s_2+1)$$

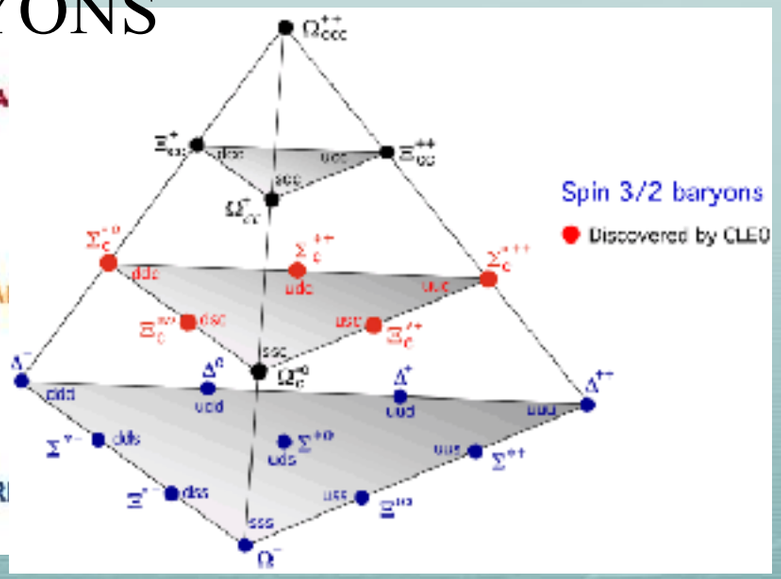
3. Spectroscopy of uncharmed and charmed Mesons and Baryons

- Particle states are displayed in a 3 dimensional space: I_3, Y and Charm (c) or Beauty (b)
- ***Mesons.*** Quark-antiquark states and fall in a 15+1 dimensional multiplet, both lowest states (spin 0-) and first resonances (spin1-)
- particles made by a pair of the same quark flavor are neutral and fall in the center of the multiplets
- B***aryons.*** Are 3 quark states, classified in two different 20-dimensional multiplets
- c and b Baryons are being observed in several different experiments

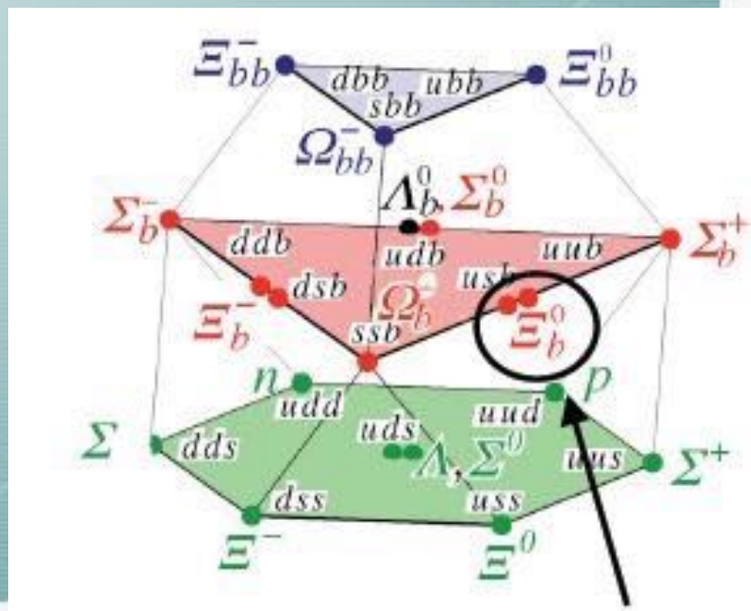




c- BARYONS



b- BARYONS



Meson masses (c=0), to warm up

- π = (q-qbar) in S wave (q=u,d), J=0; ρ, ω (q-qbar) in S wave, J=1,
- We find:

$$2(\mathbf{s}_{q_1}\cdot\mathbf{s}_{q_2}) = \mathbf{J}^2 - \mathbf{s}_{q_1}^2 - \mathbf{s}_{q_2}^2 = J(J+1) - 2s(s+1) = \begin{cases} +\frac{1}{2} & J=1 \\ & \\ -\frac{3}{2} & J=0 \end{cases}$$

$$m_\pi = 2m_q - \frac{3}{2}\kappa_{qq};\ m_\rho = m_\omega = 2m_q + \frac{1}{2}\kappa_{qq}$$

- we treat similarly the mesons with q s-bar (K and K*) and the meson with s s-bar (only φ; η is mixed with η' and is more complicated). We find
-

$$m_K = m_q + m_s - \frac{3}{2}\kappa_{sq};\ m_{K^\star} = m_q + m_s + \frac{1}{2}\kappa_{sq}$$

$$m_\phi = 2m_s + \frac{1}{2}\kappa_{ss}$$

- we have 5 parameters (2 masses and 3 kappas) and 6 masses, we get only the equality of ρ and ω masses
- but, to first order in the mass difference $m_s - m_q$: κ_{sq}-κ_{qq}= δκ, and κ_{ss}-κ_{qq}= 2δκ.
- We have one parameter less and another relation. In conclusion (masses in MeV):

$$m_\rho\ (775) = m_\omega\ (783)$$
$$m_\phi - m_{K^\star}\ (128) = m_{K^\star} - m_\rho(117)$$

Equal spacing of the Vector Meson multiplet

Baryons of the octet (N, Λ, Σ, Ξ)

- P, uud: J=1/2 and $s_{u1}+ s_{u2} =1$ (Fermi statistics, since u_1 and u_2 are antisymmetric in color !)

$$H_N = 3m_q + 2\kappa_{qq}[s_d \cdot (s_{u_1} + s_{u_2}) + s_{u_1} \cdot s_{u_2})]$$

$$\mathbf{m_N} = 3m_q + \kappa_{qq}(\frac{3}{4} - 2 - \frac{3}{4} + 2 - \frac{3}{2}) = 3m_q - \frac{3}{2}\kappa_{qq} = \mathbf{3(m_q - \frac{1}{2}\kappa_{qq})}$$

- Σ, uus: same trick, with d-> s:

$$\mathbf{m_\Sigma} = 2m_q + m_s + \kappa_{sq}(\frac{3}{4} - 2 - \frac{3}{4}) + \kappa_{qq}(2 - \frac{3}{2}) = 2m_q + m_s + \frac{1}{2}\kappa_{qq} - 2\kappa_{sq} =$$

$$= \mathbf{3(m_q - \frac{1}{2}\kappa_{qq}) + (m_s - m_q) - 2(\kappa_{sq} - \kappa_{qq})}$$

- Λ, uds: ud is in Isospin =0 -> must be $s_u+s_d = 0$ (Fermi statistics again):

$$\mathbf{m_\Lambda} = 2m_q + m_s + \kappa_{sq}(\frac{3}{4} - \frac{3}{4}) + \kappa_{qq}(-\frac{1}{2}) = 2m_q + m_s - \frac{3}{2}\kappa_{qq} =$$

$$= \mathbf{3(m_q - \frac{1}{2}\kappa_{qq}) + (m_s - m_q)}$$

- Ξ, uss, with ss in spin1: like Σ, with u⟺s

$$\mathbf{m_\Xi} = m_q + 2m_s - 2\kappa_{sq} + \frac{1}{2}\kappa_{ss} =$$

$$= \mathbf{3(m_q - \frac{1}{2}\kappa_{qq}) + 2(m_s - m_q) - 2(\kappa_{sq} - \kappa_{qq}) + \frac{1}{2}(\kappa_{ss} - \kappa_{qq})}$$

Baryons of the octet (N, Λ, Σ, Ξ) (cont'd)

- We have 4 masses (N, Λ, Σ, Ξ),
- 4 parameters: m_q-1/2κ_{qq}, m_s-m_q, κ_{sq}-κ_{qq}, κ_{ss}-κ_{qq}
- if we use the relation κ_{sq}-κ_{qq}=$\delta\kappa$, and κ_{ss}-κ_{qq}=2$\delta\kappa$ (1^{st} order in the mass difference $\delta = m_s - m_q$), we have one relation, the ***Gell-Mann, Okubo relation***:

$$\frac{N+\Xi}{2}\,(1128\ \mathrm{MeV}) \;=\; \frac{3\Lambda+\Sigma}{4}\,(1136\ \mathrm{MeV})$$

check it !!

Baryons of the decuplet

$$m_\Delta = 3m_q + \frac{3}{2}\kappa_{qq};\ \ m_P = 3(m_q - \frac{1}{2}\kappa_{qq})$$

- Δ, uuu: we can separate m_q from κ_{qq} ;
- we can compute all decuplet masses in term of parameters determined by spin 1/2 baryon masses.
- we find equally spaced masses in the decuplet, with spacing

$$\Delta M = m_s - m_q + \kappa_{sq} - \kappa_{qq} = 139 \text{ Me}$$

- experimentally

$$m(\Sigma^*) - m(\Delta) = 153$$
$$m(\Xi^*) - m(\Sigma^*) = 145$$
$$m(\Omega) - m(\Xi^*) = 142$$

Gell-Mann's prediction of Ω^- mass.

- in addition to Gell-Mann Okubo we have the prediction of the mass difference (139=153), of similar quality of the other predictions
- octet and decuplet: 8 masses, 4 parameters, 4 relations, satisfied within 20 MeV.

Parameters

	q	s
MESONS: quark mass (MeV)	308	484
BARYONS: quark mass (MeV)	362	540
difference (MeV)	54	56

Table 1: S-wave Mesons and Baryons: constituent quark masses.

MESONS	$(q\bar{q})_1$	$(q\bar{s})_1$	$(s\bar{s})_1$
κ_{ij} (MeV)	318	200	103
$\kappa_{ij} m_i m_j / \Lambda^3_{QCD}$	1.9	1.9	1.5
BARYONS	$(qq)_{\bar{3}}$	$(qs)_{\bar{3}}$	$(ss)_{\bar{3}}$
κ_{ij} (MeV)	98	59	23
$\kappa_{ij} m_i m_j / \Lambda^3_{QCD}$	0.82	0.74	0.43
Ratio: $\frac{\kappa_{MES}}{\kappa_{BAR}}$	3.2	3.4	4.5

$\Lambda_{QCD} = 250$ MeV

= Constant?

Table 1: S-wave Mesons and Baryons: spin-spin couplings.

5. Summing up

- Standard Theory with light quarks (u, d, s)…until now
  - -A brilliant description of low energy spectroscopy
  - -Scaling in deep inelastic electron and neutrino experiments and in electron-positron annhilation: explained by asymptotic freedom of QCD
  - - Weak decays of light baryons and mesons: explained by Cabibbo's theory
- What is missing?
  - -suppression of Flavour Changing Neutral Weak Currents: GIM, next lecture
  - -CP violation in K decays: 3^d quark&lepton generation (KM,1973), next lecture
  - -Higgs boson search and discovery: last lecture
- What next?????