A novel approach to access parton distribution functions

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- Quasi parton distribution
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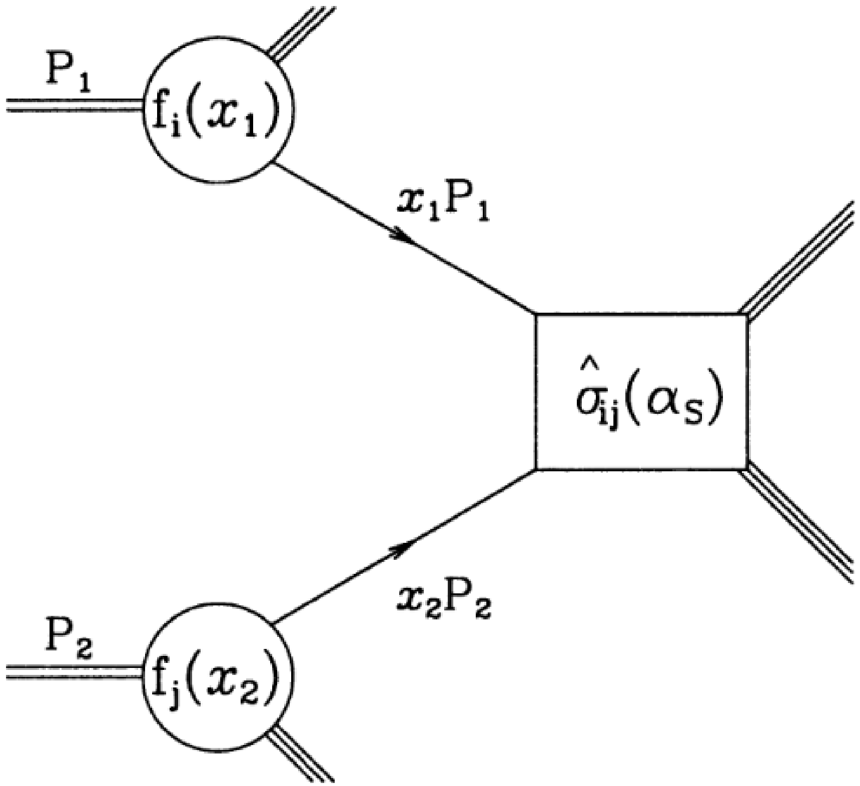
  - How to recover parton distribution from it?
- Summary

Hadronic collision

$$\sigma(P_1, P_2) = \sum_{i,j} \int dx_1 dx_2 f_i(x_1) f_j(x_2) \hat{\sigma}_{ij}(p_1, p_2)$$



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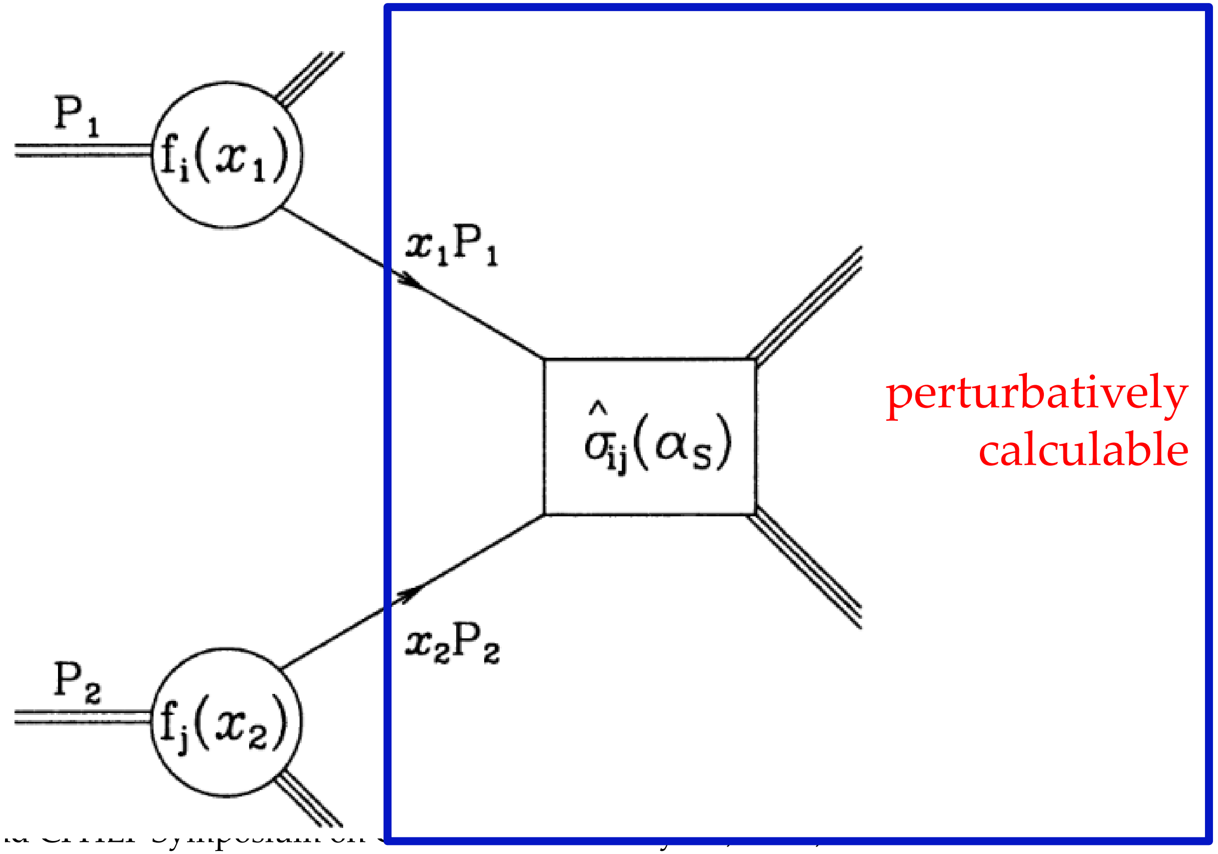

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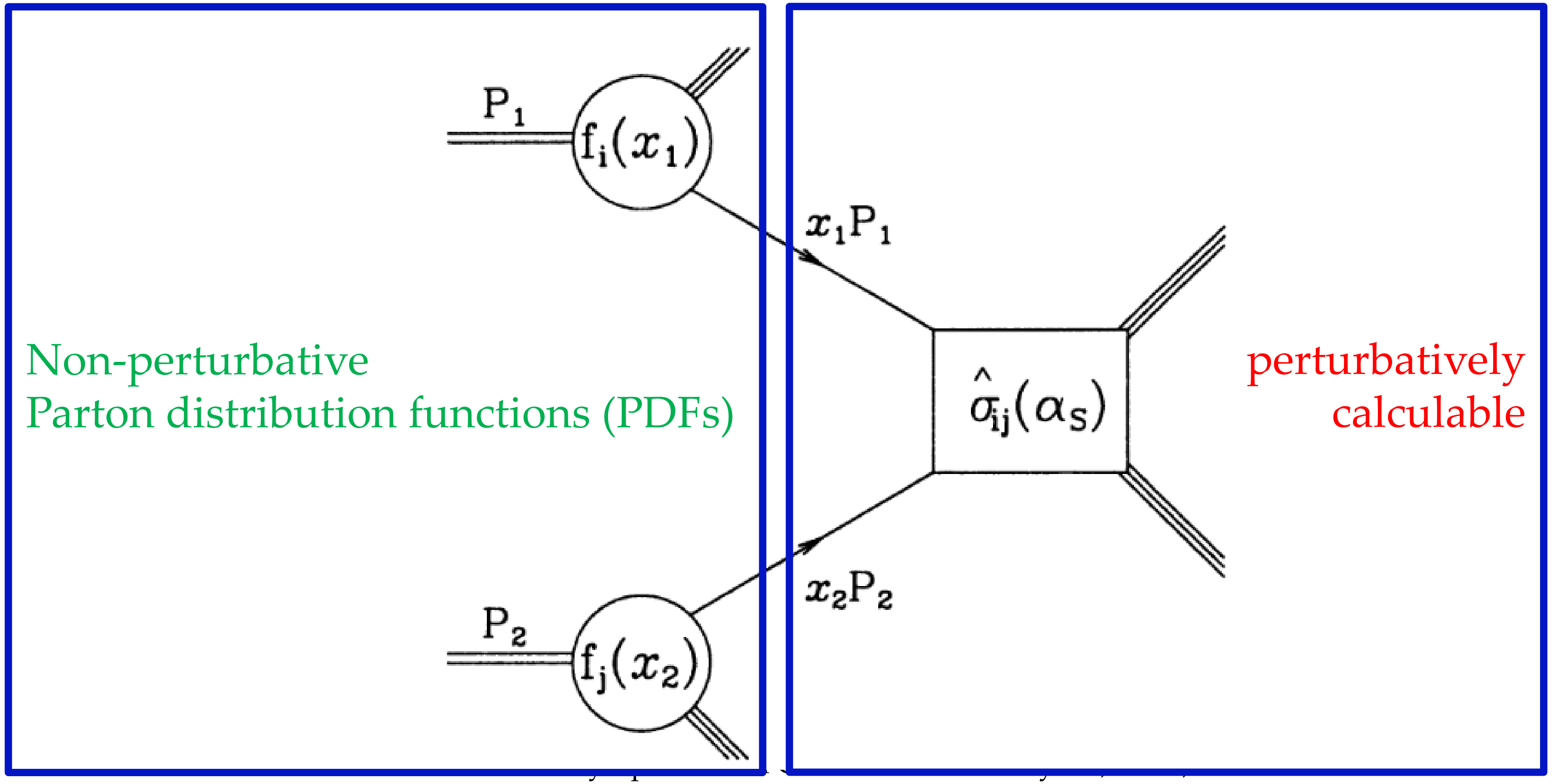

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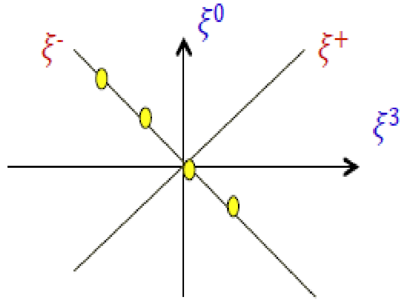
- PDFs
  - Hadrons viewed as constituted by free point-like partons in high energy collision
  - Characterize the probability of a parton having a given fraction x of the longitudinal momentum of parent hadron
  - Conveniently formulated on the light-cone

PDFs

- Operator definition of parton distribution

$$q(x,\mu^2) = \int \frac{d\xi^-}{4\pi} e^{-ix\xi^- P^+} \langle P|\overline{\psi}(\xi^-)\gamma^+ \exp\left(-ig\int_0^{\xi^-} d\eta^- A^+(\eta^-)\right)\psi(0)|P\rangle$$

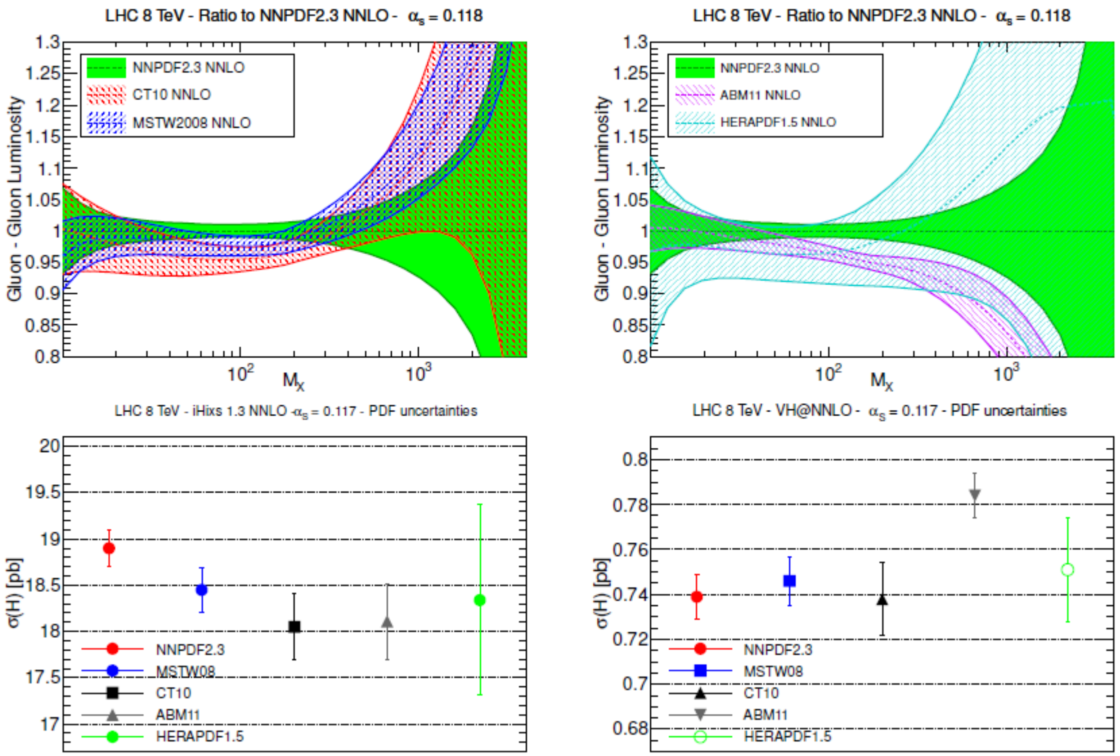
- $P^\mu = (P^0, 0, 0, P^z), \xi^\pm = \frac{t \pm z}{\sqrt{2}}, x = k^+/P^+$
- Light-cone correlation



- Parton physics is manifest in light-front quantization
  - It becomes the expectation of light-front number operator in $A^+ = 0$ gauge

PDFs

- Current strategy of determination
  - Extracted from experimental data
    - DIS, lepton pair production…
  - pQCD evolution
    - DGLAP equation
  - Different PDF sets
    - CTEQ, MSTW, NNPDF…
  - Uncertainty in theoretical predictions



PDFs

- On the lattice
  - Defined as light-cone correlation, PDFs are intrinsically Minkowskian, they cannot be directly computed using lattice QCD, which is a Euclidean approach
  - However, one can calculate their moments

$$\langle x^n(\mu^2)\rangle_q \equiv \int_0^1 dx\, x^n\, q(x,\mu^2)$$

  - Parameterization and parameters determined from lattice computed moments
  - Number of calculable moments limited

PDFs

- On the lattice

Other possibility to directly access PDFs?

Quasi parton distribution

- Recall operator definition of parton distribution

$$q(x,\mu^2) = \int \frac{d\xi^-}{4\pi} e^{-ix\xi^- P^+} \langle P|\overline{\psi}(\xi^-)\gamma^+ \exp\left(-ig\int_0^{\xi^-} d\eta^- A^+(\eta^-)\right)\psi(0)|P\rangle$$

- Look instead at an off-light-cone quasi parton distribution [Ji, 13′]

$$\tilde{q}(x,\Lambda,P^z) = \int_{-\infty}^{\infty} \frac{dz}{4\pi} e^{izk^z} \langle P|\overline{\psi}(0,0_\perp,z)\gamma^z \exp\left(-ig\int_0^{z} dz' A^z(0,0_\perp,z')\right)\psi(0)|P\rangle$$

  - Quark fields separated along z-direction, no time dependence, $x = k^z/P^z$
  - Light-cone distribution can be approached by this up to power suppressed corrections in large momentum limit

Quasi parton distribution

- From a lattice perspective, what is interesting is the quasi distribution at large but finite P^z
- It is different from the usual light-cone PDF only in finite but large or infinite momentum, therefore it shall have the same IR behavior as the light-cone PDF
- The connection can be established by a factor depending on UV physics only, and is thus perturbatively calculable

Quasi parton distribution

- How to recover PDF from the quasi one? [Ji, 13', Ji, Xiong, Zhang and Zhao, 13']

$$\tilde{q}(x, \Lambda, P^z) = \int \frac{dy}{|y|} Z\left(\frac{x}{y}, \frac{\Lambda}{P^z}, \frac{\mu}{P^z}\right) q(y, \mu)$$

  - Can be viewed as a factorization theorem
  - Soft divergences cancel on both sides
  - Collinear divergences captured by quasi distribution
  - Z factor sensitive to UV physics only, and thus perturbatively calculable

One-loop example

- LO simply a delta function
- NLO (in axial gauge $A^z = 0$)

- Remark:
  - In infinite momentum frame, support of quark density comes from requirement of positivity of cut external legs, or from integration over k^-
    - On-shell partons cannot have negative plus-momentum fraction, 0<x<1
  - In finite momentum frame, support of quasi quark density comes from integration over k^0, and the momentum fraction $-\infty < x < \infty$

One-loop example

- One-loop quasi distribution

$$\tilde{q}(x,\Lambda,P^z) = (1+\tilde{Z}_F^{(1)}(\Lambda,P^z))\delta(x-1) + \tilde{q}^{(1)}(x,\Lambda,P^z) + \dots$$

$$\tilde{q}^{(1)}(x,\Lambda,P^z) = \frac{\alpha_S C_F}{2\pi}\begin{cases} \frac{1+x^2}{1-x}\ln\frac{x}{x-1} + 1 + \frac{\Lambda}{(1-x)^2 P^z}\,, & x>1\,, \\ \frac{1+x^2}{1-x}\ln\frac{(P^z)^2}{m^2} + \frac{1+x^2}{1-x}\ln\frac{4x}{1-x} - \frac{4x}{1-x} + 1 + \frac{\Lambda}{(1-x)^2 P^z}\,, & 0<x<1\,, \\ \frac{1+x^2}{1-x}\ln\frac{x-1}{x} - 1 + \frac{\Lambda}{(1-x)^2 P^z}\,, & x<0\,, \end{cases}$$

$$\tilde{Z}_F^{(1)}(\Lambda,P^z) = \frac{\alpha_S C_F}{2\pi}\int dy \begin{cases} -\frac{1+y^2}{1-y}\ln\frac{y}{y-1} - 1 - \frac{\Lambda}{(1-y)^2 P^z}\,, & y>1\,, \\ -\frac{1+y^2}{1-y}\ln\frac{(P^z)^2}{m^2} - \frac{1+y^2}{1-y}\ln\frac{4y}{1-y} + \frac{4y^2}{1-y} + 1 - \frac{\Lambda}{(1-y)^2 P^z}\,, & 0<y<1\,, \\ -\frac{1+y^2}{1-y}\ln\frac{y-1}{y} + 1 - \frac{\Lambda}{(1-y)^2 P^z}\,, & y<0\,. \end{cases}$$

No logarithmic UV divergence, but ln P^z instead in 0<x<1 region
Momentum fraction not restricted to [0,1]

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$$Z_F^{(1)}(\Lambda) = \frac{\alpha_S C_F}{2\pi}\int dy \begin{cases} 0\,, & y>1 \text{ or } y<0\,, \\ -\frac{1+y^2}{1-y}\ln\frac{\Lambda^2}{m^2} + \frac{1+y^2}{1-y}\ln(1-y)^2 + \frac{2y}{1-y}\,, & 0<y<1\,, \end{cases}$$

Logarithmic UV divergence in 0<x<1 region
Momentum fraction restricted to [0,1]
Same mass singularity as the quasi distribution

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Same mass singularity as the quasi distribution

One-loop example

- Matching between quasi and light-cone quark distribution

$$\tilde{q}(x,\Lambda,P^z) = \int \frac{dy}{|y|} Z\left(\frac{x}{y},\frac{\Lambda}{P^z},\frac{\mu}{P^z}\right) q(y,\mu)$$

- Z factor $Z\left(\xi,\frac{\Lambda}{P^z},\frac{\mu}{P^z}\right) = \delta(\xi-1) + \frac{\alpha_s}{2\pi} Z^{(1)}\left(\xi,\frac{\Lambda}{P^z},\frac{\mu}{P^z}\right) + \ldots$

$$Z^{(1)}(\xi)/C_F = \left(\frac{1+\xi^2}{1-\xi}\right)\ln\frac{\xi}{\xi-1} + 1 + \frac{1}{(1-\xi)^2}\frac{\Lambda}{P^z}, \qquad \xi > 1$$

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One-loop example

- Matching between quasi and light-cone quark distribution

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One-loop example

- Matching between quasi and light-cone quark distribution

$$\tilde{q}(x, \Lambda, P^z) = \int \frac{dy}{|y|} Z\left(\frac{x}{y}, \frac{\Lambda}{P^z}, \frac{\mu}{P^z}\right) q(y, \mu)$$

- Time-independent quasi distribution and perturbatively calculable matching factor allow a direct computation of parton distribution

- Infinite momentum result as an effective theory of finite momentum result, similar strategy can be applied to other quantities defined on the light-cone, can be formulated in terms of a large momentum effective theory (LaMET) [Ji, 14', Ji, Zhang and Zhao, to appear]

How does a LaMET work?

- Construct a frame-dependent, Euclidean quasi operator $\tilde{O}$ that becomes in the infinite momentum limit the light-cone operator under investigation, and can be computed on the lattice
    - Choice of quasi operator is not necessarily unique
    - e.g. both P^0 and P^z approach P^+ in the infinite momentum limit

- The matrix element of the quasi operator $\tilde{O}$ depends on external momentum P and UV cutoff Λ of the theory, $\tilde{O}(P, \Lambda)$

How does a LaMET work?

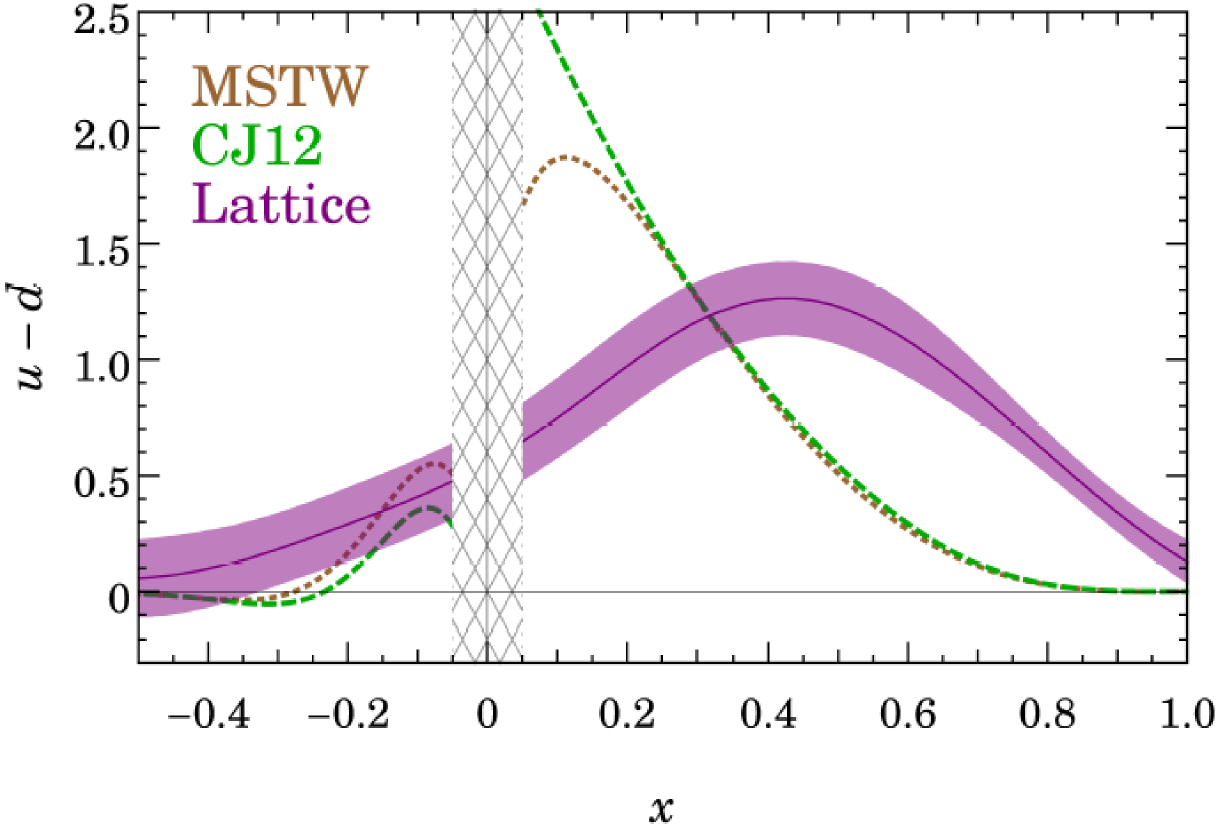
- Extract the infinite momentum (or light-cone) result $O(\mu)$ from $\tilde{O}(P,\Lambda)$ at large P through a factorization formula or matching condition

$$\tilde{O}(P,\Lambda) = Z(P/\Lambda, \mu/\Lambda)O(\mu) + \frac{c_2}{P^2} + \frac{c_4}{P^4} + \cdots$$

- $\tilde{O}(P,\Lambda)$ has the same IR behavior as $O(\mu)$
- Z factor depends on UV physics only
- c_2, c_4 denote corrections suppressed by power of external momentum

A first attempt

- [Lin, Chen, Cohen and Ji, 14’]



Summary

- Light-cone parton distribution or other quantities can be studied by investigating a related time-independent quantity at large momentum
  - Space-like correlation for parton distribution
- Allows calculation of parton distribution and related quantities on the lattice
- Matching required to connect quasi and light-cone distributions, but it depends on UV physics only, and therefore is perturbatively calculable

BACKUP SLIDES