Lattice Optimization for Top-Off Injection

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Abstract

- Consider injection into the circular electron-positron Higgs factory.
- High average luminosity with significant particle loss rates demands full energy, top-off injection.
- Vertical injection and the possibility of kicker-free, bunch-by-bunch injection occuring concurrently with physics running are considered.
- Injector and/or collider lattices should be designed to maximize injection efficiency.
- Focusing should be stronger in injector than in collider.
- Scaling formulas are derived for the most important lattice parameters.
- Injection efficiency increases with increasing ring circumference.
- Recommendation: adjust L_c for maximum luminosity and $L_i < L_c$ for best injection.

4 Injection Strategy: Strong Focusing Injector, Weak Focusing Collider

- It is a given that full energy top-off injection will be required.
- There is one disadvantage. The cost in energy of losing a full energy particle due to injection inefficiency is the same as the cost of losing a circulating particle to beamstrahlung or any other mechanism.
- Injection efficiency of 50% is equivalent to doubling the irreducible circulating beam loss rate.
- To make this degradation unimportant one should therefore try for 90% injection efficiency.

- Achieving high efficiency injection justifies optimizing injector and/or collider lattices to improve injection.
- One can shrink the injector beam emittances and expand the collider beam acceptances by using stronger focusing in the injector than in the collider.
- The dynamic-aperture/beam-width ratio increases as $R^{1/2}$ for both injector and collider. Before addressing this optimization other injection issues will be surveyed.

6 Kicker-Free, Vertical Top-Up Injection

- Handling the synchrotron radiation at a Higgs Factory is difficult and replenishing the power loss is expensive.
- Otherwise the RF power loss is purely beneficial, especially for injection. Betatron damping decrements δ can be 1% or greater, and increasing with increasing energy.
- According to Liouville's theorem, increasing the beam particle density by injection is impossible for a Hamiltonian system.
- The damping decrement δ measures the degree to which the system is *not* Hamiltonian.
- If δ is large enough bumpers and kickers may not be needed to keep the already stored beam captured while the injected beam has time to damp.

7 Why Vertical Injection?

- The most fundamental parameter limiting injection efficiency is the emittance of the injected beam.
- The vertical emittance in the booster accelerator can be very small, perhaps $\epsilon_y < 10^{-10}$ m, which can be taken to be effectively zero. This may require a brief flat top at full energy.
- The next most important injector parameter is the septum thickness. For vertical injection, with angular deflection not necessarily required, the septum can be very thin, even zero.
- The remaining (and most important) injection uncertainty is whether the ring dynamic aperture extends out to the septum.
- If not, it may be possible to improve the situation by moving the closed orbit closer to the wall using DC bumpers, but not kickers. (However, vertical emittance growth makes even vertical bumpers undesirable.)

- With top-off injection the linear part of the beam-beam tune shift can be designed into the linear lattice optics. One beam "looks like a lens" to a particle in the other beam.
- Large octupole moments makes the lens far from ideal. But the octupole component, though nonlinear, does not necessarily limit the luminosity severely.
- With injection continuing during data collection there would be no need for cycling between injection mode and data collection mode.
- This would improve both average luminosity and data quality.
- Let $n_{\mathrm{inj.}}$ be the small integer equal to the number of turns following injection before the injected beam scrapes the injection septum. Careful choice of vertical, horizontal, and synchrotron tunes may allow $n_{\mathrm{inj.}}$ of 10 or more.
- The fractional shrinkage of the Courant-Snyder invariant after $n_{\mathrm{inj.}}$ turns is $n_{\mathrm{inj.}}\delta$, and correspondingly high injection efficiency.

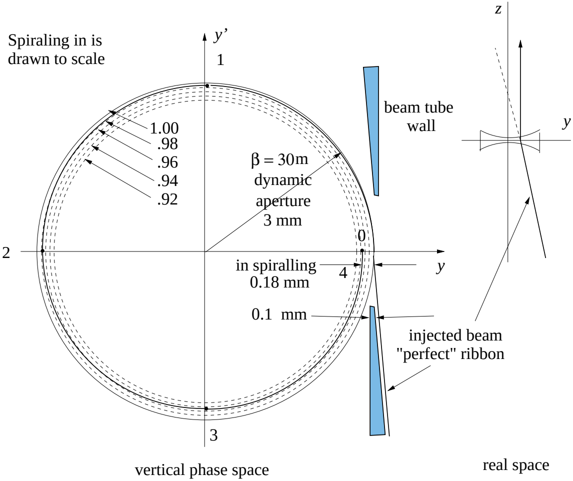

Figure: A cartoon of kicker-free, vertical injection. The dashed line shows the Courant-Snyder amplitude of the injected particle with the fractional shrinking per turn drawn more or less to scale.

10 Constant Dispersion Scaling with Bend Radius R

Scale Higgs factory parameters directly from LEP values, which are: phase advance per cell $\mu_x = \pi/2$, full cell length $L_c = 79\,\mathrm{m}$. At constant phase advance, the beta function β_x scales as L_c and dispersion D scales as bend angle per cell $\phi = L_c/R$ multiplied by cell length L_c;

$$D \propto \frac{L_c^2}{R}. \tag{1}$$

Holding L_c constant as R is increased would decrease the dispersion, impairing our ability to control chromaticity. Let us therefore *tentatively adopt the scaling*

$$L_c \propto R^{1/2}, \quad \text{corresponding to} \quad \phi \propto R^{-1/2}. \tag{2}$$

This holds dispersion D constant. These quantities and "Sands curly H" $\mathcal{H}$ then scale as

$$\beta_x \propto R^{1/2}, \quad D \propto 1, \quad \mathcal{H} \propto \frac{D^2}{\beta_x} \propto \frac{1}{R^{1/2}}. \tag{3}$$

 Copied from Jowett[1], the fractional energy spread is given by

$$\sigma_\epsilon^2 = \frac{55}{32\sqrt{3}} \frac{\hbar}{m_e c} \gamma_e, \quad \text{where}$$

$$F_\epsilon = \frac{< 1/R^3 >}{J_x < 1/R^2 >} \propto \frac{1}{R}, \tag{4}$$

and the horizontal emittance is given by

$$\epsilon_x = \frac{55}{32\sqrt{3}} \frac{\hbar}{m_e c} \gamma_e F_x, \quad \text{where}$$

$$F_x = \frac{< \mathcal{H}/R^3 >}{J_x < 1/R^2 >} \propto \frac{1}{R^{3/2}}. \tag{5}$$

The betatron contribution to beam width scales as

$$\sigma_{x,\mathrm{betatron}} \propto \sqrt{\beta_x \epsilon_x} \propto 1/R^{1/2}. \tag{6}$$

Similarly, at fixed beam energy, the fractional beam energy (or momentum) spread σ_δ scales as

$$\sigma_\delta \propto \sqrt{B} \propto 1/R^{1/2}. \tag{7}$$

Scaling with R of Arc Sextupole Strengths and Dynamic Aperture

- It remains uncertain whether the IP-induced chromaticity can be cancelled locally, giving a large increase in luminosity, but requires strong bends close to the IP.
- For now assume the IP chromaticity is cancelled in the arcs and take the IP chromaticity equal to the arc chromaticity.
- With dispersion $D \propto 1$, quad strength $q \propto 1/R^{1/2}$, and $S^{\text{arc chr.}} \propto q/D$, one obtains the scaling of sextupole strengths and dynamic aperture scaling;

$$S \propto \frac{1}{R^{1/2}}, \quad \text{and} \quad x^{\text{dyn. ap.}} \propto \frac{q}{S^{\text{arc chr.}}} \propto 1. \tag{8}$$

- The most appropriate measure of dynamic aperture is

$$\frac{x^{\text{dyn. ap.}}}{\sigma_x} \propto \frac{1}{1/R^{1/2}} \propto R^{1/2}. \tag{9}$$

- (Reservation: the chromatic mismatch between IP and arc is thought to be more important in limiting the dynamic aperture than is the simple compensation of total chromaticity.)

Parameter	Symbol	Proportionality	Scaling
phase advance per cell	μ		1
cell length	L		$R^{1/2}$
bend angle per cell	ϕ	$= L/R$	$R^{-1/2}$
quad strength $(1/f)$	q	$1/L$	$R^{-1/2}$
dispersion	D	ϕL	1
beta	β	L	$R^{1/2}$
tunes	Q_x, Q_y	R/β	$R^{1/2}$
Sands's "curly H"	$\mathcal{H}$	$= D^2/\beta$	$R^{-1/2}$
partition numbers	$J_x/J_y/J_\epsilon$	$= 1/1/2$	1
horizontal emittance	ϵ_x	$\mathcal{H}/(J_x R)$	$R^{-3/2}$
fract. momentum spread	σ_δ	$\sqrt{B}$	$R^{-1/2}$
arc beam width-betatron	$\sigma_{x,\beta}$	$\sqrt{\beta\epsilon_x}$	$R^{-1/2}$
-synchrotron	$\sigma_{x,synch.}$	$D\sigma_\delta$	$R^{-1/2}$
sextupole strength	S	q/D	$R^{-1/2}$
dynamic aperture	$x^{\max}$	q/S	1
relative dyn. aperture	$x^{\max}/\sigma_x$		$R^{1/2}$
pretzel amplitude	x_p	σ_x	$R^{-1/2}$

Table: *Constant dispersion* scaling is the result of choosing cell length $L \propto R^{1/2}$. This is emphasized by the shaded row, where the 1 in the final column indicates constancy as the ring radius is changed.

14 Injection-Optimized Parameters

- Discussed so far has been "constant dispersion scaling".
- But we want to make injector emittances smaller and collider acceptances larger.
- Shorten injector length L_i and lengthen collider cell length L_c. The next tables shows the scaling and, followng that, numerical values for 50 km and 100 km rings.

Parameter	Symbol	Proportionality	$L \propto R^{1/4}$ injector	$L \propto R^{1/2}$	$L \propto R^{3/4}$ collider
phase advance per cell	μ_x		90°	90°	90°
cell length	L		$R^{1/4}$	$R^{1/2}$	$R^{3/4}$
			110 m	153 m	213 m
bend angle per cell	ϕ	$= L/R$	$R^{-3/4}$	$R^{-1/2}$	$R^{-1/4}$
momentum compaction		ϕ^2	$R^{-3/2}$	R^{-1}	$R^{-1/2}$
quad strength $(1/f)$	q	$1/L$	$R^{-1/4}$	$R^{-1/2}$	$R^{-3/4}$
dispersion	D	ϕL	$R^{-1/2}$	1	$R^{1/2}$
beta	β	L	$R^{1/4}$	$R^{1/2}$	$R^{3/4}$
tune	Q_x	R/β	$R^{3/4}$	$R^{1/2}$	$R^{1/4}$
			243.26	174.26	125.26
tune	Q_y	R/β	$R^{3/4}$	$R^{1/2}$	$R^{1/4}$
			205.19	147.19	106.19
Sands's "curly H"	$\mathcal{H}$	$= D^2/\beta$	$R^{-5/4}$	$R^{-1/2}$	$R^{1/4}$
partition numbers	$J_x/J_y/J_\epsilon$	1/1/2	1/1/2	1/1/2	1/1/2
horizontal emittance	ϵ_x	$\mathcal{H}/(J_x R)$	$R^{-9/4}$	$R^{-3/2}$	$R^{-3/4}$
fract. momentum spread	σ_δ	$\sqrt{B}$	$R^{-1/2}$	$R^{-1/2}$	$R^{-1/2}$
arc beam width-betatron	$\sigma_{x,\beta}$	$= \sqrt{\beta\epsilon_x}$	R^{-1}	$R^{-1/2}$	1
-synchrotron	$\sigma_{x,synch.}$	$= D\sigma_\delta$	R^{-1}	$R^{-1/2}$	1
sextupole strength	S	q/D	$R^{1/4}$	$R^{-1/2}$	$R^{-5/4}$
dynamic aperture	x^{da}	q/S	$R^{-1/2}$	1	$R^{1/2}$
relative dyn. aperture	x^{da}/σ_x		$R^{1/2}$	$R^{1/2}$	$R^{1/2}$
separation amplitude	x_p	σ_x	N/A	$R^{-1/2}$	1

Table: To improve injection efficiency (compared to constant dispersion scaling) the injector cell length can increase more weakly, for example $L_i \propto R^{1/4}$, and the collider cell length can increase more strongly, for example $L_i \propto R^{3/4}$. The shaded entries assume circumference $\mathcal{C}$=100 km, R/R_{LEP}=3.75.

Parameter	Symbol	LEP(sc)	Unit	Injector		Collider	
mean bend radius	R	**3026**	m	**5675**	**11350**	**5675**	**11350**
beam Energy		120	GeV	120	120	120	120
circumference	$\mathcal{C}$	26.7	km	50	100	50	100
cell length	L	79	m	92.4	110	127	213
momentum compaction	α_c	1.85e-4	m	0.72e-4	0.25e-4	1.35e-4	0.96e-4
tunes	Q_x	90.26		144.26	243.26	105.26	125.26
	Q_y	76.19		122.19	205.19	89.19	106.19
partition numbers	$J_x/J_y/J_\epsilon$	1/1.6/1.4		1/1/2	1/1/2	1/1/2	1/1/2
main bend field	B_0	0.1316	T	0.0702	0.0351	0.0702	0.0351
energy loss per turn	U_0	6.49	GeV	3.46	1.73	3.46	1.73
radial damping time	τ_x	0.0033	s	0.0061	0.0124	0.0061	0.0124
	τ_x/T_0	37	turns	69	139	69	139
fractional energy spread	σ_δ	0.0025		0.0018	0.0013	0.0018	0.0013
emittances (no BB), x	ϵ_x	**21.1**	nm	**5.13**	**1.08**	**13.2**	**7.82**
y	ϵ_y	1.0	nm	0.25	0.05	0.66	0.39
max. arc beta functs	$\beta_x^{\max}$	125	m	146	174	200	337
max. arc dispersion	$D^{\max}$	0.5	m	0.37	0.26	0.68	0.97
quadrupole strength	$q \approx \pm 2.5/L_p$	0.0316	1/m	0.027	0.0227	0.0197	0.0117
max. beam width (arc)	$\sigma_x = \sqrt{2\beta_x^{\max}\epsilon_x}$	$1.6\sqrt{2}$	mm	$0.865\sqrt{2}$	$0.433\sqrt{2}$	$1.62\sqrt{2}$	$1.62\sqrt{2}$
(ref) sextupole strength	$S = q/D$	0.0632	1/m^2	0.0732	0.0873	0.0290	0.0121
(ref) dynamic aperture	$x^{\mathrm{da}} \sim q/S$	∼0.5	m	∼0.370	∼0.260	∼0.679	∼0.967
(rel-ref) dyn.ap.	x^{da}/σ_x	∼0.313		∼0.428	∼0.600	∼0.417	∼0.621
separation amplitude	$\pm 5\sigma_x$	$\pm 8.0\sqrt{2}$	mm			$\pm 8.1\sqrt{2}$	$\pm 7.8\sqrt{2}$

Table: Lattice parameters for improved injection efficiency. The shaded row indicates how successfully the injector emittance has been reduced relative to the collider emittance. The factor of seven improvement, 7.82/1.08, in this ratio for a 100 km ring, seems unnecessarily large, indicating that less radical scaling should be satisfactory.

17 Implications of Changing Lattices for Improved Injection

- There is substantial advantage and little disadvantage to strengthening the injector focusing and weakening the collider focusing.
- This has been achieved by shortening the injector cell length L_i and increasing the collider cell length L_c. Weakening the collider focusing has the effect of increasing the transverse beam sizes
- The improvement in the injector emittance/collider acceptance ratio is probably unnecessaily large—seven times for a 100 km ring.

- But *another constraint needs to be met.* The beam aspect ratio at the IP has to be adjusted maximum luminosity. This constrains horizontal emittance ϵ_x.
- The preferred method for controlling ϵ_x is cell length L_c.
- According to my WG 2 paper, "Ring Circumference and Two Rings vs One Ring", with $\beta_y^* = 5\,\mathrm{mm}$ ϵ_x is 3.98 nm. The value found here is $\epsilon_x = 7.82\,\mathrm{nm}$.
- This can be "close enough for now", or calls for furthrer parameter adjustment (which is obvious in any case). But the suggestion is that the $L_c = 213\,\mathrm{m}$ collider cell is too long.
- Unfortunately the optimal value of ϵ_x depends strongly on the optimal value of β_y^*, which is presently unkown. These considerations show that the arc and intersection region designs cannot be separately optimized. Rather a full complex optimization is required.

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- R. Talman, *Accelerator X-Ray Sources*, Wiley-VCH Verlag, 2006