Neutrino Mixing, Oscillations, $(\beta\beta)_{0\nu}$-Decay, Leptonic CP-Violation and Leptogenesis

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Plan of Lectures

1. Introduction.

2. Neutrino Mixing: Current Status.

3. Determining the Type of Neutrino Mass Spectrum.

4. High Precision Measurement of $\Delta m^2_{\odot}$ and $\sin^2\theta_{\odot}$.

5. Neutrino Physics Prospects of $(\beta\beta)_{0\nu}$−Decay.

6. Dirac and Majorana CP-Violation and Leptogenesis.

7. Conclusions.

Compelling Evidences for $\nu -$Oscillations

$-\nu_{\text{atm}}$: SK UP-DOWN ASYMMETRY

θ_Z-, $L/E-$ **dependences of $\mu-$like events**

Dominant $\nu_\mu \longrightarrow \nu_\tau$ K2K, MINOS; CNGS (OPERA)

$-\nu_\odot$: **Homestake, Kamiokande, SAGE, GALLEX/GNO**

Super-Kamiokande, SNO, BOREXINO; KamLAND

Dominant $\nu_e \longrightarrow \nu_{\mu,\tau}$ **BOREXINO; KamLAND..., LowNu**

$-$ LSND

Dominant $\bar{\nu}_\mu \longrightarrow \bar{\nu}_e$; **MiniBOONE 11/04/07: negative result**

$$\nu_{l\text{L}} = \sum_{j=1} U_{lj}\, \nu_{j\text{L}} \qquad l = e, \mu, \tau.$$

B. Pontecorvo, 1957; 1958; 1967;

Z. Maki, M. Nakagawa, S. Sakata, 1962;

Compelling Evidences for $\nu-$Oscillations: 3-ν mixing

$$\nu_{l\mathsf{L}} = \sum_{j=1} U_{lj}\, \nu_{j\mathsf{L}} \qquad l = e, \mu, \tau.$$

Three Neutrino Mixing

$$\nu_{l\mathrm{L}} = \sum_{j=1}^{3} U_{lj}\, \nu_{j\mathrm{L}} \ .$$

U is the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) neutrino mixing matrix,

$$U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}$$

- U - $n \times n$ unitary:

	n	2	3	4
mixing angles:	$\frac{1}{2}n(n-1)$	1	3	6
CP-violating phases:				
• $\nu_j -$ Dirac:	$\frac{1}{2}(n-1)(n-2)$	0	1	3
• $\nu_j -$ Majorana:	$\frac{1}{2}n(n-1)$	1	3	6

$n = 3$: 1 Dirac and
2 additional CP-violating phases, Majorana phases

S.M. Bilenky, J. Hosek, S.T.P., 1980;
J. Schechter, J.W.F. Valle, 1980.

Majorana Neutrinos

Can be defined in QFT using fields or states.

Fields: $\chi_k(x)$ - 4 component (spin 1/2), complex, m_k

Majorana condition:

$$C\ (\bar{\chi}_k(x))^{\mathsf{T}} = \xi_k \chi_k(x), \quad |\xi_k|^2 = 1$$

– Invariant under proper Lorentz transformations.
– Reduces by 2 the number of components in $\chi_k(x)$.

Implications:

$$U(1):\ \chi_k(x) \to e^{i\alpha}\chi_k(x) - \text{ impossible}$$

– $\chi_k(x)$ cannot absorb phases.
– $Q_{U(1)} = 0:\ Q_{\mathrm{el}} = 0,\ L_l = 0,\ L = 0, ...$
– $\chi_k(x)$: 2 spin states of a spin 1/2 absolutely neutral particle
– $\chi_k \equiv \bar{\chi}_k$

Propagators: $\Psi(x)-$Dirac, $\chi(x)-$Majorana

$$< 0|T(\Psi_\alpha(x)\bar{\Psi}_\beta(y))|0 >= S^F_{\alpha\beta}(x-y) \ ,$$

$$< 0|T(\Psi_\alpha(x)\Psi_\beta(y))|0 >= 0 \ , \quad < 0|T(\bar{\Psi}_\alpha(x)\bar{\Psi}_\beta(y))|0 >= 0 \ .$$

$$< 0|T(\chi_\alpha(x)\bar{\chi}_\beta(y))|0 >= S^F_{\alpha\beta}(x-y) \ ,$$

$$< 0|T(\chi_\alpha(x)\chi_\beta(y))|0 >= -\xi^* S^F_{\alpha\kappa}(x-y)C_{\kappa\beta} \ ,$$

$$< 0|T(\bar{\chi}_\alpha(x)\bar{\chi}_\beta(y))|0 >= \xi \ C^{-1}_{\alpha\kappa}S^F_{\kappa\beta}(x-y)$$

$$U_{CP} \ \chi(x) \ U^{-1}_{CP} = \eta_{CP} \ \gamma_0 \ \chi(x'), \quad \eta_{CP} = \pm i \ .$$

PMNS Matrix: Standard Parametrization

$$U = V \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\frac{\alpha_{21}}{2}} & 0 \\ 0 & 0 & e^{i\frac{\alpha_{31}}{2}} \end{pmatrix}$$

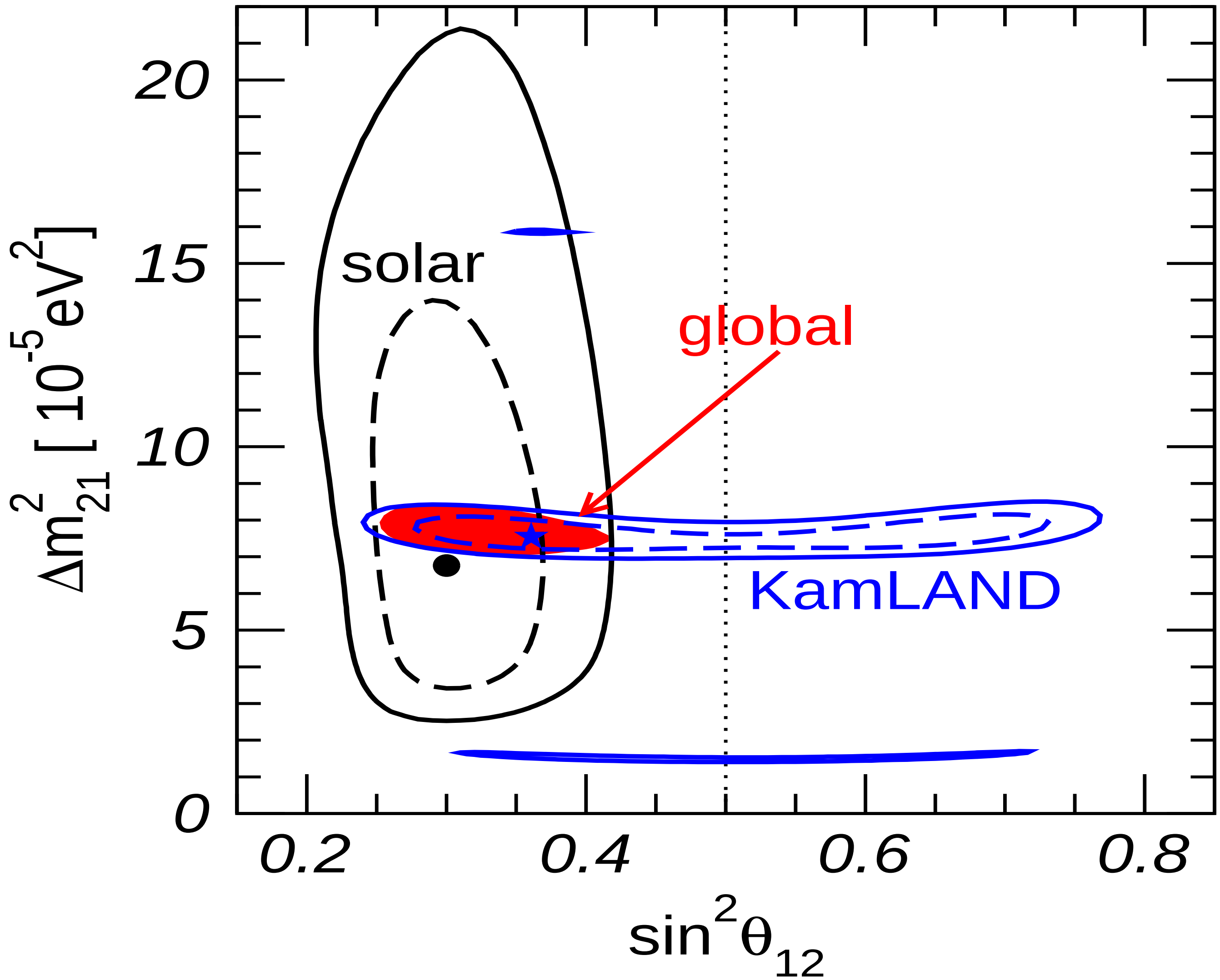
$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

- $s_{ij} \equiv \sin\theta_{ij}$, $c_{ij} \equiv \cos\theta_{ij}$, $\theta_{ij} = [0, \frac{\pi}{2}]$,
- δ - Dirac CP-violation phase, $\delta = [0, 2\pi]$,
- α_{21}, α_{31} - the two Majorana CP-violation phases.
- $\Delta m^2_{\odot} \equiv \Delta m^2_{21} \cong 7.6 \times 10^{-5}$ eV$^2 > 0$, $\sin^2\theta_{12} \cong 0.32$, $\cos 2\theta_{12} \gtrsim 0.26$ (2σ),
- $|\Delta m^2_{\rm atm}| \equiv |\Delta m^2_{31}| \cong 2.4\ (2.5) \times 10^{-3}$ eV^2, $\sin^2 2\theta_{23} \cong 1$,
- θ_{13} - the CHOOZ angle: $\sin^2\theta_{13} < 0.033\ (0.050\ (0.063))$ 2σ (3σ).

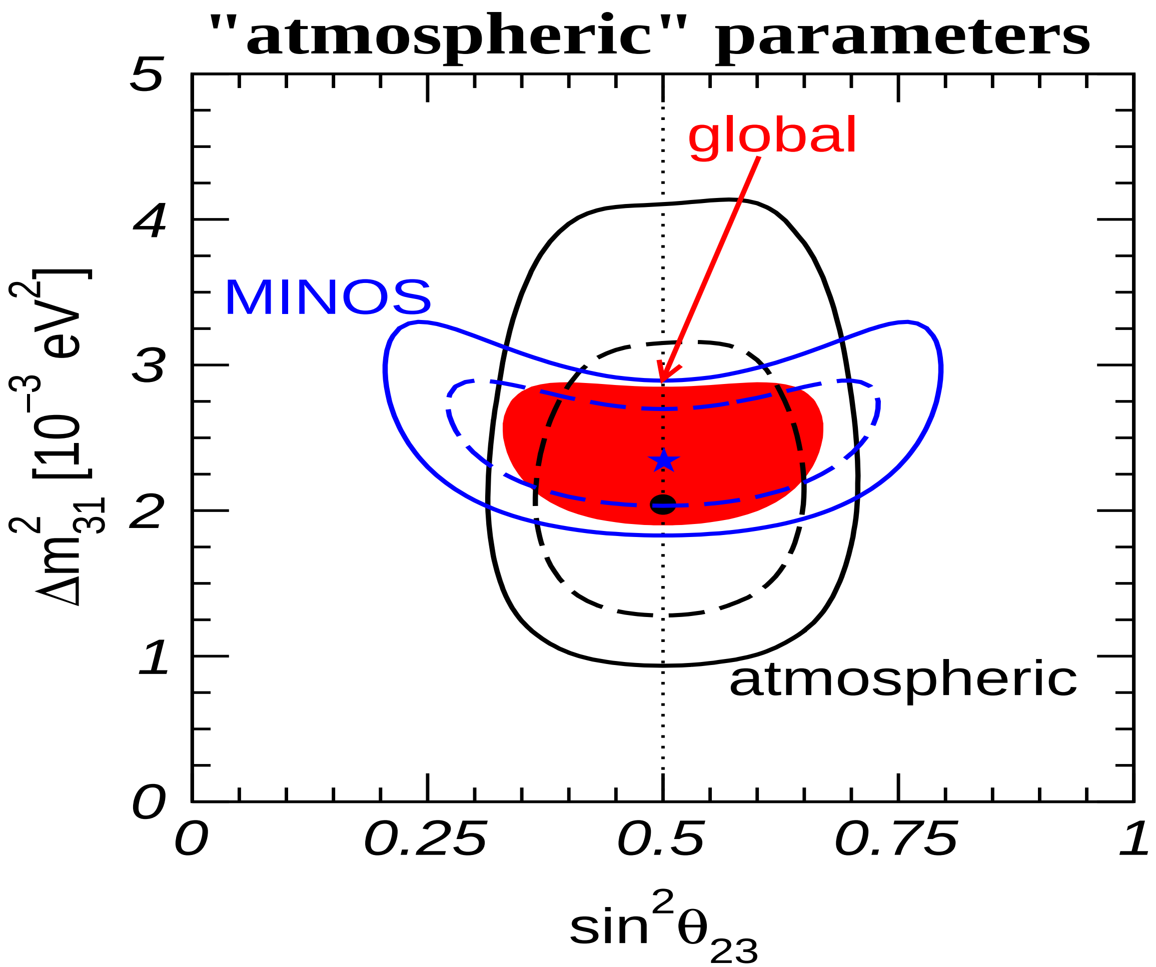
A.Bandyopadhyay, S.Choubey, S.Goswami, S.T.P., D.P.Roy, arXiv:0804.4857;

T. Schwetz, arXiv:0710.5027

"solar" parameters



T. Schwetz, arXiv:0710.5027[hep-ph]



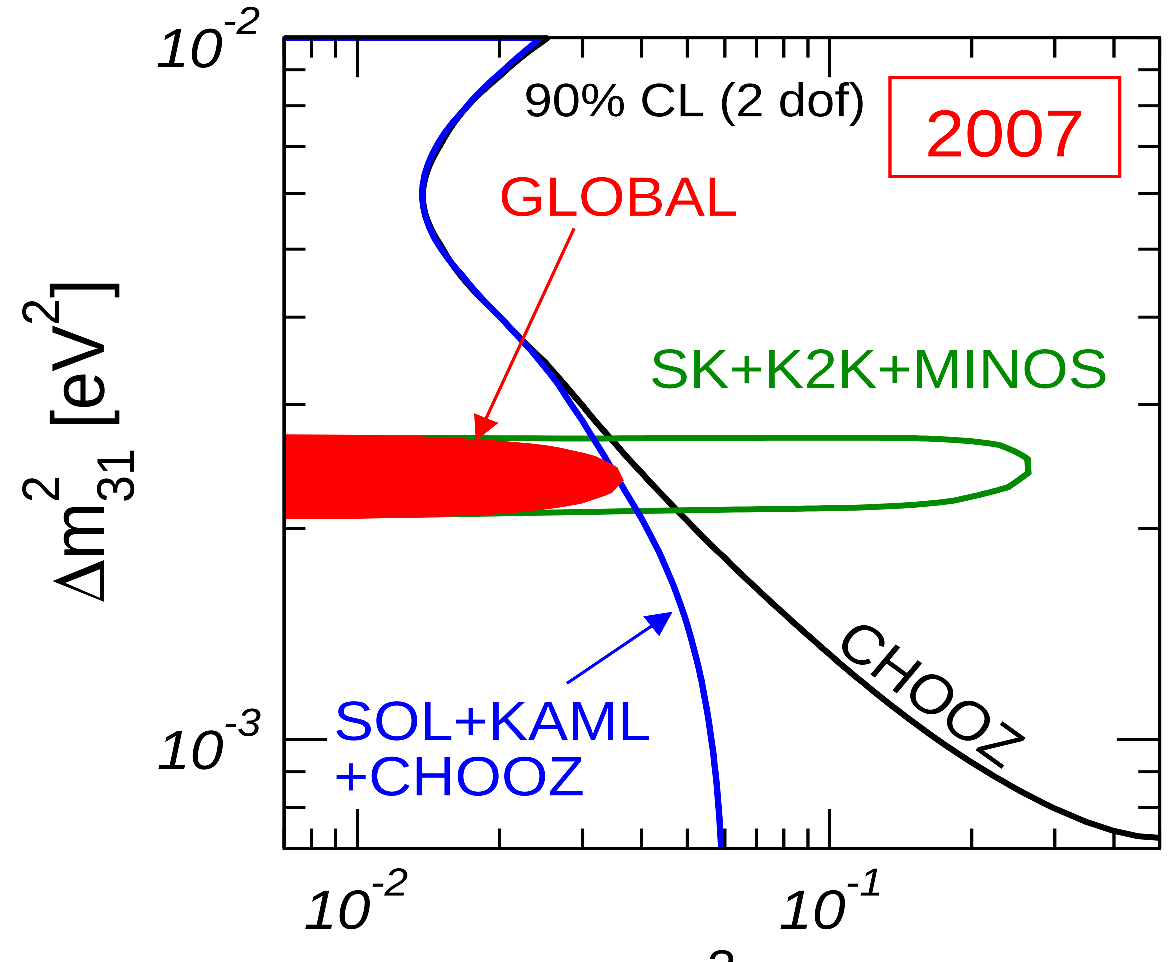
T. Schwetz, arXiv:0710.5027[hep-ph]

- sign of $\Delta m^2_{\rm atm}$ not determined;

3-ν mixing: $\Delta m^2_{31} > 0$, $m_1 < m_2 < m_3$ (normal ordering (NO));

$\Delta m^2_{31} < 0$, $m_3 < m_1 < m_2$ (inverted ordering (IO)).

- If $\theta_{23} \neq \frac{\pi}{4}$: θ_{23}, $(\frac{\pi}{4} - \theta_{23})$ ambiguity.



- $\sin^2\theta_{13} < 0.033$ (0.050) at 95% (99.73%) C.L.

Neutrino Oscillation Parameters

parameter	bf	1σ acc.	2σ range	3σ range
Δm^2_{21} [10^{-5} eV^2]	7.6	3%	$7.3 - 8.1$	$7.1 - 8.3$
$\|\Delta m^2_{31}\|$ [10^{-3} eV^2]	2.4	6%	$2.1 - 2.7$	$2.0 - 2.8$
$\sin^2\theta_{12}$	0.32	9%	$0.28 - 0.37$	$0.26 - 0.40$
$\sin^2\theta_{23}$	0.50	16%	$0.38 - 0.63$	$0.34 - 0.67$
$\sin^2\theta_{13}$	–	–	≤ 0.033	≤ 0.050

Best fit values (bf), relative accuracies at 1σ, and 2σ and 3σ allowed ranges of three-flavor neutrino oscillation parameters from a combined analysis of global data.

T. Schwetz, arXiv:0710.5027[hep-ph]

3−ν Mixing Analysis: $\Delta m^2_\odot \ll |\Delta m^2_{\rm atm}|$

$P^{3\nu}_\odot \cong \sin^4\theta_{13} + \cos^4\theta_{13}\; P^{2\nu}_\odot$,

$P^{2\nu}_\odot = \bar{P}^{2\nu}_\odot + P^{2\nu}_{\odot}{}_{\rm osc}$,

$\bar{P}^{2\nu}_\odot = \frac{1}{2} + (\frac{1}{2} - P')\cos 2\theta^m_{12}(t_0)\cos 2\theta_{12} \qquad (\theta_{12} \equiv \theta_\odot)$,

$P' = 0$: S. Mikheyev, A. Smirnov, 1985;

$P' \neq 0$: S. Parke, W. Haxton, 1986;

$P^{2\nu}_{\odot}{}_{\rm osc}$: S.T.P., 1988

$N_e \rightarrow N_e \cos^2\theta_{13}$,

$$P' \;=\; \frac{e^{-2\pi r_0 \frac{\Delta m^2}{2E}\sin^2\theta} - e^{-2\pi r_0 \frac{\Delta m^2}{2E}}}{1 - e^{-2\pi r_0 \frac{\Delta m^2}{2E}}}$$

S.T.P., 1988

LMA: $P' \ll 1, \quad < P^{2\nu}_{\odot}{}_{\rm osc} > \cong 0$

J. Rich, S.T.P., 1988

$$P^{3\nu}_{\rm KL} \cong \sin^4\theta_{13} + \cos^4\theta_{13}\left[1 - \sin^2 2\theta_{12}\sin^2(\frac{\Delta m^2_\odot}{4E}L)\right]$$

$$P^{3\nu}_{\rm CHOOZ} \cong 1 - \sin^2 2\theta_{13}\sin^2(\frac{\Delta m^2_{\rm atm}}{4E}L)$$

MSW Transitions of Solar Neutrinos in the Sun and the Hydrogen Atom

$$i\frac{d}{dt}\begin{pmatrix} A_\alpha(t,t_0) \\ A_\beta(t,t_0) \end{pmatrix} = \begin{pmatrix} -\epsilon(t) & \epsilon'(t) \\ \epsilon'(t) & \epsilon(t) \end{pmatrix} \begin{pmatrix} A_\alpha(t,t_0) \\ A_\beta(t,t_0) \end{pmatrix} \qquad (1)$$

where $\alpha = \nu_e$, $\beta = \nu_{\mu(\tau)}$,

$\epsilon(t) = \frac{1}{2}\,[\frac{\Delta m^2}{2E}\cos 2\theta - \sqrt{2}G_F N_e(t)]$,

$\epsilon'(t) = \frac{\Delta m^2}{4E}\sin 2\theta$, with $\Delta m^2 = m_2^2 - m_1^2$.

- Standard Solar Models

$N_e(t) = N_e(t_0)\exp\left\{-\frac{t-t_0}{r_0}\right\}$, $r_0 \sim 0.1R_\odot$, $R_\odot = 6.96\times 10^5$km

Introducing the dimensionless variable

$$Z = ir_0\sqrt{2}G_F N_e(t_0)e^{-\frac{t-t_0}{r_0}}, \quad Z_0 = Z(t = t_0),$$

and making the substitution

$$A_e(t,t_0) = (Z/Z_0)^{c-a}\ e^{-(Z-Z_0)+i\int_{t_0}^{t}\epsilon(t')dt'}\ A'_e(t,t_0),$$

$A'_e(t,t_0)$ satisfies the confluent hypergeometric equation (CHE):

$$\left\{\ Z\frac{d^2}{dZ^2} + (c - Z)\ \frac{d}{dZ} - a\right\} A'_e(t,t_0) = 0,$$

where

$$a = 1 + ir_0\ \frac{\Delta m^2}{2E}\sin^2\theta, \qquad c = 1 + ir_0\ \frac{\Delta m^2}{2E}.$$

The confluent hypergeometric equation describing the ν_e oscillations in the Sun, coincides in form with the Schroedinger (energy eigenvalue) equation obeyed by the radial part, $\psi_{kl}(r)$, of the non-relativistic wave function of the hydrogen atom,

$$\Psi(\overrightarrow{r}) = \frac{1}{r}\psi_{kl}(r) Y_{lm}(\theta', \phi'),$$

r, θ' and ϕ' are the spherical coordinates of the electron in the proton's rest frame, l and m are the orbital momentum quantum numbers ($m = -l, ..., l$), k is the quantum number labeling (together with l) the electron energy (the principal quantum number is equal to $(k+l)$), E_{kl} ($E_{kl} < 0$), and $Y_{lm}(\theta', \phi')$ are the spherical harmonics. The function

$$\psi'_{kl}(Z) = Z^{-c/2}\ e^{Z/2}\ \psi_{kl}(r)$$

satisfies the confluent hypergeometric equation in which the variable Z and the parameters a and c are in this case related to the physical quantities characterizing the hydrogen atom:

$$Z = 2\,\frac{r}{a_0}\sqrt{-E_{kl}/E_I},\ a \equiv a_{kl} = l+1-\sqrt{-E_I/E_{kl}},\ c \equiv c_l = 2(l+1),$$

$a_0 = \hbar/(m_e e^2)$ is the Bohr radius and $E_I = m_e e^4/(2\hbar^2) \cong 13.6\ eV$ is the ionization energy of the hydrogen atom.

Quite remarkably, the behavior of such different physical systems as solar neutrinos undergoing MSW transitions in the Sun and the non-relativistic hydrogen atom are governed by one and the same differential equation.

Any solution - linear combination of two linearly independent solutions:

$\Phi(a,c;Z)$, $Z^{1-c}\ \Phi(a-c+1,2-c;Z)$; $\Phi(a',c';Z=0)=1$, $a',c'\neq 0,-1,-2,\ldots$

$$A(\nu_e \rightarrow \nu_{\mu(\tau)}) = \frac{1}{2}\ \sin 2\theta\ \left\{\Phi(a-c,2-c;Z_0) - e^{i(t-t_0)\frac{\Delta m^2}{2E}}\ \Phi(a-1,c;Z_0)\ \right\}.$$

Sun: $N_e(x) \cong N_e(x_0)e^{-\frac{x}{r_0}}$, $r_0 \cong 0.1 R_\odot$, $R_\odot \cong 7\times 10^5$ km

The region of $\nu_\odot$ production:

$20\ N_A\ cm^{-3} \lesssim N_e(x_0) \lesssim 100\ N_A\ cm^{-3}$: $|Z_0| > 500$ (!)

The solar ν_e survival probability:

$$\bar{P}(\nu_e \rightarrow \nu_e) = \tfrac{1}{2} + (\tfrac{1}{2} - P')\cos 2\theta_m^0 \cos 2\theta,$$

$$P' = \frac{e^{-2\pi r_0 \frac{\Delta m^2}{2E}\sin^2\theta} - e^{-2\pi r_0 \frac{\Delta m^2}{2E}}}{1 - e^{-2\pi r_0 \frac{\Delta m^2}{2E}}}$$

S.T.P., 1988

- sgn($\Delta m^2_{\rm atm}$) = sgn(Δm^2_{31}) not determined

$$\Delta m^2_{\rm atm} \equiv \Delta m^2_{31} > 0, \quad \text{normal mass ordering}$$

$$\Delta m^2_{\rm atm} \equiv \Delta m^2_{32} < 0, \quad \text{inverted mass ordering}$$

Convention: $m_1 < m_2 < m_3$ - NMO, $m_3 < m_1 < m_2$ - IMO

$$m_1 \ll m_2 < m_3, \qquad \text{NH},$$

$$m_3 \ll m_1 < m_2, \qquad \text{IH},$$

$$m_1 \cong m_2 \cong m_3, \ m^2_{1,2,3} >> \Delta m^2_{\rm atm}, \ \text{QD}; \ m_j \gtrsim 0.10 \text{ eV}.$$

- Dirac phase δ: $\nu_l \leftrightarrow \nu_{l'}$, $\bar{\nu}_l \leftrightarrow \bar{\nu}_{l'}$, $l \neq l'$; $A^{(l,l')}_{\rm CP} \propto J_{\rm CP} \propto \sin\theta_{13} \sin\delta$
- Majorana phases α_{21}, α_{31}:
  - $\nu_l \leftrightarrow \nu_{l'}$, $\bar{\nu}_l \leftrightarrow \bar{\nu}_{l'}$ not sensitive;

S.M. Bilenky, J. Hosek, S.T.P.,1980;
P. Langacker, S.T.P., G. Steigman, S. Toshev, 1987

  - $\left| < m > \right|$ in $(\beta\beta)_{0\nu}$−decay depends on α_{21}, α_{31};
  - $\Gamma(\mu \to e + \gamma)$ etc. in SUSY theories depend on $\alpha_{21,31}$;
  - BAU, leptogenesis scenario: $\alpha_{21,31}$!

Neutrino Mixing Parameters

$\theta_{12},\ \theta_{23},\ \theta_{13}$

ν_j Dirac Majorana

δ $\delta,\ \alpha_{21},\ \alpha_{31}$

$m_1,\quad m_2,\quad m_3$

$m_1,\ m_2,\ m_3$ - in terms of $\Delta m^2_{\odot},\ \Delta m^2_{\text{atm}}$ and $\min(m_j)$

Conventions

A. $m_1 < m_2 < m_3$ (NO) or $m_3 < m_1 < m_2$ (IO)

- $\Delta m^2_{\odot} = \Delta m^2_{21} > 0$
- $\Delta m^2_{\text{atm}} = \Delta m^2_{31} > 0$ (NO), $\Delta m^2_{\text{atm}} = \Delta m^2_{32} < 0$ (IO)
- $m_2 = \sqrt{m_1^2 + \Delta m^2_{21}},\ \ m_3 = \sqrt{m_1^2 + \Delta m^2_{31}}$

B. $m_1 < m_2 < m_3$

- $\Delta m^2_{\text{atm}} = \Delta m^2_{31} > 0$
- $\Delta m^2_{\odot} = \Delta m^2_{21} > 0$, NO; $\Delta m^2_{\odot} = \Delta m^2_{32} > 0$, IO

Absolute Neutrino Mass Measurements

The Troitzk and Mainz ^{3}H β-decay experiments

$$m_{\nu_e} < 2.3 \text{ eV} \qquad (95\% \text{ C.L.})$$

There are prospects to reach sensitivity

$$\text{KATRIN}: \qquad m_{\nu_e} \sim 0.2 \text{ eV}$$

Cosmological and astrophysical data: the WMAP result combined with data from large scale structure surveys (2dFGRS, SDSS)

$$\sum_j m_j \equiv \Sigma < (0.4 - 1.4) \text{ eV}$$

The WMAP and future PLANCK experiments can be sensitive to

$$\sum_j m_j \cong 0.4 \text{ eV}$$

Data on weak lensing of galaxies by large scale structure, combined with data from the WMAP and PLANCK experiments may allow to determine

$$\sum_j m_j : \qquad \delta \cong 0.04 \text{ eV}.$$

Future Progress

• Determination of the nature - Dirac or Majorana, of ν_j .

• Determination of sgn(Δm^2_{atm}), type of $\nu-$ mass spectrum

$$m_1 \ll m_2 < m_3, \qquad \text{NH},$$

$$m_3 \ll m_1 < m_2, \qquad \text{IH},$$

$$m_1 \cong m_2 \cong m_3,\ m^2_{1,2,3} >> \Delta m^2_{\text{atm}},\ \text{QD};\ m_j \gtrsim 0.10 \text{ eV}.$$

• Determining, or obtaining significant constraints on, the absolute scale of ν_j-masses, or min(m_j).

• Status of the CP-symmetry in the lepton sector: violated due to δ (Dirac), and/or due to α_{21}, α_{31} (Majorana)?

• Measurement of, or improving by at least a factor of (5 - 10) the existing upper limit on, $\sin^2\theta_{13}$.

• High precision determination of $\Delta m^2_{\odot}$, $\theta_{\odot}$, Δm^2_{atm}, θ_{atm}.

• Searching for possible manifestations, other than ν_l-oscillations, of the non-conservation of L_l, $l = e, \mu, \tau$, such as $\mu \to e + \gamma$, $\tau \to \mu + \gamma$, etc. decays.

• Understanding at fundamental level the mechanism giving rise to the $\nu-$ masses and mixing and to the L_l-non-conservation. Includes understanding

– the origin of the observed patterns of ν-mixing and ν-masses ;

– the physical origin of CPV phases in $U_{\rm PMNS}$;

– Are the observed patterns of ν-mixing and of $\Delta m^2_{21,31}$ related to the existence of a new symmetry?

– Is there any relations between $q-$mixing and $\nu-$ mixing? Is $\theta_{12} + \theta_c = \pi/4$?

– Is $\theta_{23} = \pi/4$, or $\theta_{23} > \pi/4$ or else $\theta_{23} < \pi/4$?

– Is there any correlation between the values of CPV phases and of mixing angles in $U_{\rm PMNS}$?

• Progress in the theory of ν-mixing might lead to a better understanding of the origin of the BAU.

– Can the Majorana and/or Dirac CPVP in $U_{\rm PMNS}$ be the leptogenesis CPV parameters at the origin of BAU?

HOW?

- $\nu_{\odot}-$, $\nu_{\text{atm}}-$ experiments

  SK (ν_{atm});

  INO (ν_{atm}); MEMPHYS (projects)

  MINOS (ν_{μ}^{atm}); ATLAS, CMS (ν_{μ}^{atm}) (?)

  SNO (2006)

  SAGE

  BOREXINO

  LowNu (XMASS, LENS,...) projects

- Experiments with Reactor $\bar{\nu}_e$, $\sim (1 - 180)$ km (SKGd)

- Accelerator Experiments

  MINOS 732 km

  CNGS (OPERA) 732 km

- Super Beams

  T2K, SK (HK) 295 km

  NOνA $\sim$800 km

  SPL+$\beta-$beams, MEMPHYS (0.5 megaton):
  CERN-Frejus $\sim$140 km

  $\nu-$Factories $\sim$ 3000, 7000 km

- $(\beta\beta)_{0\nu}-$Decay, ^{3}H $\beta-$Decay
- Astrophysics, Cosmology

$$\Delta m^2_\odot = \Delta m^2_{21},\ \theta_\odot = \theta_{12}$$

Data from $\nu_\odot$- experiments

- **SNO**: $A_{\mathrm{D-N}} < 4.3\%$

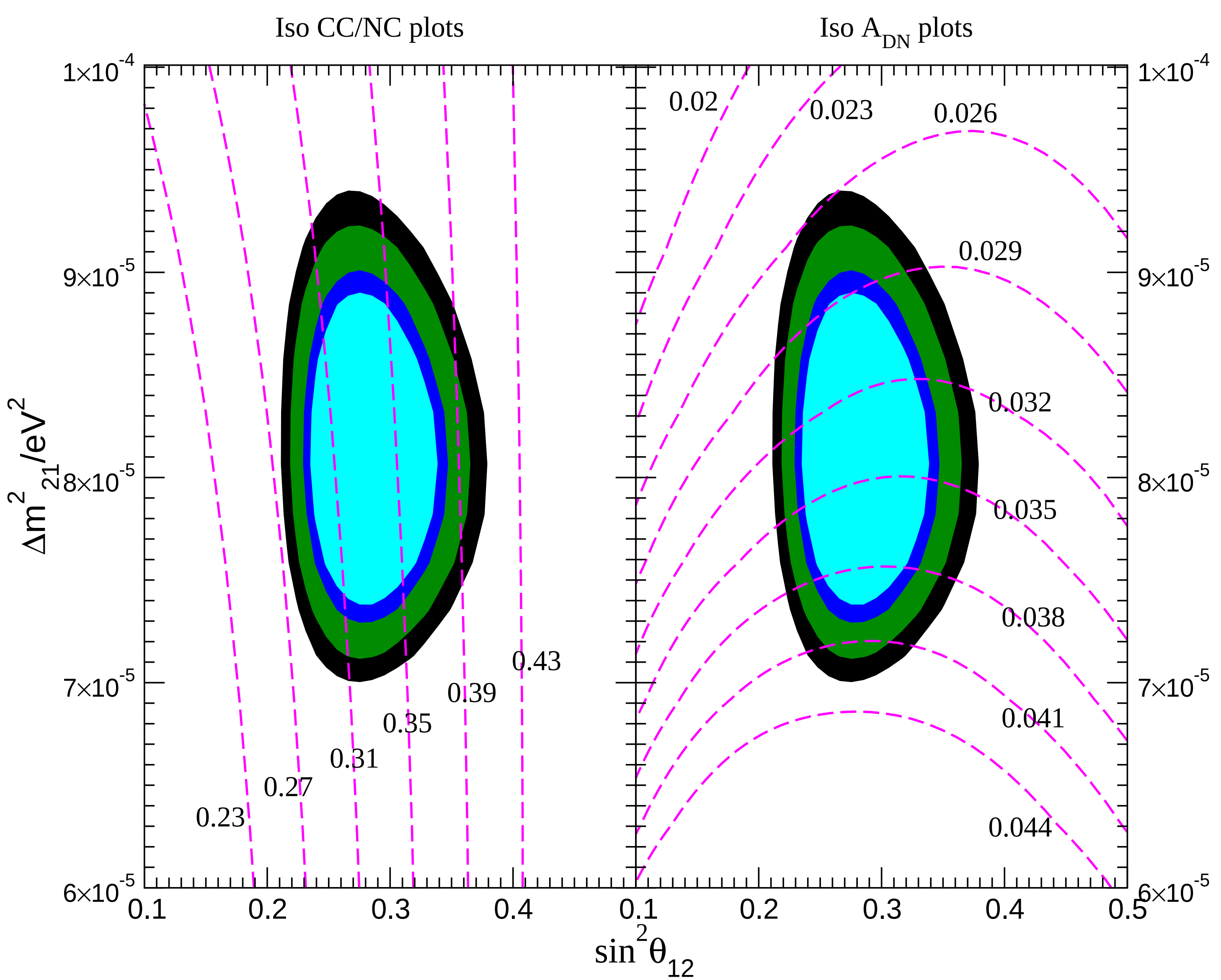
  would restrict further Δm^2_{21} from below

  $R_{\mathrm{CC/NC}} = 0.306 \pm 0.035$, **reducing the error**

  would restrict further

  the range of $\sin^2\theta_{12}$

- **BOREXINO**
- **LowNu (pp neutrinos) - LENS, XMASS**: $\sin^2 2\theta_{12}$



A. Bandyopadhyay et al, hep-ph/0406328

LowNu: generic $\nu - e^-$ ES experiment

pp: $E_\nu \leq 0.42$ **MeV**, $\bar{E}_\nu = 0.286$ **MeV**

Assume $T_e \geq 50$ **keV**

$R_{pp} \cong \bar{P} + r_{pp}(1 - \bar{P})$, $\bar{P} \cong \cos^4\theta_{13}(1 - \frac{1}{2}\sin^2 2\theta_{12})$, $r_{pp} \cong 0.3$

$R_{CC/NC}(SNO) \cong \sin^2\theta_{12}\cos^4\theta_{13}$

$\Delta(\sin^2\theta_{12}) \sim 0.5\Delta(R_{pp})/(\cos 2\theta_{12}(1 - r_{pp}))$

$\Delta(R_{pp}) < \Delta(R_{CC/NC})$ **to reduce** $\Delta(\sin^2\theta_{12})$**; SNO3:** $\sim 6\%$

BP04: $R_{pp} \cong 0.71$ **(**3σ**: 0.67 - 0.76)**

With $\Delta(R_{pp}) = 2\%$, $\Delta(\sin^2\theta_{12}) \gtrsim 15\%$ **at** 3σ

Dedicated reactor experiment with $L \sim 60$ **km:**

$\Delta(\sin^2\theta_{12}) = (6 - 9)\%$ **at** 3σ

A. Bandyopadhyay et al., hep-ph/0302243 and hep-ph/0410283;

H. Minakata et al., hep-ph/0407326

Reactor Experiments

Future more precise KamLAND data: Δm^2_{21} **with higher precision**

$\sin^2\theta_{12}$ **cannot be determined with a high precision**

("wrong distance")

even with SHIKA-2 reactor when operative

("right distance", L = 88 km, but signal too weak (3.926 GW))

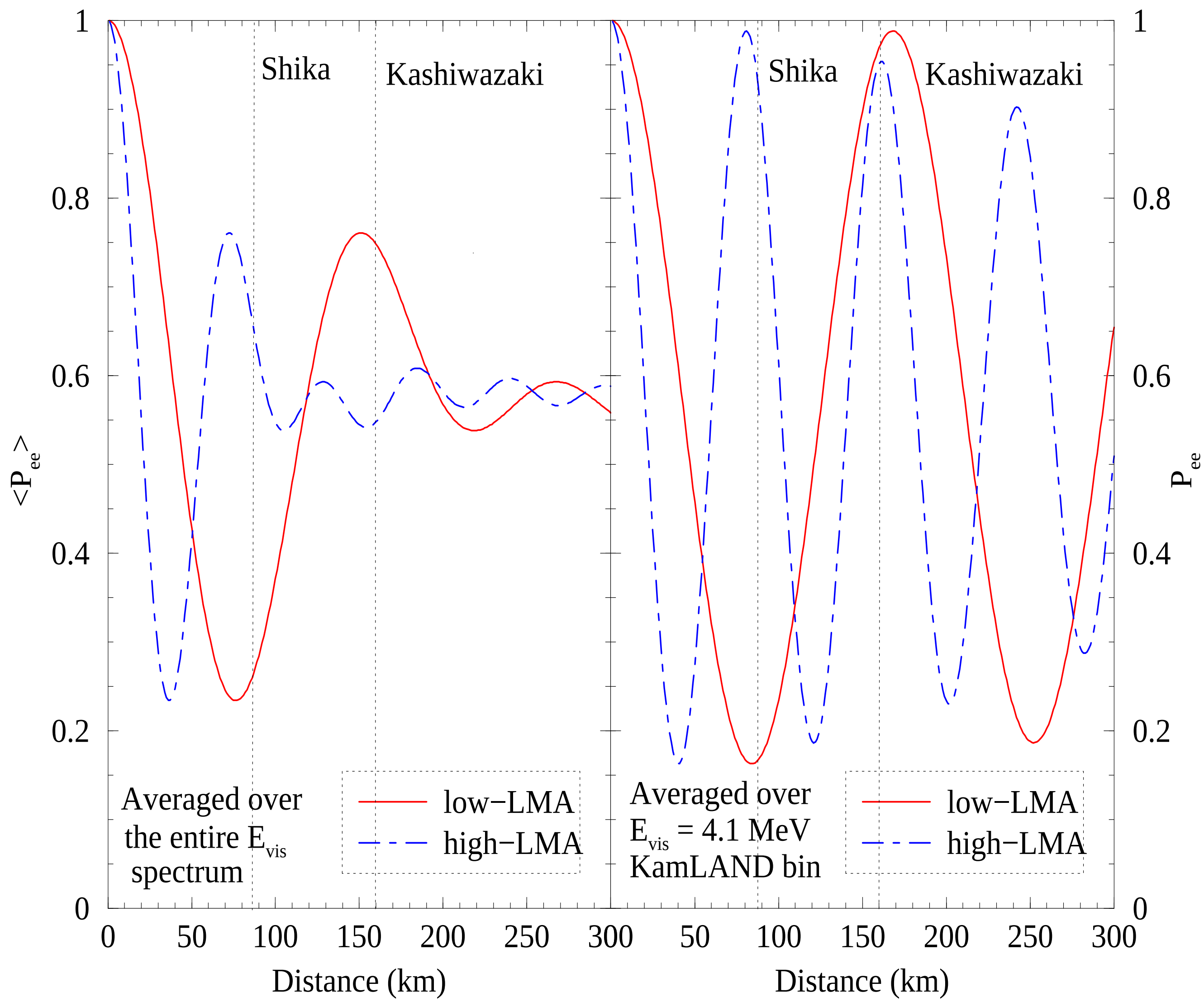
$$P^{3\nu}_{\mathrm{KL}} \cong \sin^4\theta_{13} + \cos^4\theta_{13}\left[1 - \sin^2 2\theta_{12}\sin^2\left(\frac{\Delta m^2_{\odot}}{4E}L\right)\right]$$

$\sin^2(\frac{\Delta m^2_{\odot}}{4E}L) \cong 0$ **(SPMAX; KamLAND):**

strong sensitivity to $\Delta m^2_{\odot} \equiv \Delta m^2_{21}$**, weak sensitivity to** $\sin^2\theta_{12}$

$\sin^2(\frac{\Delta m^2_{\odot}}{4E}L) \cong 1$ **(SPMIN): E = 4 MeV,** $L \cong 60$ **km,**

strong sensitivity to $\sin^2\theta_{12}$



Shika
Kashiwazaki
Shika
Kashiwazaki
Averaged over
the entire E_{vis}
spectrum
Averaged over
E_{vis} = 4.1 MeV
KamLAND bin
low−LMA
high−LMA
low−LMA
high−LMA
$<P_{ee}>$
P_{ee}
Distance (km)
Distance (km)
0
0.2
0.4
0.6
0.8
1
0
50
100
150
200
250
300

SK + 0.1% Gd

J.F. Beacom and M.R. Vagins, hep-ph/0309300

- **SK-Gd reactor $\bar{\nu}_e$ rate $\sim$ 43 times KamLAND rate**

3 years statistics in SK-Gd, 99% C.L.:

$$\Delta m^2_{21} = (8.01 - 8.61) \times 10^{-5}\text{eV}^2; \quad \text{spread} = 3.6\%$$
$$\sin^2\theta_{12} = (0.22 - 0.34); \quad \text{spread} = 21\%$$

5 years statistics in SK-Gd, 99% C.L.:

$$\Delta m^2_{21} = (8.07 - 8.53) \times 10^{-5}\text{eV}^2; \quad \text{spread} = 2.8\%$$
$$\sin^2\theta_{12} = (0.22 - 0.32); \quad \text{spread} = 18\%$$

$$\text{spread} = \frac{a_{max} - a_{min}}{a_{max} + a_{min}}, \;\; \mathbf{a} \equiv \mathbf{\Delta m^2_{21}} \text{ or } \sin^2\theta_{12}$$

Comment: SK-Gd data simulated at $\Delta m^2_{21} = 8.3 \times 10^{-5}$ eV^2, $\sin^2\theta_{12} = 0.27$ (the "old" global best-fit point). The precision on Δm^2_{21} and $\sin^2\theta_{12}$ for a given statistics remains approximately the same for $\Delta m^2_{21} = 7.6 \times 10^{-5}$ eV^2, $\sin^2\theta_{12} = 0.30$ (the new global best-fit point).

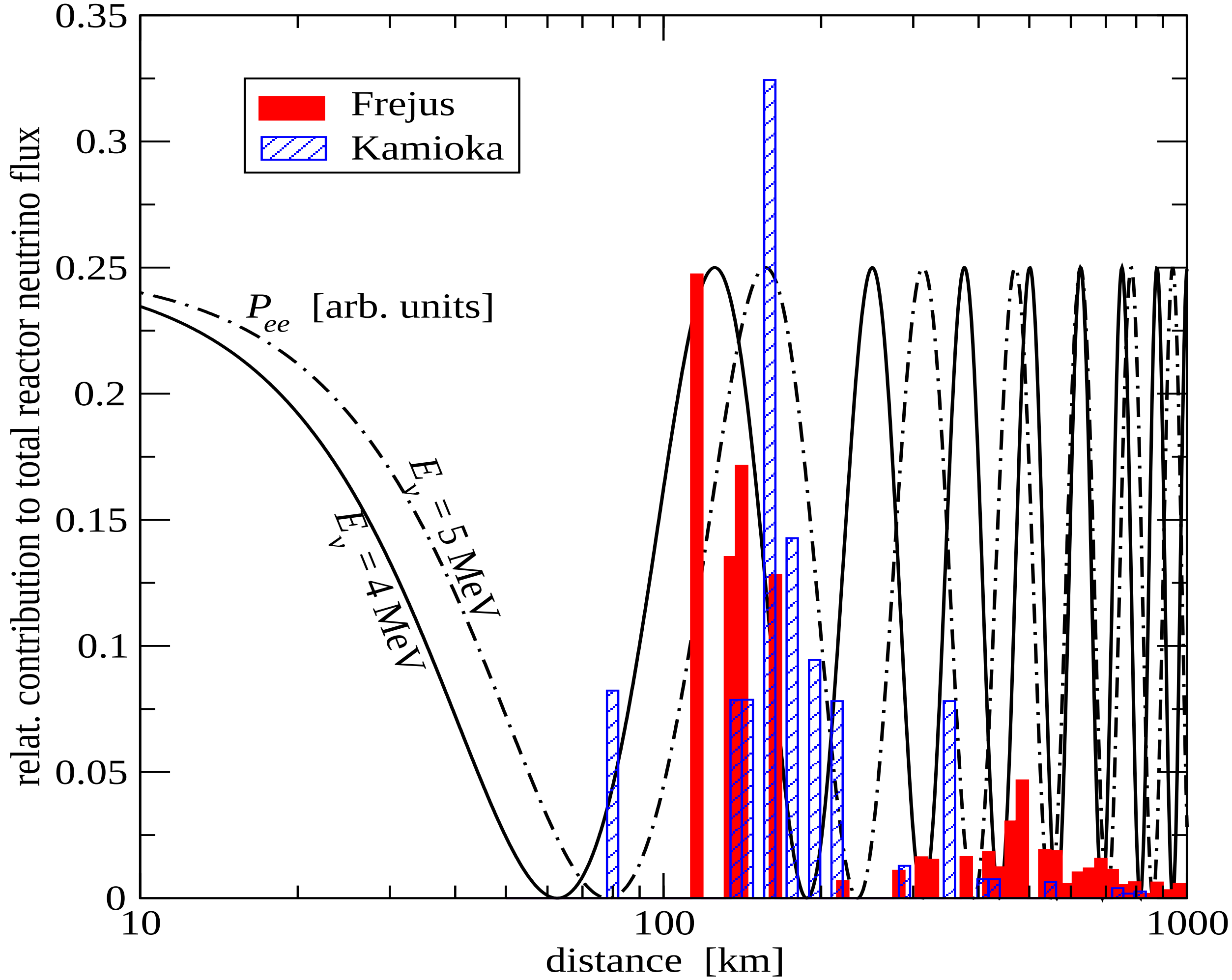
Sensitivity to Δm^2_{21} and $\sin^2\theta_{12}$

Data set used	99% CL range of $\Delta m^2_{21} \times 10^{-5}\text{eV}^2$	99% CL spread of Δm^2_{21}	99% CL range of $\sin^2\theta_{12}$	99% CL spread in $\sin^2\theta_{12}$
only solar	3.2 - 14.9	65%	0.22 – 0.37	25%
solar with future SNO	3.3 – 11.9	57%	2.2 – 0.34	21%
solar+1 kTy KL(low-LMA)	6.5 - 8.0	10%	0.23 – 0.37	23%
solar+2.6 kTy KL(low-LMA)	6.7 – 7.7	7%	0.23 – 0.36	22%
solar with future SNO+1.3 kTy KL(low-LMA)	6.7 – 7.8	8%	0.24 – 0.34	17%
3 yrs SK-Gd	7.0 - 7.4	3%	0.25 – 0.37	19%
5 yrs SK-Gd	7.0 – 7.3	2%	0.26 – 0.35	15%
solar+3 yrs SK-Gd(low-LMA)	7.0 – 7.4	3%	0.25 – 0.34	15%
solar with future SNO+3 yrs SK-Gd(low-LMA)	7.0 – 7.4	3%	0.25 – 0.335	14%
7 yrs SK-Gd with *only* Shika-2 "up"	7.0 – 7.3	2%	0.28 – 0.32	6.7%

Future SNO: 5% on R_{CC}, 6% on R_{NC}

S.T.P. and S. Choubey, hep-ph/0404103

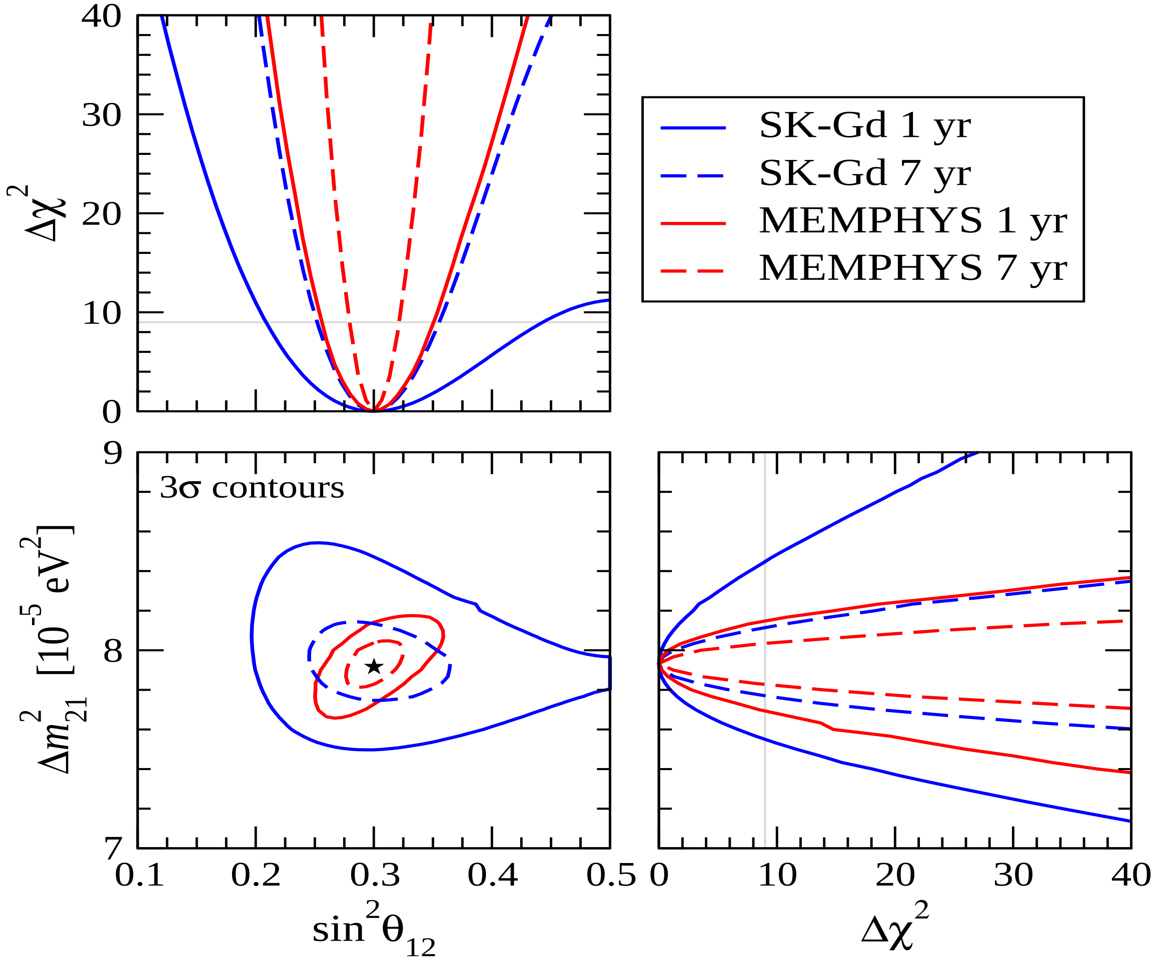
MEMPHYS (Frejus) + 0.1% Gd



MEMPHYS (Frejus): 147 kt water-Čerenkov detector, $\sim 6.5\times$SK

56 reactors within 1000 km; 65% of the flux from reactors within 160 km

MEMPHYSGd vs SKGd



1 year MEMGd $\cong$ 7 years SKGd: $3\sigma(\Delta m^2_{21}) \cong 3\%$, $3\sigma(\sin^2_{21}) \cong 20\%$

7 years MEMPHYSGd: $3\sigma(\Delta m^2_{21}) \cong 1.4\%$, $3\sigma(\sin^2_{21}) \cong 13\%$

S.T.P. and T. Schwetz, hep-ph/0607155

Dedicated Reactor Experiment on $\sin^2 2\theta_{12}$

$$P_{KL}^{3\nu} \cong \sin^4\theta_{13} + \cos^4\theta_{13}\left[1 - \sin^2 2\theta_{12}\sin^2(\frac{\Delta m^2_\odot}{4E}L)\right]$$

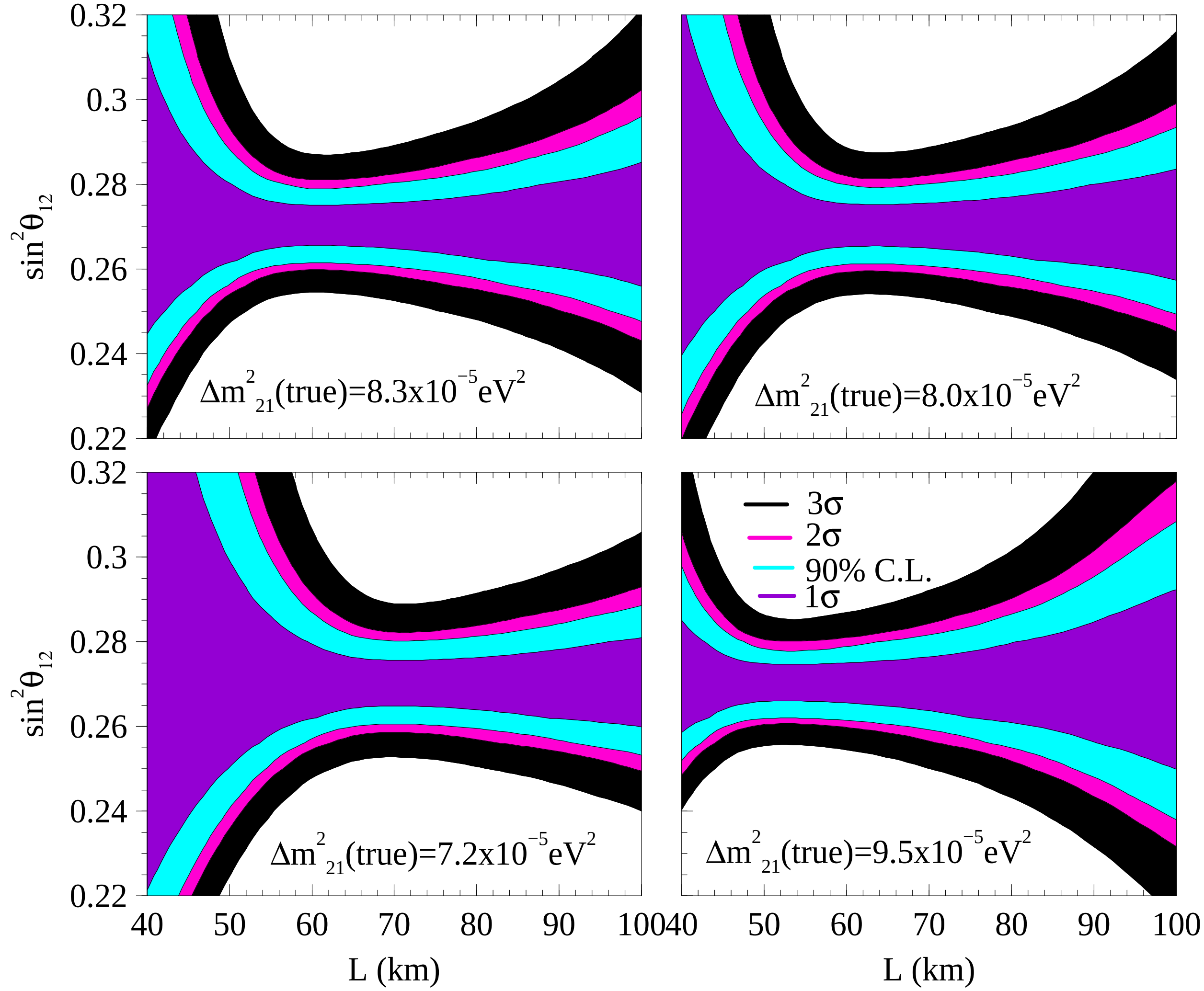
SPMIN: $L \sim 60$ **km:** $\sin^2 2\theta_{12}$

$\Delta(\sin^2\theta_{12}) = (6-9)\%$ **at** 3σ

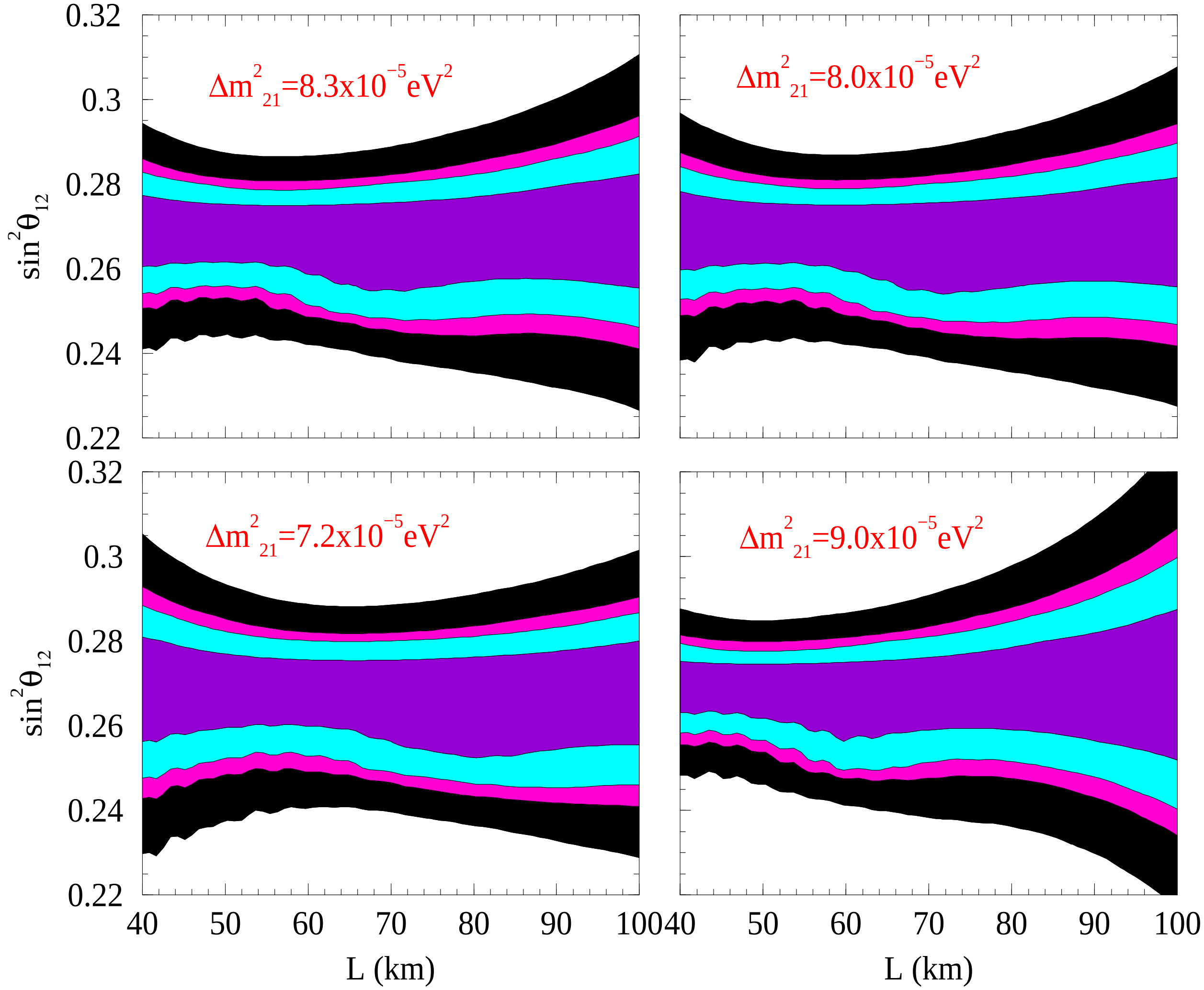
A. Bandyopadhyay, S. Choubey, S. Goswami, hep-ph/0302243;

A. Bandyopadhyay et al., hep-ph/0410283;

H. Minakata et al., hep-ph/0407326



Systematic uncertainty 2%; statistics 73 GWkTy; KamLAND-like detector



$\sin^2\theta_{13}$ - **free within the 3σ allowed range**

SPMIN: $\delta(\sin^2 2\theta_{12}) \approx 2\Delta P_{ee} \sin^2\theta_{13} + 2\ \cos^2 2\theta_{12}\ \Delta(\sin^2\theta_{13})$

Oscillation Parameters

$$\Delta m^2_\odot = 8.0\ (7.6) \times 10^{-5}\ \text{eV}^2\ , \quad 3\sigma(\Delta m^2_\odot) = 9\%\ ,$$

$$\sin^2\theta_\odot = 0.30\ , \quad 3\sigma(\sin^2\theta_\odot) = 24\%\ ,$$

$$|\Delta m^2_{\text{atm}}| = 2.5 \times 10^{-3}\ \text{eV}\ , \quad 3\sigma(|\Delta m^2_{\text{atm}}|) = 18\%.$$

Future:

SNO III: $3\sigma(\sin^2\theta_\odot) = 21\%$ **;**

3 kTy KamLAND: $3\sigma(\Delta m^2_\odot) = 7\%\ , \quad 3\sigma(\sin^2\theta_\odot) = 18\%$ **;**

SK-Gd (0.1% Gd: 43×(KL $\bar{\nu}_e$ rate)), 3y: $3\sigma(\Delta m^2_\odot) \cong 4\%$

KL type reactor $\bar{\nu}_e$ detector, $L \sim 60$ km, ~ 60 GW kTy:

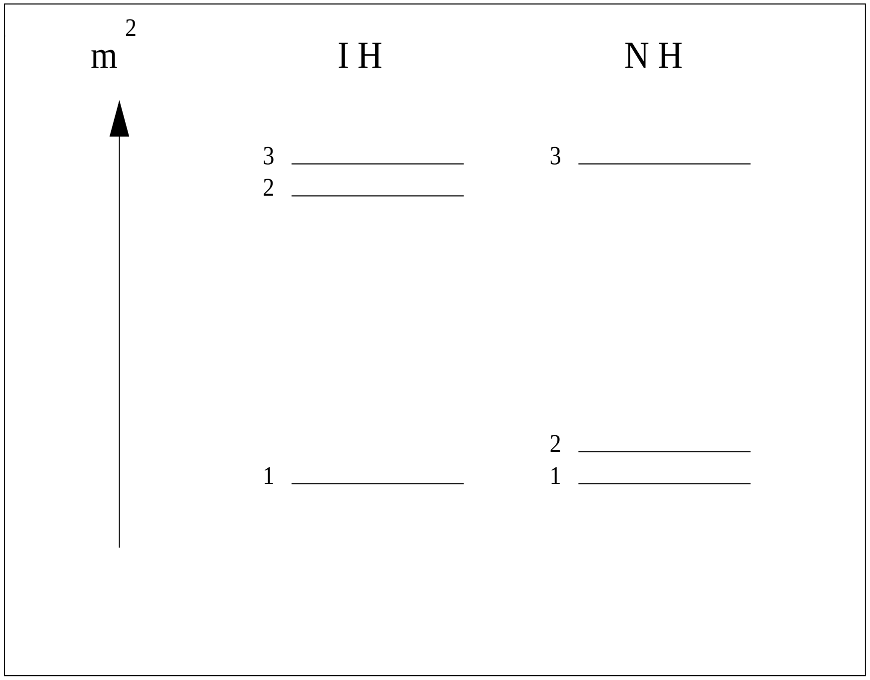
$3\sigma(\sin^2\theta_\odot) \cong 6\%\ (9\%)$ **for 2% (5%) syst. error; +** $\delta(\sin^2\theta_{13}) : 9\%\ (11\%)$

A. Bandyopadhyay, et al., hep-ph/0410283

T2K (SK): $3\sigma(|\Delta m^2_{\text{atm}}|) \cong 12\%$

P. Huber et al., hep-ph/0403068

Determining the $\nu-$Mass Hierarchy (sgn($\Delta m^2_{\rm atm}$)



- **Reactor $\bar{\nu}_e$ Oscillations in vacuum.**

- **Atmospheric ν experiments: subdominant $\nu_{\mu(e)} \rightarrow \nu_{e(\mu)}$ and $\bar{\nu}_{\mu(e)} \rightarrow \bar{\nu}_{e(\mu)}$ oscillations (matter effects).**

- **LBL $\nu-$oscillation experiments (T2KK, NOνA); $\nu-$factory.**

- ^{3}H β**-decay Experiments (sensitivity to** 5×10^{-2} **eV).**

- $(\beta\beta)_{0\nu}-$**Decay Experiments (ν_j- Majorana particles).**

Reactor $\bar{\nu}_e$ Oscillations in vacuum

$$P_{\rm NH}(\bar{\nu}_e \to \bar{\nu}_e) = 1 - \frac{1}{2}\sin^2 2\theta_{13}\left(1 - \cos\frac{\Delta m^2_{\rm A}\,L}{2\,E_\nu}\right) - \frac{1}{2}\cos^4\theta_{13}\,\sin^2 2\theta_\odot\,\left(1 - \cos\frac{\Delta m^2_\odot\,L}{2\,E_\nu}\right) + \sin^2 2\theta_{13}\,\sin^2\theta_\odot\,\sin\frac{\Delta m^2_\odot\,L}{4\,E_\nu}\,\sin\left(\frac{\Delta m^2_{\rm A}\,L}{2\,E_\nu} - \frac{\Delta m^2_\odot\,L}{4\,E_\nu}\right)\,,$$

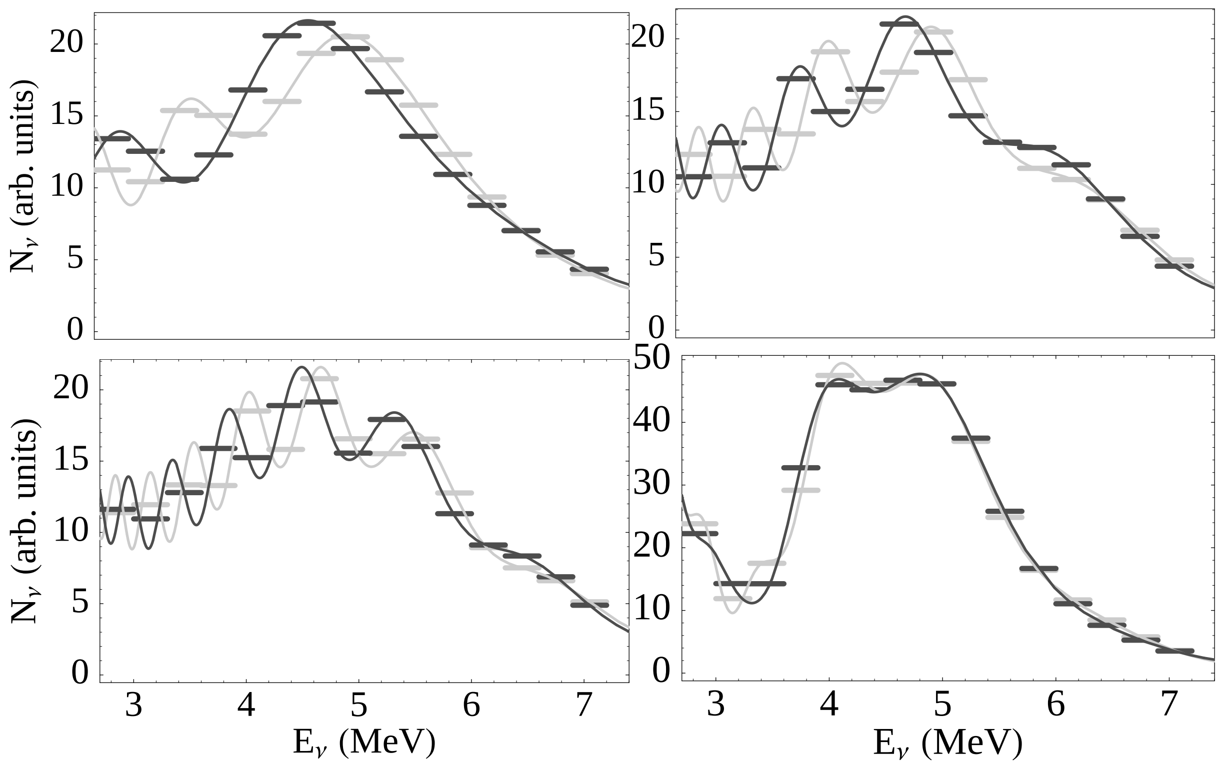
$$P_{\rm IH}(\bar{\nu}_e \to \bar{\nu}_e) = 1 - \frac{1}{2}\sin^2 2\theta_{13}\left(1 - \cos\frac{\Delta m^2_{\rm A}\,L}{2\,E_\nu}\right) - \frac{1}{2}\cos^4\theta_{13}\,\sin^2 2\theta_\odot\,\left(1 - \cos\frac{\Delta m^2_\odot\,L}{2\,E_\nu}\right) + \sin^2 2\theta_{13}\,\cos^2\theta_\odot\,\sin\frac{\Delta m^2_\odot\,L}{4\,E_\nu}\,\sin\left(\frac{\Delta m^2_{\rm A}\,L}{2\,E_\nu} - \frac{\Delta m^2_\odot\,L}{4\,E_\nu}\right)\,,$$

$\theta_\odot = \theta_{12}\,,\Delta m^2_\odot = \Delta m^2_{21} > 0\,;\;\; \sin^2\theta_{12} = 0.30\;(b.f.); \sin^2\theta_{12} \leq 0.38$ at 3σ;

$\Delta m^2_{\rm A} = \Delta m^2_{31} > 0\,,$ NH spectrum,

$\Delta m^2_{\rm A} = \Delta m^2_{23} > 0\,,$ IH spectrum

S.M. Bilenky, D. Nicolo, S.T.P., hep-ph/0112216;
M. Piai, S.T.P., hep-ph/0112074;

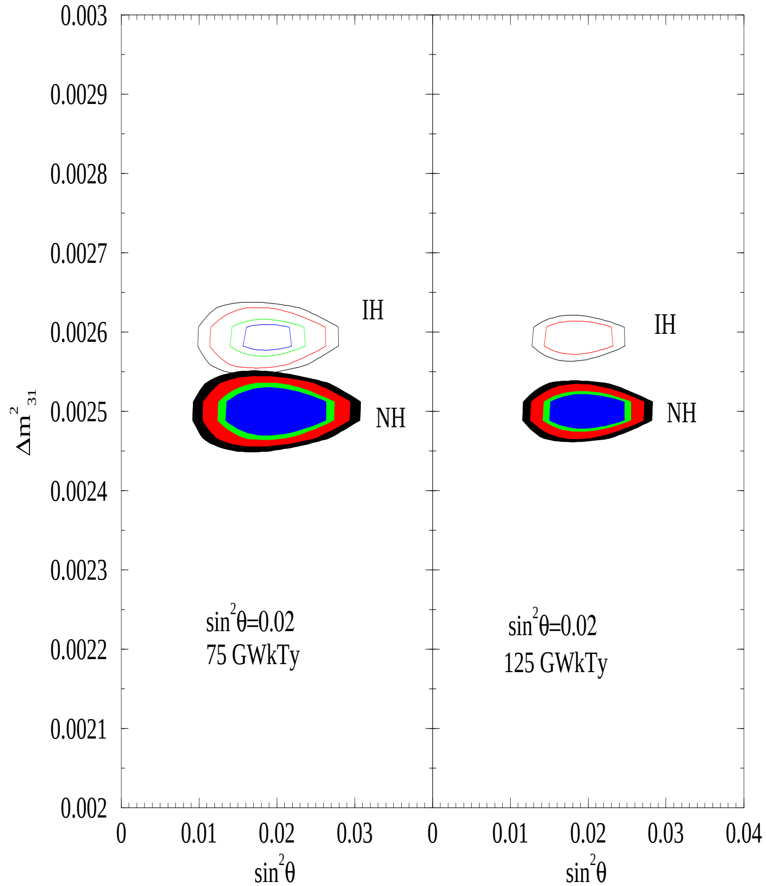


M. Piai, S.T.P., 2001

$\sin^2\theta_{13} = 0.05$, $\Delta m^2_{21} = 2\times 10^{-4}$ **eV**^{2}; $\Delta m^2_{\rm A} = 1.3;\ 2.5;\ 3.5\times 10^{-3}$ **eV**^{2}

$L = 20$ **km**; $\Delta E_\nu = 0.3$ **MeV**

NH − light grey; IH − dark grey



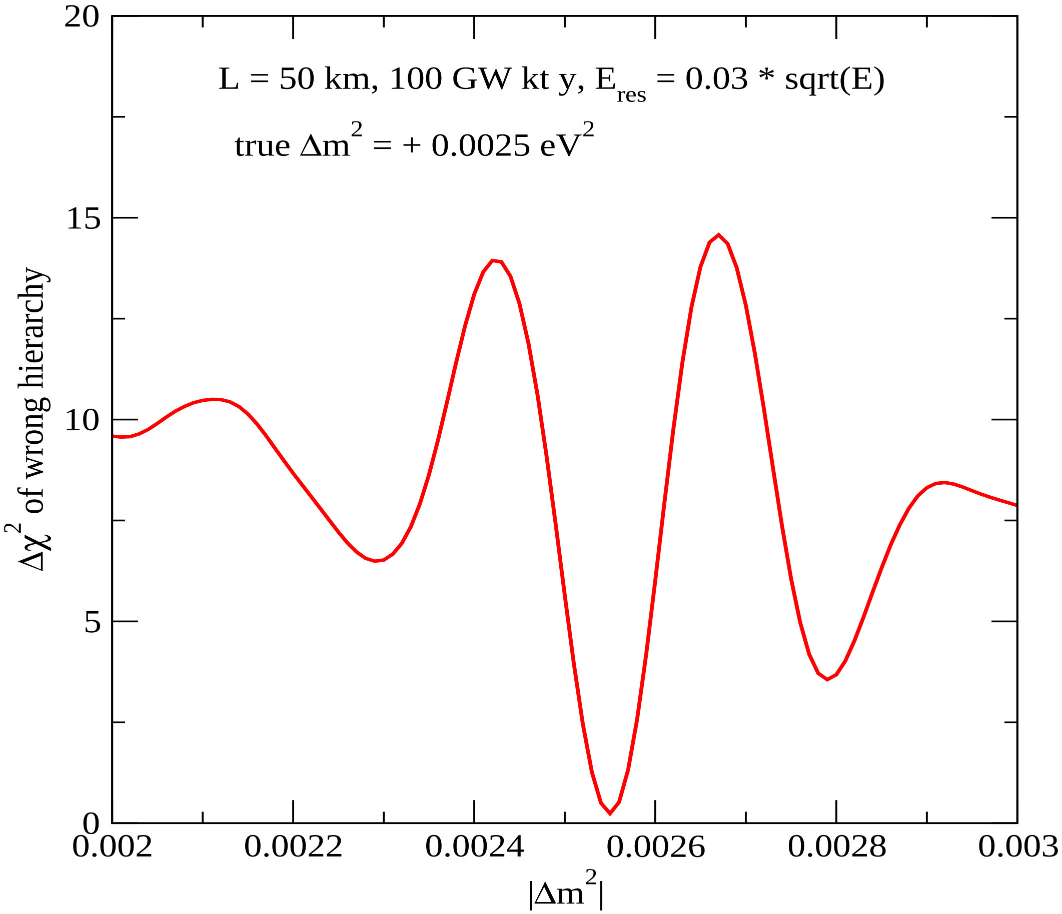
S. Choubey, S.T.P., 2003

$\sin^2\theta_\odot = 0.30$, $\Delta m^2_{21} = 1.5 \times 10^{-4}$ **eV^2**, $\Delta m^2_{\rm A} = 2.5 \times 10^{-3}$ **eV^2**

$L = 20$ **km; $\Delta E_\nu = 0.1$ MeV; syst. error 2%**

"True": NH; 90%, 95%, 99% and 99.73% solution regions

J. Learned et al., 2006 (Hanohano project)



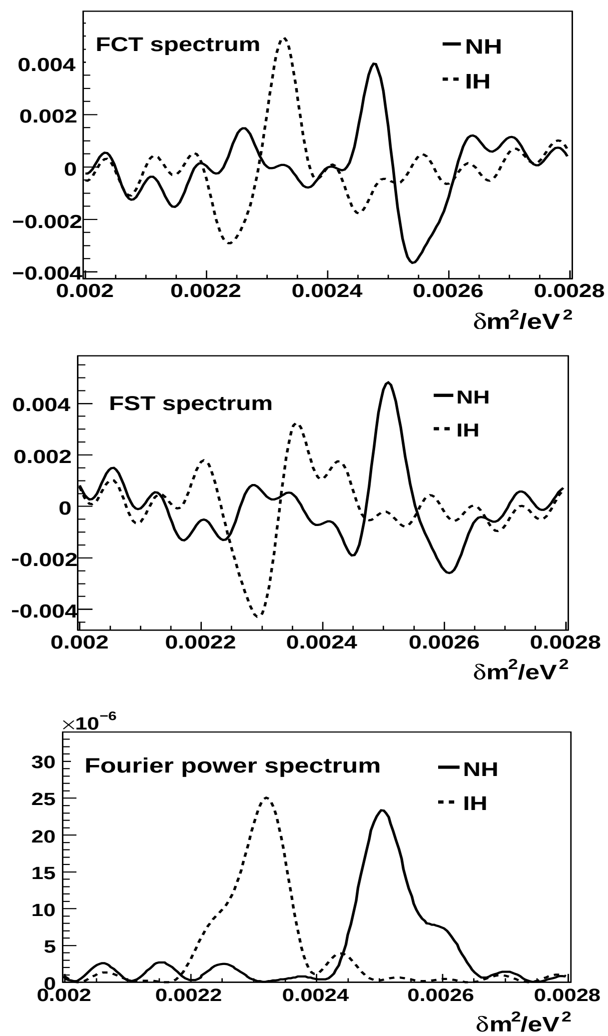
T. Schwetz, September 2006

$\sin^2\theta_\odot = 0.30$, $\Delta m^2_{21} = 8 \times 10^{-5}$ **eV^2; "true"** $\Delta m^2_{\rm A} = 2.50 \times 10^{-3}$ **eV^2 (NH)**

Minimum at $\Delta m^2_{\rm A} = -\ 2.55 \times 10^{-3}$ **eV^2 (IH)**

Precision of $\sim$ 1% on $|\Delta m^2_{\rm A}|$ required

J. Learned et al., 2006 (Hanohano project): can achieve it.



L. Zhan, Y. Wang, J. Cao, L. Wen, August 2008

Estimated sensitivity to the hierarchy for $\sin^2 2\theta_{13} > 0.005$

Atmospheric ν experiments

Subdominant $\nu_{\mu(e)} \rightarrow \nu_{e(\mu)}$ and $\bar{\nu}_{\mu(e)} \rightarrow \bar{\nu}_{e(\mu)}$ oscillations in the Earth.

$$P_{3\nu}(\nu_e \rightarrow \nu_\mu) \cong P_{3\nu}(\nu_\mu \rightarrow \nu_e) \cong s^2_{23}\, P_{2\nu}\,, P_{3\nu}(\nu_e \rightarrow \nu_\tau) \cong c^2_{23}\, P_{2\nu}\,,$$

$$P_{3\nu}(\nu_\mu \rightarrow \nu_\mu) \cong 1 - s^4_{23}\; P_{2\nu} - 2c^2_{23}s^2_{23}\; \left[1 - Re\; (e^{-i\kappa} A_{2\nu}(\nu_\tau \rightarrow \nu_\tau))\right]\,,$$

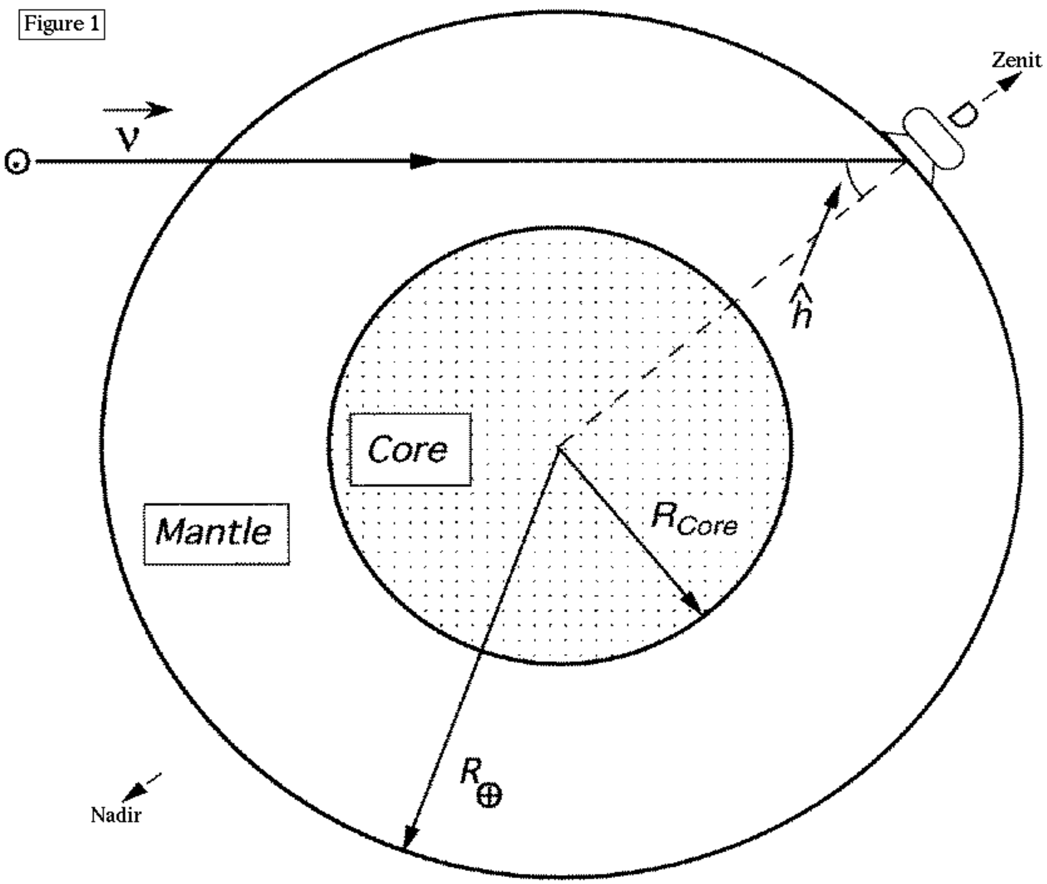
$P_{2\nu} \equiv P_{2\nu}(\Delta m^2_{31}, \theta_{13}; E, \theta_n; N_e)$: **2-$\nu$ $\nu_e \rightarrow \nu'_\tau$ oscillations in the Earth,**
$\nu'_\tau = s_{23}\, \nu_\mu + c_{23}\, \nu_\tau$;

κ **and** $A_{2\nu}(\nu_\tau \rightarrow \nu_\tau) \equiv A_{2\nu}$ **are known phase and 2-ν amplitude.**

NH: $\nu_{\mu(e)} \rightarrow \nu_{e(\mu)}$ **matter enhanced,** $\bar{\nu}_{\mu(e)} \rightarrow \bar{\nu}_{e(\mu)}$ **- suppressed**

IH: $\bar{\nu}_{\mu(e)} \rightarrow \bar{\nu}_{e(\mu)}$ **matter enhanced,** $\nu_{\mu(e)} \rightarrow \nu_{e(\mu)}$**−suppressed**

The Earth



Earth: $R_{core} = 3446$ **km,** $R_{man} = 2885$ **km;**

Neutrino trajectories crossing the Earth core: Nadir angle $\theta_n \leq 33.17^0$**;**

Earth: $\bar{N}_e^{man} \sim 2.3\ N_A\ cm^{-3}$, $\bar{N}_e^{core} \sim 6.0\ N_A\ cm^{-3}$

The Earth

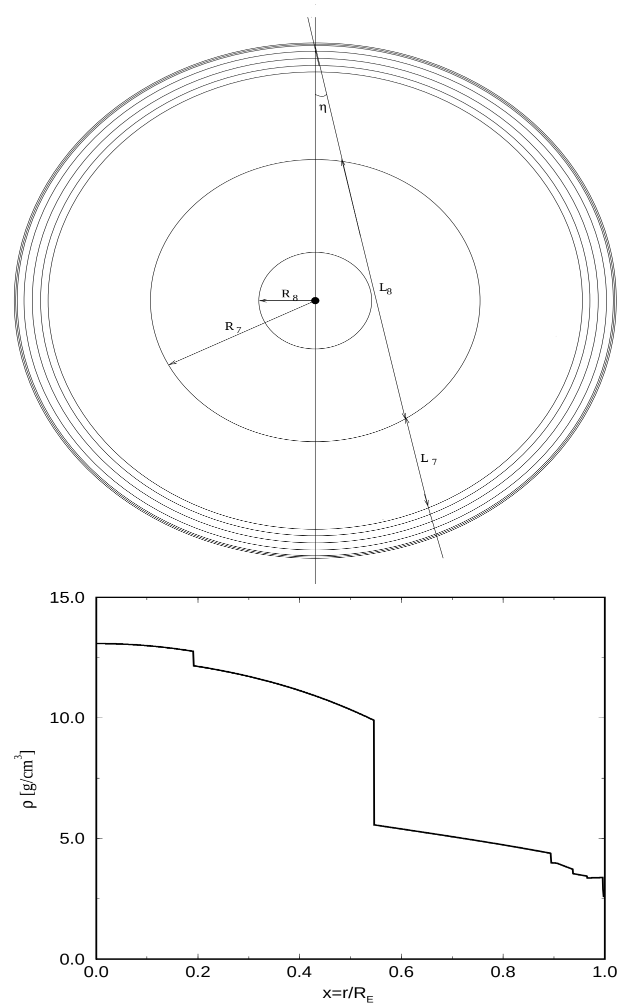
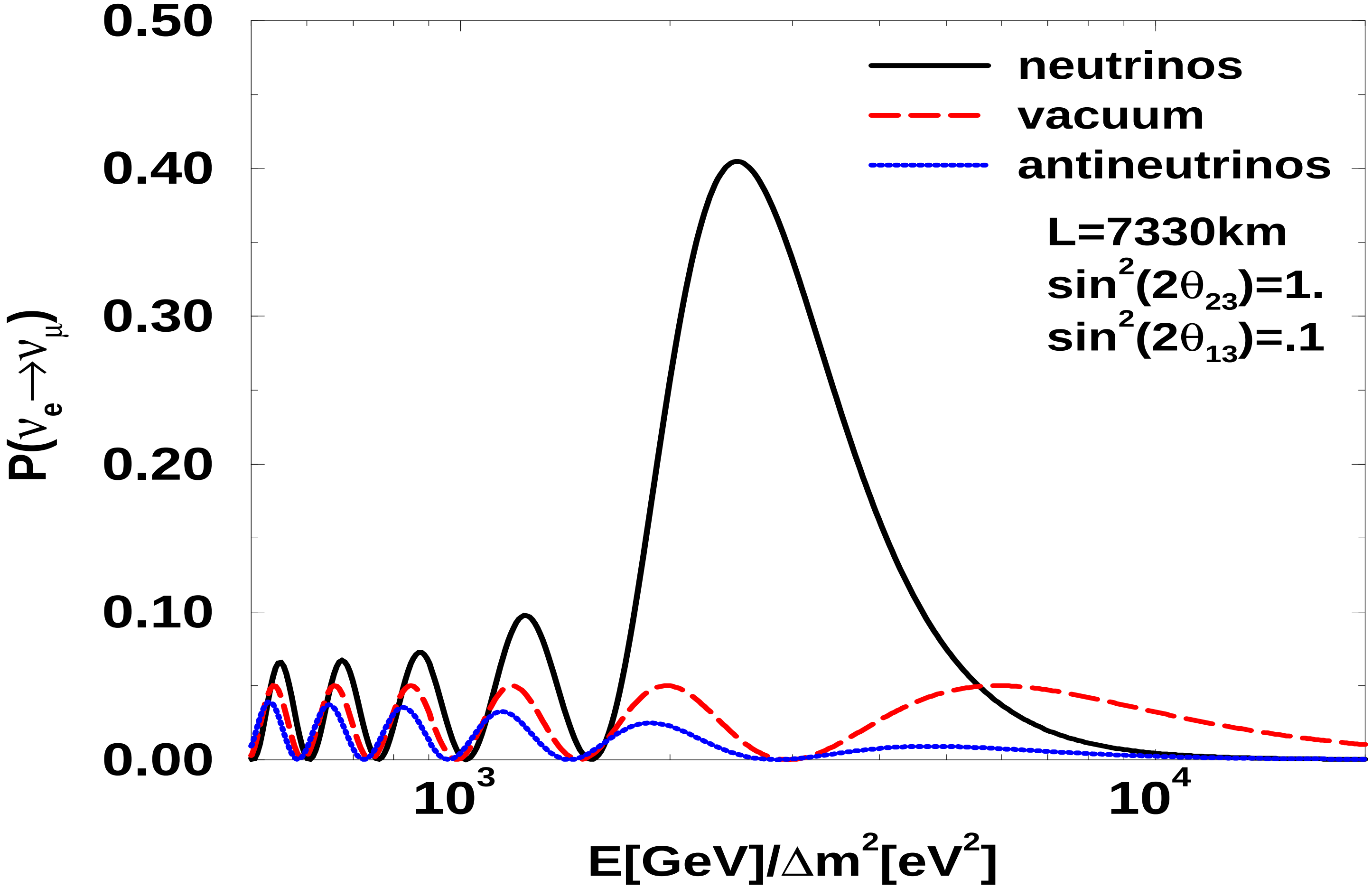


FIG. 1. Density profile of the Earth.

16

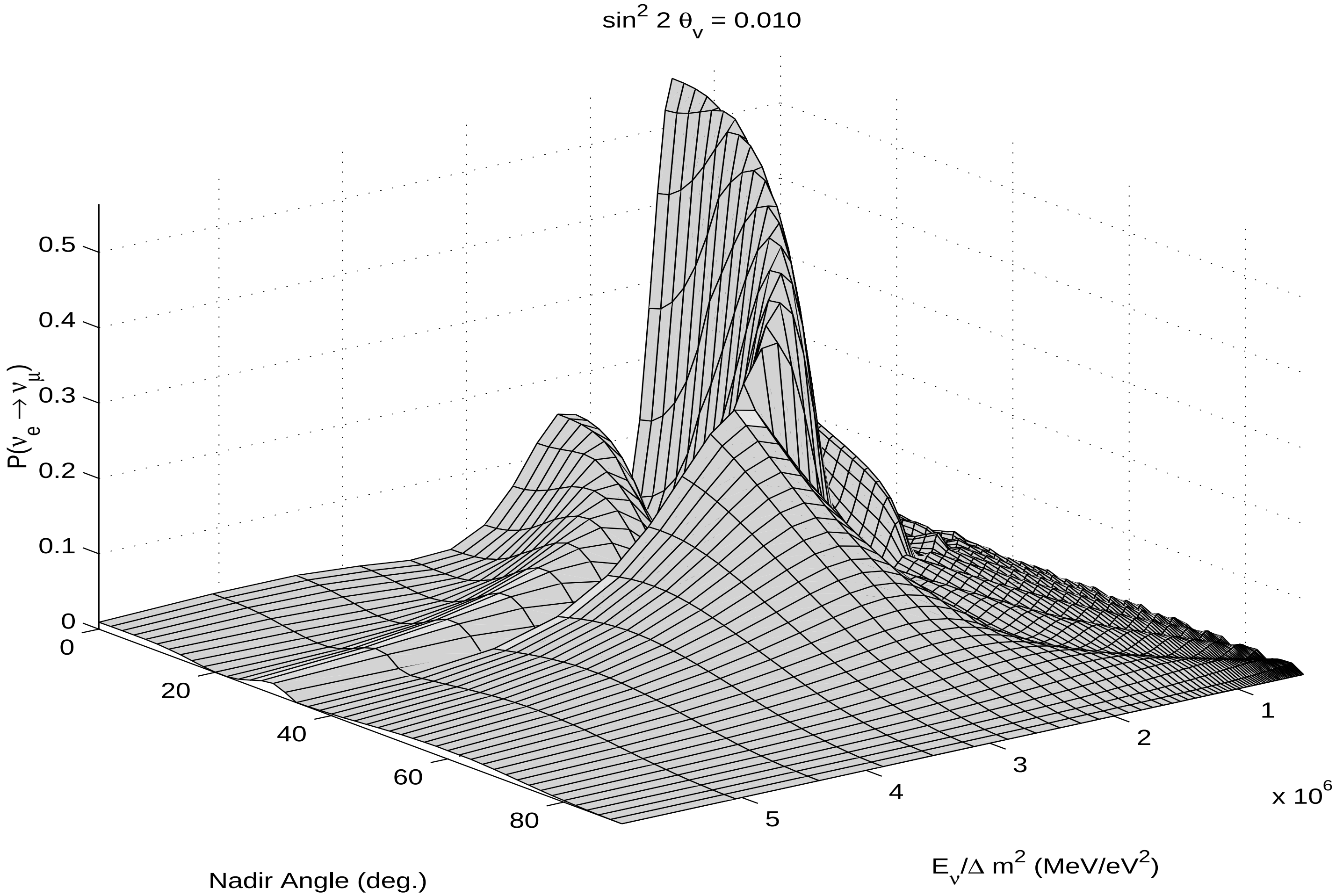
$R_{core} = 3446$ **km,** $R_{man} = 2886$ **km;** $\bar{N}_e^{man} \sim 2.3\ N_A\ cm^{-3}$, $\bar{N}_e^{core} \sim 6.0\ N_A\ cm^{-3}$

Earth matter effect in $\nu_\mu \to \nu_e$, $\bar{\nu}_\mu \to \bar{\nu}_e$ (MSW)



I. Mocioiu, R. Shrock, 2000

Earth matter effects in $\nu_\mu \to \nu_e$, $\bar{\nu}_\mu \to \bar{\nu}_e$ (NOLR)



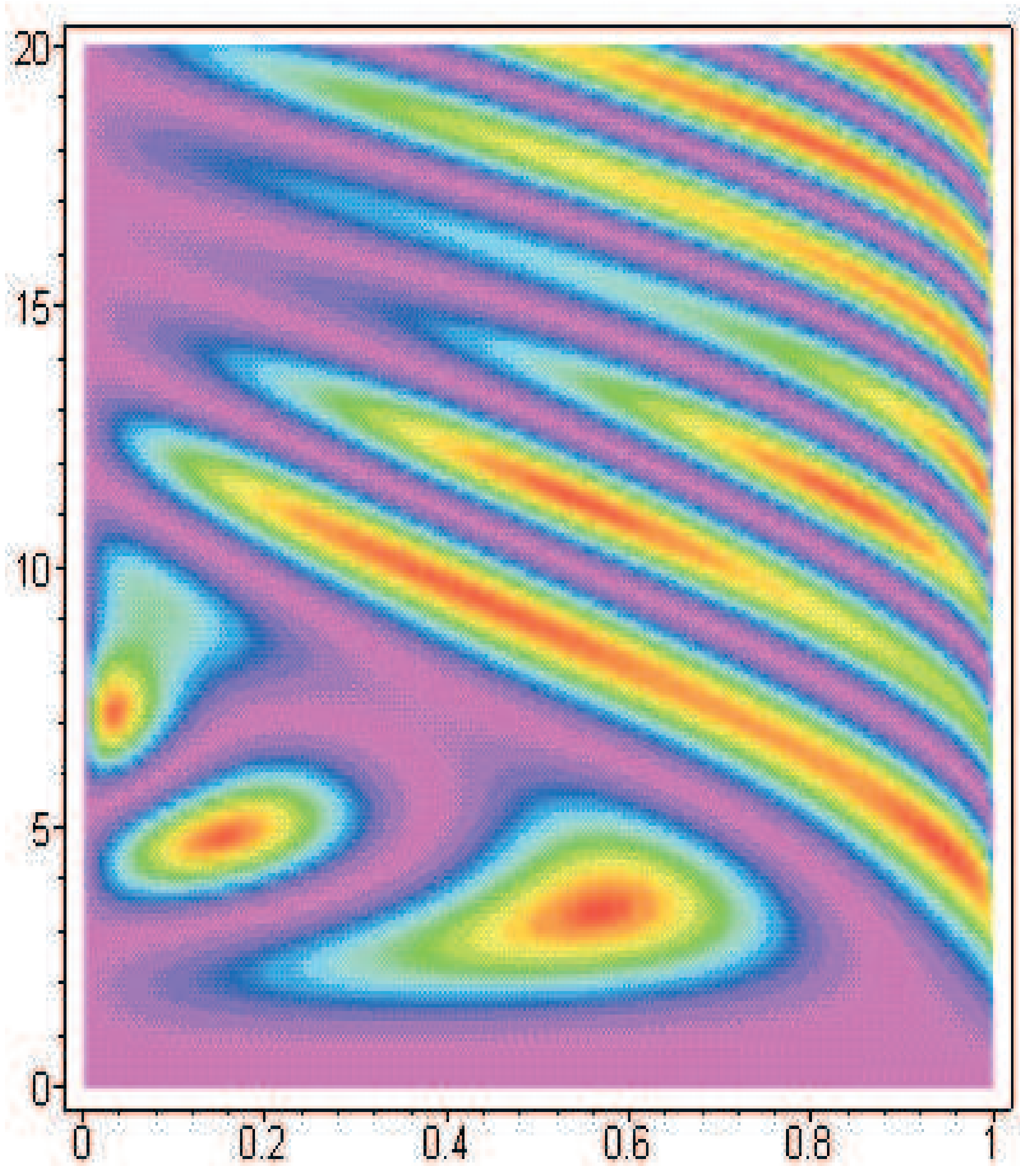
S.T.P., 1998;

M. Chizhov, M. Maris, S.T.P., 1998; M. Chizhov, S.T.P., 1999

$P(\nu_e \to \nu_\mu) \equiv P_{2\nu} \equiv (s_{23})^{-2} P_{3\nu}(\nu_{e(\mu)} \to \nu_{\mu(e)})$, $\theta_\mathrm{v} \equiv \theta_{13}$, $\Delta m^2 \equiv \Delta m^2_{\mathrm{atm}}$;

Absolute maximum: Neutrino Oscillation Length Resonance (NOLR);

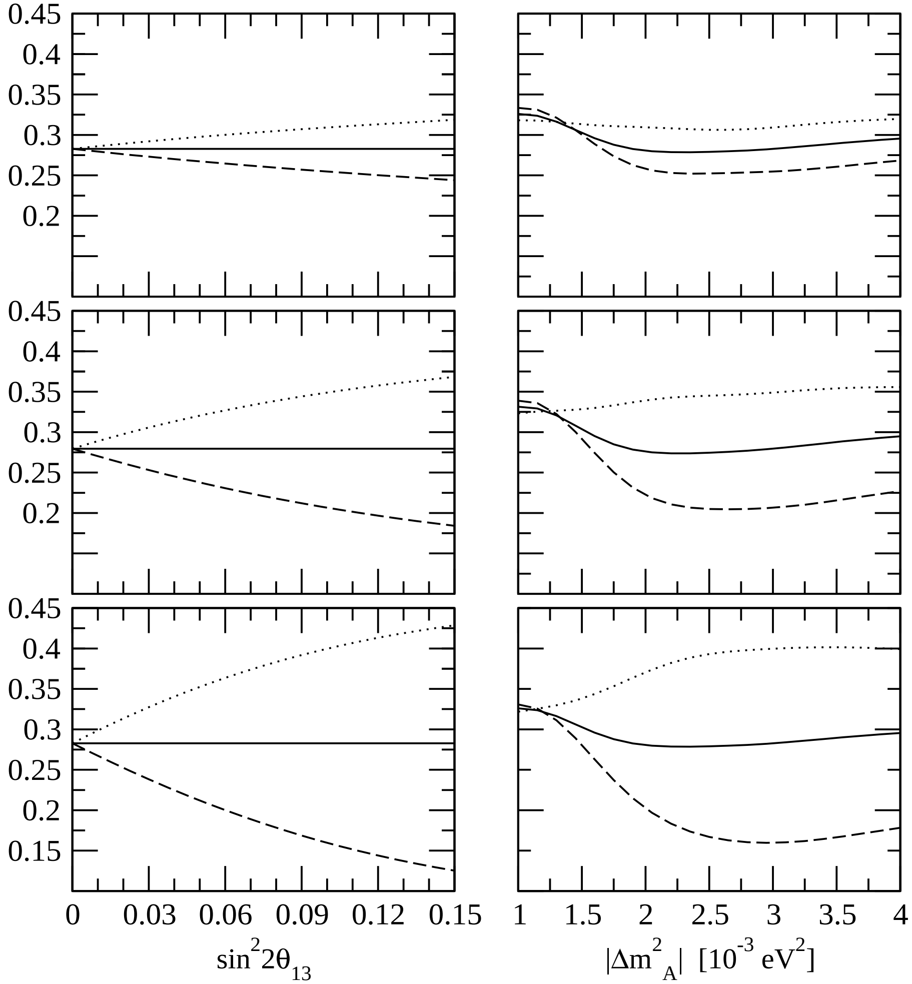
Local maxima: MSW effect in the Earth mantle or core.



$(s_{23})^{-2}P_{3\nu}(\nu_{e(\mu)} \to \nu_{\mu(e)}) \equiv P_{2\nu}$; **NOLR: "Dark Red Spots"**, $P_{2\nu} = 1$;

Vertical axis: $\Delta m^2/E$ $[10^{-7}eV^2/MeV]$; **horizontal axis:** $\sin^2 2\theta_{13}$; $\theta_n = 0$

M. Chizhov, S.T.P., 1999 (hep-ph/9903399,9903424)

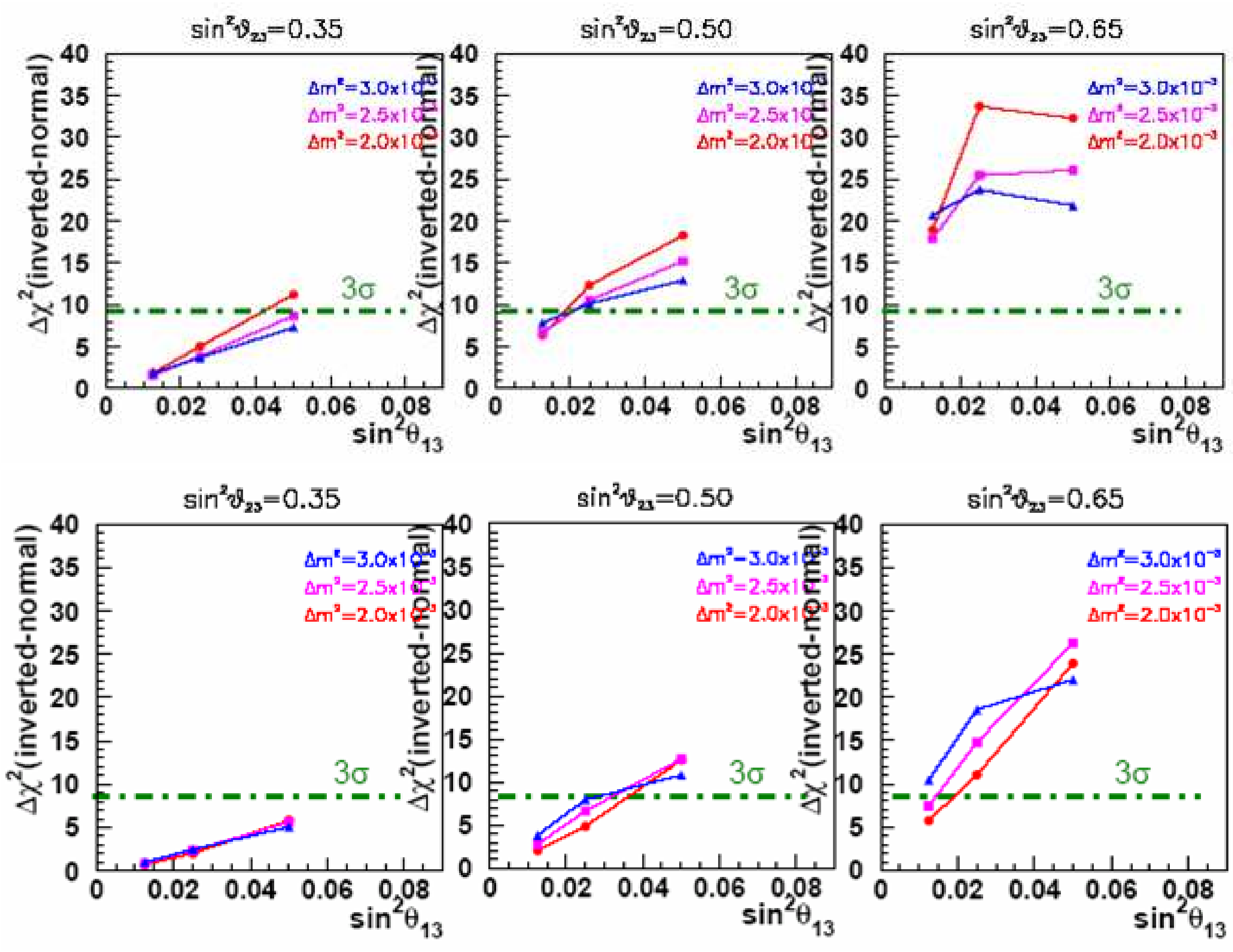


Iron Magnetized Detectors (MINOS, INO): multi-GeV μ^- and μ^+ event rates, N_{μ^-} **and** N_{μ^+}; $\cos\theta_n = (0.30 - 0.84)$ **mantle bin, E = [5,20] GeV**

$A \equiv \frac{U-D}{U+D}$ **in the θ_n- dependence of $\frac{N_{\mu^-}}{N_{\mu^+}}$**

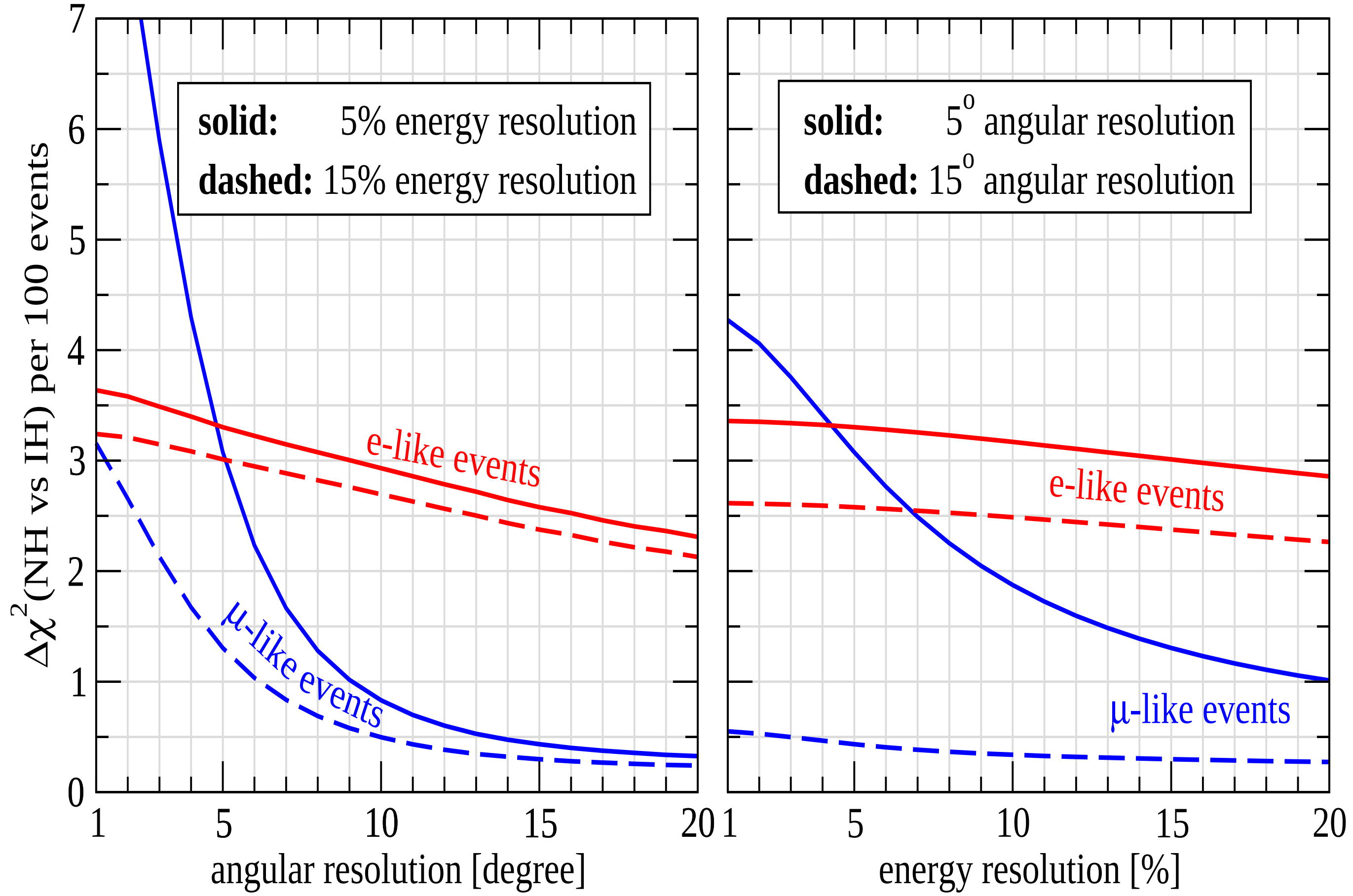
- $|\Delta m^2_{31}| = 3 \times 10^{-3}$ eV^2, $\sin^2\theta_{23}$ =**0.36, 0.50, 0.64**
- $\Delta m^2_{31} > 0$-**NH (dashed)**, $\Delta m^2_{31} < 0$-**IH (dotted)**, $2-\nu$ **(solid)**

S.T.P., S. Palomares-Ruiz, hep-ph/0406



Water-Cerenkov detector, 1.8 MTy

T. Kajita et al., 2004



INO; ATLAS, CMS (?)

T. Schwetz, S.T.P., 2005

$\sin^2 2\theta_{13} = 0.10$, $\sin^2\theta_{23} = 0.50$, $|\Delta m^2_{\rm A}| = 2.4 \times 10^{-3}$ **eV**^{2}

$E_\nu = (2 - 10)$ **GeV;** $0.1 \leq \cos\theta_n \leq 1.0$

^{3}H β-decay : 3H $\rightarrow^3$ He + e$^-$ + $\bar{\nu}_e$

$$\frac{d\,\Gamma}{d\,E_e} = \sum_i |U_{ei}|^2\ \frac{d\,\Gamma(m_i)}{d\,E_e}\,,$$

$$\frac{d\,\Gamma(m_i)}{d\,E_e} = C\,p_e\,(E_e + m_e)\,(E_0 - E_e)\,\sqrt{(E_0 - E_e)^2 - m_i^2}\,F(E_e)\,\theta(E_0 - E_e - m_i)\,.$$

NH: $m_1 << m_2 < m_3$, $m_2 \cong \sqrt{\Delta m^2_{21}} \cong 9 \times 10^{-3}$ **eV**, $m_3 \cong \sqrt{\Delta m^2_{31}} \cong 5 \times 10^{-2}$ eV

IH: $m_3 << m_1 \cong m_2$, $m_{1,2} \cong \sqrt{\Delta m^2_{23}} \cong 5 \times 10^{-2}$ eV

Assume sensitivity to 5×10^{-2} eV.

- **NH:** m_1, m_2 - **below the sensitivity; the effect of** m_3 - **unobservable, suppressed by** $\sin^2\theta_{13}$:

$$\frac{d\,\Gamma}{d\,E_e} \cong \frac{d\,\Gamma(m_i = 0)}{d\,E_e}$$

- **IH:** m_3 - **below the sensitivity;** $m_2 - m_1 \cong 1.6 \times 10^{-3}$ **eV - unobservable:**

$$\frac{d\,\Gamma}{d\,E_e} \cong \frac{d\,\Gamma(m_{1,2})}{d\,E_e} \cong \frac{d\,\Gamma(\sqrt{\Delta m^2_{23}})}{d\,E_e}$$

No e^-spectrum deformation observed: NH spectrum.

Deformations observed:

1) spectrum with inverted neutrino mass ordering, $\Delta m^2_{23} < 0$**,**
a) inverted hierarchical (IH), $m_3 \ll m_1 < m_2$**, or**
b) partial inverted hierarchy, $m_3 < m_1 < m_2$**;**

2) spectrum with normal neutrino mass ordering, $\Delta m^2_{23} > 0$**, but with partial neutrino mass hierarchy,** $m_1 < m_2 < m_3$**.**

Example (hypothetical) of the possibility 2): $m_1 = 5.0 \cdot 10^{-2}$ **eV,**

$m_2 = \sqrt{m_1^2 + \Delta m^2_{12}} \cong 5.1 \cdot 10^{-2}$ **eV,** $m_3 = \sqrt{m_1^2 + \Delta m^2_{13}} \cong 6.9 \cdot 10^{-2}$ **eV**

$m_1 + m_2 + m_3 \cong 0.17$ **eV**

$$\frac{d\,\Gamma}{d\,E_e} \cong (1 - |U_{e3}|^2)\,\frac{d\,\Gamma(m_{1,2})}{d\,E_e} + |U_{e3}|^2\,\frac{d\,\Gamma(m_3)}{d\,E_e} \cong \frac{d\,\Gamma(m_{1,2})}{d\,E_e}$$

S.M. Bilenky, M. Mateyev, S.T.P., 2006

Neutrino Physics Potential of

$(\beta\beta)_{0\nu}$–Decay Experiments

If $\nu_j -$ Majorana particles, $U_{\rm PMNS}$ **contains (3-ν mixing)**

δ-Dirac, α_{21}, α_{31} - Majorana **physical CPV phases**

ν-oscillations $\nu_l \leftrightarrow \nu_{l'}$, $\bar{\nu}_l \leftrightarrow \bar{\nu}_{l'}$, $l, l' = e, \mu, \tau$,

- **are not sensitive to the nature of ν_j,**

S.M. Bilenky et al.,1980;
P. Langacker et al., 1987

- **provide information on $\Delta m^2_{jk} = m^2_j - m^2_k$, but not on the absolute values of ν_j masses.**

The Majorana nature of ν_j can manifest itself in the existence of $\Delta L = \pm 2$ processes:

$$K^+ \rightarrow \pi^- + \mu^+ + \mu^+$$
$$\mu^- + (\mathrm{A}, \mathrm{Z}) \rightarrow \mu^+ + (\mathrm{A}, \mathrm{Z}-2)$$

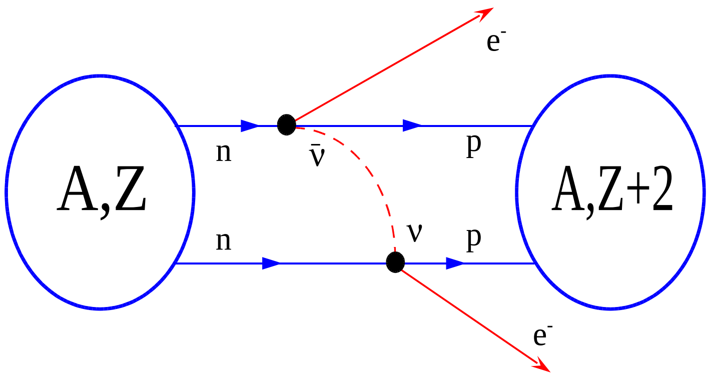
The process most sensitive to the possible Majorana nature of ν_j - $(\beta\beta)_{0\nu}$-decay

$$(\mathrm{A}, \mathrm{Z}) \rightarrow (\mathrm{A}, \mathrm{Z}+2) + e^- + e^-$$

of even-even nuclei, ^{48}Ca, ^{76}Ge, ^{82}Se, ^{100}Mo, ^{116}Cd, ^{130}Te, ^{136}Xe, ^{150}Nd.

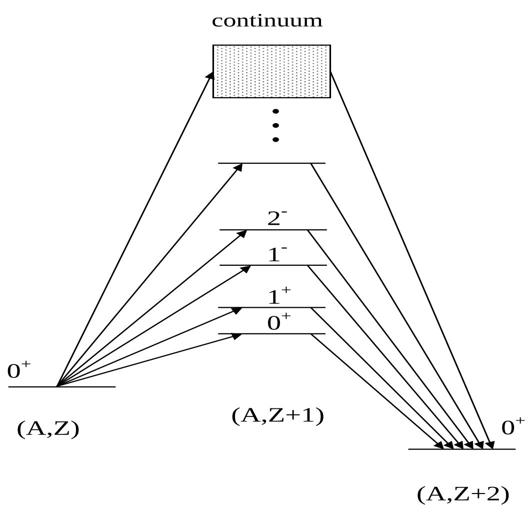
$2n$ from (A,Z) exchange a virtual Majorana ν_j (via the CC weak interaction) and transform into $2p$ of (A,Z+2) and two free e^-.

Nuclear $0\nu\beta\beta$-decay



strong in-medium modification of the basic process

$$dd \rightarrow uue^- e^- (\overline{\nu_e}\overline{\nu_e})$$



virtual excitation
of states of all multipolarities
in (A,Z+1) nucleus

V. Rodin, talk at Gran Sasso, 2006

$(\beta\beta)_{0\nu}-$Decay Experiments:

- Majorana nature of ν_j
- Type of $\nu-$mass spectrum (NH, IH, QD)
- Absolute neutrino mass scale

^{3}H β-decay , cosmology: m_ν (QD, IH)

- CPV due to Majorana CPV phases

ν_j- Dirac or Majorana particles, fundamental problem

ν_j-Dirac: conserved lepton charge exists, $L = L_e + L_\mu + L_\tau$, $\nu_j \neq \bar{\nu}_j$

ν_j-Majorana: no lepton charge is exactly conserved, $\nu_j \equiv \bar{\nu}_j$

The observed patterns of $\nu-$mixing and of $\Delta m^2_{\rm atm}$ and $\Delta m^2_\odot$ can be related to Majorana ν_j and an approximate symmetry:

$$L' = L_e - L_\mu - L_\tau$$

S.T.P., 1982

See-saw mechanism: ν_j- Majorana

Establishing that ν_j are Majorana particles would be as important as the discovery of $\nu-$ oscillations.

$A(\beta\beta)_{0\nu} \sim <m>$ **M(A,Z),** **M(A,Z) - NME,**

$$|<m>| = \left| m_1 |U_{e1}|^2 + m_2 |U_{e2}|^2 \ e^{i\alpha_{21}} + m_3 |U_{e3}|^2 \ e^{i\alpha'_{31}} \right|$$

$$= \left| m_1 \ c_{12}^2 \ c_{13}^2 + m_2 \ s_{12}^2 \ c_{13}^2 \ e^{i\alpha_{21}} + m_3 \ s_{13}^2 \ e^{i\alpha'_{31}} \right|, \quad \theta_{12} \equiv \theta_{\odot}, \ \theta_{13}\text{- CHOOZ}$$

α_{21}, α_{31} **- the two Majorana CPVP of the PMNS matrix;** $\alpha'_{31} \equiv \alpha_{31} - 2\delta$

CP-invariance: $\alpha_{21} = 0, \pm\pi$, $\alpha_{31} = 0, \pm\pi$;

$$\eta_{21} \equiv e^{i\alpha_{21}} = \pm 1, \quad \eta_{31} \equiv e^{i\alpha_{31}} = \pm 1$$

relative CP-parities of ν_1 and ν_2, and of ν_1 and ν_3 .

L. Wolfenstein, 1981;

S.M. Bilenky, N. Nedelcheva, S.T.P., 1984;

B. Kayser, 1984.

$|\!<\!m\!>\!|$ **:** m_j, $\theta_\odot \equiv \theta_{12}$, θ_{13}, $\alpha_{21,31}$

$m_{1,2,3}$ **- in terms of min(m_j),** $\Delta m^2_{\rm atm}$, $\Delta m^2_\odot$

S.T.P., A.Yu. Smirnov, 1994

Convention: $m_1 < m_2 < m_3$ - **NMO,** $m_3 < m_1 < m_2$ - **IMO**

$$\Delta m^2_\odot \equiv \Delta m^2_{21}, \quad m_2 = \sqrt{m_1^2 + \Delta m^2_\odot},$$

while either

$$\Delta m^2_{\rm atm} \equiv \Delta m^2_{31} > 0, \quad m_3 = \sqrt{m_1^2 + \Delta m^2_{\rm atm}}, \quad \text{normal mass ordering,} \quad \text{or}$$

$$\Delta m^2_{\rm atm} \equiv \Delta m^2_{32} < 0, \quad m_1 = \sqrt{m_3^2 + |\Delta m^2_{\rm atm}| - \Delta m^2_\odot}, \quad \text{inverted mass ordering}$$

The neutrino mass spectrum –

Normal hierarchical (NH) **if** $m_1 << m_2 << m_3$,

Inverted hierarchical (IH) **if** $m_3 << m_1 \cong m_2$,

Quasi-degenerate (QD) **if** $m_1 \cong m_2 \cong m_3 = m$, $m_j^2 >> |\Delta m^2_{\rm atm}|$; $m_j \gtrsim 0.1$ **eV**

Given $|\Delta m^2_{\rm atm}|$, $\Delta m^2_\odot$, $\theta_\odot$, θ_{13},

$$|\!<\!m\!>\!| \;= |\!<\!m\!>\!|\;(\mathrm{m_{min}}, \alpha_{21}, \alpha_{31}; \mathrm{S}), \; \mathrm{S} = \mathrm{NO(NH), IO(IH)}.$$

$A(\beta\beta)_{0\nu} \sim <m>$ **M(A,Z),** **M(A,Z) - NME,**

$$\left|<m>\right| \cong \left|\sqrt{\Delta m^2_\odot}\sin^2\theta_{12} e^{i\alpha} + \sqrt{\Delta m^2_{31}}\sin^2\theta_{13} e^{i\beta}\right|, \quad m_1 \ll m_2 \ll m_3 \text{ (NH)},$$

$$\left|<m>\right| \cong \sqrt{m_3^2 + \Delta m^2_{13}}\left|\cos^2\theta_{12} + e^{i\alpha}\ \sin^2\theta_{12}\right|, \quad m_3 < (\ll) m_1 < m_2 \text{ (IH)},$$

$$\left|<m>\right| \cong m\left|\cos^2\theta_{12} + e^{i\alpha}\ \sin^2\theta_{12}\right|, \quad m_{1,2,3} \cong m \gtrsim 0.10 \text{ eV (QD)},$$

$\theta_{12} \equiv \theta_\odot$, θ_{13}-CHOOZ; $\alpha \equiv \alpha_{21}$, $\beta + 2\delta \equiv \alpha_{31}$.

CP-invariance: $\alpha = 0, \pm\pi$, $\beta_M = 0, \pm\pi$;

$\left|<m>\right| \lesssim 5 \times 10^{-3}$ eV, **NH;**

$\sqrt{\Delta m^2_{13}}\cos 2\theta_{12} \cong 0.013$ eV $\lesssim \left|<m>\right| \lesssim \sqrt{\Delta m^2_{13}} \cong 0.055$ eV, **IH;**

$m\cos 2\theta_{12} \lesssim \left|<m>\right| \lesssim m$, $m \gtrsim 0.10$ eV, **QD** .

Solar neutrino and KamLAND data:

$\cos 2\theta_{\odot} = 0.0$ **excluded at** $>$ **6 s.d.**

Best fit value: $\cos 2\theta_{\odot} \simeq 0.40$

$\cos 2\theta_{\odot} \gtrsim 0.28$, 95% C.L.

Normal hierarchical spectrum:

$$(|<m>|\ )_{\text{max}} \lesssim 0.005 \text{ eV}$$

Inverted hierarchical spectrum:

$$(|<m>|\ )_{\text{min}} \simeq \sqrt{|\Delta m^2_{\text{atm}}|} \cos 2\theta_{\odot} \cos^2\theta_{13} \gtrsim 0.01 \text{ eV}$$

$$(|<m>|\ )_{\text{max}} \simeq \sqrt{|\Delta m^2_{\text{atm}}|} \cos^2\theta_{13} \lesssim 0.055 \text{ eV}$$

Quasi-degenerate spectrum:

$$(|<m>|\ )_{\text{min}} \simeq m\ (\cos 2\theta_{\odot} \cos^2\theta_{13} - \sin^2\theta_{13}) \gtrsim 0.03 \text{ eV}$$

Normal Hierarchical $\nu-$Mass Spectrum

$$m_1 \ll m_2 \ll m_3.$$

This implies:

$$m_2 \simeq \sqrt{\Delta m^2_{\odot}}, \qquad m_3 \simeq \sqrt{\Delta m^2_{\rm atm}}\ .$$

One has

$$|{<}m{>}| = \left|(m_1 \cos^2\theta_{\odot} + \sqrt{m_1^2 + \Delta m^2_{\odot}} \sin^2\theta_{\odot})(1 - |U_{e3}|^2) e^{i\alpha_{21}} + \sqrt{m_1^2 + \Delta m^2_{\rm atm}} |U_{e3}|^2 e^{i\alpha_{31}}\right|$$

$$\simeq \left|\sqrt{\Delta m^2_{\odot}}(1 - |U_{e3}|^2) \sin^2\theta_{\odot} + \sqrt{\Delta m^2_{\rm atm}} |U_{e3}|^2 e^{i(\alpha_{31} - \alpha_{21})}\right|$$

Even if $m_1 = 0$, $|{<}m{>}|$ **depends on** $\alpha_{32} = \alpha_{31} - \alpha_{21}$.

$|{<}m{>}| \lesssim 6 \times 10^{-3}$ **eV at 3**σ**; at 2**σ**:**
$\sqrt{\Delta m^2_{\rm atm}} |U_{e3}|^2 \lesssim 1.5$ **meV,** $\sqrt{\Delta m^2_{\odot}} \sin^2\theta_{\odot} \cong (2.1 - 3.2)$ **meV,**
$|{<}m{>}| \gtrsim 0.6$ **meV.**

Inverted Hierarchical $\nu-$Mass Spectrum

$$m_3 \ll m_1 \simeq m_2.$$

We can identify

$$\Delta m^2_{\odot} \equiv \Delta m^2_{21}, \quad \Delta m^2_{\rm atm} \equiv \Delta m^2_{32} \simeq \Delta m^2_{31},$$

$$|U_{\rm e3}|^2 \; = \sin^2\theta_{13} < 0.04 \qquad (\text{CHOOZ } + \nu_{\rm A} + \nu_{\odot} + \text{ KL}),$$

$$|U_{\rm e1}|^2 \; = \cos^2\theta_{\odot}(1 - \; |U_{\rm e3}|^2 \;), \quad |U_{\rm e2}|^2 \; = \sin^2\theta_{\odot}(1 - \; |U_{\rm e3}|^2 \;),$$

$$m_1 \simeq m_2 \simeq \sqrt{|\Delta m^2_{\rm atm}|}.$$

$\cos 2\theta_{\odot} \gg \sin^2\theta_{13}$: $m_3 \sin^2_{13}|$ **negligible in** $|<m>|$ **,**

$$|<m>| \; \cong \sqrt{|\Delta m^2_{\rm atm}|}(1 - s^2_{13})\sqrt{1 - \sin^2 2\theta_{\odot} \sin^2\left(\frac{\alpha_{21}}{2}\right)},$$

$$\sqrt{|\Delta m^2_{\rm atm}|} \; c^2_{13}|\cos 2\theta_{\odot}| \leq |<m>| \; \leq \sqrt{|\Delta m^2_{\rm atm}|} \; c^2_{13}.$$

$$0.01 \text{ eV} \lesssim |<m>| \; \lesssim 0.055 \text{ eV}.$$

The max, min values: $\alpha_{21} = 0$, $\alpha_{21} = \pm\pi$ - **CP-conserving.**

$$\sin^2\frac{\alpha_{21}}{2} = \left(1 - \frac{|<m>|^{\;2}}{|\Delta m^2_{\rm atm}|(1 - \;|U_{\rm e3}|^2\;)^2}\right)\frac{1}{\sin^2 2\theta_\odot}.$$

Three Quasi-Degenerate Neutrinos

$$m_1 \simeq m_2 \simeq m_3 \equiv m, \quad m^2 \gg |\Delta m^2_{\rm atm}|.$$

We have:

$$\Delta m^2_{\odot} \equiv \Delta m^2_{21}, \quad \Delta m^2_{\rm atm} \equiv \Delta m^2_{31},$$

$$|U_{\rm e1}|^2 = \cos^2\theta_\odot (1 - |U_{\rm e3}|^2), \quad |U_{\rm e2}|^2 = \sin^2\theta_\odot (1 - |U_{\rm e3}|^2),$$

$$|U_{\rm e3}|^2 = \sin^2\theta_{13} < 0.05 \quad ({\rm CHOOZ} + \nu_{\rm A} + \nu_\odot + {\rm KL}).$$

The mass scale m effectively coincides with the $\bar{\nu}_e$ mass $m_{\bar{\nu}_e}$ measured in the current ^{3}H β-decay experiments:

$$m \cong m_{\bar{\nu}_e}.$$

Thus, $m < 2.3$ eV. Cosmology: $m \lesssim (0.7 - 1.8)$ eV.

The QD spectrum - realized for m, which can be measured in the ^{3}H $\beta-$decay experiment KATRIN, $m_{\bar{\nu}_e} \gtrsim (0.2 - 0.3)$ eV.

$$|<m>| \cong m \left|\cos^2\theta_\odot (1 - |U_{\rm e3}|^2) + \sin^2\theta_\odot (1 - |U_{\rm e3}|^2) e^{i\alpha_{21}} + |U_{\rm e3}|^2 e^{i\alpha_{31}}\right|$$

$$\cong m \left|\cos^2\theta_\odot + \sin^2\theta_\odot e^{i\alpha_{21}}\right|;$$

$m\,|\cos 2\theta_\odot| \lesssim |<m>| \lesssim m$; **limits:** $\alpha_{21} = 0; \ \pm\pi$ - **CPC**

$$\sin^2 \frac{\alpha_{21}}{2} \cong \left(1 - \frac{|<m>|^{\ 2}}{m(1 - |U_{e3}|^2)^2}\right) \frac{1}{\sin^2 2\theta_\odot}.$$

Best sensitivity: Heidelberg-Moscow ^{76}Ge experiment.

Claim for a positive signal at $> 3\sigma$:

H. Klapdor-Kleingrothaus et al., PL B586 (2004),

$|{<}m{>}| = (0.1 - 0.9)$ eV **(99.73% C.L.).**

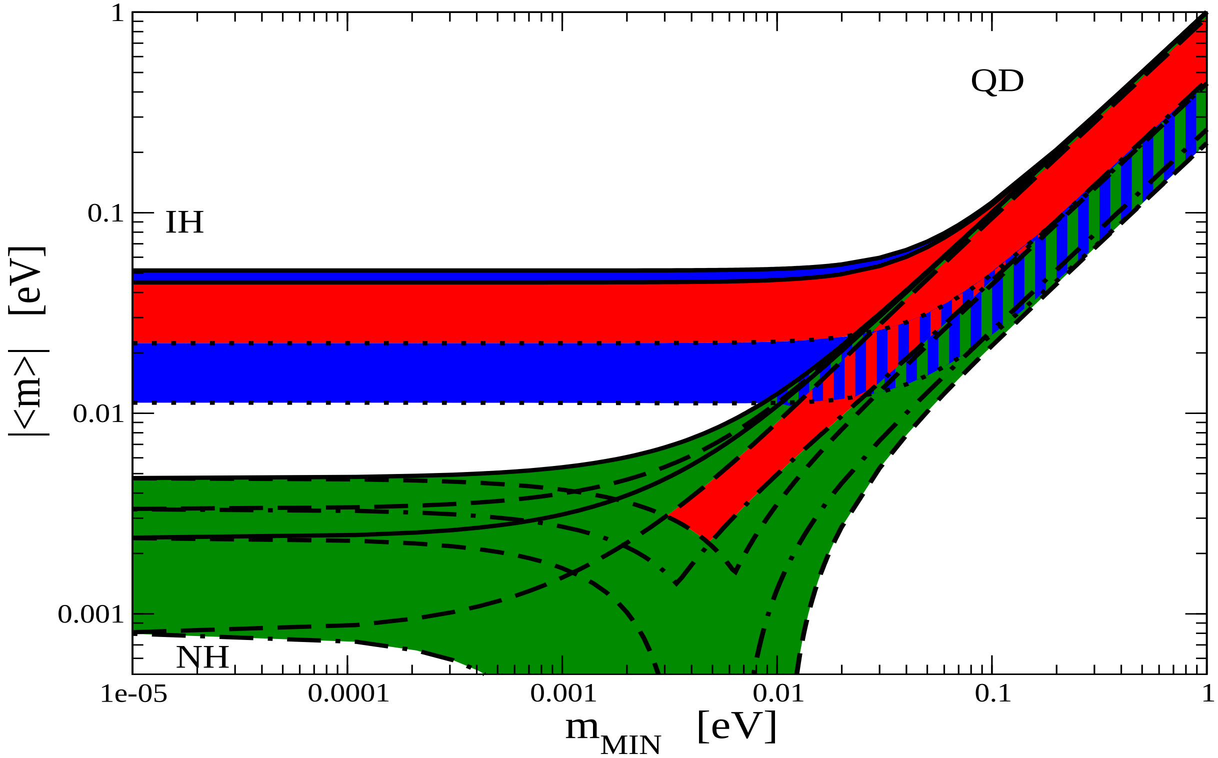
IGEX ^{76}Ge: $|{<}m{>}| < (0.33 - 1.35)$ eV **(90% C.L.).**

Taking data - NEMO3 (^{100}Mo), CUORICINO (^{130}Te):

$|{<}m{>}|$ **<(0.7–1.2) eV,** $|{<}m{>}|$ **<(0.18–0.90) eV (90% C.L.).**

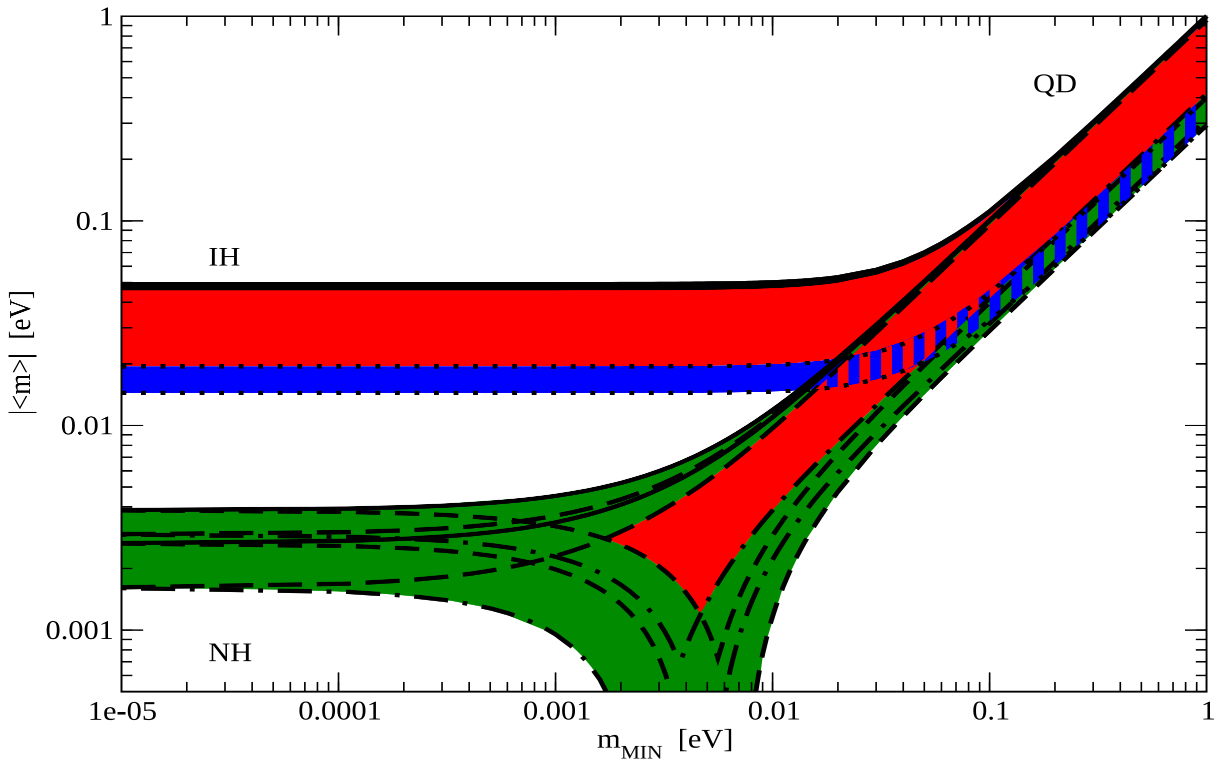
Large number of projects: $|{<}m{>}| \sim (0.01 - 0.05)$ **eV**

CUORE - ^{130}Te;
GERDA - ^{76}Ge;
SuperNEMO - ^{82}Se,...;
COBRA - ^{116}Cd;
EXO - ^{136}Xe;
MAJORANA - ^{76}Ge;
MOON - ^{100}Mo;
CANDLES - ^{48}Ca;
XMASS - ^{136}Xe.



S. Pascoli, S.T.P., 2007

The current 2σ ranges of values of the parameters used.



$\sin^2\theta_{13} = 0.01 \pm 0.006$; $1\sigma(\Delta m^2_\odot) = 2\%$, $1\sigma(\sin^2\theta_\odot) = 4\%$, $1\sigma(|\Delta m^2_{\rm atm}|) = 2\%$;

$2\sigma(|<m>|)$ **used.**

Nuclear Matrix Element Uncertainty

$$|<m>| = \zeta \left((|<m>|_{\text{exp}})_{\text{MIN}} \pm \Delta \right) , \quad \zeta \geq 1,$$

$(|<m>|_{\text{exp}})_{\text{MIN}}-$ **obtained with the maximal physically allowed value of NME.**

A measurement of the $(\beta\beta)_{0\nu}$-decay half-life time

$$(|<m>|_{\text{exp}})_{\text{MIN}} - \Delta \leq |<m>| \leq \zeta ((|<m>|_{\text{exp}})_{\text{MIN}} + \Delta) \ .$$

The estimated range of ζ^2:

^{48}**Ca,** $\zeta^2 \simeq 3.5$

^{76}**Ge,** $\zeta^2 \simeq 10$

^{82}**Se,** $\zeta^2 \simeq 10$

^{130}**Te,** $\zeta^2 \simeq 38.7$

S. Elliot, P. Vogel, 2002

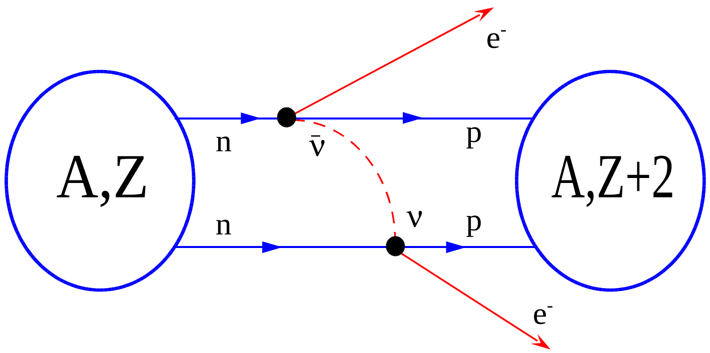
NH vs IH (QD):

$$\zeta \ |<m>|^{\text{NH}}_{\text{max}} < |<m>|^{\text{IH(QD)}}_{\text{min}} \ , \ \zeta \geq 1 \ .$$

IH vs QD:

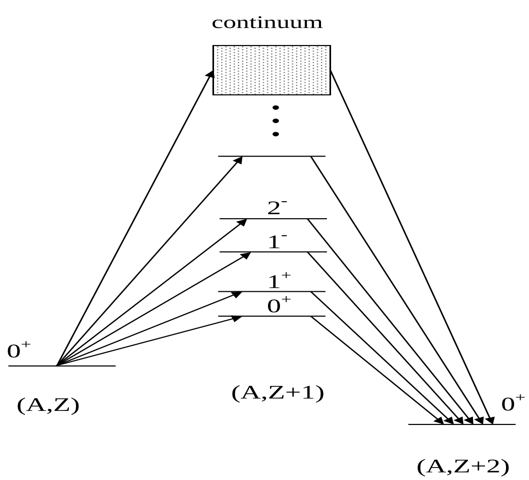
$$\zeta \ |<m>|^{\text{IH}}_{\text{max}} < |<m>|^{\text{QD}}_{\text{min}} \ , \ \zeta \geq 1 \ .$$

Nuclear $0\nu\beta\beta$-decay



strong in-medium modification of the basic process

$$dd \rightarrow uue^{-}e^{-}(\overline{\nu}_e\overline{\nu}_e)$$



virtual excitation
of states of all multipolarities
in (A,Z+1) nucleus

V. Rodin, talk at Gran Sasso, 2006

On the NME Uncertainties

The $(\beta\beta)_{0\nu}$-decay half-life

$$(T^{0\,\nu}_{1/2}(A,Z))^{-1} = |{<}\,m\,{>}|^{\,2}\,|M^{0\,\nu}(A,Z)|^2\, G^{0\,\nu}(E_0,Z),$$

$G^{0\nu}(E_0,Z)$, E_0 - known phase-space factor and energy release.

If we use a model M of the calculation of NME,

$$|{<}\,m\,{>}|^{\,2}_{\,M}(A,Z) = \frac{1}{T^{0\,\nu}_{1/2}(A,Z)\,|M^{0\,\nu}_{M}(A,Z)|^2\, G^{0\,\nu}(E_0,Z)}.$$

Suppose $(\beta\beta)_{0\nu}-$decay of several nuclei is observed.

$|{<}\,m\,{>}|$ cannot depend on parent nucleus (A_j, Z_j).

If the light Majorana ν-exchange - dominant mechanism of $(\beta\beta)_{0\nu}$-decay,
model M for NME can be correct only if

$$|{<}\,m\,{>}|^{\,2}_{\,M}(A_1,Z_1) \simeq |{<}\,m\,{>}|^{\,2}_{\,M}(A_2,Z_2) = ...$$

For different models and the same nucleus (A,Z),

$$|{<}\,m\,{>}|^{\,2}_{\,M_1}(A,Z)\;|M^{0\nu}_{M_1}(A,Z)|^2 = |{<}\,m\,{>}|^{\,2}_{\,M_2}(A,Z)\;|M^{0\nu}_{M_2}(A,Z)|^2 = ...,$$

$$|{<}\,m\,{>}|^{\,2}_{\,M_2}(A,Z) = \eta^{M_2;M_1}(A,Z)\,|{<}\,m\,{>}|^{\,2}_{\,M_1}(A,Z)\;,$$

$$\eta^{M_2;M_1}(A,Z) = \frac{|M^{0\,\nu}_{M_1}(A,Z)|^2}{|M^{0\,\nu}_{M_2}(A,Z)|^2}\;.$$

Nucleus	$\eta^{M_2;M_1}$	$\eta^{M_3;M_1}$	$\eta^{M_2;M_3}$
^{76}Ge	**0.37**	**0.19**	**1.93**
^{82}Se	—	**0.38**	—
^{100}Mo	—	—	**6.56**
^{130}Te	**0.74**	**0.10**	**7.32**
^{136}Xe	**0.53**	**0.02**	**22.42**

M_1 **(SM): E. Caurier et al., 1999;** M_2 **(QRPA): V. Rodin et al., 2003;**
M_3 **(QRPA): O. Civatarese and J. Suhonen, 2003.**

The observation of $(\beta\beta)_{0\nu}$-decay of at least 3 nuclei would be important for the solution of the problem of NME.

Table 2 suggests: ^{76}Ge, ^{130}Te, ^{136}Xe .

If for some model M

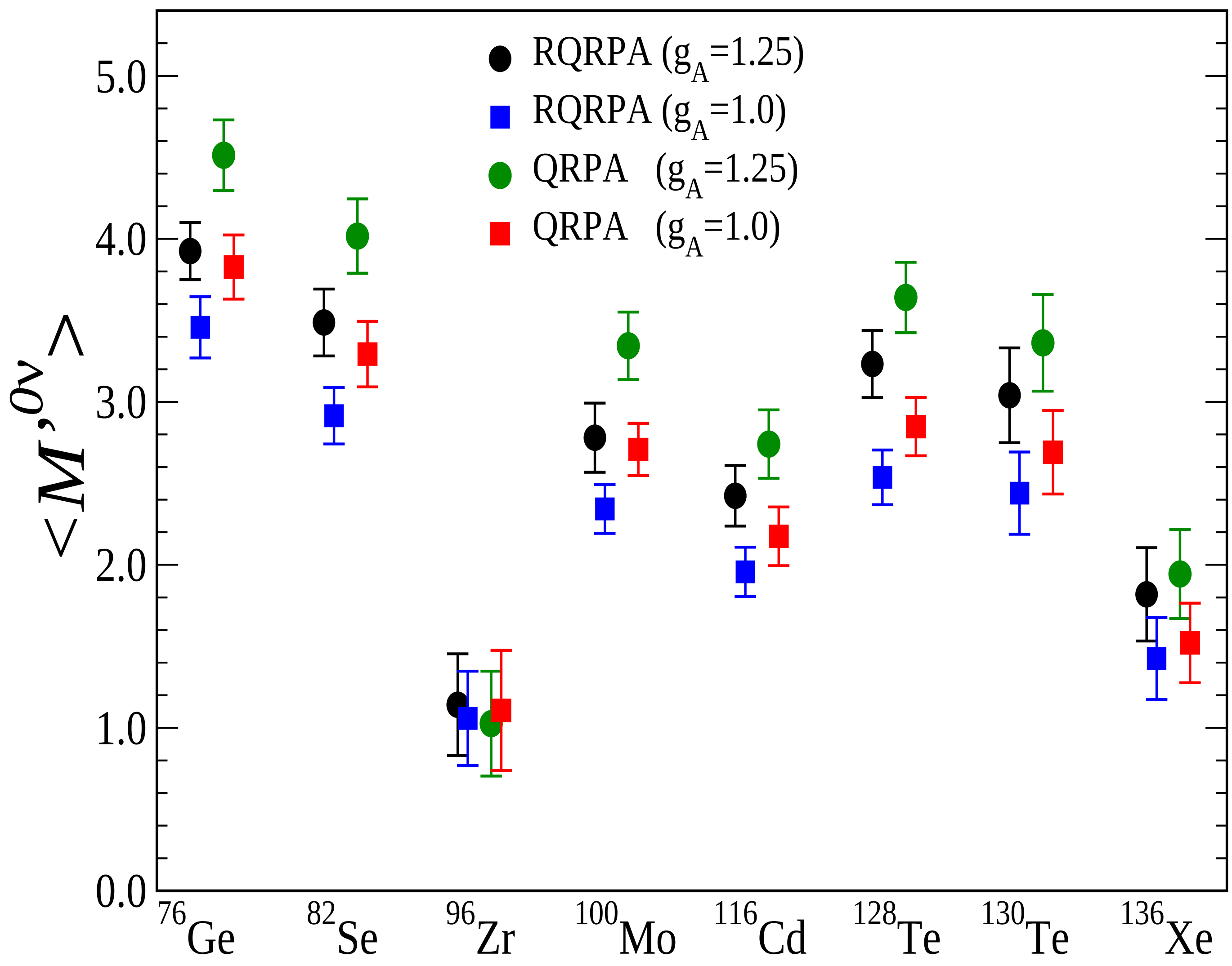
$$|<m>|\ ^2_M(A_1, Z_1) \simeq |<m>|\ ^2_M(A_2, Z_2) = ... \equiv |<m>|\ ^2_0\ ,$$

$|<m>|\ _0$ **- the true value (most likely).**
Strong dependence of NME on (A, Z) - crucial for the test.

S. M. Bilenky, S.T.P., 2004

Encouraging results on the problem of calculating the NME ($\xi \lesssim 1.5$) have been obtained recently in

V. A. Rodin, A. Faessler, F. Simkovic, P. Vogel, nucl-th/0503063



V. A. Rodin *et al.*, nucl-th/0503063

The errors have no statistical origin, just illustrate the degree of the variation of the results by changing the basis size. The "systematic error" of the QRPA (due to neglecting many-particle configurations): $(3 \div 5) \times 10\%$, can vary from one nucleus to another.

Majorana CPV Phases and $|<m>|$

CPV can be established provided

– $|<m>|$ **measured with** $\Delta \lesssim 15\%$ **;**

– Δm^2_{atm} **(IH) or** m_0 **(QD) measured with** $\delta \lesssim 10\%$ **;**

– $\xi \lesssim 1.5$ **;**

– α_{21} **(QD): in the interval** $\sim [\frac{\pi}{4} - \frac{3\pi}{4}]$**, or** $\sim [\frac{5\pi}{4} - \frac{3\pi}{2}]$ **;**

– $\tan^2\theta_\odot \gtrsim 0.40$.

S. Pascoli, S.T.P., W. Rodejohann, 2002

S. Pascoli, S.T.P., L. Wolfenstein, 2002

S. Pascoli, S.T.P., T. Schwetz, hep-ph/0505226

No "No-go for detecting CP-Violation via $(\beta\beta)_{0\nu}$-decay"

V. Barger *et al.*, 2002

Alternative Mechanisms of $(\beta\beta)_{0\nu}$-Decay

- Light neutrino exchange

- R-parity violating SUSY

- Heavy neutrino exchange

- Right-handed weak currents

Conclusions

Experiments with reactor $\bar{\nu}_e$ have remarkable physics potential:

- **Can provide high precision determination of** $\sin^2\theta_{12}$**,** Δm^2_{21}**,** $|\Delta m^2_{31}|$

- **Can provide important constraint or measure** $\sin^2\theta_{13}$

- **Can determine the type of** ν **mass spectrum**

Conclusions (contd.)

The $(\beta\beta)_{0\nu}$-decay experiments:

– Can establish the Majorana nature of ν_j

– Can provide unique information on the ν mass spectrum

– Can provide unique information on the absolute scale of ν masses

– Can provide information on the Majorana CPV phases

The knowledge of the values of the relevant $(\beta\beta)_{0\nu}-$decay NME with a sufficiently small uncertainty is crucial for obtaining quantitative information on the neutrino mass and mixing parameters from a measurement of $\Gamma(\beta\beta)_{0\nu}$.

The precision in the measurement of $\Gamma(\beta\beta)_{0\nu}$ will also be very important for the quantitative interpretation of the data.

SUPPORTING SLIDES

2001− **Remarkable progress in the studies of $\nu-$ mixing and oscillations**

- **June, 2001: SNO CC data + SK data →** $\nu_{\mu,\tau}$ **and/or** $\bar{\nu}_{\mu,\tau}$ **in** $\Phi_E(\nu_\odot)$

- **April, 2002: SNO NC data → evidence for** $\nu_{\mu,\tau}$ **and/or** $\bar{\nu}_{\mu,\tau}$ **in** $\Phi_E(\nu_\odot)$ **strengthen**

- **December, 2002: KamLAND**

  – First compelling evidence for $\nu-$oscillations in an experiment with terrestrial ν's

  – Evidence for ν_e-mixing in vacuum

  – $\nu_\odot$: LMA solution (CPT)

  – KamLAND "massacre":

  VO, QVO, LOW, SMA MSW, RSFP, FCNC, WEPV, LIV,...

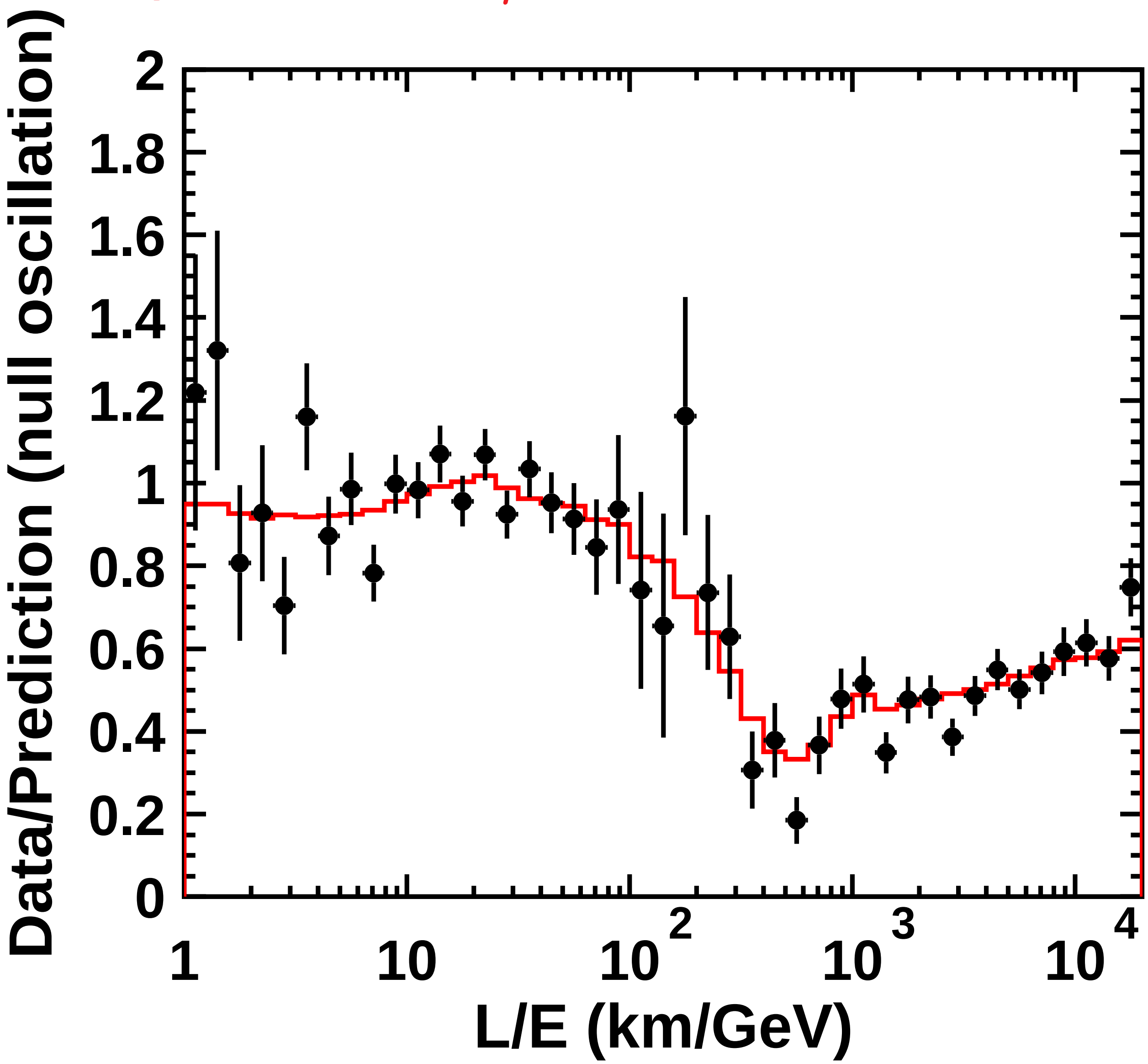
- **September, 2003: SNO salt phase data,**

$\Phi_B(\nu_\odot)$ - **higher precision**

- **2004: KamLAND, e^+-spectrum; K2K, $\nu_\mu-$spectrum**

  SK, L/E; SNO

SK: L/E Dependence, μ−Like Events



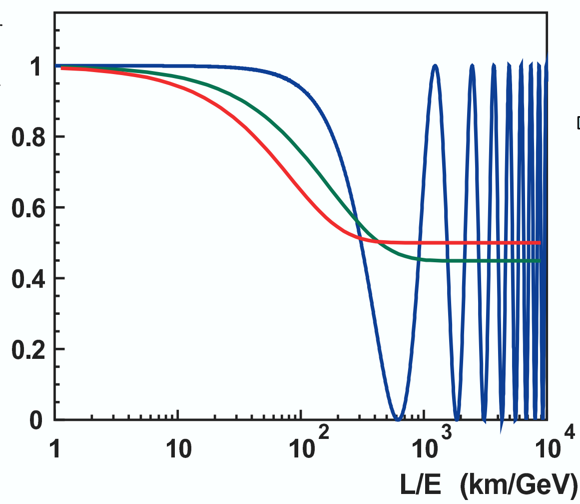
L/E dependence of $P_{\rm osc}$: V. Gribov, B. Pontecorvo, 1969

L/E analysis

Neutrino oscillation : $P_{\mu\mu} = 1 - \sin^2 2\theta \sin^2(1.27 \frac{\Delta m^2 L}{E})$

Neutrino decay : $P_{\mu\mu} = (\cos^2\theta + \sin^2\theta \times \exp(-\frac{m}{2\tau}\frac{L}{E}))^2$

Neutrino decoherence : $P_{\mu\mu} = 1 - \frac{1}{2}\sin^2 2\theta \times (1 - \exp(-\gamma_0 \frac{L}{E}))$



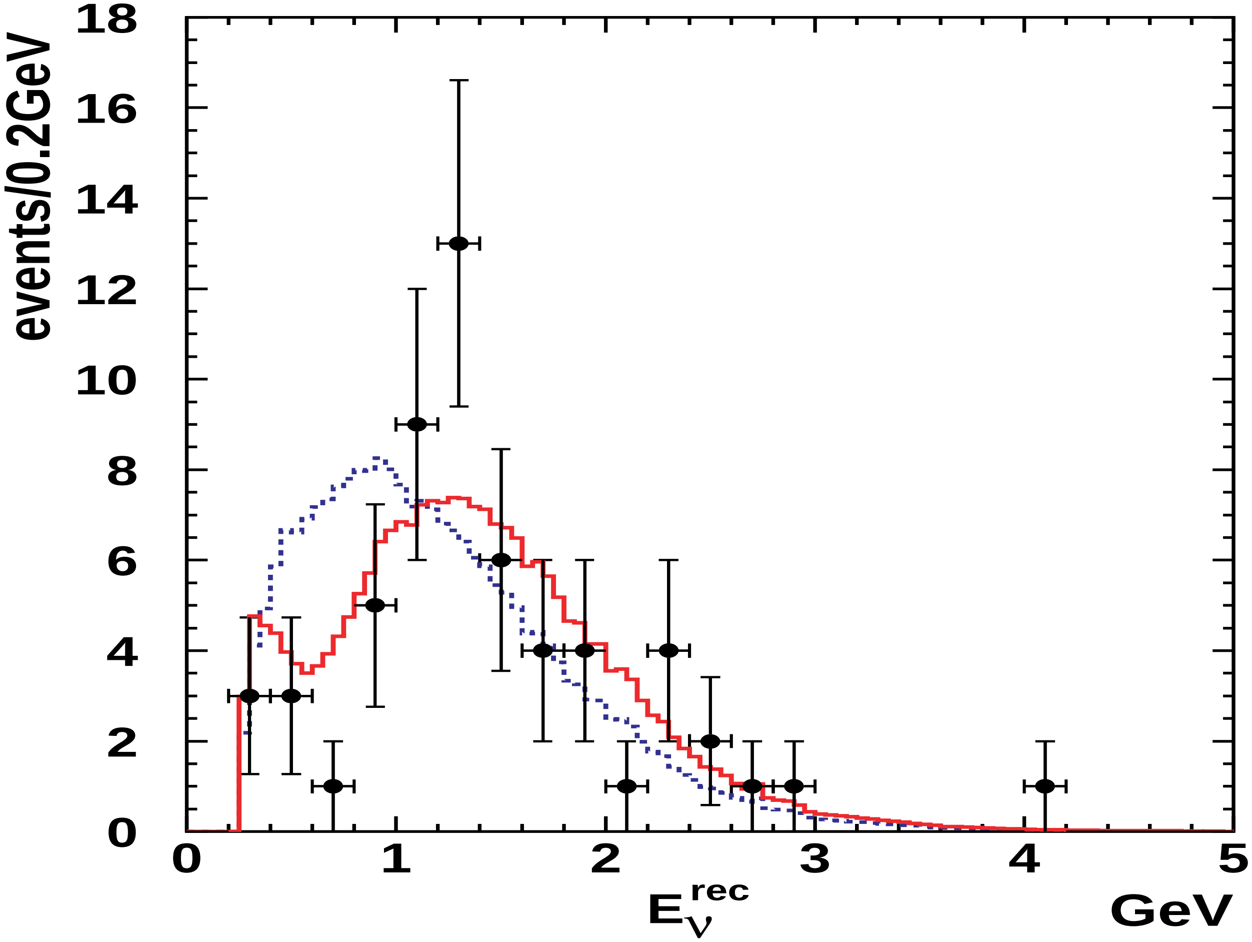
Use events with high resolution in L/E

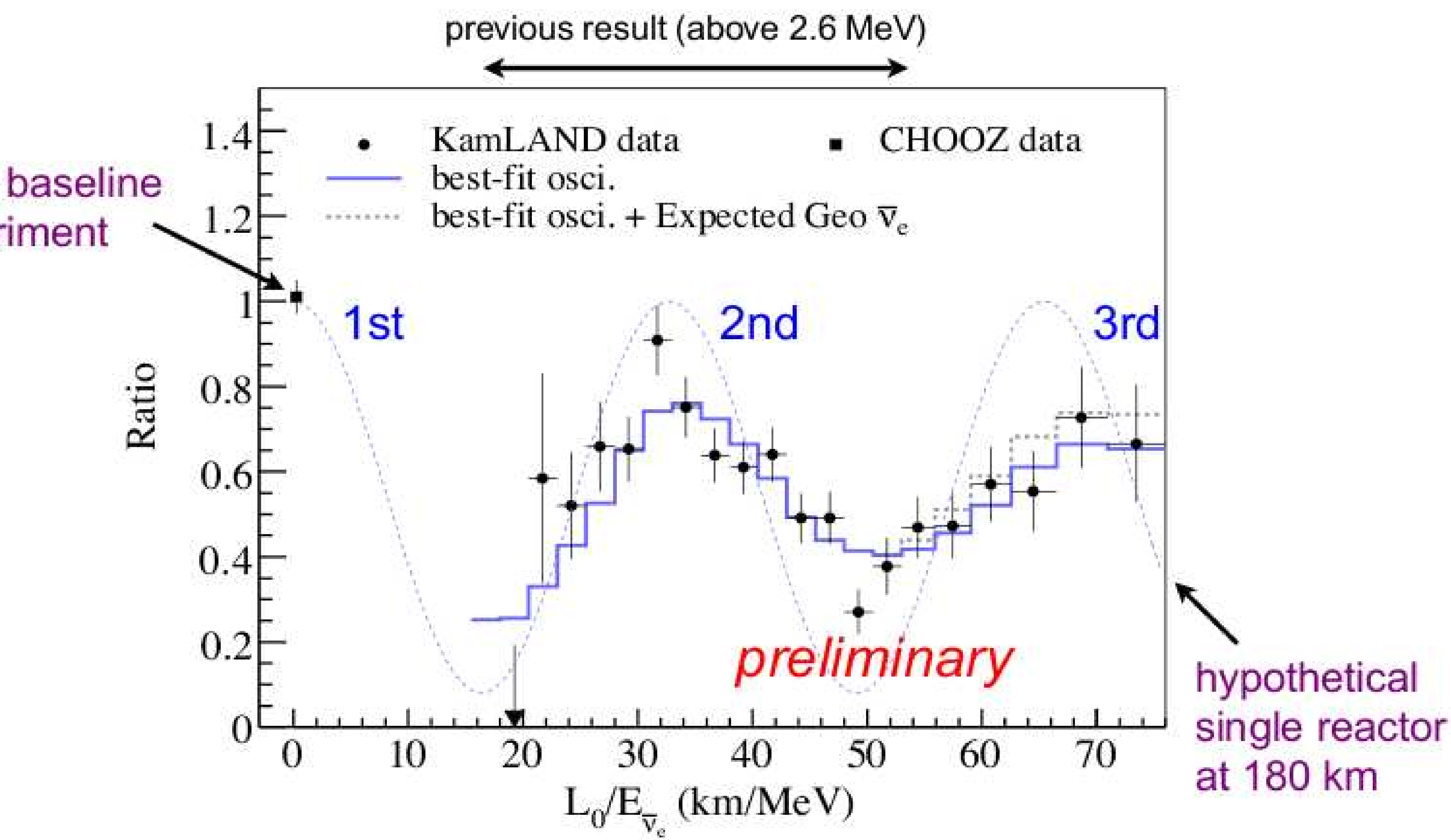
The first dip can be observed

→ Direct evidence for oscillations

→ Strong constraint to oscillation parameters, especially Δm^2 value

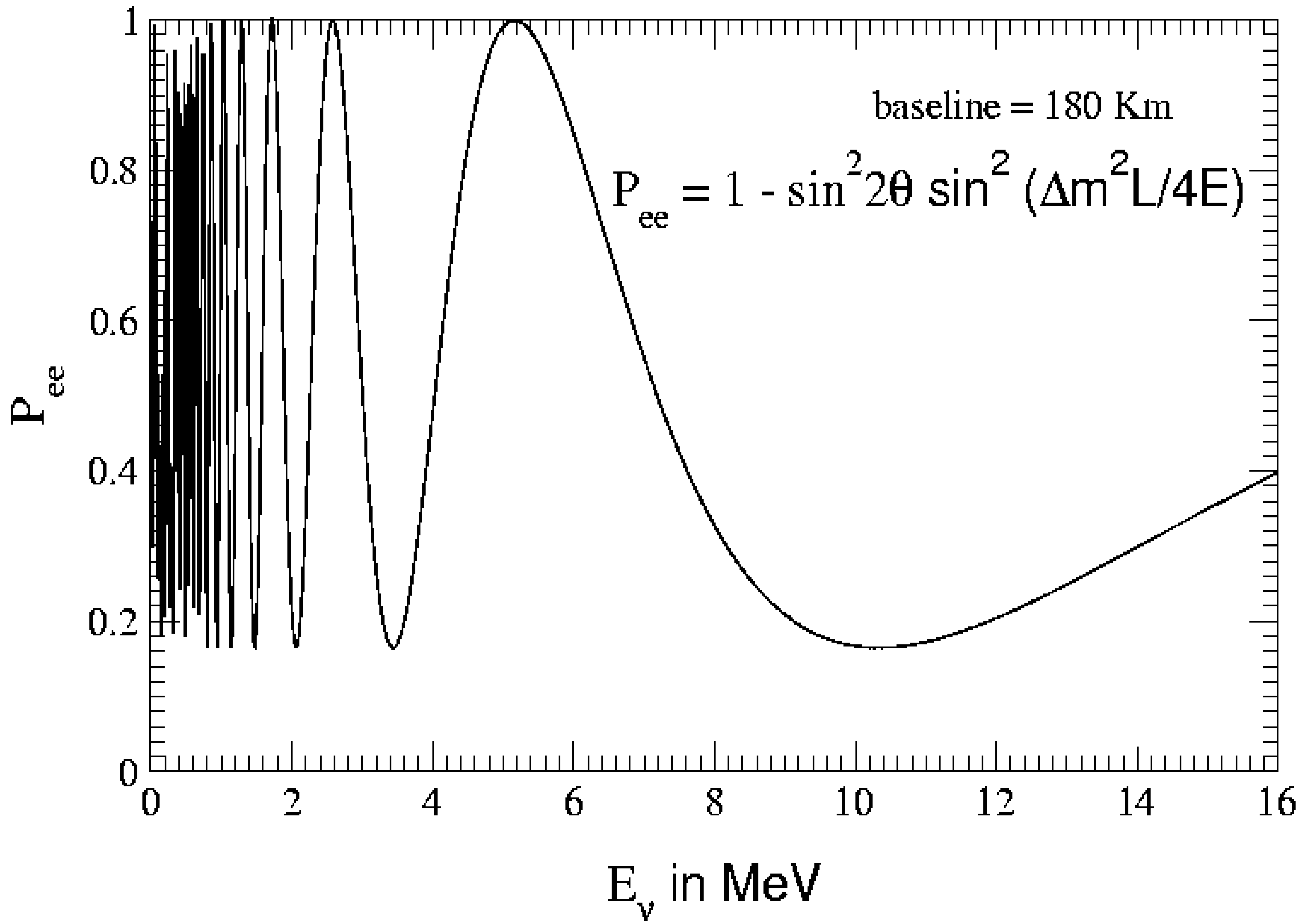
K2K: ν_μ Spectrum





KamLAND: $L/E-$Dependence

$\bar{\nu}_e \longrightarrow \bar{\nu}_e$



MINOS: ν_μ Spectrum