Beamstrahlung and energy acceptance

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Thanks to Y. Zhang and D. Shatilov

Introduction

- Beamstrahlung is synchrotron orbit radiation during the beam-beam interaction.
- Energy loss due to beamstrahlung results a tail of momentum spread and bunch length.
- The bunch lengthening make worse the beam-beam performance.
- The energy tail results beam lifetime shortening: i.e. a large momentum acceptance is required.
- We estimate longitudinal beam tail and lifetime using simulations.

Beamstrahlung

Synchrotron radiation emitted by the beam-beam interaction.

- Beam-beam force

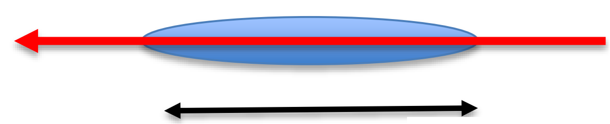
$$\Delta p_x = \frac{N r_e}{\gamma} \frac{x}{(\sigma_x + \sigma_y)\sigma_x} \qquad \Delta p_y = \frac{N r_e}{\gamma} \frac{y}{(\sigma_x + \sigma_y)\sigma_y}$$

- Typical $x \sim \sigma_x$ $\quad y \sim \sigma_y$

$$\Delta p_{x,y} \approx \frac{N r_e}{\gamma} \frac{1}{\sigma_x + \sigma_y}$$

- Interaction length of collision

$$\Delta s = \sqrt{\pi/2}\,\sigma_z$$



$\sqrt{2\pi}\sigma_z$

- Curvature of the beam traiectory

$$\frac{1}{\rho_{xy}} = \frac{\Delta p_{x,y}}{\Delta s} = \frac{4 N r_e}{\sqrt{2\pi}\gamma} \frac{1}{(\sigma_x + \sigma_y)\sigma_z}$$

Synchrotron radiation

- Number of photon $$N_\gamma = \frac{5\sqrt{3}\alpha\gamma}{6\rho}\Delta s$$
- Characteristic energy $$u_c = \frac{3\hbar c\gamma^3}{2\rho}$$

$$\frac{1}{\rho_{xy}} = \frac{4Nr_e}{\sqrt{2\pi}\gamma}\frac{1}{(\sigma_x+\sigma_y)\sigma_z}$$

- ρ=23.5m << ρ_{bend}= 6,094m (CEPC)
- P=19.7m << ρ_{bend}= 11,000m (TLEP)

- u_c= 164 MeV(CEPC) 194 MeV(TLEP-H) << E_0
- N_γ=0.21(CEPC) 0.092(TLEP-H) <<1

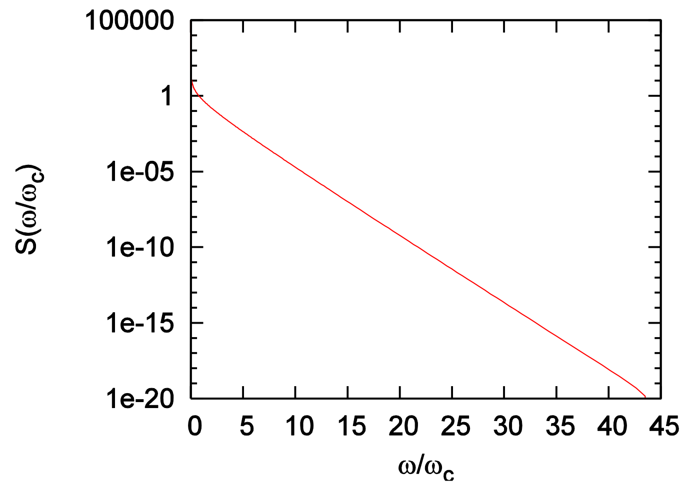
Synchrotron radiation

- photon number spectrum

$$\omega_c = \frac{3c\gamma^3}{2\rho}$$

$$\frac{dn_\gamma(\omega)}{d\omega} = \frac{\sqrt{3}\alpha\gamma\Delta s}{2\pi\rho\omega_c} S(\omega/\omega_c)$$

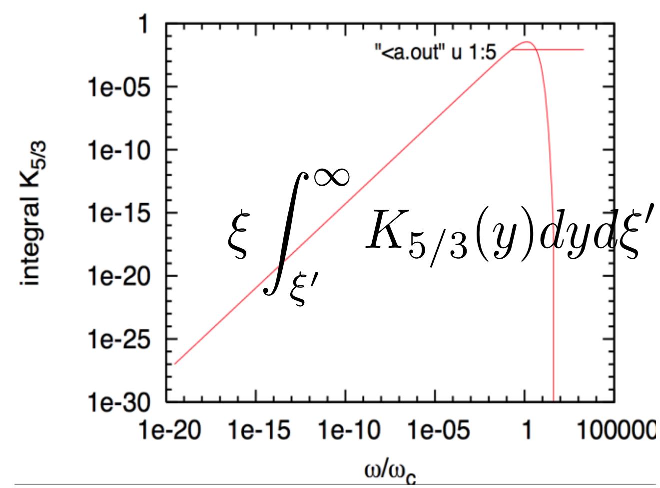
$$S(\xi) = \int_\xi^\infty K_{5/3}(y)dy.$$



- Power spectrum

$$P(\omega) = \hbar\omega \frac{dn_\gamma(\omega)}{d\omega}$$

$$= \frac{\sqrt{3}\alpha\hbar\gamma\Delta s}{2\pi\rho} \Delta s \frac{\omega}{\omega_c} S(\omega/\omega_c)$$



Energy loss due to beamstrahlung

- Averaged photon energy

$$\langle u \rangle = \frac{\int \hbar\omega n_\gamma(\omega) d\omega}{\int n\gamma(\omega) d\omega} = \frac{3}{5\pi}\hbar\omega_c \int_0^\infty \xi S(\xi) d\xi = \frac{8}{15\sqrt{3}}\hbar\omega_c$$

$$\langle \Delta E \rangle = n_\gamma \langle u \rangle \Delta s = \frac{2\hbar\alpha c \gamma^4}{3\rho^2}$$

$$\frac{\langle \Delta E \rangle}{E_0} = \frac{16\sqrt{\pi/2}}{3\pi} r_e^3 \gamma \left(\frac{N_e}{\sigma_z(\sigma_x + \sigma_y)} \right) \sigma_z$$

$$\langle d\delta_{BS} \rangle = \frac{\langle \Delta E \rangle}{E_0} = 0.864 r_e^3 \gamma \left(\frac{N_e}{\sigma_z(\sigma_x + \sigma_y)} \right)^2 \sigma_z$$

Diffusion due to beamstrahlung

Diffusion due to photon energy deviation

$$\langle u^2 \rangle = \frac{\int \hbar^2 \omega^2 n_\gamma(\omega) d\omega}{\int n\gamma(\omega) d\omega} = \frac{3}{5\pi}(\hbar\omega_c)^2 \int_0^\infty \xi^2 S(\xi) d\xi = \frac{11}{27}(\hbar\omega_c)^2$$

$$\frac{\langle \Delta E^2 \rangle}{E_0^2} = \frac{n_\gamma \langle u^2 \rangle}{E_0^2} = n_\gamma \frac{11}{27}(\hbar\omega_c)^2 = \frac{275}{64 n_\gamma}\left(\frac{\langle \Delta E \rangle}{E_0}\right)^2$$

Diffusion due to n_γ<1

Combined and integrate realistic particle orbit.

$$\sqrt{\langle d\delta_{BS}^2 \rangle} = \langle d\delta_{BS} \rangle \sqrt{0.1639 + \frac{5.129}{n_\gamma}}$$

$$\sqrt{\langle d\delta_{BS}^2 \rangle} = \langle d\delta_{BS} \rangle \sqrt{0.333 + \frac{4.583}{n_\gamma}}.$$

Equilibrium bunch length

- The diffusion is accumulated during radiation damping time, and equilibrium spread is achieved.

$$\sigma_{\delta,BS} = \frac{1}{2}\sqrt{\tau_z N_{IP}}\sqrt{\langle d\delta_{BS}^2\rangle}$$

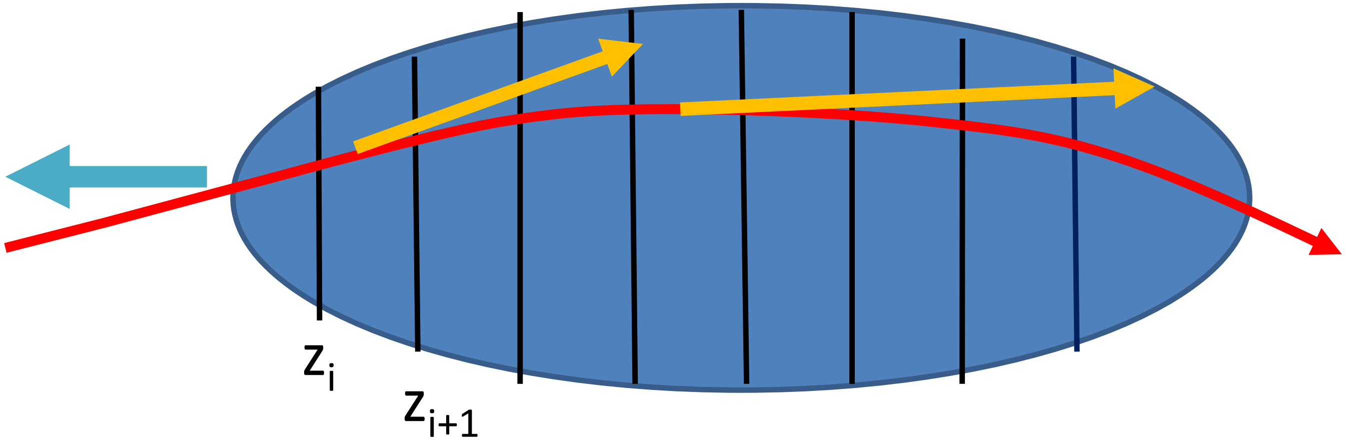
$$\sigma_{\delta,tot} = \sqrt{\sigma_{\delta,SR}^2 + \sigma_{\delta,BS}^2}$$

$$\sigma_{z,tot} = \sigma_{z,SR}\sigma_{\delta,tot}/\sigma_{\delta,SR}$$

- Self-consistent solution for both beams.

Schematic view of the simulation

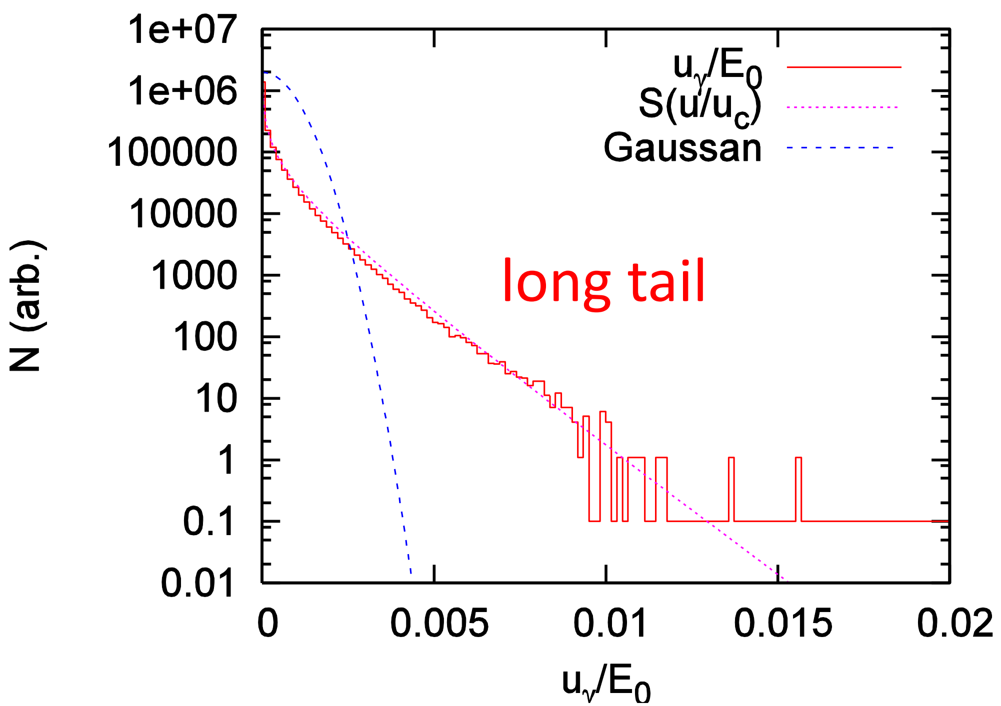
- Calculate trajectory interacting with colliding beam.
- Particles emit synchrotron radiation due to the momentum kick $dp/ds=1/\rho$.



$$ds = \frac{z_i - z_{i+1}}{2} \qquad \frac{1}{\rho_{x,y}} = \frac{dp_{x,y}}{ds}$$

Beamstrahlung spectrum

- γ energy spectrum, $N_e = 10^7$, $N_\gamma \sim 2\times10^6$

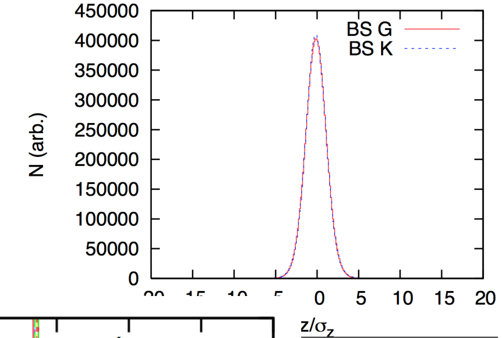


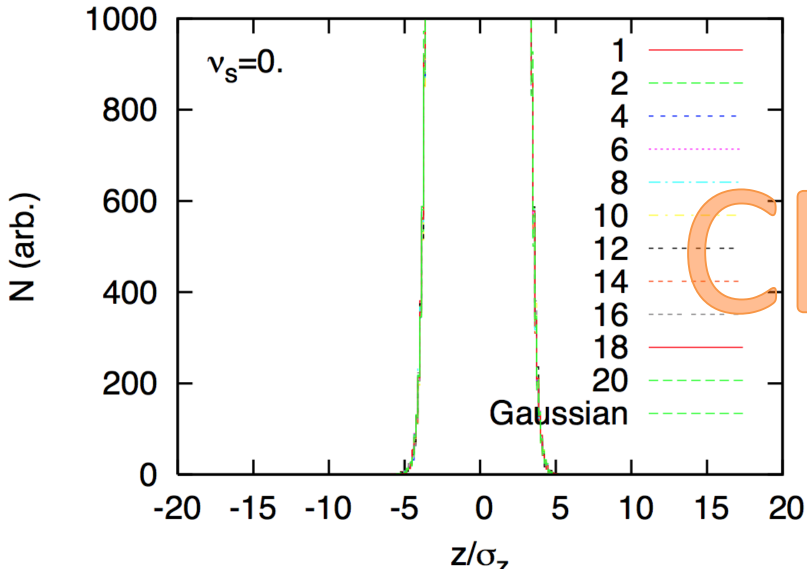
Careful treatment for energy tail is necessary.

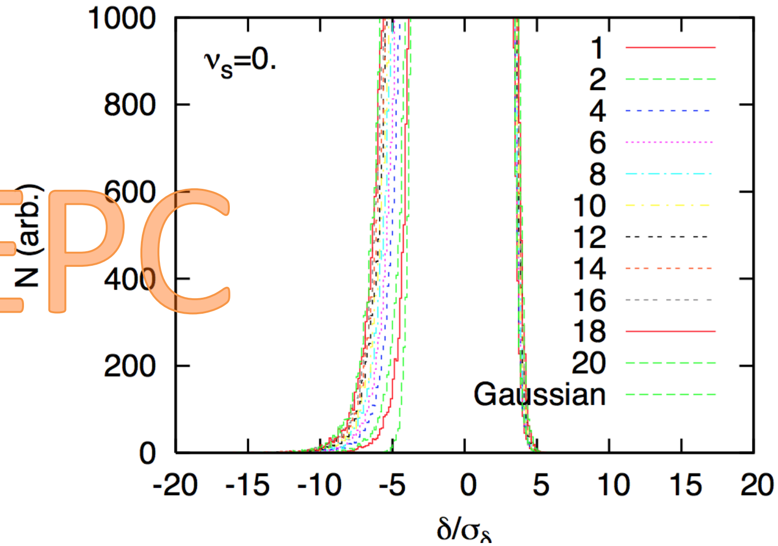
Gaussan $\exp\left(-\frac{27E_0^2}{11u_c^2}\frac{u^2}{E_0^2}\right) \approx \exp\left(-10^6\frac{u^2}{E_0^2}\right)$ No tail

Evolution of energy distribution in First 10 turn (20 interactions)

- No synchrotron motion
- $N_{tot}=10^7$
- $\sigma_z=2.1\text{mm}$, $\sigma_\delta=0.13\%$





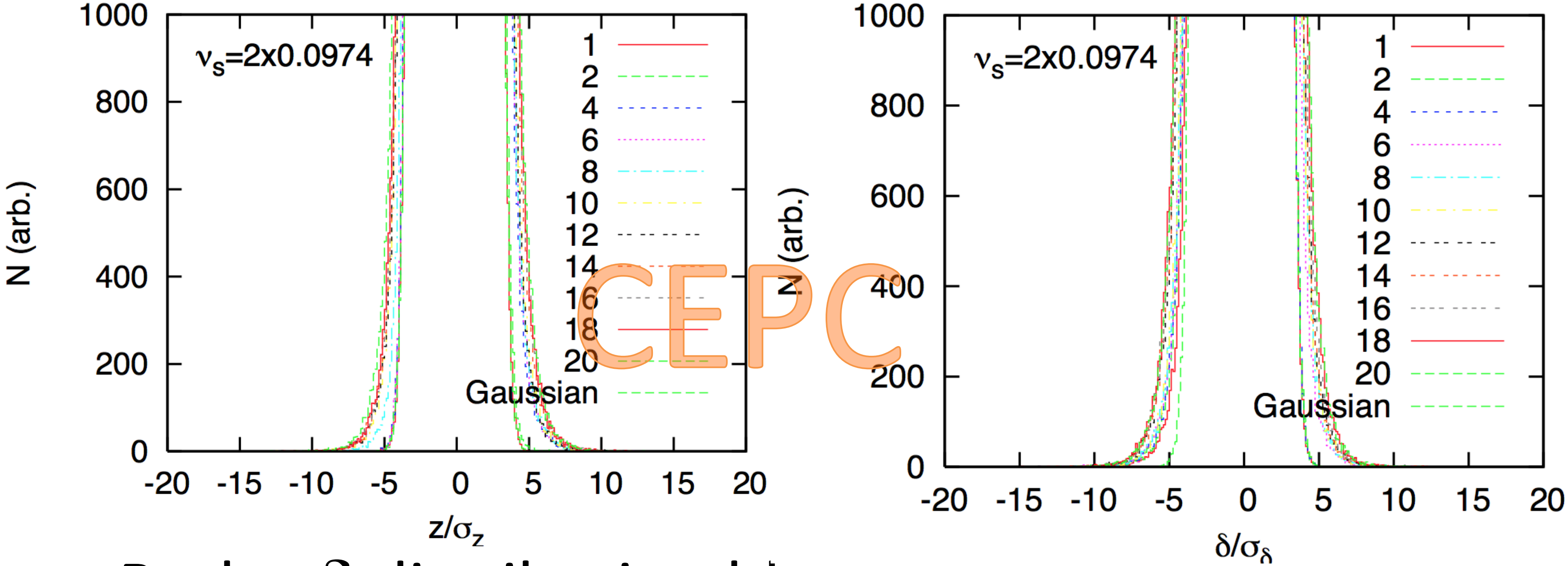


CEPC

- Energy deviation is accumulated lower energy part.

First 10 turn (20 interactions)

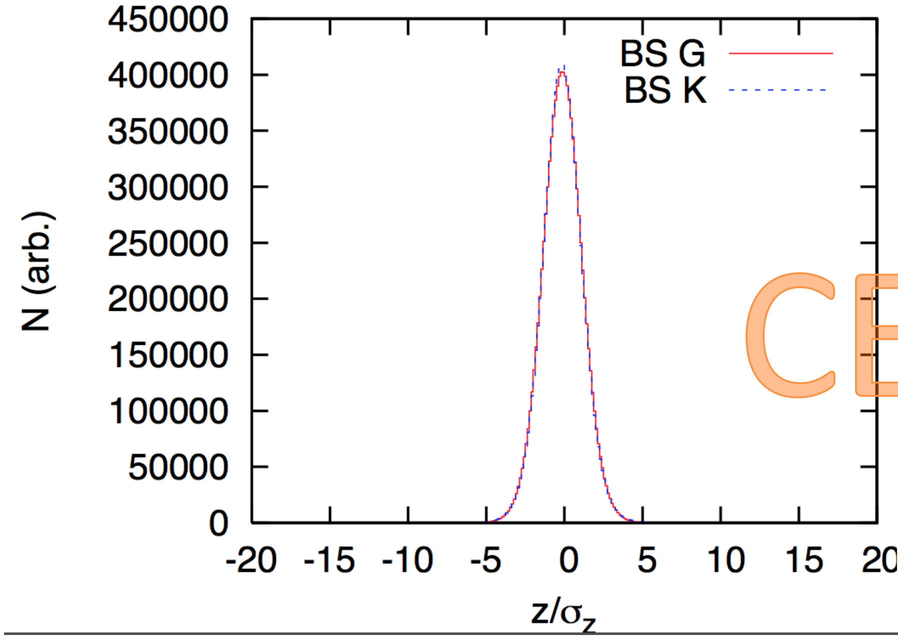
- With Synchrotron motion

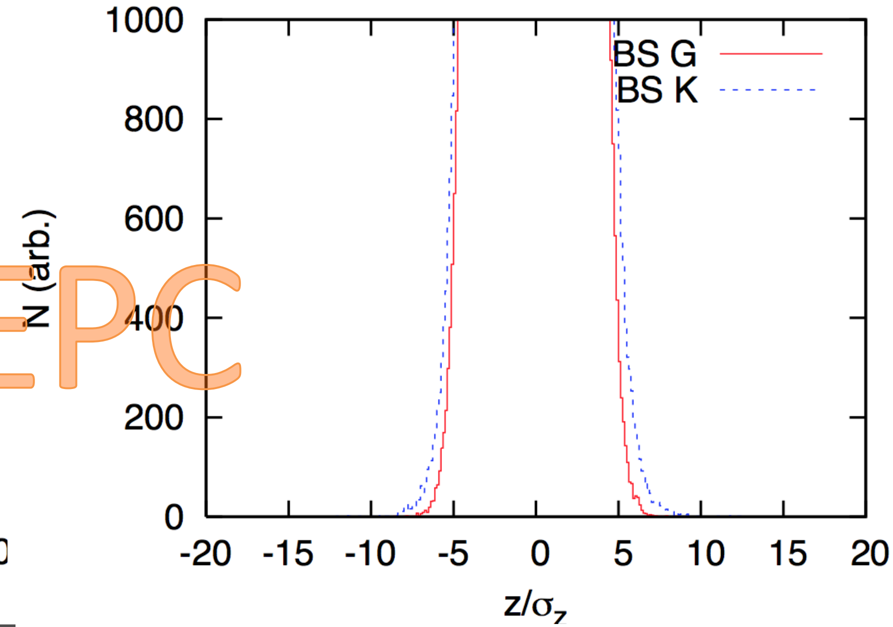


- Both z,δ distribution blow up
- Energy deviation is less accumulated.

Equilibrium distribution-z

- After 1000 turn (1000/80=12 damping time)



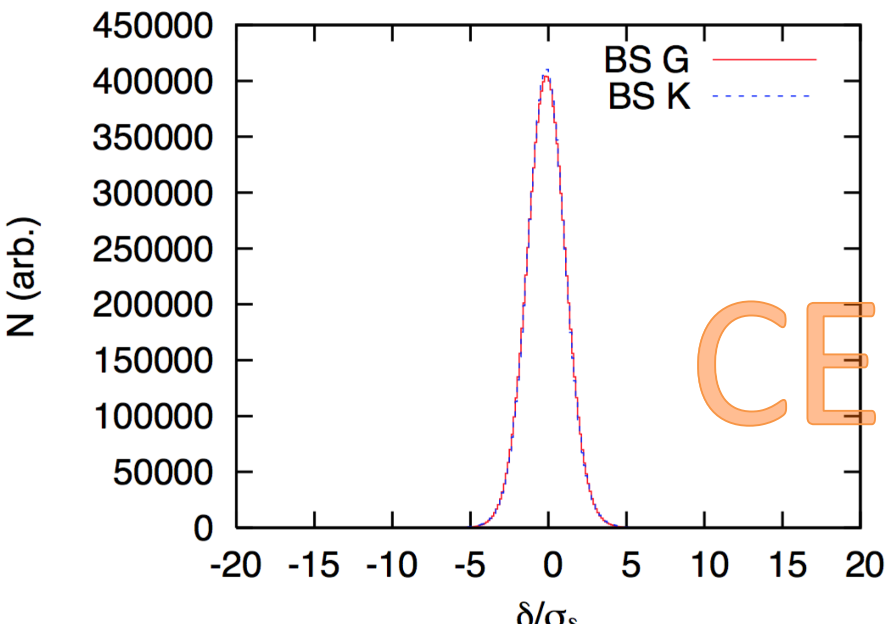


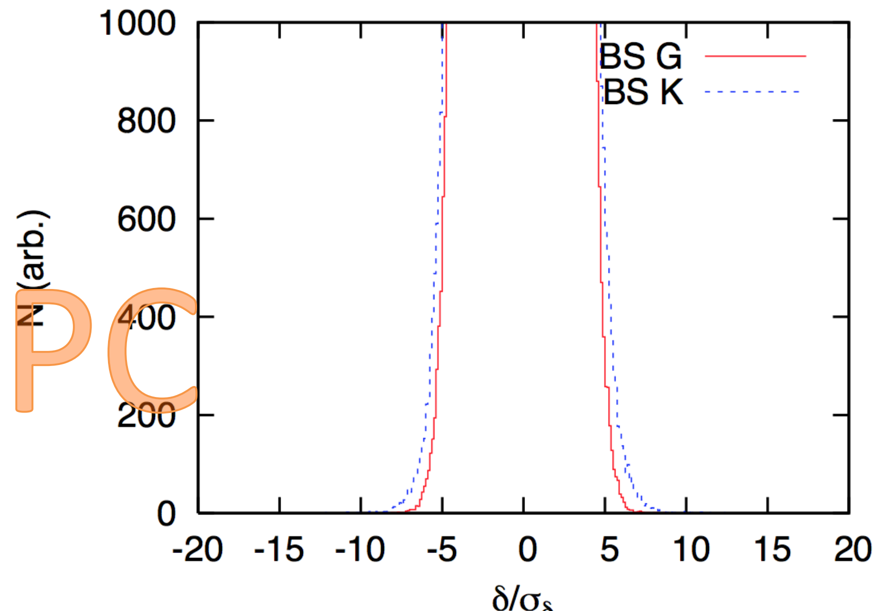
CEPC

- Tail is enhanced a little for K Bessel formula.
- σ_z=2.1mm

Equilibrium distribution- δ

- 1000 turn=12 damping time
- Equilibrium distribution – $\delta=\Delta p/p_0$





CEPC

- Tail is enhanced a little for K Bessel formula.
- σ_δ=0.13%

Lifetime M. Sands

- Distribution function in phase space

$$f(J_i) = \frac{1}{\varepsilon_i}\exp(-J_i/\varepsilon_i) \qquad \int_0^\infty dJ_i f(J_i) = 1$$

- Damping of amplitude

$$\frac{dJ}{dt} = -\frac{2J}{\tau_i}$$

- Loss per turn equals to density times damping at A.

$$\frac{dN}{dt} = -\frac{N}{\tau_l} = -Nf(A_i)\left.\frac{dJ_i}{dt}\right|_{J_i=A_i} = -\frac{2A_iN}{\tau_i}f(A_i)$$

- Lifetime for Gaussian distribution

$$\tau_l = \frac{\tau_i}{2A_i f(A_i)} = \frac{\tau_i}{2}\frac{\varepsilon_i}{A_i}\exp(A_i/\varepsilon) \qquad A_i = \frac{\beta_i x_{max}^2}{2}$$

Using variable $r_i = \sqrt{2J_i/\varepsilon_i}$

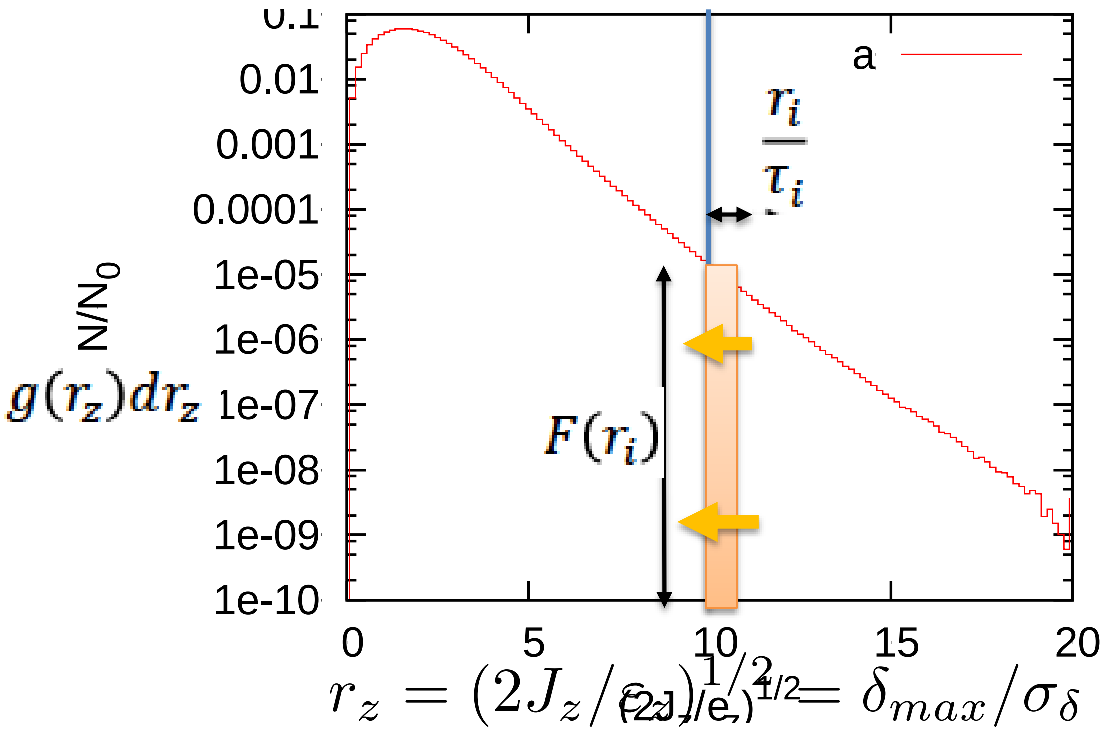
- Distribution function g(r) is given numerically
- Life time for g(r)

$$\frac{dN}{dt} = -\frac{N}{\tau_l} = Ng(r_i)\frac{dr_i}{dt} = -Ng(r_i)\frac{r_i}{\tau_i}$$

$$\tau_l = \frac{\tau_i}{r_i g(r_i)}$$

$$dr_z = \frac{20}{128}$$

$$dr_y = \frac{64}{128}$$



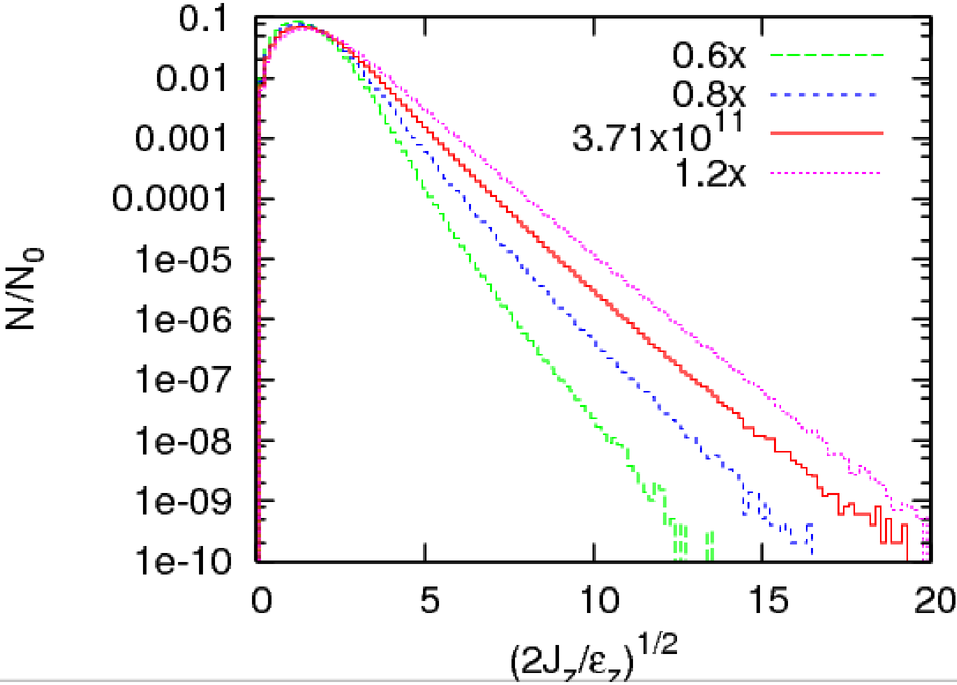
Very long term tracking using BBWS

- Self consistent σ_z, averaged every 1000 turns.
- n_p=100, n_{turn}=1x10^8/2 turns. $n_{turn}T_0$=2.3 hours.
- Distribution is accumulated every ½ turn after 10000/2 turns, n_pxn_{taccum}=9.999x10^9.

- 48 bit random number generators are used and compared. (No difference from 32bit)

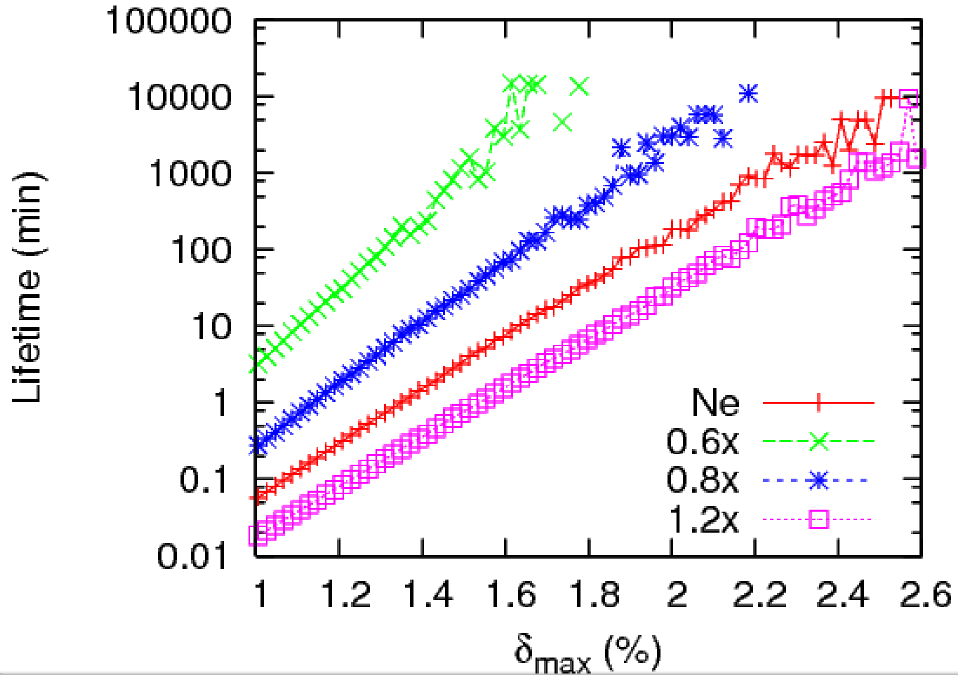
z tail distribution and lifetime (CEPC)

- r_z distribution



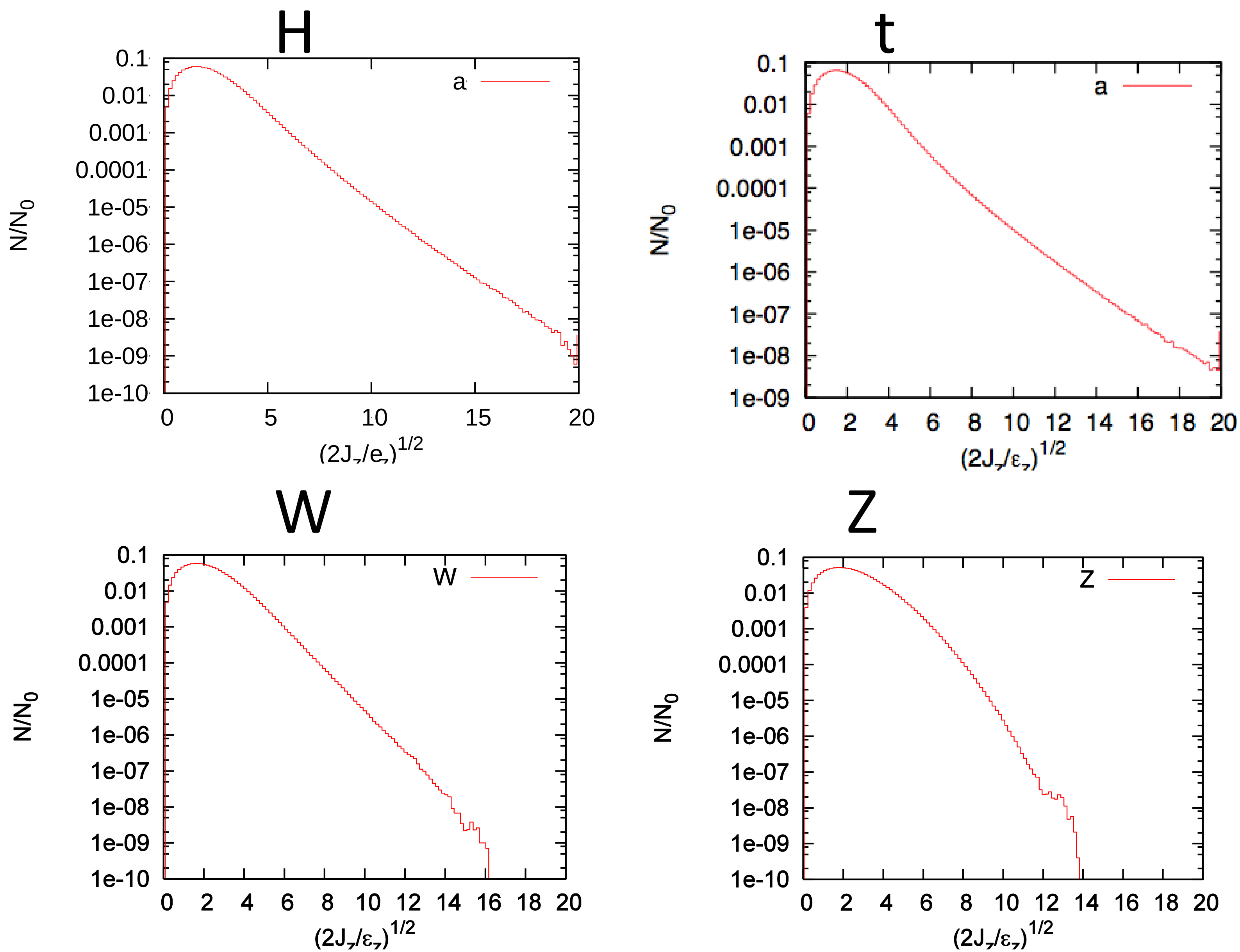
$= \delta_{max}/\sigma_\delta$

Lifetime

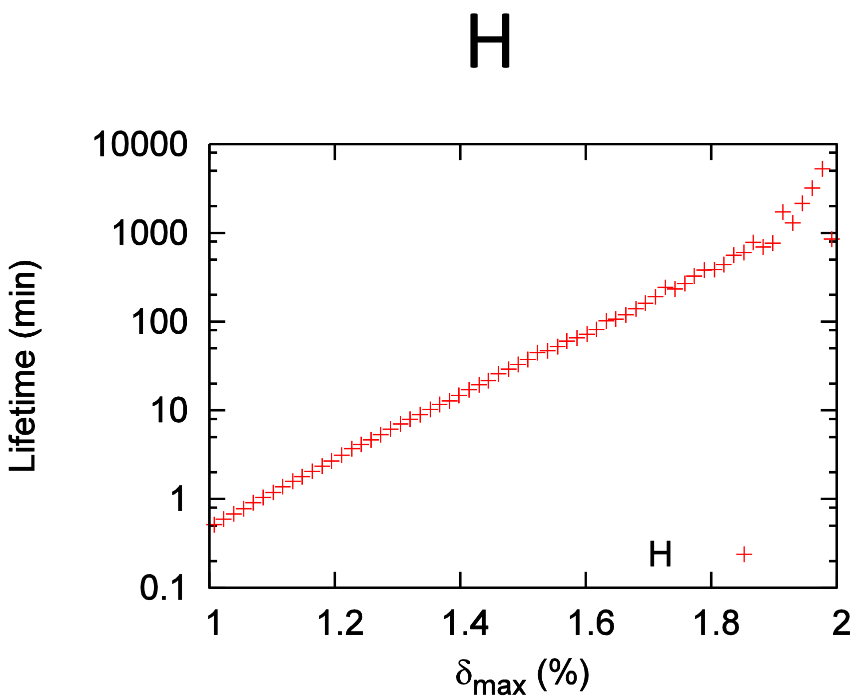


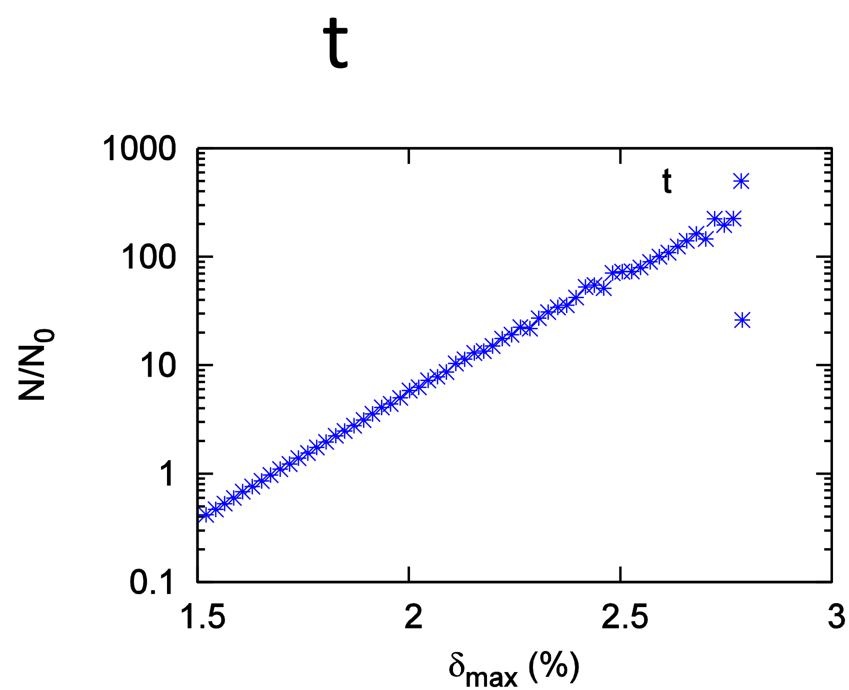
30-40min for δ_{max}=1.8%

TLEP Energy tail distribution



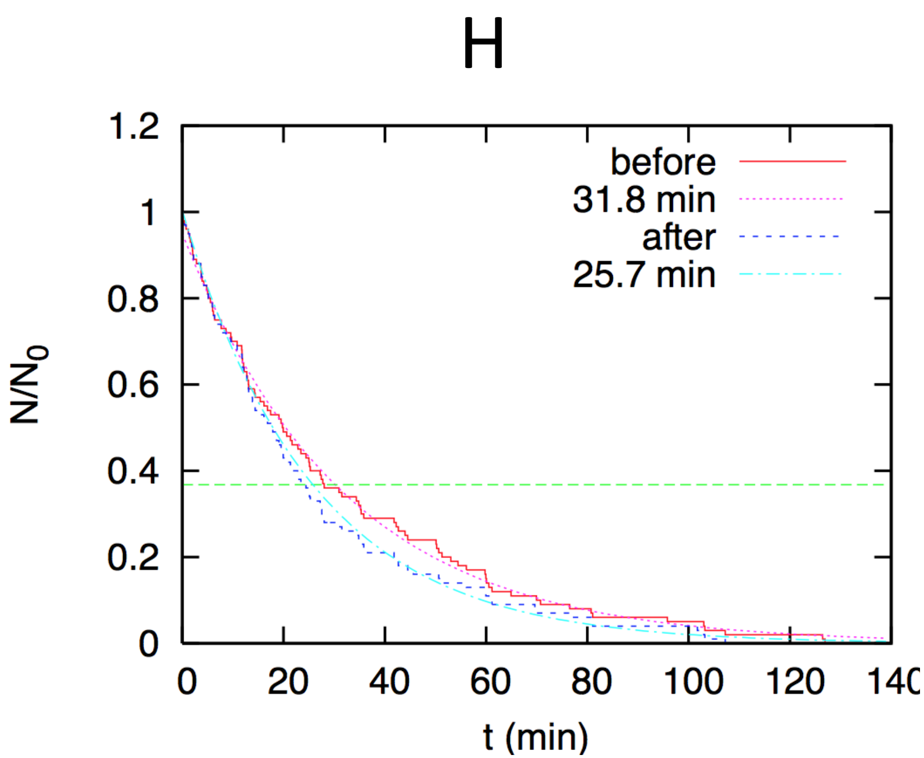
Lifetime estimation (TLEP)

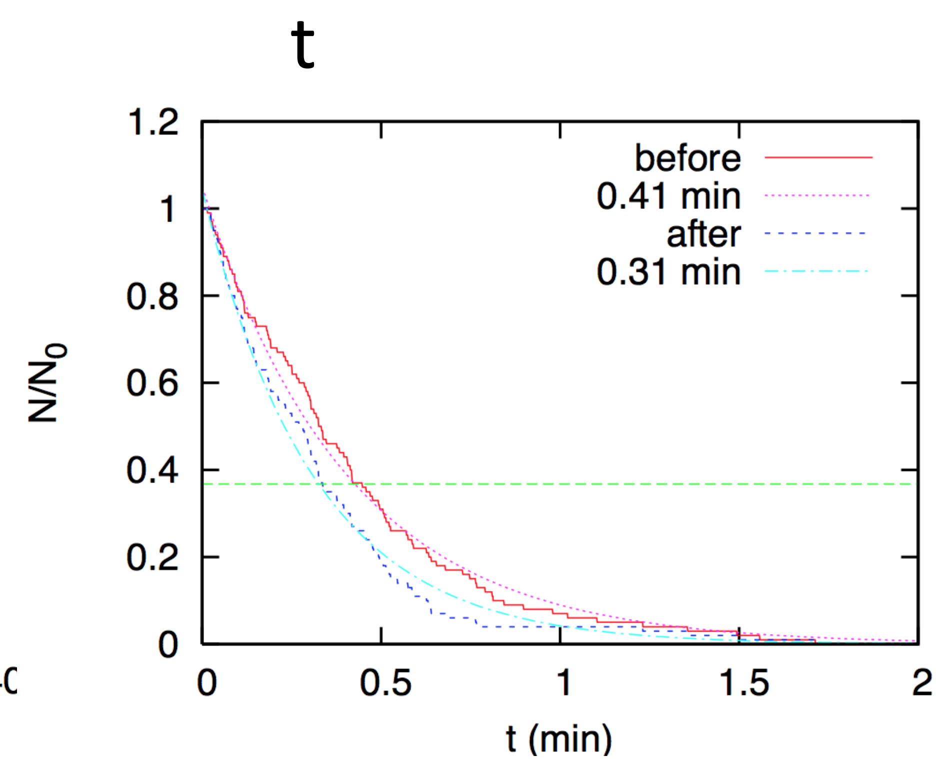




Lifetime 27 min (H) and 0.39 min (t) at δ_{max}=1.5%

Beam loss simulation





Setting 1.5% aperture, survived particles is counted.

Agree with the estimation from the damping flow.

Analytic formula (Telnov, Bogomyagkov)

$$\tau_{\mathrm{bs,Telnov}} = \frac{\mathbf{1}0}{f_0}\frac{4\sqrt{\pi}}{3}\sqrt{\frac{\eta}{\alpha r_e}} \times \exp\left(\frac{2}{3}\frac{\eta\alpha}{r_e\gamma^2}\times\frac{\gamma\sigma_x\sigma_z}{\mathbf{2}r_e N_p}\right)\frac{\mathbf{2}}{\sigma_z\gamma^2}\left(\frac{\gamma\sigma_x\sigma_z}{\mathbf{2}r_e N_p}\right)$$

1 is necessary

$$\tau_{bs} = \frac{1}{f_0}\frac{4\sqrt{\pi}}{3}\sqrt{\frac{\eta}{\alpha r_e}}\exp\left(\frac{2}{3}\frac{\eta\alpha}{r_e\gamma^2}\times\frac{\gamma\sigma_x\sigma_z}{\sqrt{\mathbf{2}}r_e N_p}\right) \times \frac{\sqrt{\mathbf{2}}}{\sqrt{\pi}\sigma_z\gamma^2}\left(\frac{\gamma\sigma_x\sigma_z}{\sqrt{\mathbf{2}}r_e N_p}\right)^{3/2},$$

$\eta = \delta_{max} = 1.5\%$

Table 2: Expected and simulated BS lifetime.

τ_{BS} [min]	TLEP-H	TLEP-t	CepC
analytical [9]	310	3.6	113
analytical [8]	1400	3.3	619
weak-strong (loss)	26	0.3	5.5
weak-strong (distr.)	33	0.3	~5

- The lifetime in simulations is shorter than analytical estimates.
- Better agreement for considering Dynamic beta.
- Detailed analysis is done by M. Koratzinos.

Summary

- Beamstrahlung in TLEP and CEPC has been treated using Gaussian and Realistic K Bessel formula.
- Longitudinal beam tail due to beamstrahlung is enhanced. It is essential for lifetime.
- Long term tracking including beamstrahlung has been done using weak strong simulation. For only longitudinal, medium term and many particle tracking is available.
- Beam lifetime was evaluated by the longitudinal and transverse distributions.
- The lifetime estimated by the damping flow agree with beam loss simulations.
- $\Delta\delta_{max}$>1.8%(CEPC and TLEP-H) and >2.6% (TLEP-t) are required in present beam parameter.

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Nest step

- Dynamic beta, which reduce σ_x, enhances energy tail. Optimize β_x.
- Precise eradiation excitation matrix to estimate correct dynamic beta/emittance.
- Optimize β_y to increase momentum acceptance.

Thank you for your attension

Parameters for CEPC

- C=53.6km, E=120 GeV
- (ν_x, ν_y, ν_s)=(0.54,0.61,0.103)x2
- $(\varepsilon_x, \varepsilon_y, \varepsilon_s)$=(6.79e-9,2.04e-11,2.938e-6) [m]
- $(\beta_x^*, \beta_y^*, \beta_s)$=(0.8, 0.0012,1.74) [m]
- $(\sigma_z, \sigma_\delta)$=(2.26(2.58)mm, 0.13(0.15)%)
- (τ_x, τ_y, τ_s)=(159.4,159.4,79.7)/2 [turn]
- $N_e=3.71\times10^{11}$, $N_b=50$
- NIP=2
- α_p=4.15e-5
- Nslice=20,ne=10000

Parameters for TLEP-H

- C=100km, E=120 GeV
- (ν_x, ν_y, ν_s)=(0.54,0.61,0.024)x4
- $(\varepsilon_x, \varepsilon_y, \varepsilon_s)$=(0.94e-9,1.9e-12,0.81e-6) [m]
- $(\beta_x^*, \beta_y^*, \beta_s)$=(0.5, 0.001,0.81) [m]
- $(\sigma_z, \sigma_\delta)$=(0.81(1.17)mm, 0.10(0.14)%) , beam strahlung
- (τ_x, τ_y, τ_s)=(568,568,284)/4 [turn]
- $N_e=0.46\text{x}10^{11}$, N_b=1360
- NIP=4
- α_p=0.5e-5
- Nslice=20,ne=10000

Lifetime formula K.Ohmi, Sep. 2 2014

- Telnov formula, PRL110, 114801(2013)

$$\omega_c = \frac{3\gamma_e^3 c}{2\rho} \qquad \rho = \frac{\sqrt{2\pi}\gamma_e(\sigma_x + \sigma_y)\sigma_z}{4N_e r_e}$$

$$u = \frac{\delta_{max} E_0}{\hbar\omega_c} \qquad n_{col} = \frac{10\sqrt{6} r_e \gamma_e u^{3/2}}{\alpha_{fsc}^2 \delta_{max} \sigma_z / 2} e^u$$

$$\tau = n_{col} \frac{C}{c}$$

Lifetime

- H
  - δ_{max}=0.015, σ_z=1.2mm, σ_x=21.7 μm, N_b=0.46x10^{11}, τ=99min
  - Dynamic beta bx 0.5m->0.345m, σ_x=18.0 μm, τ=16min
- t
  - δ_{max}=0.015, σ_z=1.6mm, σ_x=44.7 μm, N_b=1.4x10^{11}, τ=2.9min
  - Dynamic beta bx 1m->0.69m, σ_x=37 μm, τ=0.77min

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$$\tau_{bs} = \frac{1}{f_0}\frac{4\sqrt{\pi}}{3}\sqrt{\frac{\eta}{\alpha r_e}}\exp\left(\frac{2}{3}\frac{\eta\alpha}{r_e\gamma^2}\times\frac{\gamma\sigma_x\sigma_z}{\sqrt{\mathbf{2}}r_eN_p}\right) \times \frac{\sqrt{\mathbf{2}}}{\sqrt{\pi}\sigma_z\gamma^2}\left(\frac{\gamma\sigma_x\sigma_z}{\sqrt{\mathbf{2}}r_eN_p}\right)^{3/2},$$

1 is necessary

$$\eta = \delta_{max} = 1.5\%$$

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