

# Single Ring Multibunch Operation and Beam Separation

Richard Talman

Cornell University

55th ICFA Advanced Beam Dynamics Workshop  
on High Luminosity Circular  $e^+e^-$  Colliders

WG 2 "Optics" working group for HF2014  
October 9

## Abstract

- ▶ The counter-circulating electrons and positrons in a circular Higgs Factory have to be separated everywhere except at the  $N^*$  intersection points (IP).
- ▶ The separation has to be electric and, to avoid unwanted increase of vertical emittance  $\epsilon_y$ , the separation has to be horizontal.
- ▶ This paper considers only head-on collisions at  $N^* = 2$  IP's, with the beams separated by closed electric bumps everywhere else (but with nodes at RF cavities).

### 3 Outline

Electric Bump Bunch Separation

Bunch Separation at LEP

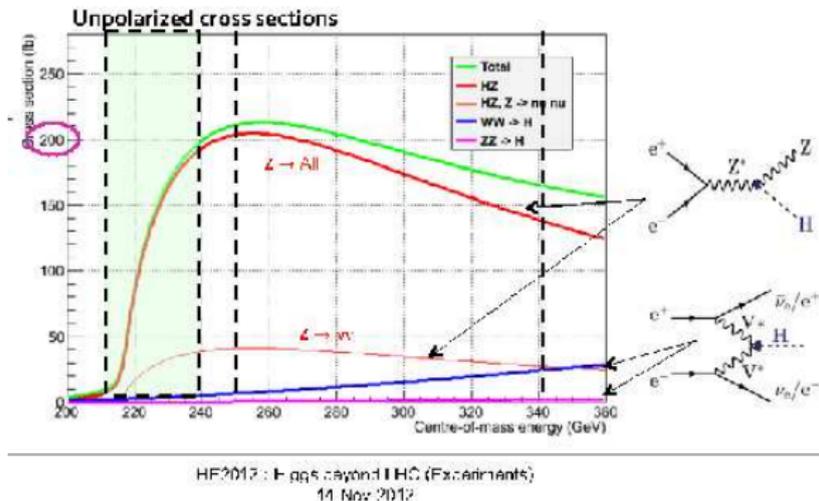
Separated Beams and RF Cavities

6 + 6 Element Closed Electric Multibump for 60 m Long Cells

Bunch Separation Partition Number Shift

Beam Separation in Long Cell Lattice

Predicted Luminosities



**Figure:** Higgs particle cross sections up to  $\sqrt{s} = 0.3 \text{ TeV}$  (copied from Patrick Janot);  $\mathcal{L} \geq 2 \times 10^{34} / \text{cm}^2 / \text{s}$ , will produce 400 Higgs per day in this range.

Refer to specific single beam energies:

- 45.6 GeV as the  $Z_0$  energy
- 80 GeV as the W-pair energy
- 100 GeV as the LEP energy
- 120 GeV as the Higgs energy
- 175 GeV as the  $t\bar{t}$  energy



- ▶ Extrapolations from LEP are based on John Jowett's article "Beam Dynamics at LEP".
- ▶ At first LEP had four bunches ( $N_b=4$ ) and four IPs ( $N^*=4$ ) operation, collisions at the 45 degree points were avoided by vertical electric separation bumps.
- ▶ It is now realized that vertical bumps are inadvisable because of their undesirable effect on vertical emittance  $\epsilon_y$ .
- ▶ I therefore consider only horizontal separation schemes.

- ▶ Various horizontal pretzel separation schemes were tried at LEP in the early 1990's. They had to be superimposed on an existing lattice and were mainly at what now would be called quite low beam energies.
- ▶ Higgs factory energies are four or five times higher. Separators have to be stronger by the same factor to obtain the same angular separation.
- ▶ But the design is not constrained by a pre-existing lattice.

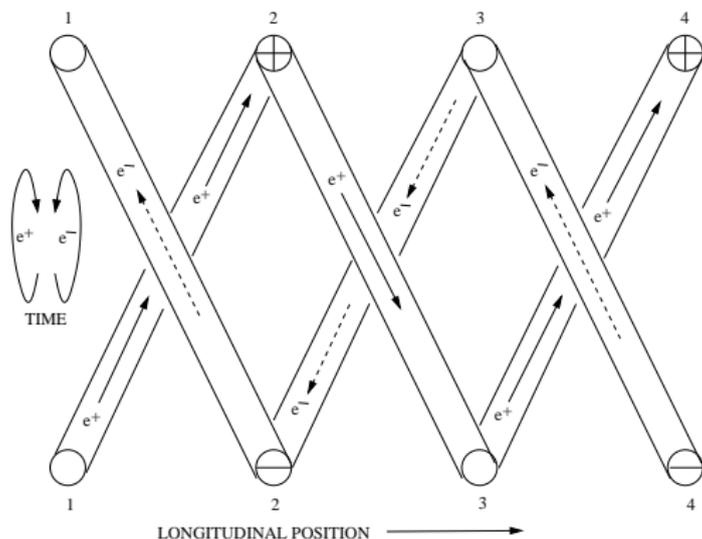
## 8 Etymology of “Pretzel Beam Separation”

- ▶ The pretzel “idea” was due to (Director) Boyce McDaniel. He realized that one could make do with a single separator, making the closed orbits of the counter-circulating beams **different everywhere**.
- ▶ At CESR there was no free space long enough, so an existing magnet had to be made shorter and stronger to free up space for an electric separator.
- ▶ Even so there are periodic “nodes” at which the orbits cross. One has only to arrange for the desired crossing points to be at nodes and the parasitic crossing points to be at “loops”.
- ▶ Raphael Littauer, the eventual inventor of workable pretzel separation, introduced the metaphorical term “pretzel” to distill this entire discussion into single word.

- ▶ At CESR the angle crossings at the collision points proved to be unacceptable. This made it necessary to use four electric separators.
- ▶ The separators were paired across North and South IR's to produce head-on collisions at the IP's.
- ▶ Strictly speaking, this invalidated the term “pretzel”, since what one had was simply separate closed electric bumps in the East and West arcs.
- ▶ The only disadvantage of this terminology is that it encourages the perception that the whole ring is one big pretzel when, in fact, the arcs are quite independent—one pretzel in each arc if one prefers.
- ▶ However the name “pretzel” stuck, and the separation scheme continues to be called “pretzel separation”.

- ▶ To emphasize this point, for this talk only, I will emphasize closing electric multibumps, arc by arc, rather than referring to an overall pretzel separation scheme.
- ▶ Separating the beam in a pre-existing ring was harder than for a not-yet-built accelerator.
- ▶ Especially by constraining the arcs to be symmetric, the electric bumps can be closed arc by arc.
- ▶ Standard closed bumps are typically  $\pi$ -bumps or  $2\pi$  bumps. But, with 4 deflectors, two at each end of a sector, bumps can easily be designed to be  $n\pi$  bumps, where  $n$  is an arbitrary integer matched to the desired number of separation points.

## 11 Jowett Toroidal Space-Time Beam Separation Plot



**Figure:** A minimal and modified “Jowett Toroidal Space-Time Beam Separation Plot” illustrating the separation of counter-circulating beams. Points with the same label at the top and the bottom of the plot are the same points (at different times). Though drawn to suggest a toroid the plot is purely two dimensional. The original McDaniel pretzel encompassed the whole ring—that is, in this figure, points 1 and 4 would also be identified. But this identification is not essential.

- ▶ In the figure, associating point 4 with point 1 would correspond to the original McDaniel pretzel scheme in which the counter-circulating orbits are different everywhere in the ring. With closed multibumps there is no such association. The separated beams are smoothly merged onto common orbits at both ends.
- ▶ (With care) the space-time plot can also be interpreted as the spatial shape of the multibump displacement pattern. A head-on collision occurs when two populated bunches pass through the same space-time point. To avoid parasitic crossings the minimum bunch separation distance is therefore twice the closed bump period.

- ▶ Another separation scheme tried at LEP was local electric bumps close to the 4 IP's and angle crossing to permit "trains" with more than one bunch per train. This permitted as many as 4 bunches per train though, in practice, more than 3 were never used. For lack of time this option is not considered in this paper.
- ▶ The primary horizontal separation scheme at LEP is illustrated in Jowett's clear, but complicated, Figure 3. The scheme used 8 primary separators and 2 trim separators with the separation bumps continuing through the 4 IP's, but with head-on collisions at all IP's. Starting from scratch in a circular collider that is still on the drawing board, one hopes for a simpler separation scheme.

14

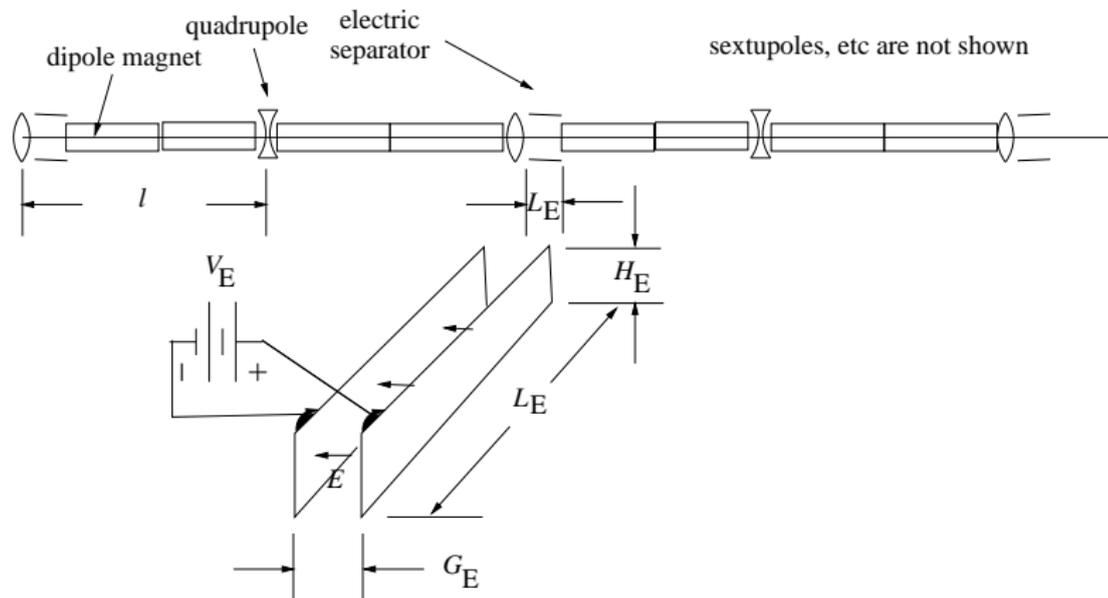
- ▶ Multibumps can be located arbitrarily without seriously perturbing any existing lattice design.
- ▶ Probably both beams should pass through the centers of the RF cavities. It seems safe to place RF cavities at bump nodes.
- ▶ Insisting on common orbits through RF cavities would allow for fewer bunches.

## 15 6 + 6 Element Closed Electric Multibump for 60 m Long Cells

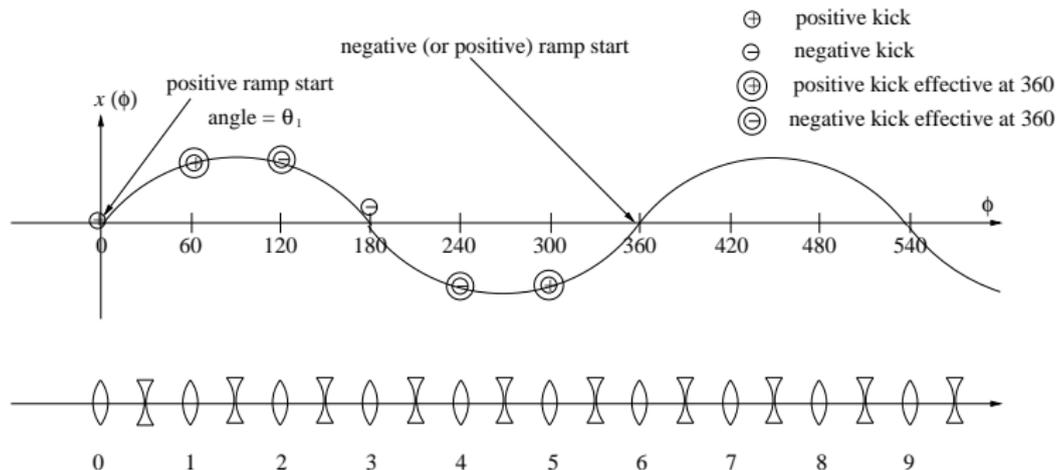
- ▶ Bunches must not collide in arcs. They should be separated by at least 10 beam width sigmas when they pass.
- ▶ A single ring is as good as dual rings if the total number of bunches can be limited to, say, less than 200.
- ▶ I discuss only the case of head-on collisions at each of the two IP's. The minimum bunch spacing is equal to the total length of the intersection region (IR).
- ▶ The half ring was shown earlier. Orbits are common only in the two IR's.
- ▶ On the exit from each IR an electric bump is started and the bump is closed just before the next IR.
- ▶ Symmetric multibumps require at least 4 controllable deflectors. Here a 12 separator multibump scheme is described.

- ▶ The design orbit spirals in significantly; this requires the RF acceleration to be distributed quite uniformly. Basically the ring is a “curved linac”.
- ▶ As with beam separation in LEP, trim separators may be required.
- ▶ Figure 3 exhibits the separation of up to 112 bunches in a 50 km ring.
- ▶ As explained earlier, to avoid head-on parasitic collisions, the bunch separations are equal to two wavelengths of the multibump pattern.

There is a (conservatively weak) electric separator in each of 6 cells at each end of each arc.



## RESONANT BUMP PHASE ADVANCES



displacement at "j" due to kick angle  $\theta_1$  at "k" =  $\hat{\beta}_x \sin(\phi_j - \phi_k) \theta_1$

$N_1$  = number of effective kicks per half bump = 4 (for 60 degree lattice)

$N_2$  = number of accumulating bump stages

$$x(120N_2) = N_1 N_2 \hat{\beta}_x \sin(\phi_j - \phi_k) \theta_1$$

$$= 4 \times 69.3\text{m} \times \theta_1 \frac{\sqrt{3}}{2} N_2 \quad (\text{for 60 degree lattice})$$

NOTE: deflections by arc quadrupoles are typically greater than electric separator deflections

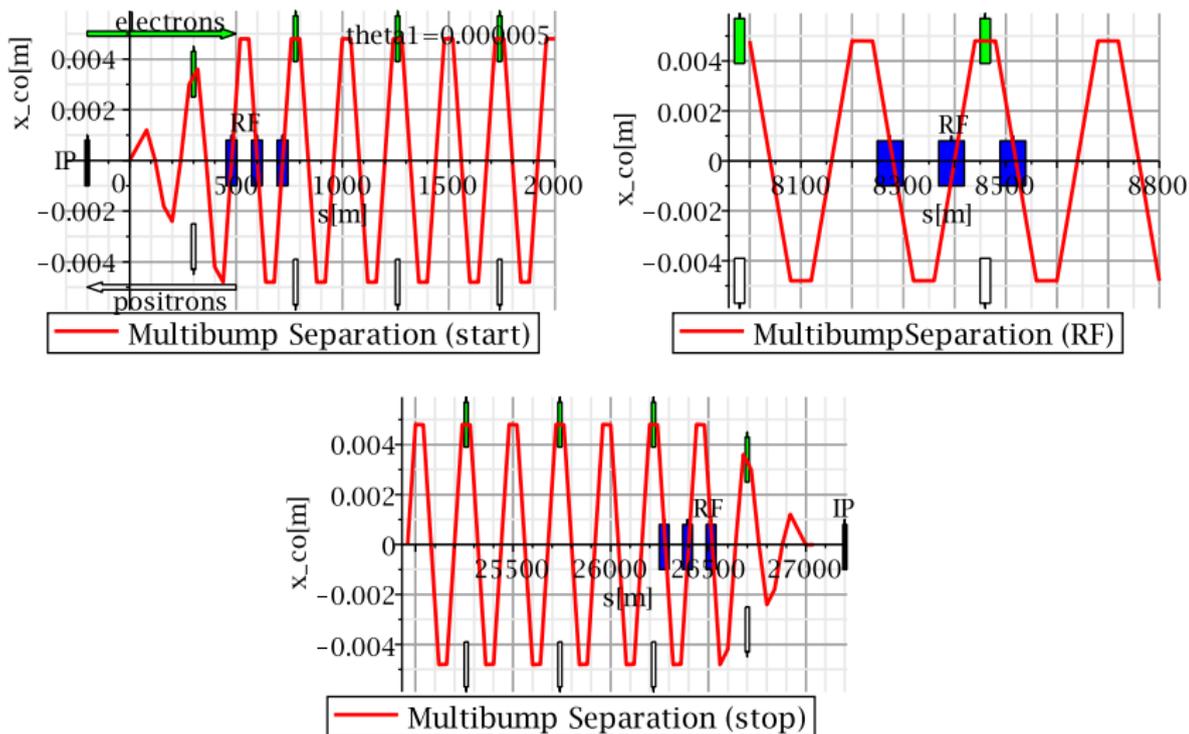


Figure: Short partial sections of the multibump beam separation.

## 20 Bunch Separation Partition Number Shift

(Mangling Jowett's careful formulation for brevity) the longitudinal partition number  $J_\epsilon$  depends on

focusing function  $K_1$ ,

dispersion  $D$ ,

fractional momentum offset,  $\delta = \bar{\delta} + \delta_{s.o.}$

separator displacement  $x_p(s)$ ;

$$J_\epsilon(\delta, x_p) = 2 + \frac{2 \oint K_1^2 D^2 ds}{\oint (1/R^2) ds} (\bar{\delta} + \delta_{s.o.}) + \frac{2 \oint K_1^2 (D(s) - D_0(s)) x_p(s) ds}{\oint (1/R^2) ds}; \quad (1)$$

here  $D/D_0$  are the separator-on/separator-off horizontal dispersion functions. The middle term here can be used to shift  $J_\epsilon$  away from 2, as proved useful at LEP, but it does not depend on  $x_p$ ; it is shown only as protection against confusing it with the final term.

Dropping the middle term and averaging,

$$J_\epsilon(|x_p|) = 2 + \frac{2 \langle K_1^2 (D - D_0) x_p \rangle}{\langle 1/R^2 \rangle}. \quad (2)$$

In spite of  $x_p$  averaging to zero, there is a non-vanishing shift of  $J_\epsilon(|x_p|)$  because  $K_1$ ,  $D$ , and  $x_p$  are correlated. At LEP this shift was observed to be significantly damaging and to be dominated by sextupoles. The factors in Eq. (2) scale as

$$\begin{aligned} x_p \propto \sigma_x \propto \frac{1}{R^{1/2}}, \quad K_1 = \frac{q}{l_q} \propto \frac{1/R^{1/2}}{R^{1/2}} \propto \frac{1}{R}, \\ D - D_0 \propto S \propto \frac{1}{R^{1/2}}, \quad \Delta J_\epsilon(|x_p|) \propto \frac{1}{R}. \end{aligned} \quad (3)$$

- ▶ These scaling formulas indicate that the seriousness of this partition number shift actually decreases with increasing  $R$ .
- ▶ Even if this were not true, should the partition number shift be unacceptably large, it can be reduced by increasing the quadrupole length  $l_q$  to decrease  $K_1$ .
- ▶ The partition number shift is due to excess radiation in the quadrupoles. Since this radiation intensity is proportional to the square of the magnetic field, doubling the quadrupole length halves the radiation and the partition number shift.

## 23 Long Cell Lattice

- ▶ The beam separation scheme shown so far has used a very short collider cell length  $L_c = 60$  m. Table 1 describes the scaling of lattice parameters obtained after redesigning both injector and collider for efficient injection.
- ▶ The resulting collider cell length is  $L_c = 213$  m. Because the cells are so long, there may be no need for multiple electrostatic separators.
- ▶ Instead one may use, for example, two or three electric kickers to launch each electric bump, with two or three matched kickers to terminate it.
- ▶ A large increase in cell length will surely also require a corresponding increase in longitudinal separation of circulating bunches. The single beam luminosity will be correspondingly reduced if the luminosity is already limited by the maximum number of bunches, as in the case of  $Z_0$  production.
- ▶ The luminosity reduction should be little affected at the Higgs energy and above.

Parameter	Symbol	Proportionality	$L \propto R^{3/4}$ collider	Values $C=100$ km	
phase advance per cell	$\mu_x$		$90^\circ$		
cell length	$L$		$R^{3/4}$	213 m	
bend angle per cell	$\phi$	$= L/R$	$R^{-1/4}$		
quad strength ( $1/f$ )	$q$	$1/L$	$R^{-3/4}$		
dispersion	$D$	$\phi L$	$R^{1/2}$		
beta	$\beta$	$L$	$R^{3/4}$		
tune	$Q_x$	$R/\beta$	$R^{1/4}$		125.26
tune	$Q_y$	$R/\beta$	$R^{1/4}$		105.19
Sands's "curly H"	$\mathcal{H}$	$= D^2/\beta$	$R^{1/4}$		
partition numbers	$J_x/J_y/J_\epsilon$	$1/1/2$	$1/1/2$		
horizontal emittance	$\epsilon_x$	$\mathcal{H}/(J_x R)$	$R^{-3/4}$		
fractional energy spread	$\sigma_\delta$	$\sqrt{B}$	$R^{-1/2}$		
arc beam width-betatron	$\sigma_{x,\beta}$	$= \sqrt{\beta\epsilon_x}$	1		
-synchrotron	$\sigma_{x,synch.}$	$= D\sigma_\delta$	1		
sextupole strength	$S$	$q/D$	$R^{-5/4}$		
dynamic aperture	$x^{\max}$	$q/S$	$R^{1/2}$		
relative dyn. aperture	$x^{\max}/\sigma_x$		$R^{1/2}$		
separator amplitude	$x_p$	$\sigma_x$	1		

Table: Parameter scaling for improved injection efficiency collider.

Parameter	Symbol	LEP-extrapolated	Unit	Collider	
mean bend radius	$R$	<b>3026</b>	m	<b>5675</b>	<b>11350</b>
beam energy		120	GeV	120	120
circumference	$C$	26.7	km	50	100
cell length	$L$	79	m	127	213
momentum compaction	$\alpha_c$	1.85e-4	m	1.35e-4	0.96e-4
tunes	$Q_x$	90.26		105.26	125.26
	$Q_y$	76.19		89.19	105.19
partition numbers	$J_x/J_y/J_z$	1/1.6/1.4		1/1/2	1/1/2
main bend field	$B_0$	0.1316	T	0.0702	0.0351
energy loss per turn	$U_0$	6.49	GeV	3.46	1.73
radial damping time	$\tau_x$	0.0033	s	0.0061	0.0124
	$\tau_x/T_0$	37	turns	69	139
fractional energy spread	$\sigma_\delta$	0.0025		0.0013	0.0009
emittances (no BB), x	$\epsilon_x$	<b>21.1</b>	nm	<b>13.2</b>	<b>7.82</b>
y	$\epsilon_y$	1.0	nm	0.66	0.39
max. arc beta functs	$\beta_x^{\max}$	125	m	200	337
max. arc dispersion	$D^{\max}$	0.5	m	0.68	0.97
quadrupole strength	$q \approx \pm 2.5/L_p$	0.0316	1/m	0.0197	0.0117
max. beam width (arc)	$\sigma_x = \sqrt{\beta_x^{\max} \epsilon_x}$	1.6	mm	1.625	1.558
(ref) sextupole strength	$S = q/D$	0.0632	1/m <sup>2</sup>	0.0290	0.0121
(ref) dynamic aperture	$x^{\text{da}} \sim q/S$	$\sim 0.5$	m	$\sim 0.679$	$\sim 0.967$
(rel-ref) dyn.ap.	$x^{\text{da}}/\sigma_x$	$\sim 0.313$		$\sim 0.417$	$\sim 0.621$
separator amplitude	$\pm 5\sigma_x$	$\pm 8.0$	mm	$\pm 8.1$	$\pm 7.8$

Table: Parameters values scaled from LEP.

With one 100 km circumference ring, the maximum number of bunches is limited to about 200. For  $N_b < 200$  the luminosity  $\mathcal{L}$  has to be reduced proportionally.  $\mathcal{L} \rightarrow \mathcal{L}_{\text{actual}} = \mathcal{L} \times N_b/200$ . Luminosities in the 100 km, 25 MW case are given in my WG2 report "Ring Circumference and Two Rings vs One Ring". Here, for comparison, and to more nearly match the separation scheme shown in Figure 3, the circumference is assumed to be  $C=50$  km, the RF power 50 MW per beam, and the number of bunches  $N_b=112$ . The results are shown in Table 3 (unlimited  $N_b$ ) and Table 4 (with  $N_b=112$ ).

The values of parameters not shown in the tables are  $\eta_{\text{Telnov}}=0.01$ ,  $\beta_y^*=5$  mm,  $x_i^{\text{typ}}/\beta_y^*=22.8$ ,  $\tau_{\text{bs}}=600$  s, Optimistic= 1.5,  $R_{\text{Gau-unif}}=0.30$ ,  $eV_{\text{rf}}=20$  GeV,  $OV_{\text{req.}}=20$  GV,  $a_{xy}=15$ ,  $r_{yz}=1$ ,  $\beta_{x,\text{arcmax}}=120$  m.

name	$E$ GeV	$\epsilon_x$ nm	$\beta_y^*$ mm	$\epsilon_y$ pm	$\xi_{\text{sat}}$	$N_{\text{tot}}$	$\sigma_y$ $\mu\text{m}$	$\sigma_x$ $\mu\text{m}$	$u_c^*$ GeV	$n_{\gamma,1}^*$	$\mathcal{L}^{\text{RF}}$ $10^{34}$	$\mathcal{L}_{\text{trans}}^{\text{bs}}$ $10^{34}$	$\mathcal{L}^{\text{bb}}$ $10^{34}$	$N_b$	$\beta_x^*$ m	$P_{\text{rf}}$ MW
Z	46	0.916	2	61.1	0.094	7.3e+14	0.35	5.24	0.000	1.97	52.5	96.8	52.513	33795	0.03	50
W	80	0.323	2	21.6	0.101	7.6e+13	0.208	3.12	0.001	2.06	9.66	16.2	9.661	5696	0.03	50
LEP	100	0.215	2	14.3	0.101	3.1e+13	0.169	2.54	0.002	2.10	4.95	8	4.947	2814	0.03	50
H	120	0.153	2	10.2	0.102	1.5e+13	0.143	2.15	0.003	2.13	2.86	4.48	2.863	1581	0.03	50
tt	175	0.077	2	5.12	0.118	3.3e+12	0.101	1.52	0.006	2.19	0.923	1.35	0.923	478	0.03	50
Z	46	16.5	5	1100	0.094	7.3e+14	2.35	35.21	0.001	2.12	21	33.2	21.005	1872	0.075	50
W	80	5.88	5	392	0.101	7.6e+13	1.4	20.99	0.003	2.22	3.86	5.52	3.864	313	0.075	50
LEP	100	3.91	5	261	0.101	3.1e+13	1.14	17.12	0.005	2.26	1.98	2.71	1.979	154	0.075	50
H	120	2.80	5	187	0.102	1.5e+13	0.966	14.50	0.007	2.30	1.15	1.52	1.145	86	0.075	50
tt	175	1.41	5	94	0.118	3.3e+12	0.686	10.28	0.016	2.38	0.369	0.455	0.369	26	0.075	50
Z	46	149	10	9900	0.094	7.3e+14	9.95	149.28	0.002	2.24	10.5	14.7	10.503	208	0.15	50
W	80	53.1	10	3540	0.101	7.6e+13	5.95	89.26	0.007	2.36	1.93	2.42	1.932	34	0.15	50
LEP	100	35.4	10	2360	0.101	3.1e+13	4.86	72.88	0.011	2.41	0.989	1.19	0.989	17	0.15	50
H	120	25.4	10	1700	0.102	1.5e+13	4.12	61.78	0.016	2.45	0.573	0.663	0.573	9.5	0.15	50
tt	175	12.9	10	857	0.118	3.3e+12	2.93	43.92	0.035	2.54	0.185	0.198	0.185	2.9	0.15	50

Table: Parameters and luminosities with unlimited number of bunches  $N_b$ , assuming 50 km circumference ring and 50MW per beam RF power.

$E$ GeV	$\beta_y^*$ m	$\xi_{\text{sat}}$	$\mathcal{L}_{\text{actual}}$ $10^{34}$	$N_{\text{actual}}$	$P_{\text{rf}}$ MW
46	0.002	0.094	0.174	112	50
80	0.002	0.1	0.190	112	50
100	0.002	0.1	0.197	112	50
120	0.002	0.1	0.203	112	50
175	0.002	0.12	0.216	112	50
46	0.005	0.094	1.256	112	50
80	0.005	0.1	1.380	112	50
100	0.005	0.1	1.434	112	50
120	0.005	0.1	1.145	86.6	50
175	0.005	0.12	0.369	26.1	50
46	0.010	0.094	5.644	112.0	50
80	0.010	0.1	1.932	34.7	50
100	0.010	0.1	0.989	17.1	50
120	0.010	0.1	0.573	9.5	50
175	0.010	0.12	0.185	2.9	50

**Table:** Luminosity influencing parameters and luminosities with the number of bunches limited to  $N_b = 112$ , assuming 50 km circumference ring and 50 MW per beam RF power.

-  J. Jowett, *Beam Dynamics at LEP*, CERN SL/98-029 (AP), 1998
-  J. Jowett, *More Bunches with Pretzel*, Private communication “pzl6.dvi” .
-  R. Talman, *Specific Luminosity Limit of  $e^+/e^-$  Colliding Rings*, Phys. Rev. ST-AB, 2002
-  R. Talman, *Accelerator X-Ray Sources*, Wiley-VCH Verlag, 2006