Center for Future High Energy Physics

高能物理前沿研究中心

Probing fundamental physics at the new scale

B+L violation & (non-)perturbative ElectroWeak dynamics at very high energy colliders

Valya Khoze

IPPP Durham

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Baryon + Lepton number violation in the Standard Model

- Electroweak vacuum has a nontrivial structure (!) [SU(2)-sector]

- The saddle-point at the top of the barrier is the *sphaleron.* New EW scale ~ 10 TeV

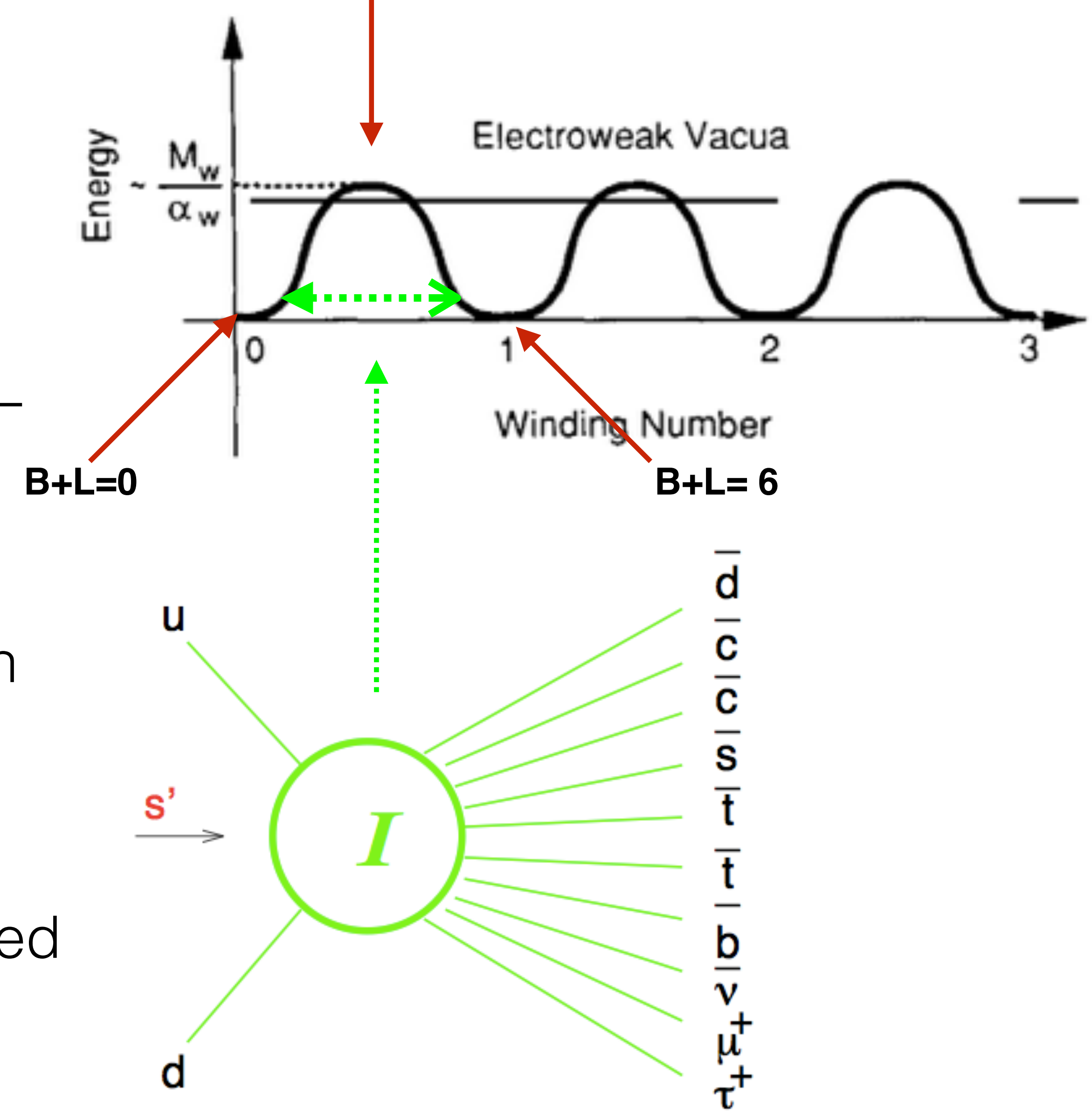
- Transitions between the vacua change B+L (result of the ABJ anomaly):
  Delta (B+L)= 3 x (1+1) ; Delta (B-L)=0

- *Instantons* are tunnelling solutions between the vacua. They mediate B+L violation

- 3 x (1 lepton + 3 quarks) = 12 fermions
  12 left-handed fermion doublets are involved

- There are EW processes which are not described by perturbation theory!

$$E_{\mathrm{sph}} = c_{\mathrm{sph}} \frac{m_W}{\alpha_W} \approx 10\,\mathrm{TeV}$$



$$\boxed{q + q \to 7\bar{q} + 3\bar{l} + n_W W + n_Z Z + n_h H}$$

- Electroweak sector of the SM is always seen as perturbative. If these instanton processes can be detected —> a truly remarkable breakthrough in realising & understanding non-perturbative EW dynamics!

- B+L processes provide the **physics programme** which is **completely unique to the very high energy pp machine**. This cannot be done anywhere else.

- The B+L processes are accompanied by ~50 EW vector bosons; charged Lepton number can also be measured —> unique experimental signature of the final state — essentially **no backgrounds** expected from *conventional perturbative processes* in the SM.

- The rate of the B+L processes is still not known theoretically. There are optimistic phenomenological models with ~pb or ~fb crossections, and there are pessimistic models with unobservable rates even at infinite energy.

- New computational methods are needed. [2014 is not 1993 (or even 2003)]

- Since the final state is essentially backgroundless, the obesrvability of the rate can be always settled experimentally (if we have the 100 or 33 TeV machine).

Instanton approach

- All instanton contributions come with an exponential suppression due to the instanton action:

$$\mathcal{A}^{\text{inst}} \propto e^{-S^{\text{inst}}} = e^{-2\pi/\alpha_w - \pi^2\rho^2 v^2}, \quad \sigma^{\text{inst}} \propto e^{-4\pi/\alpha_w} \simeq 5 \times 10^{-162}$$

- This is precisely the expected semiclassical price to pay for a quantum mechanical tunnelling process. **Are we done?**

- **Not yet.** For the B+L violating process

$$q + q \to 7\bar{q} + 3\bar{l} + n_W W + n_Z Z + n_h H$$

- at leading order, the instanton acts as a point-like vertex with a large number n of external legs => n! factors in the amplitude.

- As the number of W's, Z's and H's produced in the final state at sphaleron-like energies is allowed to be large, ~ 1/alpha, **the instanton crossection receives exponential enhancement with energy**

Ringwald 1990 => McLerran, Vainshtein, Voloshin 1990 => ….

Instanton approach

- Instanton is a classical solution in Euclidean spacetime (good for tunnelling)
  Gauge field (i.e. W's and Z's) instanton in the `singular gauge' is:

$$A^{\text{inst}\,a}_{\ \ \mu} = \frac{2}{g}\,\bar{\eta}^{a}_{\mu\nu}\,\frac{(x-x_0)^\nu\,\rho^2}{(x-x_0)^2((x-x_0)^2+\rho^2)}$$

- When the Higgs VEV is turned on, this expression gets modified at large distances so that:

$$A^{\text{inst}\,a}_{\ \ \mu} \to e^{-m_W|x-x_0|}\,, \quad \text{as } (x-x_0)^2 \gg \rho^2$$

- There is also the Higgs-field component of the instanton,

$$H^{\text{inst}} = v\left(\frac{(x-x_0)^2}{(x-x_0)^2+\rho^2}\right)^{1/2}$$

- And there are fermion components, one for each left-handed doublet (instanton fermion zero modes),

$$\psi_L^{\text{inst}} = \frac{1}{\pi}\,\frac{\rho^2}{((x-x_0)^2+\rho^2)^{3/2}}\,\frac{(x-x_0)^\mu}{|x-x_0|}\,\sigma_\mu\cdot\chi_{\text{Grassm}}$$

- And no anti-fermion solutions! B+L violation is automatic with instantons.

Instanton approach

- Start with the off-shell Green function

$$\int (D\psi)(DA)(DH)\,\psi(x_1)\ldots\psi(x_{12})\,A(y_1)\ldots A(y_{n_W+n_Z})\,H(z_1)\ldots H(z_{n_h})\times e^{-S}$$

- substituting for each field = instanton + fluctuation; integrate out the fluctuations to the leading non-vanishing order.

- To get the Amplitude: analytically continue to Minkowski space, Fourier transform instanton external legs to momentum space, go on-shell and LSZ amputate, e.g.

$$A^{\mathrm{inst}\,a}_{\ \ \mu}(x_i) \to \frac{4i\pi^2\rho^2}{g}\,\frac{\bar{\eta}^a_{\mu\nu}p_i^\nu}{p_i^2(p_i^2+m_W^2)}\,e^{ip_ix_0} \to \frac{4i\pi^2\rho^2}{g}\,\frac{\bar{\eta}^a_{\mu\nu}p_i^\nu}{p_i^2}\,e^{ip_ix_0}$$

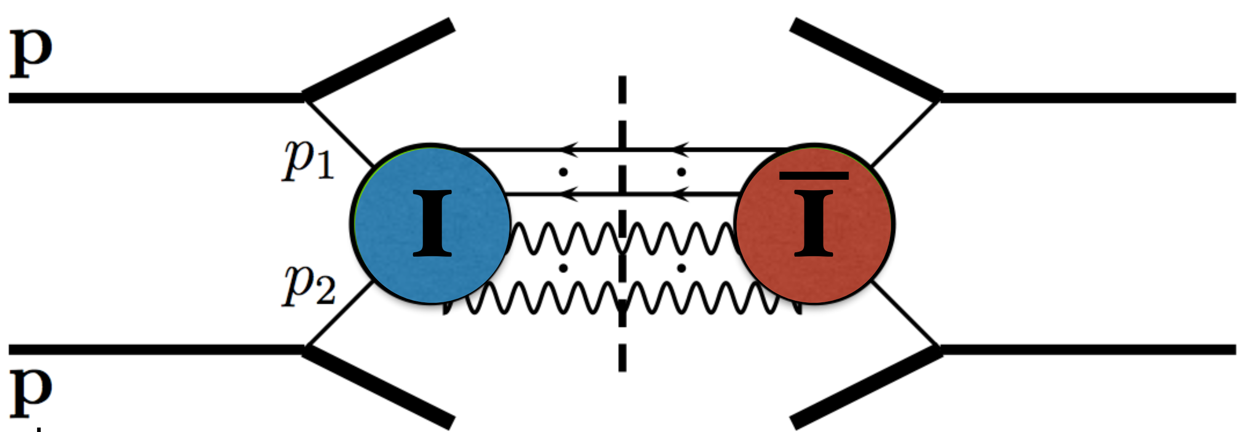
$$H^{\mathrm{inst}}(x_j) \to -\frac{2\pi^2\rho^2 v}{(p_j^2+m_H^2)}\,e^{ip_jx_0} \to -2\pi^2\rho^2 v\,e^{ip_jx_0}$$

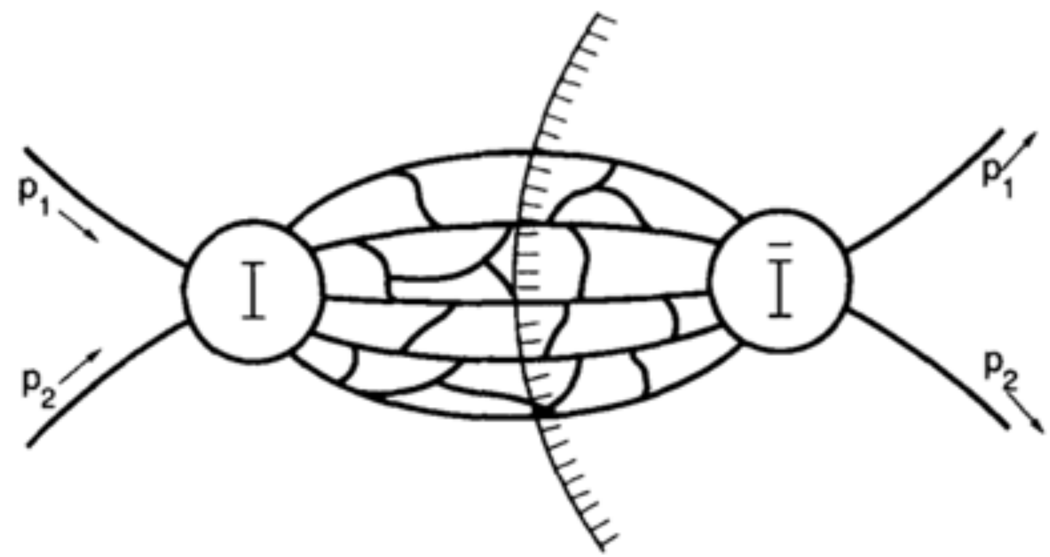
- After integrating over the instanton size of the multiple field insertions above one gets an exponential enhancement with energy.

Instanton-Antiinstanton valley

VVK & Ringwald 1991

- Crossection is obtained by |squaring| the instanton amplitude.
- Final states have been instrumental in combatting the exp. suppression.
- Now also the interactions between the final states (and the improvement on the point-like I-vertex) are taken into account.
- Use the Optical Theorem to compute *Im* part of the FES amplitude in around the **Instanton-Antiinstanton configuration**.
- Higher and higher energies correspond to shorter and shorter I-Ibar separations R. At R=0 they annihilate to perturbative vacuum.
- The suppression of the crossection is gradually reduced with energy.





Instanton-Antiinstanton valley [soft-soft interactions]

- Instanton — anti-instanton valley configuration has Q=0; it interpolates between infinitely separated instanton—anti-instanton and the perturbative vacuum at z=0

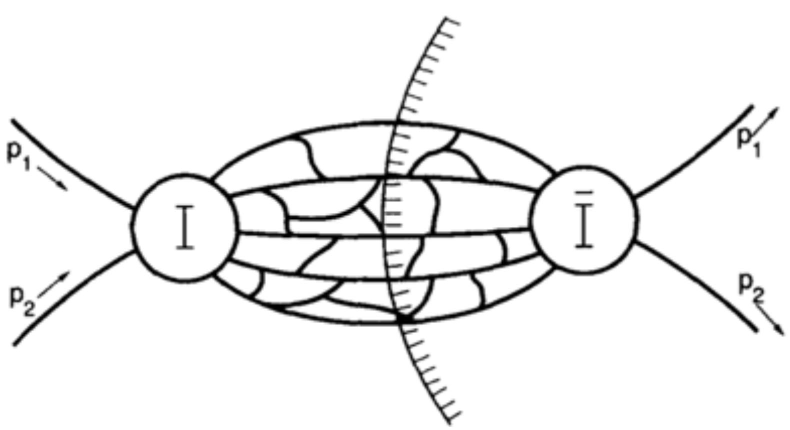
$$G_{4\,\mathrm{Eucl}} \sim \int d^4R \, d\rho_I d\rho_{\bar I} \, \ldots \, \exp\left[i(p_1+p_2)\cdot R - S_{I\bar I}(z) - \pi^2 v^2(\rho_I^2+\rho_{\bar I}^2)\right]$$

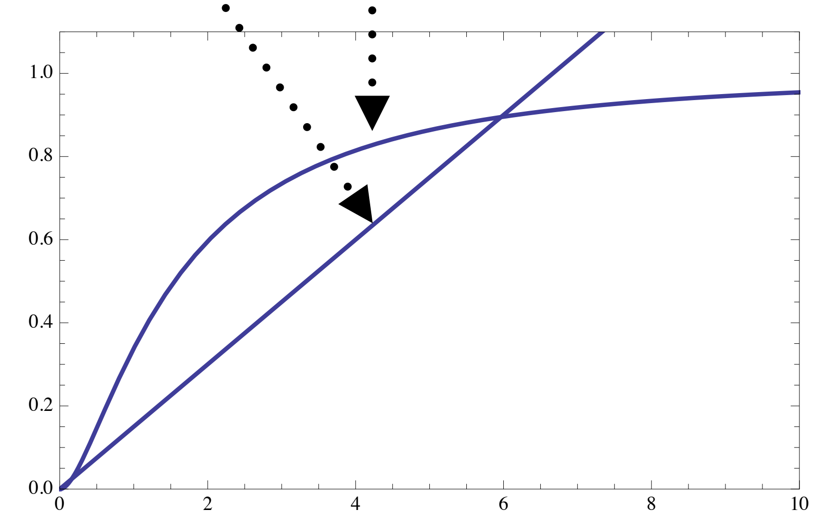
instanton separation

instanton sizes

$$z \sim \frac{R^2+\rho_I^2+\rho_{\bar I}^2}{\rho_I \rho_{\bar I}}$$

$$\sigma_{B+L} \sim \mathrm{Im} \int d^4R \, d\rho_I d\rho_{\bar I} \, \ldots \, \exp\left[ER - S_{I\bar I}(R) - \pi^2 v^2(\rho_I^2+\rho_{\bar I}^2)\right]$$





- Exponential suppression is gradually reduced with energy <= valley configuration
- no radiative corrections from hard initial states included in this approximation

Instanton-Antiinstanton optimistic estimate

VVK & Ringwald 1991

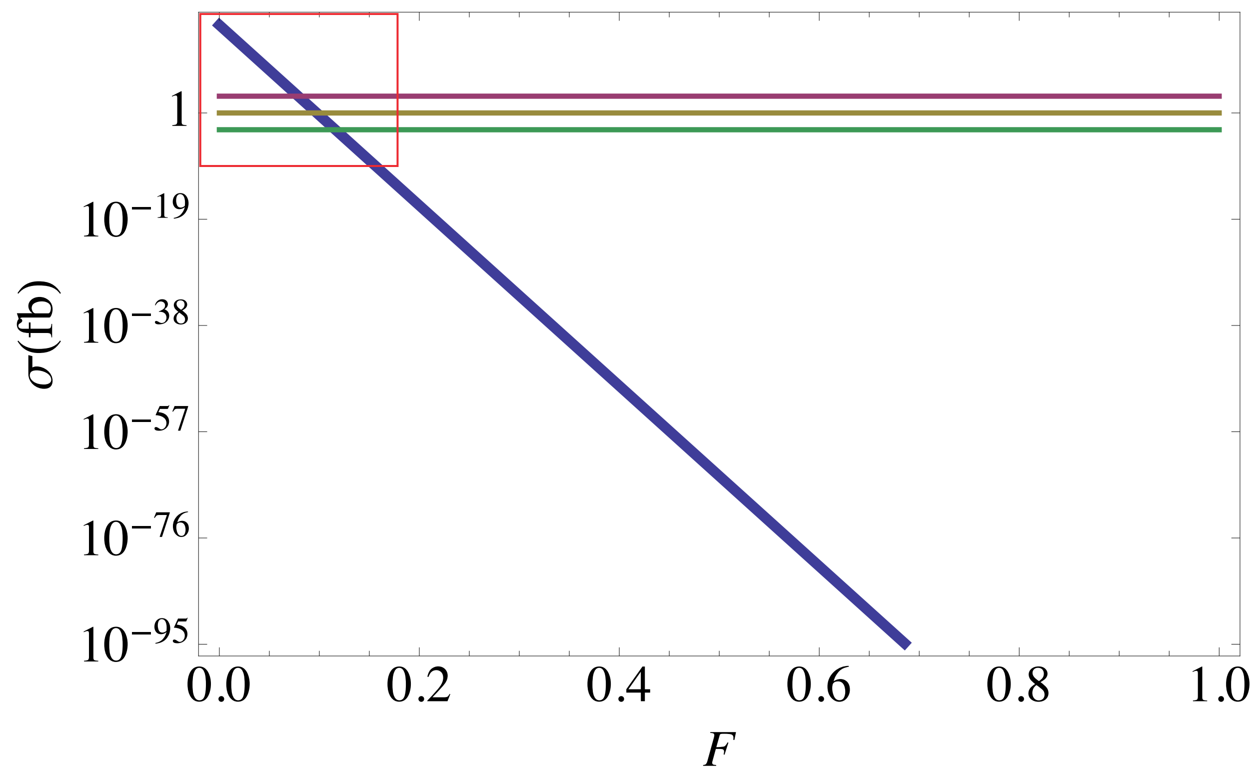
$$\hat{\sigma}_{qq}^{\rm inst} \approx \frac{1}{m_W^2}\left(\frac{2\pi}{\alpha_W}\right)^{7/2} \times \exp\left[-\frac{4\pi}{\alpha_W} F_{\rm hg}\left(\frac{\sqrt{\hat{s}}}{4\pi m_W/\alpha_W}\right)\right]$$

$$\simeq (5.28 \times 10^{15}\ {\rm fb}) \times \exp\left[-\frac{4\pi}{\alpha_W} F_{\rm hg}\left(\frac{\sqrt{\hat{s}}}{4\pi m_W/\alpha_W}\right)\right]$$

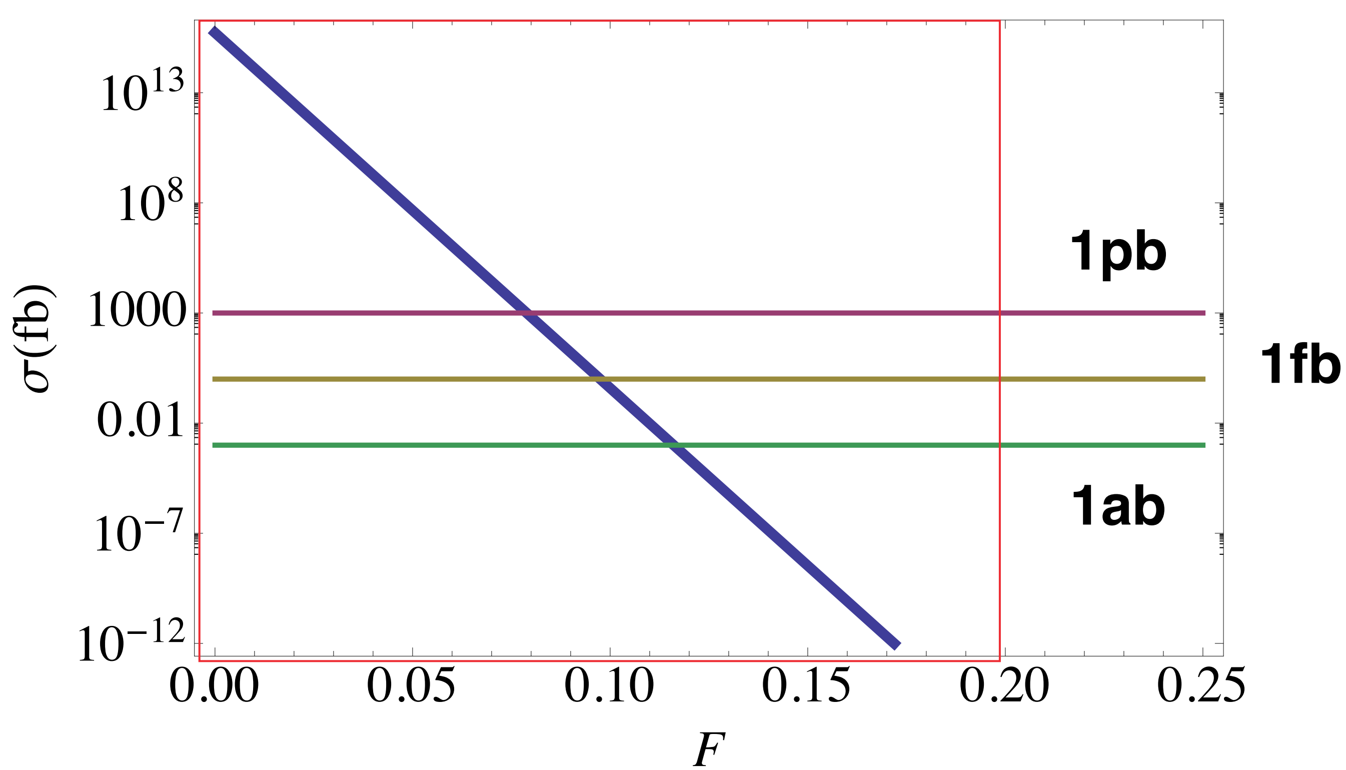
F =1 at E=0

0<F<1 at large E

The holy grail function F



The holy grail function F



The holy grail function F

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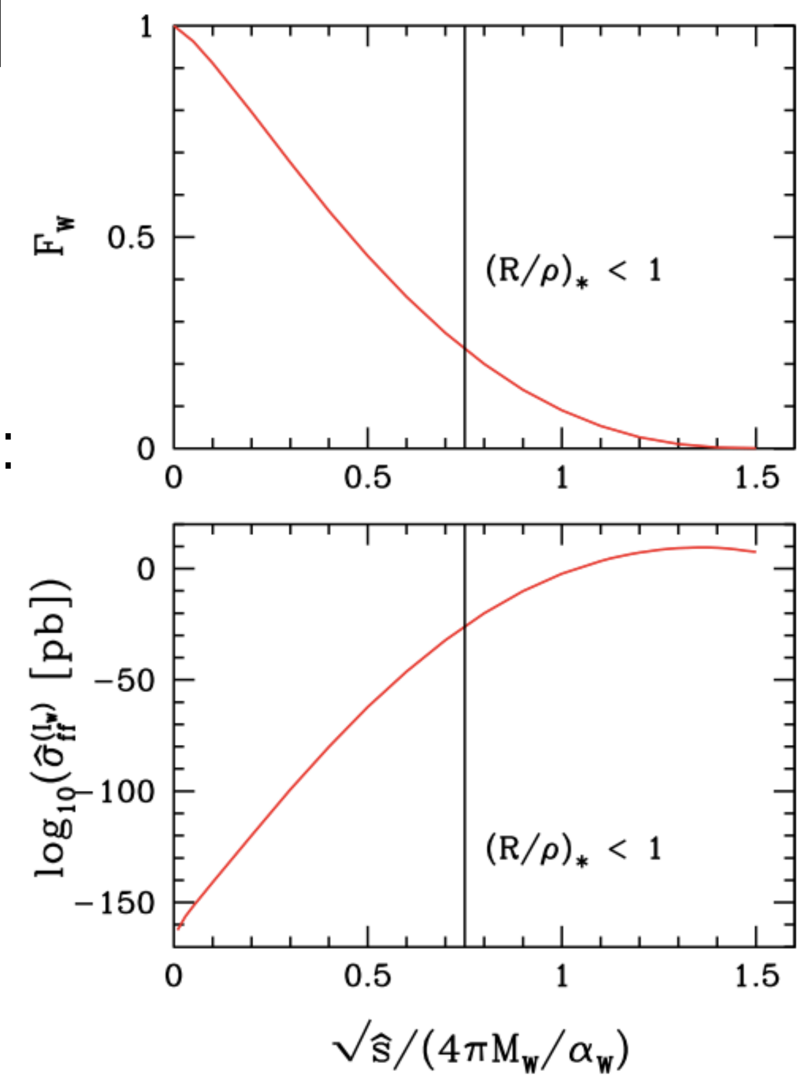
Ringwald 2002

The holy grail function F



First few terms in the energy-expansion of the holy grail:

$$F_W(\epsilon) = 1 - \frac{3^{4/3}}{2}\,\epsilon^{4/3} + \frac{3}{2}\,\epsilon^2 + \mathcal{O}(\epsilon^{8/3}) + \ldots$$

$$\epsilon = \sqrt{\hat{s}}/(4\pi m_W/\alpha_W) \simeq \sqrt{\hat{s}}/(30\ \text{TeV})$$

Mattis, Phys. Rept.1992

is a comprehensive review of the 90's literature on the holy grail

Pessimistic vs optimistic pictures

Pessimistic view:

The sphaleron is a semiclassical configuration with

$$\boxed{\mathrm{Size}_{\mathrm{sph}} \sim m_W^{-1}\,, \quad E_{\mathrm{sph}} = \mathrm{few} \times m_W/\alpha_W \simeq 10\,\mathrm{TeV}.}$$

It is 'made out' of $\sim 1/\alpha_W$ particles (i.e. it decays into $\sim 1/\alpha_W$ W's, Z's, H's).

$$\boxed{2_{\text{initial hard partons}} \;\to\; \text{Sphaleron} \;\to\; (\sim 1/\alpha_W)_{\text{soft final quanta}}}$$

The sphaleron production out of 2 hard partons is unlikely.

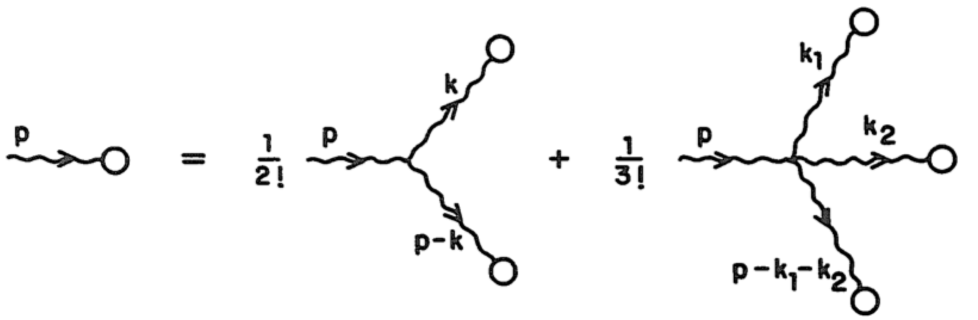
Assumptions:
(1) the intermediate state had to be the sphaleron;
(2) the initial state was a 2-particle state;
(3) that one cannot create $(\sim 1/\alpha_W)_{\text{soft final quanta}}$ from $2_{\text{initial hard partons}}$.

Pessimistic vs optimistic pictures

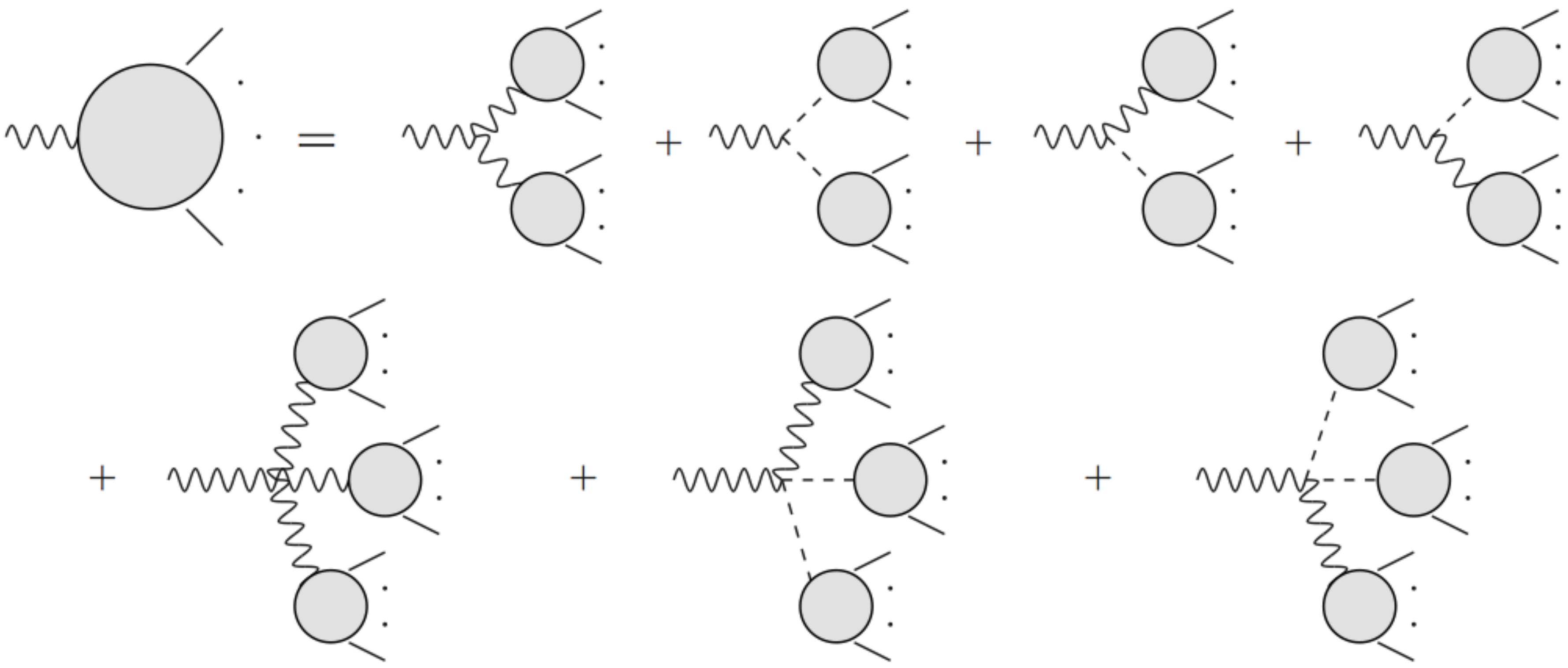
Optimistic view:

1. Use instanton, not the sphaleron as the guide. Initial hard quanta probe short distances, but are not prevented from probing larger scales as well, by emitting soft quanta. Instanton is a classical solution, thus:

Classical equation:



Berends-Giele type recursion relations for the instanton current:



The BLRRT approach (from 1/alpha to 2 initial quanta)

Construct an auxiliary solution with the initial data chosen that:

(1) the initial state has $N = \tilde{N}/\alpha_W$ particles with $\tilde{N}$ fixed and $\alpha_W \to 0$

(2) the energy also scales as $E = \tilde{E}/\alpha_W$

(3) for simplicity also assume spherical symmetry.

The probability of tunnelling from such *multiparticle* state is computed semi-classically:

$$\sigma \sim \exp\left(-\frac{4\pi}{\alpha_W} F_{\tilde{N}}(\tilde{E})\right)$$

For fixed $\tilde{N}$ and $E \sim E_{\rm sph}$ the rate will be unsuppressed. But this is not the 2-particle in-state.

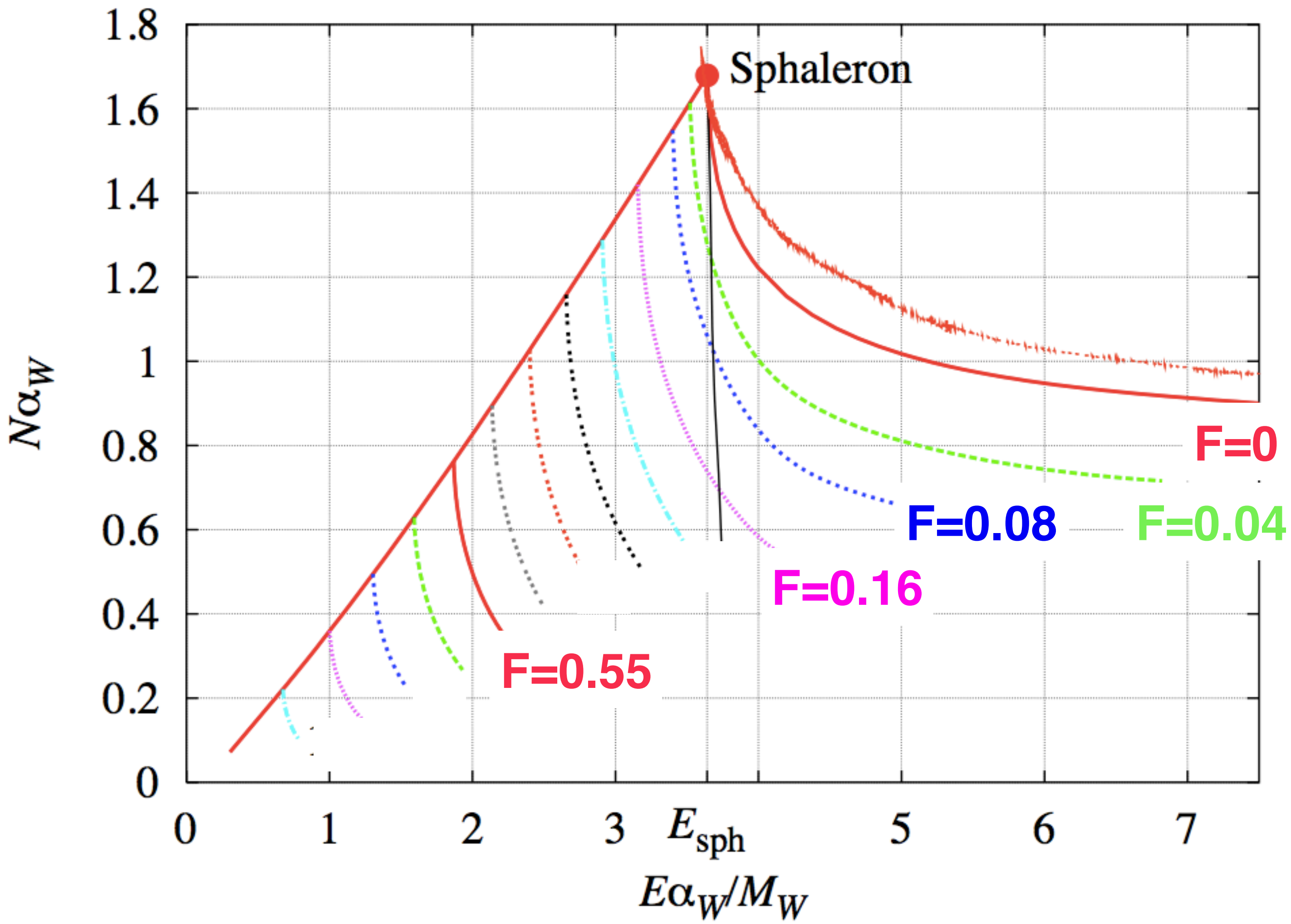
Conjecture that the holy grail function relevant for the 2-particle initial state is obtained by taking the $\tilde{N} \to 0$ limit of the overall rate,

$$\lim|_{\tilde{N}\to 0} F_{\tilde{N}}(\tilde{E}) = F_0(\tilde{E}) \quad \simeq F_{\rm hg}(\tilde{E})$$

The suppression will arise from this limit (not from the lack of Energy!)

Bezrukov, Levkov, Rebbi, Rubakov & Tinyakov 2003

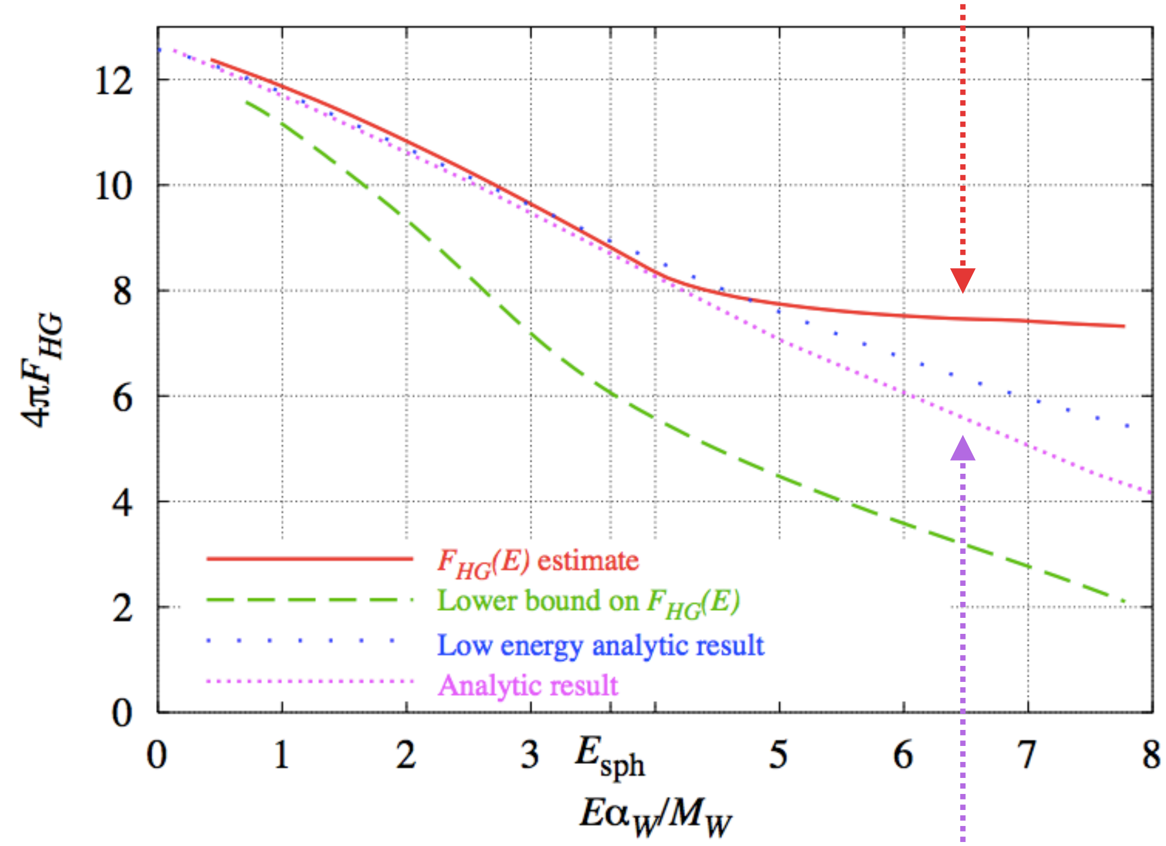
The BLRRT approach (from 1/alpha to 2 initial quanta)



So this is a pessimistic estimate

not entirely surprising, given the assumptions

BLRRT N -> 0 estimate



Instanton-Valley estimate (KR)

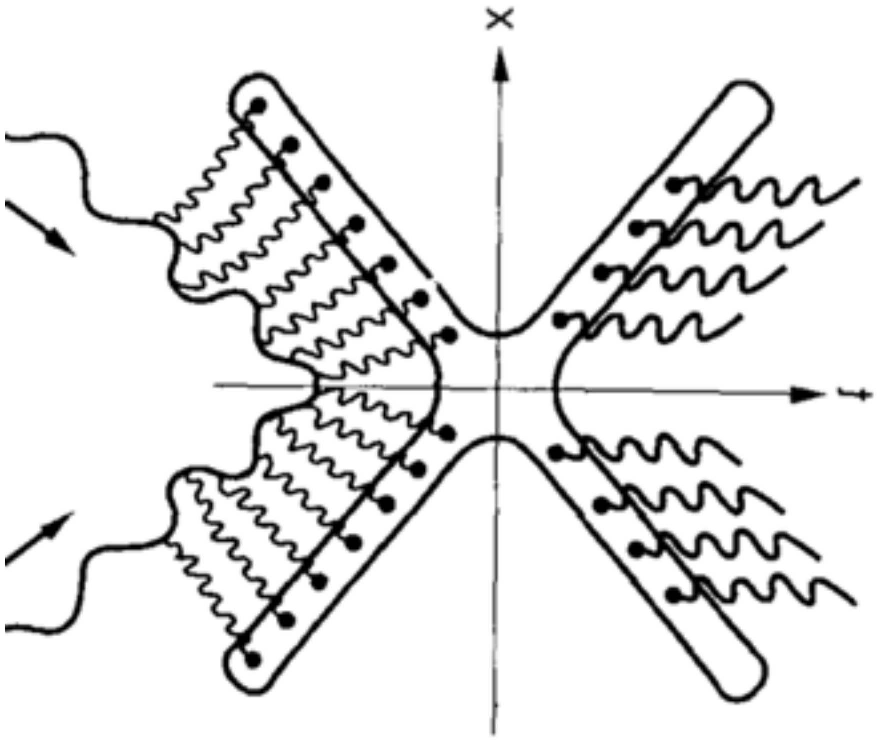
this is a pessimistic estimate, but not completely without a hope…

Bezrukov, Levkov, Rebbi, Rubakov & Tinyakov 2003

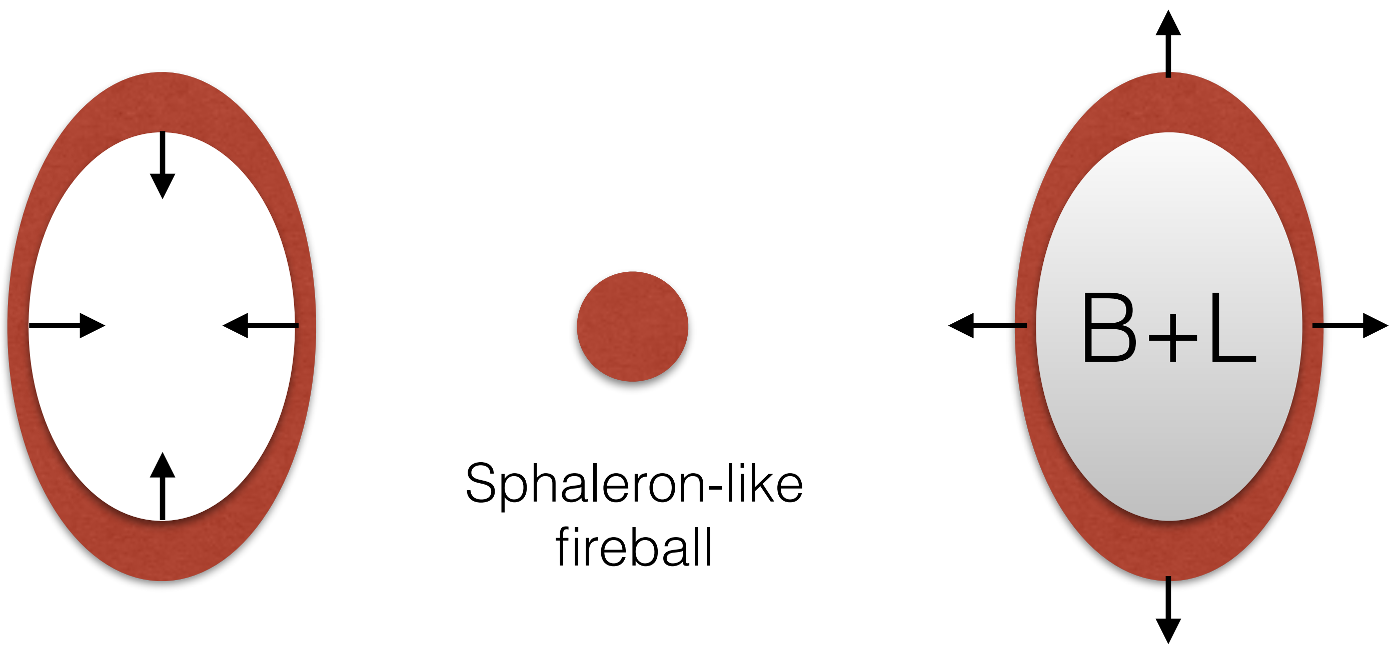
Pessimistic vs optimistic pictures

Optimistic view:

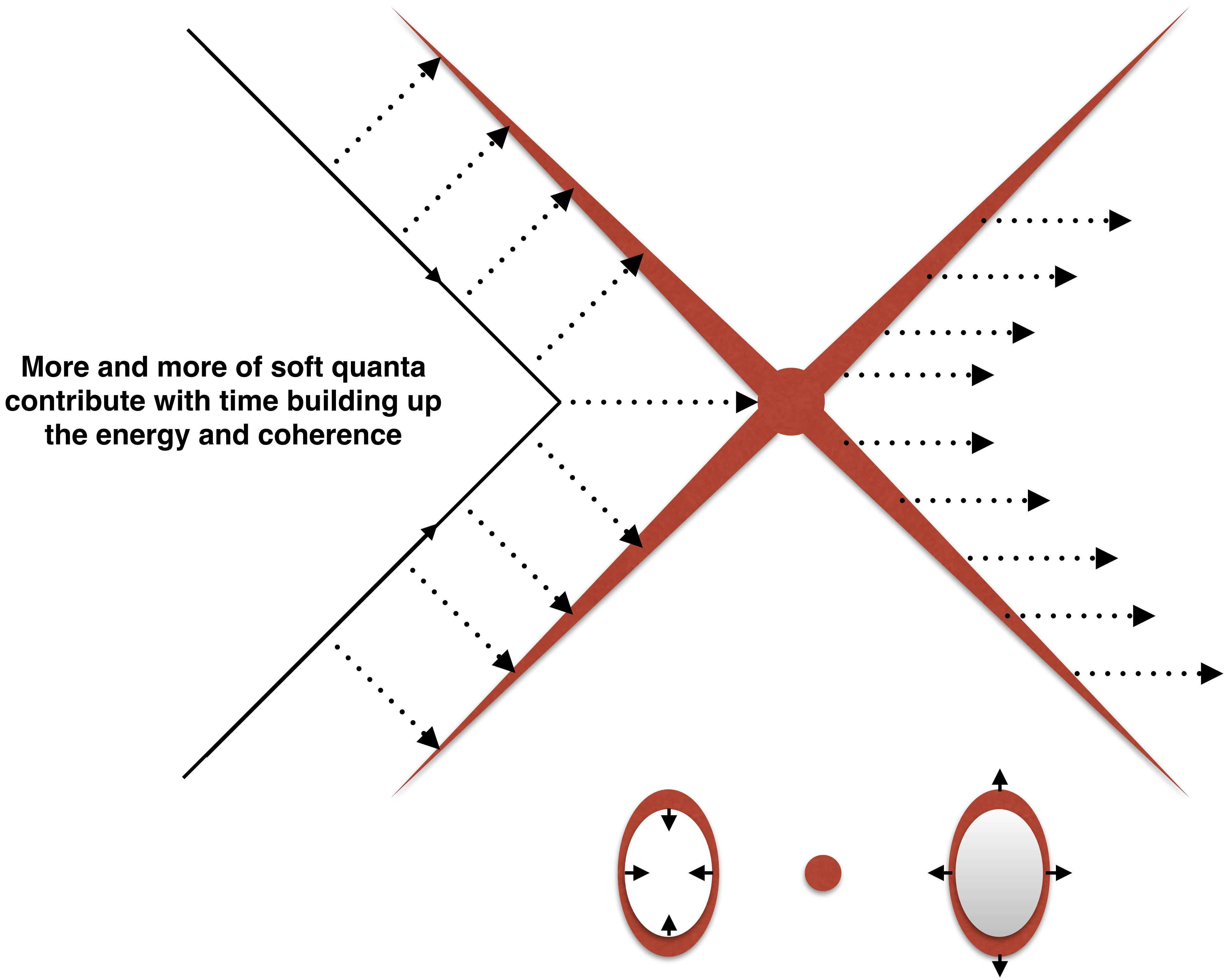
1. It is not the sphaleron which is **directly** created in the initial collision
2. Instantons in Minkwoski space are not point-like configurations; they are localized near the light-cone:



Cartoon of snapshots in time:



Initial state in the instanton approach after all may not be semiclassical



Perturbative high-n amplitudes in phi^4

review: Voloshin '1994

Tree-level $1_{\rm virtual} \to n_{\rm on-shell}(p_i = 0, E_i = m)$ in ϕ^4 without SSB:

$$A_n^{(0)} = n!\,(\lambda/(8m^2))^{\frac{n-1}{2}}$$

in ϕ^4 with SSB:

$$A_n^{(0)} = -\,n!\,(2v)^{1-n}$$

Tree-level + 1-loop in ϕ^4 with SSB:

$$A_n^{(0)+(1)} = A_n^{(0)}\left(1 + n(n-1)\frac{\sqrt{3}\lambda}{8\pi}\right)$$

Perturbation theory clearly breaks down at $n \sim 1/\sqrt{\lambda}$ at least near on-mass-shell. Need to compute at higher moment, integrate over the phase space, extend to gauge-Higgs theory...

in SU(2) gauge-Higgs theory at tree-level:

# of external gauge bosons	3	4	5	6	7	8	9
# of color-dressed diagrams	1	7	55	730	11410	226765	5230225

but there are better methods: Dai, Melnikov, Caola 2012

Conclusions

- Processes with high multiplicities of EW particles in the final state (say 50) at energies ~3 Esphaleron (>30 TeV) provide us with **physics opportunities** which are **completely unique to the very high energy pp machine**.
  This cannot be done anywhere else.

- These are not only non-perturbative B+L violating processes, but **also B+L preserving processes** with high multiplicities where at these energies (at least near the kinematic thresholds) perturbation theory breaks down — somewhat in parallel with opening up sphaleron transition channels

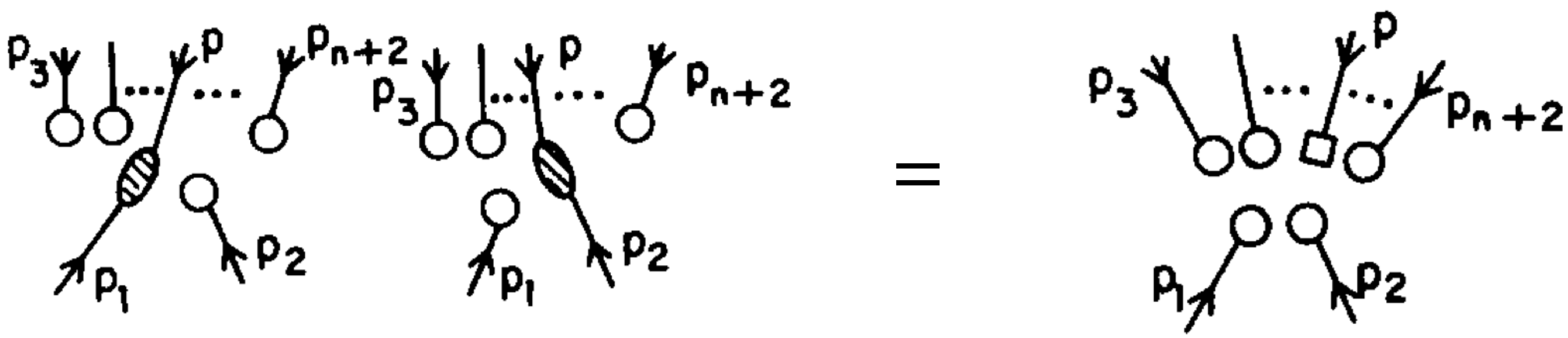
- A hard theoretical problem. Needs new computational methods. Nothing is guaranteed.

- If the rates are not suppressed => Experimental signatures should be clean.

additional slides

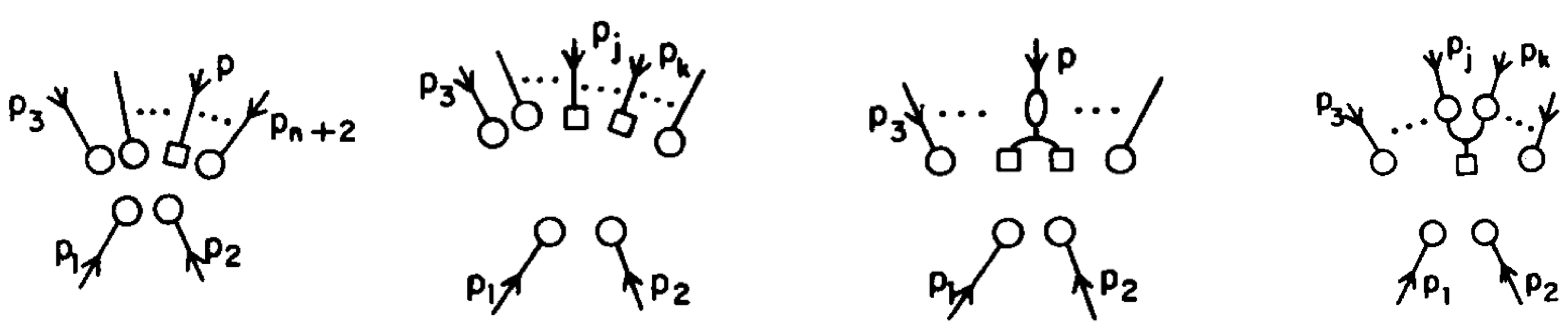
Comment: Initial states corrections [hard-soft corrs]

- Semiclassical valley estimate did not include interactions involving initial states
- Initial states corrections can exponentiate too and change the picture entirely
- However there is a possibility that these corrections can be accounted by a modified background configuration (`valley with an in-state back reaction')



Mueller 1992

- To higher orders in the coupling we have:



- Everything is again reduced to tree graphs involving a new source term. Is there a modified `classical' instanton-type configuration?

B+L at very high energies (in the last century)

- The **sphaleron** saddle-point solution in the EW sector is discovered in **1984**. 10 TeV is the new scale in the SM.

- The **1985** paper by Kuzmin, Rubakov & Shaposhnikov opens up the new research arena: electroweak baryon non-conservation and baryogenesis in the Early Universe.

- Ringwald in his **1990** paper triggers enormous interest (& controversy) in the theory community in EW baryon and lepton number violating processes at **high energy collisions.**

- **1990-1993** : The instanton calculational formalism is being developed for EW baryon and lepton number violating processes at future hadron colliders: *physics motivation* — applications to the *SSC*.

- In 1993 the SSC project is cancelled. The LHC at 14 TeV doesn't come close to the `minimal' ~30 TeV energy required to start probing the EW sphaleron barrier. This signals the end of the early golden age of B+L.