Ring Circumference and Two Rings vs One Ring

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Abstract

- The natural next future circular collider is a circular e+e- Higgs Factory and, after that, a post-LHC p,p collider in the same tunnel.
- The main Higgs factory cost-driving parameter choices include: tunnel circumference $\mathcal{C}$, whether there is to be one ring or two, what is the installed power, and what is the "Physics" for which the luminosity deserves to be maximized.
- This paper discusses some of the trade-offs among these choices,
- and attempts to show that the optimization goals for the Higgs factory and the later p,p collider are consistent.

4 General Comments

- Low Higgs mass (125 GeV) makes a circular e+e- collider ideal for producing background-free Higgs particles.
- There is also ample physics motivation for planning a proton-proton collider with center of mass energy ≈ 100 TeV.
- Two-step plan: first build a circular e+e- Higgs factory; later replace it with a post-LHC pp collider
- This paper is devoted almost entirely to the circular Higgs factory step, but preserving the p,p collider potential.
- Cost-driving parameter choices include: tunnel circumference $\mathcal{C}$, one ring or two, installed power, physics priorities.

- I favor a single ring, optimized for Higgs production at $E = 120\,\text{Gev}$, with minimum *initial* cost, and highest possible eventual p,p energy.
- I show that electron/positron and proton/proton optimization goals are consistent.
- Higgs factory power considerations and p,p collider favor a tunnel of the largest possible radius R. Obviously one ring is cheaper than two rings.
- It will be shown that one ring is both satisfactory and cheaper for Higgs production.
- But higher luminosity (by a factor of five or so) at the (45.6 GeV) Z_0 energy, requires two rings.

6 Scaling up from LEP to Higgs Factory

- Results are to be based on scaling laws with respect to bending radius R or beam energy E. Bend Radius R and tunnel circumference $\mathcal{C}$ scale the same.
- Higgs production was just barely beyond the reach of LEP by the ratio $125\,\text{GeV}/105\,\text{GeV} = 1.19$. This should make extrapolation from LEP especially reliable.
- Note that, for a ring three times the size of LEP, the ratio of E^4/R (synchrotron energy loss per turn) is $1.19^4/3 = 0.67$—i.e. *less than final LEP operation*.
- At fixed RF power $P_{\rm rf}$, the total number of stored particles is proportional to R^2—doubling the ring radius cuts in half the energy loss per turn and doubles the time interval over which the loss occurs.

7 Scaling Radius and Power Inversely Conserves Luminosity

Define

$n_1 =$ number of stored particles per MW $\propto R^2$;
$f =$ revolution frequency $\propto 1/R$;
$N_b =$ number of bunches (set by IR length) $\propto R$;
$\sigma_y^* =$ beam height at the collision point;
$\mathcal{L}_{\rm pow}^{\rm RF} =$ the maximum luminosity for given RF power $P_{\rm rf}$.

$$\mathcal{L}_{\rm pow}^{\rm RF} \propto \frac{f}{N_b}\left(\frac{n_1 P_{\rm rf}[\rm MW]}{\sigma_y^*}\right)^2. \tag{1}$$

Consider variations for which

$$P_{\rm rf} \propto \frac{1}{R}. \tag{2}$$

Then $\mathcal{L}$ is independent of R. i.e. the luminosity depends on R and $P_{\rm rf}$ only through their product $RP_{\rm rf}$.

8 Constant Dispersion Scaling with Radius

Scale from LEP: cell phase advance $\mu_x = \pi/2$, length $L_c = 79\,\text{m}$.

Parameter	Symbol	Proportionality	Scaling
phase advance per cell	μ		1
collider cell length	L_c		$R^{1/2}$
bend angle per cell	ϕ	$= L_c/R$	$R^{-1/2}$
quad strength $(1/f)$	q	$1/L_c$	$R^{-1/2}$
dispersion	D	ϕL_c	1
beta	β	L_c	$R^{1/2}$
tunes	Q_x, Q_y	R/β	$R^{1/2}$
Sands's "curly H"	$\mathcal{H}$	$= D^2/\beta$	$R^{-1/2}$
partition numbers	$J_x/J_y/J_\epsilon$	$= 1/1/2$	1
horizontal emittance	ϵ_x	$\mathcal{H}/(J_x R)$	$R^{-3/2}$
fract. momentum spread	σ_δ	$\sqrt{B}$	$R^{-1/2}$
arc beam width-betatron	$\sigma_{x,\beta}$	$\sqrt{\beta\epsilon_x}$	$R^{-1/2}$
-synchrotron	$\sigma_{x,synch.}$	$D\sigma_\delta$	$R^{-1/2}$
sextupole strength	S	q/D	$R^{-1/2}$
dynamic aperture	$x^{\max}$	q/S	1
relative dyn. aperture	$x^{\max}/\sigma_x$		$R^{1/2}$
pretzel amplitude	x_p	σ_x	$R^{-1/2}$

9 Three Optimization Stages

- To maximize both the likelihood of initial approval and the eventual p,p performance, the cost of the first step has to be minimized and the tunnel circumference maximized.
- Surprisingly, these requirements are quite consistent.
- **Stage I, e+e-:** Starting configuration. Minimize cost at "respectable" luminosity, e.g. 10^{34}. Constrain the number of rings to 1, and the number of IP's to $N^* = 2$.
- **Stage II, e+e-:** Maximize luminosity/cost for production Higgs (etc.) running. Upgrade the luminosity by some combination of: $P_{\rm rf} \rightarrow 2P_{\rm rf}$ or $4P_{\rm rf}$, one ring $\rightarrow$ two rings, increasing N^* from 2 to 4, or decreasing β_y^*.
- **Stage III, pp:** Maximize the ultimate physics reach, i.e. center of mass energy, i.e. maximize tunnel circumference.

10 Cost Optimization

Treat the cost of the 2 detectors as fixed, and let C be the cost exclusive of detectors. Express the cost as a sum of a term proportional to size and a term proportional to power;

$$C = C_R + C_P \equiv c_R R + c_P P_{\rm rf} \tag{3}$$

where c_R and c_P are unit cost coefficients. Use Eq. (2)

$$P_{\rm rf} = \frac{\mathcal{L}}{k_1 R}. \tag{4}$$

Minimizing C at fixed $\mathcal{L}$ leads to

$$R_{\rm opt} = \sqrt{\frac{1}{k_1}\,\frac{c_P}{c_R}\,\mathcal{L}}. \tag{5}$$

Conventional thinking has it that c_P is universal world wide, but, perhaps, c_R is cheaper in China than elsewhere. If so, the optimal radius should be somewhat greater in China than elsewhere.

One or Two Ring Costs and Luminosities

Use $P_{\mathrm{rf}} \propto \mathcal{L}/R$;

	R km	P_{rf} MW	C_{tun} arb.	C_{acc} arb.	Phase-I cost arb.	$\mathcal{L}^{I}$ (Higgs) 10^{34}	$\mathcal{L}^{I}$ (Z_0) 10^{34}	$\mathcal{L}^{II}$ (Higgs) 10^{34}
1	5	50	0.5	2.5	3.0	1.2	2.6	2
	10	25	1.0	2.5*	**3.5**	1.2	5.2	**5**
	10	50	1.0	4.0	5.0	2.3	10.4	5
2	5	50	0.5	4.5	5.0	1.2	21	2
	10	25	1.0	4.5*	5.5	1.2	21	5
	10	50	1.0	7.0	8.0	2.3	42	5

Table: Estimated costs (arbitrary units), one ring in the upper table, two in the lower. *A crude LEP spread sheet shows that doubling the radius and halving the power leaves the accelerator cost not very much changed. Also bending magnet costs are assumed to be proportional to stored magnetic energy.

- Doubling the radius, while cutting the power in half, increases the cost only modestly,
- and leaves generous options for upgrading to maximize Higgs luminosity,
- as well as maximizing the potential p,p physics reach.
- The shaded row in Table 1 seems like the best deal. Both Higgs factory and, later, p,p luminosities are maximized, and the initial cost is (almost) minimized.
- This optimization has been restricted to a simple choice between 50 km and 100 km circumference.

13 Luminosity Dependencies

My electron/positron beam-beam simulation[2] dead reckons the saturation tune shift $\xi_{\max}$.

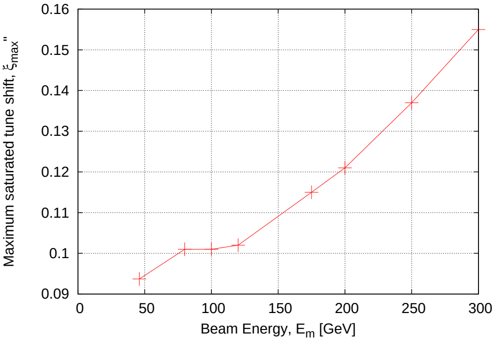

Figure: Plot of maximum tune shift $\xi_{\max}$ as a function of maximum beam energy for rings such that $E \propto R^{5/4}$. The plot assumes that the r.m.s. bunch length σ_z is equal to β_y^*, the vertical beta function at the intersection point (IP).

14 Saturated Tune Shift Simulation

- The physics of the simulation assumes there is an equilibrium established between beam-beam heating versus radiation cooling of vertical betatron oscillations.
- Under ideal single beam conditions the beam height would be $\sigma_y \approx 0$. This would give infinite luminosity in colliding beam operation —*this is unphysical*.
- In fact beam-beam forces cause the beam height to grow into a new equilibrium with normal radiation damping.
- It is parametric modulation of the vertical beam-beam force by horizontal betatron and longitudinal synchrotron oscillation that modulates the vertical force and increases the beam height.
- The resonance driving strength for this class of resonance is proportional to $1/\sigma_y$ and would be infinite if $\sigma_y = 0$—*which is also unphysical*.
- Nature, “abhoring” both zero and infinity, plays off beam-beam emittance growth against radiation dampling.
- In equilibrium the beam height is proportional to the bunch charge.

15 Luminosity Determination

- To estimate Higgs factory luminosity the tune plane is scanned for various vertical beta function values and bunch lengths, as well as other, less influential, parameters.
- The main output is the ratio $(\xi^{\mathrm{sat}}/\beta_y^*)$

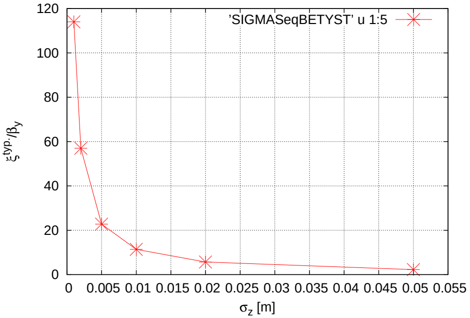

Figure: Plot of $\xi^{\mathrm{sat.}}/\beta_y$ as a function of σ_z, with $\beta_y^* = \sigma_z$, $\delta = 0.00764$, and synchrotron tune advance between collisions $Q_s = 0.0075$.

The ratio $\xi^{\rm sat.}/\beta_y^*$ determines the beam area A_{β_y} just sufficient for vertical saturation according to the formula,

$$A_{\beta_y} = \pi\sigma_x\sigma_y = \frac{N_p r_e}{2\gamma}\frac{1}{(\xi^{\rm sat.}/\beta_y)}. \tag{6}$$

- This fixes the tune-shift-saturated charge density (per unit transverse area).
- It is only the product $\sigma_x\sigma_y$ that is fixed but there is a broad optimum in luminosity for aspect ratio $a_{xy} = \sigma_x/\sigma_y \approx 15$.
- β_x^* is adjusted to make horizontal and vertical beam-beam tune shifts approximately equal.
- The lattice optics is adjusted so that the (arc-dominated) emittance ϵ_x gives the intended aspect ratio a_{xy}; $\epsilon_x = \sigma_x^2/\beta_x^*$.

17 Beamstrahlung

- "Beamstrahlung" is the same as synchrotron radiation, except that it occurs when a particle in one beam is deflected by the electric and magnetic fields of the other beam.
- Emission of the occasional single *hard* x-ray is inevitable and the lost energy has to be paid for.
- Much worse is the possibility that the reduction in momentum causes the particle itself to be lost, greatly magnifying the energy loss. It is this process that makes beamstrahlung so damaging. The damage is quantified by the beamstrahlung-dominated beam lifetime $\tau_{\rm bs}$.
- The important parameter governing beamstrahlung is the "critical energy" u_c^* which is proportional to 1/bunch-length σ_z; particle loss increases exponentially with u_c^*.
- To decrease beamstrahlung by increasing σ_z entails increasing β_y^* and reducing luminosity. A favorable compromise can be to increase charge per bunch along with β_y^*.

18 Reconciling the Luminosity Limits

- The number of electrons per bunch N_p is fixed by the available RF power and the number of bunches N_b.
- For increasing the luminosity N_b wants to be *reduced*.
- To keep beamstrahlung acceptably small N_b wants to be *increased*.
- The maximum achievable luminosity is determined by this compromise between beamstrahlung and available power.

- $\mathcal{L}^{\mathrm{RF}}_{\mathrm{pow}}$ is the RF power limited luminosity
- $\mathcal{L}^{\mathrm{bb}}_{\mathrm{sat}}$ is the beam-beam saturated luminosity;
- $\mathcal{L}^{\mathrm{bs}}_{\mathrm{trans}}$ is the beamstrahlung-limited luminosity.

Single beam dynamics gives $\sigma_y = 0$ which implies $\mathcal{L}^{\mathrm{RF}}_{\mathrm{pow}} = \infty$? Nonsense.

Recalling the earlier discussion, the resonance driving force, being proportional to $1/\sigma_y$ would also be infinite. As a result the beam-beam force expands $\sigma_y = 0$ as necessary.

Saturation is automatic (unless the single beam emittance is already too great for the beam-beam force to take control).

$$\mathcal{L}_{\rm pow}^{\rm RF} = \frac{1}{N_b}\, H(r_{yz})\, \frac{1}{a_{xy}}\, \frac{f}{4\pi} \left(\frac{n_1 P_{\rm rf}[{\rm MW}]}{\sigma_y}\right)^2, \tag{7}$$

$$\mathcal{L}_{\rm sat}^{\rm bb} = N_{\rm tot.} H(r_{yz})\, f\, \frac{\gamma}{2 r_e}\, (\xi^{\rm sat.}/\beta_y), \tag{8}$$

$$\mathcal{L}_{\rm trans}^{\rm bs} = N_b H(r_{yz})\, a_{xy} \sigma_z^2\, f \left(\frac{\sqrt{\pi}\ 1.96 \times 10^5}{28.0\,{\rm m}\,\sqrt{2/\pi}}\right)^2 \times \\ \times \frac{1}{r_e^2 \widetilde{E}^2} \left(\frac{91\eta}{\ln\left(\frac{1/\tau_{\rm bs}}{f\, n_{\gamma,1}^*\, \mathcal{R}_{\rm unif.}^{\rm Gauss}}\right)}\right)^2. \tag{9}$$

Here $H(r_{yz})$ is the hourglass reduction factor.

- If $\mathcal{L}_{\rm trans}^{\rm bs} < \mathcal{L}_{\rm sat}^{\rm bb}$ we must increase N_b.
- But $\mathcal{L}_{\rm trans}^{\rm bs} \propto N_b$, and $\mathcal{L}_{\rm pow}^{\rm RF} \propto 1/N_b$.
- We accept the compromise $N_{\rm b,new}/N_{\rm b,old} = \mathcal{L}_{\rm sat}^{\rm bb}/\mathcal{L}_{\rm trans}^{\rm bs}$.

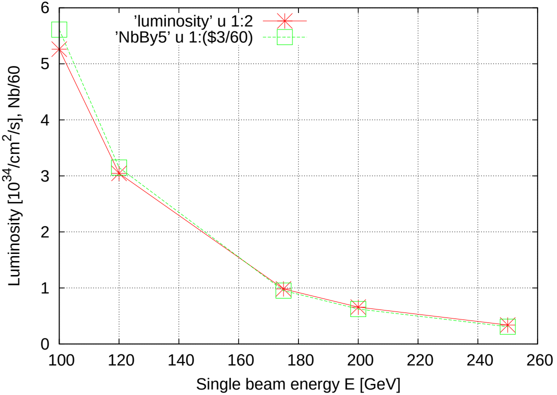

Figure: Dependence of luminosity on single beam energy (after upgrade to Stage II luminosity). The number of bunches (axis label to be read as $N_b/60$) is also shown, confirming that (as long as the optimal value of N_b is 1 or greater) the luminosity is proportional to the number of bunches. There is useful luminosity up to $E = 500\,\text{GeV}$ CM energy.

- Parameter tables, scaled up from LEP, are given for 100 km circumference Higgs factories in Tables 4 and 5.
- All but the last table assume the number of bunches N_b is unlimited.
- The last table derates the luminosity under the assumtion that N_b cannot exceed 200. Discussion of the one ring vs two rings issue can therefore be based on Table 5.
- Some parameters not given in tables are:
  Optimistic=1.5 (a shameless excuse for actual optimatization),
  $\eta_{\rm Telnov}$=0.01 (lattice fractional energy acceptance),
  $\tau_{\rm bs}$=600 s,
  $R_{\rm GauUnif}$= 0.300,
  P_{rf} = 25 MW,
  Over Voltage=20 GeV,
  aspect ratio a_{xy}=15,
  $r_{yz} = \beta^*_y/\sigma_z$=1,
  $\beta_{\rm arc\ max}$=198.2 m.

- With the exception of the final table, which is specific to the single ring option, the following tables apply equally to single ring or dual ring Higgs factories.
- The exception relates to N_b, the number of bunches in each beam.
- With N_b unlimited (as would be the case with two rings) all parameters are the same for one or two rings (according to the formulas in this paper).

name	E GeV	$\mathcal{C}$ km	R km	f KHz	U_1 GeV	$eV_{\rm excess}$ GeV	n_1 elec./MW	$\delta=\alpha_2$	u_c GeV	ϵ_x† nm	$\sigma_x^{\rm arc}$ mm
Z	46	100	10.6	3.00	0.04	20	5.81e+13	0.00020	0.00002	0.573	2
W	80	100	10.6	3.00	0.34	20	6.08e+12	0.00107	0.00011	1.771	1.19
LEP	100	100	10.6	3.00	0.83	19	2.49e+12	0.00209	0.00021	2.767	0.972
H	120	100	10.6	3.00	1.73	18	1.20e+12	0.00361	0.00036	3.984	0.824
tt	175	100	10.6	3.00	7.83	12	2.66e+11	0.01119	0.00112	8.473	0.585

Table: Single beam parameters, assuming 100 km circumference. The second last column (†) lists the value of ϵ_x appropriate only for $\beta_y^*=5\,$mm. Though determined by arc optics, ϵ_x has to be adjusted, according to the value of β_y^*, to optimize the beam shape at the IP. Other cases can be calculated from entries in other tables. U_1 is the energy loss per turn per particle. u_c is the critical energy for bending element synchrotron radiation. δ is the synchrotron radiation damping decrement.

Parameter	Symbol	Value	Unit	Energy-scaled	Radius-	scaled
Mean bend radius	R	**3026**	m	**3026**	**5675**	**11350**
	$R/3026$			1	1.875	3.751
Beam Energy	E	**45.6**/91.5	GeV	**120**	**120**	**120**
Circumference	$\mathcal{C}$	26.66	km	26.66	50	100
Cell length	L_c		m	79	108	153
Momentum compaction	α_c	1.85e-4		1.85e-4	0.99e-4	0.49e-4
Tunes	Q_x	90.26		90.26	123.26	174.26
	Q_y	76.19		76.19	104.19	147.19
Partition numbers	$J_x/J_y/J_\epsilon$	1/1/2		1/1.6/1.4 !	1/1/2	1/1/2
Main bend field	B_0	0.05/0.101	T	0.1316	0.0702	0.0351
Energy loss per turn	U_0	0.134/2.05	GeV	6.49	3.46	1.73
Radial damping time	τ_x	0.06/0.005	s	0.0033	0.0061	0.0124
	τ_x/T_0	679/56	turns	37	69	139
Fractional energy spread	σ_δ	0.95e-3/1.7e-3		0.0025	0.0018	0.0013
Emittances (no BB), x	ϵ_x	22.5/30	nm	21.1	8.2	2.9
y	ϵ_y	0.29/0.26	nm	1.0	0.4	0.14
Max. arc beta functs	$\beta_x^{\max}$	125	m	125	171	242
Max. arc dispersion	$D^{\max}$	0.5	m	0.5	0.5	0.5
Beta functions at IP	β_x^*, β_y^*	2.0,0.05	m	1.25/0.04	N/Sc.	N/Sc.
Beam sizes at IP	σ_x^*, σ_y^*	211, 3.8	μm	178/11	N/Sc.	N/Sc.
Beam-beam parameters	ξ_x, ξ_y	0.037,0.042		0.06/0.083	N/Sc.	N/Sc.
Number of bunches	N_b	8		4	N/Sc.	N/Sc.
Luminosity	$\mathcal{L}$	2e31	$\mathrm{cm}^{-2}\mathrm{s}^{-1}$	1.0e32	N/Sc.	N/Sc.
Peak RF voltage	V_{RF}	380	MV	3500	N/Sc.	N/Sc.
Synchrotron tune	Q_s	0.085/0.107		0.15	N/Sc.	N/Sc.
Low curr. bunch length	σ_z	0.88	cm	$\frac{\alpha_c R \sigma_e}{Q_s E}$	N/Sc.	N/Sc.

Table: Higgs factory parameter values for 50 km and 100 km options. The entries are mainly extrapolated from Jowett's, 45.6 Gev report[1], and educated guesses. "N/Sc." indicates (important) parameters too complicated to be estimated by scaling. Duplicate entries in the third column, such as 45.6/91.5 are from Jowett[1]; subsequent scalings are based on the 45.6 Gev values.

name	E GeV	ϵ_x nm	β_y^* mm	ϵ_y pm	ξ_{sat}	N_{tot} 10^{12}	σ_y μm	σ_x μm	u_c^* GeV	$n_{\gamma,1}^*$	$\mathcal{L}^{\mathrm{RF}}$ 10^{34}	$\mathcal{L}_{\mathrm{trans}}^{\mathrm{bs}}$ 10^{34}	$\mathcal{L}^{\mathrm{bb}}$ 10^{34}	N_b	β_x^* m	P_{rf} MW
Z	46	0.949	2	63.3	0.094	1500	0.356	5.34	0.000	2.01	52.5	103	52.5	65243	0.03	25
W	80	0.336	2	22.4	0.101	150	0.212	3.17	0.001	2.10	9.66	17.2	9.6	10980	0.03	25
LEP	100	0.223	2	14.9	0.101	62	0.172	2.59	0.002	2.13	4.95	8.46	4.94	5421	0.03	25
H	120	0.159	2	10.6	0.102	30	0.146	2.19	0.003	2.17	2.86	4.74	2.86	3044	0.03	25
tt	175	0.078	2	5.33	0.118	6.6	0.103	1.55	0.006	2.24	0.923	1.43	0.92	920	0.03	25
Z	46	17.2	5	1140	0.094	1500	2.39	35.89	0.001	2.16	21	35.1	21.	3605	0.075	25
W	80	6.11	5	408	0.101	150	1.43	21.42	0.003	2.26	3.86	5.83	3.86	602	0.075	25
LEP	100	4.07	5	271	0.101	62	1.16	17.47	0.005	2.31	1.98	2.86	1.97	296	0.075	25
H	120	2.92	5	195	0.102	30	0.987	14.80	0.008	2.35	1.15	1.6	1.14	166	0.075	25
tt	175	1.47	5	98.1	0.118	6.6	0.7	10.51	0.017	2.43	0.369	0.479	0.37	49	0.075	25
Z	46	155	10	10300	0.094	1500	10.2	152.3	0.002	2.29	10.5	15.5	10.5	400	0.15	25
W	80	55.4	10	3690	0.101	150	6.08	91.17	0.007	2.41	1.93	2.55	1.93	66	0.15	25
LEP	100	37.0	10	2470	0.101	62	4.97	74.48	0.011	2.46	0.989	1.25	0.99	32	0.15	25
H	120	26.6	10	1770	0.102	30	4.21	63.15	0.016	2.50	0.573	0.696	0.57	18.3	0.15	25
tt	175	13.5	10	898	0.118	6.6	3.0	44.94	0.036	2.60	0.185	0.207	0.19	5.5	0.15	25

Table: The major factors influencing luminosity, assuming 100 km circumference and 25 MW/beam RF power. The predicted luminosity is the smallest of the three luminosities, $\mathcal{L}^{\mathrm{RF}}$, $\mathcal{L}_{\mathrm{trans}}^{\mathrm{bs}}$, and $\mathcal{L}^{\mathrm{bb}}$. All entries in this table apply to either one ring or two rings, except where the number of bunches N_b is too great for a single ring.

- With one ring, the maximum number of bunches is limited to approximately ≤ 200.
- For $N_b > 200$ the luminosity $\mathcal{L}$ has to be de-rated accordingly; $\mathcal{L} \rightarrow \mathcal{L}_{\rm actual} = \mathcal{L} \times N_b/200$. This correction has been applied in Table 5 (showed earlier).
- When the optimal number of bunches is less than (roughly) 200, single ring operation is satisfactory, and hence favored.
- When the optimal number of bunches is much greater than 200, for example at the Z_0 energy, two rings are better.
- Note though, that the Z_0 single ring luminosities are still very healthy. In fact, with β_y^*=10 mm, which is a more conservative estimate than most others in this paper and in other FCC reports, the Z_0 single ring penalty is substantially less.

E GeV	β_y^* m	ξ_{sat}	$\mathcal{L}_{\mathrm{actual}}$ 10^{34}	N_{actual}	P_{rf} MW/beam
46	0.002	0.094	0.161	200	25
80	0.002	0.1	0.176	200	25
100	0.002	0.1	0.182	200	25
120	0.002	0.1	0.188	200	25
175	0.002	0.12	0.200	200	25
46	0.005	0.094	1.165	200	25
80	0.005	0.1	1.282	200	25
100	0.005	0.1	1.334	200	25
120	0.005	0.1	1.145	166	25
175	0.005	0.12	0.369	50	25
46	0.010	0.094	5.247	200	25
80	0.010	0.1	1.932	66.5	25
100	0.010	0.1	0.989	32.7	25
120	0.010	0.1	0.573	18.3	25
175	0.010	0.12	0.185	5.5	25

Table: Luminosites achievable with a single ring with number of bunches N_b limited to 200, 100 km circumference and 25 MW/beam RF power. The luminosity entries in (earlier) Table 1 were obtained from this table.

- J. Jowett, *Beam Dynamics at LEP*, CERN SL/98-029 (AP), 1998
- R. Talman, *Specific Luminosity Limit of e+/e- Colliding Rings*, Phys. Rev. ST-AB, 2002
- R. Talman, *Accelerator X-Ray Sources*, Wiley-VCH, 2006