Neutrino Mixing, Oscillations, $(\beta\beta)_{0\nu}$-Decay, Leptonic CP-Violation and Leptogenesis

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Plan of Lectures

1. Introduction.

2. Neutrino Mixing: Current Status.

3. Determining the Type of Neutrino Mass Spectrum.

4. High Precision Measurement of $\Delta m^2_{\odot}$ and $\sin^2\theta_{\odot}$.

5. Neutrino Physics Prospects of $(\beta\beta)_{0\nu}-$Decay.

6. Dirac and Majorana CP-Violation and Leptogenesis.

7. Conclusions.

6. Dirac and Majorana CP-Violation and Leptogenesis
Rephasing Invariants Associated with CPVP

Dirac phase δ:

$$J_{CP} = \text{Im}\left\{U_{e1}\, U_{\mu 2}\, U^{*}_{e2}\, U^{*}_{\mu 1}\right\} \ .$$

C. Jarlskog, 1985 (for quarks)

CP-, T- violation effects in neutrino oscillations

P. Krastev, S.T.P., 1988

Majorana phases α_{21}, α_{31}:

$$S_1 = \text{Im}\left\{U_{e1} U^{*}_{e3}\right\}, \quad S_2 = \text{Im}\left\{U_{e2} U^{*}_{e3}\right\} \quad \text{(not unique);} \quad \text{or}$$
$$S'_1 = \text{Im}\left\{U_{\tau 1} U^{*}_{\tau 2}\right\}, \quad S'_2 = \text{Im}\left\{U_{\tau 2} U^{*}_{\tau 3}\right\}$$

J.F. Nieves and P. Pal, 1987, 2001
G.C. Branco et al., 1986
J.A. Aguilar-Saavedra and G.C. Branco, 2000

CP-violation: both $\text{Im}\left\{U_{e1} U^{*}_{e3}\right\} \neq 0$ and $\text{Re}\left\{U_{e1} U^{*}_{e3}\right\} \neq 0$.

S_1, S_2 appear in $|{<}m{>}|$ in $(\beta\beta)_{0\nu}$-decay.

In general, J_{CP}, S_1 and S_2 are independent.

Dirac CP-Nonconservation: δ in U_{PMNS}

Observable manifestations in

$$\nu_l \leftrightarrow \nu_{l'}\ , \quad \bar{\nu}_l \leftrightarrow \bar{\nu}_{l'}\ , \quad\ l,l' = e,\mu,\tau$$

- not sensitive to Majorana CPVP $\alpha_{21},\ \alpha_{31}$

CP-invariance:

N. Cabibbo, 1978

S.M. Bilenky, J. Hosek, S.T.P.,1980;

V. Barger et al.,1980.

$$P(\nu_l \rightarrow \nu_{l'}) = P(\bar{\nu}_l \rightarrow \bar{\nu}_{l'})\ ,\quad l \neq l' = e,\mu,\tau$$

CPT-invariance:

$$P(\nu_l \rightarrow \nu_{l'}) = P(\bar{\nu}_{l'} \rightarrow \bar{\nu}_l)$$

$$l = l' : \quad P(\nu_l \rightarrow \nu_l) = P(\bar{\nu}_l \rightarrow \bar{\nu}_l)$$

T-invariance:

$$P(\nu_l \rightarrow \nu_{l'}) = P(\nu_{l'} \rightarrow \nu_l),\ l \neq l'$$

3ν — mixing:

$$A^{(l,l')}_{\text{CP}} \equiv P(\nu_l \rightarrow \nu_{l'}) - P(\bar{\nu}_l \rightarrow \bar{\nu}_{l'})\ ,\quad l \neq l' = e,\mu,\tau$$

$$A^{(l,l')}_{\text{T}} \equiv P(\nu_l \rightarrow \nu_{l'}) - P(\nu_{l'} \rightarrow \nu_l),\ l \neq l'$$

$$A^{(e,\mu)}_{\text{T}} = A^{(\mu,\tau)}_{\text{T}} = -A^{(e,\tau)}_{\text{T}}$$

P.I. Krastev, S.T.P., 1988

In vacuum: $A^{(e,\mu)}_{\mathrm{CP(T)}} = J_{\mathrm{CP}} F^{vac}_{osc}$

$$J_{CP} = \mathrm{Im}\left\{U_{e1}\, U_{\mu 2}\, U^*_{e2}\, U^*_{\mu 1}\right\} = \frac{1}{8}\sin 2\theta_{12} \sin 2\theta_{23} \sin 2\theta_{13} \cos\theta_{13} \sin\delta$$

$$F^{vac}_{osc} = \sin(\frac{\Delta m^2_{21}}{2E}L) + \sin(\frac{\Delta m^2_{32}}{2E}L) + \sin(\frac{\Delta m^2_{13}}{2E}L)$$

P.I. Krastev, S.T.P., 1988

In matter: Matter effects violate

CP : $P(\nu_l \to \nu_{l'}) \neq P(\bar{\nu}_l \to \bar{\nu}_{l'})$

CPT : $P(\nu_l \to \nu_{l'}) \neq P(\bar{\nu}_{l'} \to \bar{\nu}_l)$

P. Langacker et al., 1987

Can conserve the T-invariance **(Earth)**

$$P(\nu_l \to \nu_{l'}) = P(\nu_{l'} \to \nu_l),\ l \neq l'$$

In matter with constant density: $\mathrm{A}^{(e,\mu)}_{\mathrm{T}} = \mathrm{J}^{\mathrm{mat}}_{\mathrm{CP}} \mathrm{F}^{\mathrm{mat}}_{\mathrm{osc}}$

$$\mathrm{J}^{\mathrm{mat}}_{\mathrm{CP}} = \mathrm{J}^{\mathrm{vac}}_{\mathrm{CP}}\ \mathrm{R}_{\mathrm{CP}}$$

R_{CP} does not depend on θ_{23} and δ; $|R_{CP}| \lesssim 2.5$

P.I. Krastev, S.T.P., 1988

HOW?

- Reactor Experiments at $L \sim 1$ km: Duoble CHOOZ, Daya Bay,...;

MINOS, CNGS (OPERA), $L \sim 730$ km:

$\sin^2\theta_{13}$

- Super Beams: θ_{13}, δ, ...

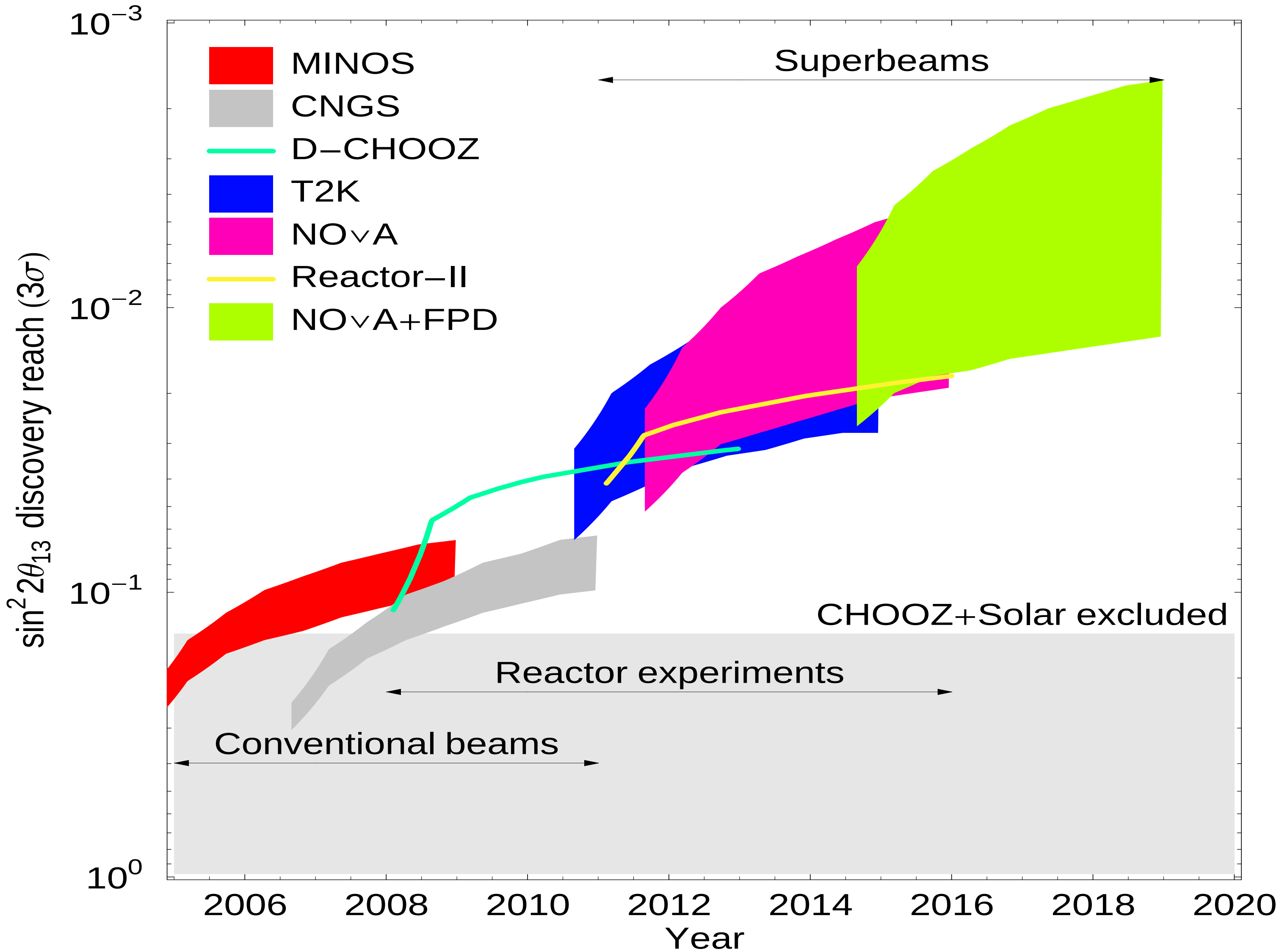
T2K, SK (HK) 295 km

NOνA $\sim$800 km

SPL+$\beta-$beams, MEMPHYS (0.5 megaton):
CERN-Frejus $\sim$140 km

$\nu-$Factories $\sim$ 3000, 7000 km


Sendai
Super Kamiokande
295km
JAERI
(Tokai)
Japan
KEK
Tokyo
Gifu
Nagoya
Kawasaki
Kyoto
Yokohama
Osaka
(c) 2000 ESRI
~ 420.0 mi / 675.8 km across



MINOS
CNGS
D−CHOOZ
T2K
NOνA
Reactor−II
NOνA+FPD
Superbeams
CHOOZ+Solar excluded
Reactor experiments
Conventional beams
$\sin^2 2\theta_{13}$ discovery reach (3σ)
10^{-3}
10^{-2}
10^{-1}
10^{0}
2006
2008
2010
2012
2014
2016
2018
2020
Year

Absolute Neutrino Mass Measurements

The Troitzk and Mainz ^{3}H β-decay experiments

$$m_{\nu_e} < 2.3 \text{ eV} \qquad (95\% \text{ C.L.})$$

There are prospects to reach sensitivity

$$\text{KATRIN}: \qquad m_{\nu_e} \sim 0.2 \text{ eV}$$

Cosmological and astrophysical data: the WMAP result combined with data from large scale structure surveys (2dFGRS, SDSS)

$$\sum_j m_j \equiv \Sigma < (0.4 - 1.4) \text{ eV}$$

The WMAP and future PLANCK experiments can be sensitive to

$$\sum_j m_j \cong 0.4 \text{ eV}$$

Data on weak lensing of galaxies by large scale structure, combined with data from the WMAP and PLANCK experiments may allow to determine

$$\sum_j m_j : \qquad \delta \cong 0.04 \text{ eV}.$$

M_ν from the See-Saw Mechanism

P. Minkowski, 1977.
M. Gell-Mann, P. Ramond, R. Slansky, 1979;
T. Yanagida, 1979;
R. Mohapatra, G. Senjanovic, 1980.

• Explains the smallness of $\nu-$masses.

• Through **leptogenesis theory** links the $\nu-$mass generation to the generation of baryon asymmetry of the Universe Y_B.

S. Fukugita, T. Yanagida, 1986.

• In SUSY GUT's with see-saw mechanism of $\nu-$mass generation, the LFV decays

$$\mu \to e + \gamma, \quad \tau \to \mu + \gamma, \quad \tau \to e + \gamma \text{ , etc.}$$

are predicted to take place with rates within the reach of present and future experiments.

F. Borzumati, A. Masiero, 1986.

• The ν_j are **Majorana particles**; $(\beta\beta)_{0\nu}-$decay is allowed.

See-Saw: Dirac ν-mass m_D + Majorana mass M_R for N_R

The See-Saw Lagrangian

$$\mathcal{L}^{\text{lep}}(x) = \mathcal{L}_{\text{CC}}(x) + \mathcal{L}_{\text{Y}}(x) + \mathcal{L}^{\text{N}}_{\text{M}}(x)\,,$$

$$\mathcal{L}_{\text{CC}} = -\frac{g}{\sqrt{2}}\,\overline{l_L}(x)\,\gamma_\alpha\,\nu_{lL}(x)\,W^{\alpha\dagger}(x) + h.c.\,,$$

$$\mathcal{L}_{\text{Y}}(x) = \lambda_{il}\,\overline{N_{iR}}(x)\,H^\dagger(x)\,\psi_{lL}(x) + Y_l\,H^c(x)\,\overline{l_R}(x)\,\psi_{lL}(x) + \text{h.c.}\,,$$

$$\mathcal{L}^{\text{N}}_{\text{M}}(x) = -\frac{1}{2}\,M_i\,\overline{N_i}(x)\,N_i(x)\,.$$

ψ_{lL} - LH doublet, $\psi^{\text{T}}_{lL} = (\nu_{lL}\;\; l_L)$, l_R - RH singlet, H - Higgs doublet.
Basis: $M_R = (M_1, M_2, M_3)$; $D_N \equiv \text{diag}(M_1, M_2, M_3)$, $D_\nu \equiv \text{diag}(m_1, m_2, m_3)$.
m_D generated by the Yukawa interaction:

$$-\mathcal{L}^{\nu}_{\text{Y}} = \lambda_{il}\overline{N_{iR}}H^\dagger(x)\psi_{lL}(x),\;\; v = 174\ \text{GeV},\;\; v\,\lambda = m_D - \text{complex}$$

For M_R - sufficiently large,

$$m_\nu \simeq v^2\;\lambda^T\,M_R^{-1}\,\lambda = U^*_{\text{PMNS}}\;m_\nu^{\text{diag}}\;U^\dagger_{\text{PMNS}}\,.$$

$Y_\nu \equiv \lambda = \sqrt{D_N}\;R\;\sqrt{D_\nu}\;(U_{\text{PMNS}})^\dagger/v_u$, all at M_R ; R-complex, $R^TR = 1$.

J.A. Casas and A. Ibarra, 2001

In GUTs, $M_R < M_X$, $M_X \sim 10^{16}$ GeV;

in GUTs, e.g., $M_R = (10^9, 10^{12}, 10^{15})$ GeV, $m_D \sim 1$ GeV.

The CP-Invarinace Constraints

Assume: $C(\overline{\nu}_j)^T = \nu_j,\ \ C(\overline{N}_k)^T = N_k,\ \ j,k = 1,2,3.$

The CP-symmetry transformation:

$$\begin{aligned} U_{\rm CP}\, N_j(x)\, U_{\rm CP}^\dagger &= \eta_j^{NCP}\, \gamma_0\, N_j(x')\,, \quad \eta_j^{NCP} = i\rho_j^N = \pm i\,, \\ U_{\rm CP}\, \nu_k(x)\, U_{\rm CP}^\dagger &= \eta_k^{\nu CP}\, \gamma_0\, \nu_k(x')\,, \quad \eta_k^{\nu CP} = i\rho_k^\nu = \pm i\,. \end{aligned}$$

CP-invariance:

$$\lambda_{jl}^* = \lambda_{jl}\,(\eta_j^{NCP})^*\,\eta^l\,\eta^{H*}\,, \quad j = 1,2,3,\ l = e,\mu,\tau,$$

Convenient choice: $\eta^l = i,\ \ \eta^H = 1 \quad (\eta^W = 1)$:

$$\begin{aligned} \lambda_{jl}^* &= \lambda_{jl}\,\rho_j^N\,,\ \ \rho_j^N = \pm 1\,, \\ U_{lj}^* &= U_{lj}\,\rho_j^\nu\,,\ \ \rho_j^\nu = \pm 1\,, \\ R_{jk}^* &= R_{jk}\,\rho_j^N\,\rho_k^\nu, \quad j,k = 1,2,3,\ l = e,\mu,\tau, \end{aligned}$$

λ_{jl}, U_{lj}, R_{jk} - either real or purely imaginary.

Relevant quantity:

$$P_{jkml} \equiv R_{jk}\,R_{jm}\,U_{lk}^*\,U_{lm}\,,\ \ k \neq m\,,$$

$$CP: \quad P_{jkml}^* = P_{jkml}\,(\rho_j^N)^2\,(\rho_k^\nu)^2\,(\rho_m^\nu)^2 = P_{jkml}\,, \quad {\rm Im}(P_{jkml}) = 0\,.$$

$$P_{jkml} \equiv R_{jk}\, R_{jm}\, U^*_{lk}\, U_{lm}\,, \;\; k \neq m\,,$$

$$CP: \quad P^*_{jkml} = P_{jkml}\,(\rho^N_j)^2\,(\rho^\nu_k)^2\,(\rho^\nu_m)^2 = P_{jkml}\,, \quad \mathrm{Im}(P_{jkml}) = 0\,.$$

Consider NH N_j, NH ν_k: $\quad P_{123\tau} = R_{12}\, R_{13}\, U^*_{\tau 2}\, U_{\tau 3}$

Suppose, CP-invrainace holds at low E: $\delta = 0$, $\alpha_{21} = \pi$, $\alpha_{31} = 0$.

Thus, $U^*_{\tau 2}\, U_{\tau 3}$ - purely imaginary.

Then real $R_{12}\, R_{13}$ corresponds to CP-violation at “high” E.

Baryon Asymmetry

$$Y_B = \frac{n_B - n_{\bar{B}}}{n_\gamma} = (6.1 \pm 0.3) \times 10^{-10}\,, \qquad \text{CMB}$$

Sakharov conditions for a dynamical generation of $Y_B \neq 0$ in the Early Universe

- B **number non-conservation.**
- **Violation of C and CP symmetries.**
- **Deviation from thermal equilibrium.**

Leptogenesis

• **The heavy Majorana neutrinos N_i are in equilibrium in the Early Universe as far as the processes which produce and destroy them are efficient.**

• **When $T < M_1$, N_1 drops out of equilibrium as it cannot be produced efficiently anymore.**

• **If $\Gamma(N_1 \to \Phi^- \ell^+) \neq \Gamma(N_1 \to \Phi^+ \ell^-)$, a lepton asymmetry will be generated.**

• **Wash-out processes, like $\Phi^+ + \ell^- \to N_1$, $\quad \ell^- + \Phi^+ \to \Phi^- + \ell^+$, etc. tend to erase the asymmetry. Under the condition of non-equilibrium, they are less efficient than the direct processes in which the lepton asymmetry is created. The final resullt is a net (non-zero) lepton asymmetry.**

• **This lepton asymmetry is then converted into a baryon asymmetry by $(B + L)$ violating but $(B - L)$ conserving sphaleron processes which exist within the SM (at $T \gtrsim M_{\mathrm{EWSB}}$).**

S. Fukugita, T. Yanagida, 1986.

In order to compute Y_B:

1. calculate the CP-asymmetry:

$$\varepsilon_1 = \frac{\Gamma(N_1 \to \Phi^- \ell^+) - \Gamma(N_1 \to \Phi^+ \ell^-)}{\Gamma(N_1 \to \Phi^- \ell^+) + \Gamma(N_1 \to \Phi^+ \ell^-)}$$

2. solve the Boltzmann equation to account for the wash-out of the asymmetry:

$$Y_{\mathsf{L}} = \kappa\, \varepsilon$$

where $\kappa = \kappa(\widetilde{m})$ is the "efficiency factor", $\widetilde{m}$ is the "the wash-out mass parameter" - determines the rate of wash-out processes;

3. the lepton asymmetry is converted into a baryon asymmetry:

$$Y_{\mathsf{B}} = \frac{c_S}{g_*} \kappa \varepsilon$$

Baryon number violation in the SM

Sphaleron processes

SU(2) instantons lead to effective 12 fermion interactions:

$O(B+L) = \Pi_i q_{Li} q_{Li} q_{Li} l_{Li}$

These would induce $\Delta B = \Delta L = 3$ processes:

$$u_L + d_L + c_L + s_L + t_L + b_L + \nu_{eL} + \nu_{\mu L} + \nu_{\tau L} \rightarrow \bar{d}_R + \bar{b}_R + \bar{s}_R$$

However, at T = 0 the probability of such processes is $\Gamma \sim e^{-4\pi/\alpha} \sim 10^{-165}$.

At finite T, the transitions proceed via thermal fluctuations with an unsuppressed probability: $\Gamma \sim \alpha T^4$.

Sphaleron processes are efficient for

$$T_{\text{EW}} \sim 100 \text{ GeV} < T < 10^{12} \text{ GeV}$$

V. Kuzmin, V. Rubakov, M. Shaposhnikov, 1985

Leptogenesis

$$Y_B = \frac{n_B - n_{\bar{B}}}{S} \sim 8.6 \times 10^{-11} \qquad (n_\gamma: \sim 6.1 \times 10^{-10})$$

$$Y_B \cong -10^{-2} \;\; \varepsilon \, \kappa$$

W. Buchmüller, M. Plümacher, 1998;
W. Buchmüller, P. Di Bari, M. Plümacher, 2004

$\kappa-$ efficiency factor; $\kappa \sim 10^{-1} - 10^{-3}$: $\varepsilon \gtrsim 10^{-7}$.

ε: $CP-$, $L-$ violating asymmetry generated in out of equilibrium $N_{Rj}-$decays in the early Universe,

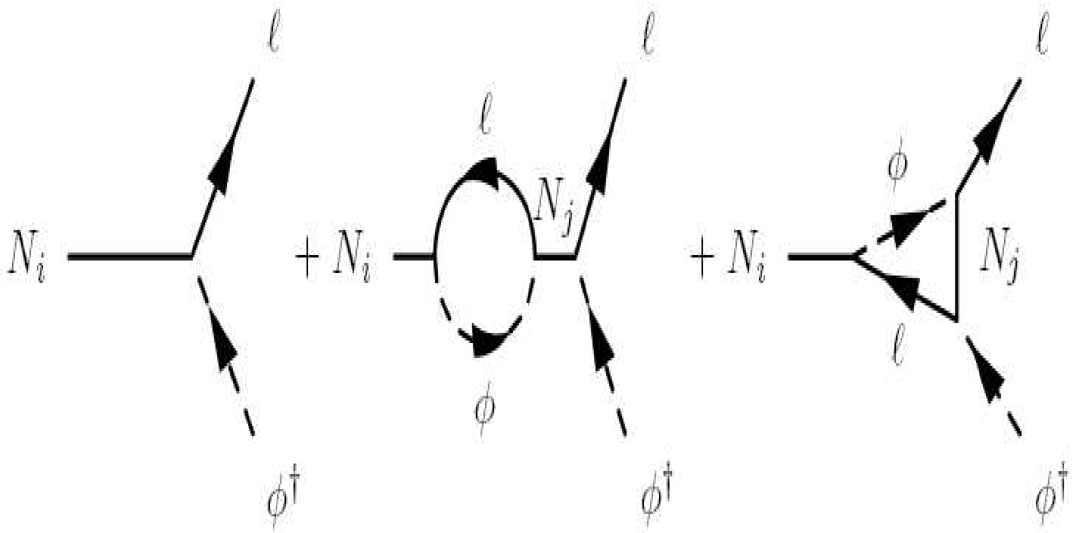
$$\varepsilon_1 = \frac{\Gamma(N_1 \to \Phi^- \, \ell^+) - \Gamma(N_1 \to \Phi^+ \, \ell^-)}{\Gamma(N_1 \to \Phi^- \, \ell^+) + \Gamma(N_1 \to \Phi^+ \, \ell^-)}$$

M.A. Luty, 1992;
L. Covi, E. Roulet and F. Vissani, 1996;
M. Flanz *et al.*, 1996;
M. Plümacher, 1997;
A. Pilaftsis, 1997.

$\kappa = \kappa(\widetilde{m})$, $\widetilde{m}$ - determines the rate of wash-out processes:

$\Phi^+ + \ell^- \to N_1$, $\;\; \ell^- + \Phi^+ \to \Phi^- + \ell^+$, etc.

W. Buchmuller, P. Di Bari and M. Plumacher, 2002;
G. F. Giudice *et al.*, 2004



N_i
ℓ
$\phi^\dagger$
$+ N_i$
ℓ
ϕ
N_j
ℓ
$\phi^\dagger$
$+ N_i$
ϕ
ℓ
N_j
ℓ
$\phi^\dagger$

Low Energy Leptonic CPV and Leptogenesis

Assume: $M_1 \ll M_2 \ll M_3$

Individual asymmetries:

$$\varepsilon_{1l} = -\frac{3M_1}{16\pi v^2} \frac{\text{Im}\left(\sum_{j,k} m_j^{1/2} m_k^{3/2} U_{lj}^* U_{lk} R_{1j} R_{1k}\right)}{\sum_j m_j \left|R_{1j}\right|^2}, \qquad v = 174 \text{ GeV}$$

$$\widetilde{m_l} \equiv \frac{|\lambda_{1l}|^2 v^2}{M_1} = \left|\sum_k R_{1k}\, m_k^{1/2}\, U_{lk}^*\right|^2, \quad l = e, \mu, \tau\,.$$

The "one-flavor" approximation - $\mathbf{Y}_{\mathbf{e},\mu,\tau}$ - "small":

Boltzmann eqn. for $n(N_1)$ and $\Delta L = \Delta(L_e + L_\mu + L_\tau)$.

$Y_l\ H^c(x)\overline{l_R}(x)\psi_{lL}$- out of equilibrium at $T \sim M_1$.

One-flavor approximation: $M_1 \sim T > 10^{12}$ GeV

$$\varepsilon_1 = \sum_l \varepsilon_{1l} = -\frac{3M_1}{16\pi v^2} \frac{\text{Im}\left(\sum_{j,k} m_j^2 R_{1j}^2\right)}{\sum_k m_k \left|R_{1k}\right|^2},$$

$$\widetilde{m_1} = \sum_l \widetilde{m_l} = \sum_k m_k \left|R_{1k}\right|^2.$$

Two-Flavour Regime

At $M_1 \sim T \sim 10^{12}$ GeV: Y_τ - in equilibrium, $Y_{e,\mu}$ - not; dynamics changes: τ_R^-, τ_L^+

$N_1 \rightarrow (\lambda_{1e}\, e_L^- + \lambda_{1\mu}\, \mu_L^- + \lambda_{1\tau}\, \tau_L^-) + \Phi^+$; $(\lambda_{1e}\, e_L^- + \lambda_{1\mu}\, \mu_L^- + \lambda_{1\tau}\, \tau_L^-) + \Phi^+ \rightarrow N_1$;

$\tau_L^- + \tau_L^+ \rightarrow \Phi^0$, $\tau_R^- + N_1 \rightarrow \nu_L + \tau_R^-$, $N_1 + \nu_L \rightarrow \tau_R^- + \tau_L^+$, etc.

$\varepsilon_{1\tau}$ and $(\varepsilon_{1e} + \varepsilon_{1\mu}) \equiv \varepsilon_2$ evolve independently.

Three-Flavour Regime

At $M_1 \sim T \sim 10^9$ GeV: Y_τ, Y_μ - in equilibrium, Y_e - not.

$\varepsilon_{1\tau}$, ε_{1e} and $\varepsilon_{1\mu}$ evolve independently.

Thus, at $M_1 \sim 10^9 - 10^{12}$ GeV: L_τ, ΔL_τ - distinguishable;

L_e, L_μ, ΔL_e, ΔL_μ - individually not distinguishable;

$L_e + L_\mu$, $\Delta(L_e + L_\mu)$

A. Abada et al., 2006; E. Nardi et al., 2006

A. Abada et al., 2006

Individual asymmetries:

Assume: $M_1 \ll M_2 \ll M_3$, $\quad 10^9 \lesssim M_1\ (\sim T) \lesssim 10^{12}$ GeV,

$$\varepsilon_{1l} = -\frac{3M_1}{16\pi v^2}\,\frac{\mathrm{Im}\left(\sum_{j,k} m_j^{1/2} m_k^{3/2} U_{lj}^* U_{lk} R_{1j} R_{1k}\right)}{\sum_j m_j\,|R_{1j}|^2}$$

$$\widetilde{m_l} \equiv \frac{|\lambda_{1l}|^2\, v^2}{M_1} = \left|\sum_k R_{1k} m_k^{1/2} U_{lk}^*\right|^2, \quad l = e, \mu, \tau\,.$$

The baryon asymmetry is

$$Y_B \simeq -\frac{12}{37 g_*}\left(\epsilon_2\,\eta\left(\frac{417}{589}\,\widetilde{m_2}\right) + \epsilon_\tau\,\eta\left(\frac{390}{589}\,\widetilde{m_\tau}\right)\right),$$

$$\eta\left(\widetilde{m_l}\right) \simeq \left(\left(\frac{\widetilde{m_l}}{8.25\times 10^{-3}\,\mathrm{eV}}\right)^{-1} + \left(\frac{0.2\times 10^{-3}\,\mathrm{eV}}{\widetilde{m_l}}\right)^{-1.16}\right)^{-1}.$$

$Y_{\mathcal{B}} = -(12/37)\,(Y_2 + Y_\tau)$,

$Y_2 = Y_{e+\mu}$, $\quad \varepsilon_2 = \varepsilon_{1e} + \varepsilon_{1\mu}$, $\quad \widetilde{m_2} = \widetilde{m_{1e}} + \widetilde{m_{1\mu}}$

A. Abada et al., 2006; E. Nardi et al., 2006

A. Abada et al., 2006

Real (Purely Imaginary) R: $\varepsilon_{1l} \neq 0$, CPV from U

$\varepsilon_{1e} + \varepsilon_{1\mu} + \varepsilon_{1\tau} = \varepsilon_2 + \varepsilon_{1\tau} = 0$,

$$\varepsilon_{1\tau} = -\frac{3M_1}{16\pi v^2}\frac{\text{Im}\left(\sum_{j,k} m_j^{1/2} m_k^{3/2} U^*_{\tau j} U_{\tau k} R_{1j} R_{1k}\right)}{\sum_j m_j \left|R_{1j}\right|^2}$$

$$= -\frac{3M_1}{16\pi v^2}\frac{\sum_{j,k>j} m_j^{1/2} m_k^{1/2} (m_k - m_j)\, R_{1j} R_{1k} \text{Im}\left(U^*_{\tau j} U_{\tau k}\right)}{\sum_j m_j \left|R_{1j}\right|^2}, R_{1j}R_{1k} = \pm\left|R_{1j}R_{1k}\right|,$$

$$= \mp\frac{3M_1}{16\pi v^2}\frac{\sum_{j,k>j} m_j^{1/2} m_k^{1/2} (m_k + m_j)\, \left|R_{1j} R_{1k}\right| \text{Re}\left(U^*_{\tau j} U_{\tau k}\right)}{\sum_j m_j \left|R_{1j}\right|^2}, R_{1j}R_{1k} = \pm i\left|R_{1j}R_{1k}\right|$$

S. Pascoli, S.T.P., A. Riotto, 2006.

CP-Violation: $\quad$ $\text{Im}\left(U^*_{\tau j} U_{\tau k}\right) \neq 0$, $\quad$ $\text{Re}\left(U^*_{\tau j} U_{\tau k}\right) \neq 0$;

$$Y_B = -\frac{12}{37}\frac{\varepsilon_{1\tau}}{g_*}\left(\eta\left(\frac{390}{589}\widetilde{m_\tau}\right) - \eta\left(\frac{417}{589}\widetilde{m_2}\right)\right)$$

$m_1 \ll m_2 \ll m_3,\ M_1 \ll M_{2,3};\quad R_{12}R_{13}$ – real; $\quad m_1 \cong 0,\ R_{11} \cong 0$ (N_3 decoupling)

$$\varepsilon_{1\tau} = -\frac{3M_1\sqrt{\Delta m_{31}^2}}{16\pi v^2}\left(\frac{\Delta m_{\odot}^2}{\Delta m_{31}^2}\right)^{\frac{1}{4}}\frac{|R_{12}R_{13}|}{\left(\frac{\Delta m_{\odot}^2}{\Delta m_{31}^2}\right)^{\frac{1}{2}}|R_{12}|^2+|R_{13}|^2}$$
$$\times\left(1-\frac{\sqrt{\Delta m_{\odot}^2}}{\sqrt{\Delta m_{31}^2}}\right)\mathrm{Im}\,(U_{\tau 2}^* U_{\tau 3})$$

$$\mathrm{Im}\,(U_{\tau 2}^* U_{\tau 3}) = -c_{13}\left[c_{23}s_{23}c_{12}\sin\left(\frac{\alpha_{32}}{2}\right) - c_{23}^2 s_{12}s_{13}\sin\left(\delta-\frac{\alpha_{32}}{2}\right)\right]$$

$\alpha_{32}=\pi,\ \delta=0$: $\quad \mathrm{Re}(U_{\tau 2}^* U_{\tau 3})=0$, $\quad$ CPV due to R

S. Pascoli, S.T.P., A. Riotto, 2006.

$M_1 \ll M_2 \ll M_3$, $m_1 \ll m_2 \ll m_3$ (NH)

Dirac CP-violation

$\alpha_{32} = 0\ (2\pi)$, $\beta_{23} = \pi\ (0)$; $\beta_{23} \equiv \beta_{12} + \beta_{13} \equiv \text{arg}(R_{12}R_{13})$.

$|R_{12}|^2 \cong 0.85$, $|R_{13}|^2 = 1 - |R_{12}|^2 \cong 0.15$ - maximise $|\epsilon_\tau|$ and $|Y_B|$:

$$|Y_B| \ \cong 2.8 \times 10^{-13}\, |\sin\delta| \left(\frac{s_{13}}{0.2}\right) \left(\frac{M_1}{10^9 \text{ GeV}}\right) .$$

$|Y_B| \gtrsim 8 \times 10^{-11}$, $\quad M_1 \lesssim 5 \times 10^{11}$ GeV imply

$$|\sin\theta_{13}\, \sin\delta| \gtrsim 0.11\,, \qquad \sin\theta_{13} \gtrsim 0.11\,.$$

The lower limit corresponds to

$$|J_{\text{CP}}| \gtrsim 2.4 \times 10^{-2}$$

FOR $\alpha_{32} = 0\ (2\pi)$, $\beta_{23} = 0\ (\pi)$:

$$|\sin\theta_{13}\, \sin\delta| \gtrsim 0.09\,, \qquad \sin\theta_{13} \gtrsim 0.09\,; \quad |J_{\text{CP}}| \gtrsim 2.0 \times 10^{-2}$$

$M_1 \ll M_2 \ll M_3$, $m_1 \ll m_2 \ll m_3$ (NH)

Majorana CP-violation

$\delta = 0$, real R_{12}, R_{13} ($\beta_{23} = \pi$ (0));

$\alpha_{32} \cong \pi/2$, $|R_{12}|^2 \cong 0.85$, $|R_{13}|^2 = 1 - |R_{12}|^2 \cong 0.15$ - maximise $|\epsilon_\tau|$ and $|Y_B|$:

$$|Y_B| \cong 2 \times 10^{-12} \left(\frac{\sqrt{\Delta m^2_{31}}}{0.05 \text{ eV}} \right) \left(\frac{M_1}{10^9 \text{ GeV}} \right) .$$

We get $|Y_B| \gtrsim 8 \times 10^{-11}$, for $M_1 \gtrsim 3.6 \times 10^{10}$ GeV

$M_1 \ll M_2 \ll M_3$, $m_3 \ll m_1 < m_2$ (IH)

$m_3 \cong 0$, $R_{13} \cong 0$ (N_3 decoupling): impossible to reproduce Y_B^{obs} for real $R_{11}R_{12}$

Dirac CP-violation, purely imaginary $R_{11}R_{12}$

$\alpha_{21} = \pi$; $R_{11}R_{12} = i\kappa|R_{11}R_{12}|$, $\kappa = 1$;

$|R_{11}| \cong 1.07$, $|R_{12}|^2 = |R_{11}|^2 - 1$, $|R_{12}| \cong 0.38$ - maximise $|\epsilon_\tau|$ and $|Y_B|$:

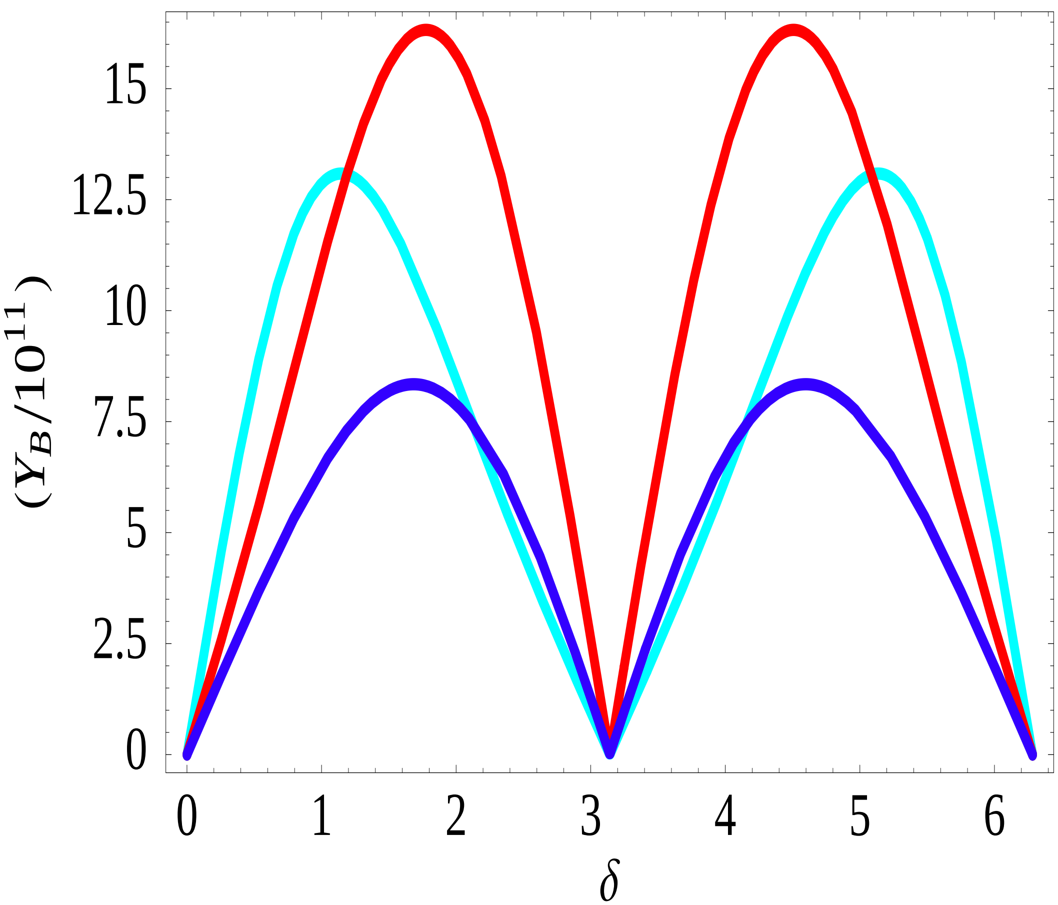
$$|Y_B| \quad \cong 8.1 \times 10^{-12} \, |s_{13} \sin\delta| \left(\frac{M_1}{10^9 \text{ GeV}} \right) .$$

$|Y_B| \gtrsim 8 \times 10^{-11}$, $\quad M_1 \lesssim 5 \times 10^{11}$ GeV imply

$$|\sin\theta_{13} \sin\delta| \gtrsim 0.02 \,, \qquad \sin\theta_{13} \gtrsim 0.02 \,.$$

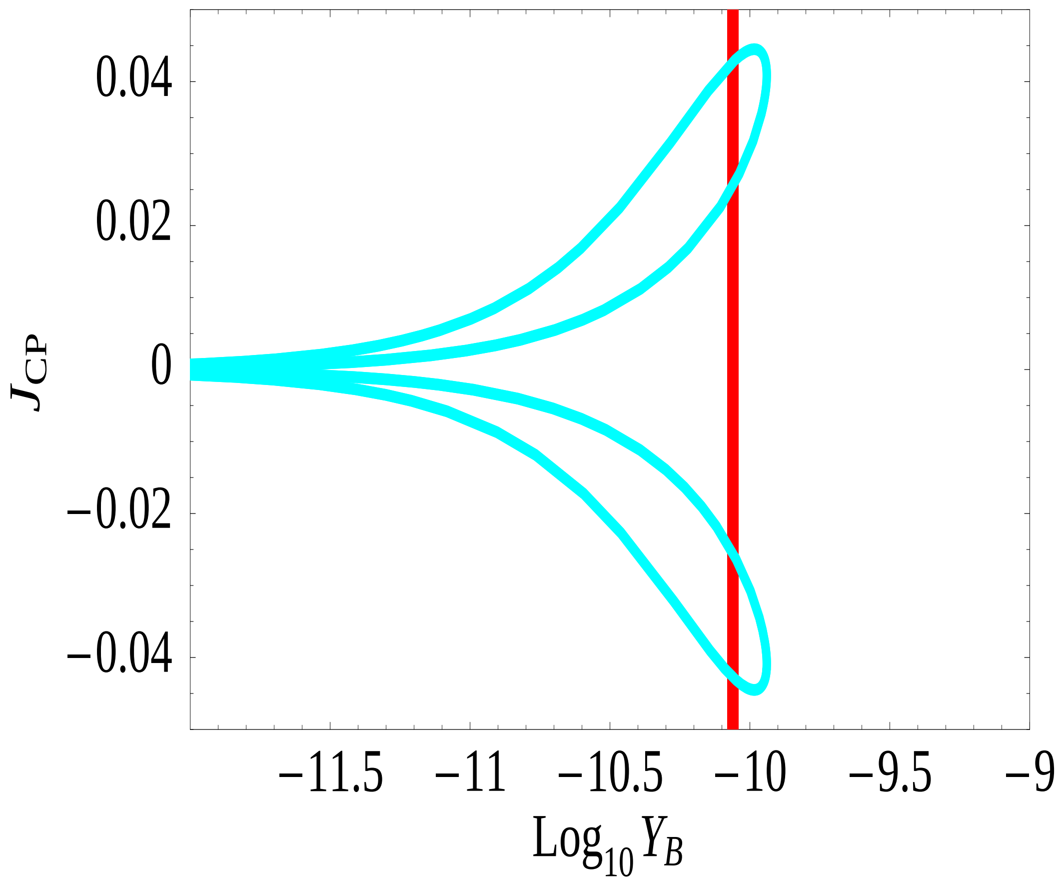
The lower limit corresponds to

$$|J_{\text{CP}}| \gtrsim 4.6 \times 10^{-3}$$



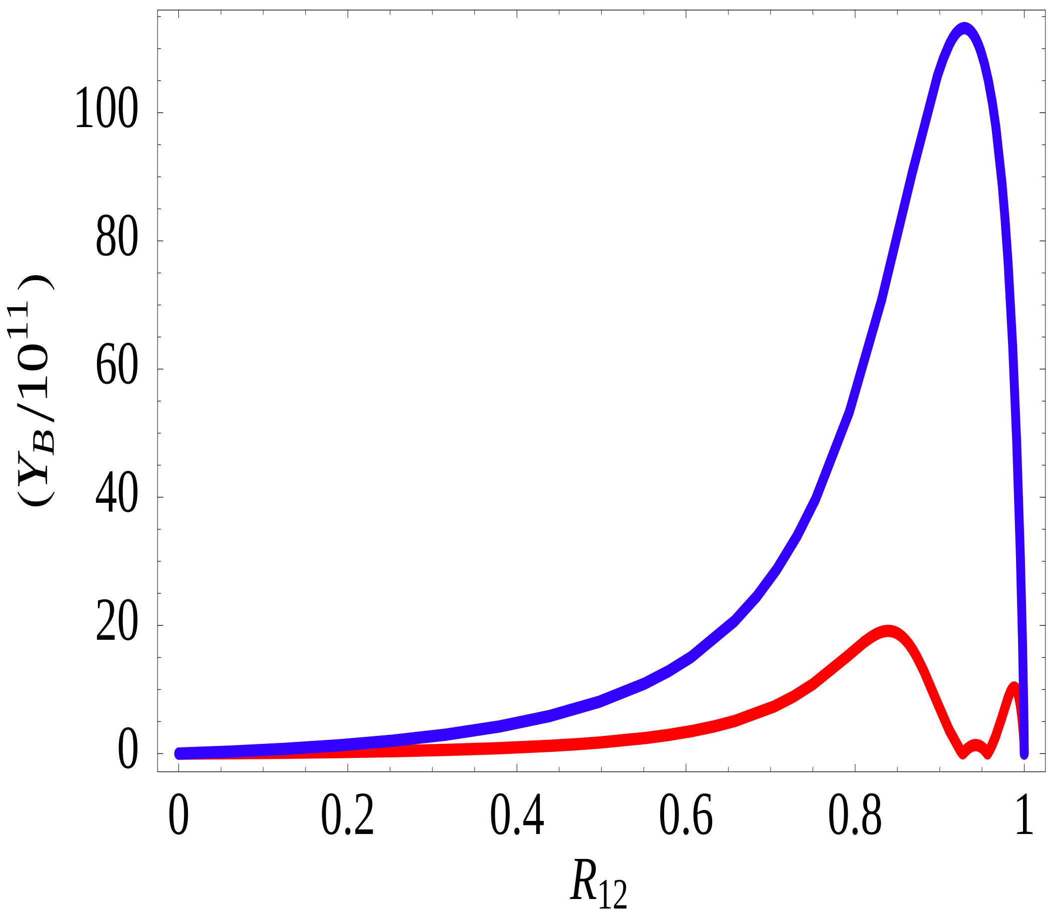
$M_1 \ll M_2 \ll M_3$, $m_1 \ll m_2 \ll m_3$; Dirac CP-violation, $\alpha_{32} = 0;\ 2\pi$;
real R_{12}, R_{13}, $|R_{12}|^2 + |R_{13}|^2 = 1$, $|R_{12}| = 0.86$, $|R_{13}| = 0.51$, sign$\,(R_{12}R_{13}) = +1$;
i) $\alpha_{32} = 0$ ($\kappa' = +1$), $s_{13} = 0.2$ (red line) and $s_{13} = 0.1$ (dark blue line);
ii) $\alpha_{32} = 2\pi$ ($\kappa' = -1$), $s_{13} = 0.2$ (light blue line);
$M_1 = 5 \times 10^{11}$ GeV.

S. Pascoli, S.T.P., A. Riotto, 2006.

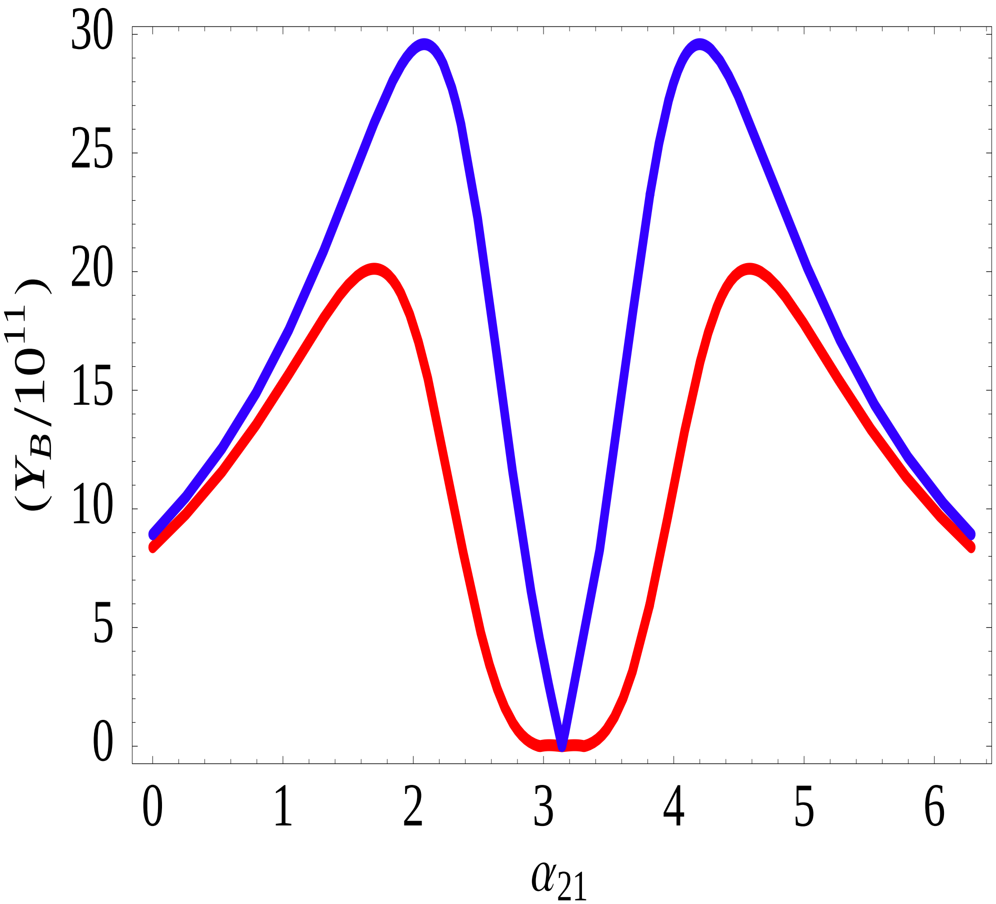


$M_1 \ll M_2 \ll M_3$, $m_1 \ll m_2 \ll m_3$; $M_1 = 5 \times 10^{11}$ GeV;
Dirac CP-violation, $\alpha_{32} = 0 \ (2\pi)$;
$|R_{12}| = 0.86$, $|R_{13}| = 0.51$, $\mathrm{sign}\,(R_{12}R_{13}) = +1 \ (-1)$ ($\beta_{23} = 0 \ (\pi)$, $\kappa' = +1$);
The red region denotes the 2σ allowed range of $Y_{\rm B}$.

S. Pascoli, S.T.P., A. Riotto, 2006.

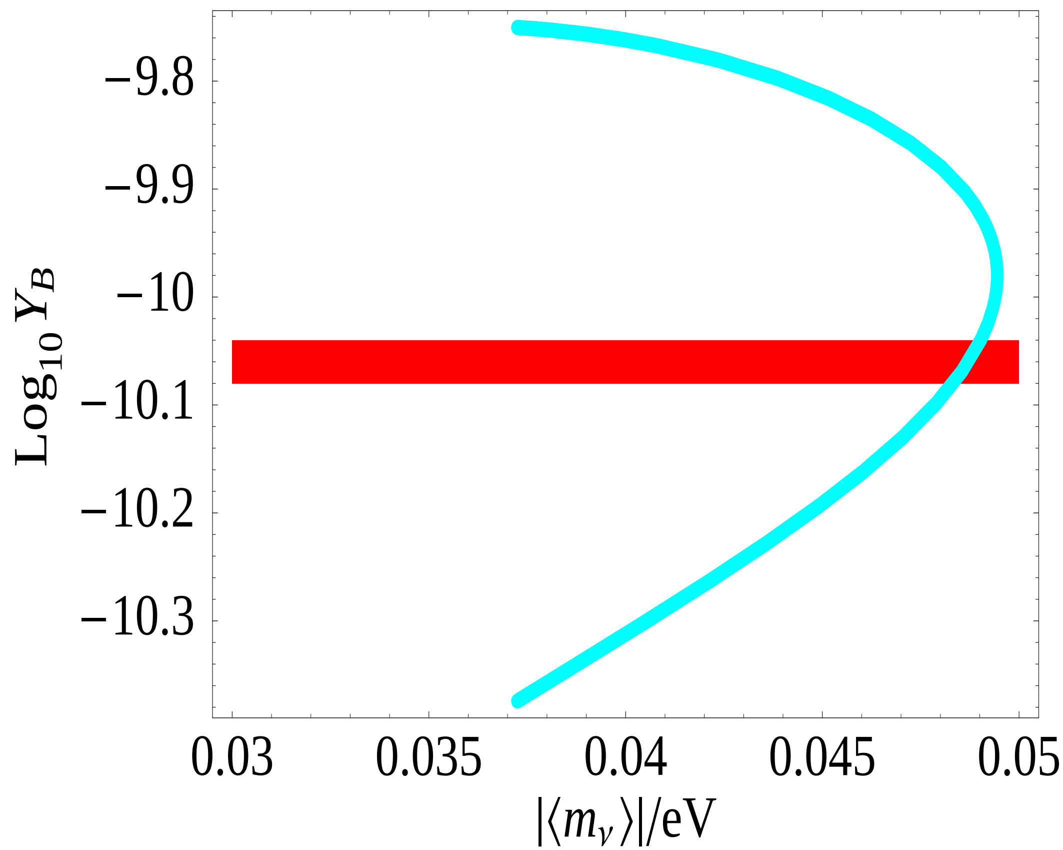


$M_1 \ll M_2 \ll M_3$, $m_1 \ll m_2 \ll m_3$; $M_1 = 5 \times 10^{11}$ GeV;
real R_{12}, R_{13}, sign $(R_{12}R_{13}) = +1$, $R_{12}^2 + R_{13}^2 = 1$, $s_{13} = 0.20$;
a) Majorana CP-violation (blue line), $\delta = 0$ and $\alpha_{32} = \pi/2$ ($\kappa = +1$);
b) Dirac CP-violation (red line), $\delta = \pi/2$ and $\alpha_{32} = 0$ ($\kappa' = +1$);
$\Delta m^2_{\odot}$, $\sin^2\theta_{12}$, Δm^2_{31}, $\sin^2 2\theta_{23}$ - fixed at their best fit values.



$M_1 \ll M_2 \ll M_3$, $m_3 \ll m_1 < m_2$; $M_1 = 2 \times 10^{11}$ GeV;
Majorana CP-violation, $\delta = 0$;
purely imaginary $R_{11}R_{12} = i\kappa|R_{11}R_{12}|$, $\kappa = -1$, $|R_{11}|^2 - |R_{12}|^2 = 1$, $|R_{11}| = 1.2$;
$s_{13} = 0$ (blue line) and 0.2 (red line).

S. Pascoli, S.T.P., A. Riotto, 2006.



$M_1 \ll M_2 \ll M_3$, $m_3 \ll m_1 < m_2$; $M_1 = 2 \times 10^{11}$ GeV;
Majorana CP-violation, $\delta = 0$, $s_{13} = 0$;
purely imaginary $R_{11}R_{12} = i\kappa|R_{11}R_{12}|$, $\kappa = +1$ $|R_{11}|^2 - |R_{12}|^2 = 1$, $|R_{11}| = 1.05$.
The Majorana phase α_{21} is varied in the interval $[-\pi/2, \pi/2]$.

S. Pascoli, S.T.P., A. Riotto, 2006.

$M_1 \ll M_2 \ll M_3$, $m_3 \ll m_1 < m_2$ (IH)

Majorana or Dirac CP-violation

$m_3 \neq 0$, $R_{13} \neq 0$, $R_{11}(R_{12}) = 0$: possible to reproduce Y_B^{obs} for real $R_{12(11)}R_{13} \neq 0$

Requires $m_3 \cong (10^{-5} - 10^{-2})$ eV; non-trivial dependence of $|Y_B|$ on m_3

Majorana CPV, $\delta = 0$ (π): requires $M_1 \gtrsim 3.5 \times 10^{10}$ GeV

Dirac CPV, $\alpha_{32(31)} = 0$: typically requires $M_1 \gtrsim 10^{11}$ GeV

$|Y_B| \gtrsim 8 \times 10^{-11}$, $\quad M_1 \lesssim 5 \times 10^{11}$ GeV imply

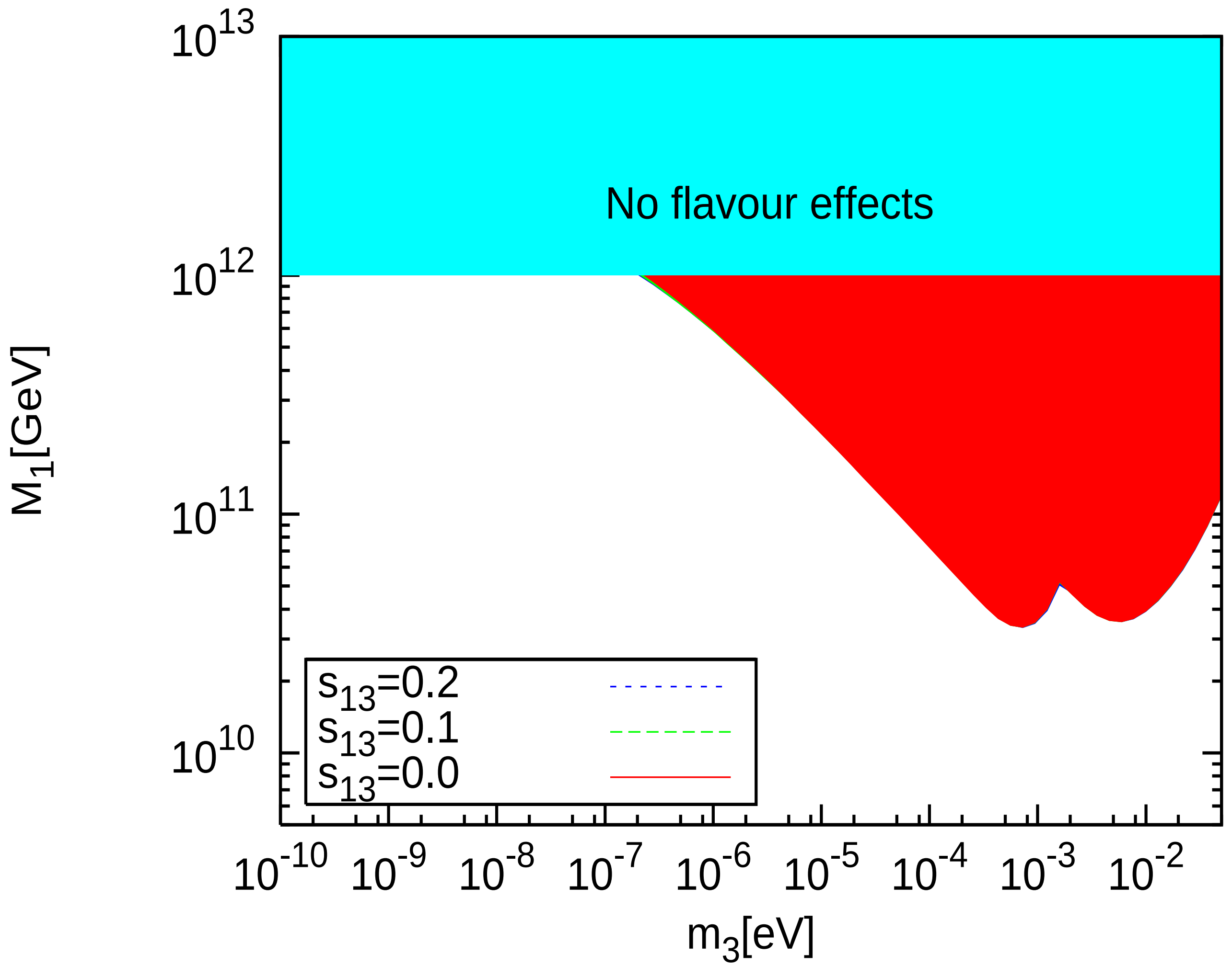
$$|\sin\theta_{13} \sin\delta|, \ \sin\theta_{13} \gtrsim (0.04 - 0.09) \,.$$

The lower limit corresponds to

$$|J_{\mathrm{CP}}| \gtrsim (0.009 - 0.02)$$

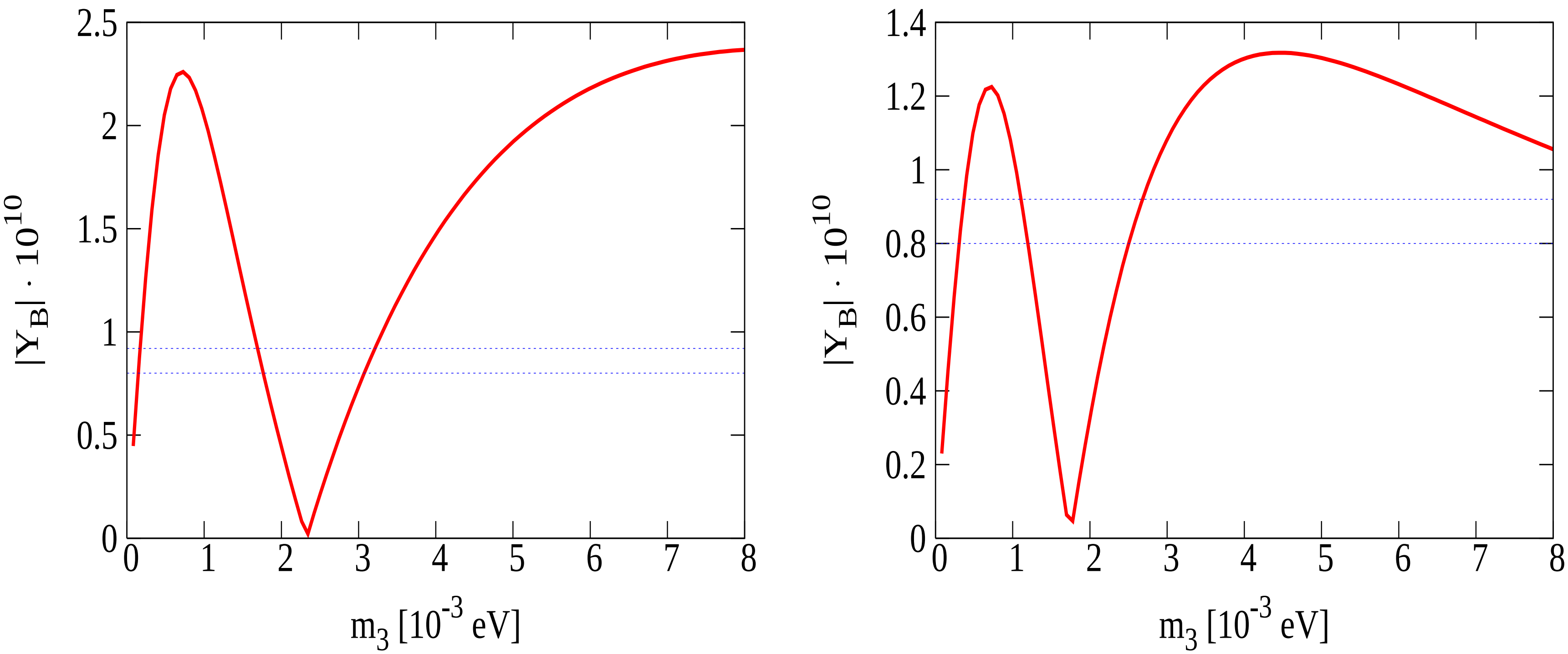
NO (NH) spectrum, $m_1 < \ (\ll) \ m_2 < m_3$: similar dependence of $|Y_B|$ on m_1 if $R_{12} = 0$, $R_{11}R_{13} \neq 0$; non-trivial effects for $m_1 \cong (10^{-4} - 5 \times 10^{-2})$ eV.

E. Molinaro, S.T.P., T. Shindou, Y. Takanishi, 2007



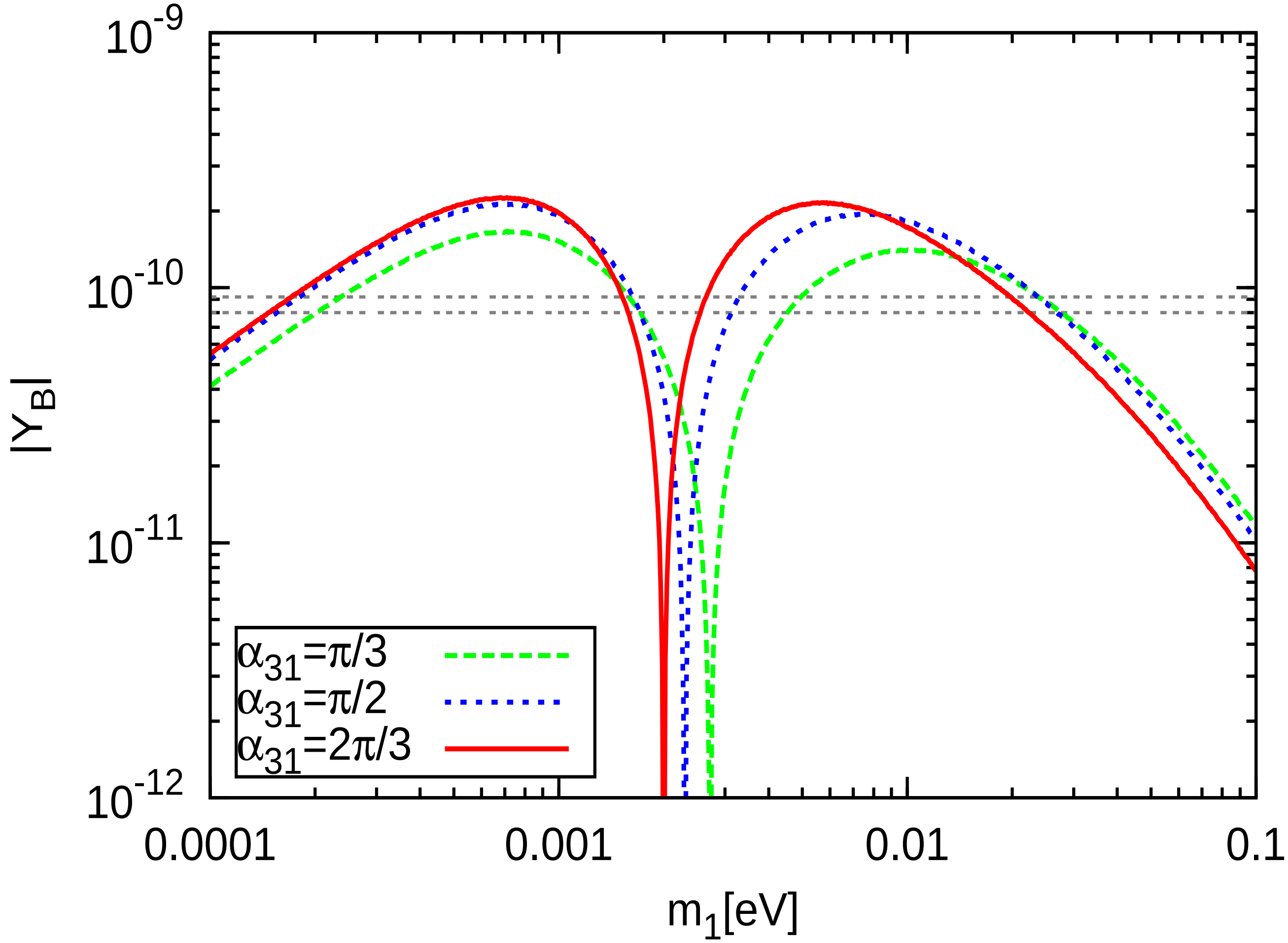
$m_3 < m_1 < m_2$, $M_1 \ll M_2 \ll M_3$, real R_{1j}; $M_1 = (10^9 - 10^{12})$ GeV, $s_{13} = 0.2; 0.1; 0$;
R_{1j} varied within $|R_{13}|^2 + |R_{12}|^2 + |R_{13}|^2 = 1$; $\alpha_{21}, \alpha_{31}, \delta$ varied in $[0, 2\pi]$;
min(M_1) for given m_3: $|Y_B| = 8.6 \times 10^{-11}$; absolute minima of M_1:
$m_3 \cong 5.5 \times 10^{-4}$; 5.9×10^{-3} eV, $\alpha_{32} \cong \pi/2$, $M_1 = 3.4\ (3.5) \times 10^{10}$ GeV.

E. Molinaro, S.T.P., T. Shindou, Y. Takanishi, 2007



$m_3 \ll m_1 \ll m_2$ (IH), $R_{11} = 0$, real $R_{12}R_{13}$, Majorana CPV;
$\alpha_{32} = \pi/2$, $s_{13} = 0$, $M_1 = 10^{11}$ GeV; $R_{12}^2/R_{13}^2 = m_3/m_2$: maximises $|\epsilon_\tau|$;
i) sgn$(R_{12}R_{13}) = +1$; ii) sgn$(R_{12}R_{13}) = -1$.

E. Molinaro, S.T.P., T. Shindou, Y. Takanishi, 2007



$m_1 < m_2 < m_3$ (NO(NH)), $R_{12} = 0$, real $R_{11}R_{13}$, Majorana CPV, $s_{13} = 0$;
sgn($R_{11}R_{13}$) $= -1$, $\sin^2\theta_{23} = 0.50$, $M_1 = 1.5 \times \times 10^{11}$ GeV;
$\alpha_{32} = 2\pi/3; \pi/2; \pi/3$ (red, blue, green lines).

E. Molinaro, S.T.P., T. Shindou, Y. Takanishi, 2007

Complex R: $\varepsilon_{1l} \neq 0$, CPV from U and R

$m_1 \ll m_2 < m_3$ (NH), $M_1 \ll M_{2,3}$; $m_1 \cong 0$, $R_{11} \cong 0$ (N_3 decoupling)

$$R_{12}^2 + R_{13}^2 = |R_{12}|^2\, e^{i2\varphi_{12}} + |R_{13}|^2\, e^{i2\varphi_{13}} = 1,$$

$$|R_{12}|^2 \sin 2\varphi_{12} + |R_{13}|^2 \sin 2\varphi_{13} = 0: \ \mathrm{sgn}(\sin 2\varphi_{12}) = -\mathrm{sgn}(\sin 2\varphi_{13})\,.$$

$$\cos 2\varphi_{12} = \tfrac{1+|R_{12}|^4-|R_{13}|^4}{2|R_{12}|^2}\,, \quad \sin 2\varphi_{12} = \pm\sqrt{1-\cos^2 2\varphi_{12}}\,,$$

$$\cos 2\varphi_{13} = \tfrac{1-|R_{12}|^4+|R_{13}|^4}{2|R_{13}|^2}\,, \quad \sin 2\varphi_{13} = \mp\sqrt{1-\cos^2 2\varphi_{13}}\,.$$

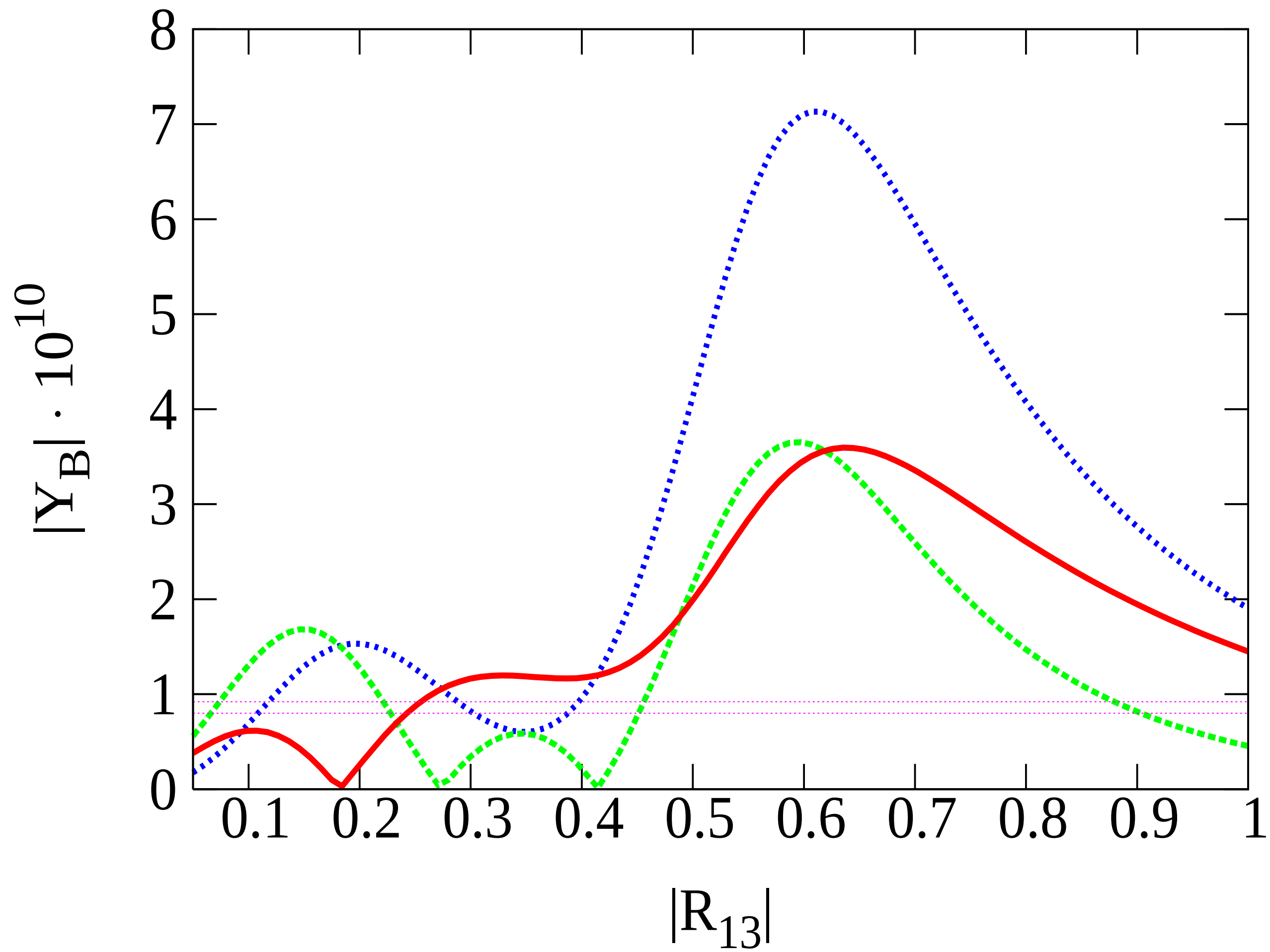
$m_3 \ll m_1 < m_2$ (IH), $M_1 \ll M_{2,3}$; $m_3 \cong 0$, $R_{13} \cong 0$ (N_3 decoupling)

$$R_{11}^2 + R_{12}^2 = |R_{11}|^2\, e^{i2\varphi_{11}} + R_{12}^2\, e^{i2\varphi_{12}} = 1,$$

$$|R_{11}|^2 \sin 2\varphi_{11} + |R_{12}|^2 \sin 2\varphi_{12} = 0\,.$$

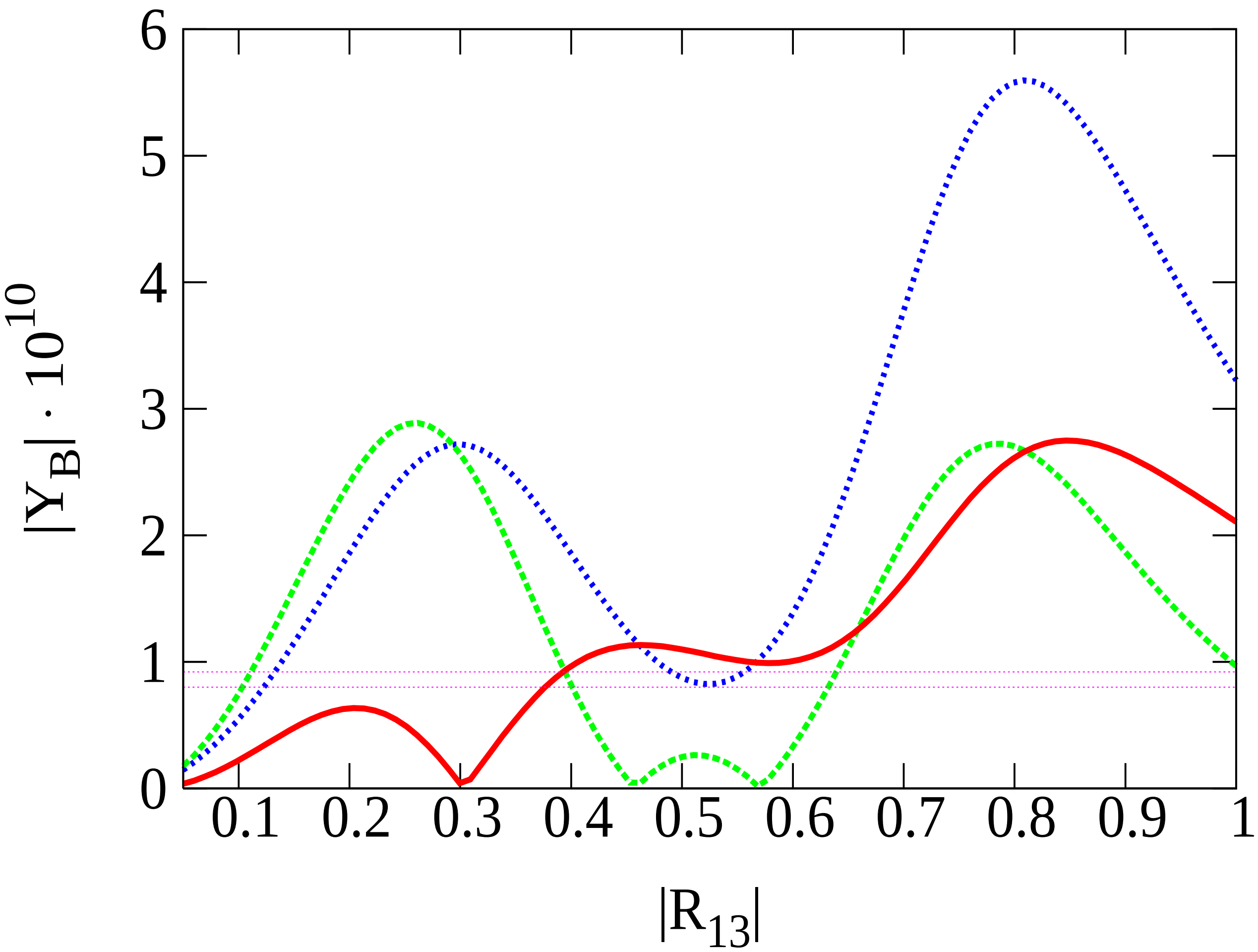
$|Y_B^0 A_{\mathrm{HE}}| \propto |R_{11}|^2 \sin(2\varphi_{11})\,(|U_{\tau 1}|^2 - |U_{\tau 2}|^2)$ - can be suppressed:

$$|U_{\tau 1}|^2 - |U_{\tau 2}|^2 \cong (s_{12}^2 - c_{12}^2)s_{23}^2 - 4s_{12}c_{12}s_{23}c_{23}s_{13}\cos\delta \cong -0.20 - 0.92\, s_{13}\,\cos\delta\,.$$



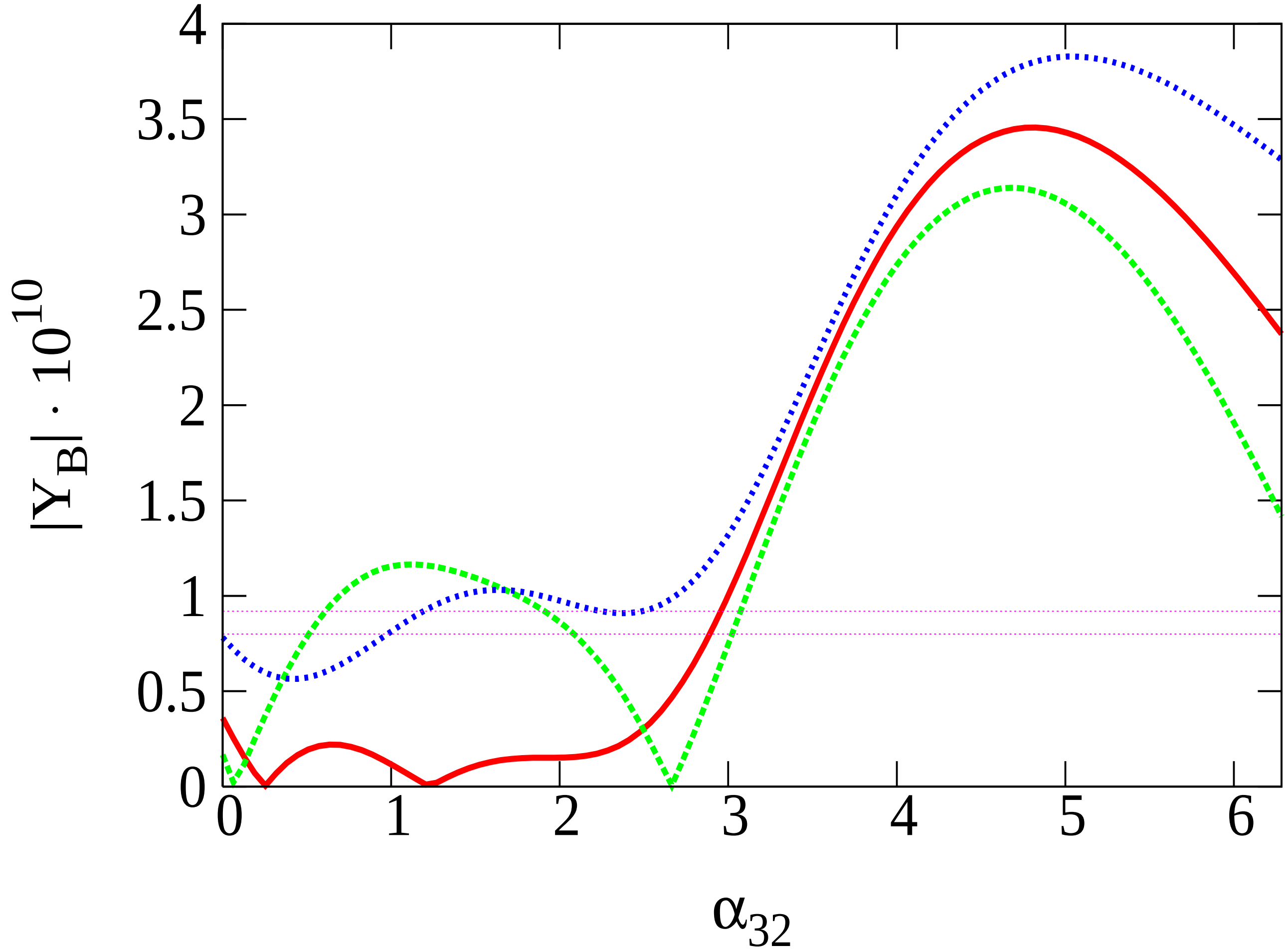
$m_1 < m_2 < m_3$ (NO(NH)), $R_{11} = 0$, CPV due to R and U,
$\alpha_{32} = \pi/2$, $s_{13} = 0$, $\sin^2\theta_{23} = 0.50$, $M_1 = 10^{11}$ GeV;
$|Y_B^0 A_{\mathrm{HE}}|$ (R CPV, blue), $|Y_B^0 A_{\mathrm{MIX}}|$ (U CPV, green), total $|Y_B|$ (red line)

E. Molinaro, S.T.P., 2008

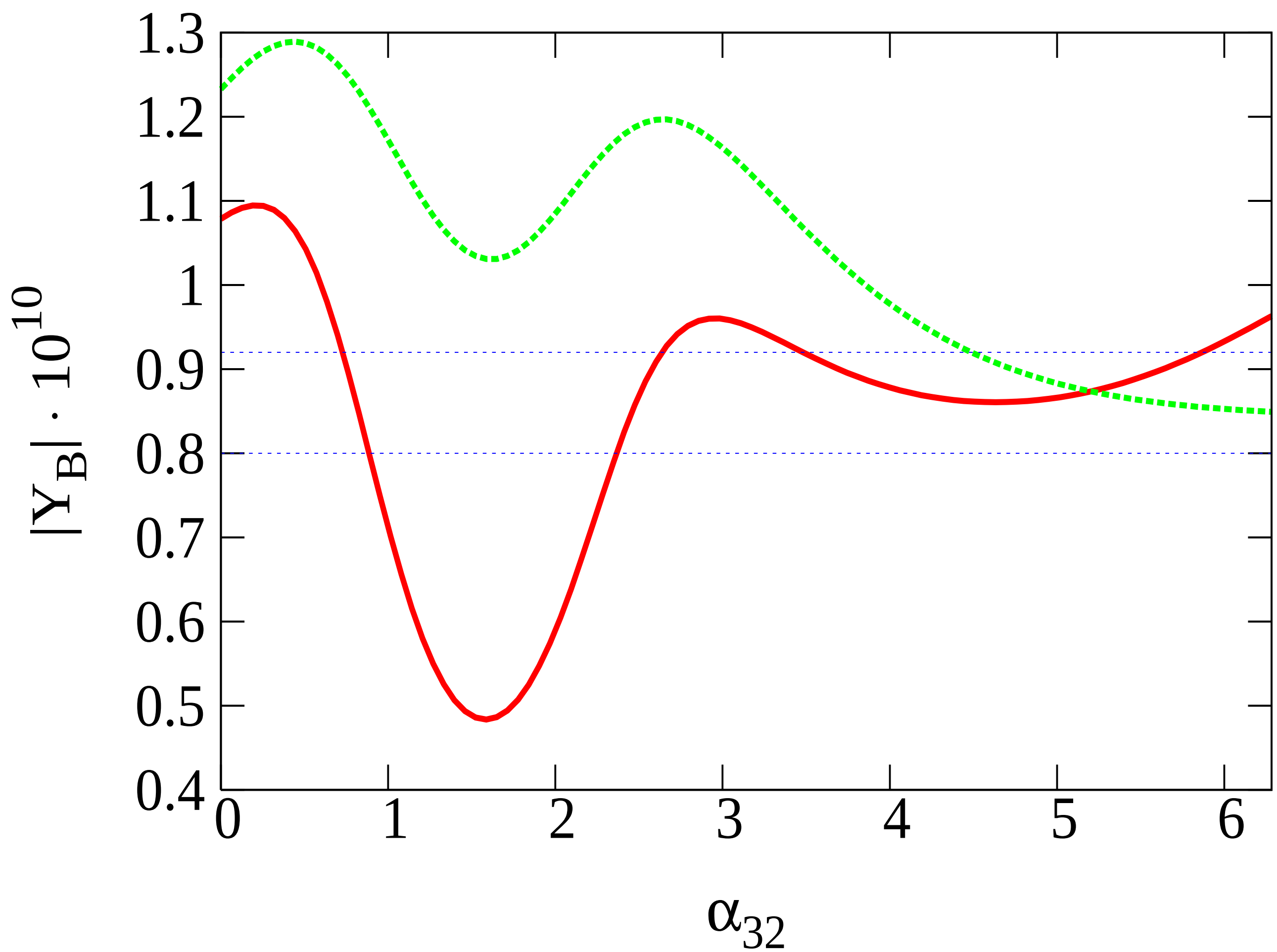


$m_1 < m_2 < m_3$ (NO(NH)), $R_{11} = 0$, CPV due to R and U,
$\alpha_{32} = \pi/2$, $s_{13} = 0$, $\sin^2\theta_{23} = 0.64$, $M_1 = 10^{11}$ GeV;
$|Y_B^0 A_{\text{HE}}|$ (R CPV, blue), $|Y_B^0 A_{\text{MIX}}|$ (U CPV, green), total $|Y_B|$ (red line)

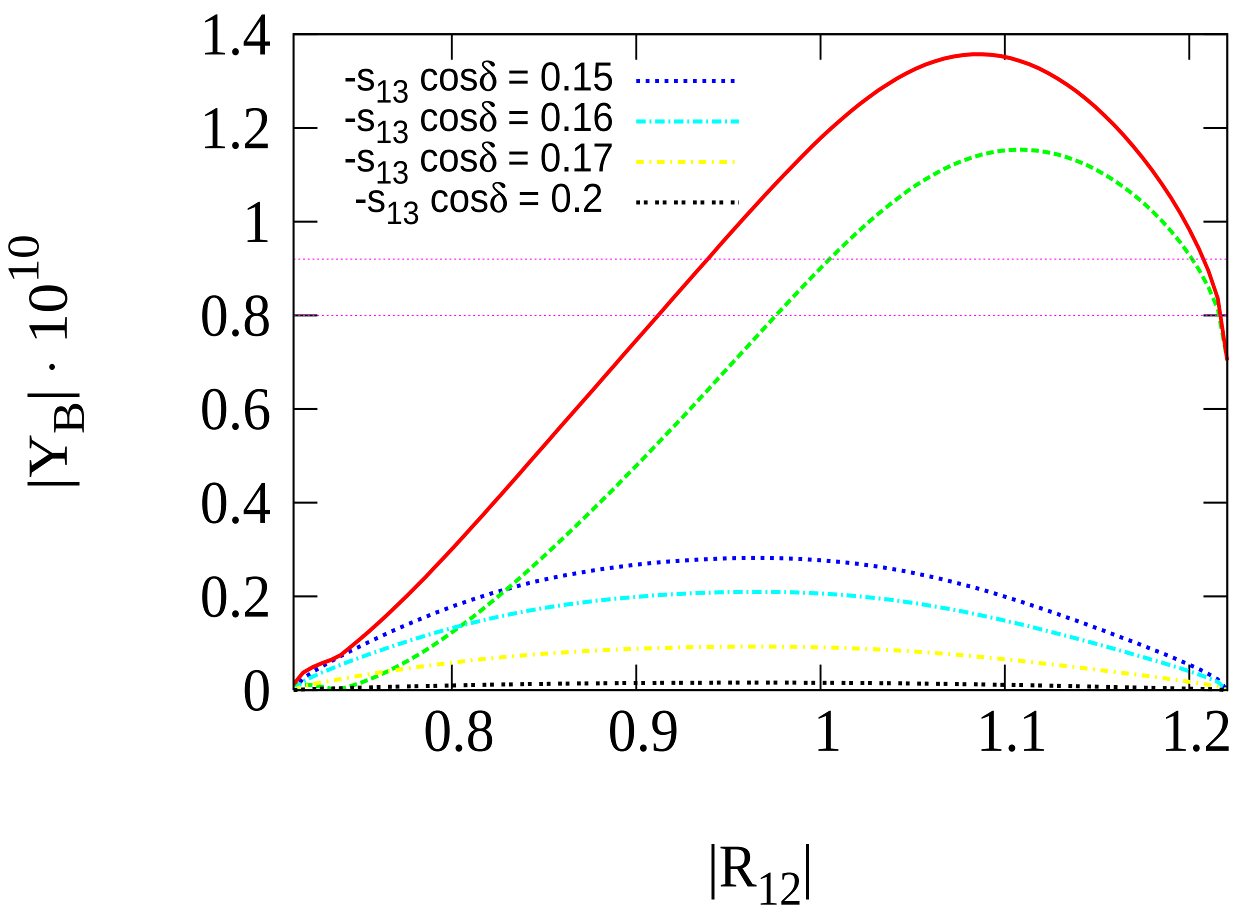
E. Molinaro, S.T.P., 2008



$m_1 \ll m_2 \ll m_3$ (NH), $R_{11} = 0$, Majorana and R-matrix CPV ;
$|R_{12}| = 1$, $|R_{13}| = 0.19$;
i) $s_{13} = 0$ (red line) , ii) $s_{13} = 0.2$, $\delta = 0$ (π) (green (blue) line) ,
$s_{23}^2 = 0.5$, $M_1 = 2 \times 10^{11}$ GeV.

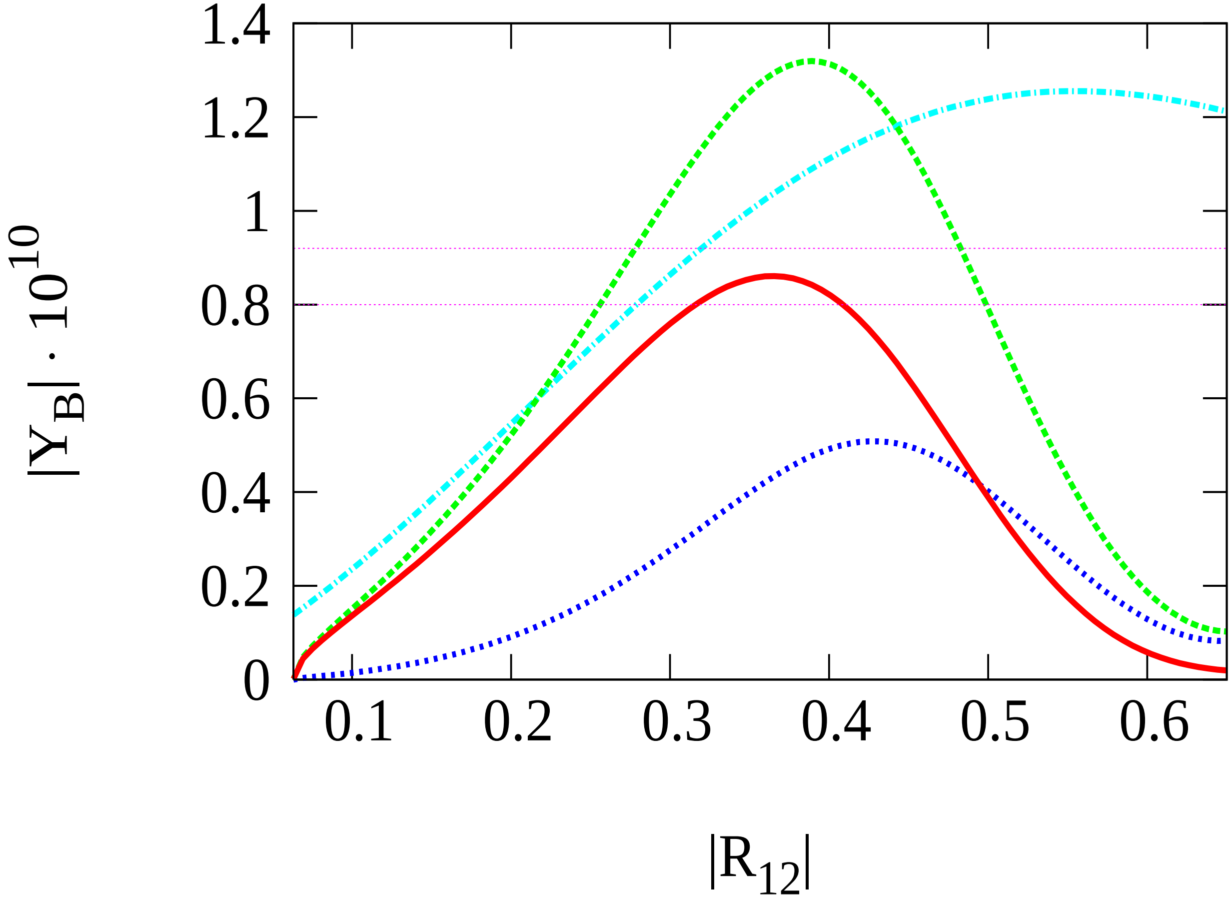


$m_1 \ll m_2 \ll m_3$ (NH), $R_{11} = 0$, Majorana and R-matrix CPV ;
$|R_{12}| = 1$, $|R_{13}| = 0.51$, $s_{13} = 0.2$, $\delta = 0$ (π) (red (green) line) ,
$s^2_{23} = 0.5$, $M_1 = 3.5 \times 10^{10}$ GeV.



$m_3 \ll m_1 < m_2$ (IH)), $R_{13} = 0$, Majorana and R-matrix CPV ,
$\alpha_{21} = \pi/2$, $(-s_{13} \cos\delta) = 0.15$, $|R_{11}| = 1.2$, $M_1 = 10^{11}$ GeV;
$|Y_B^0 A_{\mathrm{HE}}|$ (R CPV, blue), $|Y_B^0 A_{\mathrm{MIX}}|$ (U CPV, green), total $|Y_B|$ (red line) .

E. Molinaro, S.T.P., 2008



$m_3 \ll m_1 < m_2$ (IH)), $R_{13} = 0$, Majorana and R-matrix CPV ,
$\alpha_{21} = \pi/2$, $s_{13} = 0$, $|R_{11}| \cong 1$, $M_1 = 10^{11}$ GeV;
$|Y_B^0 A_{\mathrm{HE}}|$ (R CPV, blue), $|Y_B^0 A_{\mathrm{MIX}}|$ (U CPV, green), total $|Y_B|$ (red line) .
Light-blue line: CP-conserving R, $R_{11}R_{12} \equiv ik|R_{11}R_{12}|$, $k = -1$ $|R_{11}|^2 - |R_{12}|^2 = 1$.

E. Molinaro, S.T.P., 2008

Low Energy Leptonic CPV and Leptogenesis: Summary

Leptogenesis: see-saw mechanism; N_j - heavy RH ν's;
N_j, ν_k - Majorana particles

N_j: $M_1 \ll M_2 \ll M_3$

The observed value of the baryon asymmetry of the Universe can be generated

A. CP-violation due to the Dirac phase δ in U_{PMNS}, no other sources of CPV (Majorana phases in U_{PMNS} equal to 0, etc.); requires $M_1 \gtrsim 10^{11}$ GeV.

$m_1 \ll m_2 \ll m_3$ (NH):

$$|\sin\theta_{13}\sin\delta| \gtrsim 0.09\,, \qquad \sin\theta_{13} \gtrsim 0.09\,; \quad |J_{\text{CP}}| \gtrsim 2.0 \times 10^{-2}$$

$m_3 \ll m_1 < m_2$ (IH):

$$|\sin\theta_{13}\sin\delta| \gtrsim 0.02\,, \qquad \sin\theta_{13} \gtrsim 0.02\,; \quad |J_{\text{CP}}| \gtrsim 4.6 \times 10^{-3}$$

B. CP-violation due to the Majorana phases in U_{PMNS}, no other sources of CPV (Dirac phase in U_{PMNS} equal to 0, etc.); requires $M_1 \gtrsim 3.5 \times 10^{10}$ GeV.

C. CP-violation due to both Dirac and Majorana phases in U_{PMNS}.

D. Y_B can depend non-trivially on $\min(m_j) \sim (10^{-5} - 10^{-2})$ eV.

S. Pascoli, S.T.P., A. Riotto, 2006 (A-C);
E. Molinaro, S.T.P., T. Shindou, Y. Takanishi, 2007 (D).

Conclusions

Determining the nature - Dirac or Majorana, of massive neutrinos is of fundamental importance for understanding the origin of neutrino masses.

The see-saw mechanism provides a link between ν-mass generation and BAU.
Majorana CPV phases in U_{PMNS}: $(\beta\beta)_{0\nu}$-decay, Y_{B}.
Any of the CPV phases in U_{PMNS} can be the leptogenesis CPV parameters.

Obtaining information on Dirac and Majorana CPV is a remarkably challenging problem.

Dirac and Majorana CPV may have the same source.

Low energy leptonic CPV can be directly related to the existence of BAU.

Understanding the status of the CP-symmetry in the lepton sector is of fundamental importance.

These results underline further the importance of the experiments aiming to measure the CHOOZ angle θ_{13} and of the experimental searches for Dirac and/or Majorana leptonic CP-violation at low energies.

We are at the beginning of the Road...

And the future of Neutrino Physics is bright.