Relativistic Heavy Ion Collisions

Theoretical Perspectives

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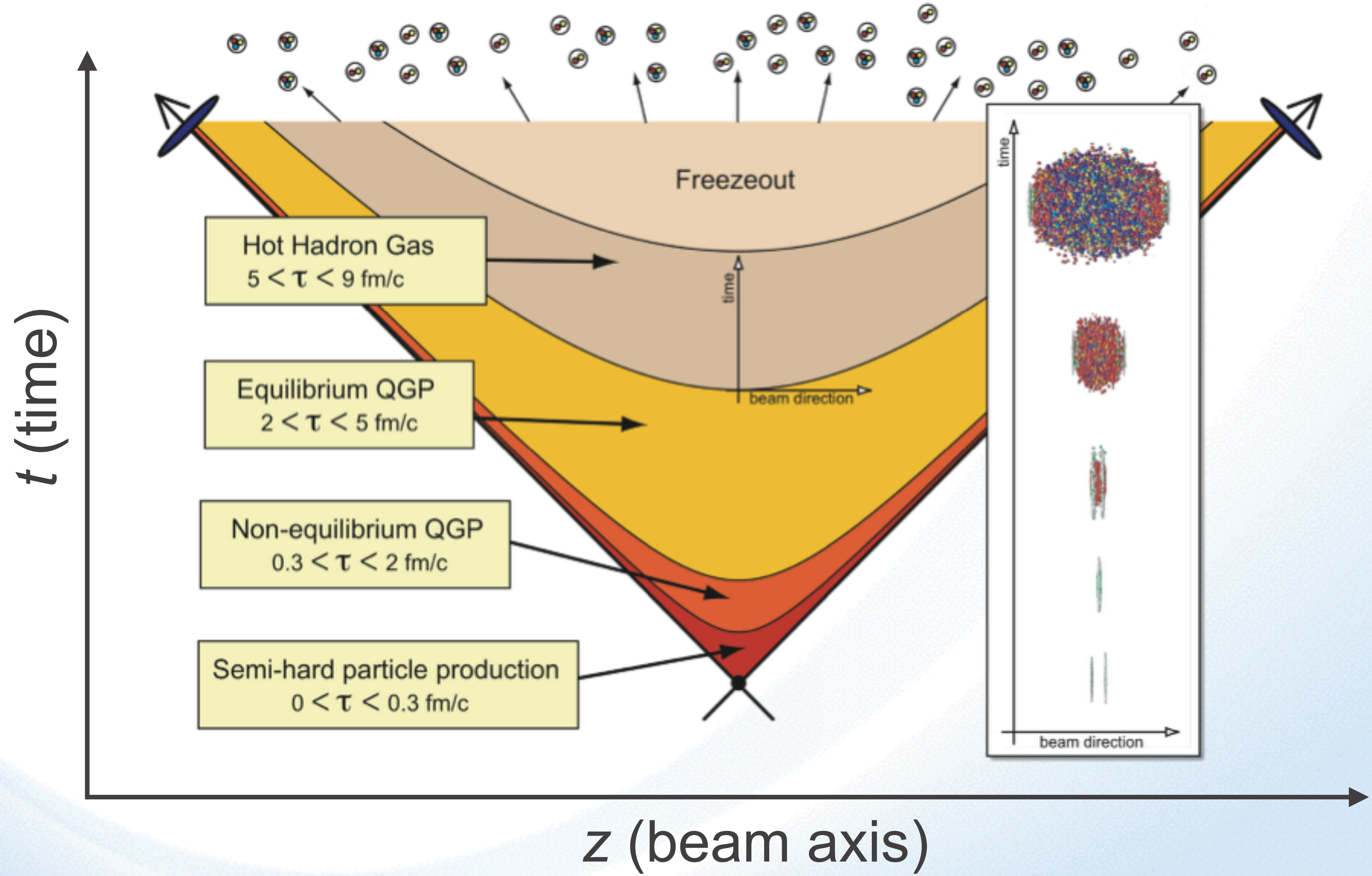
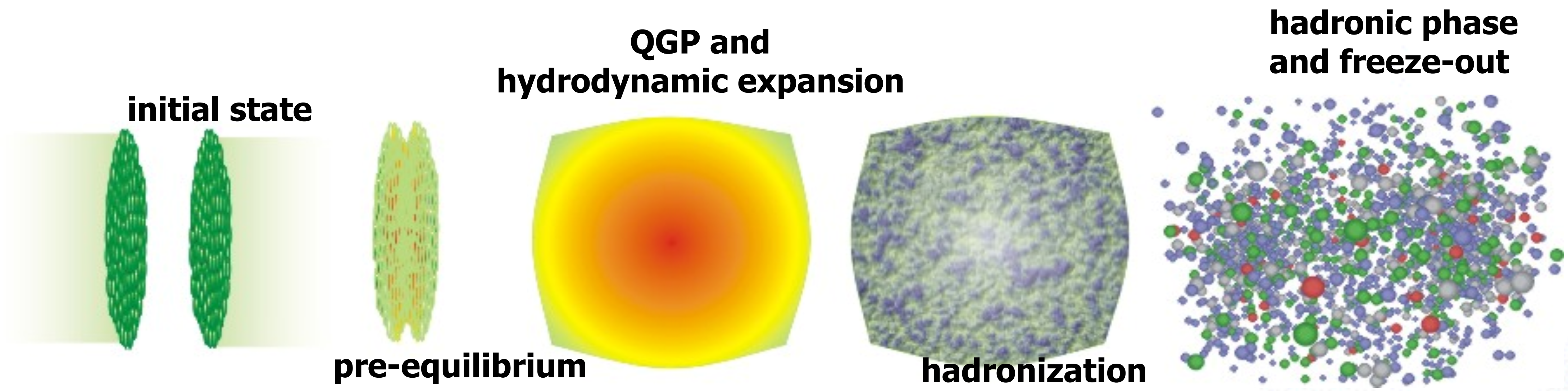
Space-time evolution



Standard model of the "Little Bang"



CGC "Glasma" Hydrodynamics Hadronic gas

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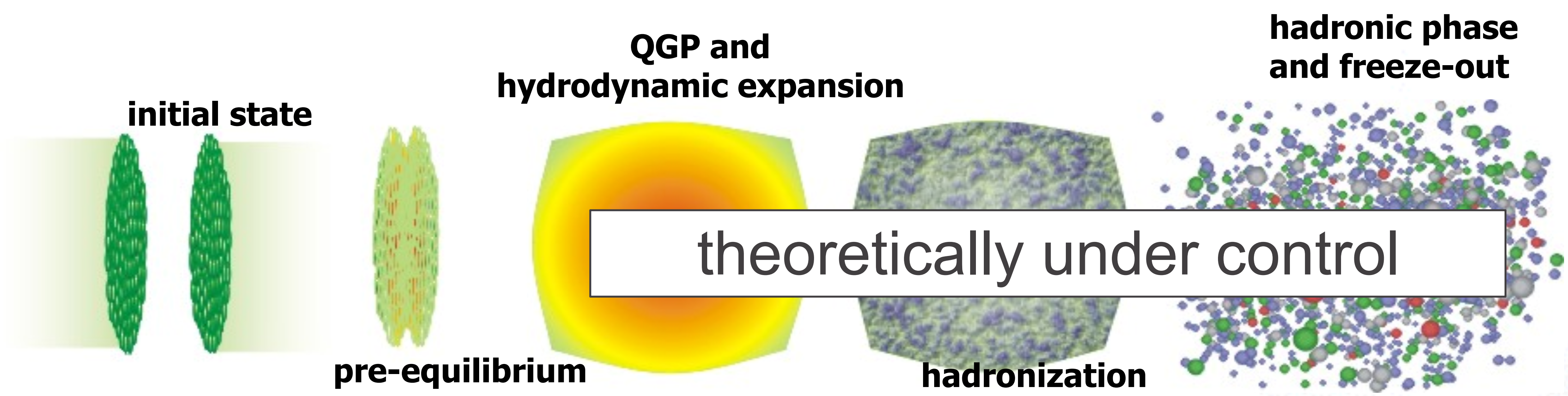

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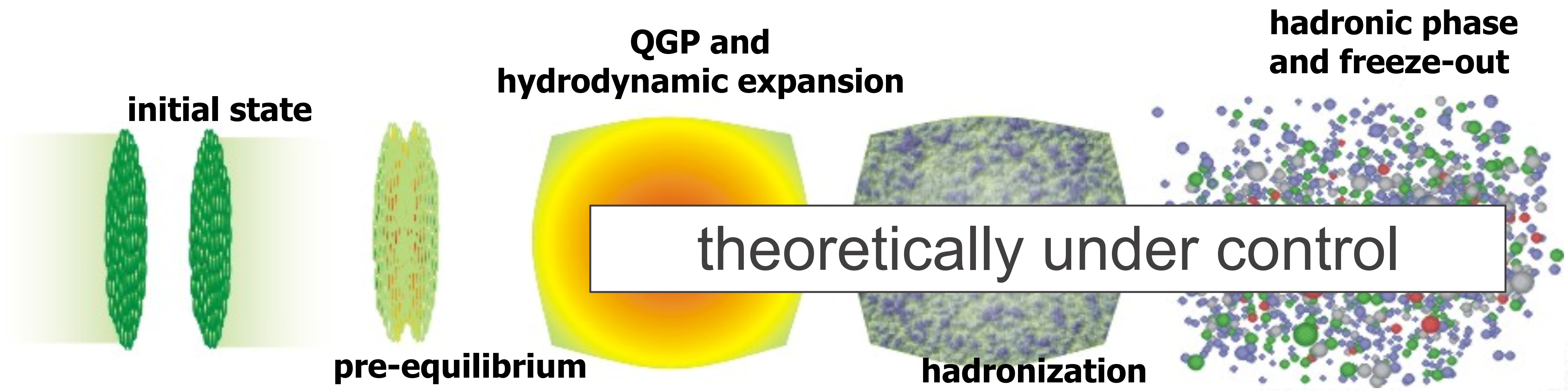
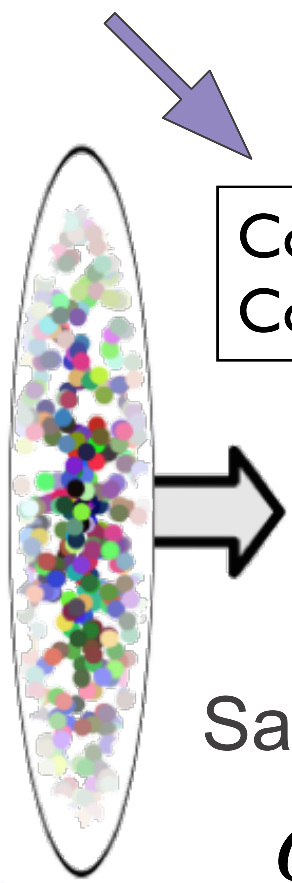


theoretically under control

CGC | "Glasma" | Hydrodynamics | Hadronic gas



Color Glass Condensate

Saturation scale

$$Q_s^2 \sim x^{-\lambda} A^{1/3}$$

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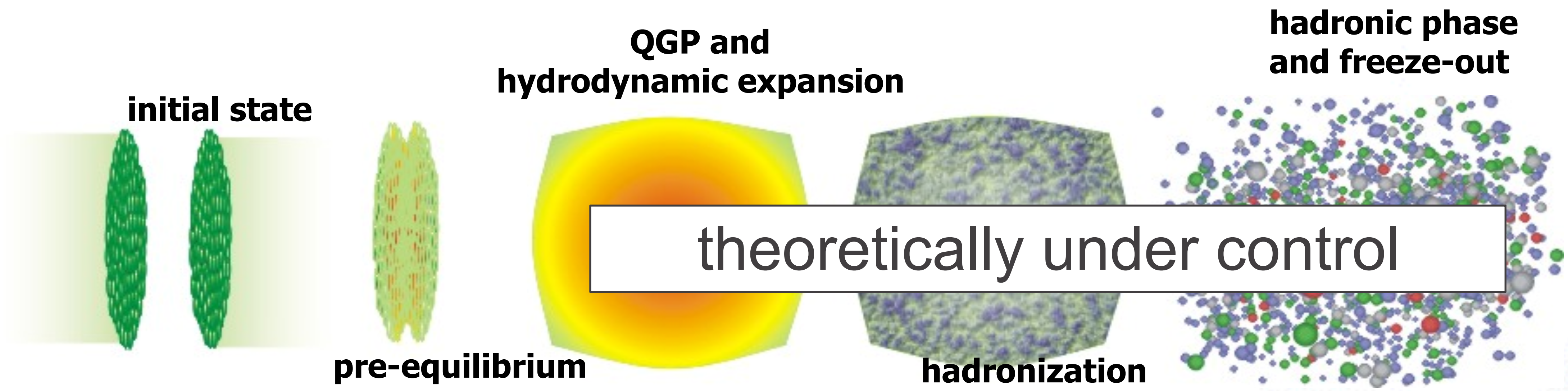

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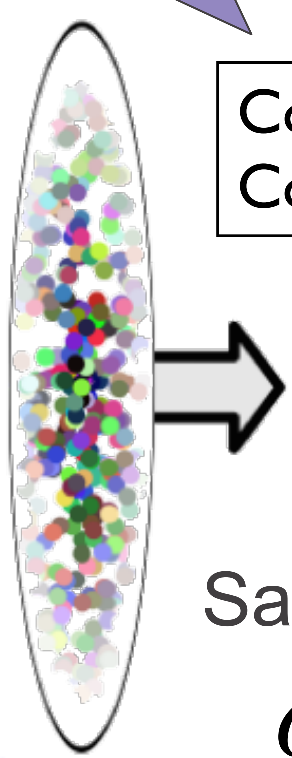

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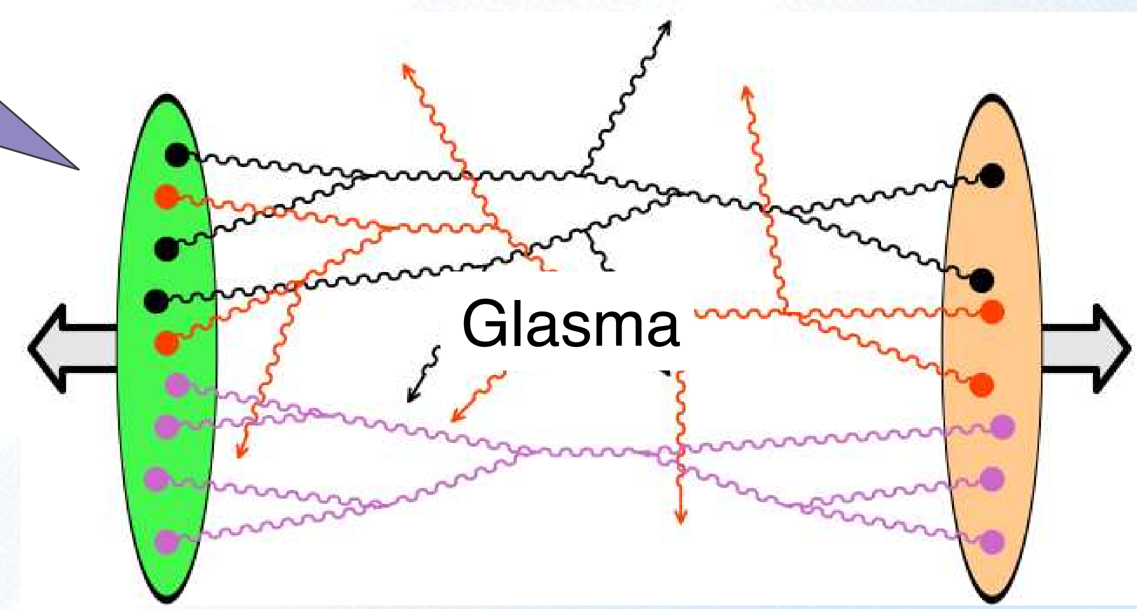

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So what have we discovered?

Imagine....

...heating a liquid (nuclear matter) until it turns into vapor (nucleon/hadron gas) at approximately 100 billion degrees.

But when you heat it to 20 times this temperature (2 trillion degrees) you find that it suddenly turns into a **liquid** again, in fact, into the **most perfect liquid** ever observed.

What happens at even higher temperatures? [LHC tells us]

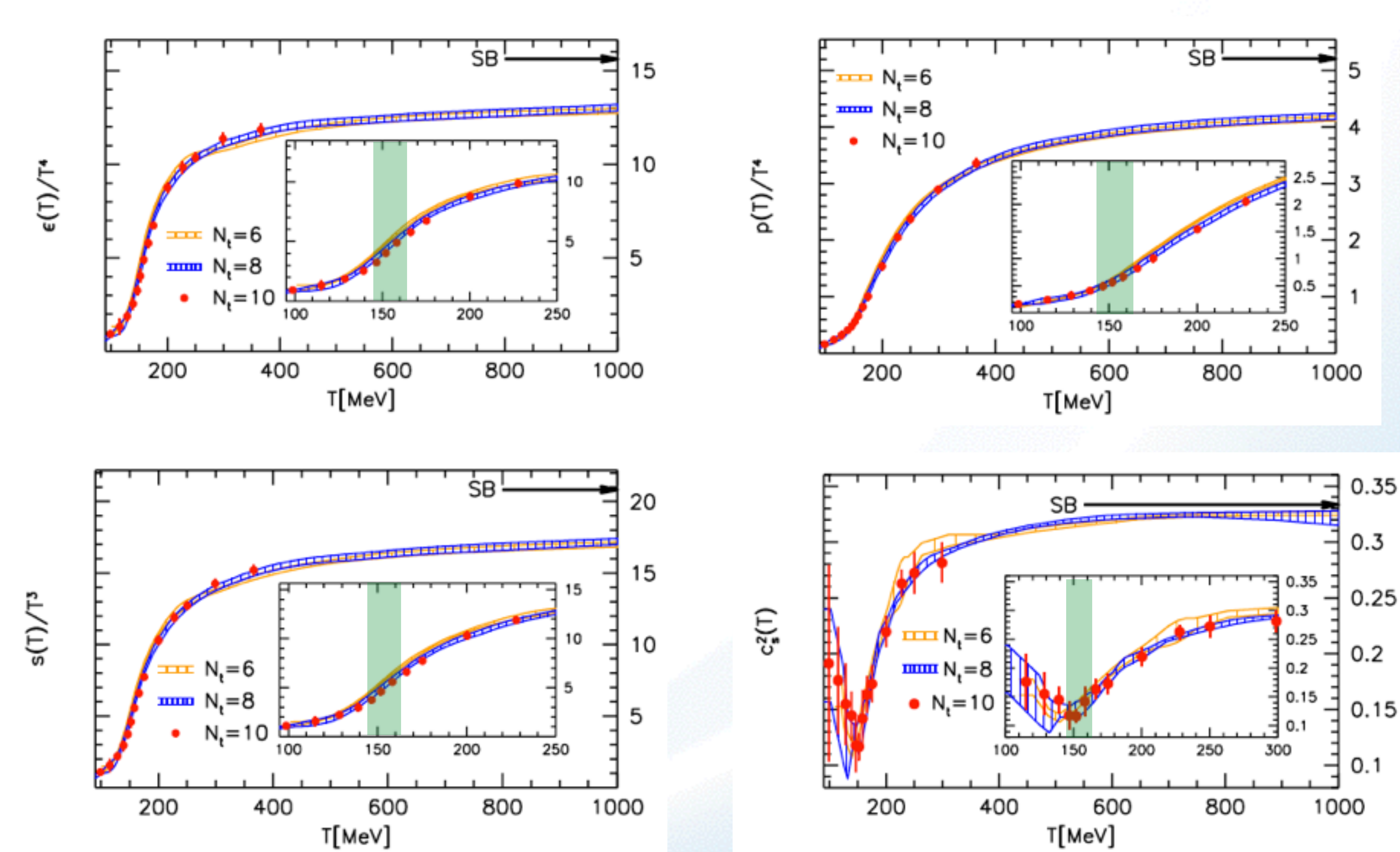
Where does the transition occur? [RHIC will tell us]

How is this possible? [Still a mystery - we may nee an EIC]

Equation of State of QCD Matter

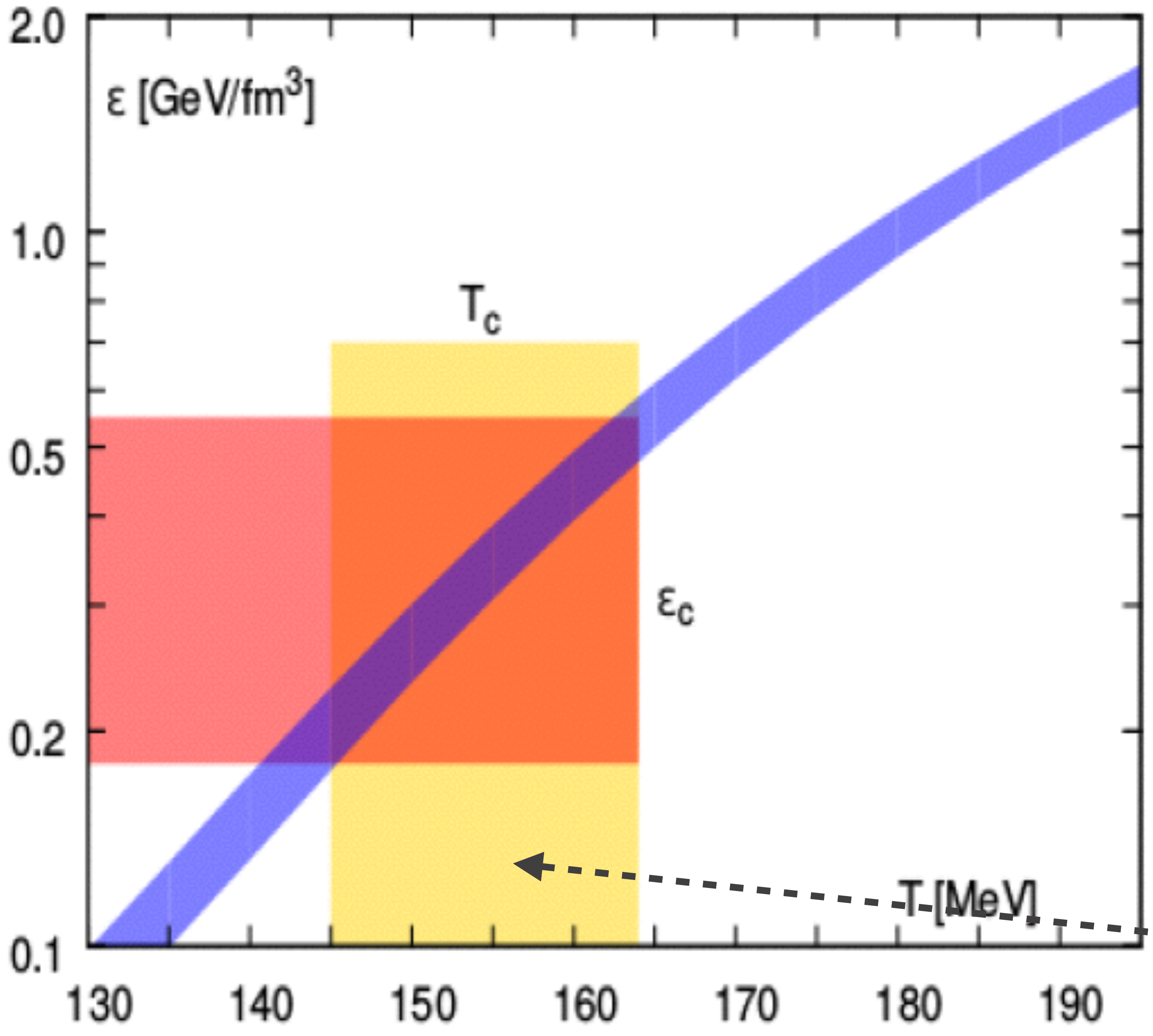
QCD EOS at $\mu_B = 0$

Results (true quark masses, continuum extrapolated) have converged; full agreement found between groups (HotQCD, Wuppertal-Budapest) using different quark actions.



(Pseudo-) Critical temperature

Transition between hadron gas and quark-gluon plasma is a **cross-over** at $\mu_B = 0$ and for small μ_B. Precise value of T_c depends on the quantity used to define it.



HotQCD: arXiv:1407.6387

Pseudo-critical temperature from chiral susceptibility peak:

T_c = 154 ± 9 MeV

critical energy density:

$\epsilon_c = 0.18 - 0.50 \text{ GeV/fm}^3$

$\epsilon_c = (1.2 - 3.3)\rho_{nuclear}$

Uncertainty in T_c, not width of cross-over region!

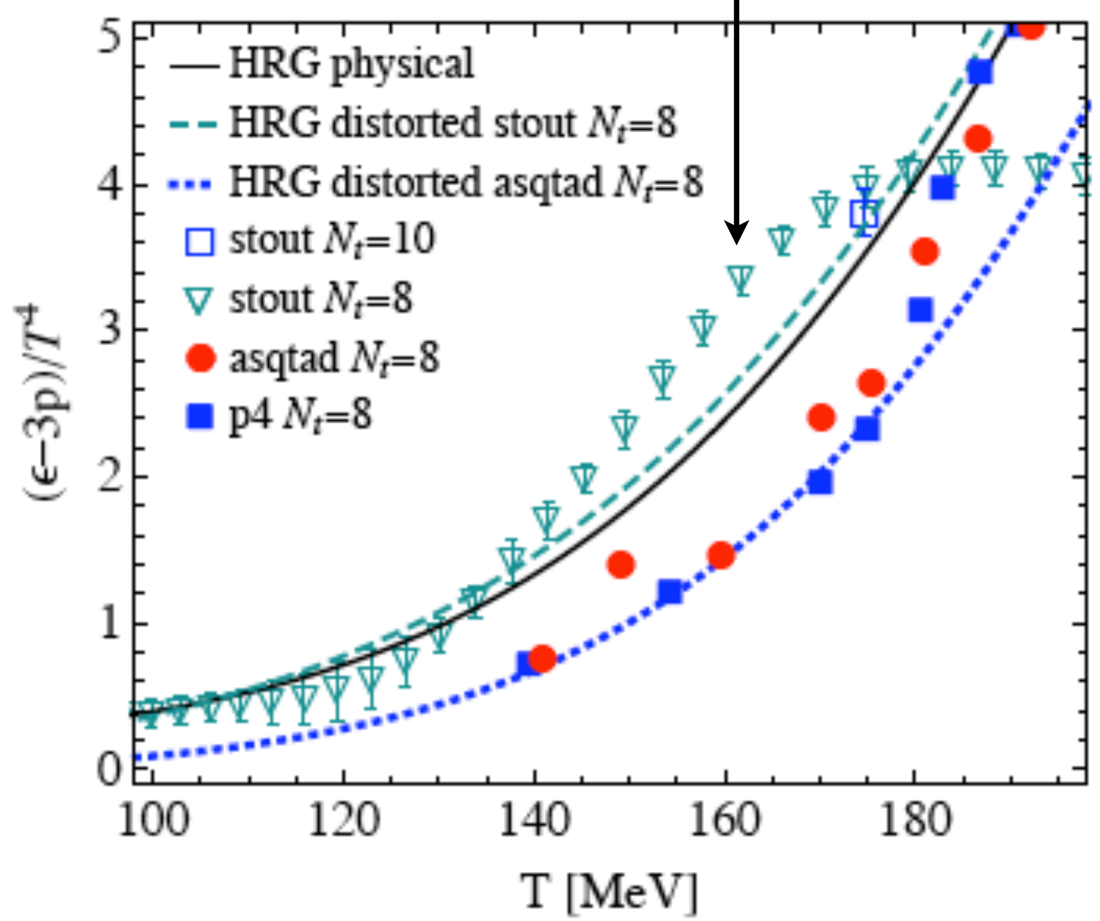
Hadron mass spectrum

Below T_c, the quantity $(\varepsilon - 3p)/T^4$ measures the level density of massive hadronic excitations of the QCD vacuum.

Hagedorn spectrum ($T_H \approx 180$ MeV):

$$\rho_H(m) = \frac{A\, e^{m/T_H}}{\left(m^2 + m_0^2\right)^{5/4}}$$

Lines: Hadron resonance gas using only PDG resonances
Data points: Lattice QCD
LQCD lies above HRG for T > 140 MeV
Indicates additional hadron resonances



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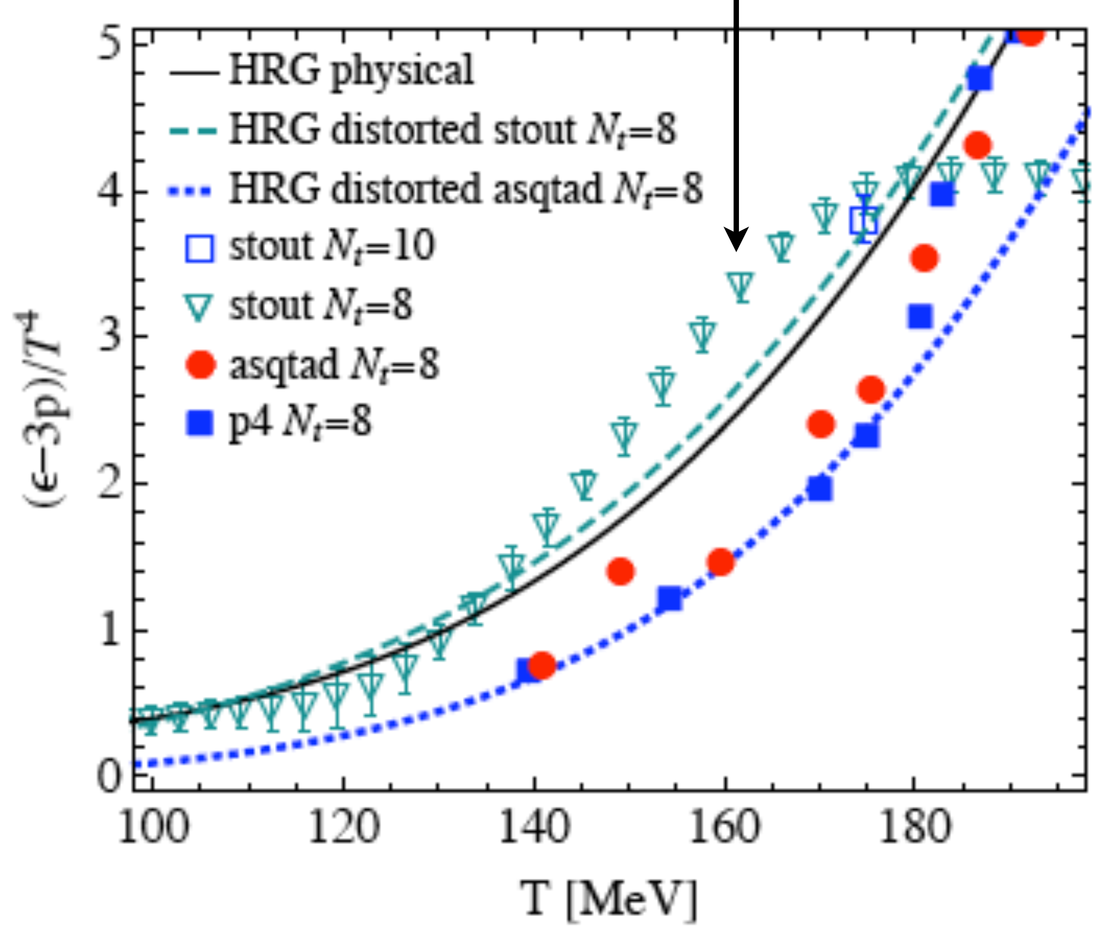
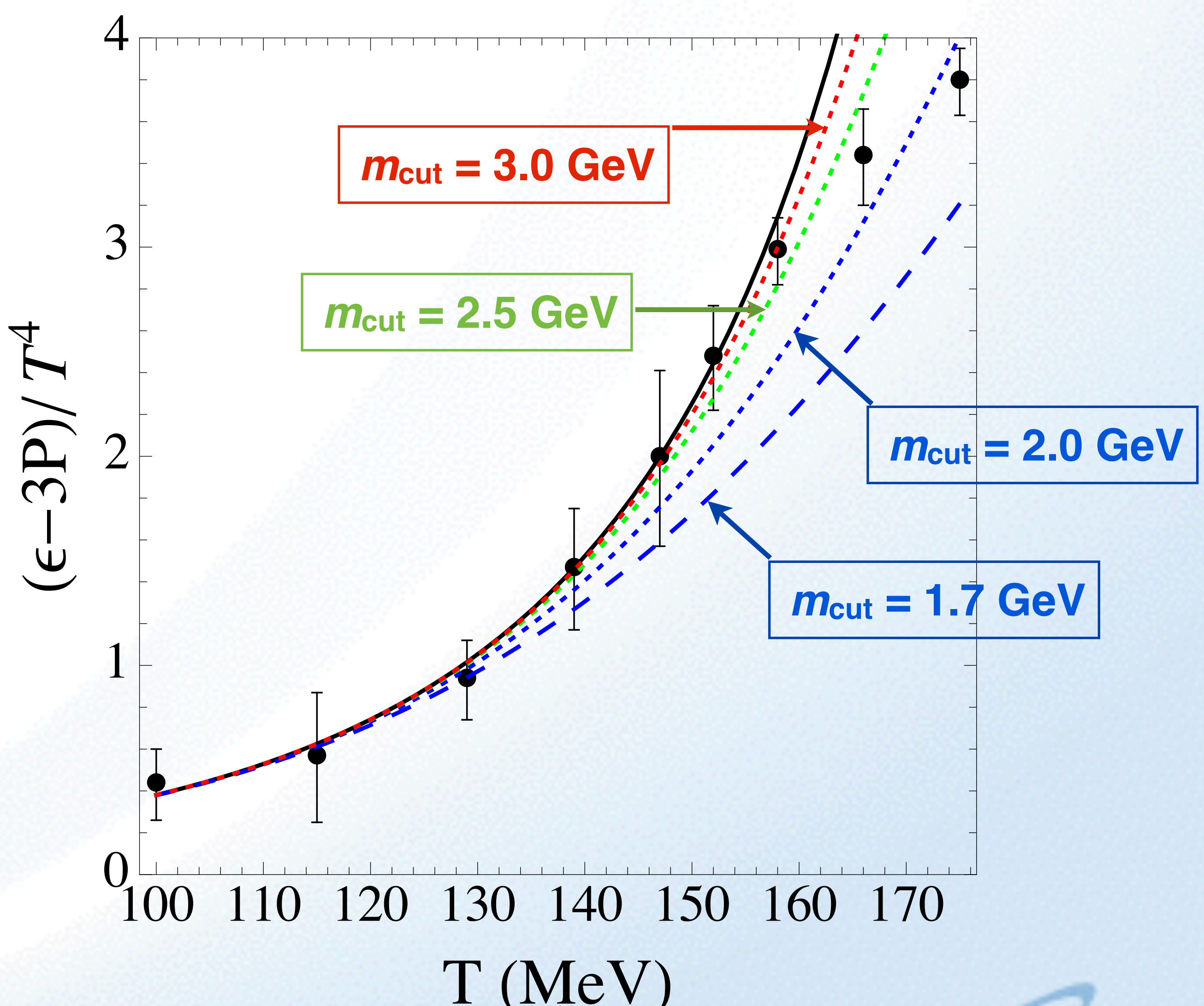


Hagedorn spectrum ($T_H \approx$ 180 MeV):

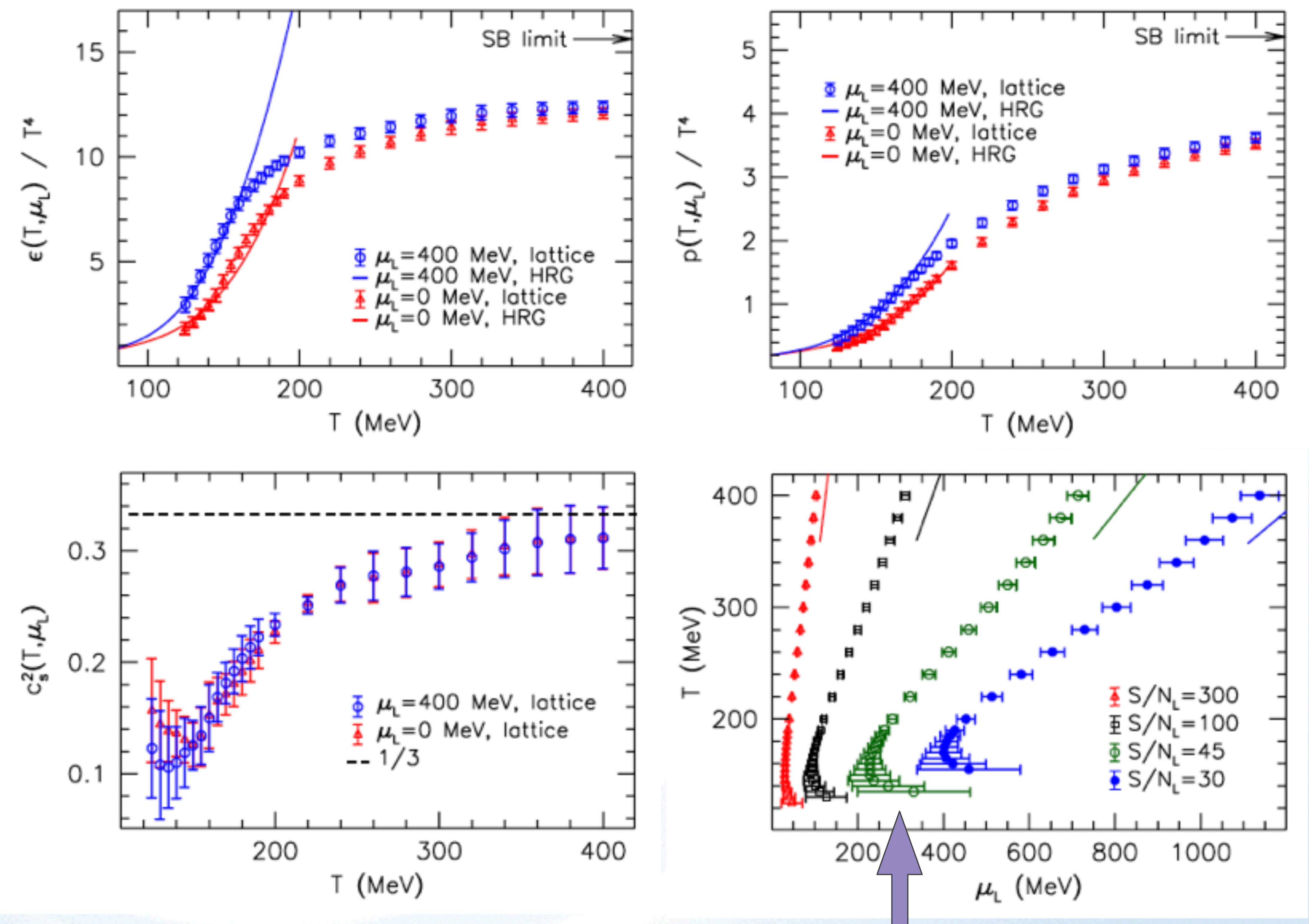
$$\rho_H(m) = \frac{A\, e^{m/T_H}}{\left(m^2 + m_0^2\right)^{5/4}}$$

In good agreement with lattice results
Hadrons up to 3 GeV mass contribute



QCD EOS at $\mu_B \neq 0$

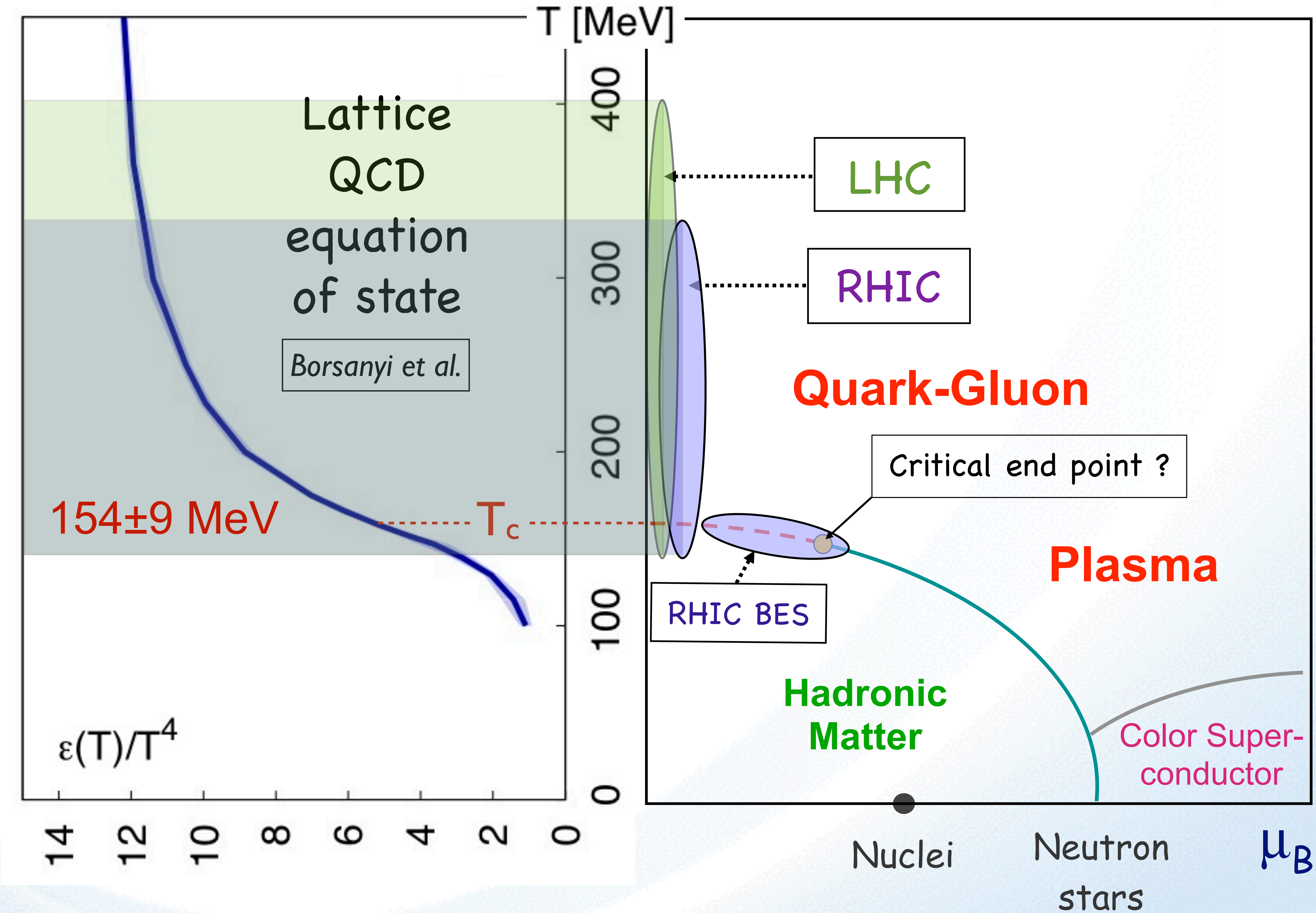
Borszanyi et al., arXiv:1204:6710



Approximate trajectories in QCD phase diagram

Probing the QCD Phase Boundary

QCD Phase Diagram



Thermodynamic fluctuations

Susceptibilities measure thermodynamic fluctuations.
Interesting because they exhibit singularities at a critical point.
Fluctuations of conserved quantities (charge *Q*, baryon number *B*,…) cannot be changed by local final-state processes.

Expt.: mean: M_Q
variance: σ_Q^2
skewness: S_Q
kurtosis: κ_Q

$$\boxed{\sqrt{s} \Leftrightarrow (T, \mu_B)}$$

Lattice gauge theory:

$$\chi_n^X(T, \mu_X) = \frac{\partial^n \left(p(T, \mu_X)/T^4\right)}{\partial (\mu_X/T)^n}$$

Ratios are independent of the (unknown) freeze-out volume:

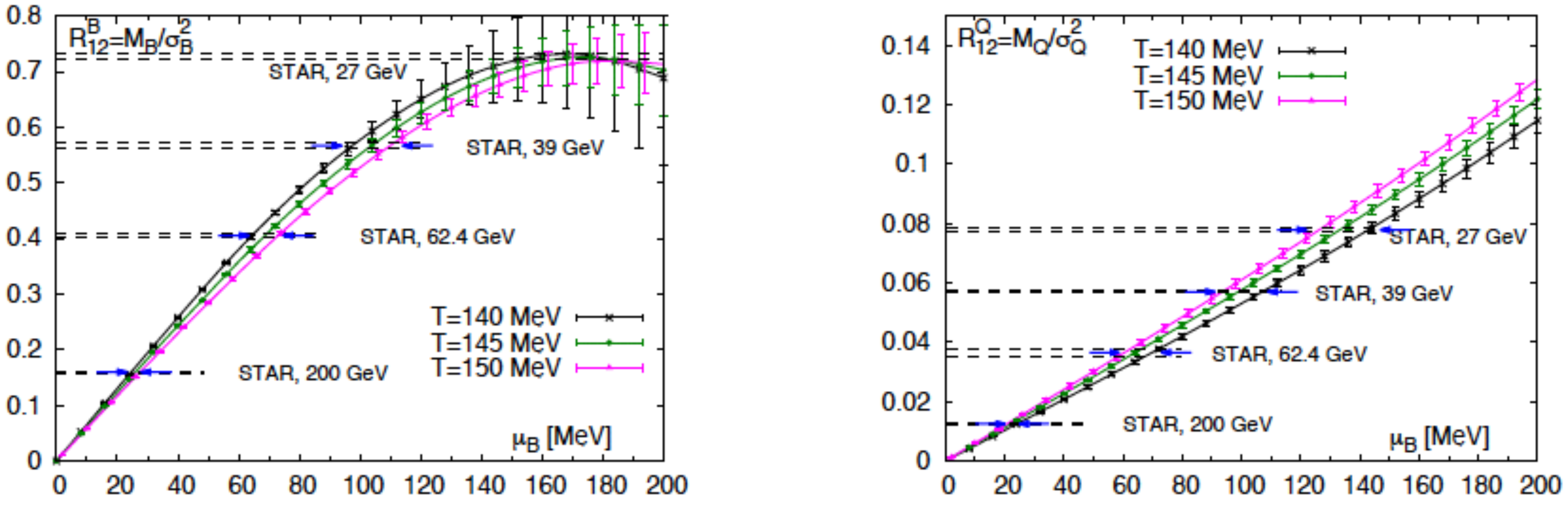
$$\frac{M_Q(\sqrt{s})}{\sigma_Q^2(\sqrt{s})} = \frac{\chi_1^Q(T, \mu_B)}{\chi_2^Q(T, \mu_B)} \qquad \frac{S_Q(\sqrt{s})\,\sigma_Q^3(\sqrt{s})}{M_Q(\sqrt{s})} = \frac{\chi_3^Q(T, \mu_B)}{\chi_1^Q(T, \mu_B)}$$

Chemical freeze-out

… from fluctuations of conserved quantum numbers (Q, B):

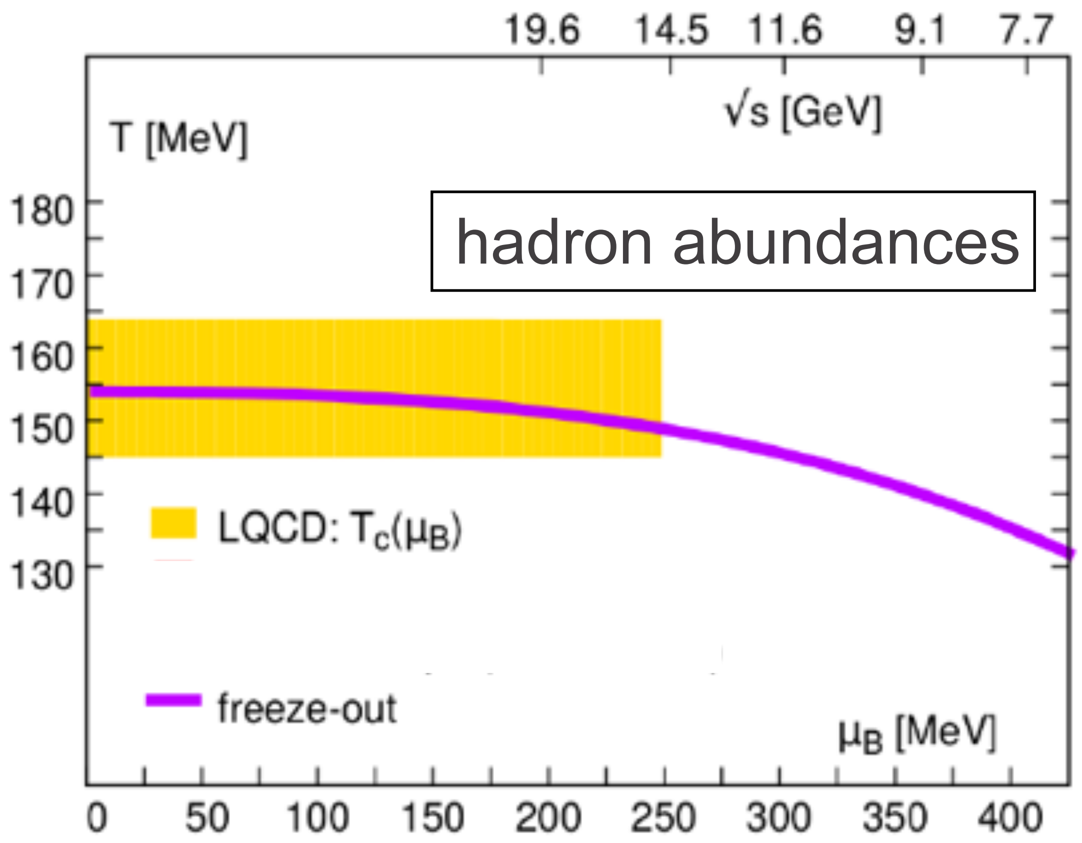
Borsanyi et al. Wuppertal-Budapest Coll. Phys.Rev.Lett. 111, 062005 (2013); Phys.Rev.Lett. 113, 052301 (2014)

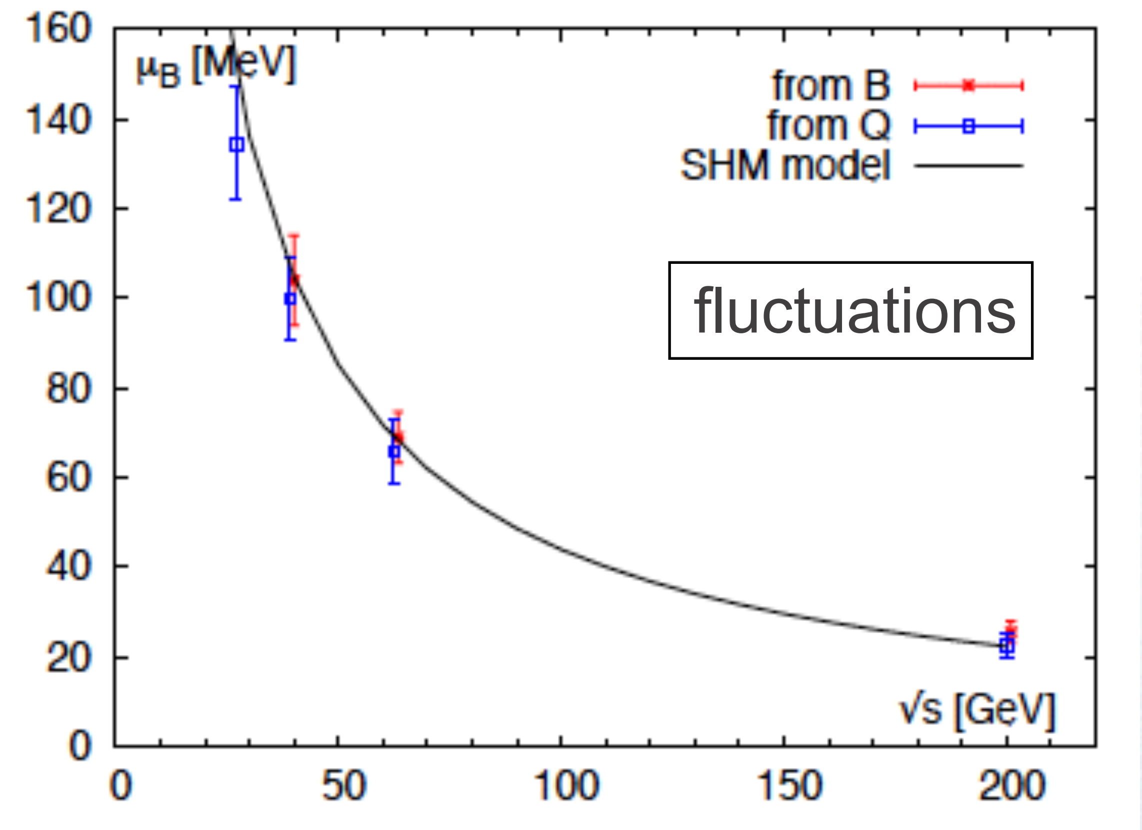
use M/σ^2 both in the baryon and in the charge sector



Compare lattice results with the STAR data for the fluctuation ratios in the temperature range 140−150 MeV permits to read off μ_B.
Both methods are consistent with each other and with the measured baryon/antibaryon ratios, if additional strange baryon states beyond those in the PDG tables (e.g. in the quark model) are accounted for.

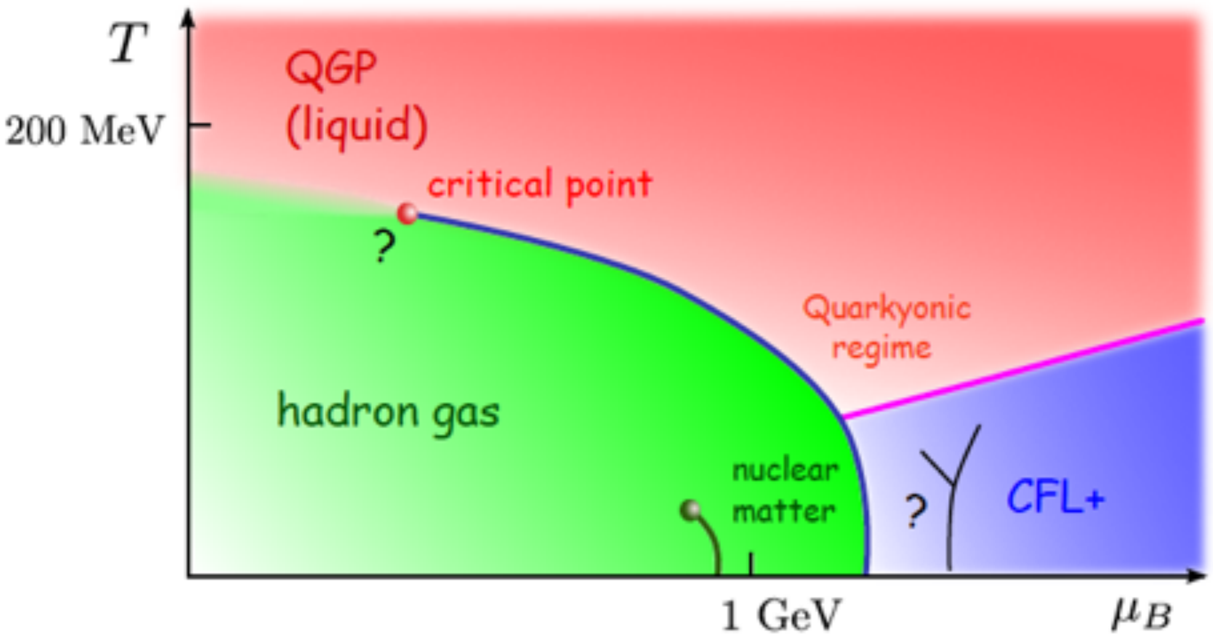
Chemical freeze-out



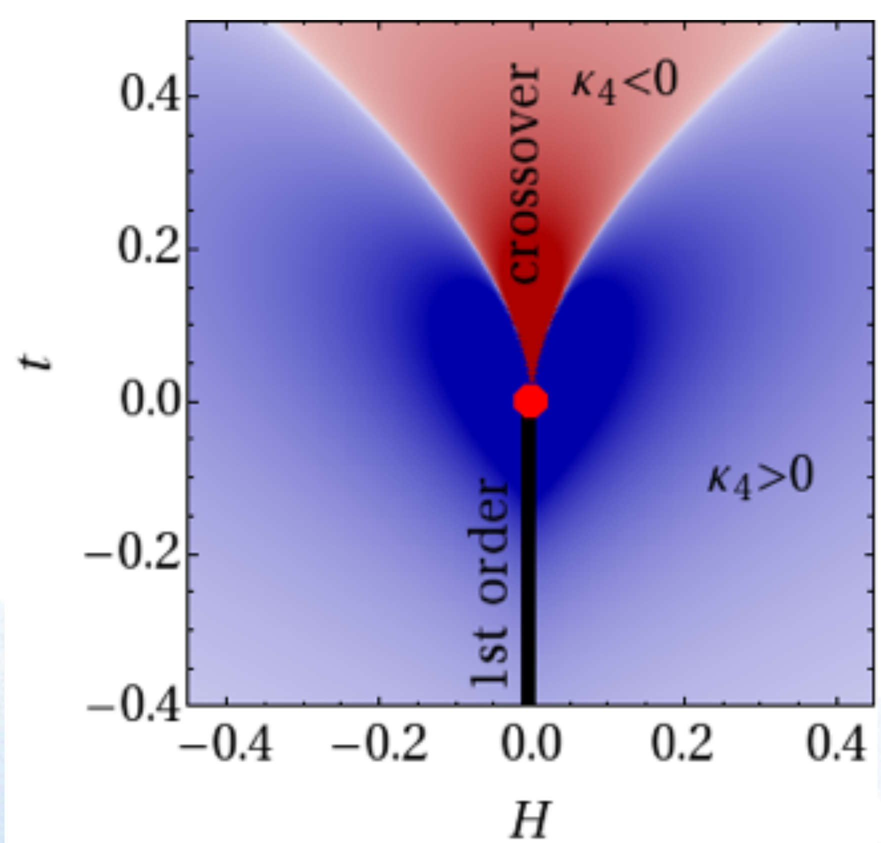


Consistency of freeze-out parameters from mean hadron abundances and from fluctuations (*Q*, *B*) opens the door to search for a critical point in the QCD phase diagram by looking for enhanced critical fluctuations as function of beam energy.

Toward critical fluctuations



Model independent structure of kurtosis



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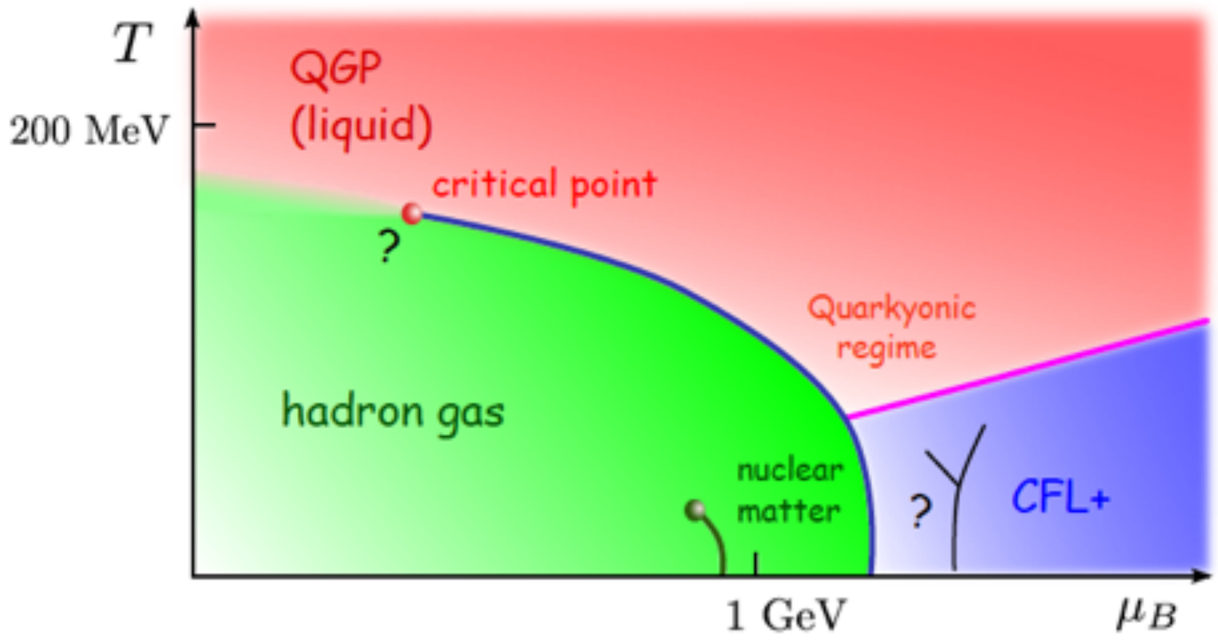

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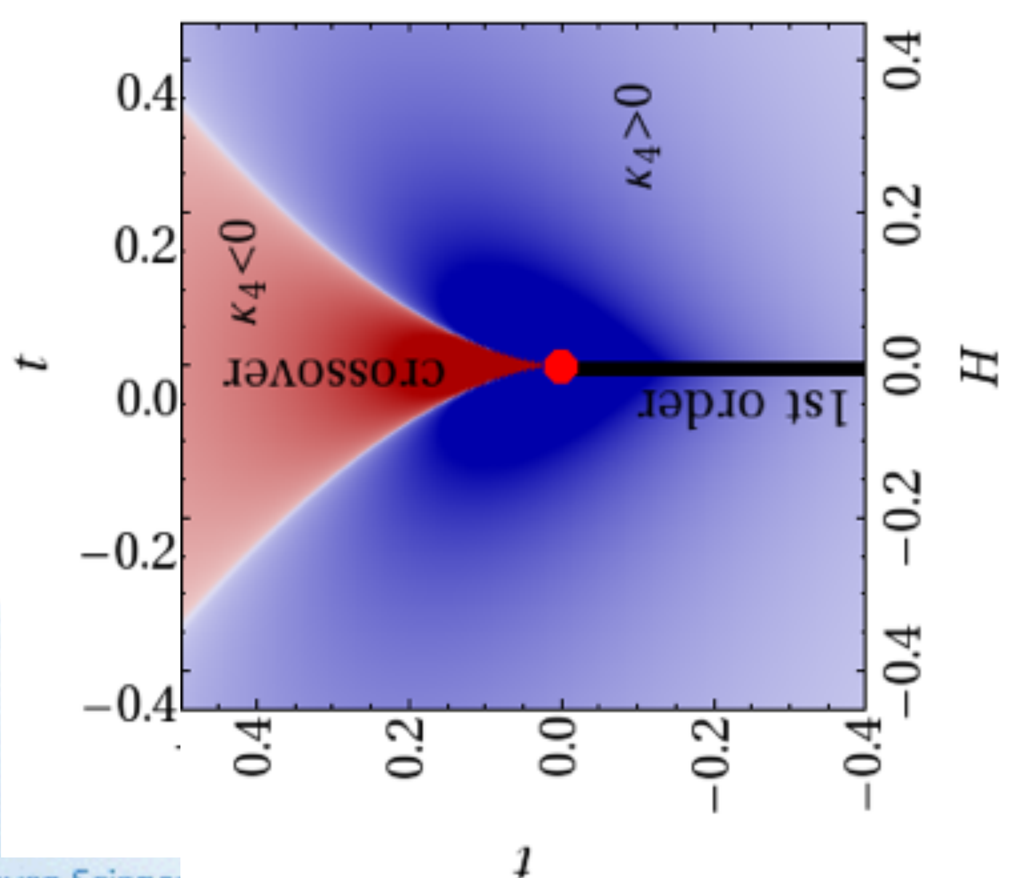

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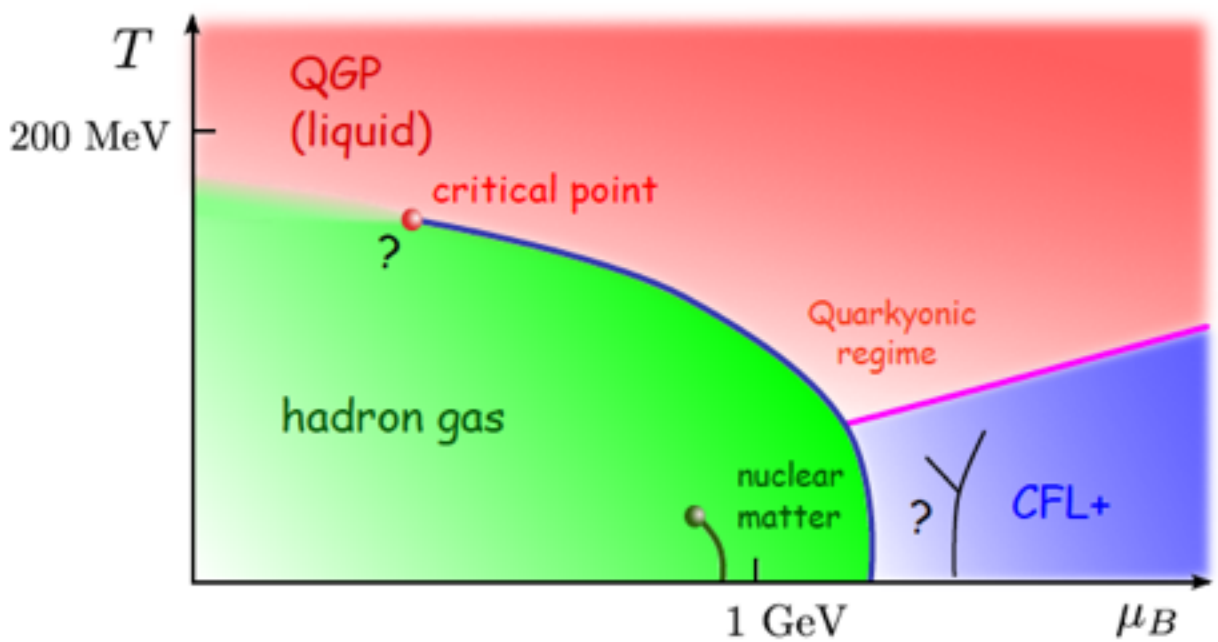

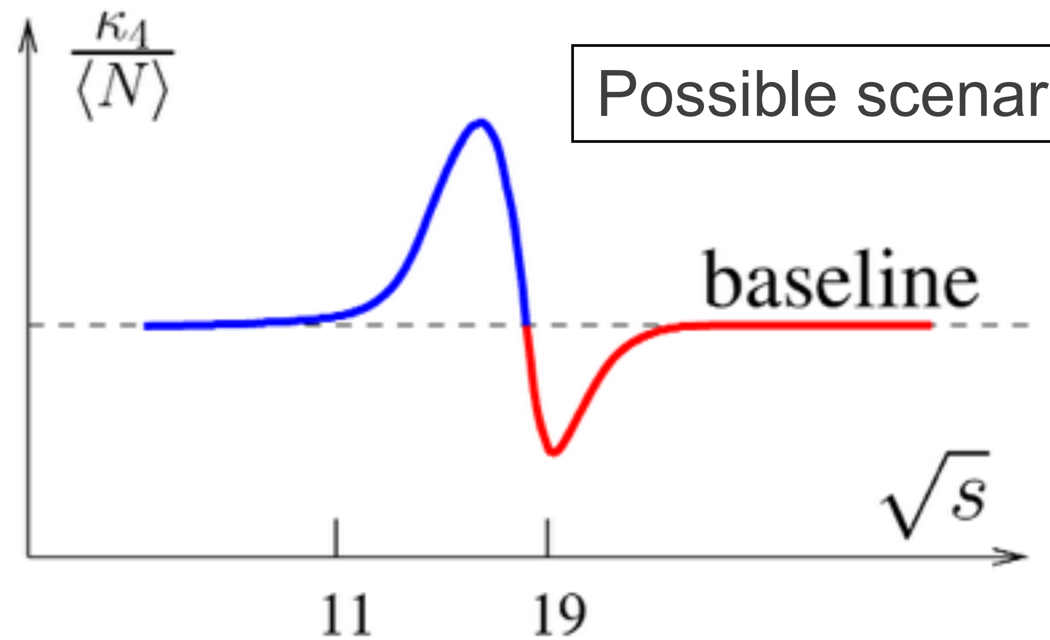

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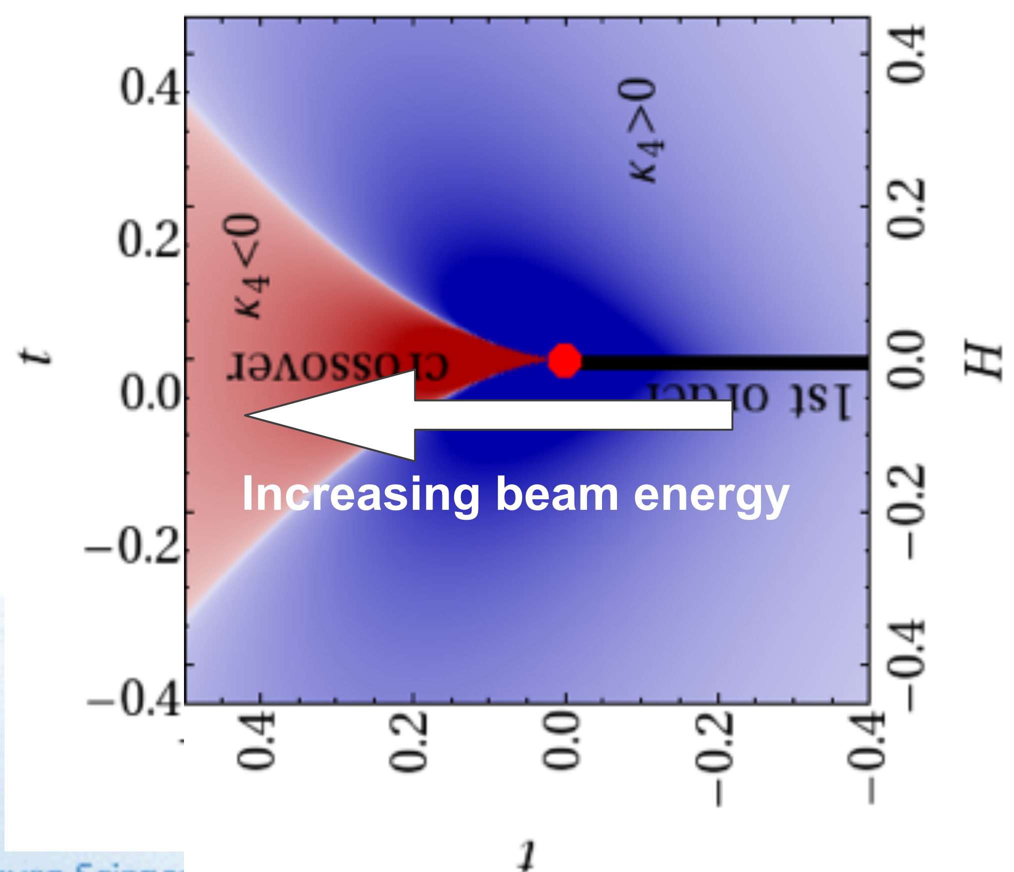

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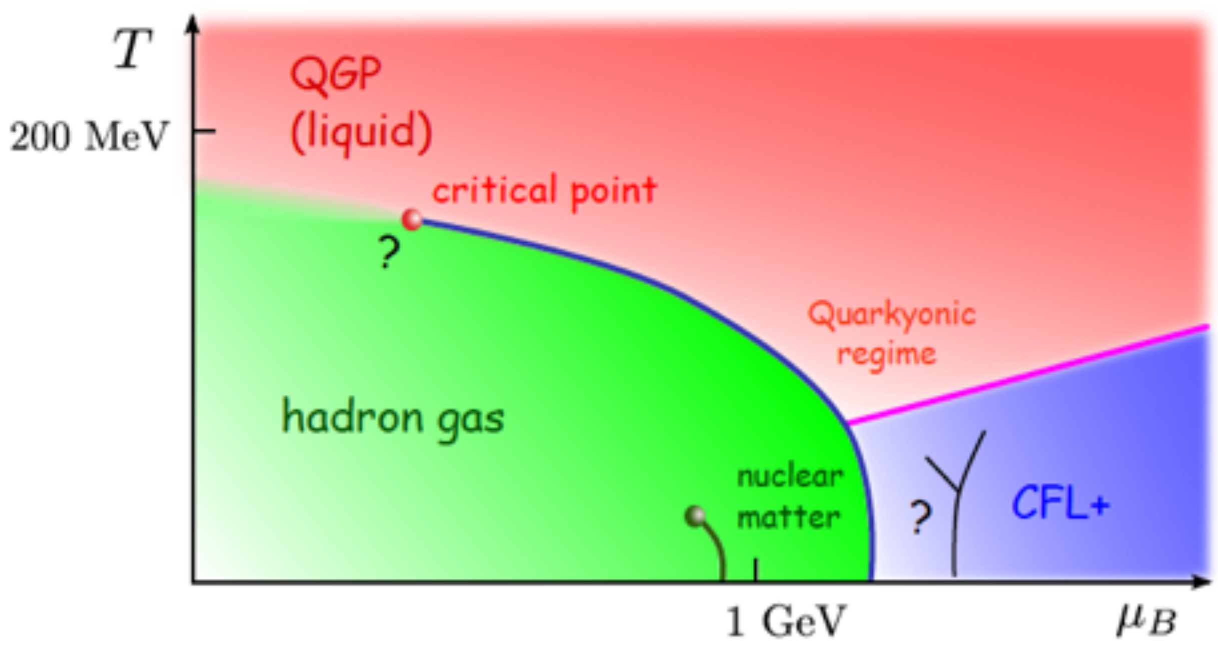

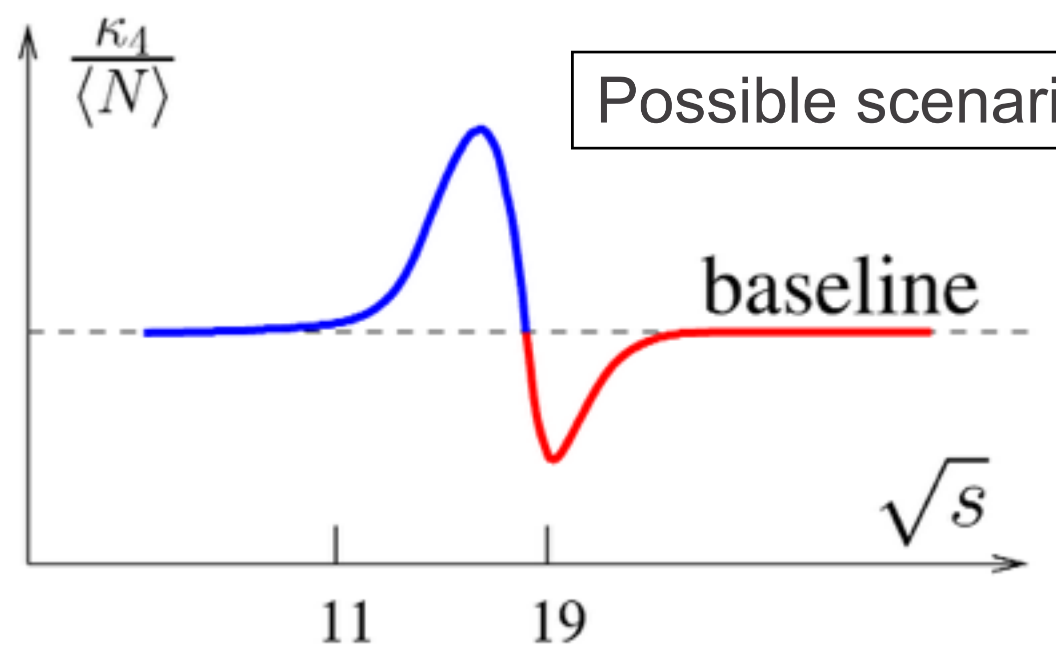

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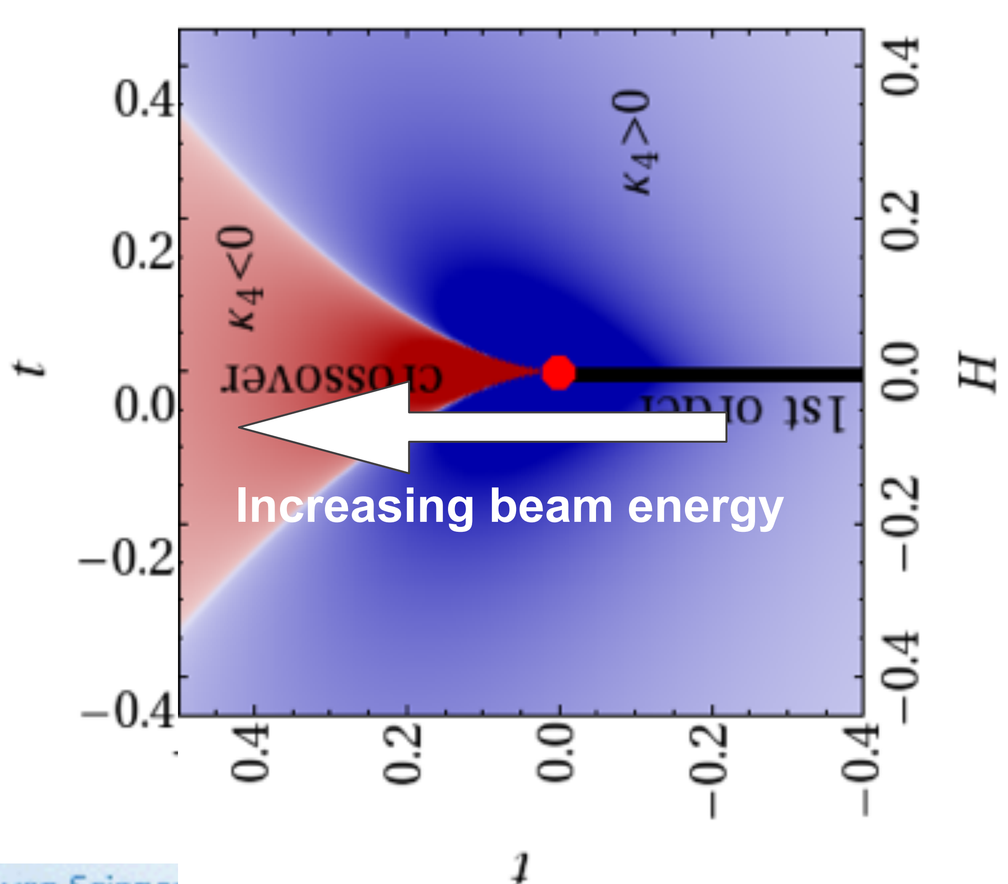

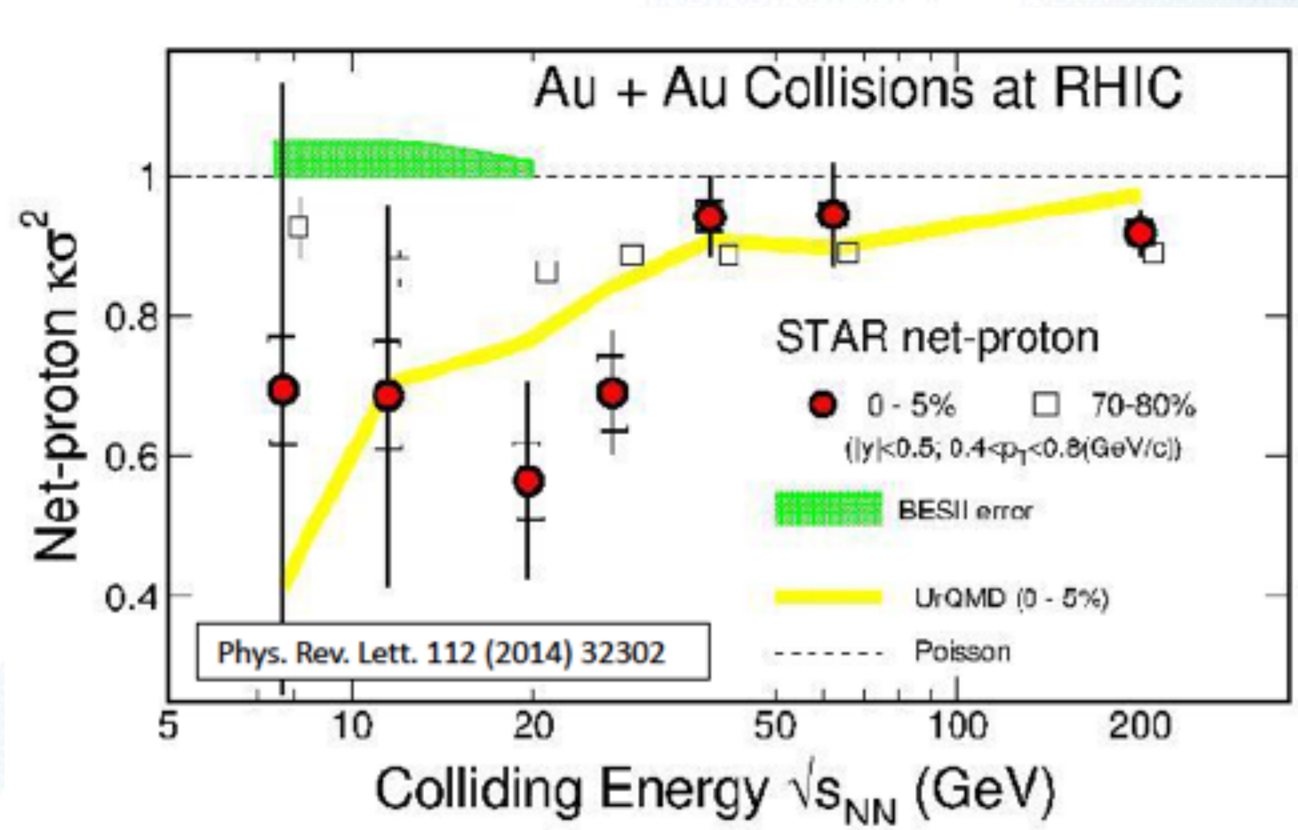

Probing the Quark-Gluon Plasma

Hot QCD matter properties

Which **properties of hot QCD matter** can we hope to determine and how ?

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$$\eta = \frac{1}{T}\int d^4x \left\langle T_{xy}(x) T_{xy}(0) \right\rangle$$

Shear viscosity: anisotropic collective flow

Very Hard for LQCD

$$\hat{q} = \frac{4\pi^2 \alpha_s C_R}{N_c^2 - 1} \int dy^- \left\langle U^\dagger F^{a+i}(y^-) U F_i^{a+}(0) \right\rangle$$

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Momentum/energy diffusion:
parton energy loss, jet fragmentation

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QGP Radiance: Lepton pairs, photons

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$$m_D = -\lim_{|x|\to\infty} \frac{1}{|x|} \ln \left\langle U^\dagger E^a(x) U E^a(0) \right\rangle$$

Color screening: Quarkonium states

The “perfect” fluid

Viscous hydrodynamics

Hydrodynamics = effective theory of energy and momentum conservation

energy-momentum tensor = **ideal fluid** + **dissipation**

$$\partial_\mu T^{\mu\nu} = 0 \quad \text{with} \qquad T^{\mu\nu} = (\varepsilon + P)u^\mu u^\nu - P g^{\mu\nu} + \Pi^{\mu\nu}$$

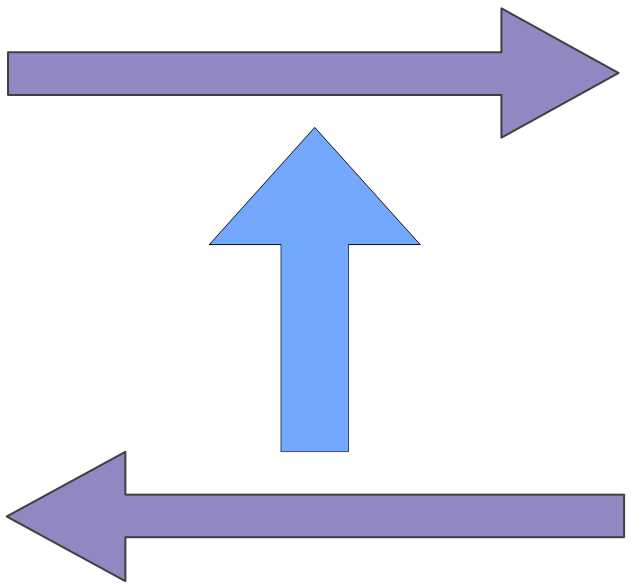
$$\tau_\Pi \left[\frac{d\Pi^{\mu\nu}}{d\tau} + \left(u^\mu \Pi^{\nu\lambda} + u^\nu \Pi^{\mu\lambda} \right) \frac{du^\lambda}{d\tau} \right] = \eta \left(\partial^\mu u^\nu + \partial^\nu u^\mu - \text{trace} \right) - \Pi^{\mu\nu}$$

Input: Equation of state P(ε), shear viscosity, initial conditions ε(x,0), u^μ(x,0)

Shear viscosity **η** is normalized by density: **kinematic viscosity η/ρ**.

Relativistically, the appropriate normalization factor is the **entropy density** s = (ε+P)/T, because the particle density is not conserved: **η/s**.

Shear viscosity



Shear viscosity describes a material's ability to transport momentum across flow gradients! Kinetic theory:

$$\eta \approx \frac{1}{3} n \bar{p} \lambda_f \qquad \lambda_f = \frac{1}{n\sigma} \quad \rightarrow \quad \eta \approx \frac{\bar{p}}{3\sigma}$$

$$\sigma \leq \frac{4\pi}{\bar{p}^2} \quad \rightarrow \quad \eta \geq \frac{\bar{p}^3}{12\pi}$$

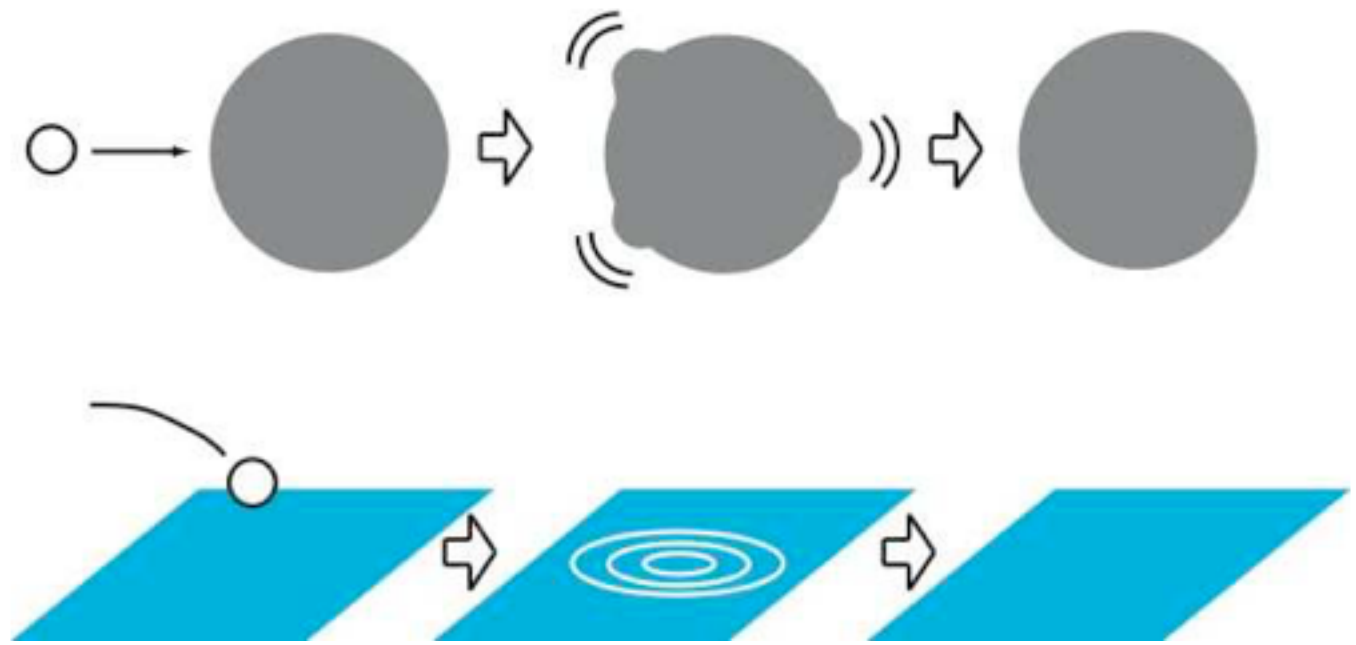
For relativistic system of massless particles: $\bar{p}^3 \sim T^3 \sim s$

Lower shear viscosity bound $\eta / s \geq O[1]$

Should be attained at **strong coupling** unless structure reorganizes via polymerization, solidification, etc. (e.g. hadronization).

The Black Hole connection

Dynamics of hot QCD matter can be mathematically (holography) mapped onto black hole dynamics in 4+1 dimensions (AdS$_5$ space).



BH swallowing matter

⇩

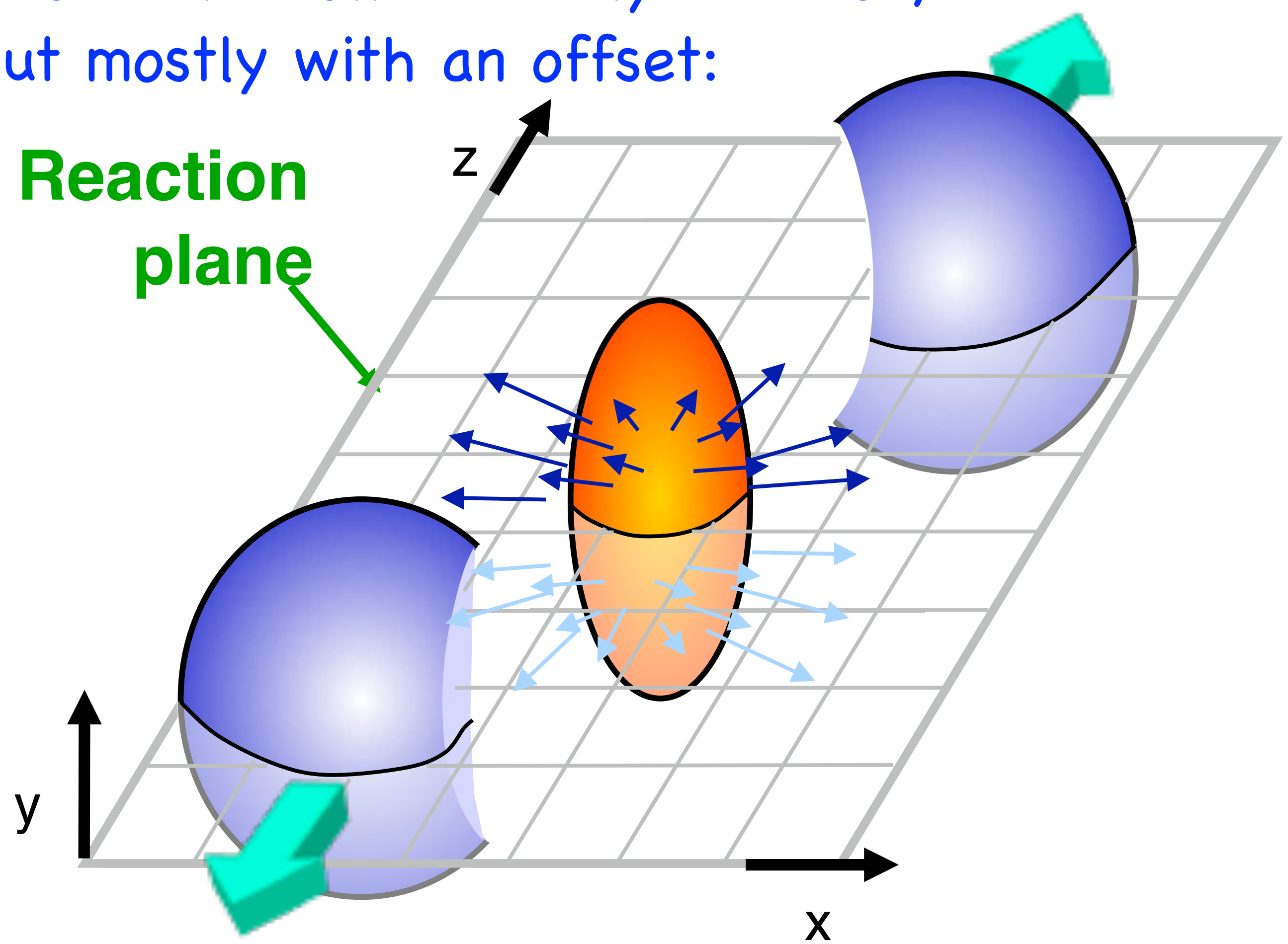
Perfect fluid hydrodynamics

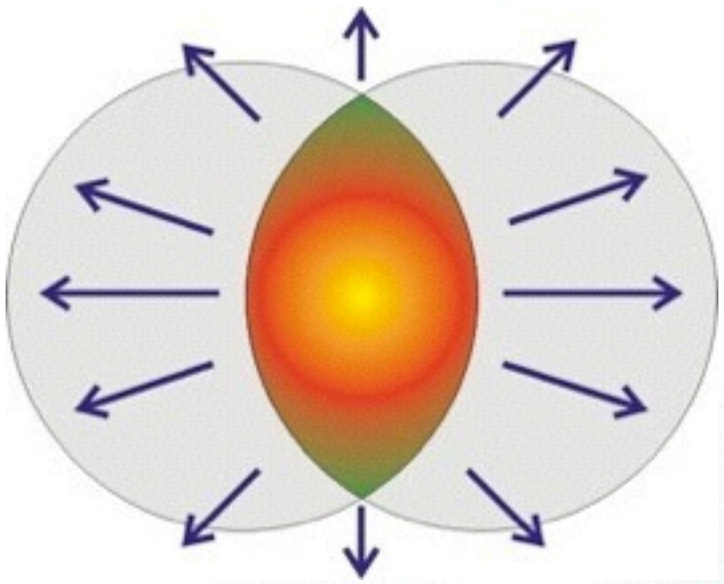
Formation of hot QCD matter at RHIC is similar to formation of a black hole, tied to information loss. Relies on the notion that ‘t Hooft coupling $g^2 N_c \sim 12$ is large enough to apply the classical limit of the dual theory:

$$\boxed{\eta / s \geq 1/4\pi}$$

Elliptic flow

- two nuclei collide rarely head-on, but mostly with an offset:





only matter in the overlap area gets compressed and heated

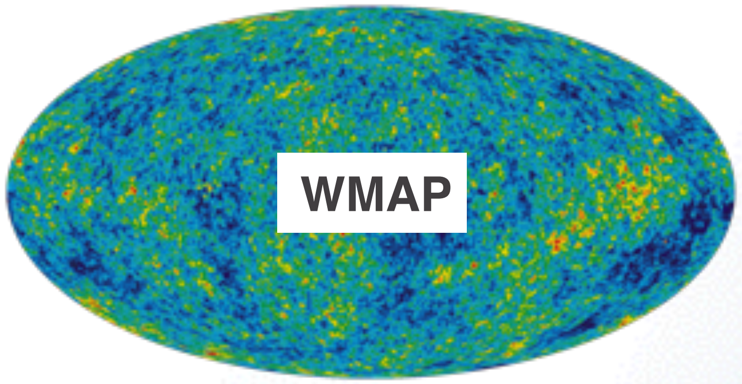
$$2\pi \frac{dN}{d\phi} = N_0 \left(1 + 2\sum_n \mathrm{v}_n(p_T,\eta) \cos n\left(\phi - \psi_n(p_T,\eta)\right)\right)$$

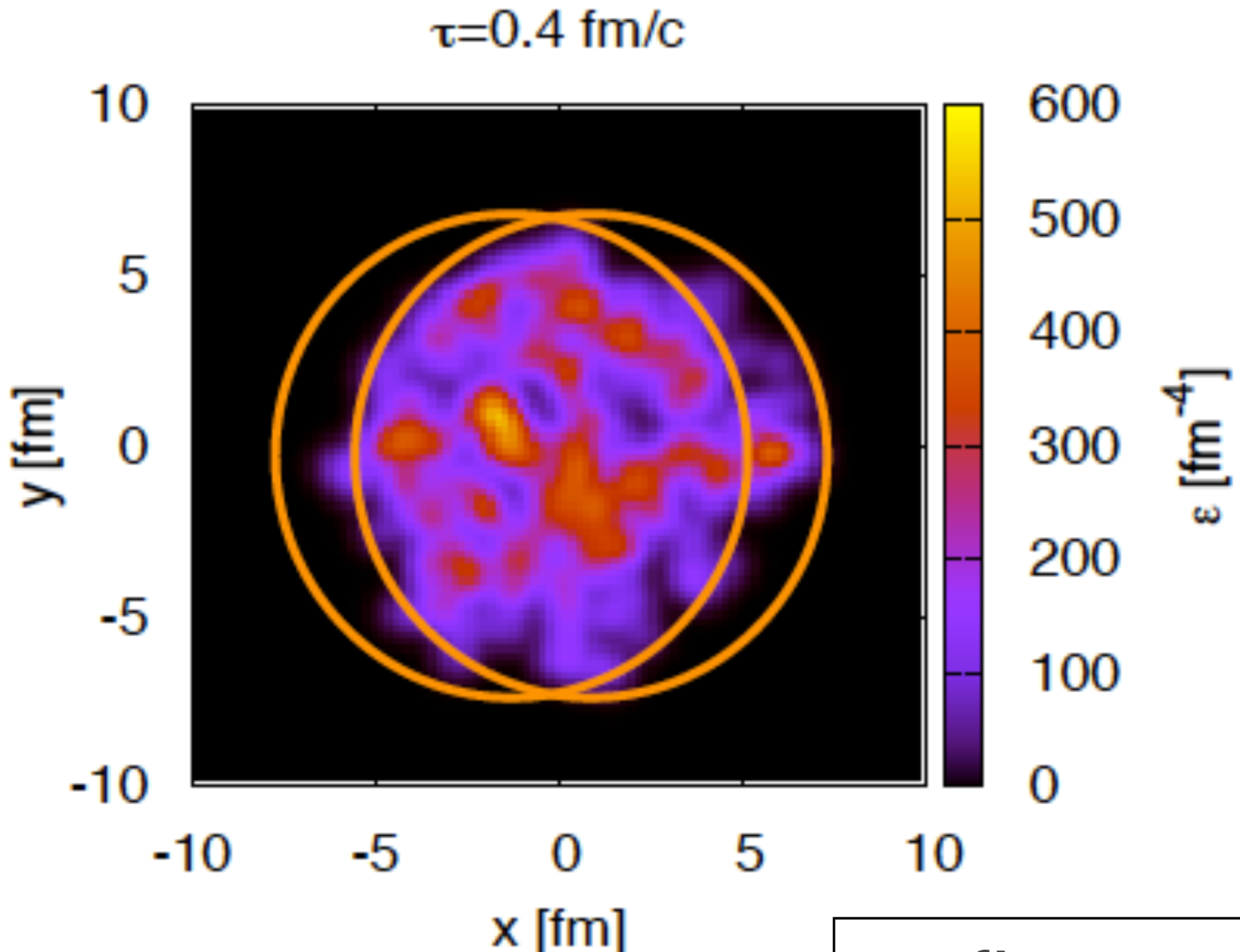
anisotropic flow coefficients

event plane angle

Event-by-event fluctuations

Initial state generated in A+A collision is grainy
event plane ≠ reaction plane
⇒ eccentricities ε_1, ε_2, ε_3, ε_4, etc. ≠ 0





Idea: Energy density fluctuations in transverse plane from initial state quantum fluctuations. These thermalize to different temperatures locally and then propagate hydrodynamically to generate angular flow velocity fluctuations in the final state.

⇒ flows v_1, v_2, v_3, v_4,…

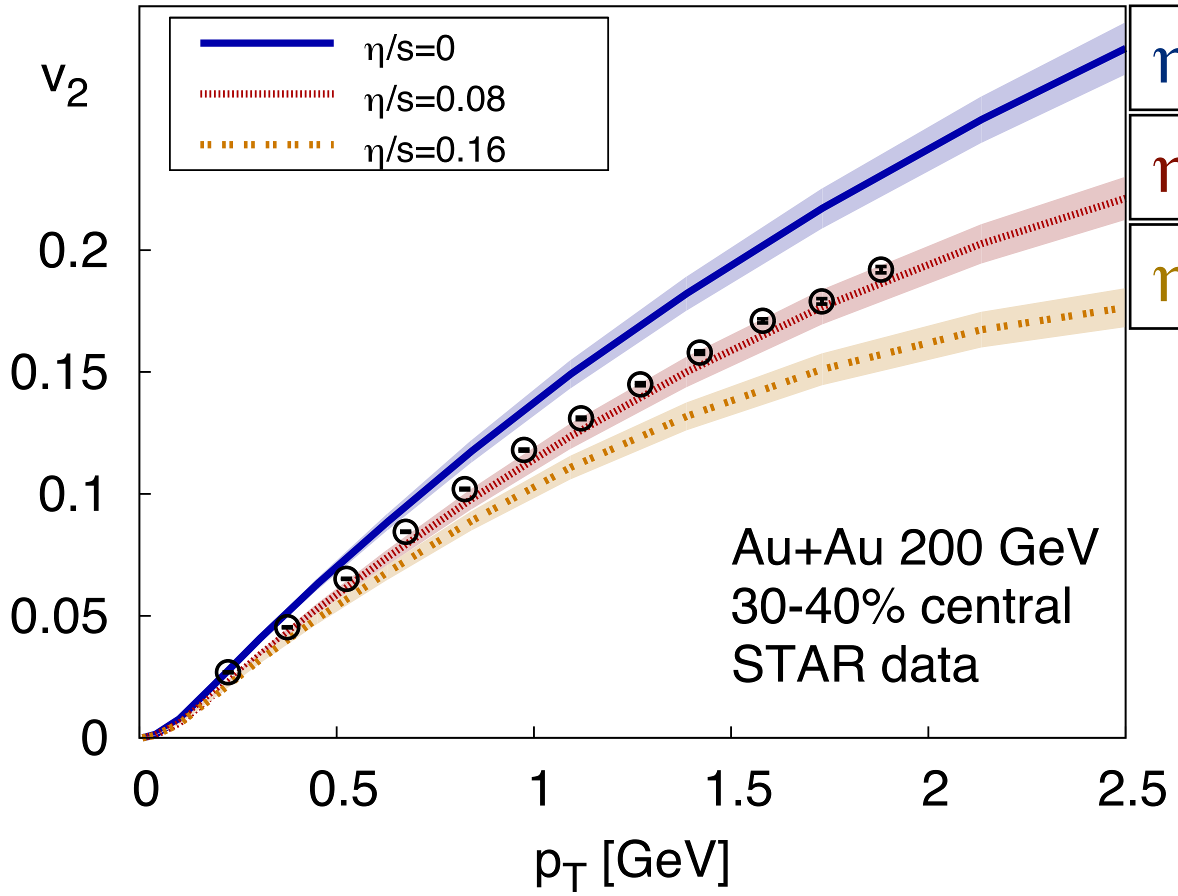
Elliptic flow "measures" η_{QGP}

Universal strong coupling limit of non-abelian gauge theories with a gravity dual:

$\eta/s \to 1/4\pi$

aka: the "perfect" liquid

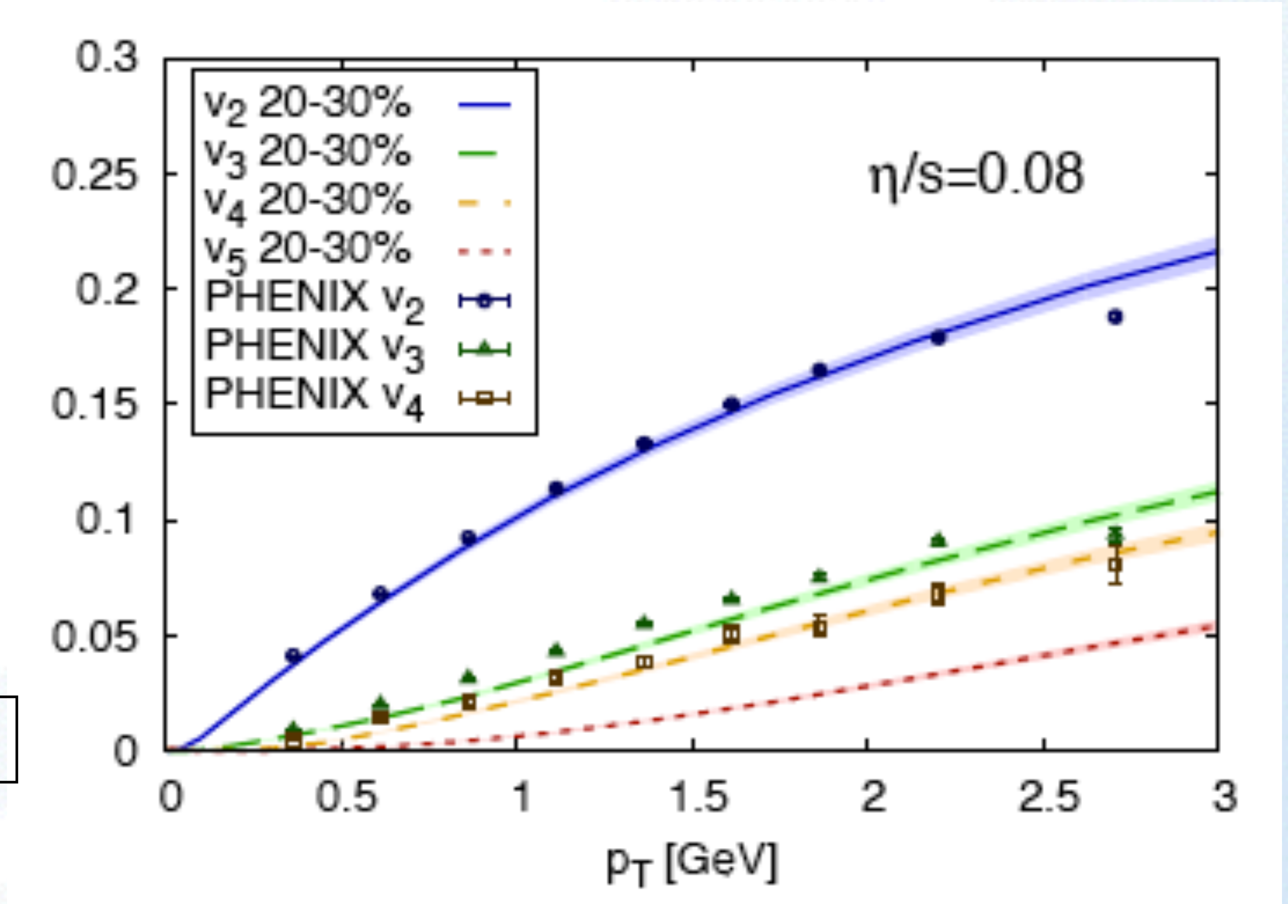
Schenke, Jeon, Gale, PRL 106 (2011) 042301



$\eta/s = 0$

$\eta/s = 1/4\pi$

$\eta/s = 2/4\pi$

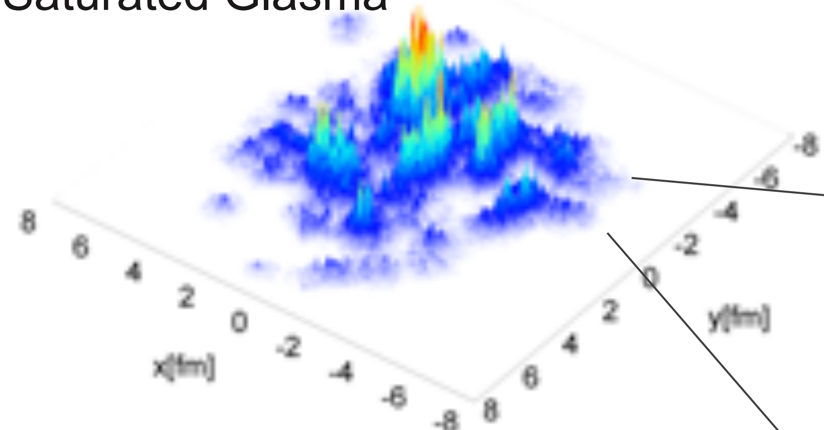


Schenke, Jeon, Gale, PRC 85 (2012) 024901

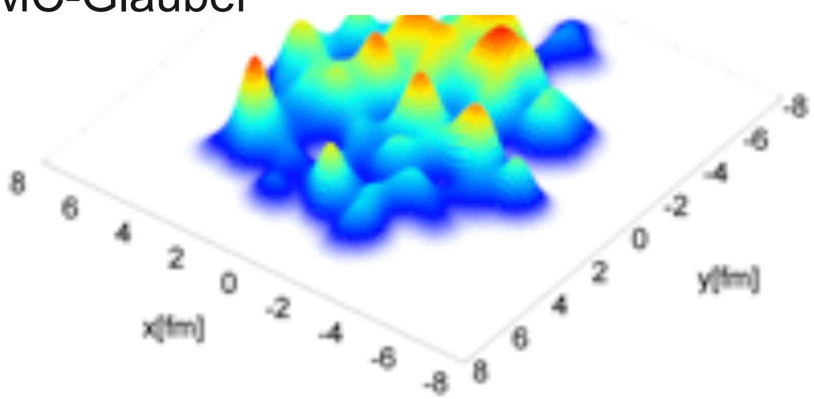
RHIC vs. LHC

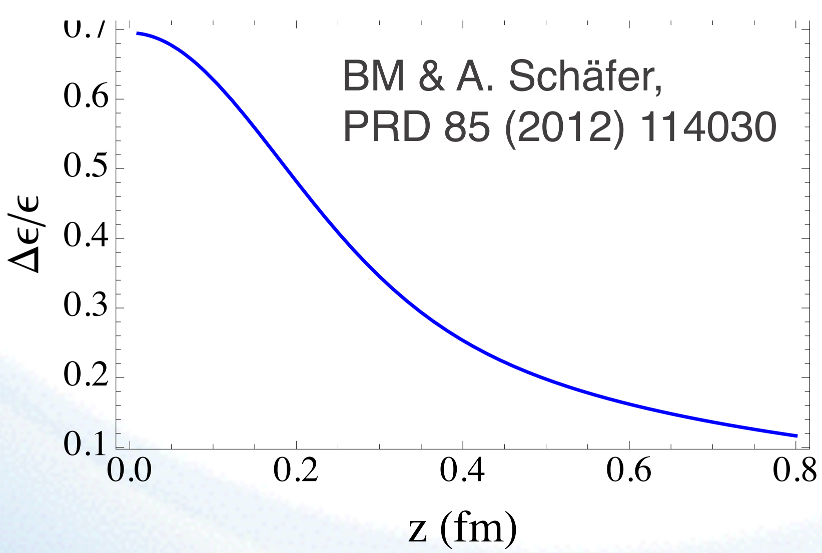
Gale, Jeon, Schenke, Tribedy, Venugopalan, arXiv:1209.6330

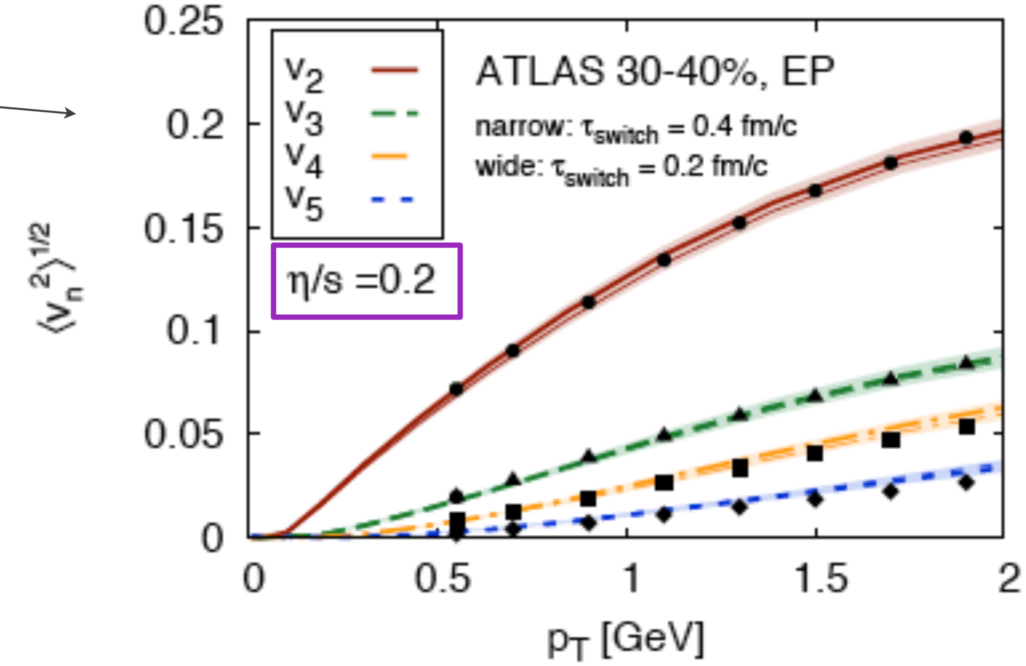
Saturated Glasma



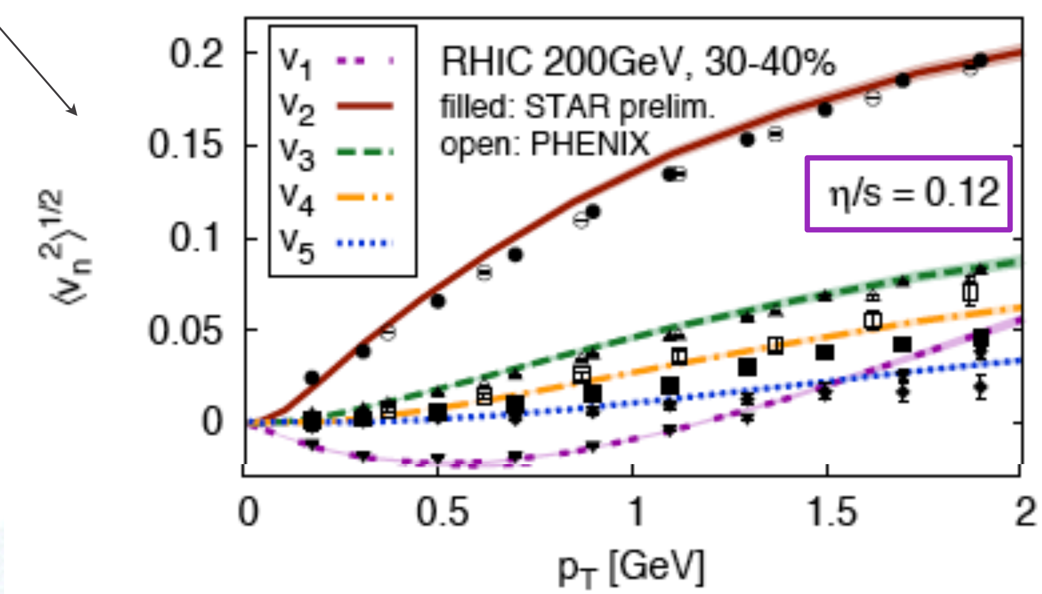
MC-Glauber







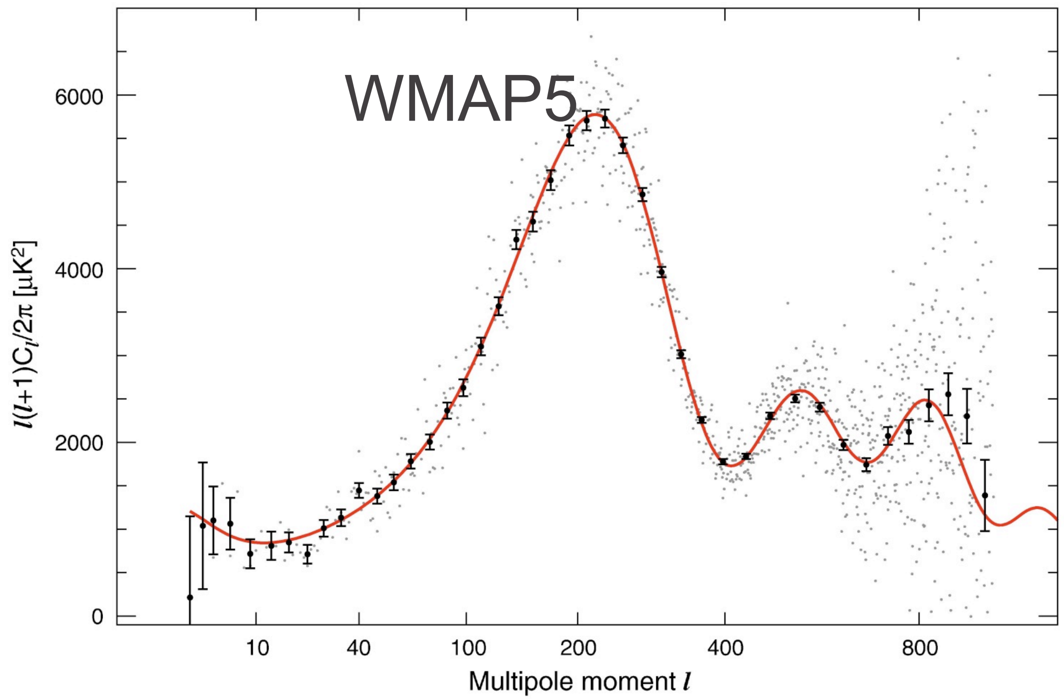
LHC



RHIC

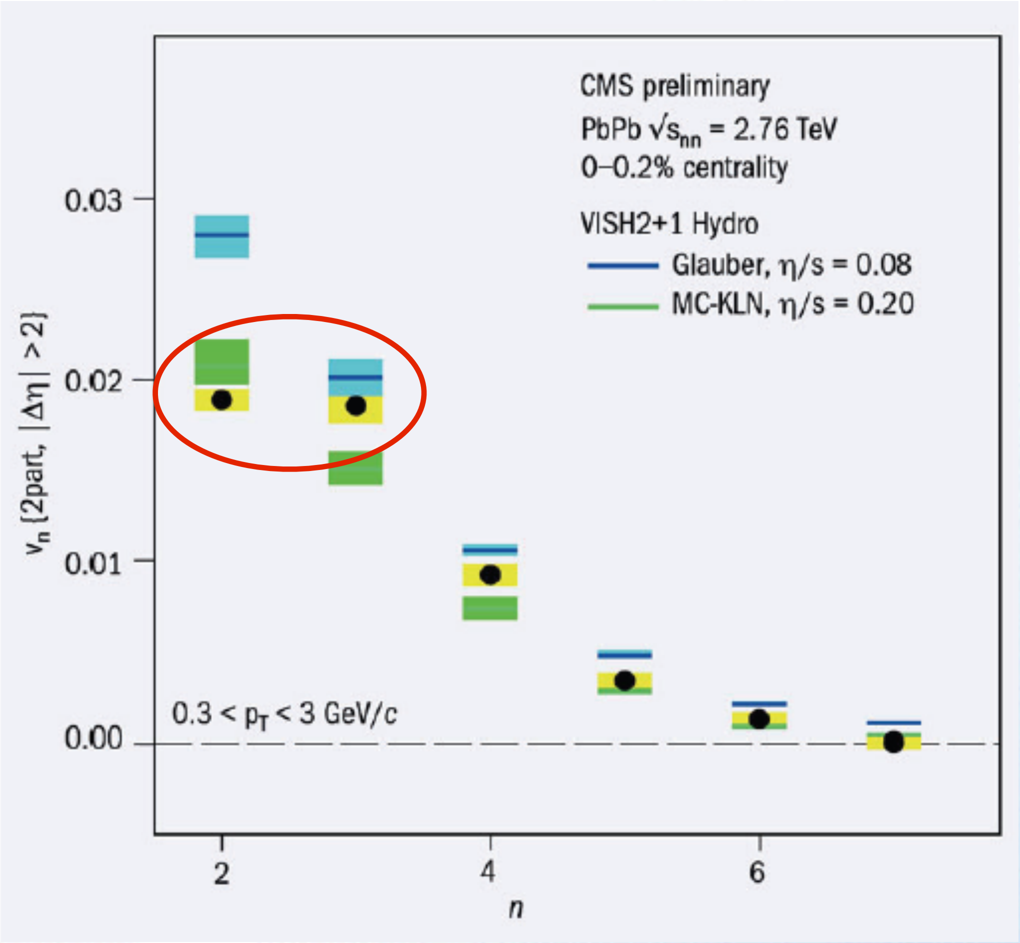
Fluctuation spectrum

Can the power spectrum of v_n be used to determine η/s and v_{sound} ?



The RHIC/LHC advantage:
There are many knobs to turn, not just a single universe to observe.

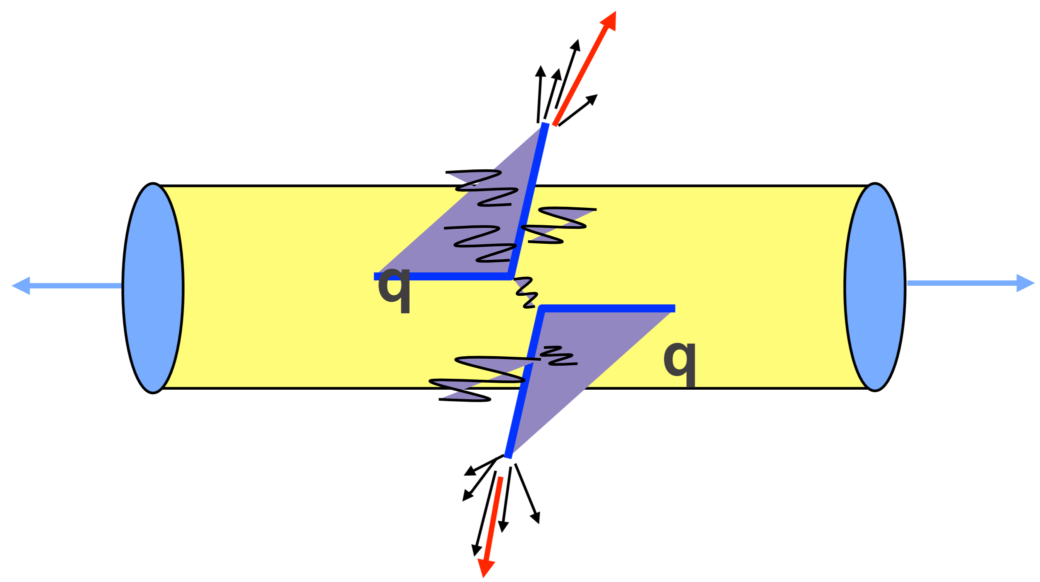
Power spectrum in ultracentral Pb+Pb collisions
Data: CMS. Theory: U. Heinz, arXiv:1304.3634



Data (v_3/v_2) indicate more fluctuations relative to global geometric effects than predicted by nucleon-scale granularity of initial state.

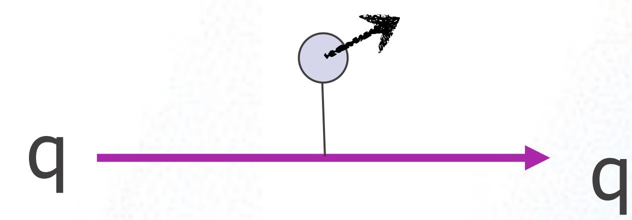
Color opacity

Parton energy loss

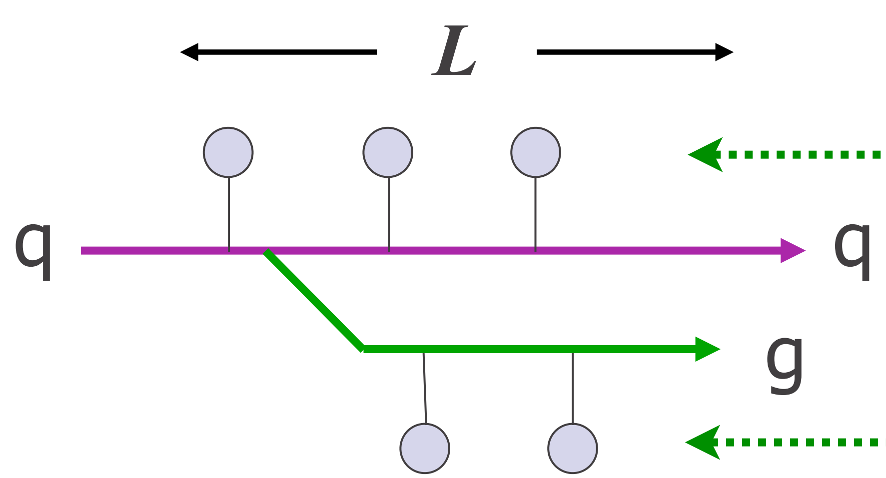


Elastic energy loss:

$$\frac{dE}{dx} = -C_2\,\hat{e}$$



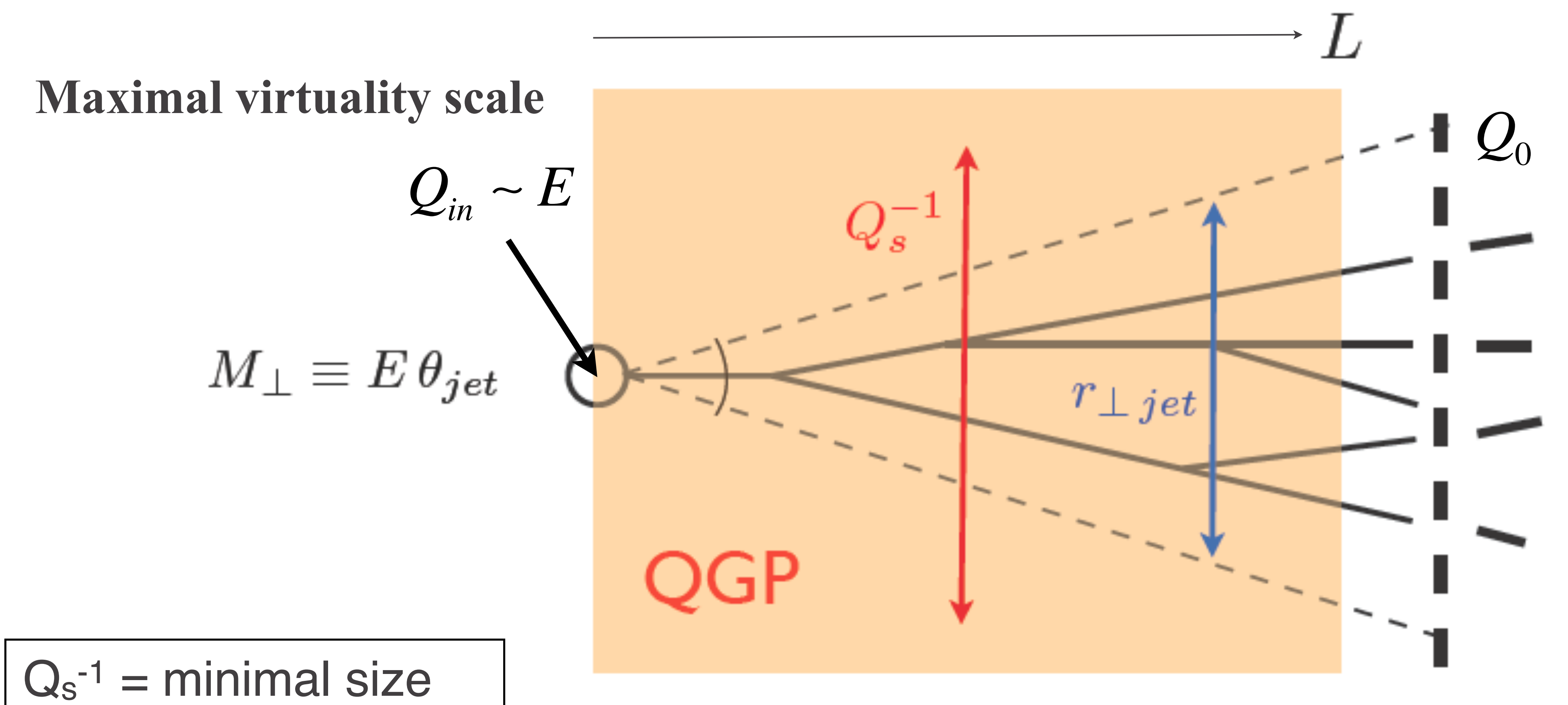
Radiative energy loss:



Scattering centers
$\Leftrightarrow$ color charges

$$\frac{dE}{dx} = -C_2\,\hat{q}\,L \qquad \hat{q} = \rho\int q^2\,dq^2\,\frac{d\sigma}{dq^2} = \int dx^-\left\langle F_i^{+}(x^-)F^{+i}(0)\right\rangle$$

Jet scales in the medium



Q_s^{-1} = minimal size of probe to which the medium look opaque

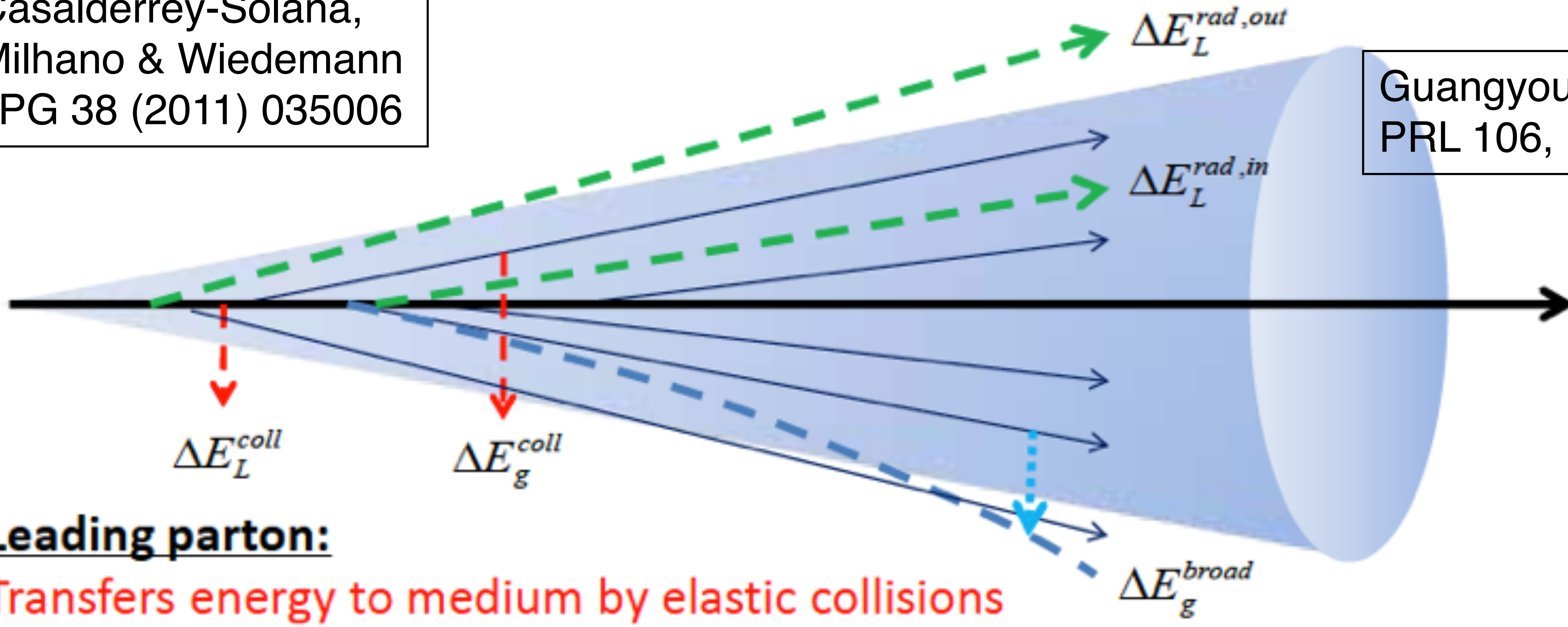
Opacity scale of medium $Q_s = \sqrt{qL} \approx m_D \sqrt{N_{\text{scatt}}}$

Transverse size of jet $r_{\perp\,\text{jet}} = \theta_{\text{jet}} L$

Jet collimation

Casalderrey-Solana, Milhano & Wiedemann JPG 38 (2011) 035006

Guangyou Qin & BM PRL 106, 162302 (2011)



Leading parton:

Transfers energy to medium by elastic collisions

Radiates gluons scattering in the medium (*inside* and *outside* jet cone)

$$E_L(t) = E_L(t_i) - \int \hat{e}_L dt - \int \omega d\omega dk_\perp^2 dt \frac{dN_g^{med}}{d\omega dk_\perp^2 dt}$$

Radiated gluons (*vacuum* & *medium-induced*):

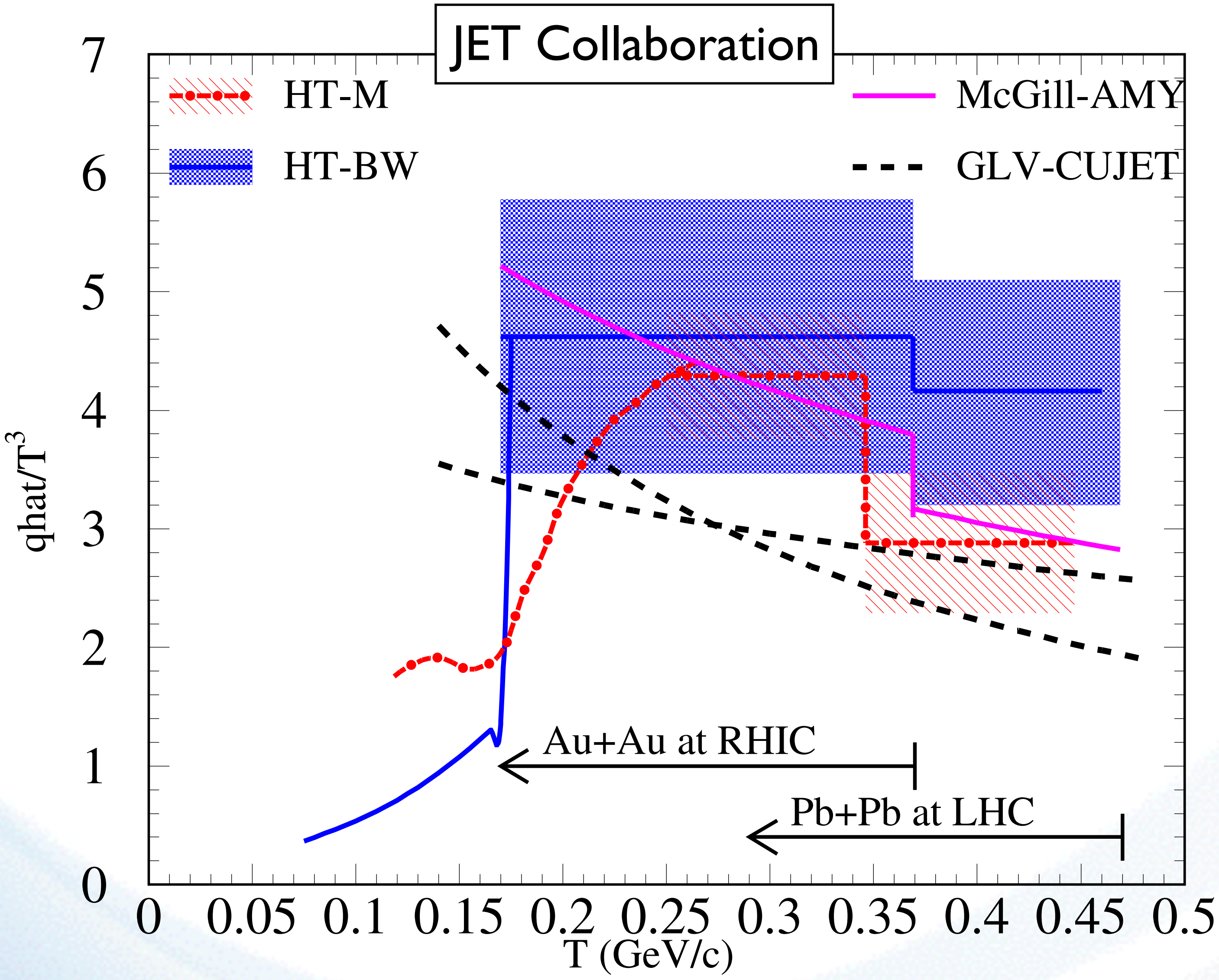
Transfer energy to medium by elastic collisions

Be kicked out of the jet cone by multiple scatterings after emission

$$\frac{df_g(\omega,k_\perp^2,t)}{dt} = \hat{e}\frac{\partial f_g}{\partial \omega} + \frac{1}{4}\hat{q}\nabla_{k_\perp}^2 f_g + \frac{dN_g^{med}}{d\omega dk_\perp^2 dt}$$

Jet quenching vs. η/s

Collaborative theoretical efforts are making extraction of qhat and ehat from data possible.



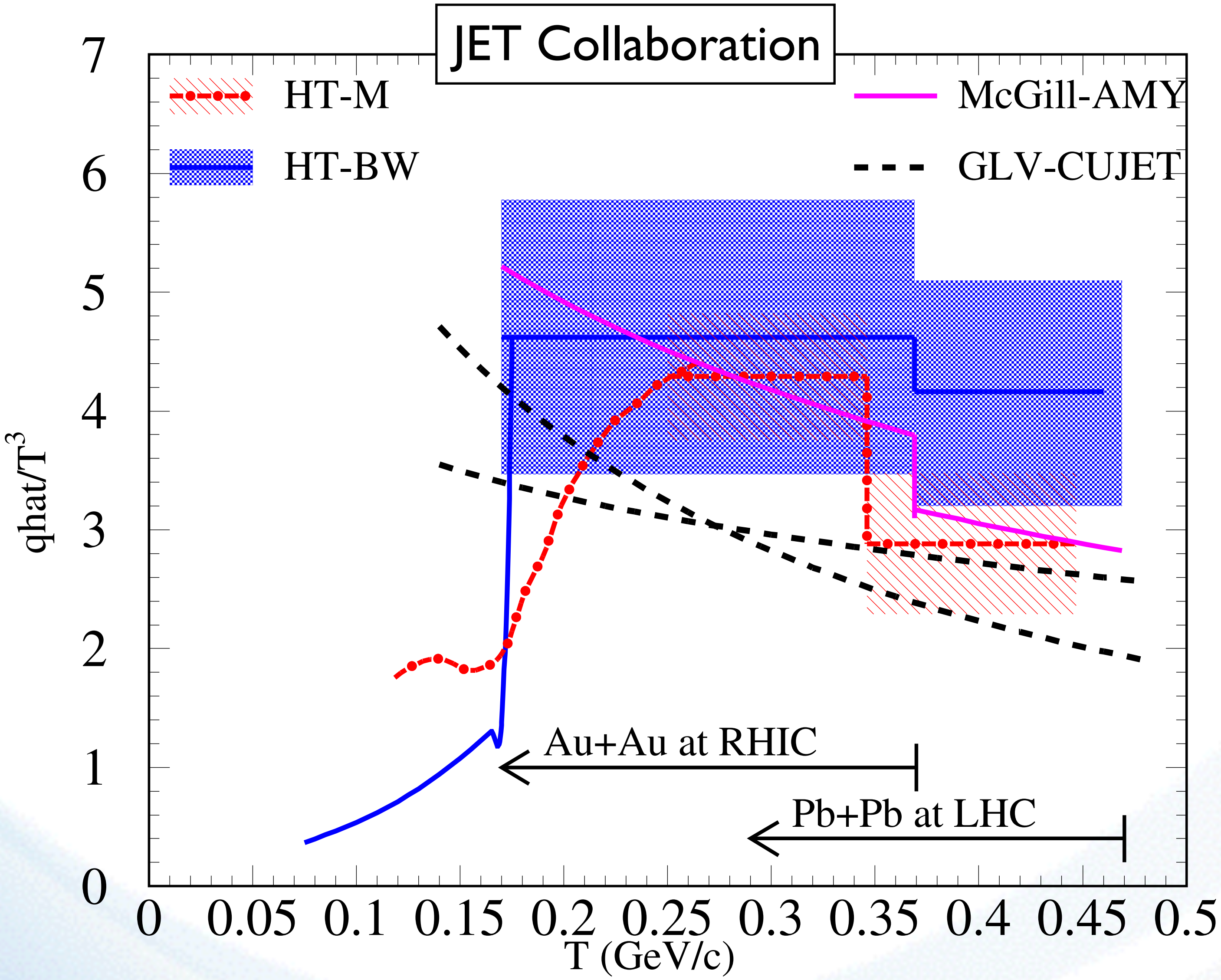
Jet quenching vs. η/s

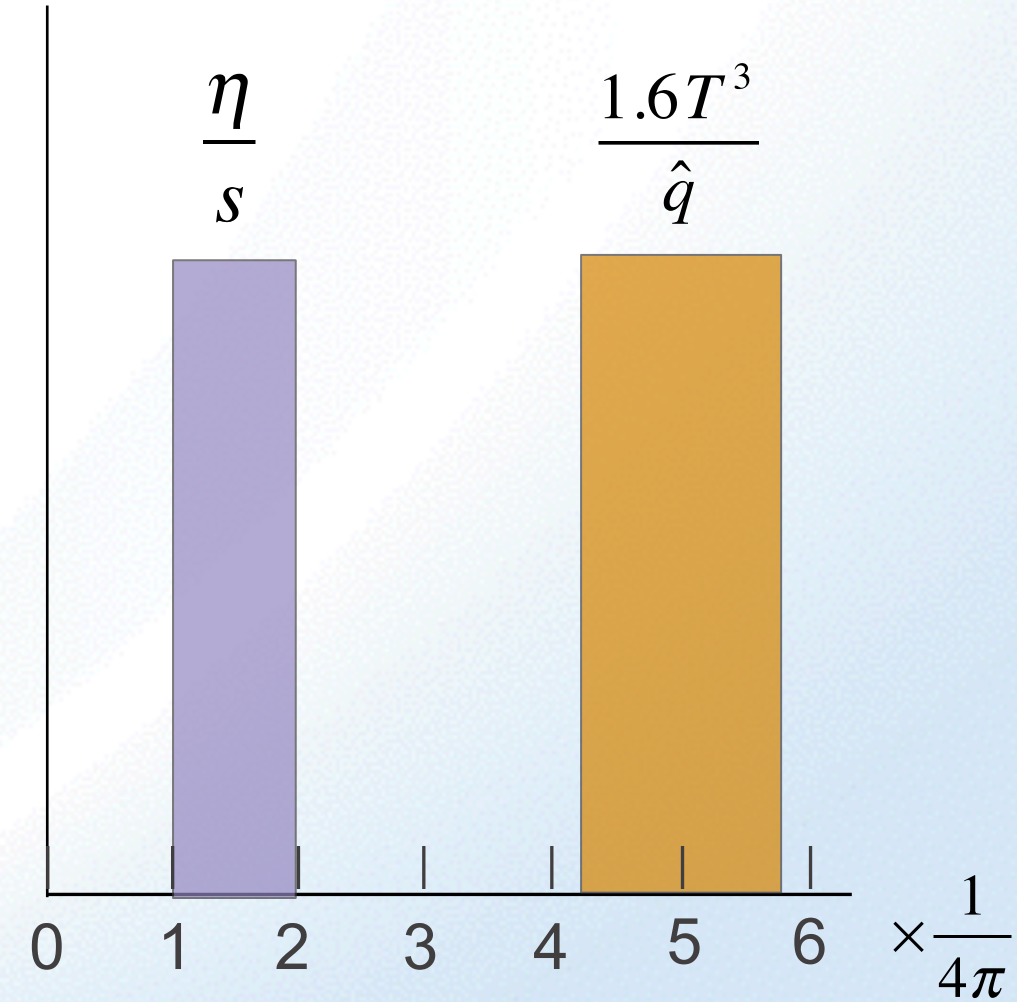
Majumder, BM, Wang

Collaborative theoretical efforts are making extraction of qhat and ehat from data possible.

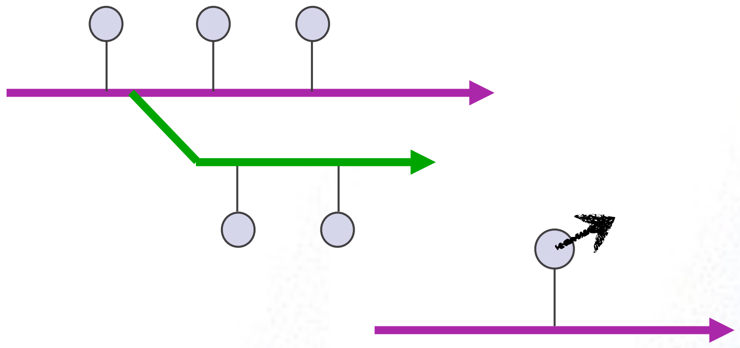
$$\eta / s \approx \frac{0.065}{\alpha_s^2 \ln(q_{\max}^2 / m_D^2)}$$

$$T^3 / \hat{q} \approx \frac{0.04}{\alpha_s^2 \ln(q_{\max}^2 / m_D^2)}$$



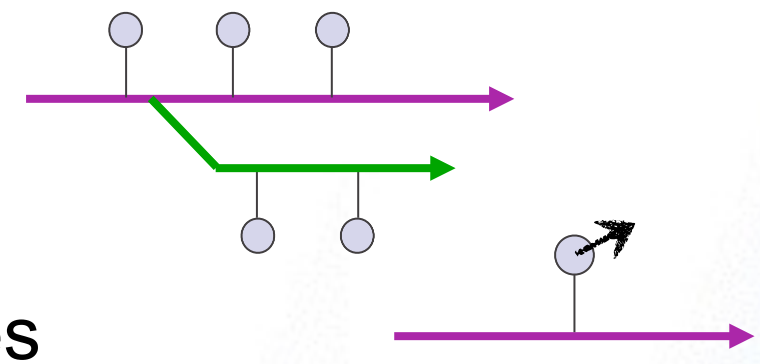


Jets quenching: Questions

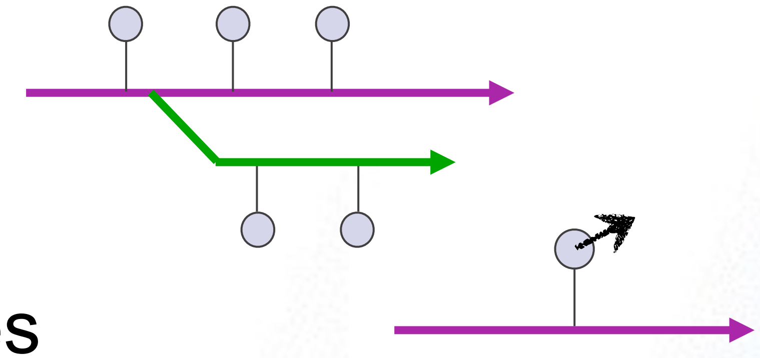


Jets quenching: Questions

- What is the mechanism of energy loss ?
  - "radiative" = into non-thermal gluon modes
  - "collisional" = directly into thermal plasma modes



Jets quenching: Questions



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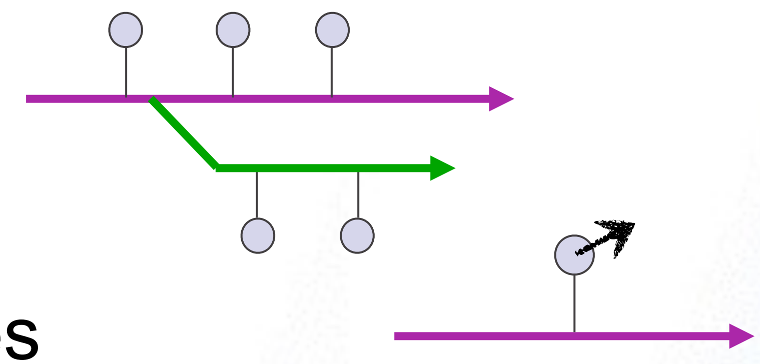

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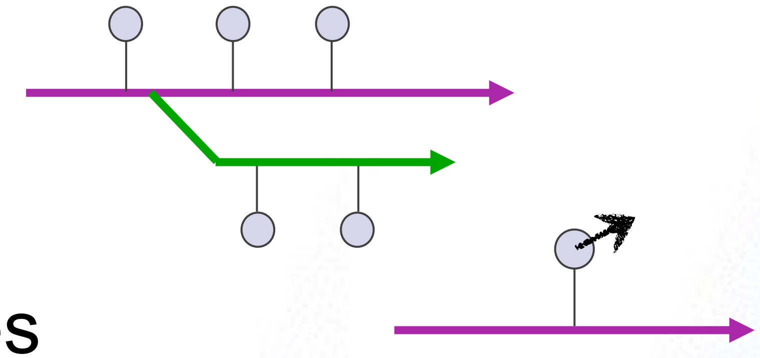
  - If EL is “radiative”, how quickly does the radiation thermalize ⇔
    What is the longitudinal momentum (z) distribution ?

Jets quenching: Questions



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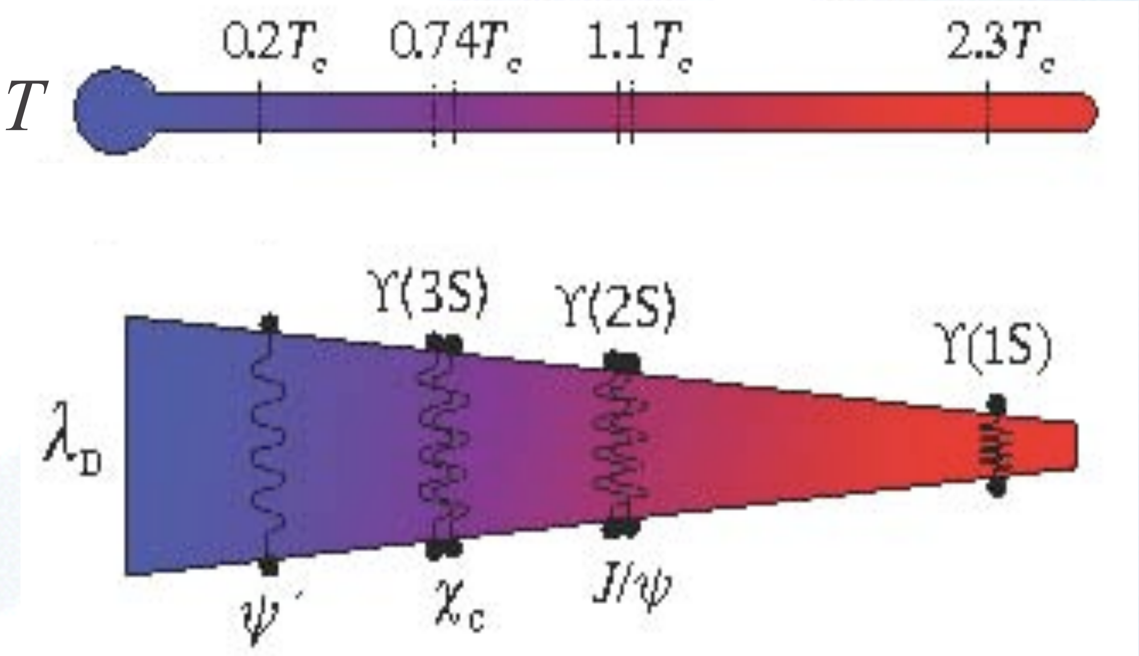
- What happens to the lost energy and momentum ?
  - If EL is “radiative”, how quickly does the radiation thermalize ⇔

    What is the longitudinal momentum (z) distribution ?
- How do the answers depend on the parton flavor ?
  - Heavy quarks (c,b) are predicted to lose a larger fraction of their energy via collisions rather than radiation.
  - Slow heavy quarks probe the chromo-electric response of the QGP.

Color screening

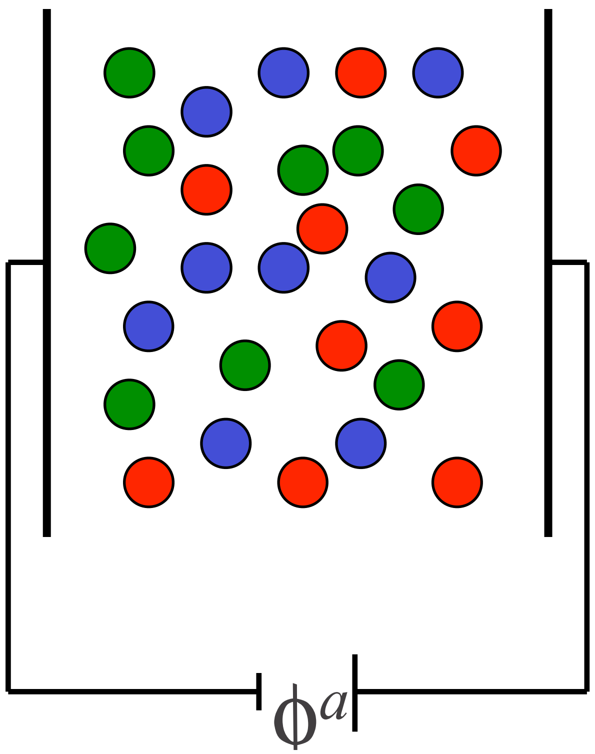
The essence of *Plasma*

- Plasma: An globally neutral state of matter with mobile charges
- Interactions among charges of many particles spread charge over a characteristic (Debye) length ➠ (chromo-) electric screening
- Strongly coupled plasmas: Only few particles in Debye sphere ➠ Nearest neighbor correlations ↔ liquid-like properties
- *Test* QGP *screening with heavy quark bound states*
  Which ones survive?
- Ideal system: Upsilon states
- Do residual correlations enhance final-state recombination?

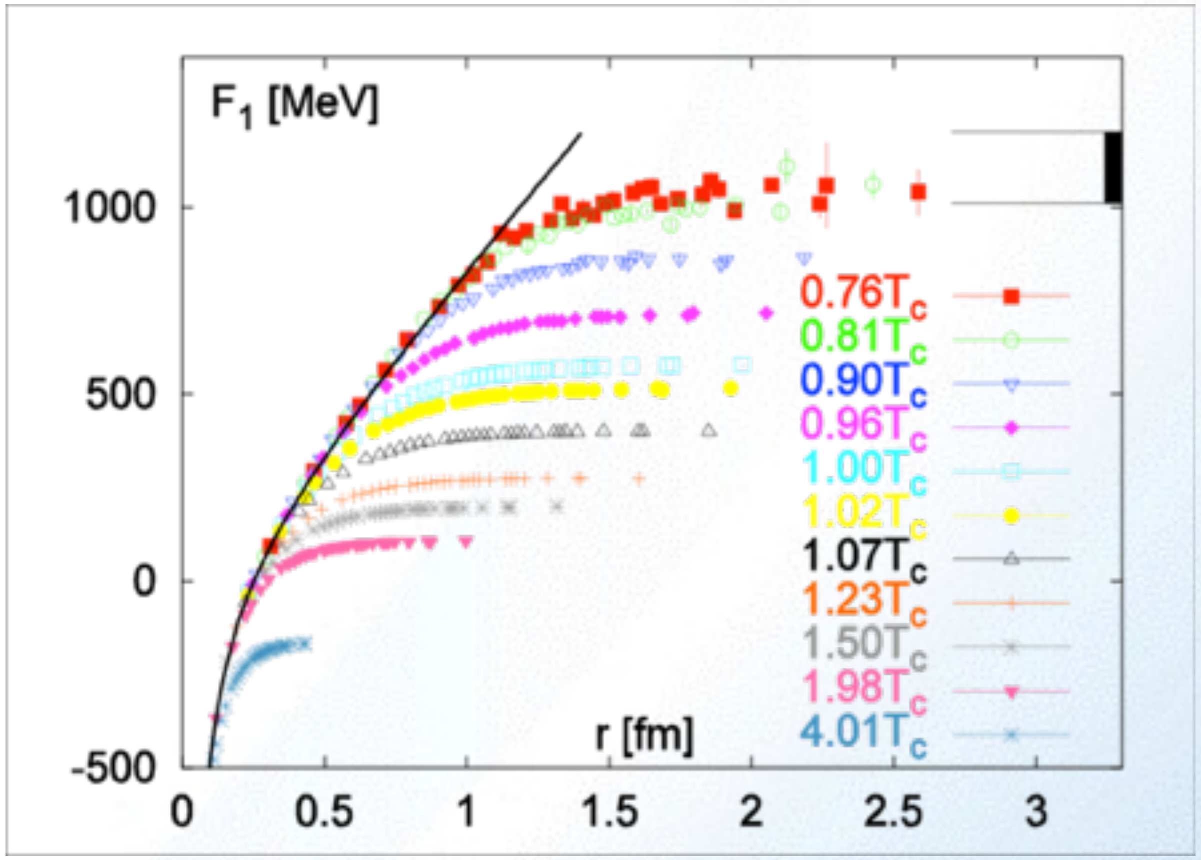


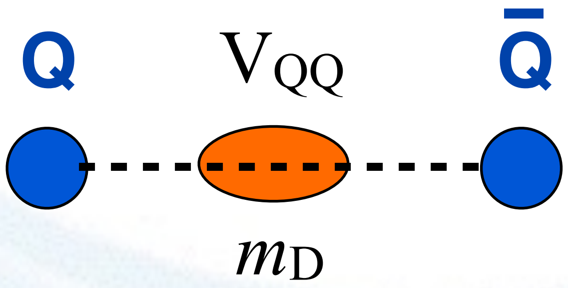
In the good old days...

... life seemed simple: It's all color screening



Lattice QCD





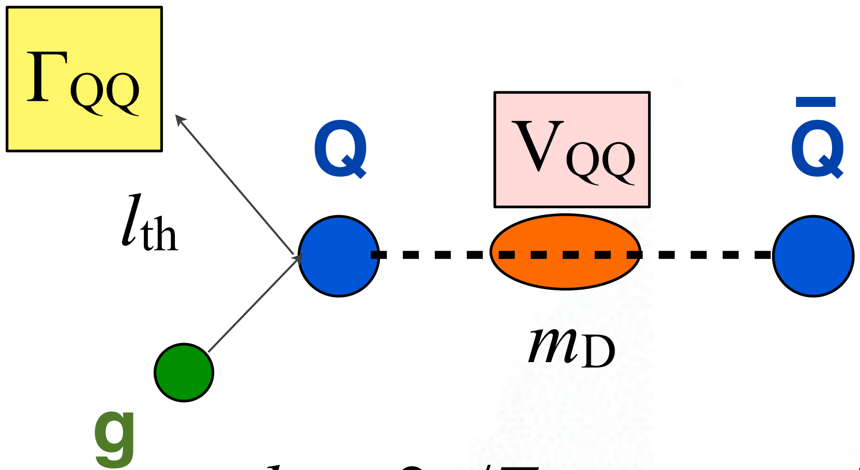
$m_D \sim gT$

Only the data did not quite fit the theory!

The real story...

...is more complicated (as usual).

Q-Qbar bound state interacts with medium elastically and inelastically!



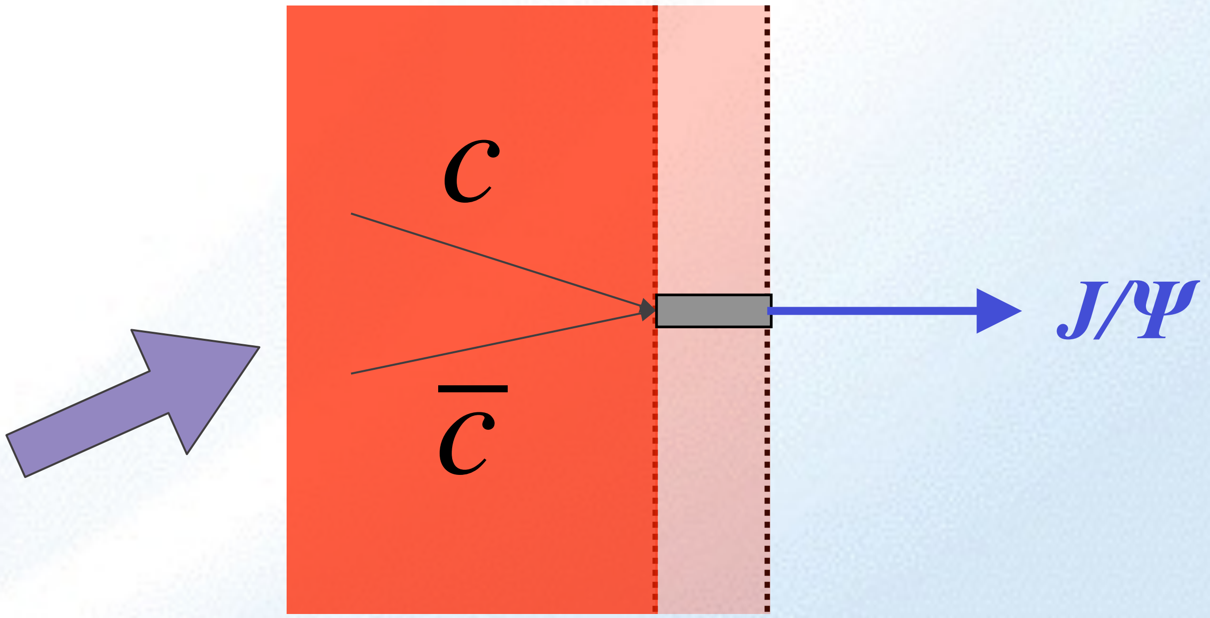
$l_{th} \sim 2\pi/T, \quad m_D \sim gT$

$$i\hbar \frac{\partial}{\partial t}\Psi_{Q\bar{Q}} = \left[\frac{p_Q^2 + p_{\bar{Q}}^2}{2M} + V_{Q\bar{Q}} - \frac{i}{2}\Gamma_{Q\bar{Q}} + \eta \right] \Psi_{Q\bar{Q}}$$

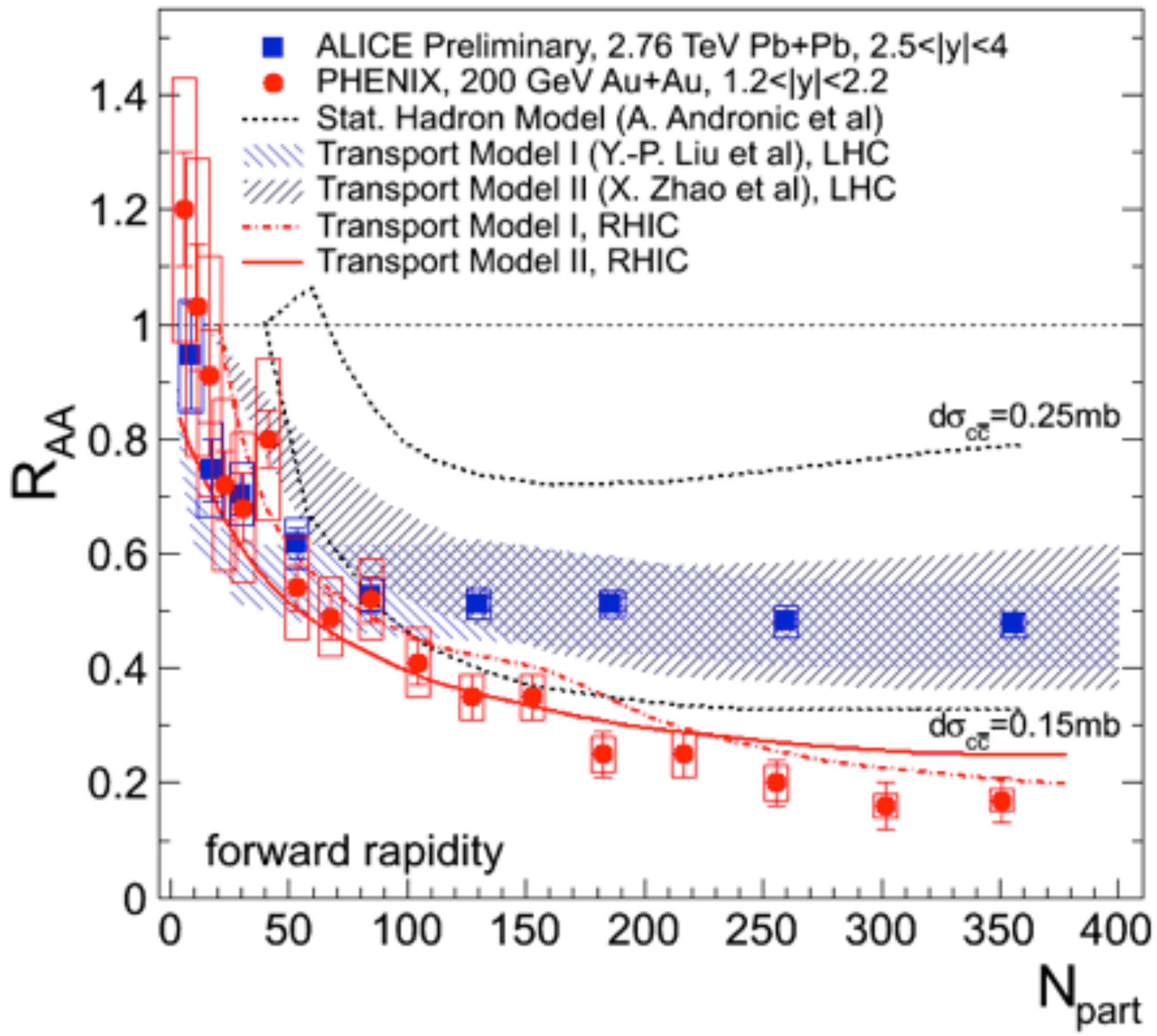
Strickland, arXiv:1106.2571, 1112.2761;
Akamatsu & Rothkopf, arXiv:1110.1203

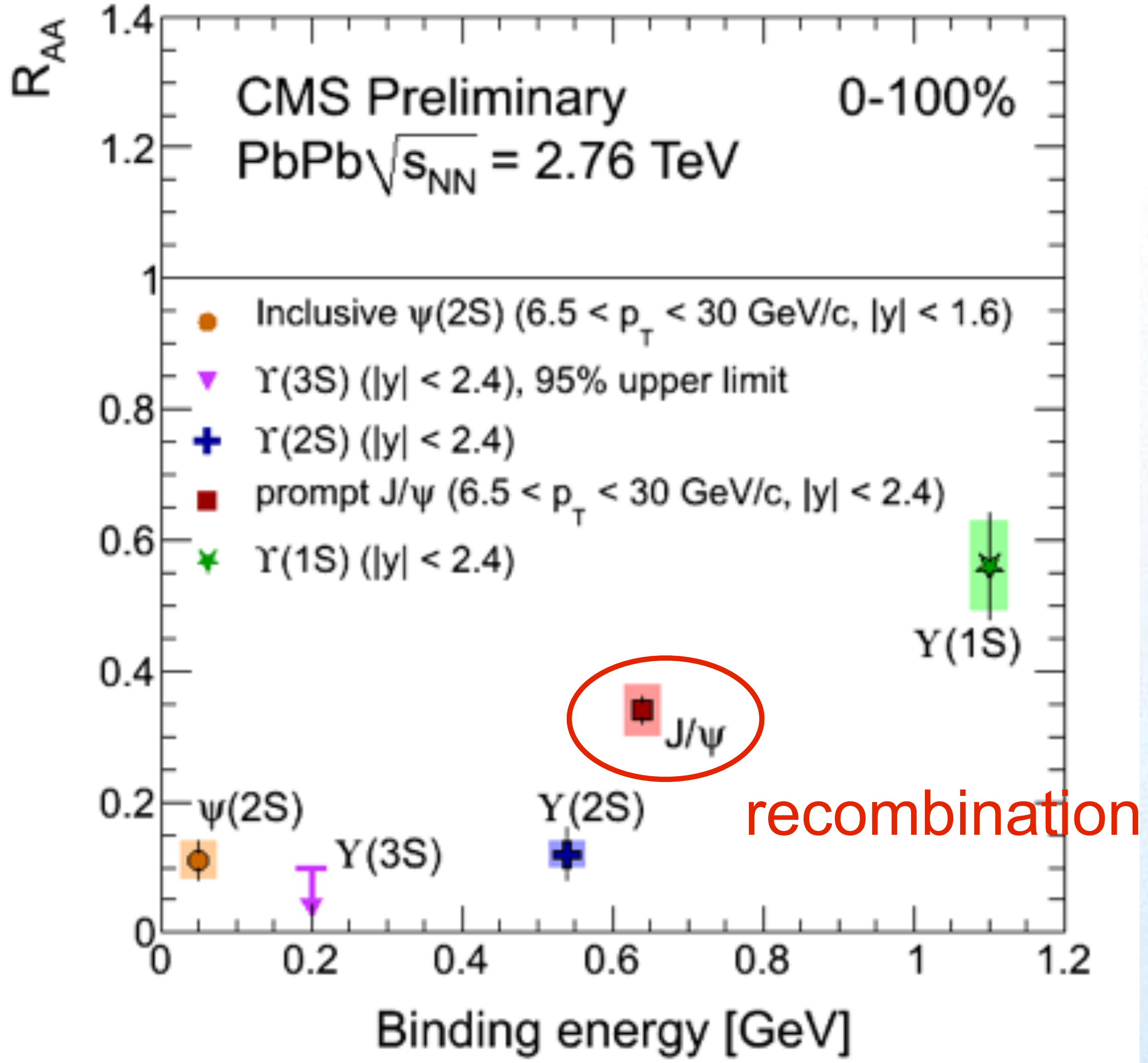
Heavy-Q energy loss and Q-Qbar suppression are closely related

Recombination can also contribute when c-quark density is high enough!



Quarkonium suppression





Less J/ψ suppression at LHC than at RHIC, at mid-rapidity and mid-forward rapidities:
c-cbar recombination explains data.

Full range of quarkonium states is becoming accessible.

Future of RHIC

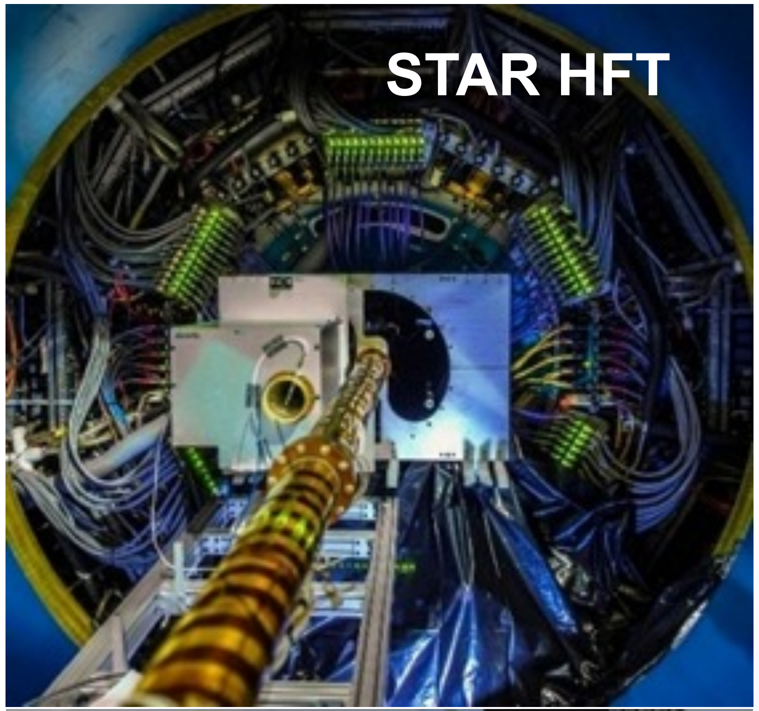
Completing the RHIC science mission

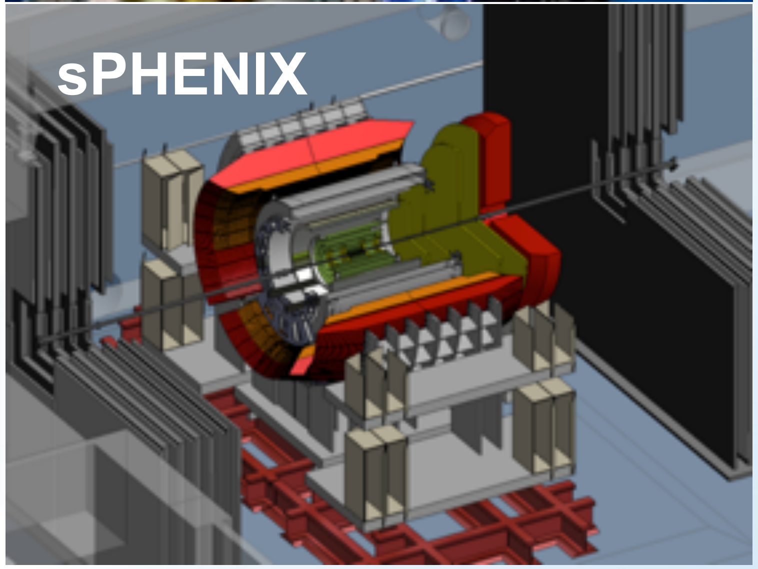
Status:

- RHIC-II configuration is complete
  - Vertex detectors in STAR (HFT) and PHENIX
  - Luminosity reaches 25 x design luminosity

Plan: Complete the RHIC mission in 3 campaigns:

- **2014–16: Heavy flavor probes of the QGP using the micro-vertex detectors**
- 2017: Install low energy e-cooling
- **2018/19: High precision scan of the QCD phase diagram & search for critical point**
- 2020: Install sPHENIX upgrade
- **2021/22: Precision measurements of jet quenching and quarkonium suppression**
- 2023-25: Transition to eRHIC

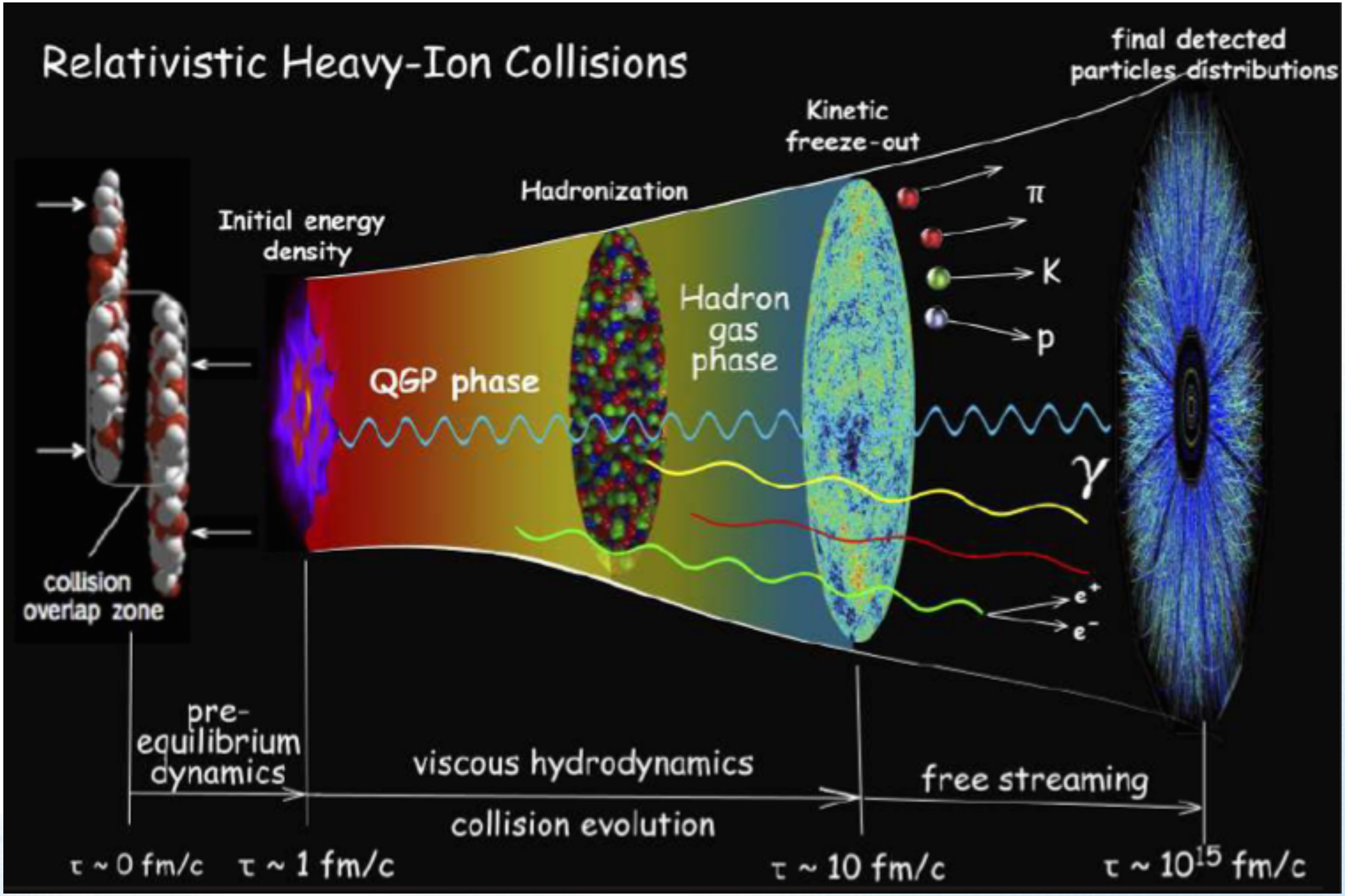




RHIC remains a unique discovery facility

Backup slides

Standard model of the “Little Bang”

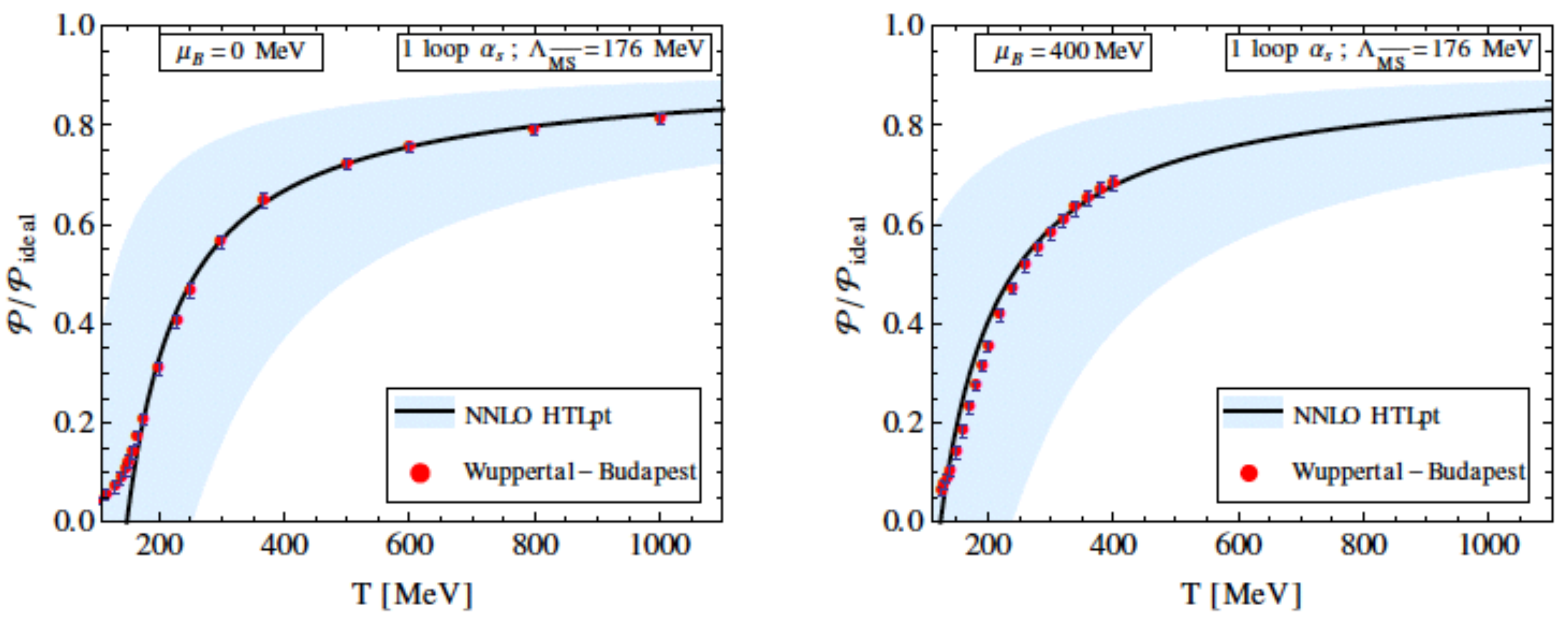


Quasiparticle QGP ?

3-loop resummed hard-thermal loop perturbation theory, uses dynamically screened quasiparticle modes as basis for a perturbative expansion.

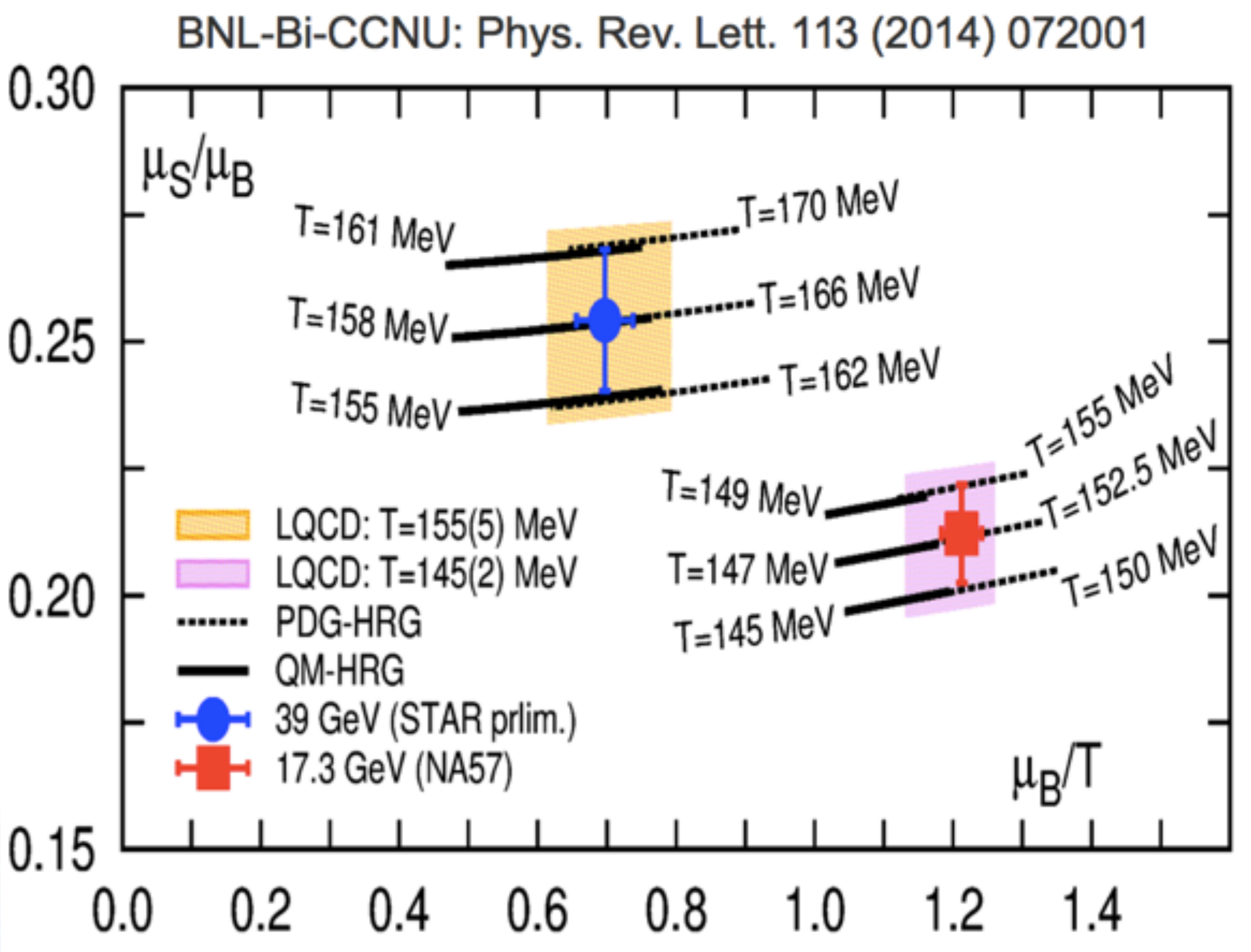
Can this approach capture the “perfext liquid” properties of the QGP near T_c ?

M. Strickland et al., arXiv:1407.3671 [hep-ph]

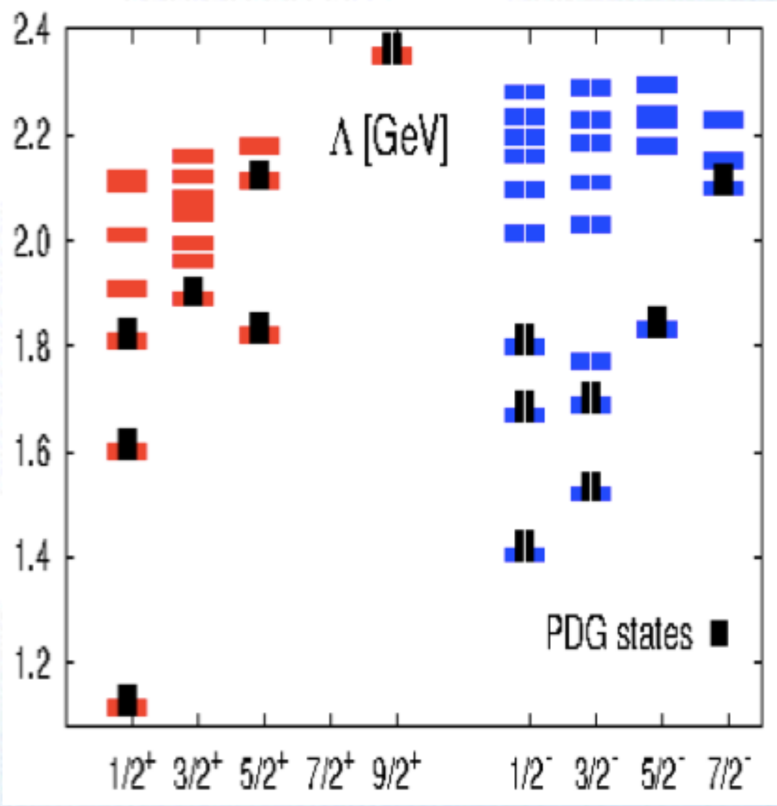


Probing the baryon spectrum

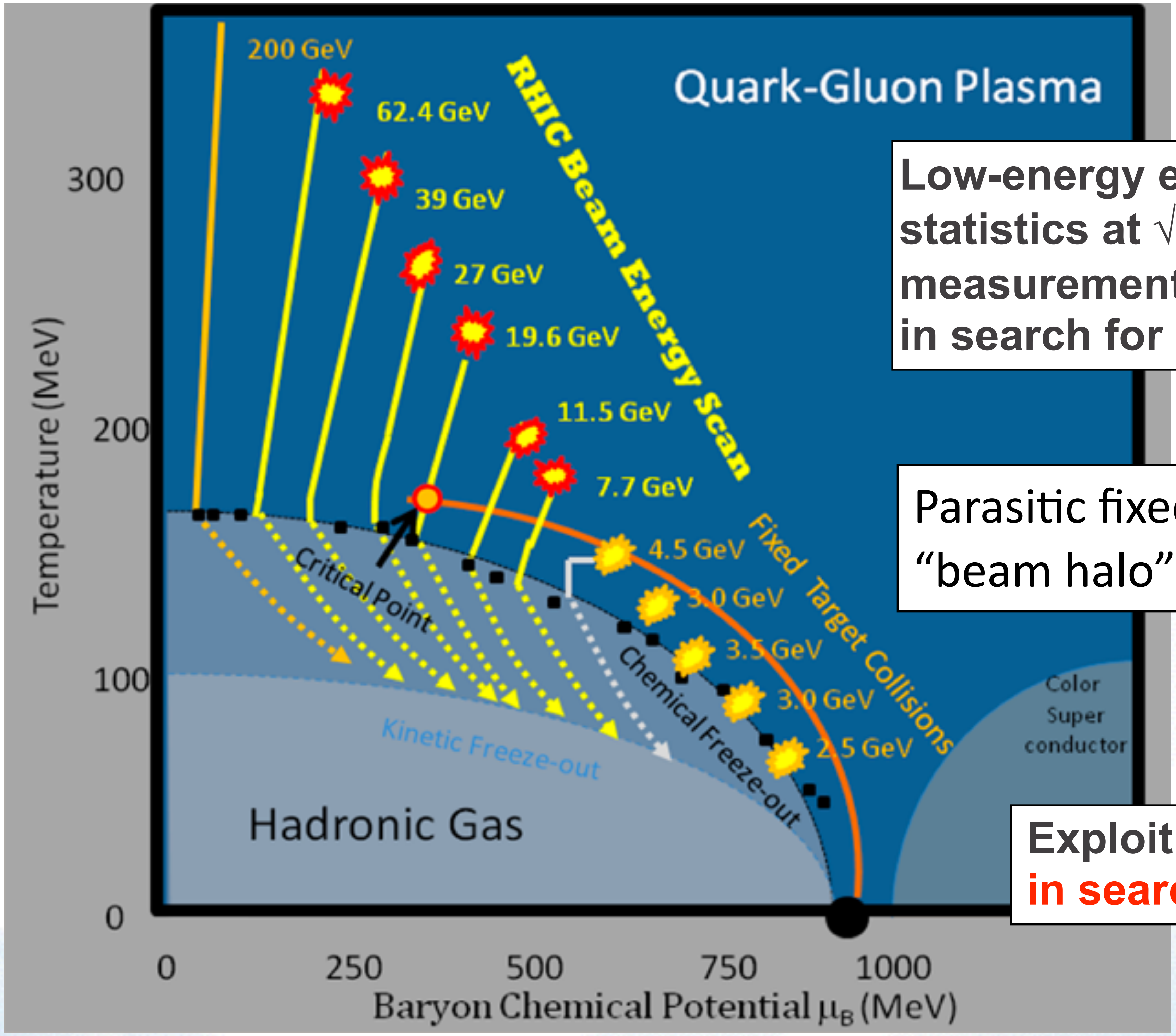
Consistency of μ_S/μ_B and μ_B/T with chemical composition of emitted hadrons and Lattice QCD requires additional strange baryon resonances beyond those in the PDG tables.



Quark model states of strange baryons



Beam energy scan II



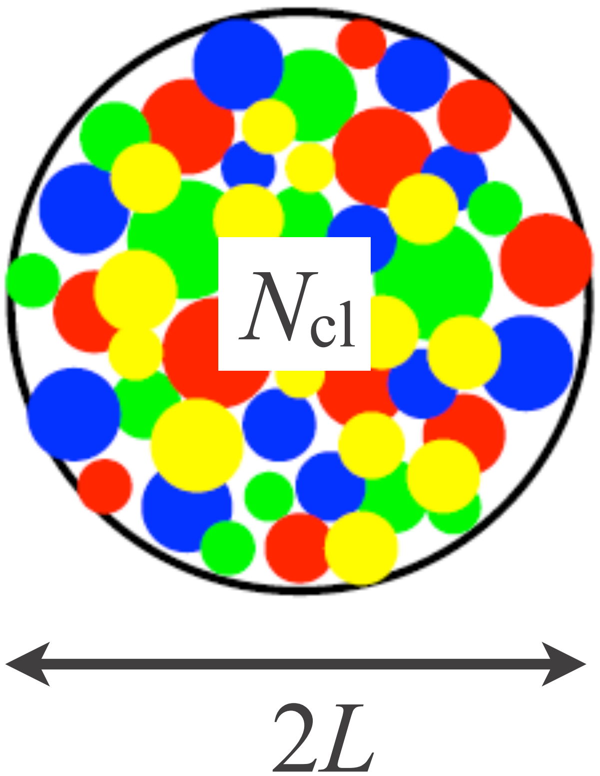
Low-energy e-cooling will improve statistics at $\sqrt{s}$ < 20 GeV for detailed measurements of sensitive quantities in search for critical point

Parasitic fixed target mode by utilizing "beam halo" inside STAR detector ?

Exploit new discovery potential in search for a QCD critical point

BROOKHAVEN NATIONAL LABORATORY

Why small QGP droplets can flow



saturation scale $$Q_s^2 \propto \frac{N_{cl}}{\pi L^2}$$

mean free path $$\ell_{mfp} \propto Q_s^{-1}$$

final multiplicity $$dN/dy \propto N_{cl}$$

Basar & Teaney, 1312.6770

Size scales out of Reynolds number:

$$\mathrm{Re} = \frac{\ell_{mfp}}{L} \propto \frac{1}{Q_s L} \propto \frac{1}{\sqrt{dN/dy}}$$

This does not imply that hydrodynamics applies for a given *dN/dy*, but it suggests that transport properties are independent of size.