$e\pm$ Collider - Polarization considerations

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Overview:

- Preamble
- A "toy" ring
- Sokolov-Ternov polarization in a 100 km ring
- Effect of wigglers
- First simulations
- Summary

HF2014, October 12, 2014

Preamble

- from M. Benedikt talk at kick-off meeting

FCC-ee parameters – starting point

Design choice: max. synchrotron radiation power set to 50 MW/beam

- Defines the maximum beam current at each energy
- 4 physics operation points (energies) foreseen *Z, WW, H, ttbar*
- Optimization at each operation point, mainly via bunch number and arc cell length

Parameter	*Z*	*WW*	*H*	*ttbar*	*LEP2*
E/beam (GeV)	45	80	120	175	105
L (10^{34} $cm^{-2}s^{-1}$)/IP	28.0	12.0	5.9	1.8	0.012
Bunches/beam	16700	4490	1330	98	4
I (mA)	1450	152	30	6.6	3
Bunch popul. [10^{11}]	1.8	0.7	0.47	1.40	4.2
Cell length [m]	300	100	50	50	79
Tune shift / IP	0.03	0.06	0.09	0.09	0.07

Polarization for high precision energy calibration at Z pole and WW

- a $e\pm$ collider in a 100 km ring: generous
- a place holder for a future p/p collider

Setting the geometry

Assuming: B_{max}=16 T, E^p_{beam}=50 TeV and L_{tot}=100 Km

$$\rho_b = \frac{p}{e}B = 10423.6 \text{ m} \quad L_{bends} = 2\pi\rho_b = 65493.5 \text{ m} \quad \frac{L_{bends}}{L_{tot}} = 0.655$$

Maximum dispersion (FODO):

$$\hat{D} = \frac{L_{cell}\phi_b}{2}\frac{1+0.5\sin\mu/2}{\sin^2\mu/2} \qquad 2\phi_b \equiv \text{cell bending angle}$$

ϕ_b and thus ℓ_b [a] should be large for avoiding too small dispersion (chromaticity correction!)

Attempt: ℓ_b=30 m $\phi_b = \ell_b/\rho_b$=0.00287808 rad and $\mu = 60^o$

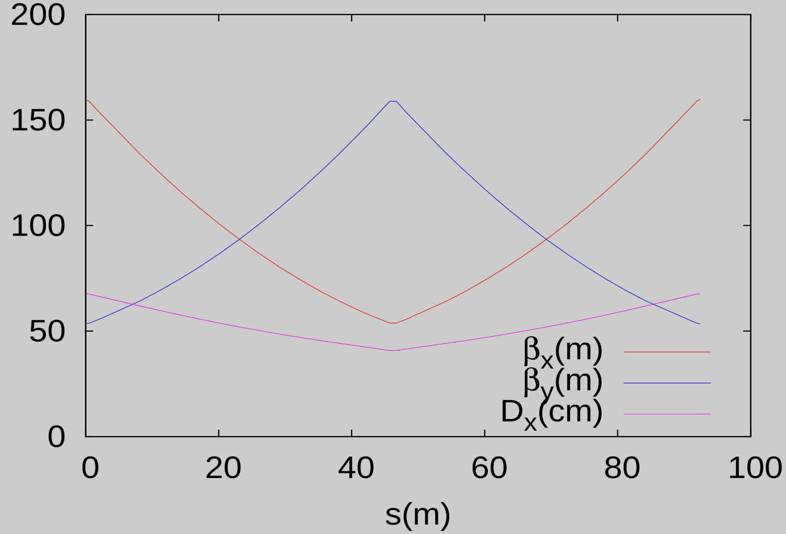
Number of cells:

$$n_{cells} = \frac{2\pi}{2\phi_b} \simeq 1090$$

[a] $L_{cell} = 0.655 L_{bends}/2\pi/\phi_b$

“Toy” ring with 1090 cells, α_p=3.2e-5.

FODO Optics



Bending radius and beam parameters

A large bending radius may be appealing for some parameters

$$U_{loss} = C_\gamma E^4/\rho \qquad (\Delta E/E)^2 = C_q \gamma^2 / J_\epsilon \rho$$

Synchrotron radiation integrals

$$\mathcal{I}_2 \equiv \oint ds \frac{1}{\rho^2}$$

$$\mathcal{I}_4 \equiv 2 \oint ds \frac{D_x K}{\rho}$$

$$\mathcal{I}_5 \equiv \oint ds \frac{\beta_x D_x'^2 + 2\alpha_x D_x D_x' + \gamma_x D_x^2}{|\rho|^3}$$

$\longrightarrow$ small equilibrium emittance: $\quad \epsilon_x = C_q \gamma^2 \dfrac{\mathcal{I}_5}{J_x \mathcal{I}_2}$

$\longrightarrow$ but large damping time: $\quad \tau_x = \dfrac{2\pi R}{C_x E^3} \dfrac{1}{\mathcal{I}_2 - \mathcal{I}_4}$

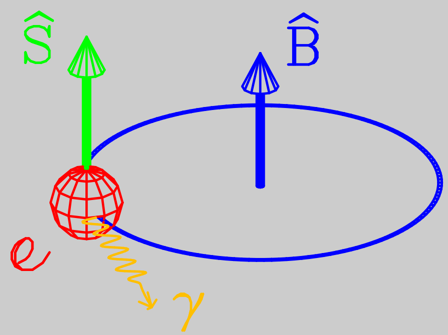
What about polarization?

1/2 spin particle in a *constant homogeneous* magnetic field.

Two stable states:

$$\vec{S} \uparrow\uparrow \vec{B}$$
$$\vec{S} \downarrow\uparrow \vec{B}$$

Sokolov-Ternov (1964): a small amount of the radiation emitted by a particle moving in such a field is accompanied by a *spin flip*, so transitions between the two states are possible.
Slightly different probabilities $\longrightarrow$ *self polarization !*



- Equilibrium polarisation:

$$P_{ST} = \frac{W_{\uparrow\downarrow} - W_{\downarrow\uparrow}}{W_{\uparrow\downarrow} + W_{\downarrow\uparrow}} = \frac{8}{5\sqrt{3}} = 92.4\%$$

- Build-up rate:

$$\frac{1}{\tau_{ST}} = W_{\uparrow\downarrow} + W_{\downarrow\uparrow} = \frac{5\sqrt{3}}{8}\frac{r_0 h}{2\pi m_0}\frac{\gamma^5}{|\rho|^3}$$

Actual ring accelerators include *quadrupoles* and their alignment is not perfect: when a particle emits a photon it starts to perform synchro-betatron oscillations around the machine *actual* closed orbit experiencing extra possibly *non vertical* fields.

The expectation value $\vec{S}$ of the spin operator moves according to the Thomas-BMT equation

$$\frac{d\vec{S}}{dt} = \vec{\Omega}(s;\vec{u}) \times \vec{S} \qquad \vec{u} \equiv \text{positions in 6D phase space}$$

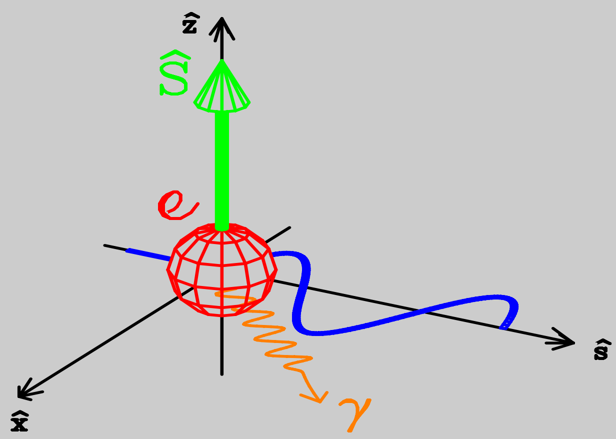
In the laboratory frame and MKS units

$$\vec{\Omega}(\vec{u};s) = -\frac{e}{m_0}\Big[\Big(a+\frac{1}{\gamma}\Big)\vec{B} - \frac{a\gamma}{\gamma+1}\vec{\beta}\cdot\vec{B}\vec{\beta} - \Big(a+\frac{1}{\gamma+1}\Big)\vec{\beta}\times\vec{E}\Big]$$

with $\vec{\beta} \equiv \vec{v}/c$ and $a = (g-2)/2$=0.0011597.

- Periodic solution along the closed orbit: $\hat{n}_0$.
- In a planar machine $n_0(s) \equiv \hat{z}$.
- Any other spin precesses around $\hat{n}_0$; spin tune (number of precession per turn): $a\gamma$ (in the rotating frame).

Planar machine

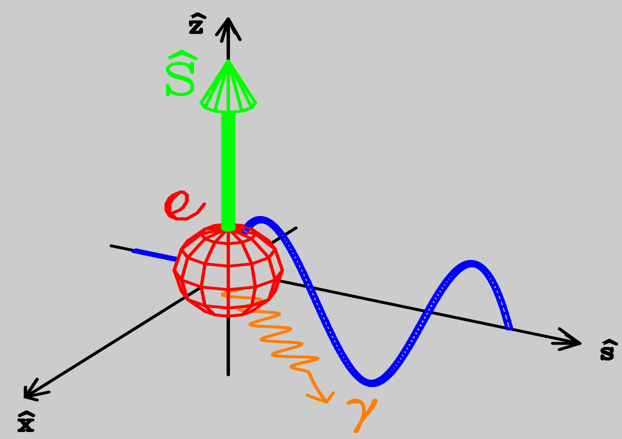


At the quads:

$$B_x = ky = 0$$
$$B_y = kx \neq 0$$

no spin diffusion

Non planar machine



$$B_x = ky \neq 0$$
$$B_y = kx \neq 0$$

spin diffusion!

Sokolov-Ternov effect
in the guiding dipole field

$\downarrow$

Polarisation

Perturbations
(v-bends, quads, sexts etc.)

$\downarrow$

Depolarisation

$\searrow$ $\swarrow$

Equilibrium polarisation $(< \boldsymbol{P_{ST}})$

General expression (Derbenev-Kondratenko, semiclassical approximation, 1973)

$$\vec{P}_{DK} = \hat{n}_0(s)\, P_{ST} \frac{\oint ds < \frac{1}{|\rho|^3}\hat{b}\cdot(\hat{n} - \frac{\partial \hat{n}}{\partial \delta}) >}{\oint ds < \frac{1}{|\rho|^3}\left[1 - \frac{2}{9}(\hat{n}\cdot\hat{s})^2 + \frac{11}{18}(\frac{\partial \hat{n}}{\partial \delta})^2\right] >}$$ [a]

with

$\hat{b} \equiv \vec{v} \times \dot{\vec{v}}/|\vec{v} \times \dot{\vec{v}}|$ $\hat{n}(\vec{u}; s) \equiv$ phase space dependent periodic solutions to T-BMT equation

$$\tau_{DK}^{-1} = P_{ST} \frac{r_e \gamma^5 \hbar}{m_0 C} \oint < \frac{1}{|\rho|^3}\left[1 - \frac{2}{9}(\hat{n}\cdot\hat{s})^2 + \frac{11}{18}(\frac{\partial \hat{n}}{\partial \gamma})^2\right] >$$

The term $\partial\hat{n}/\partial\delta$, with $\delta \equiv \delta E/E$ quantifies depolarizing effects resulting from trajectory perturbations due to the photon emission.

In a perfectly planar machine $\partial\hat{n}/\partial\delta$=0. In presence of quadrupole vertical misalignments (and/or spin rotator) $\partial\hat{n}/\partial\delta \neq$0 and large when

$$\nu_{spin} \pm mQ_x \pm nQ_y \pm pQ_s = \text{integer}$$

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[a] averages over phase space.

In *linear* orbit and spin motion approximation (Yokoya, 1982)

$$\frac{\partial \hat{n}}{\partial \delta}(\vec{u};s) = \vec{d}(s) = \frac{1}{2}\Im\left\{(\hat{m}_0 + i\hat{l}_0)^* \sum_{k=\pm x,\pm y,\pm s} \mathbf{\Delta}_k\right\} \qquad (\hat{m}_0, \hat{l}_0, \hat{n}_0) \ \text{spin basis}$$

The functions $\mathbf{\Delta}_k$ are given by

$$\mathbf{\Delta}_{\pm x,\pm y} = (a\gamma + 1)\frac{e^{\mp i\mu_{x,y}}}{e^{2i\pi(\nu \pm Q_{x,y})} - 1}\frac{[-D \pm i(\alpha D + \beta D')]_{x,y}}{\sqrt{\beta_{x,y}}} J_{\pm x,\pm y}$$

$$\mathbf{\Delta}_{\pm s} = (a\gamma + 1)\frac{e^{\pm i\mu_s}}{e^{2i\pi(\nu \pm Q_s)} - 1} J_s$$

where

$$J_{\pm x,\pm y} = \int_s^{s+L} ds'(\hat{m}_0 + i\hat{l}_0) \cdot \left\{ \begin{array}{c} \hat{y}\sqrt{\beta_x} \\ \hat{x}\sqrt{\beta_y} \end{array} \right\} K e^{\pm i\mu_{x,y}}$$

$$J_s = \int_s^{s+L} ds'(\hat{m}_0 + i\hat{l}_0) \cdot (\hat{y}D_x + \hat{x}D_y) K$$

In a flat designed perfect machine $\vec{d}(s) = 0$.

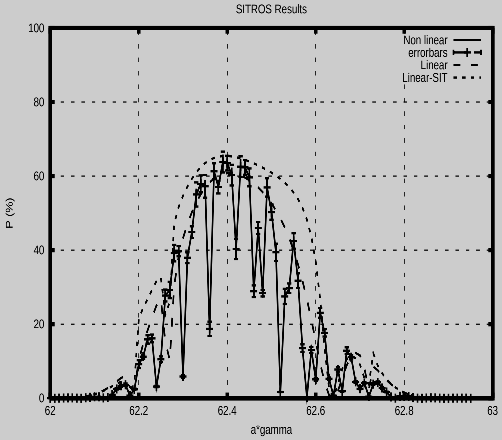
- Polarization first observed at ACO (Orsay) in 1968.
- The self polarization mechanism has been exploited at
  - HERA-e which provided *longitudinal* polarization for HERMES, H1 and ZEUS by using *spin rotators*.
  - LEP for energy calibration through RF resonant depolarization.

High level of polarization was obtained through

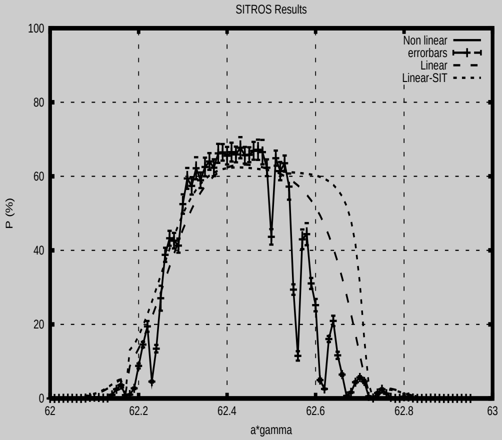
- Beam energy optimization: with ν_s half-integer the working point is halfway from all resonances.

HERA-e

- Small fractional part of orbital tunes



q_x=.2, q_y=.3, q_s=.05



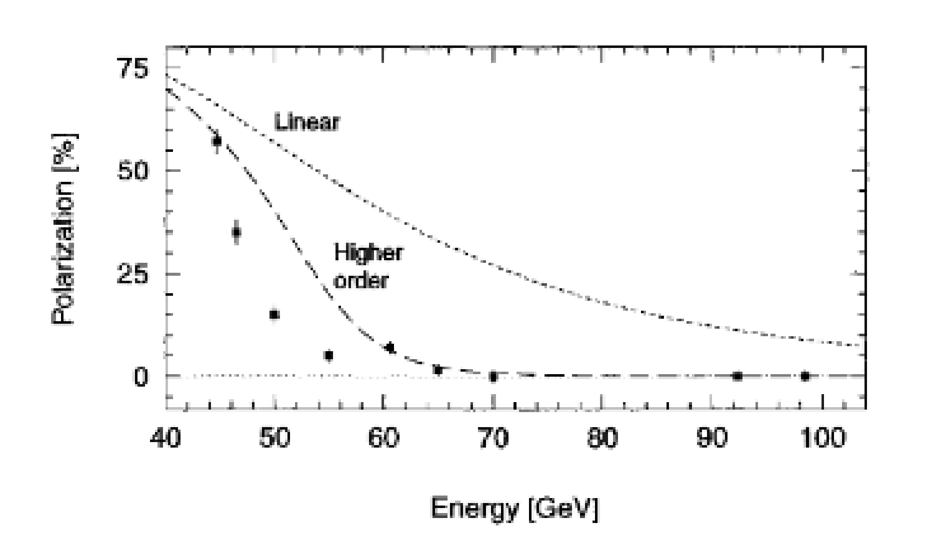
q_x=.1, q_y.=2, q_s=.05

- Orbit correction in order to minimize vertical emittance and to reduce the distortion, $\delta\hat{\boldsymbol{n}}_o$, of the T-BMT equation periodic solution from the design one (vertical in the arcs).

- Dedicated $\delta\hat{\boldsymbol{n}}_0$ correction

LEP operated between 40 and 100 GeV.

LEP measured polarization



(R. Assmann et al., SPIN2000, Osaka)

Due to larger spin diffusion polarization strongly decreased with energy!

Polarization at the $e^{\pm}$ FCC

For a 100 km long machine with ρ_b=10423.6 m

E (GeV)	U_{loss} (MeV)	$\Delta E/E$ ‰	ϵ_x (μm)	τ_x (ms)	τ_{pol} (h)
45	35	0.38	0.85e-3	868	256
80	349	0.67	0.27e-2	218	14

In presence of imperfections the actual polarization and polarization time are reduced

$$\tau_p = \tau_{ST}\frac{P}{P_{ST}}$$

For instance τ_p=30' corresponds to

E (GeV)	P
45	0.2
80	3.3

Useful level of polarization for energy calibration: $\sim$ 5-10%
$\rightarrow$ Not an option, at least at 45 GeV ...

Polarization in presence of wigglers

For decreasing the polarization time the obvious recipe is increasing synchrotron radiation emission by introducing *wiggler* magnets.

Polarization rate in a perfect planar machine, with fields possibly pointing in different directions (from DK equation on the design closed orbit):

$$\tau_p^{-1} = \frac{5\sqrt{3}}{8}\frac{r_e \gamma^5 \hbar}{m_0 C} \oint \frac{ds}{|\rho|^3} \equiv F\Big[\int_{dip} \frac{ds}{|\rho_d|^3} + \int_{wig} \frac{ds}{|\rho_w|^3}\Big]$$

Any wiggler decreases τ_p. Polarization:

$$\vec{P} = \hat{n}_0 \, P_{ST} \frac{\oint ds \frac{\hat{B}\cdot\hat{n}_0}{|\rho|^3}}{\oint ds \frac{1}{|\rho|^3}}$$

$\hat{n}_0 \equiv$ periodic solution to T-BMT equation on the *design* orbit

$$P \propto \tau_p \oint ds \, \frac{\hat{B}\cdot\hat{n}_0}{|\rho|^3} \quad \rightarrow \quad \text{Small } \tau_p\text{, small } P\ ???$$

Not necessarily...

$$\oint \boldsymbol{ds}\, \frac{\hat{\boldsymbol{B}} \cdot \hat{\boldsymbol{n}}_0}{|\boldsymbol{\rho}|^3} = \int_{dip} \boldsymbol{ds}\, \frac{\hat{\boldsymbol{B}}_d \cdot \hat{\boldsymbol{n}}_0}{|\boldsymbol{\rho}_d|^3} + \int_{wig} \boldsymbol{ds}\, \frac{\hat{\boldsymbol{B}}_w \cdot \hat{\boldsymbol{n}}_0}{|\boldsymbol{\rho}_w|^3}$$

The wiggler does not change $\hat{\boldsymbol{n}}_0$ which in a perfectly planar ring is vertical:

$$\int_{wig} \boldsymbol{ds}\, \frac{\hat{\boldsymbol{B}}_w \cdot \hat{\boldsymbol{n}}_0}{|\boldsymbol{\rho}_w|^3} = \frac{1}{\boldsymbol{ep}} \int_{wig} \boldsymbol{ds}\, \boldsymbol{B}_w^3$$

This term must be large in order to preserve a high level of polarization. For instance an antisymmetric wiggler, $\boldsymbol{B}(\boldsymbol{s}) = -\boldsymbol{B}(-\boldsymbol{s})$, would results in very small polarization.

To the wiggler field constraints for an unperturbed orbit outside the wiggler

$$\int_{wig} ds\, B_w = 0 \Rightarrow x' = 0 \quad \text{outside wiggler}$$

$$\int_{wig} ds\, sB_w = 0 \Rightarrow x = 0 \quad \text{outside wiggler}$$

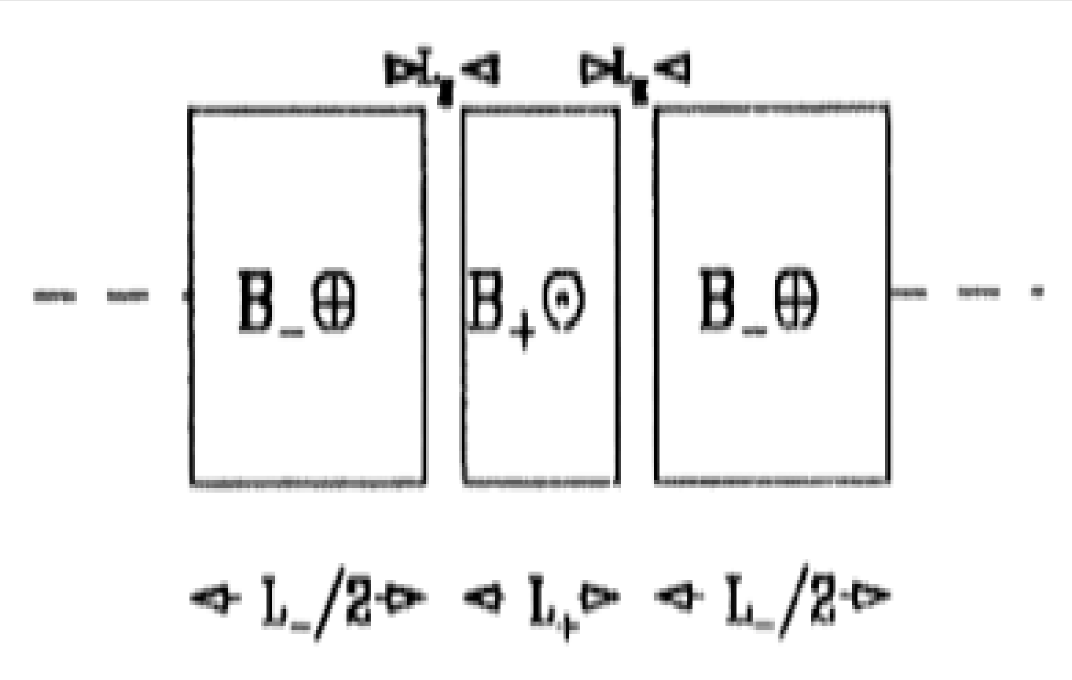
we must therefore add

$$\int_{wig} ds\, B_w^3 \neq 0 \quad \text{(large)}$$

For a symmetric wiggler the condition for $x = 0$ is automatically fulfilled.

If in addition the field integral vanishes thus also $x' = 0$.

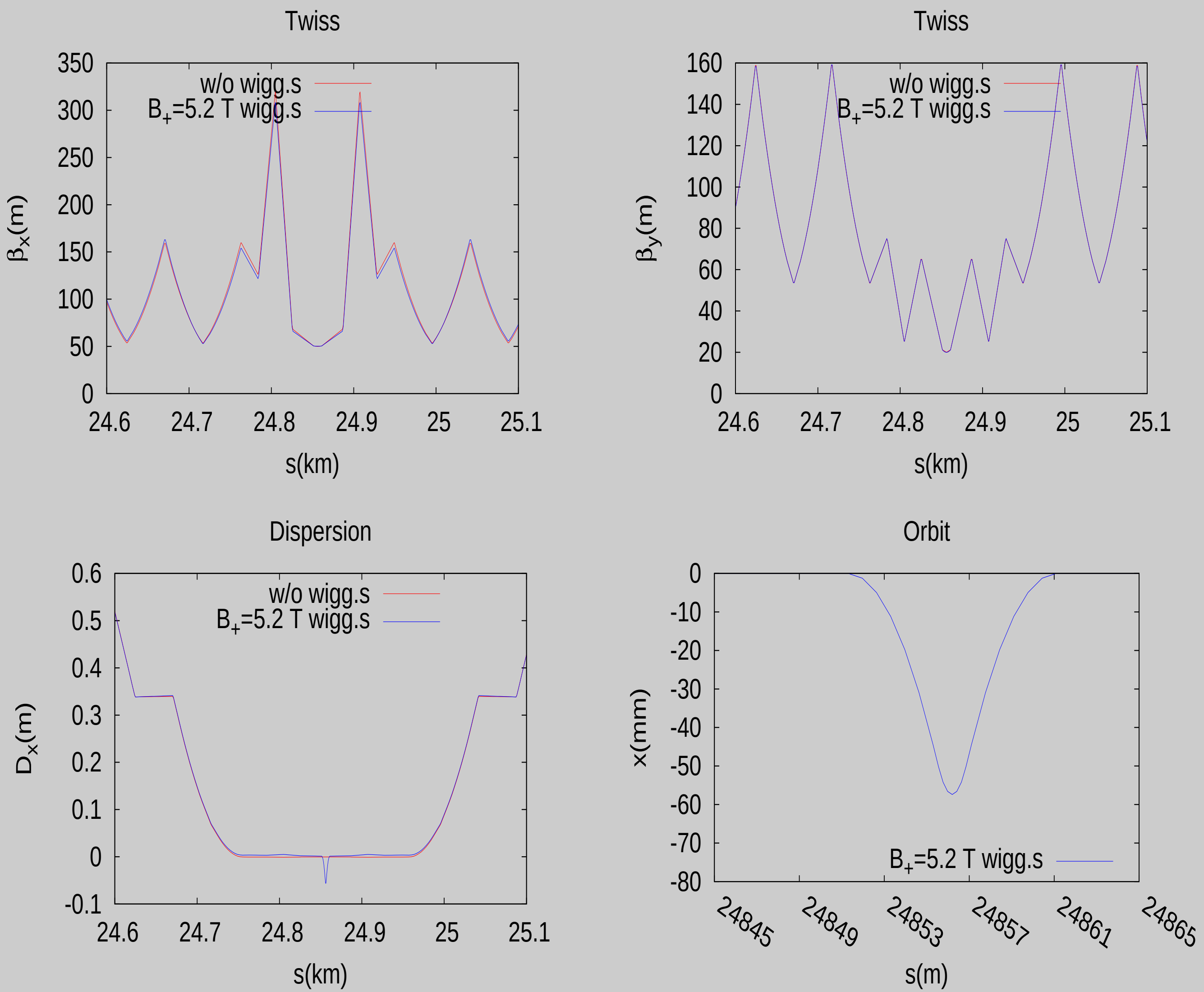
LEP polarization wiggler unit (Blondel-Jowett):



with B_+/B_-=6.

4 wigglers with L_+= 8 m introduced in dispersion free sections.

Optics in the dispersion free section w/o and w wiggler



Some beam parameters in presence of wigglers [a]

B_+ (T)	U_{loss} (MeV/turn)	$\Delta E/E$ (‰)	ϵ_x (MADX) (μm)	ϵ_x (SLIM) (μm)	τ_x (s)	P (%)	τ_{pol} (min)
0	37	0.38	0.868e-3	0.867e-3	0.82	92.4	14e3
1.3	64	2.2	0.55e-2	0.54e-2	0.48	87.6	247
2.6	144	4.1	0.072	0.070	0.21	87.6	31
3.9	278	5.5	0.281	0.274	0.11	87.6	9
5.2	466	6.5	0.708	0.691	0.06	87.6	4

Horizontal emittance and energy spread increase: potentially harmful for polarization!

[a] Implications on luminosity, beam-beam etc not investigated!

Usually the dominant higher order resonances are the *synchrotron sidebands* of the first order ones.

Distance between *imperfection* (or zeroth) order resonances: 440 MeV independently of energy! In presence of the wigglers and at 45 GeV:

B_+ (T)	$\Delta E/E$ (‰)	ΔE MeV	τ_{pol} (min)
0	0.38	17	14e3
1.3	2.2	99	247
2.6	4.1	184	31
3.9	5.5	247	9
5.2	6.5	292	4

For comparison:

	E (GeV)	$\Delta E/E$ (%)	ΔE (MeV)
HERA-e	27	0.1	27
LEP	40	0.06	26
LEP	100	0.16	160

Importance of being Q_s

Derbenev-Kondratenko-Skrinsky predict a resurrection of polarization at high energy when the condition

$$\frac{a\gamma T_{rev}}{\tau_p Q_s^3} \ll 1$$

is satisfied.

Synchrotron sidebands originate from the spin precession frequency modulation due to synchrotron oscillations.

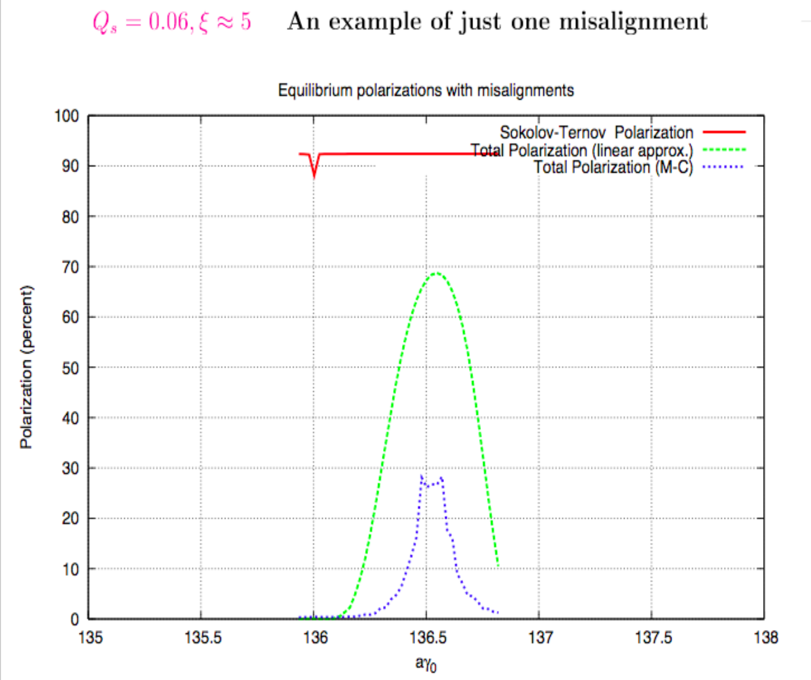
Depolarization enhancement factor due to energy spread (Yokoya, Mane)

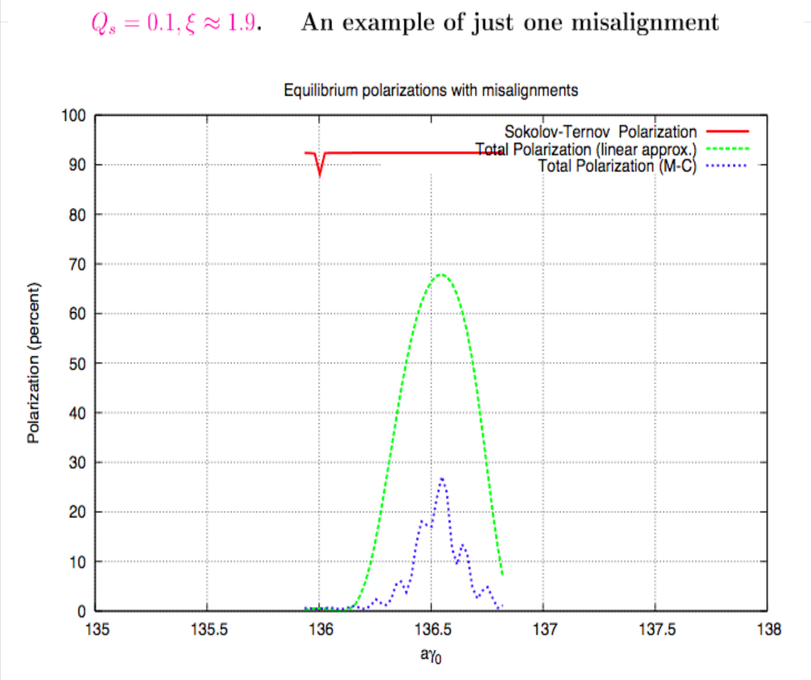
$$\xi = \left(\frac{a\gamma}{Q_s}\frac{\Delta E}{E}\right)^2$$

- Unlike the case when energy spread is small, a *large* Q_s could counteract the larger energy spread due to the high beam energy and/or to presence of wigglers
- These predictions are obtained under some assumptions and should be verified by simulations
- If confirmed they could have consequences on the the design of the optics (large α_p) and/or the choice of the RF parameters.

LHeC Ring-Ring scenario

60 GeV, ΔE ~56 MeV





(D. P. Barber et al., SPIN2010, Jülich)

What is good for polarization?

Of course resonances do not manifest themselves in a perfect machine.

- Planarity by design. Distortions to $\hat{\boldsymbol{n}}_0$ from the vertical direction must be local and spin-matched.
- Extremely well aligned magnets: it is realized in now day synchrotron radiation machines, but over 100 km?
- Non planarity due to errors must be well compensated: harmonic bumps and BBA alignment techniques should be planned. The latter requires a trim+BPM+corrector per each quadrupole.
- Space for anti-solenoids for compensating experimental solenoids must be provided.

Available codes for radiative polarization computation

- SLIM by A. Chao: analytical, linear orbit and spin motion; poor description of machine errors.
- SMILE by S. R. Mane: perturbartive, convergence problem at high energy (HERA-e and beyond).
- SITROS by J. Kewisch: tracking non-linear orbit (2th order) and spin motion; accurate description of machine errors.
- SLICKTRACK by D. P. Barber: tracking non-linear orbit and spin motion, based on a thick lenses version of SLIM formalism.

Available to me now: SLIM and SITROS, but quite some work needed to get them running again!

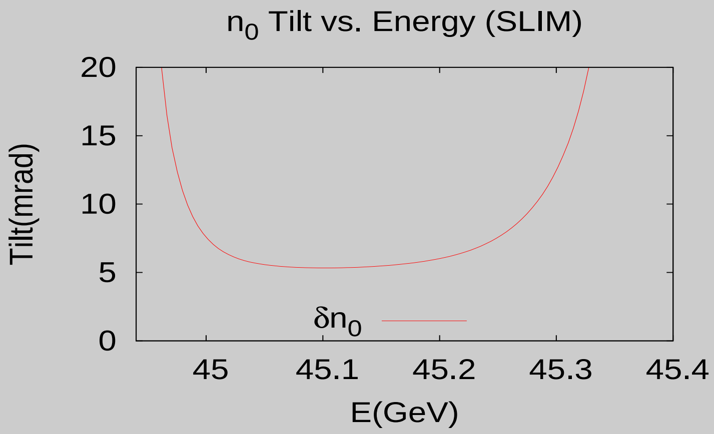
SLICKTRACK soon available from Desmond and collaborators!

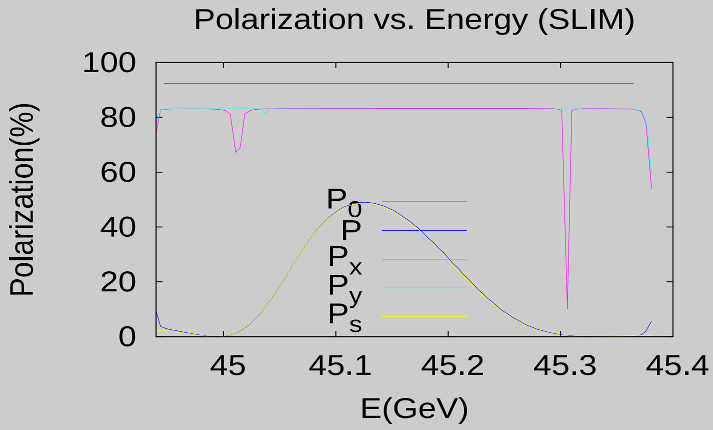
SLIM: Polarization in presence of vertical misalignments (no corrections!)

SLIM results for the "toy" machine with Q_x=181.185, Q_y=183.227 and Q_s=0.09

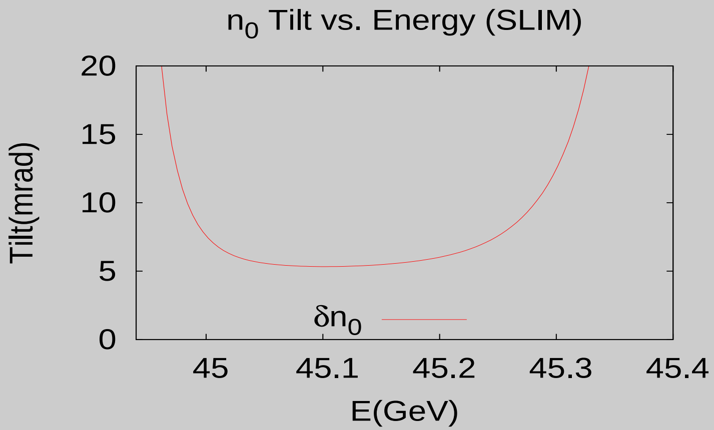
δ_y^Q=0.15 mm → x_{rms}=0.15 mm ,y_{rms}=5.4 mm

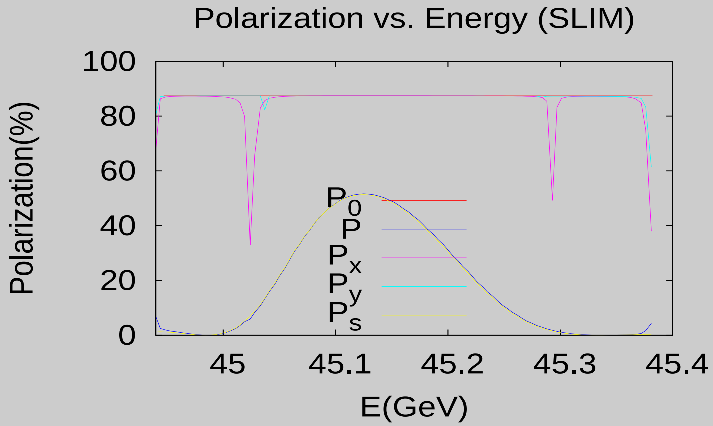
w/o wigglers





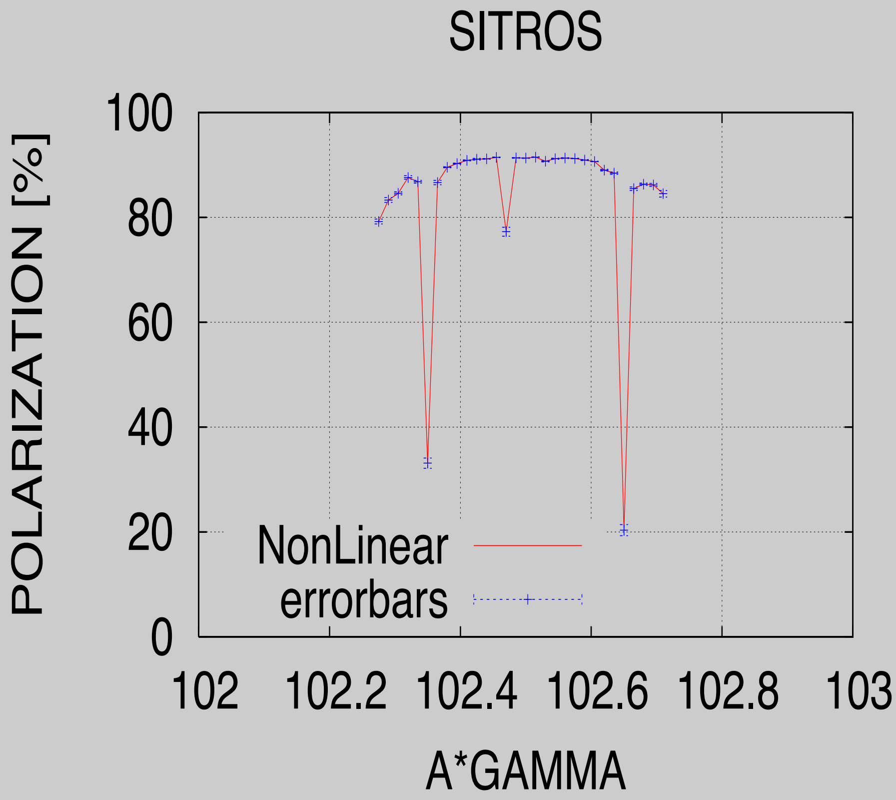
B_+=5.2 T



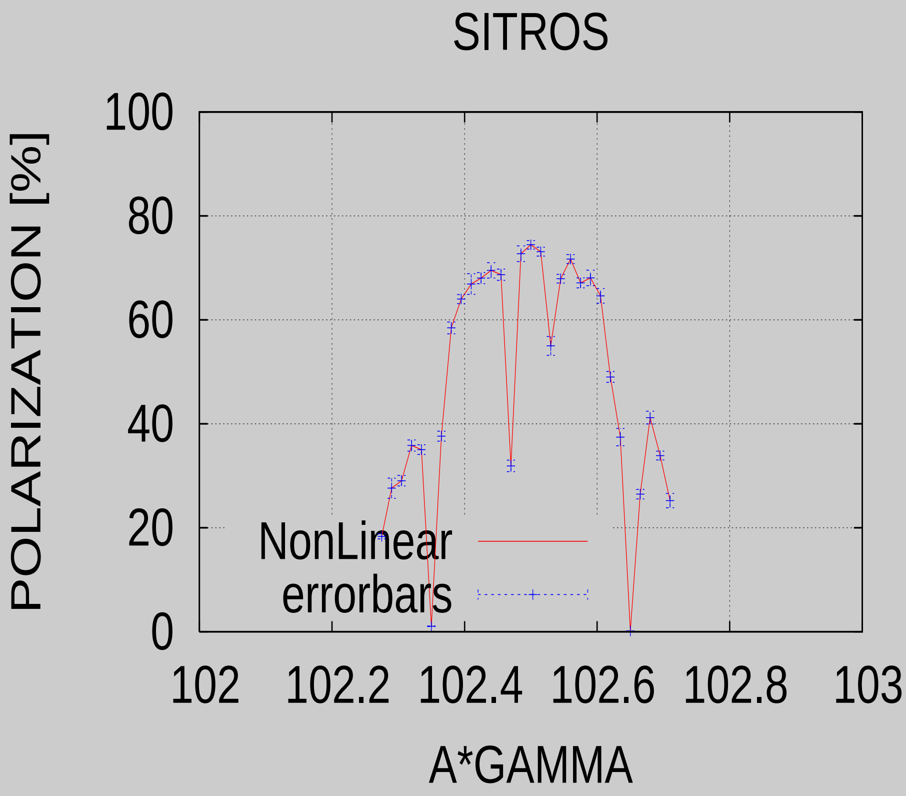


SITROS: Polarization in presence of vertical misalignments w/o wigglers (no corrections!)

B_+=0, δ_y^Q=10 μm, y_{rms} 0.4 mm



B_+=0, δ_y^Q=50 μm, y_{rms} 2 mm



Summary

45.5 GeV scenario the only here considered

- The large bending radius inflates the polarization time
- Wigglers may reduce it but at the cost of a large beam energy spread
- Polarization greatly depends upon machine planarity
- *Linear* calculations (SLIM) still foresee a useful level of polarization in presence of errors
- The large absolute energy spread however requires *higher order* calculations to assess how much, if any, polarization survives!

80 GeV scenario

- The larger energy enhances the polarization process
- Wigglers may further reduce the polarization time
- However the larger absolute energy spread enhances depolarization due to synchrotron motion.