

Effects of imaginary and real rotations on QCD matters

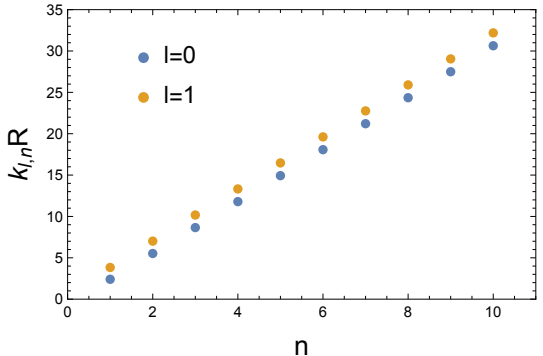
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The partition function of the rotating system is given by:

$$Z(\Omega) = \mathrm{Tr} \exp\left[-\beta(H - \Omega J_z)\right], \tag{1}$$

the total angular momentum :$J_z = L_z + S_z$, and solution is the Bessel function of the first kind, $J_l(k_l r)$, where l is the orbital angular momentum quantum number and $k_l r$ is the transverse momentum.



The n-th eigenenergies $k_{l,n}$ that satisfy $J_l(k_{l,n}R) = 0$ for $l = 0$ (blue) and $l = 1$ (yellow), with the ordering $k_{0,n} < k_{1,n} < k_{0,n+1} < k_{1,n+1}$.

In the pure SU(3) gauge theory, we consider a constant and diagonal background gauge field:

$$\boxed{g\bar{A}_4 = T\begin{pmatrix} q_1 & 0 & 0 \\ 0 & q_2 & 0 \\ 0 & 0 & q_3 \end{pmatrix} = T\,\mathrm{diag}(q_1, q_2, q_3)}$$

with

$$\boxed{[g\bar{A}_4, E_{ij}] = T(q_i - q_j)E_{ij}}$$

In the background field method, the covariant derivative of the gluon field is given by

$$\bar{D}_\mu a_\nu = \partial_\mu a_\nu - ig[\bar{A}_\mu, a_\nu]$$

Therefore, the eigenvalue can be written as

$$\lambda_{n,ij} = (\omega_n \pm Tq_{ij})^2 + \boldsymbol{p}^2$$

Ghost Field in the Diagonal Background

The quadratic ghost action in the background field method is

$$S^{(2)}_{\rm gh} = \int d^4x\, \bar{c}(-\bar{D}^2)c, c(x) = \sum_{i\neq j} c^{ij}(x)E_{ij} + c_{\rm diag}(x)$$

where the background covariant derivative is

$$\bar{D}_\mu c = \partial_\mu c - ig[\bar{A}_\mu, c].$$

Using the commutation relation,

$$\boxed{[g\bar{A}_4, E_{ij}] = T(q_i - q_j)E_{ij}}$$

$$\bar{D}_4 = \partial_4 - iTq_{ij}.$$

Using the bosonic Matsubara frequency $\omega_n = 2\pi nT$, we obtain the ghost inverse propagator eigenvalue:

$$\boxed{\lambda^{\rm gh}_{n,ij} = (\omega_n \pm Tq_{ij})^2 + \boldsymbol{p}^2}$$

Rotational Effects on Ghost Fields

The ghost field is a Lorentz scalar and therefore has no spin contribution to the total angular momentum. Its rotational effects arise solely from orbital angular momentum ℓ.
In the presence of imaginary angular velocity Ω_I, the color-dependent frequency is shifted as

$$\boxed{q_{ij} \longrightarrow q_{ij} + \ell \frac{\Omega_I}{T}}$$

Consequently, the ghost dispersion relation becomes

$$\boxed{\lambda^{\mathrm{gh}}_{n,ij,\ell} = \left(\omega_n \pm T q_{ij} + \ell \Omega_I\right)^2 + p_z^2 + k_{\perp,\ell}^2}$$

where ℓ is the orbital angular momentum quantum number, and $\omega_n = 2\pi n T$ is the bosonic Matsubara frequency.

Rotational Effects in the Deconfined Phase

Perturbative calculations show that imaginary rotation can be introduced through the transformation

$$q_{ij} \longrightarrow q_{ij} + s\frac{\Omega_I}{T}, \qquad s = \pm 1.$$

Ghost Contributions in the Confined Phase

Although ghost fields are Lorentz scalars, they carry orbital angular momentum ℓ and can be affected by rotation.
For a finite cylindrical system, the radial modes satisfy

$$J_\ell(k_{\ell,n}R) = 0.$$

Nonzero angular momentum modes may contribute to spatially averaged quantities.

The rotational transformation obtained in the deconfined phase is extended to the full Polyakov-loop potential in the confined phase.

$$V^{\Omega_I}(q_{ij}) = \frac{1}{2}\sum_{s=\pm 1} V\left(q_{ij} + s\frac{\Omega_I}{T}\right).$$

Fukushima Potential (V_1)

The logarithmic Polyakov-loop potential is given by

$$\boxed{\frac{V_1(L,L^*)}{T^4} = -\frac{a_1(\widetilde{T})}{2}|L|^2 - \frac{1.75}{\widetilde{T}^3}\ln H(L,L^*)}$$

with

$$a_1(\widetilde{T}) = 3.51 - \frac{2.47}{\widetilde{T}} + \frac{15.2}{\widetilde{T}^2}.$$

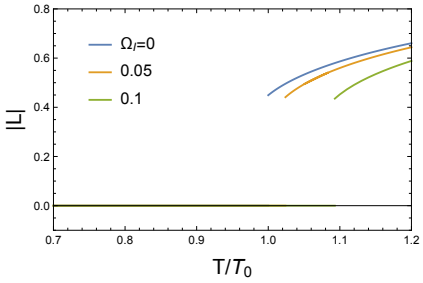
Munich Polynomial Potential (V_2)

The polynomial Polyakov-loop potential is

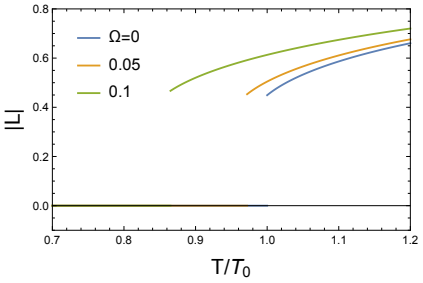
$$\boxed{\frac{V_2(L,L^*)}{T^4} = -\frac{a_2(\widetilde{T})}{2}|L|^2 - \frac{0.75}{6}(L^3 + L^{*3}) + \frac{7.5}{4}|L|^4}$$

with

$$a_2(\widetilde{T}) = 6.75 - \frac{1.95}{\widetilde{T}} + \frac{2.625}{\widetilde{T}^2} - \frac{7.44}{\widetilde{T}^3}.$$



Polyakov loop versus temperature under imaginary rotation



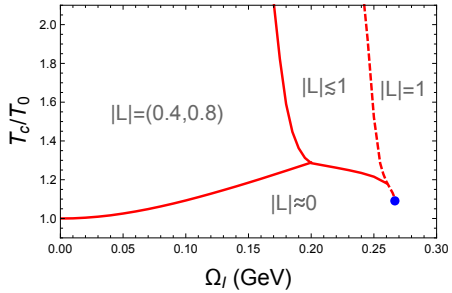
Polyakov loop versus temperature under real rotation

$$\underbrace{T_c(\Omega_I)\uparrow}_{\text{Imaginary rotation}} \xrightarrow{\Omega_I \to -i\Omega} \underbrace{T_c(\Omega)\downarrow}_{\text{Real rotation}}$$

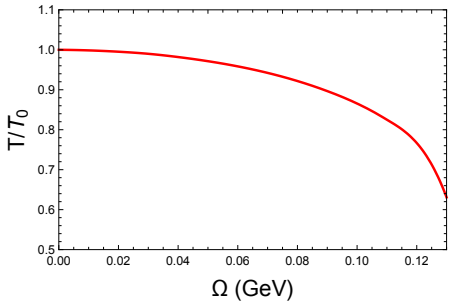
Since this branch corresponds to the deconfinement transition

$$|L| \approx 0 \longrightarrow |L| \approx 0.4$$

V_1: Phase Diagrams under Rotation



T–Ω_I phase diagram under imaginary rotation

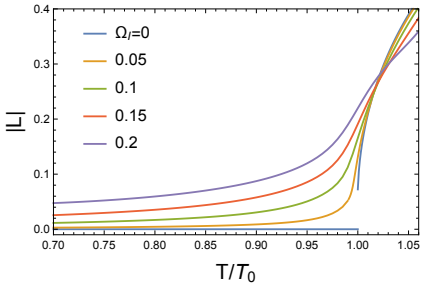


T–Ω phase diagram under real rotation

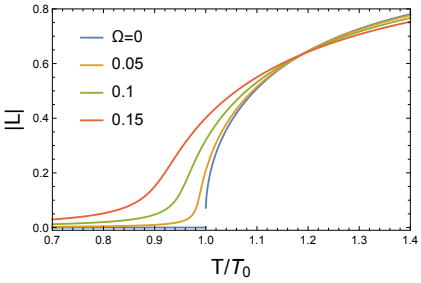
Imaginary rotation: Three transition branches appear due to multiple local minima of the effective potential.

$$
\begin{aligned}
|L| \approx 0 \to |L| \approx 0.4 \quad & (T_c \uparrow), \\
|L| \approx 0.8 \to |L| \lesssim 1 \quad & (T_c \downarrow), \\
|L| \lesssim 1 \to |L| = 1 \quad & (T_c \downarrow).
\end{aligned}
$$

Real rotation: Only one first-order transition branch remains, with T_c decreasing as Ω increases.



Polyakov loop versus temperature under imaginary rotation

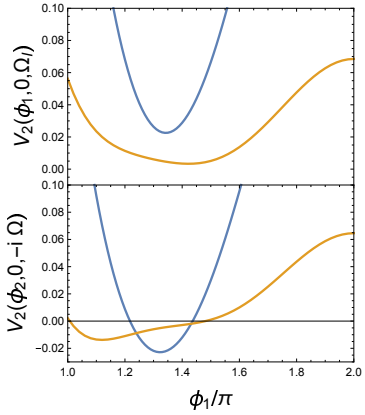


Polyakov loop versus temperature under real rotation

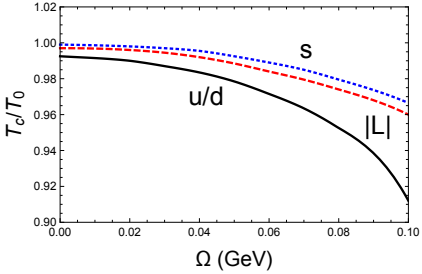
$$\underbrace{T_c(\Omega_I)\downarrow}_{\text{Imaginary rotation}} \xrightarrow{\Omega_I \to -i\Omega} \underbrace{T_c(\Omega)\downarrow}_{\text{Real rotation}}$$

Since this branch corresponds to the deconfinement transition

$$|L| \approx 0 \longrightarrow |L| \approx 0.4?$$



Assume $\phi_2 = 0$, the potential V_2 is demonstrated as a function of ϕ_1 for $\Omega_I = 0.05\,\mathrm{GeV}$ (upper panel) and $\Omega = 0.05\,\mathrm{GeV}$ (lower panel) with $T/T_0 = 0.9$ (blue) and 1 (yellow).



The pseudocritical temperatures T_c as functions of re rotation Ω for $m_{u/d}$, m_s and $|L|$ in the PNJL model

SU(3) Yang–Mills theory in a constant $\bar{A}_0$ background

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Yang–Mills Lagrangian

$$\mathcal{L}_{\rm YM} = \frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu}, \qquad F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu.$$

The gauge field is decomposed into a background and quantum fluctuations:

$$A^a_\mu = B^a_\mu + Q^a_\mu, \qquad \mathcal{L}_{\rm gf} = \frac{1}{2\xi} \left[D^{ab}_\mu(B) Q^b_\mu \right]^2, \qquad \xi = 1.$$

Constant temporal background and Polyakov loop

$$\boxed{\bar{A}_0 = \bar{A}^3_0 T^3 + \bar{A}^8_0 T^8}$$

$$\boxed{\Omega_g = 2T \int \frac{d^3p}{(2\pi)^3} \, \mathrm{tr}_A \ln \left(1 - \hat{L}_A e^{-E_g/T} \right).}$$

Two method

Cao

$$\Omega_g = 2T \int \frac{d^3p}{(2\pi)^3} \operatorname{tr}_A \ln\left(1 - \hat{L}_A e^{-E_g/T}\right)$$

with

Eigenvalues of $\hat{L}_A$:

$$\left\{1, 1, e^{iq_{12}}, e^{-iq_{12}}, e^{iq_{13}}, e^{-iq_{13}}, e^{iq_{23}}, e^{-iq_{23}}\right\}.$$

Using $q_{ij} \to \frac{1}{2}(q_{ij} - s\Omega_I)$

Chen shi et.al

$$\Omega_g = 2T \int \frac{d^3p}{(2\pi)^3} \operatorname{tr}_A \ln\left(1 - \hat{L}_A e^{-E_g/T}\right)$$

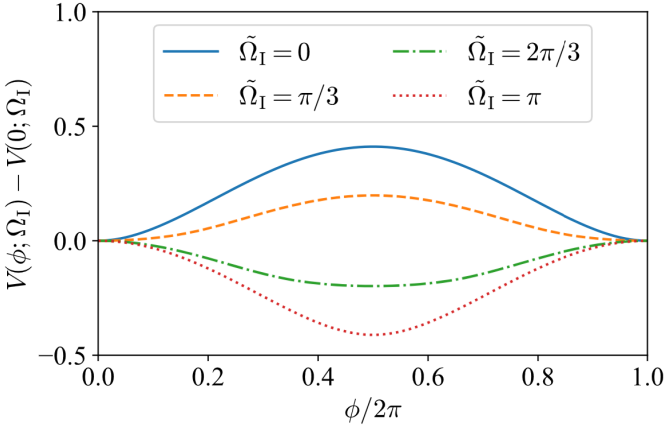
Under rotation, the gluon energy $E_g = |\mathbf{k}|$ is modified by the coupling between the angular velocity and the total angular momentum, giving

$$E_g^{\Omega} = |\mathbf{k}| - \Omega(l + s_z). \tag{2}$$

when $r = 0$:

$$E_g^{\Omega} = |\mathbf{k}| - \Omega(s). \tag{3}$$

Ω_g=Weiss potential



Evolution of the Polyakov loop potential (made dimensionless with T^4) for $\tilde{\Omega}_I = 0,\ \pi/3,\ 2\pi/3,\ \pi$ in the color SU(2) case at $\tilde{r} = 0$. Chen, S., Phys. Rev. Lett. **129**, 242002 (2022).

Fukushima Polyakov-loop Effective Potential (V_1)

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Effective potential

$$V_1 = \underbrace{-\frac{a(\widetilde{T})}{2} T^4 |L|^2}_{\text{Weiss term}} + \underbrace{-1.75\, T T_0^3 \ln H(L, L^*)}_{\text{Non-Weiss term}}$$

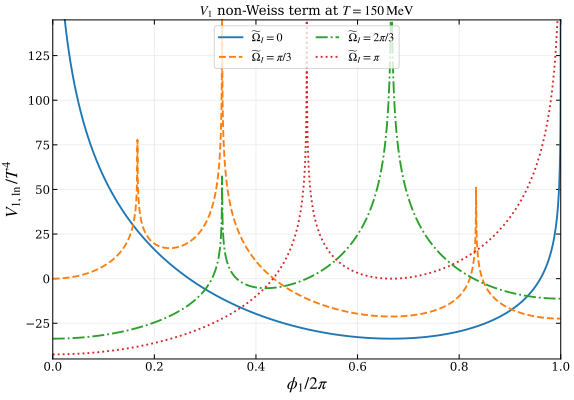
where

$$a(\widetilde{T}) = 3.51 - \frac{2.47}{\widetilde{T}} + \frac{15.2}{\widetilde{T}^2}, \qquad \widetilde{T} = \frac{T}{T_0}.$$

$$H(L, L^*) = 27\left[1 - 6|L|^2 + 4(L^3 + L^{*3}) - 3|L|^4\right].$$

The effective potential consists of a quadratic Polyakov-loop term and a logarithmic Haar-measure term.

non-Weiss term



Adding rotation

Polyakov loop

$$L(\phi_1) = \frac{1 + 2\cos\left(\frac{\phi_1}{2}\right)}{3}.$$

rotation

$$\Omega_I \uparrow .$$

$$L : 0 \to 1.$$

Weiss term

$$\Omega_I \uparrow .$$

$$L : 1 \to 0.$$

Effective potential

$$V = \underbrace{2T \int \frac{d^3p}{(2\pi)^3} \, \mathrm{tr}_A \ln\left(1 - \hat{L}_A e^{-E_g/T}\right)}_{\text{Weiss term}} \underbrace{-B(T) \ln H(L, L^*)}_{\text{Non-Weiss term}}$$

CF model/Matrix model

$$V = \underbrace{2T \int \frac{d^3p}{(2\pi)^3} \, \mathrm{tr}_A \ln\left(1 - \hat{L}_A e^{-E_g/T}\right)}_{\text{Weiss term}} \underbrace{+V}_{\text{Non-Weiss term}}$$

Other model

$$V = \underbrace{2T \int \frac{d^3p}{(2\pi)^3} \, \mathrm{tr}_A \ln\left(1 - \hat{L}_A e^{-E_g/T}\right)}_{\text{Weiss term}} \underbrace{+V}_{\text{Vaccum term}}$$

Thank you for your attention!