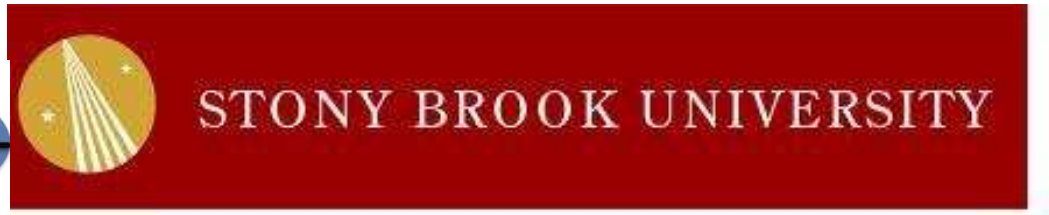
Causal Limits to Neutron Star Properties

J. M. Lattimer







Institute of High Energy Physics
Chinese Academy of Sciences
Beijing, China

16 November 2026

Acknowledgements

Funding Support:

DOE - Nuclear Physics

NASA - Neutron Star Interior Composition ExploreR (NICER)

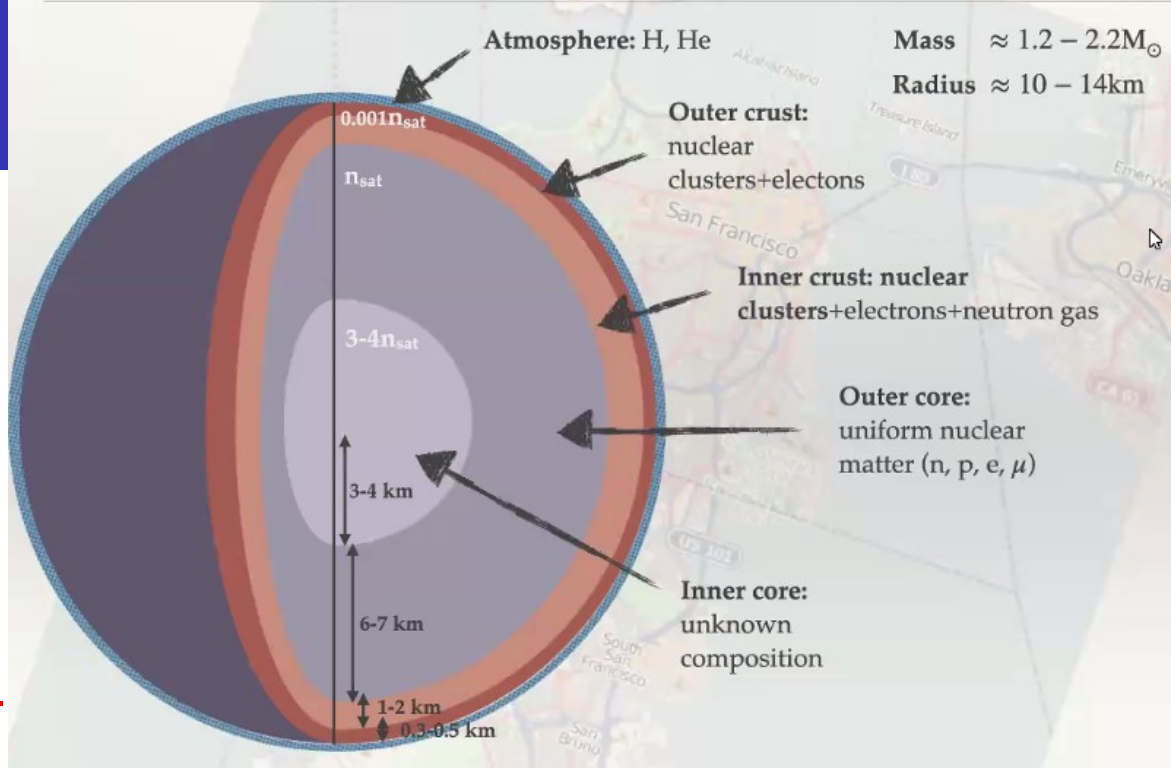
NSF - Neutrinos, Nuclear Astrophysics and Symmetries (PFC - N3AS)

Recent Collaborators:

Boyang Sun (Stony Brook), Tianqi Zhao (Berkeley), Probit Kalita and Bharat Kumar (Rourkela, NIT), Tuhin Malik(Coimbra U.)

Neutron Stars Basics

- Nearly all known NSs are pulsars (rapidly rotating and highly magnetized) that emit X-ray, optical or radio beams from their poles, like a lighthouse.



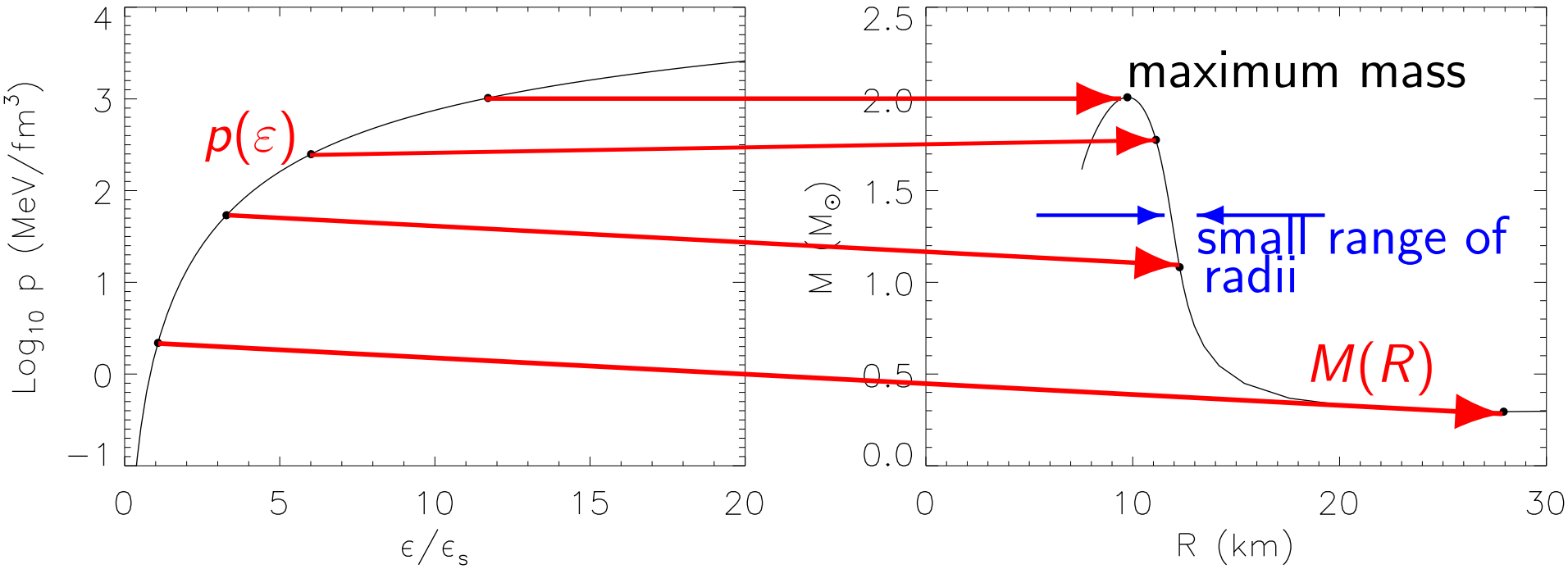
- The radii of most NSs are about 12 km.
- NSs are believed to be solely formed in the gravitational collapse of massive stars at the ends of their lives; some of those collapses produce black holes instead.
- The minimum theoretical NS mass is $0.1M_\odot$ but the smallest that can be made in supernovae are about $1.1M_\odot$; none are observed with $\lesssim 1M_\odot$.
- The maximum possible NS mass is probably less than $2.5M_\odot$; the largest observed mass is 2.08 ± 0.07 (PSR J0437-4715).

Neutron Star Structure

Tolman-Oppenheimer-Volkov equations

$$\frac{dp}{dr} = -\frac{G}{c^4}\frac{(mc^2 + 4\pi pr^3)(\varepsilon + p)}{r(r - 2Gm/c^2)}$$

$$\frac{dm}{dr} = 4\pi\frac{\varepsilon}{c^2}r^2$$



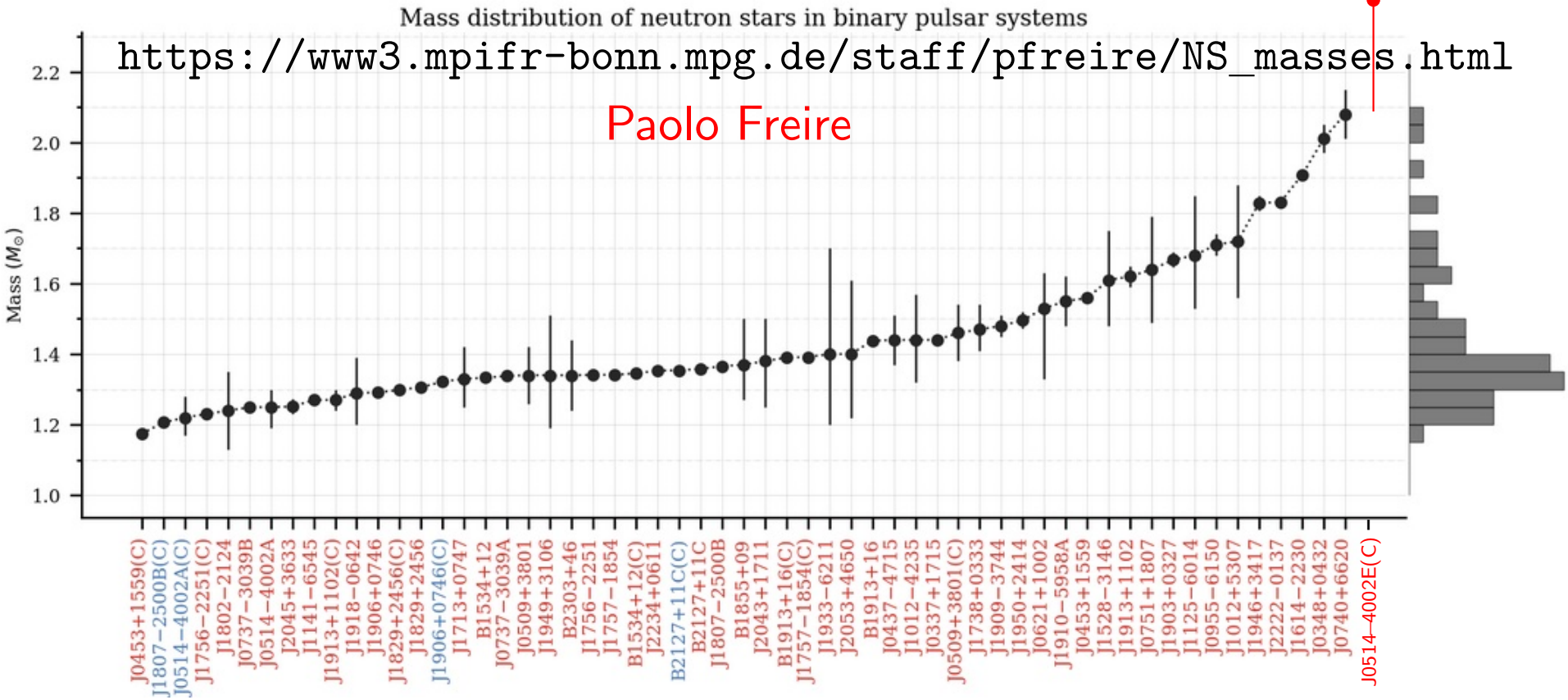
Equation of State ⟵ Observations

Importance of Mass-Radius

Crucial constraints:

- M_{lower}, the lower limit to the neutron star maximum mass M_{max}, observationally estimated by the most massive, accurately-measured, pulsar mass.
- $R_{1.4}$, the radius of a $1.4 M_\odot$ star, highly correlated with the neutron matter pressure near the saturation density $n_s \simeq 0.16$ fm^{-3}, $\rho_s \simeq 2.7 \times 10^{14}$ g cm^{-3}, $\mathcal{E}_s \simeq 150$ MeV fm^{-3}.
- $\Delta R = R_{2.0} - R_{1.4}$, an indicator of stiffening/softening of the EOS for $\mathcal{E} \gtrsim 2\mathcal{E}_s$, possibly due to a first-order or crossover transition to quark matter.

Masses of Pulsars in Binaries from Pulsar Timing



Largest: 2.08 ± 0.07 $M_\odot$
Smallest: 1.174 ± 0.004 $M_\odot$

Many NSs measured by other means include some estimated with $> 2M_\odot$ (e.g., black widow pulsars like PSRs J0514-4002E [$2.40 \pm 0.31 M_\odot$] and J0952-0607 [$2.35 \pm 0.17 M_\odot$]) and $< 1M_\odot$ (HESS J1731-347), but have large uncertainties or disputed values.

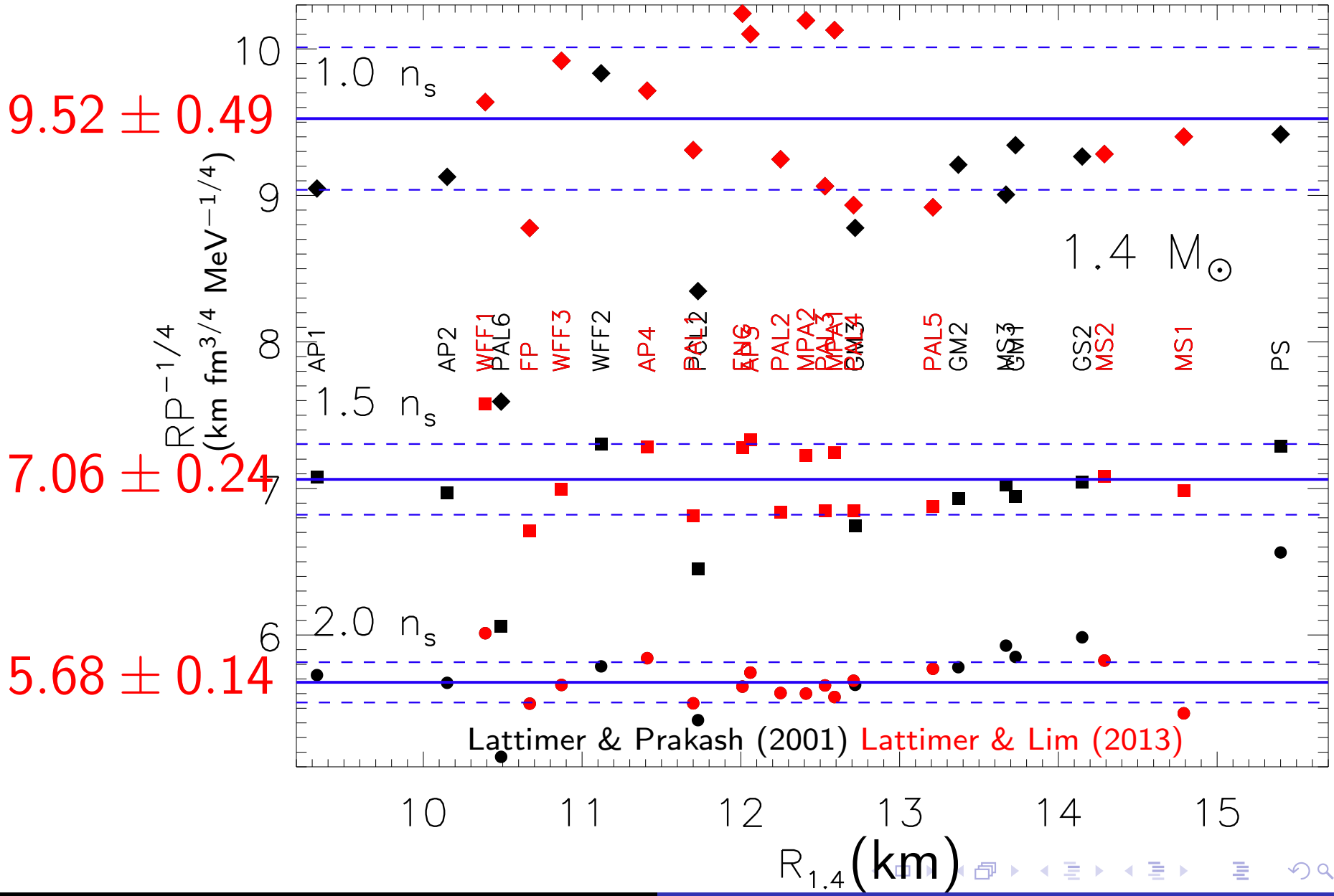
What is the Maximum Mass?

Minimum Maximum Mass

- PSR B1516+02B (Freire et al. 2008) $M = 2.08 \pm 0.19\ M_\odot$
- PSR J1614+2230 (Arzoumanian et al. 2018) $M = 1.908 \pm 0.016 M_\odot$
- PSR J1748-2021B (Clifford 2019) $M = 2.53^{+0.05}_{-0.08}\ M_\odot$ (total mass is $2.675 \pm 0.022 M_\odot$, but inclination is unknown)
- PSR J0348+0432 (Saffer et al. 2025) $M = 1.806 \pm 0.037\ M_\odot$
- PSR 1957+20 (van Kerkwijk 2010) $M = 2.4 \pm 0.3\ M_\odot$; black widow pulsar (BWP)
- PSR J1311-3430 (Romani et al. 2012) $M = 2.22 \pm 0.10 M_\odot$ BWP
- PSR J1544+4937 (Tang et al. 2014) $M = 2.06 \pm 0.56! M_\odot$ BWP
- PSR 2FGL J1653.6-0159 (Romani et al. 2014) $M > 1.96\ M_\odot$
- PSR J1227-4859 (de Martino et al. 2014) $M = 2.2 \pm 0.8\ M_\odot$ redback pulsar.
- PSR J0740+6620 (Fonseca et al. 2021) $M = 2.08 \pm 0.07\ M_\odot$
- PSR J0952-0607 (Romani et al. 2022) $M = 2.35 \pm 0.17 M_\odot$ BWP
- PSR J0514-4002E(C) (Barr et al. 2024) $M = 2.40 \pm 0.31\ M_\odot$ companion to $1.49 \pm 0.31\ M_\odot$ pulsar

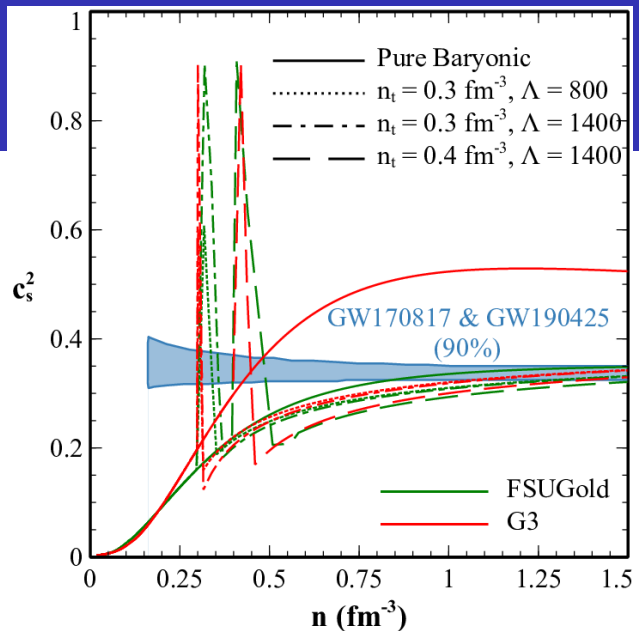
Maximum mass upper limit from GW170817: $\simeq$ 2.2-2.3 $M_\odot$?

The Radius – Pressure Correlation

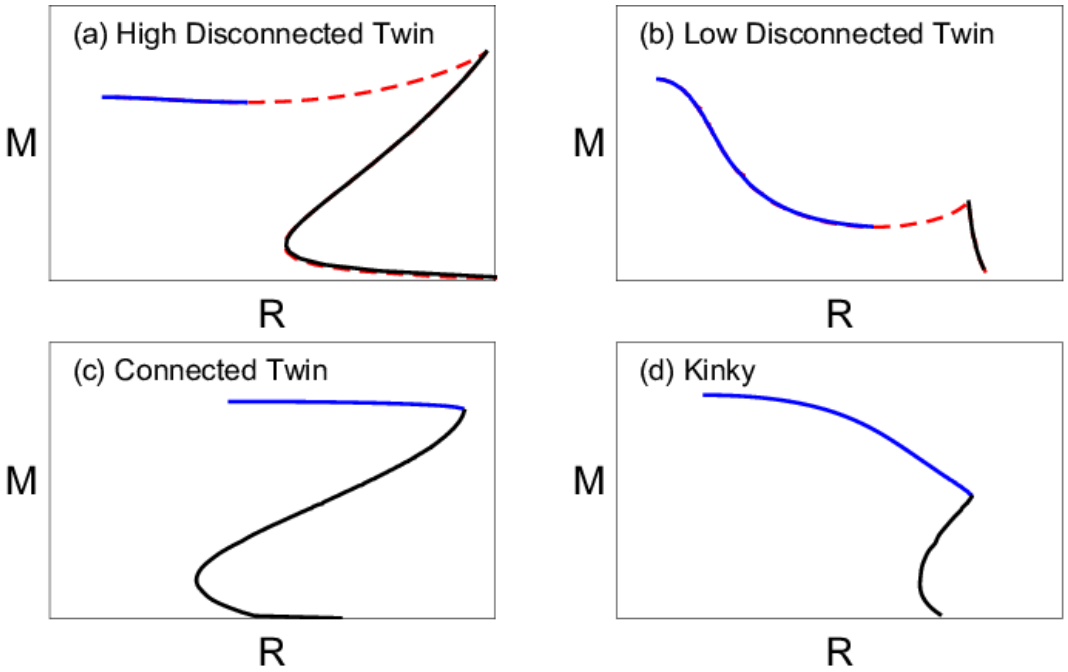


Signatures of Phase Transitions?

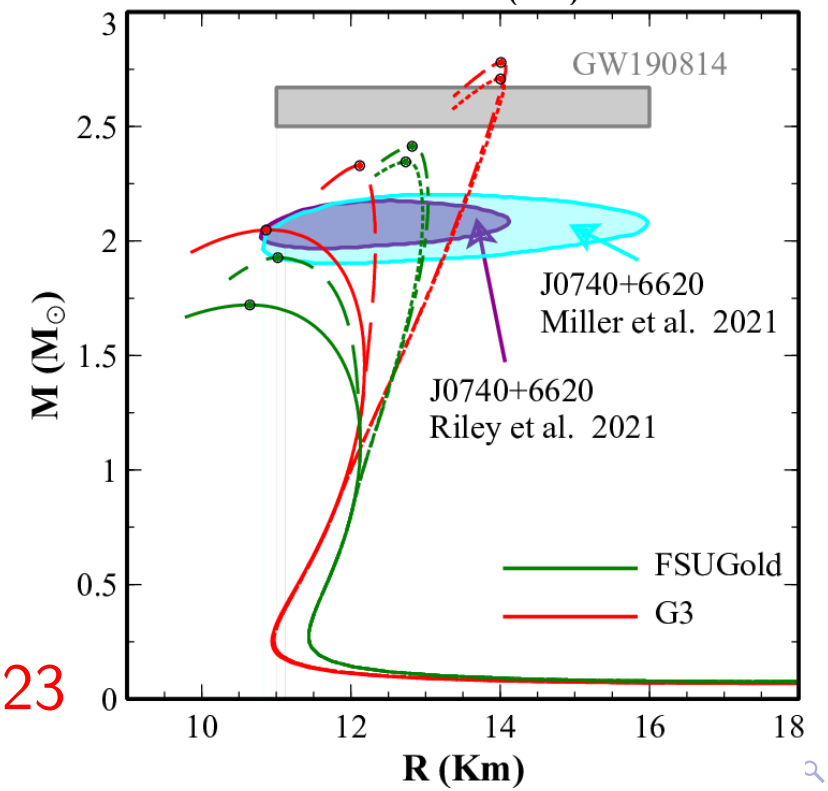
crossover (quarkyonic) [$\Delta R > 0$?]



Tan et al. 2021



first-order [$\Delta R < 0$?]



Kumar et al. 2023

How Can a Neutron Star's Radius Be Measured?

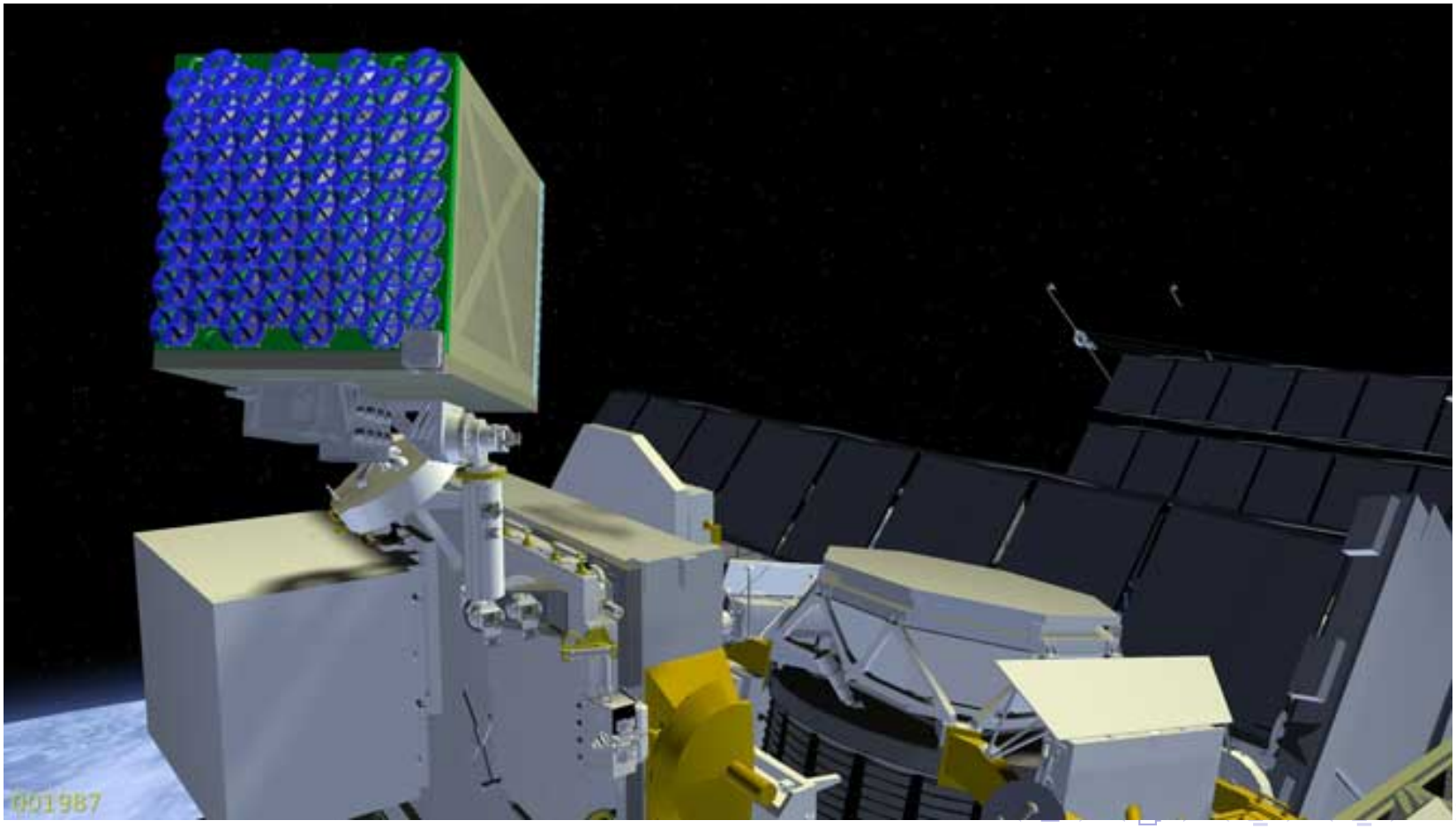
- Flux $= \frac{\text{Luminosity}}{4\pi D^2} = \frac{4\pi R^2 \sigma_B T_s^4}{4\pi D^2} = \left(\frac{R}{D}\right)^2 \sigma_B T_s^4$
  X-ray observations of quiescent neutron stars in low-mass X-ray binaries measure the flux and surface temperature T_s. Distance D somewhat uncertain; GR effects introduce an M dependence.
- $F_{Edd} = \frac{GMc}{\kappa D^2}$ X-ray observations of bursting neutron stars in accreting systems measure the Eddington flux F_{Edd}. κ is the poorly-known opacity; GR effects introduce an R dependence.
- X-ray phase-resolved spectroscopy of millisecond pulsars with nonuniform surface emissions (hot spots). NICER: PSR J0030+0451, PSR J0437-4715 (closest and brightest millisecond pulsar) and PSR J0740+6620 (most massive pulsar).
- $R_{1.4} \simeq (11.5 \pm 0.3) \frac{\mathcal{M}}{M_\odot} \left(\frac{\tilde{\Lambda}}{800}\right)^{1/6} \text{km}, \quad \mathcal{M} = \frac{(M_A M_B)^{3/5}}{(M_A+M_B)^{1/5}}$
  GW observations of neutron star mergers measure the chirp mass $\mathcal{M}$ and binary tidal deformability $\tilde{\Lambda}$ (GW170817).
- $I_A \propto M_A R_A^2$ Radio observations of extremely relativistic binaries show spin-orbit coupling from which moment of inertia is inferred.
- Some gamma-ray bursts show quasi-periodic flux oscillations consistent with GW frequency peaks seen in merger simulations.

Neutron Star Interior Composition ExploreR (NICER)

Launched aboard a SpaceX Falcon 9 rocket on June 3, 2017.

It is installed aboard the International Space Station.

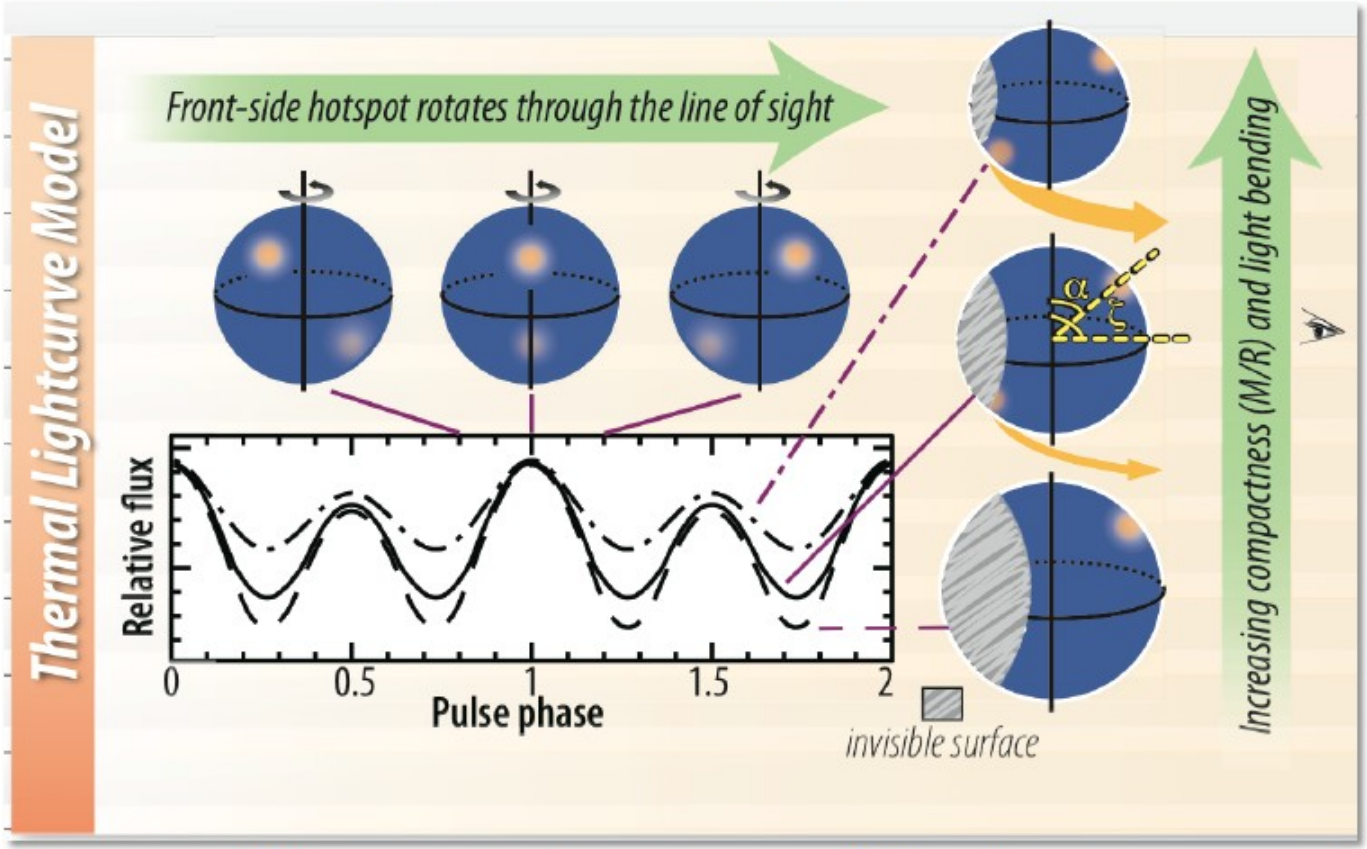
Dedicated to the study of neutron stars through soft X-ray timing.



Science Measurements

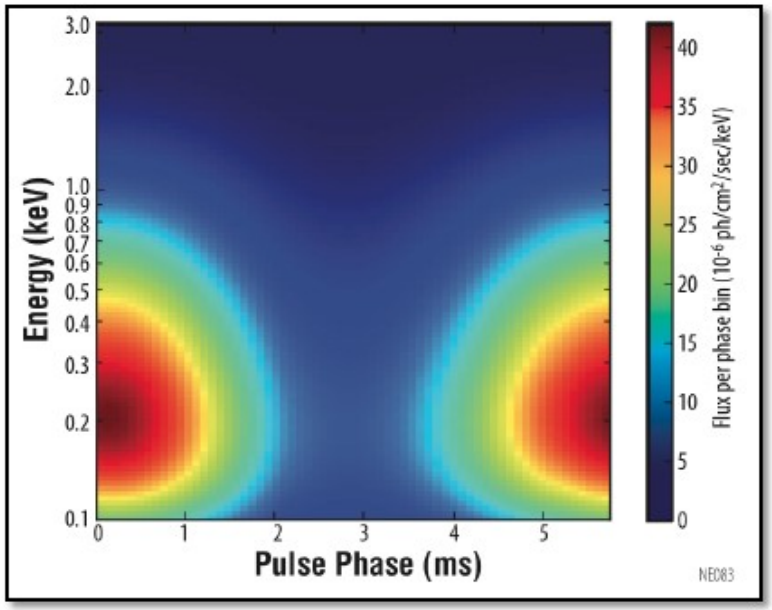
NICER

Reveal stellar structure through lightcurve modeling, long-term timing, and pulsation searches

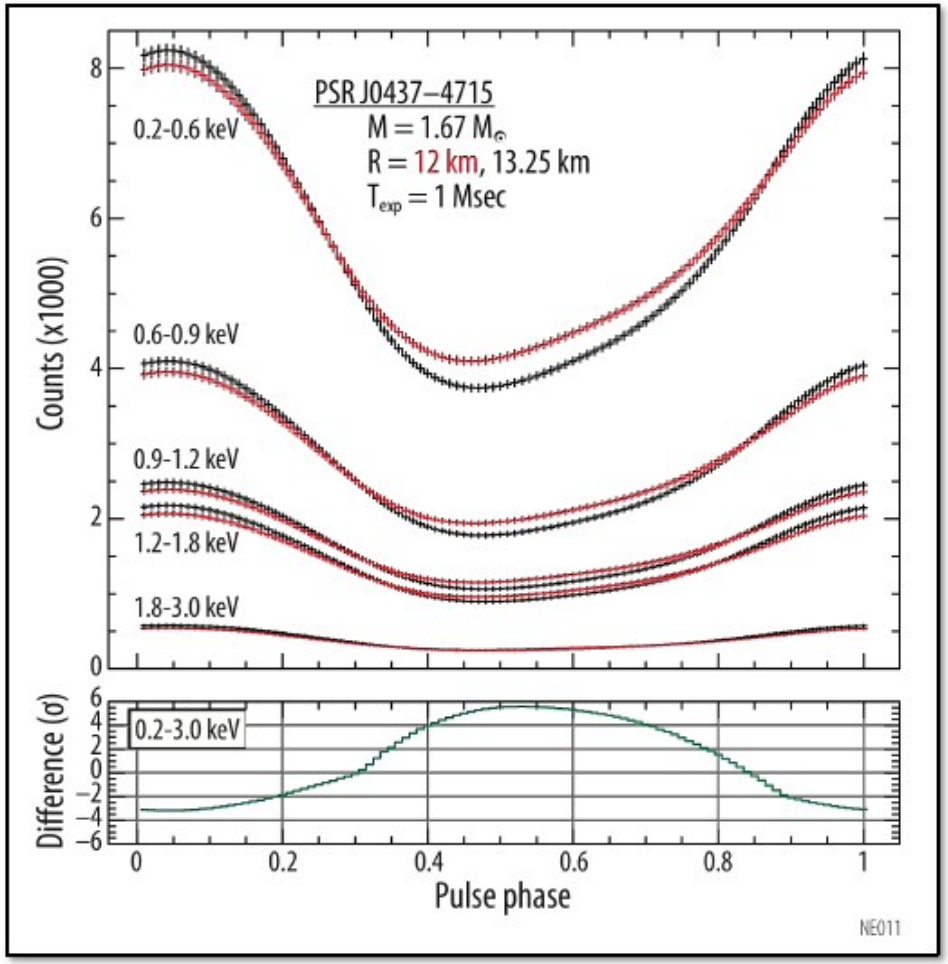


Lightcurve modeling constrains the compactness (M/R) and viewing geometry of a non-accreting millisecond pulsar through the depth of modulation and harmonic content of emission from rotating hot-spots, thanks to gravitational light-bending…

Science Measurements (cont.)



… while phase-resolved spectroscopy promises a direct constraint of radius *R*.



Equation of State up to n_s is Understood

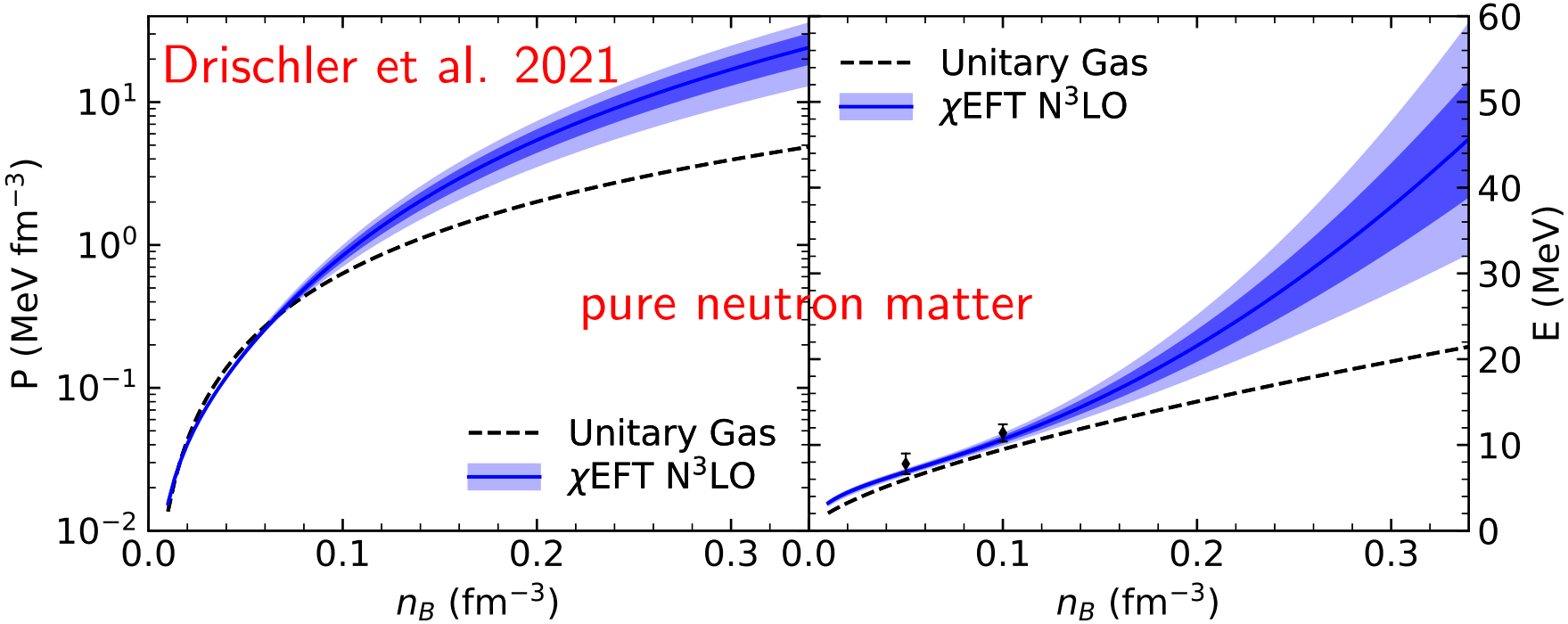
The pressure of matter in the neutron star crust ($n < n_s/2$) is dominated by a relativistic degenerate non-interacting electron gas and a dilute weakly interacting non-relativistic degenerate neutron fluid. Most of the baryons are in nuclei, which have approximately zero pressure individually, and a small negative pressure collectively due to their being a Coulomb lattice.

At the core-crust boundary, it becomes energetically favorable for the nuclei to dissolve into a nucleon fluid, so that the positive surface and Coulomb energies vanish. When $n \approx n_s$, in beta equilibrium $\partial E/\partial x = \mu_n - \mu_p - \mu_e = 0$, where $x \simeq 0.04$ is the proton fraction. So the pressure of matter for $n_s/2 < n < 2n_s$ is nearly that of pure neutron matter.

Theoretical calculations of neutron matter have progressed substantially in recent years.

Theoretical Neutron Matter Studies

Recent advances in chiral effective field theory allow a systematic expansion of nuclear forces at low energies based on the symmetries of quantum chromodynamics. It exploits the gap between the pion mass (the pseudo-Goldstone boson of chiral symmetry-breaking) and the energy scale of short-range nuclear interactions established from experimental phase shifts. It provides the only known consistent framework for estimating energy uncertainties.



Extremal Properties of Neutron Stars

Rhoades & Ruffini (1974) examined the problem of determining the maximum neutron star mass from the TOV equations using optimal control theory. The star is divided into two regions:
1) a low-density outer region with $\mathcal{E} < \mathcal{E}_{low}$ having a mass M_{low}, which includes the crust and the region close to the nuclear saturation density where the EOS is well-known;
2) the neutron star core with mass $M - M_{low}$.

They chose $\mathcal{E}$ as the independent variable, and identified an EOS control variable $u = c_s^2/c^2 \leq 1$ to satisfy causality. The structure equations are linear in u:

$$\frac{dm}{d\mathcal{E}} = \frac{4\pi r^2 \mathcal{E}}{A} u, \quad \frac{dr}{d\mathcal{E}} \equiv \frac{u}{A} = -\frac{c^4}{G} \frac{r(r - 2GM/c^2)}{(\mathcal{E} + P)(mc^2 + 4\pi r^3 P)} u, \quad \frac{dP}{d\mathcal{E}} = u.$$

$$M = M_{low} + \int_{\mathcal{E}_{low}}^{\mathcal{E}_c} \frac{B(\mathcal{E}, r)}{A(\mathcal{E}, P, m, r)} u d\mathcal{E}$$

Optimal Control Theory

The quantity J is to be extremized, where

$$J = \Phi(x(t_0), t_0) + \int_{t_0}^{t_f} \phi(\mathbf{x}(t), u(t), t) dt$$

$\mathbf{x}$ is the state variable vector, ϕ is the running payoff, u is the control variable with constraint $u \in \mathcal{U}$ and t the independent running variable. The necessary condition for δJ=0 is the extremization of the Hamiltonian

$$H(\mathbf{x}(t), u(t), \mathbf{y}(t), t) = \mathbf{y}(t) \cdot \mathbf{f}(\mathbf{x}(t), u(t)) + \phi(\mathbf{x}(t), u(t), t)$$

where $\mathbf{f}(t) = d\mathbf{x}(t)/dt$ is the evolution of the system and $\mathbf{y}(t)$ is the Lagrange multiplier (costate) vector. $\mathbf{x}$ and $\mathbf{y}$ satisfy

$$\frac{d\mathbf{x}(t)}{dt} = \frac{\partial H}{\partial \mathbf{y}} = \mathbf{f}(x, u), \qquad \frac{d\mathbf{y}(t)}{dt} = -\frac{\partial H}{\partial \mathbf{x}}.$$

The boundary condition for $\mathbf{y}$ is $\mathbf{y}(t_0) = (\partial\Phi/\phi\mathbf{x})_{t=t_0}$.

Pontryagin's Principle Gives a Necessary Condition

For extremization of $J = M$ for fixed $t_f = \mathcal{E}_c$ and $t_0 = \mathcal{E}_{low}$, identify $t = \mathcal{E}$, $\mathbf{x}^T = [P, m, r]$,

$$\phi = -\frac{dm}{d\mathcal{E}} = -\frac{uB}{A}, \quad \mathbf{f}^T = \frac{d\mathbf{x}^T}{d\mathcal{E}} = \left[\frac{dP}{d\mathcal{E}}, \frac{dm}{d\mathcal{E}}, \frac{dr}{d\mathcal{E}}\right]$$

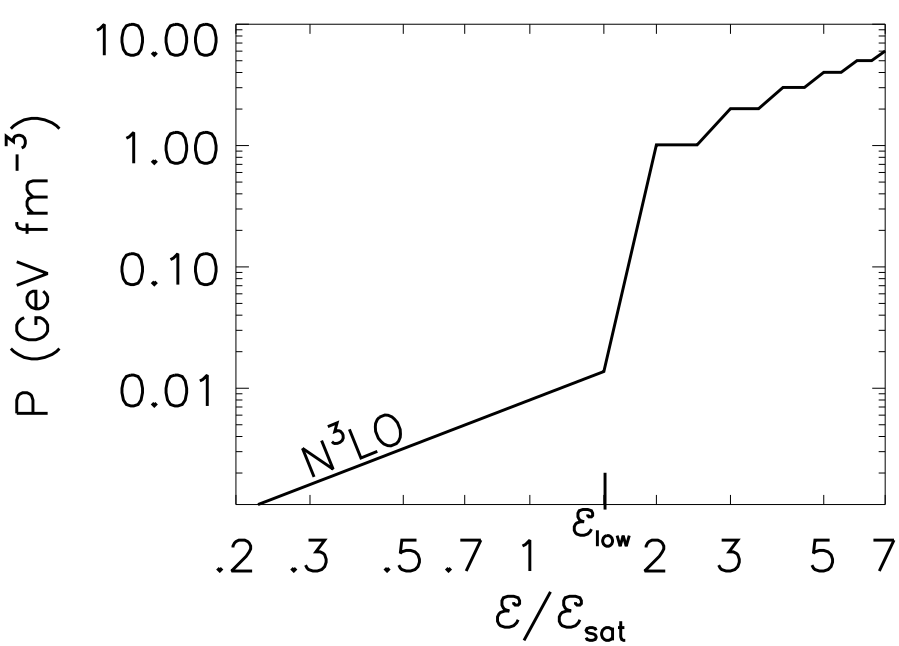
$$H_M(\mathcal{E}) = \mathbf{y}\cdot\mathbf{f} + \phi = \left[y_P + (y_m - 1)\frac{B}{A} + y_r\frac{1}{A}\right] u = \sigma_M(\mathcal{E})u(\mathcal{E}).$$

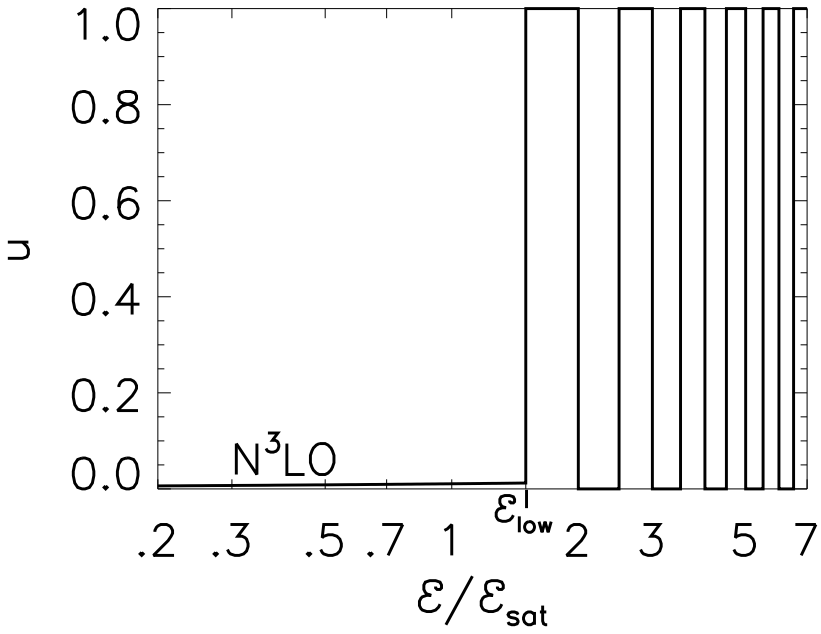
$$\begin{aligned}
\frac{dy_P}{d\mathcal{E}} &= -\frac{\partial H_M}{\partial P} = -\left[(y_m - 1)B + y_r\right]\frac{\partial A^{-1}}{\partial P}u, \\
\frac{dy_m}{d\mathcal{E}} &= -\frac{\partial H_M}{\partial m} = -\left[(y_m - 1)B + y_r\right]\frac{\partial A^{-1}}{\partial m}u, \\
\frac{dy_r}{d\mathcal{E}} &= -\frac{\partial H_M}{\partial r} = \left[(y_m - 1)\frac{\partial (B/A)}{\partial r} + y_r\frac{\partial A^{-1}}{\partial r}\right]u.
\end{aligned}$$

$\sigma_M(\mathcal{E})$ is the switching function. Pontryagin's Principle: A *necessary* condition to extremize H_M is that u must lie on the boundary of $\mathcal{U}$, i.e., $u = 0$ or $u = 1$. This is the so-called *bang-bang* solution.

The Sufficient Condition

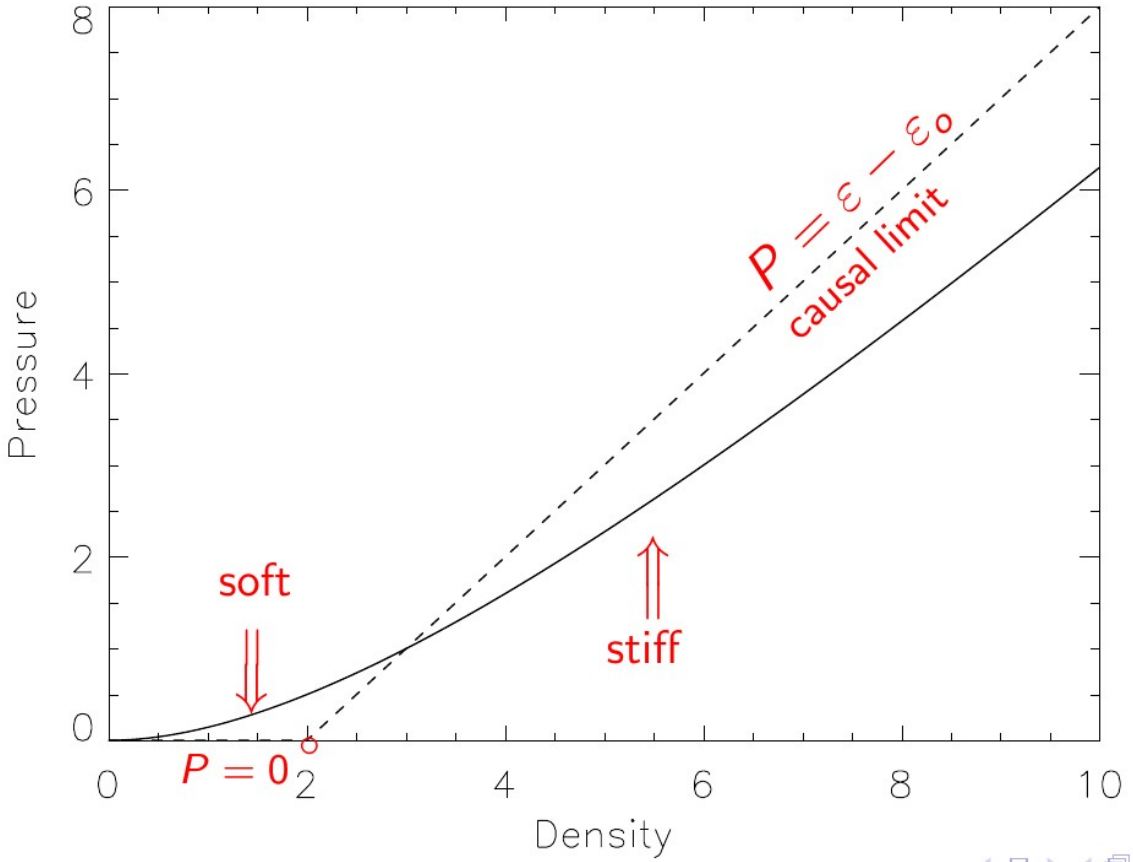
However, there are infinitely many possible such solutions, creating stair-like P–$\mathcal{E}$ relations. Rhoades & Ruffini (1974) conjectured that $u = 1$ for $\mathcal{E}_{low} \leq \mathcal{E} \leq \mathcal{E}_c$ was optimal.





The *sufficient* condition to maximize (minimize) H_M is that u must be 1 (0) where the switching function $\sigma_M(\mathcal{E}) > (<)\ 0$. We showed $\sigma_M > 0$ for $\mathcal{E} > \mathcal{E}_{low}$, proving R & R's conjecture.

The absolute maximum occurs when $P = 0$ for $\mathcal{E} \leq \mathcal{E}_{low} = \mathcal{E}_0$ producing a self-bound star.



ε_o is the only EOS parameter

The TOV solutions scale with ε_o

$w = \varepsilon/\varepsilon_o$
$y = P/\varepsilon_o = w - 1$
$x = r\sqrt{G\varepsilon_o}/c^2$
$z = m\sqrt{G^3\varepsilon_o}/c^2$

Prakash & Lattimer (2011) showed the maximum mass occurs in the self-bound case ($P_o = 0$) when $\mathcal{E}_c = 3.034\mathcal{E}_o$ or $P_c = 2.034\mathcal{E}_o$.

Maximum possible compactness is $\beta = \frac{GM}{Rc^2} = \frac{1}{2.824} = 0.354$.

Maximum possible baryon chemical potential is

$$\mu_c = \mu_o \ln(\mathcal{E}_c/\mathcal{E}_o + P/\mathcal{E}_o) = \mu_o \ln(5.068) \simeq 2.09 \text{ GeV},$$

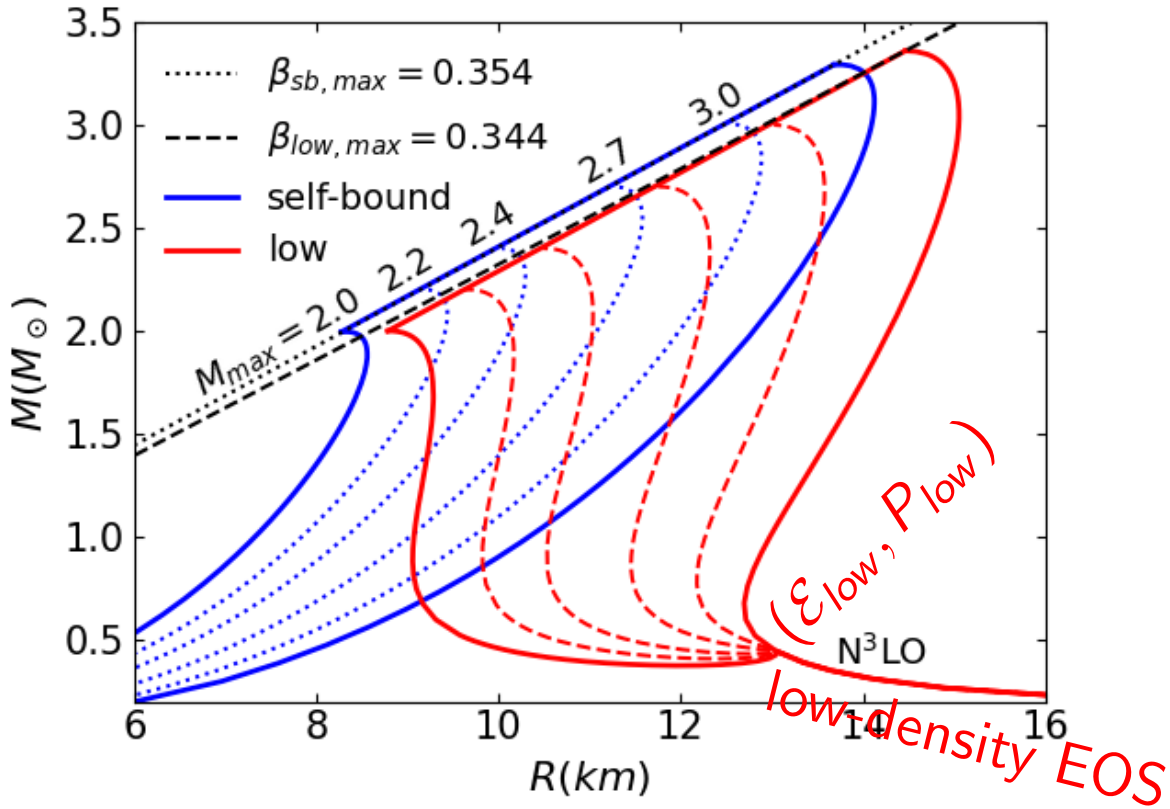
where $\mu(P = 0) = \mu_o = 0.93$ GeV (iron).

Maximum and Minimum of Radius

$$\frac{dr}{d\mathcal{E}} = \frac{u}{A}, \qquad R = \Delta R + \int_{\mathcal{E}_{low}}^{\mathcal{E}_c} \frac{u}{A} d\mathcal{E},$$

where $\Delta = R - r(\mathcal{E}_{low})$ is thickness of low-density region. Optimal control theory can be applied; again Hamiltonian $H_R = \sigma_R(\mathcal{E})u$ is linear in u, so *bang-bang* solution applies.

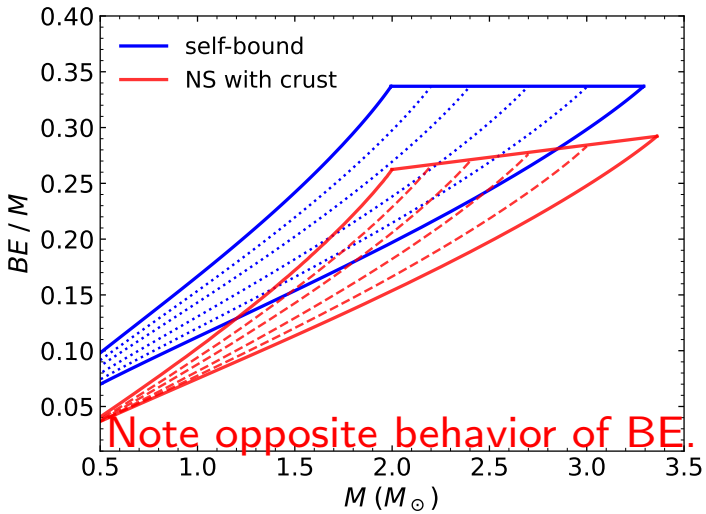
Analysis shows $\sigma_R > 0$ for $\mathcal{E} > \mathcal{E}_{low}$, so $u = 1$ maximizes R for a given $\mathcal{E}_c$, or, equivalently, M.



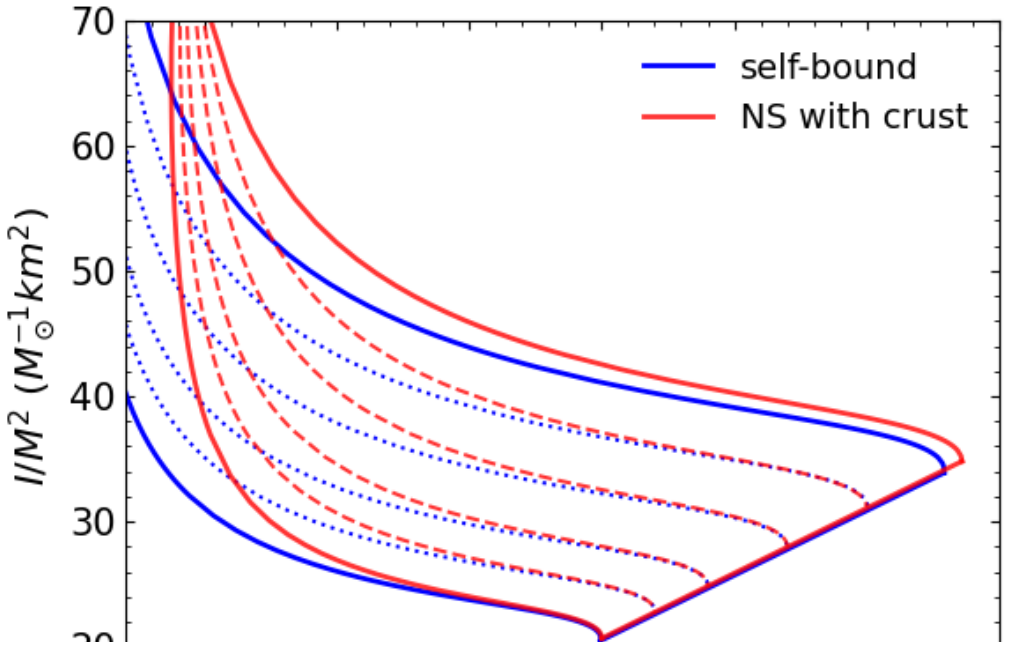
The minimum R for each M is realized by setting $u = 0$ for $\mathcal{E}_{low} \leq \mathcal{E} \leq \mathcal{E}_k$ and $u = 1$ for $\mathcal{E} \geq \mathcal{E}_k$. $\mathcal{E}_k$ is set by requiring $M_{max} \geq M_{obs}$, the largest accurately-measured pulsar mass.

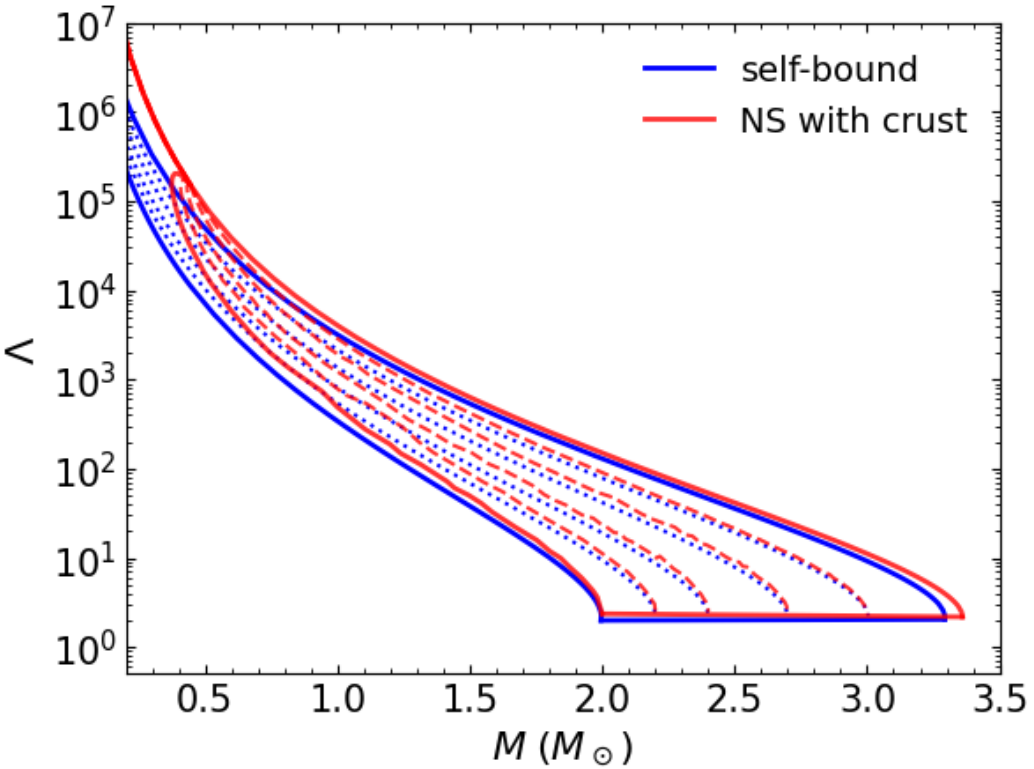
Structural Properties

Moment of inertia I, total baryon number $\mathcal{N}$, and tidal deformability Λ, are all maximized by the bang-bang solution with $u = 1$, and minimized, like R, by the solution with the combination $u = 0$ for $\mathcal{E}_{low} < \mathcal{E} < \mathcal{E}_t$ and $u = 1$ for $\mathcal{E} > \mathcal{E}_t$. BE$= \mathcal{N} m_b c^2 - M$.

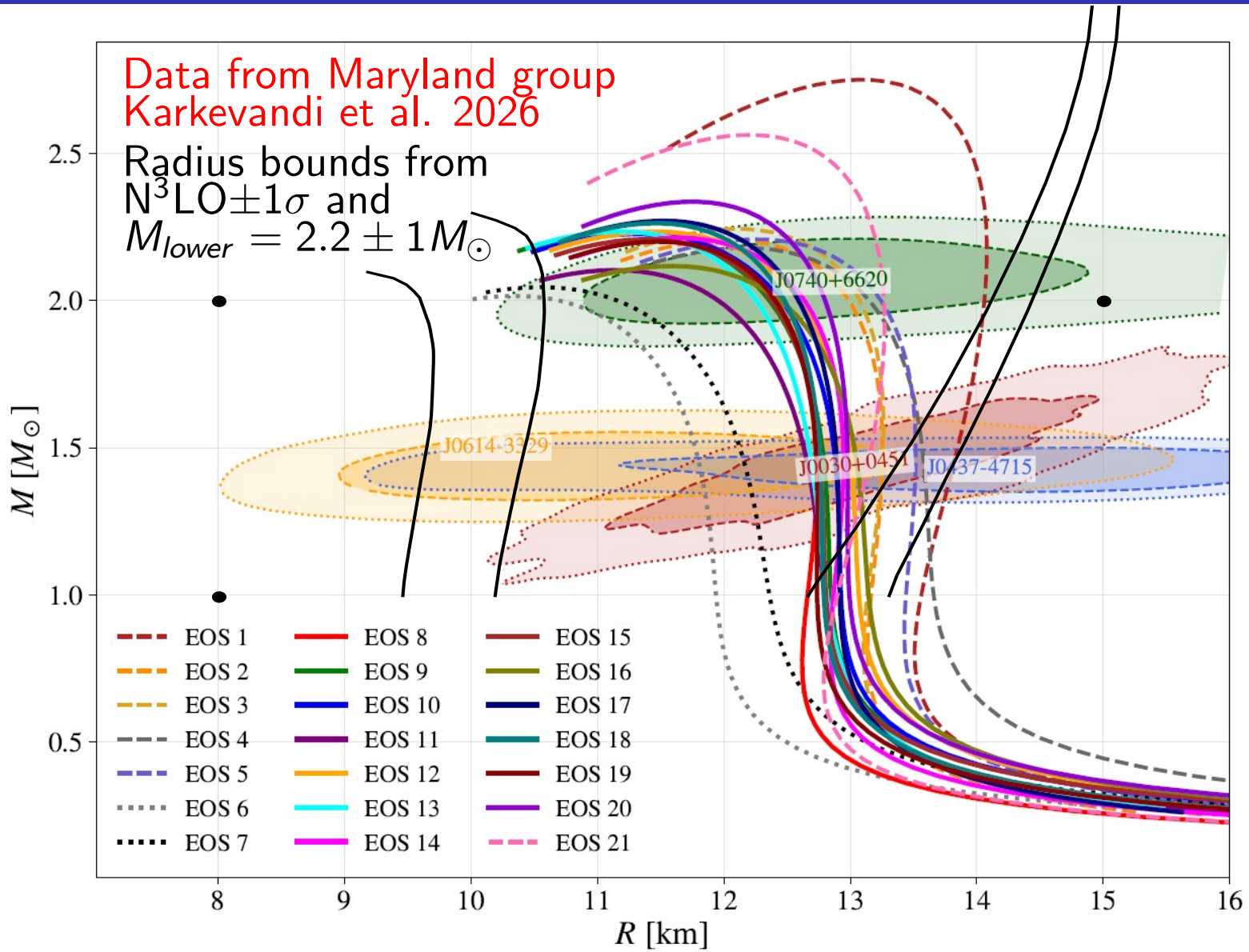


Note opposite behavior of BE.

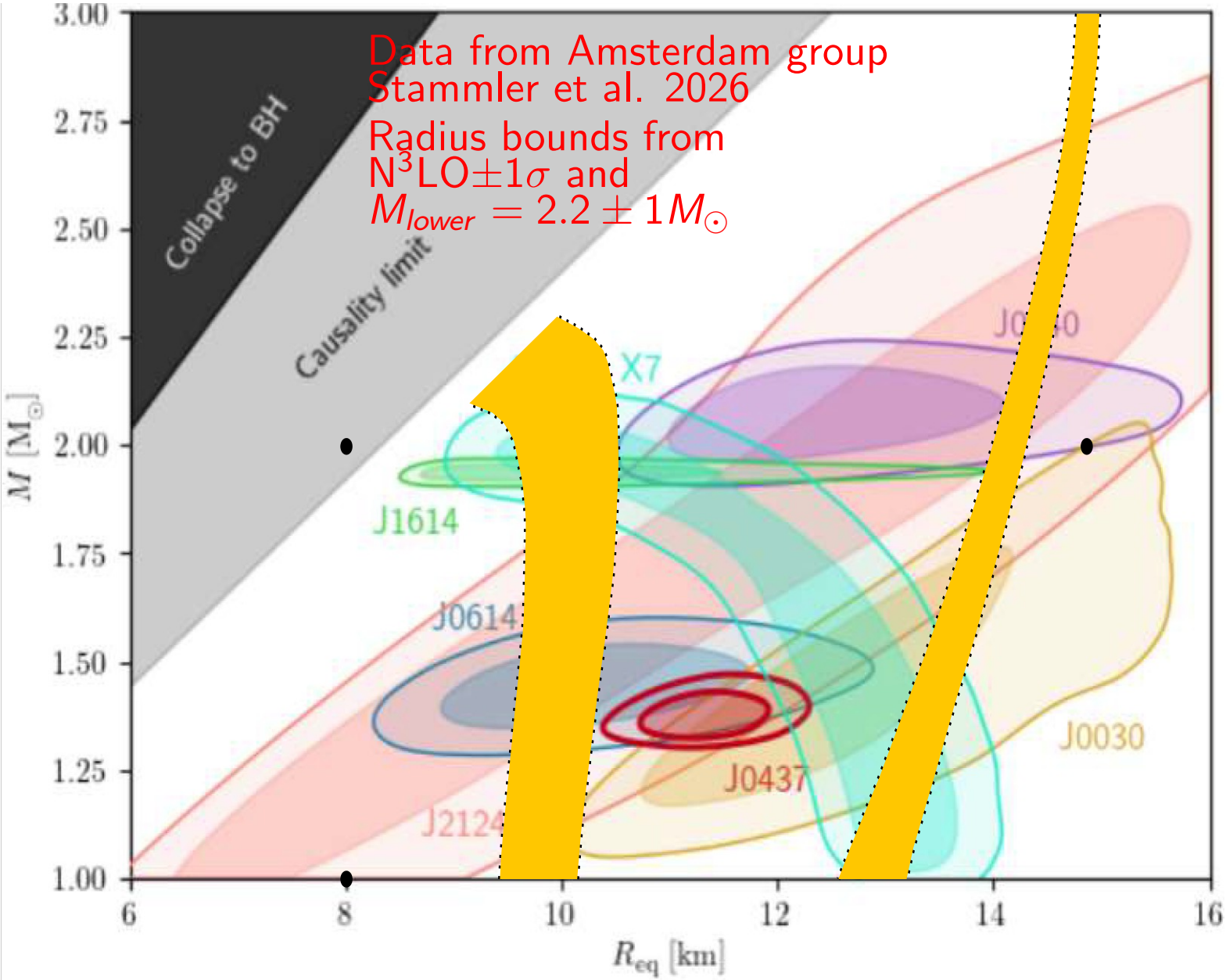




Constraining Radius Priors

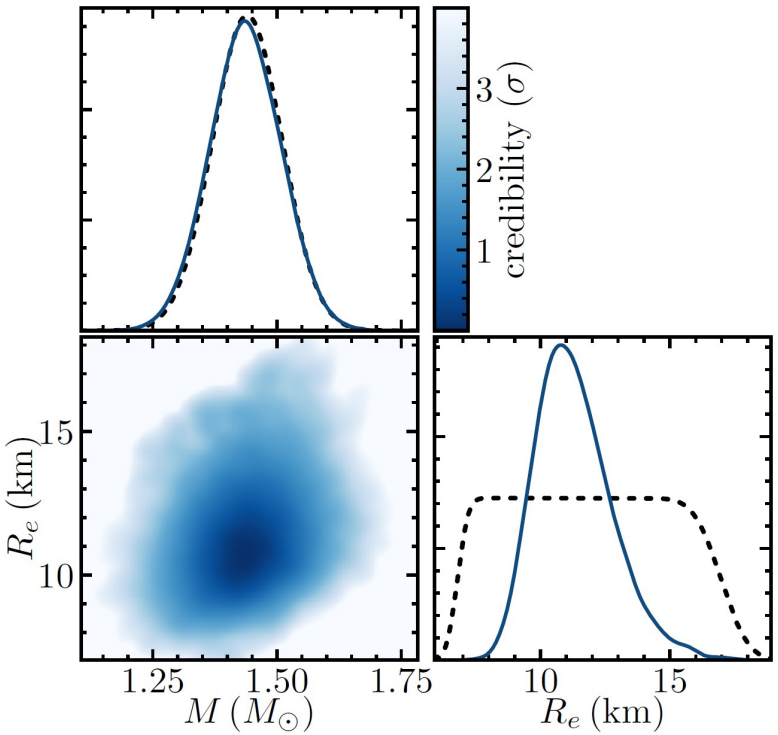


Constraining Radius Priors

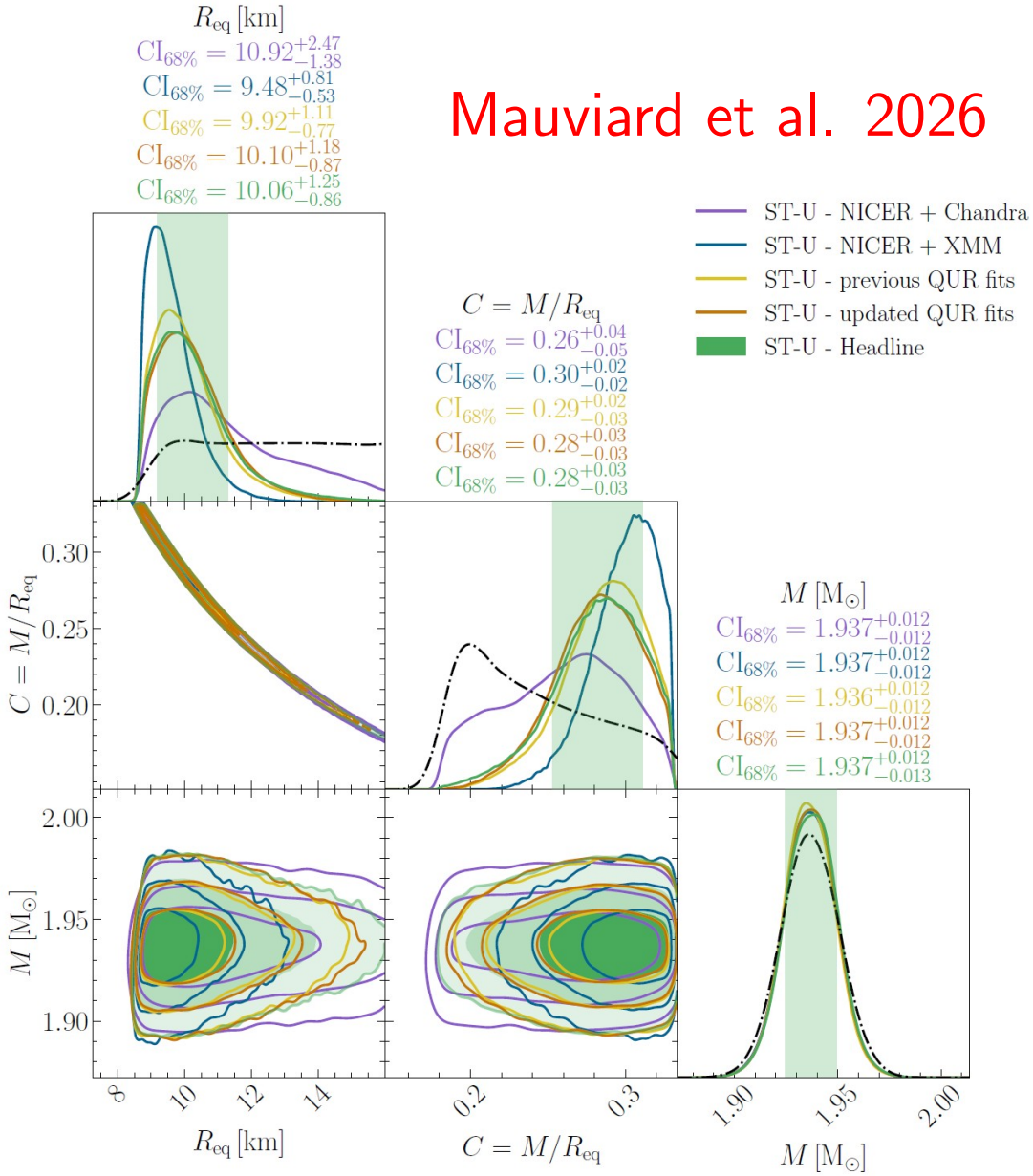


Existing Priors

Miller et al. 2026



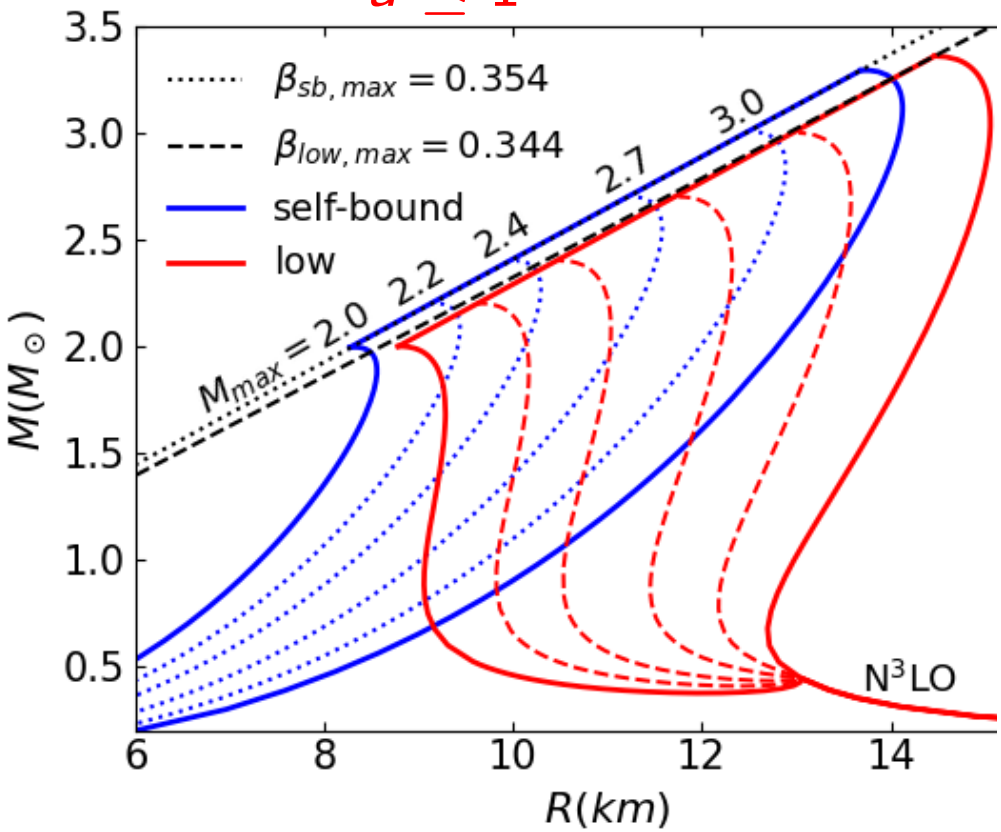
Mauviard et al. 2026



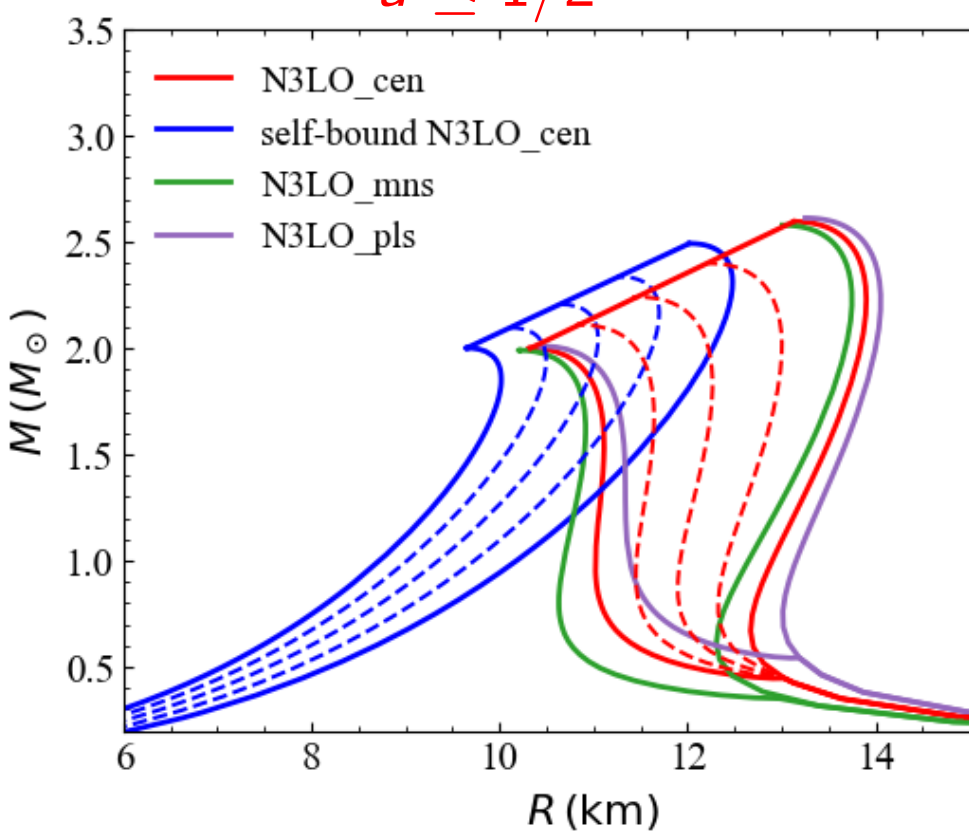
Additional QCD Constraint?

What if QCD could set an upper bound to u?

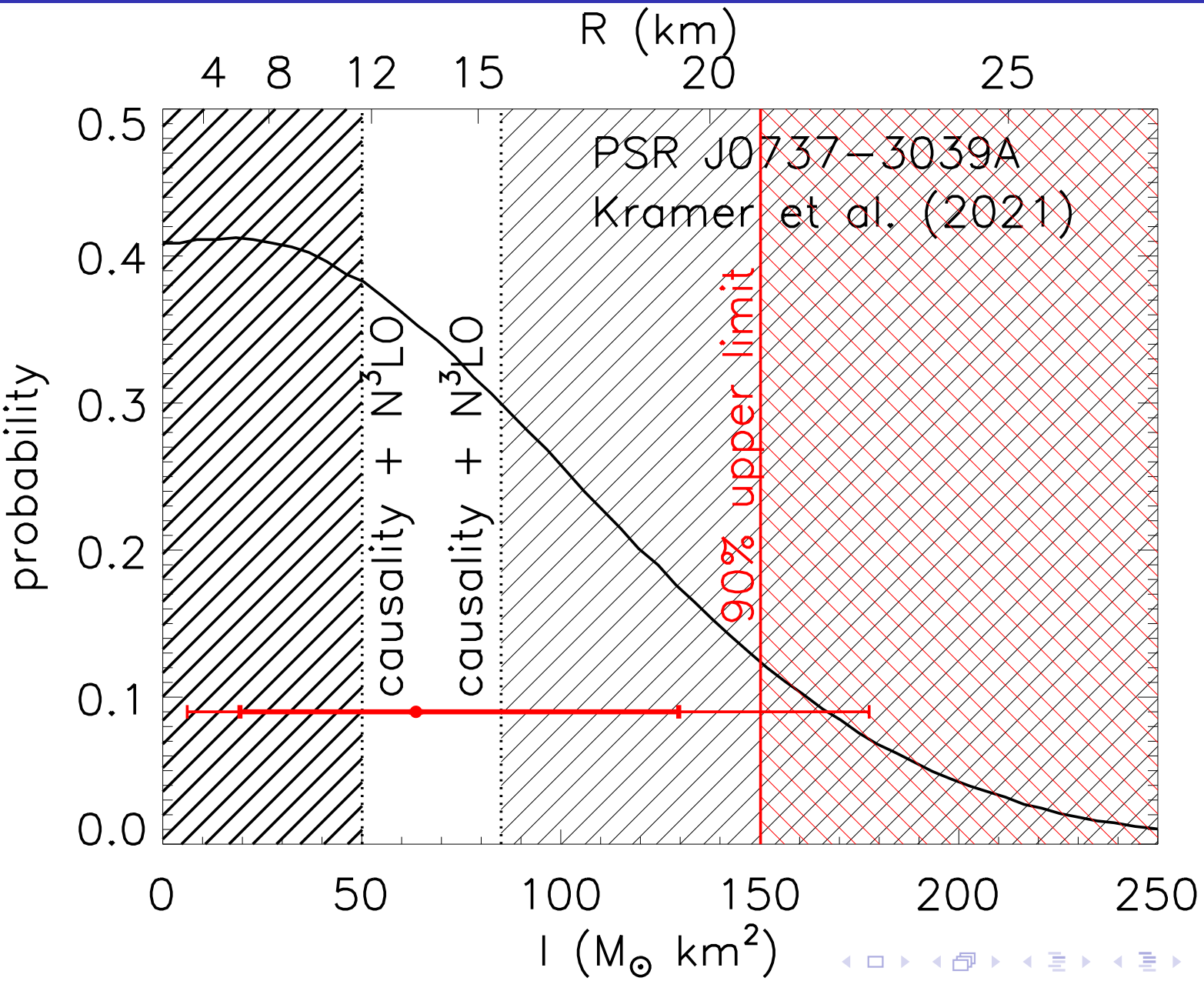
$u \leq 1$



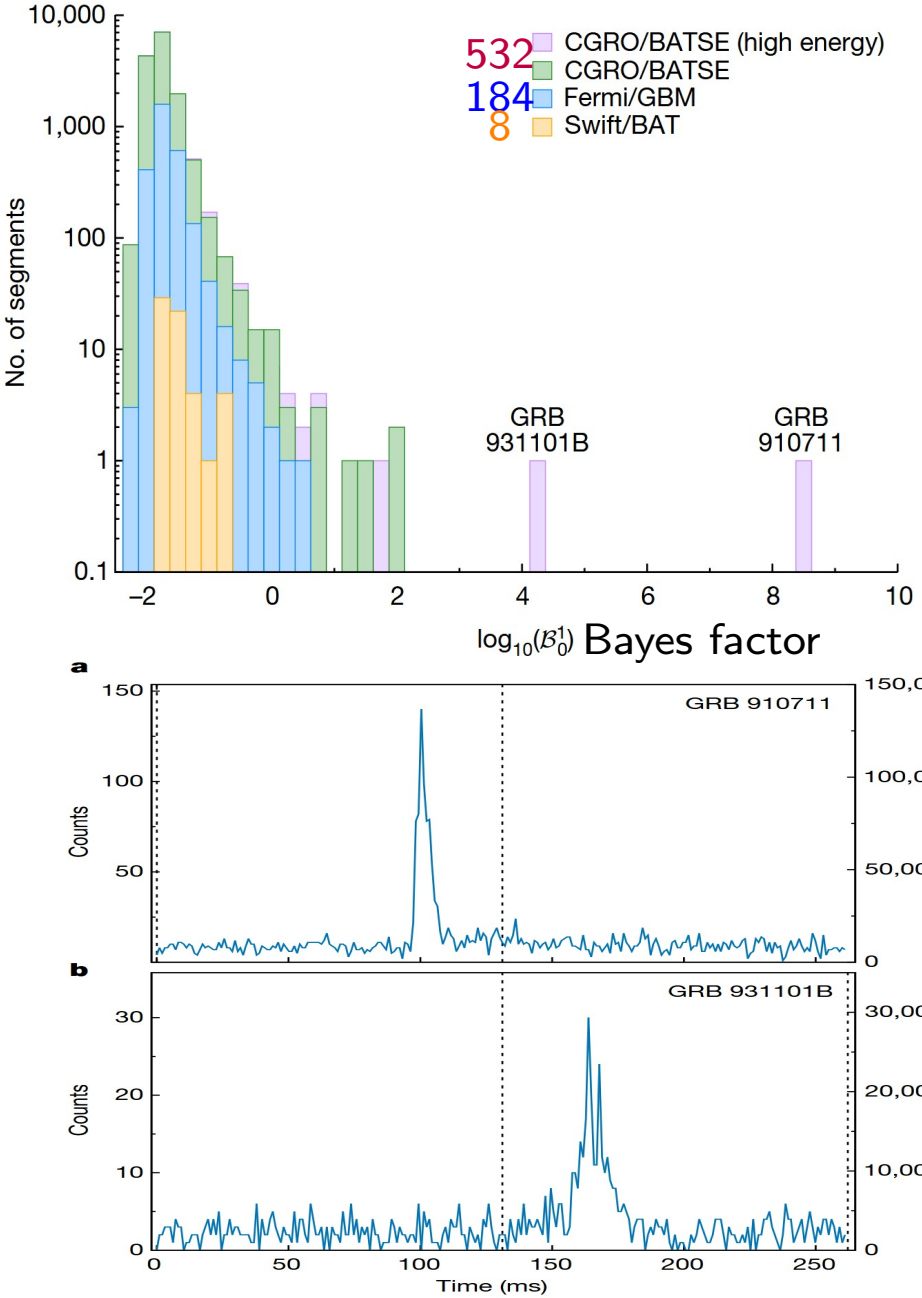
$u \leq 1/2$



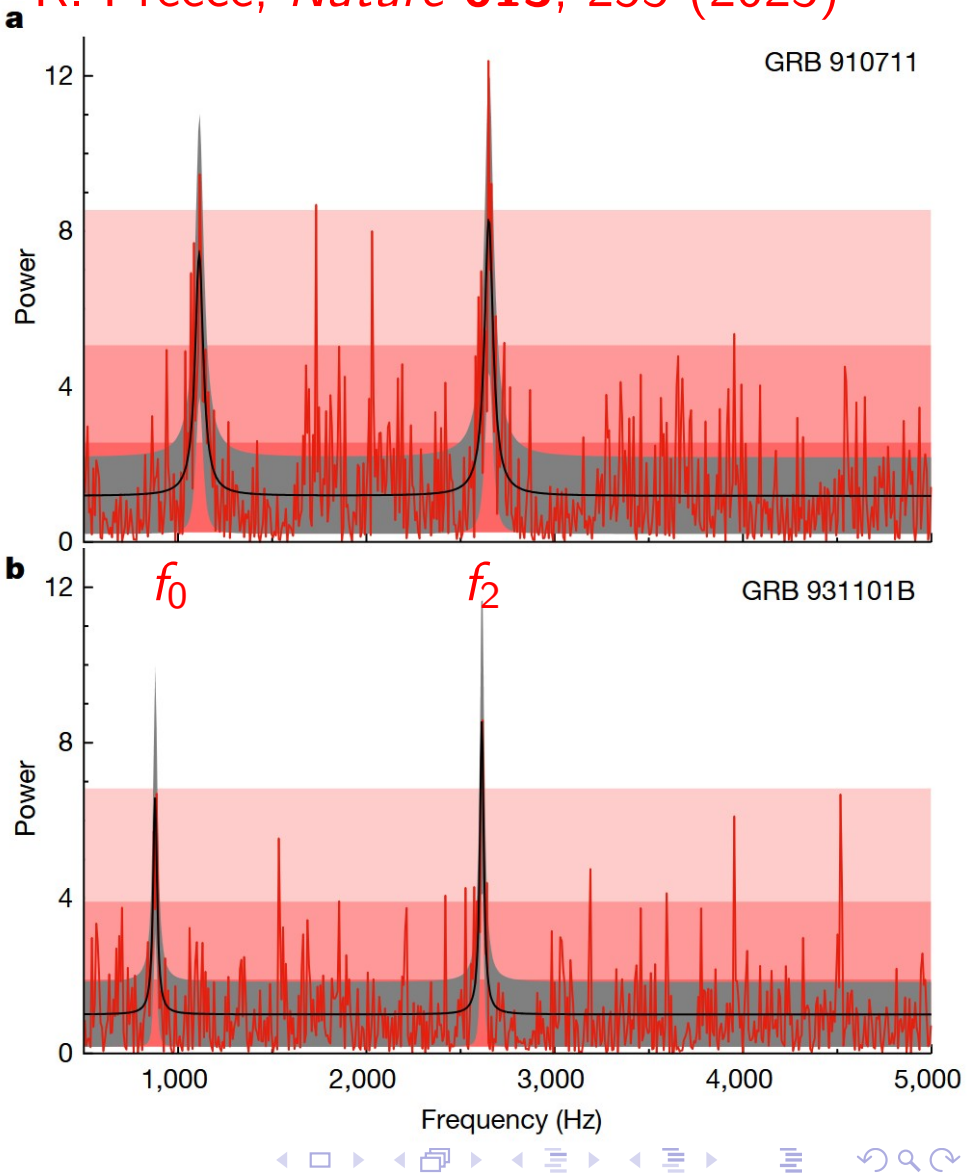
Current Moment of Inertia Measurement



GRB QPOs



C. Chirenti, S. Dichiari, A. Lien, M. C. Miller, R. Preece, *Nature* **613**, 253 (2023)

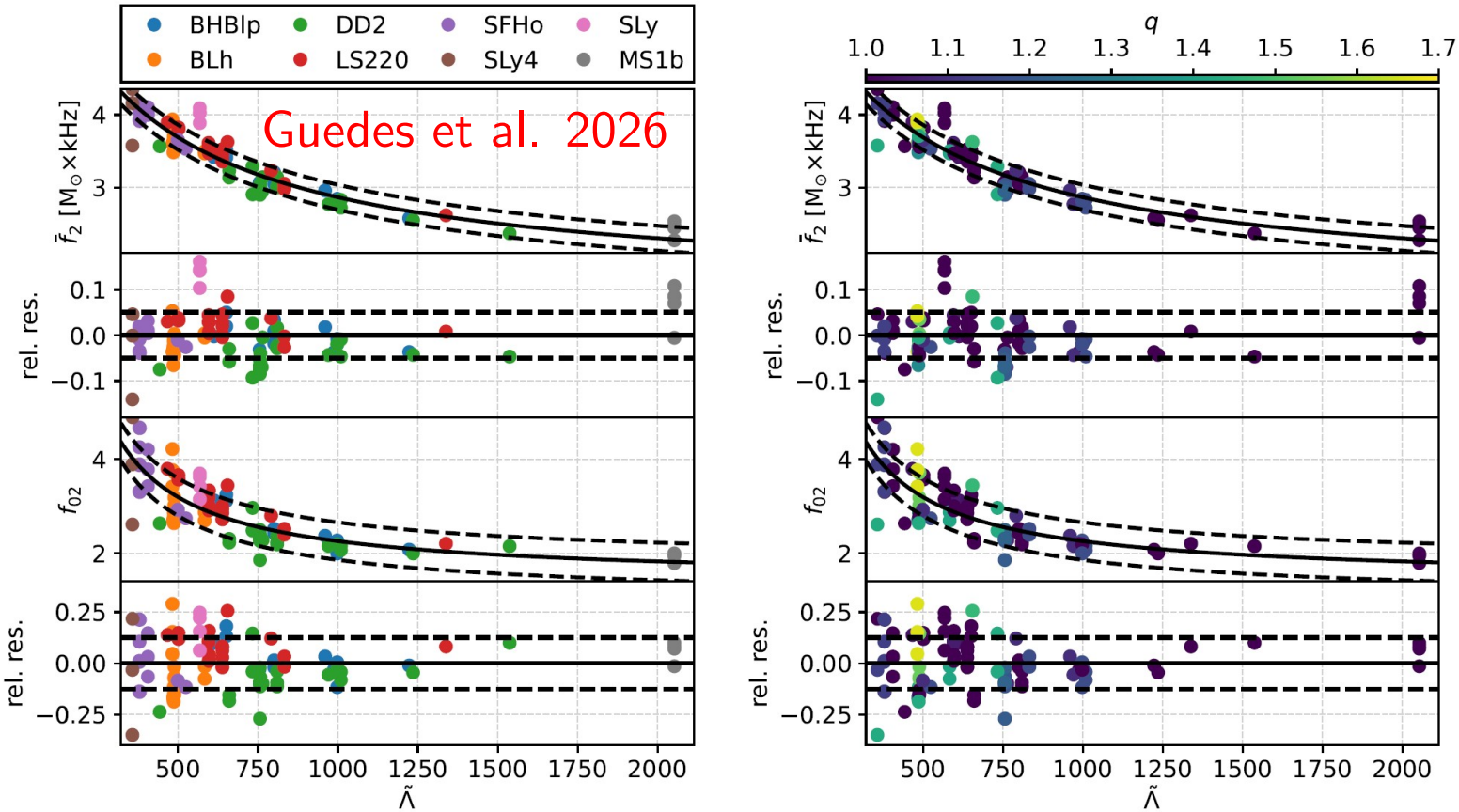


Neutron Star Constraints from Gamma-Ray Bursts

Quasi-periodic oscillations (QPOs) observed in short GRBs are consistent with radial and quadrupolar vibrations of hypermassive neutron stars (HMNS) formed in the aftermath of BNS mergers (Bauswein & Janka 2012; Takami et al. 2015; Rezzolla & Takami 2016). The GW peak frequencies f_0, f_2 obey quasi-universal (little EOS dependence) correlations with the pre-merger chirp mass $\mathcal{M}$ and binary tidal deformability $\tilde{\Lambda}$.

$\tilde{f}_2 = f_2 \mathcal{M}$

$f_{02} = f_2 / f_0$

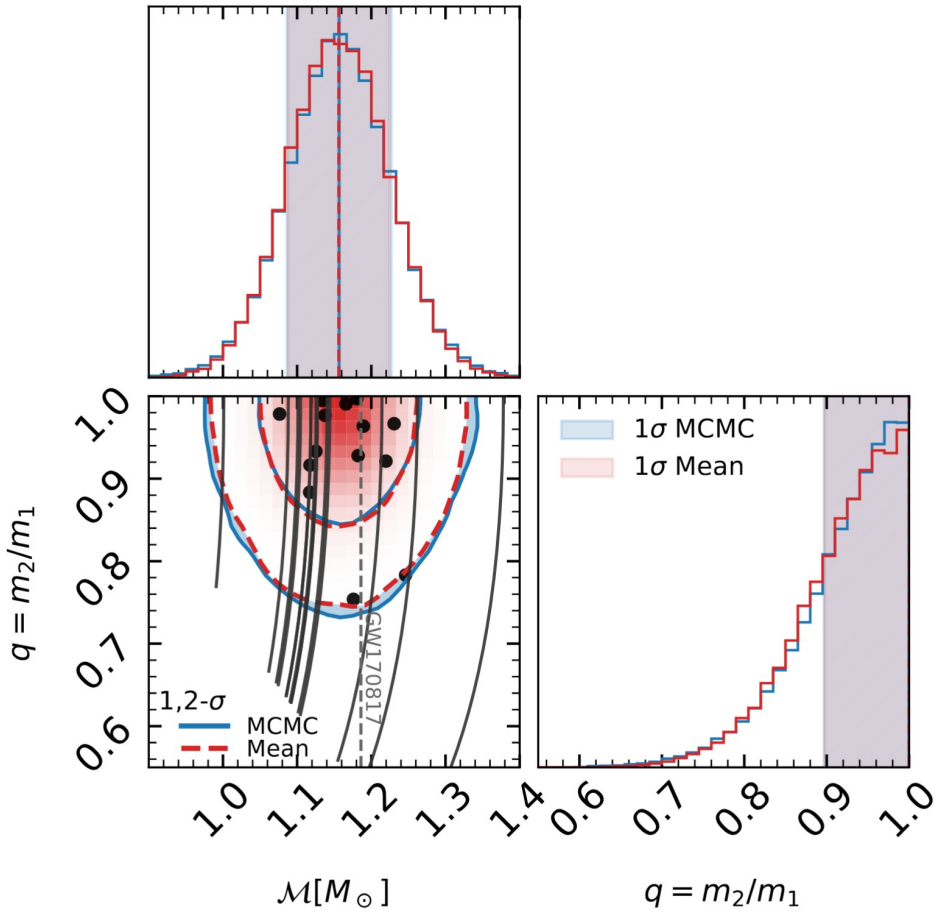


GRB QPOs (modified from Guedes et al. 2025)

Although observed frequencies are redshifted, their ratio is not. The correlations give for GRB 910711 (931101B) $\tilde{\Lambda} = 1022 \pm 607$ (595 ± 204) and $\mathcal{M}(1+z) = 1.14 \pm 0.23 M_\odot$ $(1.36 \pm 0.23 M_\odot)$.

$\mathcal{M}$ matches galactic BNS range.

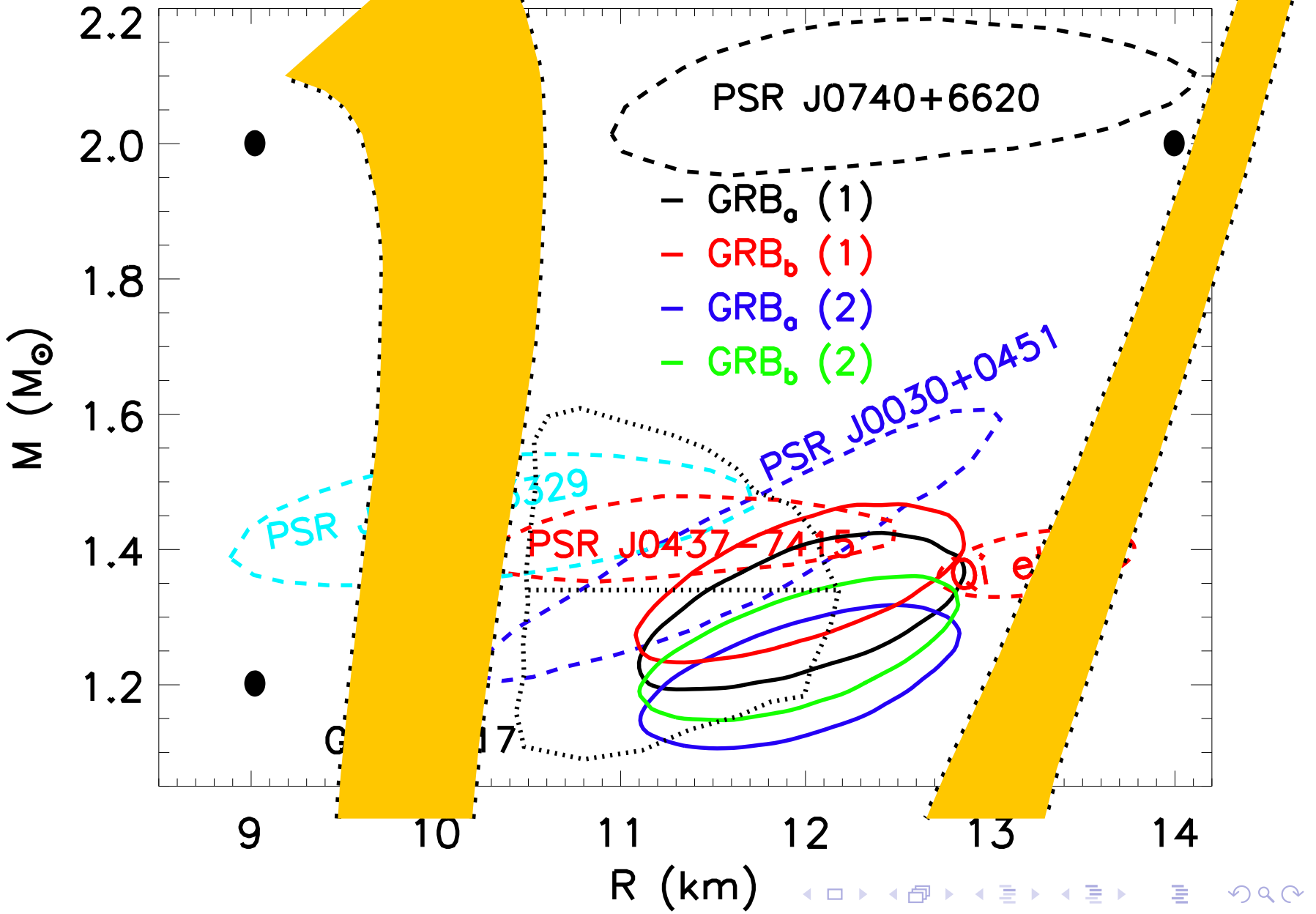
Priors for q, z and $\mathcal{M}$ taken from galactic BNS and s-GRBs [$\mathcal{P}(z) \propto z^3$, $\mathcal{P}(q) = \exp(-[q/0.2]^2)$, $\mathcal{M} \in [1.0, 1.30] M_\odot$]. Long-lived hypermassive neutron stars can't form if $\mathcal{M}$ is too large or too small, which could be used to set even tighter constraints.



We assume all 4 neutron stars have a common radius R.

The semi-universal relation (Zhao & Lattimer 2018) valid for $1.2 \leq M/M_\odot \leq 1.6$ is $\Lambda \simeq (0.0088 \pm 0.0008)(Rc^2/GM)^6$, which implies $R = 12.00 \pm 0.55$ km and masses $1.15 < M/M_\odot < 1.43$.

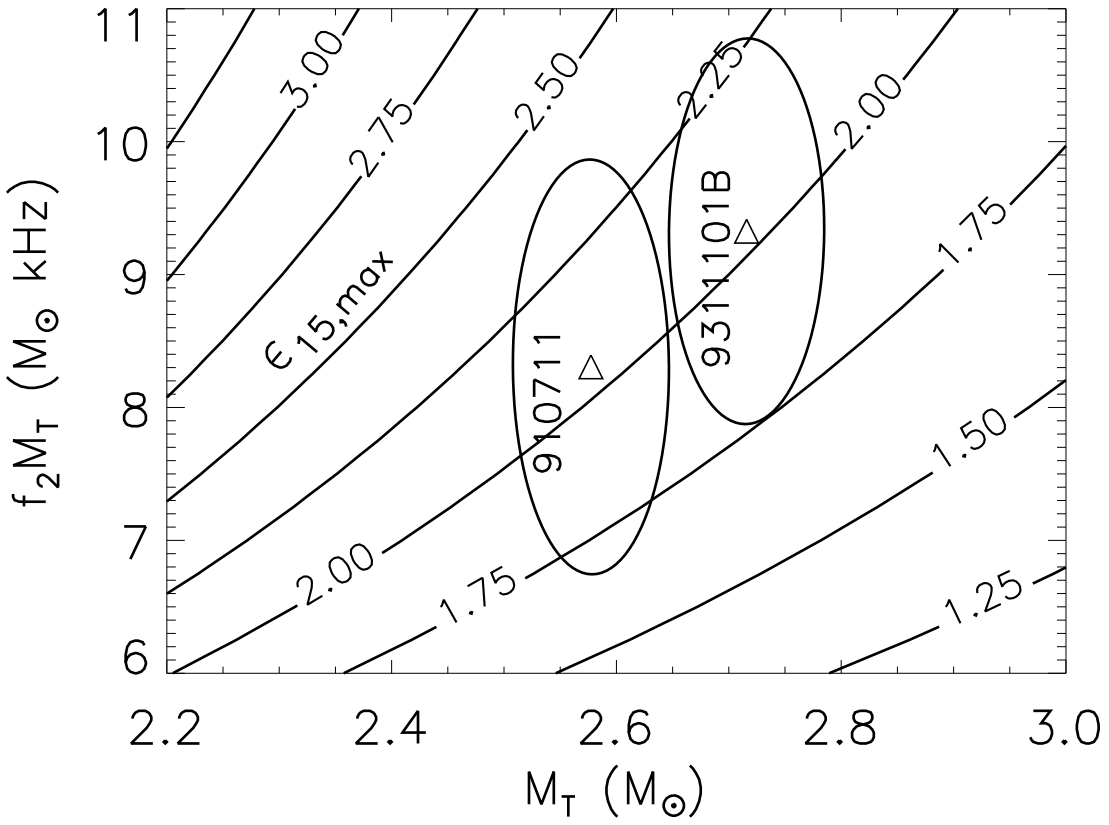
GRB QPO constraints



A Taste of the Future?

If the GRB QPO hypothesis is correct, we get a glimpse of what might be learned from post-merger gravitational waves.

Since the total mass M_T and the frequency f_2 can be inferred, one may estimate the central density of the maximum mass non-rotating star from the Breschi et al. (2022) fit (uncertainty of about 6%):



$$\mathcal{E}_{max} \simeq \frac{a_0 c^6}{G^2 M_T^2}\left[1 + a_1\left(\frac{c^2}{\pi G M_T f_2}\right)^{1/6}\right], \quad M_{max} < 2.6 M_\odot.$$