Fermion Mass Hierarchies and Flavor Mixing from T' Symmetry

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Outline

- Current experimental data on fermion masses and flavor mixings and its parameterization
- Short review on A_4 flavor model
- Basic properties of T' group
- SUSY model based on $T' \otimes Z_3 \otimes Z_9$ and its predictions
- Summary

Current experimental data on quark masses and its parameterizations

- The up type quark masses

$$\begin{aligned} m_u &\simeq 1.5 - 3\text{MeV} \\ m_c &\simeq 1.16 - 1.34\text{GeV} \\ m_t &\simeq 170.9.1 - 177.5\text{GeV} \end{aligned}$$

- The down type quark masses

$$\begin{aligned} m_d &\simeq 3 - 7\text{MeV} \\ m_s &\simeq 70 - 120\text{MeV} \\ m_b &\simeq 4.13 - 4.27\text{GeV} \end{aligned}$$

- Parameterization of the masses hierarchies in terms of $\lambda \simeq 0.22$(including renormalization evolution)

$$\begin{aligned} & m_t : m_c : m_u \sim 1 : \lambda^4 : \lambda^8 \\ & m_b : m_s : m_d \sim 1 : \lambda^2 : \lambda^4 \\ & m_t : m_b \sim 1 : \lambda^3 \end{aligned}$$

Quark mixing and CKM matrix

- Recent precise measurements(Babar and Belle) have greatly improved the knowledge of the CKM matrix, the experimental constraints on the CKM mixing parameters are

$$|V^{\text{Exp}}_{\text{CKM}}| \simeq \begin{pmatrix} 0.97377 \pm 0.00027 & 0.2257 \pm 0.0021 & (4.31 \pm 0.30) \times 10^{-3} \\ 0.230 \pm 0.011 & 0.957 \pm 0.095 & (41.6 \pm 0.6) \times 10^{-3} \\ (7.4 \pm 0.8) \times 10^{-3} & (40.6 \pm 2.7) \times 10^{-3} & > 0.78 \ at \ 95\% \ CL \end{pmatrix}$$

- Wolfenstein's parameterization of the CKM matrix to $\mathcal{O}(\lambda^4)$

$$V_{\text{CKM}} \simeq \begin{pmatrix} 1 - \frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

where

$$\lambda = 0.2272 \pm 0.0010, \quad A = 0.818^{+0.007}_{-0.017}$$

$$\bar{\rho} = 0.221^{+0.064}_{-0.028}, \quad \bar{\eta} = 0.340^{+0.017}_{-0.045}$$

Therefore the magnitudes of the CKM matrix elements are

$$|V_{us}| \sim \lambda\,, \quad |V_{cb}| \sim \lambda^2\,, \quad |V_{td}| \sim \lambda^3, \quad |V_{ub}| \sim \lambda^4$$

Lepton mass hierarchies and neutrino mixing

- Charged lepton mass hierarchies and its parameterization

$$m_e \simeq 0.511\text{MeV}$$
$$m_\mu \simeq 105.7\text{MeV}$$
$$m_\tau \simeq 1777\text{MeV}$$
$$m_\tau : m_\mu : m_e \sim 1 : \lambda^2 : \lambda^4$$

- Current knowledge about neutrino mainly comes from neutrino oscillation, and the neutrino mass spectrum can be normal hierarchical, inverted hierarchical or degenerate. Best-fit values, 2σ and 3σ intervals for the three–flavour neutrino oscillation parameters from global data including solar, atmospheric, reactor (KamLAND and CHOOZ) and accelerator (K2K and MINOS) experiments (From T.Schwetz, arXiv:0710.5027 and G.L. Fogli et al, arXiv:0806.2649.)

parameter	best fit	2σ	3σ
Δm^2_{21} [10^{-5}eV^2]	7.6	7.3–8.1	7.1–8.3
Δm^2_{31} [10^{-3}eV^2]	2.4	2.1–2.7	2.0–2.8
$\sin^2\theta_{12}$	0.32	0.28–0.37	0.26–0.40
$\sin^2\theta_{23}$	0.50	0.38–0.63	0.34–0.67
$\sin^2\theta_{13}$	0.007	≤ 0.033	≤ 0.050

- The current data within 1σ is well approximated by the so-called Tri-Bimaximal mixing

$$U_{\rm TB} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

TB mixing predicts $\sin^2\theta_{12,\rm TB} = \frac{1}{3}$, $\sin^2\theta_{23,\rm TB} = \frac{1}{2}$ and $\sin^2\theta_{13,\rm TB} = 0$. TB mixing seems to indicate an underlying symmetry.

Family symmetry, fermion mass hierarchies and flavor mixing

- Family symmetry is a well-known mechanism to understanding the hierarchies in fermion masses and flavor mixing, it assumes that the patterns in fermion mass and flavor mixing come from certain family symmetry between generations and its spontaneous broken.
- Many models with family symmetries gauged or global, continuous or discrete, Abelian or non-Abelian, have been suggested so far. E.g. U(2) flavor symmetry model by L.J. Hall et al successfully accounts for the quark masses and CKM mixing angles.
- Discrete family symmetries(S_3, S_4 and A_4 etc) seem suitable to produce the Tri-Bimaximal mixing in the lepton sector, especially the A_4 symmetry.

- **It is challenging to build a family symmetry model, which can naturally produce the masses and mixing angles in both the quark and the lepton sectors.**

Short review on A_4 model by Altarelli and Feruglio

- A_4 assignments of the matter fields

$$\begin{array}{rl} l_i (i = 1,2,3) & -------\mathbf{3} \\ e^c, \mu^c, \tau^c & -------\mathbf{1}, \mathbf{1}'', \mathbf{1}' \end{array}$$

- Flavon fields and A_4 spontaneous breaking

$$\varphi_T \quad ----\langle\varphi_T\rangle \propto (1,0,0) --- Z_3 \textit{ in the charged lepton sector}$$
$$\varphi_S \quad ----\langle\varphi_S\rangle \propto (1,1,1) --- Z_2 \textit{ in the neutrino sector}$$

- Mass matrices of charged lepton and neutrino at the leading order

$$M_l = v_d \frac{v_T}{\Lambda} \begin{pmatrix} y_e & 0 & 0 \\ 0 & y_\mu & 0 \\ 0 & 0 & y_\tau \end{pmatrix}$$
$$M_\nu = \frac{v_u^2}{\Lambda} \begin{pmatrix} a + 2b/3 & -b/3 & -b/3 \\ -b/3 & 2b/3 & a - b/3 \\ -b/3 & a - b/3 & 2b/3 \end{pmatrix}$$

- TB mixing is derived naturally, and the subleading corrections don't spoil the successful leading order results

$$U_{TB}^{T} M_{\nu} U_{TB} = \frac{v_u^2}{\Lambda} diag(a+b, a, -a+b)$$

- (Challenge)If the same classification scheme in A_4 is extended from leptons to quarks, the CKM matrix is a unit matrix at leading order, and the non-leading corrections in the up and down quark sector almost cancel in the mixing matrix. It seems difficult to derive the observed quark mixing via A_4 flavor symmetry.

Basic properties of T' group

- Geometrically, T' are proper rotations leaving a regular tetrahedron invariant in the SU(2) space.

- From group theory, T' is the double cover of A_4, which is the even permutation of 4 objects, and the order of T' is 24. It is by two generators S and T with the multiplication rules

$$S^4 = T^3 = \mathbf{1}, \quad TS^2 = S^2 T, \quad ST^{-1}S = TST$$

- Character table of the group T', ω is the third root of unity, i.e. $\omega = e^{\frac{2\pi i}{3}} = -\frac{1}{2} + i\frac{\sqrt{3}}{2}$. C_i are the classes of the group, $^{\circ}C_i$ is the order of the ith class, i.e. the number of distinct elements contained in this class, $^{\circ}h_{C_i}$ is the order of the elements A in the class C_i, i.e. the smallest integer (> 0) for which the equation $A^{^{\circ}h_{C_i}} = 1$ holds.

	C_1	C_2	C_3	C_4	C_5	C_6	C_7
$^\circ C_i$	1	1	6	4	4	4	4
$^\circ h_{C_i}$	1	2	4	6	3	3	6
1	1	1	1	1	1	1	1
1′	1	1	1	ω	ω^2	ω	ω^2
1″	1	1	1	ω^2	ω	ω^2	ω
2	2	-2	0	1	-1	-1	1
2′	2	-2	0	ω	$-\omega^2$	$-\omega$	ω^2
2″	2	-2	0	ω^2	$-\omega$	$-\omega^2$	ω
3	3	3	-1	0	0	0	0

- In addition to the singlet representations and triplet representation, T' has three doublet representations. T' can replicate the success of A_4 model building, and it allows us to treat the first two generation quarks and the third generation quark differently.

SUSY model based on $T' \otimes Z_3 \otimes Z_9$ flavor symmetry

- Symmetry of the model

$$SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \quad ----> \textit{gauge symmetry}$$
$$T' \otimes Z_3 \otimes Z_9 \quad ----> \textit{global flavor symmetry}$$

- Auxiliary symmetry Z_3 and Z_9

$Z_3 \quad ----> \textit{distinguishing } \varphi_T \textit{ from } \varphi_S$

$Z_9 \quad ----> \textit{hierarchies in the fermion masses and mixing angles}$

- (MSSM)Matter fields and their transformation rules

Fields	ℓ	e^c	μ^c	τ^c	Q_L	U^c	D^c	Q_3	t^c	b^c	$h_{u,d}$
T'	3	$\mathbf{1}$	$\mathbf{1}''$	$\mathbf{1}'$	$\mathbf{2}'$	$\mathbf{2}$	$\mathbf{2}$	$\mathbf{1}''$	$\mathbf{1}'$	$\mathbf{1}'$	$\mathbf{1}$
Z_3	α	α^2	α^2	α^2	α	α^2	α^2	α	α^2	α^2	$\mathbf{1}$
Z_9	$\mathbf{1}$	$\mathbf{1}$	β^6	β^8	β^3	β^3	β	$\mathbf{1}$	$\mathbf{1}$	β^7	$\mathbf{1}$

with $\alpha = \exp[i2\pi/3]$ and $\beta = \exp[i2\pi/9]$

Comment: The above assignments are free from discrete anomaly(communication with C.Luhn.)

Flavor symmetry spontaneous breaking

- Flavon fields which are responsible for flavor symmetry breaking, and their transformation properties under the global flavor symmetry

Fields	φ_T	φ_S	$\xi,\tilde{\xi}$	ϕ	θ''	θ'	Δ	$\bar{\Delta}$	χ
T'	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{2'}$	$\mathbf{1''}$	$\mathbf{1'}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
Z_3	1	α	α	1	1	1	1	1	1
Z_9	β	$\mathbf{1}$	$\mathbf{1}$	β^6	β	β	β^2	β^4	β

- The vacuum expectation values(VEV) of the flavon fields

$$\begin{aligned}
&\langle\varphi_T\rangle=(v_T,0,0), \quad \langle\varphi_S\rangle=(v_S,v_S,v_S), \quad \langle\phi\rangle=(v_1,0),\\
&\langle\xi\rangle=u_\xi, \quad \langle\tilde{\xi}\rangle=0, \quad \langle\theta'\rangle=u'_\theta, \quad \langle\theta''\rangle=u''_\theta,\\
&\langle\Delta\rangle=u_\Delta, \quad \langle\bar{\Delta}\rangle=\bar{u}_\Delta, \quad \langle\chi\rangle=u_\chi
\end{aligned}$$

- The magnitudes of VEVs

$$|\frac{v_T}{\Lambda}|\simeq|\frac{v_S}{\Lambda}|\simeq|\frac{v_1}{\Lambda}|\sim\lambda^2, \qquad |\frac{u''_\theta}{\Lambda}|\simeq|\frac{u_\Delta}{\Lambda}|\simeq|\frac{\bar{u}_\Delta}{\Lambda}|\sim\lambda^3$$

where Λ is the cut off scale of the theory.

Charged lepton sector

- The leading order ($\frac{1}{\Lambda^3}$) Yukawa interactions responsible for the charged lepton masses, which are invariant under the gauge symmetry of the standard model and the flavor symmetry $T'\otimes Z_3\otimes Z_9$

$$
\begin{aligned}
w_e &= y_e e^c(\ell\varphi_T)\bar{\Delta}^2 H_d/\Lambda^3 + h_{e1}e^c(\ell\varphi_S)(\varphi_S\varphi_S)H_d/\Lambda^3 \\
&+h_{e2}e^c(\ell\varphi_s)'(\varphi_S\varphi_S)''H_d/\Lambda^3 + h_{e3}e^c(\ell\varphi_S)''(\varphi_S\varphi_S)'H_d/\Lambda^3 \\
&+h_{e4}e^c(\ell\varphi_S)\xi^2 H_d/\Lambda^3 + y_{\mu1}\mu^c(\ell\phi\phi)'H_d/\Lambda^2 + y_{\mu2}\mu^c(\ell\varphi_T)'\Delta H_d/\Lambda^2 \\
&+h_{\mu1}\mu^c(\ell\varphi_T)'(\varphi_T\varphi_T)H_d/\Lambda^3 + h_{\mu2}\mu^c((\ell\varphi_T)_{\mathbf{3}_S}(\varphi_T\varphi_T)_{\mathbf{3}_S})'H_d/\Lambda^3 \\
&+h_{\mu3}\mu^c((\ell\varphi_T)_{\mathbf{3}_A}(\varphi_T\varphi_T)_{\mathbf{3}_S})'H_d/\Lambda^3 + h_{\mu4}\mu^c(\ell\varphi_T\varphi_T)'\chi H_d/\Lambda^3 \\
&+h_{\mu5}\mu^c(\ell\varphi_T\varphi_T)\theta' H_d/\Lambda^3 + h_{\mu6}\mu^c(\ell\varphi_T\varphi_T)''\theta'' H_d/\Lambda^3 \\
&+h_{\mu7}\mu^c(\ell\varphi_T)'\chi^2 H_d/\Lambda^3 + h_{\mu8}\mu^c(\ell\varphi_T)'\theta'\theta'' H_d/\Lambda^3 + h_{\mu9}\mu^c(\ell\varphi_T)\chi\theta' H_d/\Lambda^3 \\
&+h_{\mu10}\mu^c(\ell\varphi_T)\theta''\theta'' H_d/\Lambda^3 + h_{\mu11}\mu^c(\ell\varphi_T)''\chi\theta'' H_d/\Lambda^3 \\
&+h_{\mu12}\mu^c(\ell\varphi_T)''\theta'\theta' H_d/\Lambda^3 + y_\tau\tau^c(\ell\varphi_T)''H_d/\Lambda
\end{aligned}
$$

- After electroweak(EW) symmetry breaking and flavor symmetry breaking, the charged leptons obtain mass. The mass matrix is

$$M^e = \begin{pmatrix} y_e\frac{\bar{u}_\Delta^2 v_T}{\Lambda^3} + y'_e\frac{v_S^3}{\Lambda^3} & y'_e\frac{v_S^3}{\Lambda^3} & y'_e\frac{v_S^3}{\Lambda^3} \\ y_{\mu e}\frac{u'_\theta v_T^2}{\Lambda^3} & y_\mu\frac{v_1^2}{\Lambda^2} & y_{\mu\tau}\frac{u''_\theta v_T^2}{\Lambda^3} \\ 0 & 0 & y_\tau\frac{v_T}{\Lambda} \end{pmatrix} v_d$$

where $y'_e = 3(h_{e1} + h_{e2} + h_{e3}) + h_{e4}\frac{u_\xi^2}{v_S^2}$, $y_\mu \simeq iy_{\mu 1} + y_{\mu 2}\frac{u_\Delta v_T}{v_1^2}$, $y_{\mu e} = \frac{2}{3}h_{\mu 5} + h_{\mu 9}\frac{u_\chi}{v_T} + h_{\mu 10}\frac{u''^2_\theta}{u'_\theta v_T}$ and $y_{\mu\tau} = \frac{2}{3}h_{\mu 6} + h_{\mu 11}\frac{u_\chi}{v_T} + h_{\mu 12}\frac{u'^2_\theta}{u''_\theta v_T}$.

- The above charged lepton mass matrix M^e can be diagonalized through biunitary transformation $V_R^{e\dagger} M^e V_L^e = diag(m_e, m_\mu, m_\tau)$, standard perturbation diagonalization procedure gives

$$V_L^e \simeq \begin{pmatrix} 1 & s_{12}^e & 0 \\ -s_{12}^{e*} & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

with $s_{12}^e = (\frac{y_{\mu e}}{y_\mu}\frac{u'_\theta v_T^2}{v_1^2\Lambda})^* + \frac{|y'_e|^2}{|y_\mu|^2}\frac{|v_S|^6}{|v_1|^4\Lambda^2} \sim \lambda^3$.

The charged lepton masses are,

$$\begin{aligned} m_e &\simeq |(y_e \frac{\bar{u}_\Delta^2 v_T}{\Lambda^3} + y'_e \frac{v_S^3}{\Lambda^3}) v_d| \\ m_\mu &\simeq |y_\mu \frac{v_1^2}{\Lambda^2} v_d| \\ m_\tau &\simeq |y_\tau \frac{v_T}{\Lambda} v_d| \end{aligned}$$

- Realistic hierarchies among the charged lepton masses are generated

$$\frac{m_e}{m_\tau} \simeq |\frac{y_e}{y_\tau}\frac{\bar{u}_\Delta}{\Lambda^2} + \frac{y'_e}{y_\tau}\frac{v_S^3}{v_T \Lambda^2}| \simeq |\frac{y'_e}{y_\tau}\frac{v_S^3}{v_T \Lambda^2}| \sim \lambda^4$$

$$\frac{m_\mu}{m_\tau} \simeq |\frac{y_\mu}{y_\tau}\frac{v_1^2}{v_T \Lambda}| \sim \lambda^2$$

Neutrino sector

- The Yukawa interactions for the neutrino masses are

$$w_\nu = (y_\xi \xi + \tilde{y}_\xi \tilde{\xi})(\ell\ell) h_u h_u/\Lambda^2 + y_S(\varphi_S \ell\ell) h_u h_u/\Lambda^2 + ...$$

Here the neutrino mass comes from 5-dimensional operators, and the see-saw mechanism can be implemented as well.

- The neutrino mass matrix at the leading order is

$$M^\nu = \begin{pmatrix} 2y_\xi \frac{u_\xi}{\Lambda} + \frac{4}{3} y_S \frac{v_S}{\Lambda} & -\frac{2}{3} y_S \frac{v_S}{\Lambda} & -\frac{2}{3} y_S \frac{v_S}{\Lambda} \\ -\frac{2}{3} y_S \frac{v_S}{\Lambda} & \frac{4}{3} y_S \frac{v_S}{\Lambda} & 2y_\xi \frac{u_\xi}{\Lambda} - \frac{2}{3} y_S \frac{v_S}{\Lambda} \\ -\frac{2}{3} y_S \frac{v_S}{\Lambda} & 2y_\xi \frac{u_\xi}{\Lambda} - \frac{2}{3} y_S \frac{v_S}{\Lambda} & \frac{4}{3} y_S \frac{v_S}{\Lambda} \end{pmatrix} \frac{v_u^2}{\Lambda}$$

which is exactly diagonalized by the TB mixing matrix

$$V_L^{\nu T} M^\nu V_L^\nu = diag(2y_\xi \frac{u_\xi}{\Lambda} + 2y_S \frac{v_S}{\Lambda}, 2y_\xi \frac{u_\xi}{\Lambda}, -2y_\xi \frac{u_\xi}{\Lambda} + 2y_S \frac{v_S}{\Lambda}) \frac{v_u^2}{\Lambda}$$

with $V_L^\nu = U_{TB}$, and the neutrino mass spectrum is normal hierarchical

MNSP matrix and comparision with A_4 model

- The lepton mixing matrix(MNS matrix)

$$V_{\mathrm{MNS}} = V_L^{e\,\dagger} V_L^{\nu} \simeq \begin{pmatrix} \sqrt{\frac{2}{3}} + \frac{1}{\sqrt{6}} s_{12}^e & \frac{1}{\sqrt{3}} - \frac{1}{\sqrt{3}} s_{12}^e & \frac{1}{\sqrt{2}} s_{12}^e \\ -\frac{1}{\sqrt{6}} + \sqrt{\frac{2}{3}} s_{12}^{e*} & \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{3}} s_{12}^{e*} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

  MNS matrix deviates from the TB mixing pattern because of the corrections from the charged lepton sector, which is of order $\mathcal{O}(\lambda^3)$.

- As for the lepton sector, the difference between our model and A_4 model are mainly in the symmetry broken chain and the origin of mass hierarchies in the charged lepton.

	Symmetry broken chain	Source of mass hierarchies
A_4 model	$A_4 \to Z_3 \to$ nothing	FN mechanism
our model	$T' \to$ nothing	Z_9

Quark sector

- Yukawa interactions in the up and down quark sector

$$\begin{aligned}w_q &= y_{u1}(\varphi_T Q_L U^c)\Delta H_u/\Lambda^2 + y_{u2}((Q_L U^c)_{\mathbf{3}}(\phi\phi)_{\mathbf{3}})H_u/\Lambda^2\\ &+y_{u3}(Q_L U^c)'\theta''\Delta H_u/\Lambda^2 + y_{u4}(Q_L\phi)''t^c H_u/\Lambda + y_{u5}Q_3(U^c\phi)'H_u/\Lambda\\ &+y_t Q_3 t^c H_u + y_{d1}(\varphi_T Q_L D^c)\bar{\Delta}H_d/\Lambda^2 + y_{d2}(Q_L D^c)'\theta''\bar{\Delta}H_d/\Lambda^2\\ &+y_{d3}(Q_L\phi)''b^c\Delta H_d/\Lambda^2 + y_{d4}Q_3(D^c\phi)'\Delta H_d/\Lambda^2 + +y_{b1}Q_3 b^c\Delta H_d/\Lambda\\ &+y_{b2}Q_3 b^c(\varphi_T\varphi_T)h_d/\Lambda^2 + y_{b3}Q_3 b^c\chi^2 h_d/\Lambda^2 + y_{b4}Q_3 b^c\theta'\theta''/\Lambda^2 \ldots\end{aligned}$$

- After electroweak and flavor symmetry breaking, quark acquire mass

$$\begin{aligned}w_q &= y_{u1}\frac{u_\Delta v_T}{\Lambda^2}v_u cc^c + iy_{u2}\frac{v_1^2}{\Lambda^2}v_u cc^c + y_{u3}\frac{u''_\theta u_\Delta}{\Lambda^2}v_u(uc^c - cu^c) + y_{u4}\frac{v_1}{\Lambda}v_u ct^c\\ &+y_{u5}\frac{v_1}{\Lambda}v_u tc^c + y_t v_u tt^c + y_{d1}\frac{\bar{u}_\Delta v_T}{\Lambda^2}v_d ss^c + y_{d2}\frac{u''_\theta\bar{u}_\Delta}{\Lambda^2}v_d(ds^c - sd^c)\\ &+y_{d3}\frac{u_\Delta v_1}{\Lambda^2}v_d sb^c + y_{d4}\frac{u_\Delta v_1}{\Lambda^2}v_d bs^c + y_b\frac{u_\Delta}{\Lambda}v_d bb^c\end{aligned}$$

with $y_b = y_{b1} + y_{b2}\frac{v_T^2}{u_\Delta\Lambda} + y_{b3}\frac{u_\chi^2}{u_\Delta\Lambda} + y_{b4}\frac{u'_\theta u''_\theta}{u_\Delta\Lambda}$.

Textures of quark mass matrices

$$
M^u = \begin{pmatrix} 0 & -y_{u3}\frac{u''_\theta u_\Delta}{\Lambda^2} & 0 \\ y_{u3}\frac{u''_\theta u_\Delta}{\Lambda^2} & y_{u1}\frac{u_\Delta v_T}{\Lambda^2} + i y_{u2}\frac{v_1^2}{\Lambda^2} & y_{u5}\frac{v_1}{\Lambda} \\ 0 & y_{u4}\frac{v_1}{\Lambda} & y_t \end{pmatrix} v_u
$$

$$
M^d = \begin{pmatrix} 0 & -y_{d2}\frac{u''_\theta \bar{u}_\Delta}{\Lambda^2} & 0 \\ y_{d2}\frac{u''_\theta \bar{u}_\Delta}{\Lambda^2} & y_{d1}\frac{\bar{u}_\Delta v_T}{\Lambda^2} & y_{d4}\frac{u_\Delta v_1}{\Lambda^2} \\ 0 & y_{d3}\frac{u_\Delta v_1}{\Lambda^2} & y_b\frac{u_\Delta}{\Lambda} \end{pmatrix} v_d
$$

- The above mass matrices exactly have the same textures as those in the well-known U(2) flavor model.

- Predictions for quark masses

$$m_u \simeq \left|\frac{y_{u3}^2 y_t u_\theta''^2 u_\Delta^2}{(i y_{u2} y_t - y_{u4} y_{u5}) v_1^2 \Lambda^2} v_u\right| \sim \lambda^8 v_u$$

$$m_c \simeq \left|\left(i y_{u2} - \frac{y_{u4} y_{u5}}{y_t}\right)\frac{v_1^2}{\Lambda^2} v_u\right| \sim \lambda^4 v_u$$

$$m_t \simeq |y_t v_u| \sim 1 v_u$$

$$m_d \simeq \left|\frac{y_{d2}^2 u_\theta''^2 \bar{u}_\Delta}{y_{d1} v_T \Lambda^2} v_d\right| \sim \lambda^7 v_d$$

$$m_s \simeq \left|y_{d1}\frac{\bar{u}_\Delta v_T}{\Lambda^2} v_d\right| \sim \lambda^5 v_d$$

$$m_b \simeq \left|y_b \frac{u_\Delta}{\Lambda} v_d\right| \sim \lambda^3 v_d$$

with $\tan\beta \equiv \frac{v_u}{v_d} \sim 1$

- Predictions for the CKM matrix

$$V_{ud} \simeq V_{cs} \simeq V_{tb} \simeq 1$$

$$V_{us}^* \simeq -V_{cd} \simeq \frac{y_{d2}}{y_{d1}}\frac{u''_\theta}{v_T} - \frac{y_{u3}y_t u''_\theta u_\triangle}{(iy_{u2}y_t - y_{u4}y_{u5})v_1^2} \sim \lambda$$

$$V_{cb}^* \simeq -V_{ts} \simeq (\frac{y_{d3}}{y_b} - \frac{y_{u4}}{y_t})\frac{v_1}{\Lambda} \sim \lambda^2$$

$$V_{ub}^* \simeq -\frac{y_{u3}y_t}{iy_{u2}y_t - y_{u4}y_{u5}}(\frac{y_{d3}}{y_b} - \frac{y_{u4}}{y_t})\frac{u''_\theta u_\triangle}{v_1\Lambda} + \frac{y_{d2}y_{d4}^*}{|y_b|^2}\frac{u''_\theta \bar{u}_\triangle v_1^*}{u_\triangle \Lambda^2} \sim \lambda^4$$

$$V_{td} \simeq \frac{y_{d2}}{y_{d1}}(\frac{y_{d3}}{y_b} - \frac{y_{u4}}{y_t})\frac{u''_\theta v_1}{v_T\Lambda} - \frac{y_{d2}y_{d4}^*}{|y_b|^2}\frac{u''_\theta \bar{u}_\triangle v_1^*}{u_\triangle \Lambda^2} \sim \lambda^3$$

- Two interesting relations between the quark masses and CKM elements

$$|\frac{V_{td}}{V_{ts}}| \simeq \sqrt{\frac{m_d}{m_s}}, \qquad |\frac{V_{ub}}{V_{cb}}| \simeq \sqrt{\frac{m_u}{m_c}}$$

Vacuum alignment

- Following Altarelli and Feruglio, we exploit a global continuous $U(1)_R$ symmetry to simplify the vacuum alignment problem. $+1$ R-charge is assigned to the matter fields, and 0 R-charge to the Higgs and flavon supermultiplets. Driving fields carrying $+2$ R-charge are introduced.

Fields	φ_T^R	φ_S^R	ξ^R	ϕ^R	θ''^R	Δ^R	$\bar{\Delta}^R$	χ^R
T'	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{2''}$	$\mathbf{1''}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
Z_3	1	α	α	1	1	1	1	1
Z_9	β^6	1	1	β^2	β^7	β^7	β^5	β^7

- At the leading order, the superpotential depending on the driving fields is

$$
\begin{aligned}
w_v = {} & g_1(\varphi_T^R\phi\phi) + g_2(\varphi_T^R\varphi_T)\Delta + g_3(\phi^R\phi)\chi + g_4(\varphi_T\phi^R\phi) + g_5\chi^R\chi^2 \\
& + g_6\chi^R\theta'\theta'' + g_7\chi^R(\varphi_T\varphi_T) + g_8\theta''^R\theta''^2 + g_9\theta''^R\theta'\chi + g_{10}\theta''^R(\varphi_T\varphi_T)' \\
& + M_\Delta\Delta^R\Delta + g_{11}\Delta^R\chi^2 + g_{12}\Delta^R\theta'\theta'' + g_{13}\Delta^R(\varphi_T\varphi_T) + \bar{M}_\Delta\bar{\Delta}^R\bar{\Delta} \\
& + g_{14}\bar{\Delta}^R\Delta^2 + g_{15}(\varphi_S^R\varphi_S\varphi_S) + g_{16}(\varphi_S^R\varphi_S)\tilde{\xi} + g_{17}\xi^R(\varphi_S\varphi_S) + g_{18}\xi^R\xi^2 \\
& + g_{19}\xi^R\xi\tilde{\xi} + g_{20}\xi^R\tilde{\xi}^2
\end{aligned}
$$

- The scalar potential of the model is

$$V = \sum_i |\frac{\partial w_v}{\partial \mathcal{S}_i}|^2 + \sum_i m^2_{\mathcal{S}_i} |\mathcal{S}_i|^2 + \ldots$$

- The driving fields have zero VEV. In the SUSY limit, the minimization is

$$\begin{aligned}
\frac{\partial w_v}{\partial \varphi^R_{T1}} &= i g_1 \phi_1^2 + g_2 \varphi_{T1} \Delta = 0 \\
\frac{\partial w_v}{\partial \varphi^R_{T2}} &= (1-i) g_1 \phi_1 \phi_2 + g_2 \varphi_{T3} \Delta = 0 \\
\frac{\partial w_v}{\partial \varphi^R_{T3}} &= g_1 \phi_2^2 + g_2 \varphi_{T2} \Delta = 0 \\
\frac{\partial w_v}{\partial \phi^R_1} &= g_3 \phi_2 \chi + g_4' (\varphi_{T1} \phi_2 - (1-i) \varphi_{T3} \phi_1) = 0 \\
\frac{\partial w_v}{\partial \phi^R_2} &= -g_3 \phi_1 \chi + g_4' (\varphi_{T1} \phi_1 + (1+i) \varphi_{T2} \phi_2) = 0 \\
\frac{\partial w_v}{\partial \chi^R} &= g_5 \chi^2 + g_6 \theta' \theta'' + g_7 (\varphi^2_{T1} + 2 \varphi_{T2} \varphi_{T3}) = 0
\end{aligned}$$

$$
\begin{aligned}
\frac{\partial w_v}{\partial \theta''^R} &= g_8 \theta''^2 + g_9 \theta' \chi + g_{10}(\varphi_{T3}^2 + 2\varphi_{T1}\varphi_{T2}) = 0 \\
\frac{\partial w_v}{\partial \Delta^R} &= M_\Delta \Delta + g_{11}\chi^2 + g_{12}\theta'\theta'' + g_{13}(\varphi_{T1}^2 + 2\varphi_{T2}\varphi_{T3}) = 0 \\
\frac{\partial w_v}{\partial \bar{\Delta}^R} &= \bar{M}_\Delta \bar{\Delta} + g_{14}\Delta^2 = 0 \\
\frac{\partial w_v}{\partial \varphi_{S1}^R} &= \frac{2}{3} g_{15}(\varphi_{S1}^2 - 2\varphi_{S2}\varphi_{S3}) + g_{16}\varphi_{S1}\tilde{\xi} = 0 \\
\frac{\partial w_v}{\partial \varphi_{S2}^R} &= \frac{2}{3} g_{15}(\varphi_{S2}^2 - \varphi_{S1}\varphi_{S2}) + g_{16}\varphi_{S3}\tilde{\xi} = 0 \\
\frac{\partial w_v}{\partial \varphi_{S3}^R} &= \frac{2}{3} g_{15}(\varphi_{S3}^2 - \varphi_{S1}\varphi_{S2}) + g_{16}\varphi_{S2}\tilde{\xi} = 0 \\
\frac{\partial w_v}{\partial \xi^R} &= g_{17}(\varphi_{S1}^2 + 2\varphi_{S2}\varphi_{S3}) + g_{18}\xi^2 + g_{19}\xi\tilde{\xi} + g_{20}\tilde{\xi}^2 = 0
\end{aligned}
$$

- These sets of equations admit the solutions

$$
\begin{aligned}
\langle\chi\rangle &= u_\chi \\
\langle\theta'\rangle &= u'_\theta = -[\frac{(g_3^2 g_7 + g_4^2 g_5)^2 g_8}{g_4^4 g_6^2 g_9}]^{1/3} u_\chi \\
\langle\theta''\rangle &= u''_\theta = [\frac{(g_3^2 g_7 + g_4^2 g_5) g_9}{g_4^2 g_6 g_8}]^{1/3} u_\chi \\
\langle\Delta\rangle &= u_\Delta = \frac{g_3^2(g_7 g_{12} - g_6 g_{13}) + g_4^2(g_5 g_{12} - g_6 g_{11})}{g_4^2 g_6} \frac{u_\chi^2}{M_\Delta} \\
\langle\bar\Delta\rangle &= \bar u_\Delta = -\frac{[g_3^2(g_7 g_{12} - g_6 g_{13}) + g_4^2(g_5 g_{12} - g_6 g_{11})]^2 g_{14}}{g_4^4 g_6^2} \frac{u_\chi^4}{M_\Delta^2 \bar M_\Delta} \\
\langle\phi\rangle &= (v_1, 0), \\
v_1 &= (\frac{i g_2 g_3 [g_3^2(g_7 g_{12} - g_6 g_{13}) + g_4^2(g_5 g_{12} - g_6 g_{11})]}{g_1 g_4^3 g_6})^{1/2} M_\Delta^{-1/2} u_\chi^{3/2} \\
\langle\varphi_T\rangle &= (v_T, 0, 0) \text{ with } v_T = \frac{g_3}{g_4} u_\chi, \quad \langle\tilde\xi\rangle = 0, \quad \langle\xi\rangle = u_\xi \\
\langle\varphi_S\rangle &= (v_S, v_S, v_S), \qquad v_S = (-\frac{g_{18}}{3 g_{17}})^{1/2} u_\xi
\end{aligned}
$$

Corrections to the leading order predictions

- Two sources of corrections to the leading order results

  1. Higher dimensional operators in the driving superpotential w_v—-revising the vacuum alignment

  2. Higher dimensional operators in the Yukawa superpotentials w_e,w_ν,w_u and w_d—-modifying the Yukawa couplings after the electroweak and flavor symmetry breaking

- The subleading corrections have been studied in details. All observables get a correction of order $1/\Lambda$, and the leading order predictions are not spoiled.

Testing this model experimentally(in progress)

- The most promising are the FCNC(flavor changing neutral current) processes, such as lepton flavor violation $\mu \longrightarrow e\gamma$ and $\mu - e$ conversion in atom, electric dipole moments of the electron and neutron, proton decay and so on.

- The constraints and the implication form leptogenesis.

Summary

- A SUSY model based on $T' \otimes Z_3 \otimes Z_9$ is built.

- TB mixing with small corrections is derived naturally, and the correct pattern in the charged lepton masses are generated. In the charged lepton sector, T' is broken completely at the leading order, and it is broken to Z_4 in the neutrino sector

- The up type quark and down type quark mass matrices have the same textures as the U(2) flavor theory at the leading order, realistic hierarchies in quark masses and CKM matrix elements are produced.

Thank you!