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Comprehensive study of eta and eta' physics in chiral effective field theory

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Outline:

1. Introduction

2. Decay processes involving eta and eta'

3. ChPT calculation of their mixing

4. Summary

1. Introduction

- η & η' : a rather old subject

- New revival for multiple interests (Th & Exp)

Invisible decays (DM,BSM),

Spontaneous chiral symmetry breaking of QCD

U(1) axial anomaly of QCD

SU(3)-flavor symmetry breaking ……

BESIII: more than one million eta'

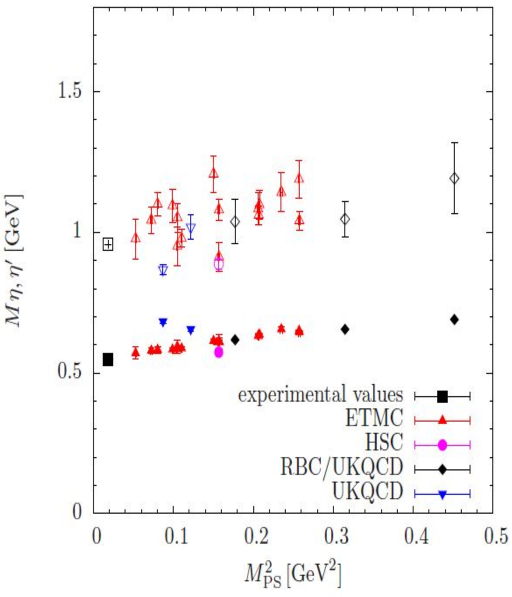
KLOE-2: around 25 millions eta

WASA-COSY, Crystal Ball/TAPS MAMI,

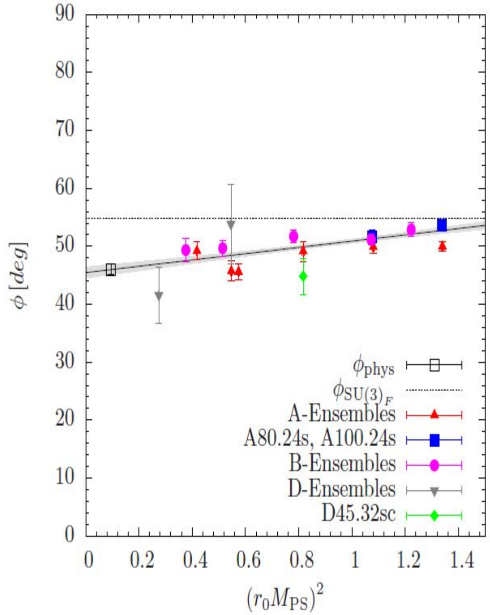
Jefferson-Lab, ……

Lattice QCD starts to make precise simulations on eta/eta'

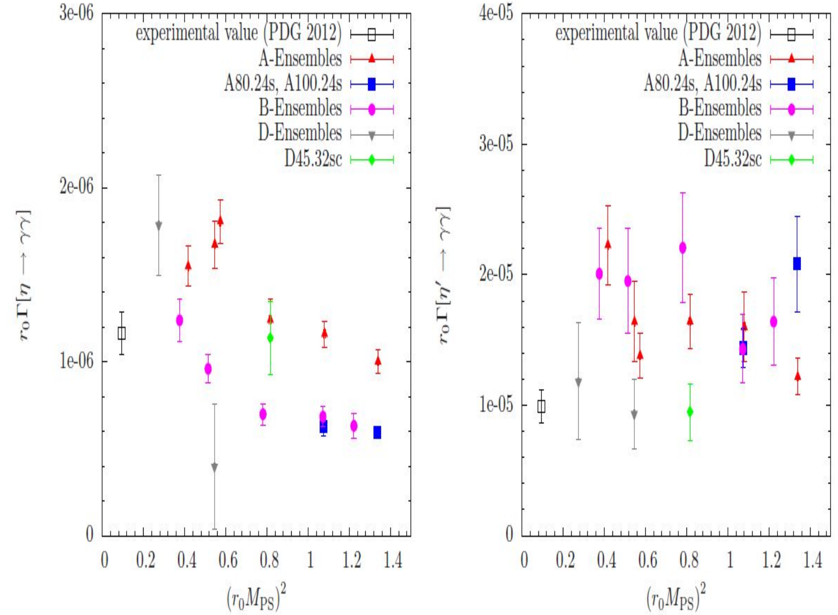
- **ETMC:** C.Michael, et al., Phys.Rev.Lett 111,181602 (2013); PoS(Lattice 2013) 253
- **UKQCD:** E.B.Gregory, et al., Phys.Rev.D 86, 014504 (2012)
- **HSC:** J.J.Dudek, et al., Phys.Rev.D 83, 111502 (2011)
- **RBC/UKQCD:** Christ, et al., Phys.Rev.Lett.105,241601 (2010)



$M_{\eta,\eta'}$ v.s. m_π^2



φ v.s. m_π^2



$\Gamma_{\eta/\eta' \to \gamma\gamma}$ v.s. m_π^2

Our aim: a global study of Exp data and Lattice simulations

2. VP$\gamma^{(*)}$, P$\gamma\gamma^{(*)}$ and J/psi -> VP, P$\gamma^{(*)}$

--- eta/eta' in decay processes involving light vectors and J/psi

Processes available from Exp:

(1) light flavor radiative decays involving η / η'

$$\omega\to\eta\gamma \quad \phi\to\eta'\gamma \quad \rho^0\to\eta\gamma \quad \eta'\to\omega\gamma \quad \eta\to\gamma\mu^-\mu^+ \quad \eta'\to\gamma^*\gamma \quad \ldots$$

(2) J/psi decays involving η / η'

$$\psi\to\omega\eta \quad \psi\to\phi\eta' \quad \psi\to\eta'\gamma \quad J/\psi\to\eta l^+l^- \quad \ldots$$

- **Resonance Chiral Theory to handle light flavor resonances**

The roadmap to build RChT: CCWZ formalism: **Ecker et al., NPB'89**

Chiral group: $G = SU(3)_L \times SU(3)_R$, $H = SU(3)_V$, $u(\phi) = G/H$

pNGB and external sources : $X = u_\mu,\ \chi_\pm,\ f_\pm^{\mu\nu},\ h_{\mu\nu}, \qquad X \stackrel{G}{\Longrightarrow} h\,X\,h^\dagger$

$$\text{Resonances :} \qquad R \stackrel{G}{\Longrightarrow} h\,R\,h^\dagger, \qquad h \in H$$

$$\nabla_\mu R \stackrel{G}{\Longrightarrow} h\,(\nabla_\mu R)\,h^\dagger, \qquad \nabla_\mu R = \partial_\mu R + [\Gamma_\mu, R]$$

$$\Gamma_\mu = \frac{1}{2}\{u^+[\partial_\mu - i(v_\mu + a_\mu)]u + u[\partial_\mu - i(v_\mu - a_\mu)]u^+\}$$

$$V_{\mu\nu} = \begin{pmatrix} \frac{1}{\sqrt{2}}\rho^0 + \frac{1}{\sqrt{6}}\omega_8 + \frac{1}{\sqrt{3}}\omega_0 & \rho^+ & K^{*+} \\ \rho^- & -\frac{1}{\sqrt{2}}\rho^0 + \frac{1}{\sqrt{6}}\omega_8 + \frac{1}{\sqrt{3}}\omega_0 & K^{*0} \\ K^{*-} & \overline{K}^{*0} & -\frac{2}{\sqrt{6}}\omega_8 + \frac{1}{\sqrt{3}}\omega_0 \end{pmatrix}_{\mu\nu},$$

$$\Phi = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta_8 + \frac{1}{\sqrt{3}}\eta_0 & \pi^+ & K^+ \\ \pi^- & \frac{-1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta_8 + \frac{1}{\sqrt{3}}\eta_0 & K^0 \\ K^- & \bar{K}^0 & \boxed{\frac{-2}{\sqrt{6}}\eta_8 + \frac{1}{\sqrt{3}}\eta_0} \end{pmatrix}$$

Modern recipe: the two-mixing angle scheme

$$\begin{pmatrix} \eta \\ \eta' \end{pmatrix} = \frac{1}{F} \begin{pmatrix} F_8 \cos\theta_8 & -F_0 \sin\theta_0 \\ F_8 \sin\theta_8 & F_0 \cos\theta_0 \end{pmatrix} \begin{pmatrix} \eta_8 \\ \eta_0 \end{pmatrix}$$

Leutwyler, '97

Phenomenologically explored by many groups: Escribano, Feldmann, Goity, Pennington, Pham,Thomas, … …

Relevant operators in our study: VVP and VJP types

$$\mathcal{L}_{VJP} = \frac{\tilde{c}_1}{M_V}\varepsilon_{\mu\nu\rho\sigma}\langle\{V^{\mu\nu}, \tilde{f}_+^{\rho\alpha}\}\nabla_\alpha\tilde{u}^\sigma\rangle + \frac{\tilde{c}_2}{M_V}\varepsilon_{\mu\nu\rho\sigma}\langle\{V^{\mu\alpha}, \tilde{f}_+^{\rho\sigma}\}\nabla_\alpha\tilde{u}^\nu\rangle + \frac{i\tilde{c}_3}{M_V}\varepsilon_{\mu\nu\rho\sigma}\langle\{V^{\mu\nu}, \tilde{f}_+^{\rho\sigma}\}\tilde{\chi}_-\rangle$$

$$+ \frac{i\tilde{c}_4}{M_V}\varepsilon_{\mu\nu\rho\sigma}\langle V^{\mu\nu}[\tilde{f}_-^{\rho\sigma}, \tilde{\chi}_+]\rangle + \frac{\tilde{c}_5}{M_V}\varepsilon_{\mu\nu\rho\sigma}\langle\{\nabla_\alpha V^{\mu\nu}, \tilde{f}_+^{\rho\alpha}\}\tilde{u}^\sigma\rangle + \frac{\tilde{c}_6}{M_V}\varepsilon_{\mu\nu\rho\sigma}\langle\{\nabla_\alpha V^{\mu\alpha}, \tilde{f}_+^{\rho\sigma}\}\tilde{u}^\nu\rangle$$

$$+ \frac{\tilde{c}_7}{M_V}\varepsilon_{\mu\nu\rho\sigma}\langle\{\nabla^\sigma V^{\mu\nu}, \tilde{f}_+^{\rho\alpha}\}\tilde{u}_\alpha\rangle - i\tilde{c}_8 M_V\sqrt{\frac{2}{3}}\varepsilon_{\mu\nu\rho\sigma}\langle V^{\mu\nu}\tilde{f}_+^{\rho\sigma}\rangle \ln(\det\tilde{u}),$$

$$\mathcal{L}_{VVP} = \tilde{d}_1\varepsilon_{\mu\nu\rho\sigma}\langle\{V^{\mu\nu}, V^{\rho\alpha}\}\nabla_\alpha\tilde{u}^\sigma\rangle + i\tilde{d}_2\varepsilon_{\mu\nu\rho\sigma}\langle\{V^{\mu\nu}, V^{\rho\sigma}\}\tilde{\chi}_-\rangle + \tilde{d}_3\varepsilon_{\mu\nu\rho\sigma}\langle\{\nabla_\alpha V^{\mu\nu}, V^{\rho\alpha}\}\tilde{u}^\sigma\rangle$$

$$+ \tilde{d}_4\varepsilon_{\mu\nu\rho\sigma}\langle\{\nabla^\sigma V^{\mu\nu}, V^{\rho\alpha}\}\tilde{u}_\alpha\rangle - i\tilde{d}_5 M_V^2\sqrt{\frac{2}{3}}\varepsilon_{\mu\nu\rho\sigma}\langle V^{\mu\nu}V^{\rho\sigma}\rangle \ln(\det\tilde{u}).$$

[Ruiz-Femenia, Pich and Portolés, '03]

[Chen, ZHG, Zheng, PRD'12]

An efficient way to constrain the resonance couplings:

the high energy behaviors dictated by QCD [Ecker, et al., PLB'89]

Important objects: VVP, VAP, AAP,… (order parameters of chiral symmetry, free of pQCD corrections in chiral limit)

VVP Green-functions

$$\int d^4x \int d^4y e^{i(p\cdot x+q\cdot y)}\langle 0|T[V^a_\mu(x)V^b_\nu(y)P^c(0)]|0\rangle$$
$$= d^{abc}\epsilon_{\mu\nu\alpha\beta}p^\alpha q^\beta \Pi_{\rm VVP}(p^2,q^2,r^2),$$

$\Pi_{\rm VVP}(p^2,q^2,r^2)$ can be calculated in OPE and RChT

$$\lim_{\lambda\to\infty}\Pi^{(8)}_{\rm VVP}[(\lambda p)^2,(\lambda q)^2,(\lambda p+\lambda q)^2]$$
$$= \lim_{\lambda\to\infty}\Pi^{(0)}_{\rm VVP}[(\lambda p)^2,(\lambda q)^2,(\lambda p+\lambda q)^2]$$
$$= -\frac{\langle\bar\psi\psi\rangle_0}{2\lambda^4}\frac{p^2+q^2+r^2}{p^2q^2r^2}[1+\mathcal{O}(\alpha_s)]+O\left(\frac{1}{\lambda^6}\right)$$

$$\begin{aligned}
\Pi^{(0)}_{\rm VVP}(p^2,q^2,r^2) =& \\
-\frac{\langle\bar\psi\psi\rangle_0}{F^2}\Big\{&-4F_V^2\frac{(\tilde d_1-\tilde d_3)r^2+\tilde d_3(p^2+q^2)}{(M_V^2-p^2)(M_V^2-q^2)(M_0^2-r^2)}\\
&+2\sqrt2\frac{F_V}{M_V}\frac{(\tilde c_1+\tilde c_2-\tilde c_5)r^2+(\tilde c_2+\tilde c_5-\tilde c_1-2\tilde c_6)p^2+(\tilde c_1-\tilde c_2+\tilde c_5)q^2}{(M_0^2-r^2)(M_V^2-p^2)}\\
&+2\sqrt2\frac{F_V}{M_V}\frac{(\tilde c_1+\tilde c_2-\tilde c_5)r^2+(\tilde c_2+\tilde c_5-\tilde c_1-2\tilde c_6)q^2+(\tilde c_1-\tilde c_2+\tilde c_5)p^2}{(M_0^2-r^2)(M_V^2-q^2)}\\
&+\frac{32F_V^2\tilde d_2}{(M_V^2-p^2)(M_V^2-q^2)}-\frac{16\sqrt2F_V\tilde c_3}{M_V}\left(\frac{1}{M_V^2-p^2}+\frac{1}{M_V^2-q^2}\right)+\frac{N_C}{8\pi^2(M_0^2-r^2)}\\
&+4\sqrt3\tilde c_8M_VF_V\left[\frac{1}{(M_V^2-q^2)(M_0^2-r^2)}+\frac{1}{(M_V^2-p^2)(M_0^2-r^2)}\right]\\
&-2\sqrt6\frac{\tilde d_5F_V^2M_V^2}{(M_V^2-p^2)(M_V^2-q^2)(M_0^2-r^2)}\Big\}.
\end{aligned}$$

To match the VVP Green function in OPE and RChT:

$$4\tilde{c}_3 + \tilde{c}_1 = 0,$$

$$\tilde{c}_1 - \tilde{c}_2 + \tilde{c}_5 = 0,$$

$$\tilde{c}_5 - \tilde{c}_6 = \frac{N_C}{64\pi^2}\frac{M_V}{\sqrt{2}F_V},$$

$$\tilde{d}_1 + 8\tilde{d}_2 - \tilde{d}_3 = \frac{F^2}{8F_V^2},$$

$$\tilde{d}_3 = -\frac{N_C}{64\pi^2}\frac{M_V^2}{F_V^2} + \frac{F^2}{8F_V^2} - \frac{\sqrt{3}M_V}{F_V}\tilde{c}_8 - \frac{\sqrt{2}M_0^2}{M_V F_V}\tilde{c}_1.$$

$$\tilde{c}_8 = -\frac{\sqrt{2}M_0^2}{\sqrt{3}M_V^2}\tilde{c}_1.$$

Similar procedure also applied to form factors $P\gamma\gamma^*$

$$\tilde{c}_1 - \tilde{c}_2 + \tilde{c}_5 = 0,$$

$$\tilde{c}_5 - \tilde{c}_6 = \frac{F_V}{\sqrt{2}M_V}\tilde{d}_3 + \frac{N_C M_V}{32\sqrt{2}\pi^2 F_V},$$

$$\tilde{c}_5 - \tilde{c}_6 = -\frac{F_V}{\sqrt{2}M_V}\tilde{d}_3.$$

$$\tilde{c}_5 - \tilde{c}_6 = \frac{N_C}{64\pi^2}\frac{M_V}{\sqrt{2}F_V},$$

$$\tilde{d}_3 = -\frac{N_C}{64\pi^2}\frac{M_V^2}{F_V^2} - \frac{\sqrt{3}M_V}{F_V}\tilde{c}_8 - \frac{\sqrt{2}M_0^2}{M_V F_V}\tilde{c}_1,$$

[Ruiz-Femenia, Pich and Portolés, '03]

[Chen, ZHG, Zheng, PRD'12] [Roig, Sanz-Cillero, PLB'14]

The combined high energy constraints used in our global-fit analyses:

$$4\tilde{c}_3 + \tilde{c}_1 = 0,$$

$$\tilde{c}_1 - \tilde{c}_2 + \tilde{c}_5 = 0,$$

$$\tilde{c}_5 - \tilde{c}_6 = \frac{N_C}{64\pi^2}\frac{M_V}{\sqrt{2}F_V},$$

$$\tilde{d}_1 + 8\tilde{d}_2 - \tilde{d}_3 = \frac{F^2}{8F_V^2},$$

$$\tilde{d}_3 = -\frac{N_C}{64\pi^2}\frac{M_V^2}{F_V^2}$$

$$\tilde{c}_8 = -\frac{\sqrt{2}M_0^2}{\sqrt{3}M_V^2}\tilde{c}_1.$$

$$\tilde{d}_5 = 4.4\tilde{d}_2 - 0.06.$$

[Chen, ZHG, Zou, PRD'15]

Effective Lagrangian for the interactions of J/Psi and light mesons

Strong interactions:

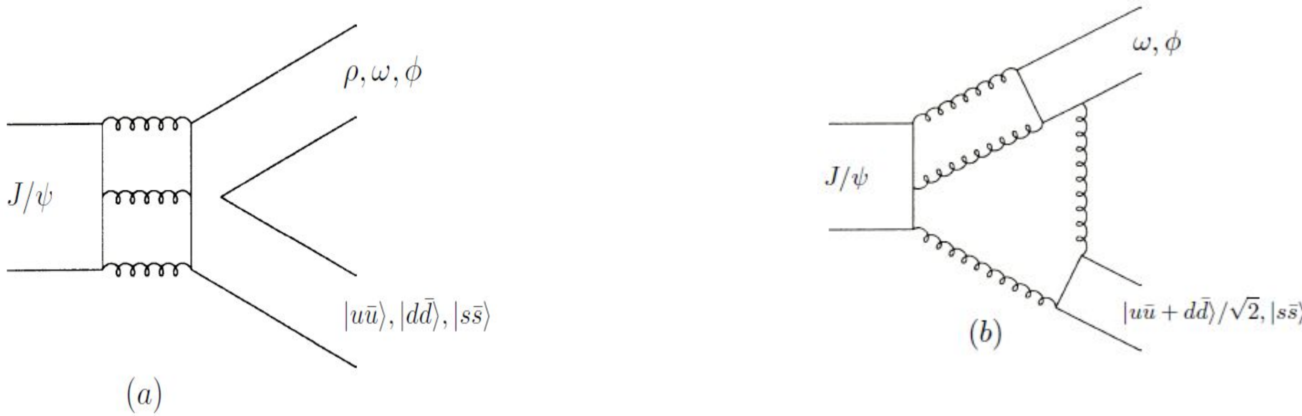
$$\mathcal{L}_{\psi VP} = M_\psi h_1 \varepsilon_{\mu\nu\rho\sigma}\psi^\mu \langle \tilde{u}^\nu V^{\rho\sigma}\rangle + \frac{1}{M_\psi} h_2 \varepsilon_{\mu\nu\rho\sigma}\psi^\mu \langle \{\tilde{u}^\nu, V^{\rho\sigma}\}\tilde{\chi}_+\rangle + M_\psi h_3 \varepsilon_{\mu\nu\rho\sigma}\psi^\mu \langle \tilde{u}^\nu\rangle\langle V^{\rho\sigma}\rangle,$$

[Y.H.Chen, ZHG, B.S.Zou, PRD'15]

Notice: this is NOT chiral EFT, but more like a phenomenological model.

The chiral building blocks are employed to incorporate the light flavor mesons.

Merits: systematical way to include the SU(3)-flavor symmetry breaking and the OZI rules (large Nc, by counting the number of traces)



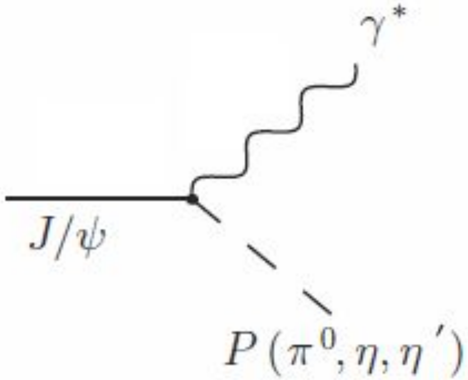
[Seiden, et al., PRD'88]

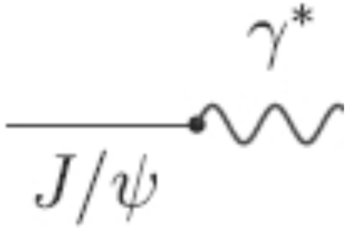
EM interactions:

$$\mathcal{L}_{\psi P\gamma} = g_1 \varepsilon_{\mu\nu\rho\sigma}\psi^\mu \langle \tilde{u}^\nu \tilde{f}_+^{\rho\sigma}\rangle$$

$$+ \frac{1}{M_\psi^2} g_2 \varepsilon_{\mu\nu\rho\sigma}\psi^\mu \langle \{\tilde{u}^\nu, \tilde{f}_+^{\rho\sigma}\}\tilde{\chi}_+\rangle,$$

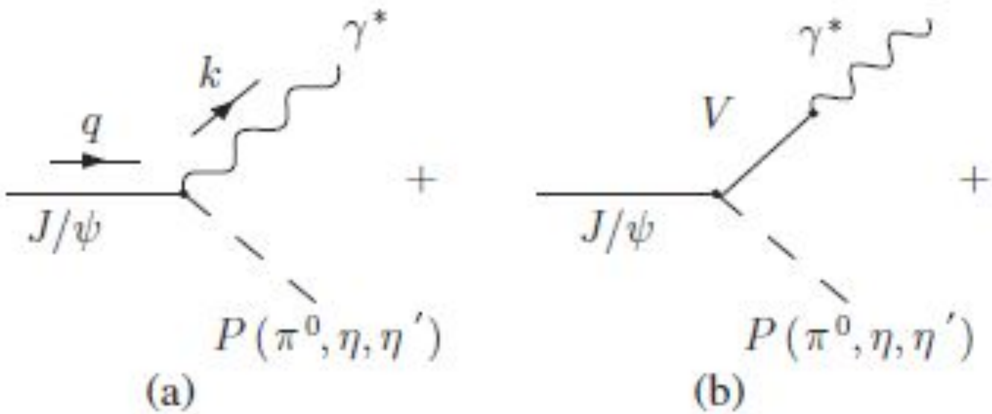
$$\mathcal{L}_2^\psi = \frac{-1}{2\sqrt{2}}\frac{f_\psi}{M_\psi}\langle \hat{\psi}_{\mu\nu}\tilde{f}_+^{\mu\nu}\rangle,$$

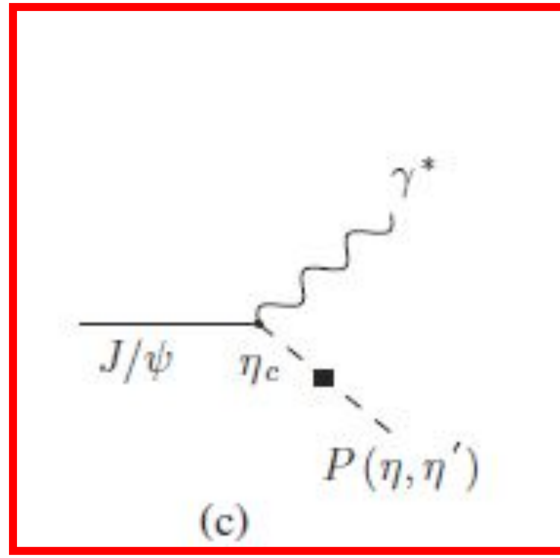




$$f_\psi = \left(\frac{27 M_\psi \Gamma_{\psi\to e^+e^-}}{32\pi\alpha^2}\right)^{\frac{1}{2}}$$

Feynman Diagrams for J/Psi -> P γ $^{(*)}$





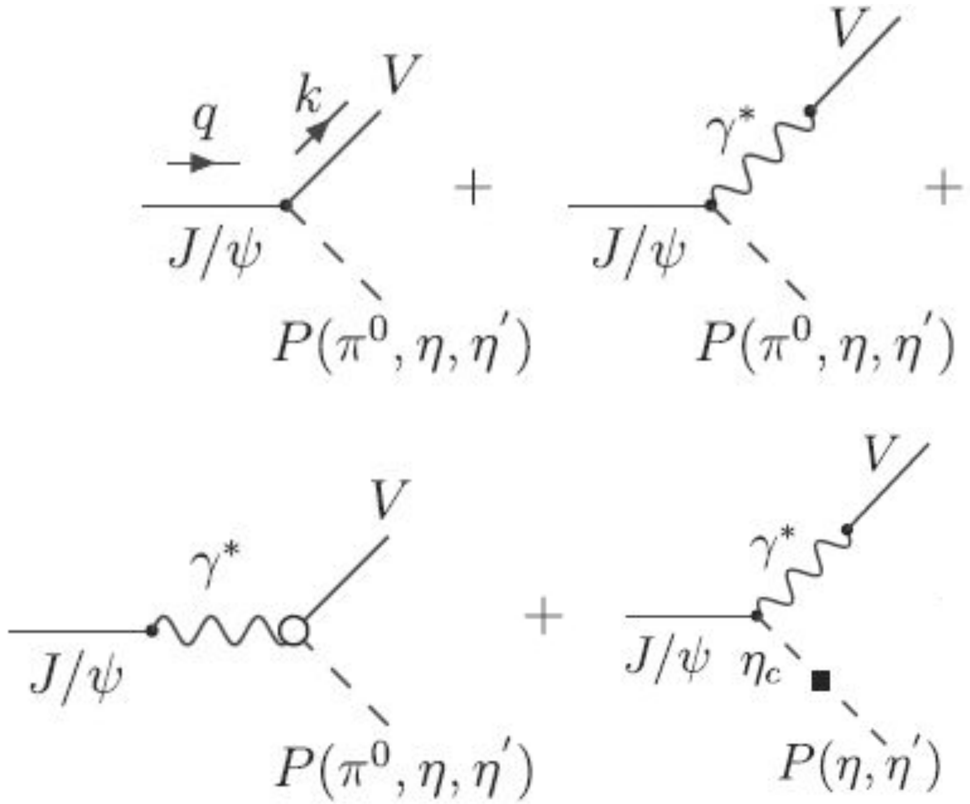
We confirm the mechanism proposed by K.T.Chao, NPB'90

$$iM_{\psi\to\eta_c\gamma^*} = ie\varepsilon_{\mu\nu\rho\sigma}\epsilon_\psi^\mu \epsilon_{\gamma^*}^\nu q^\rho k^\sigma g_{\psi\eta_c\gamma}(s),$$

$$g_{\psi\eta_c\gamma}(s) = g_{\psi\eta_c\gamma}(0)e^{\frac{s}{16\beta^2}},$$ Lattice: [Dudek, et al., PRD'06]

$$|g_{\psi\eta_c\gamma}(0)| = \left(\frac{24M_\psi^3\Gamma_{\psi\to\eta_c\gamma}}{\alpha(M_\psi^2 - m_{\eta_c}^2)^3}\right)^{\frac{1}{2}}$$

Feynman Diagrams for J/Psi -> VP



Phenomenological Discussions

Data included in the fit

$$J/\psi \to P\gamma \qquad J/\psi \to VP \qquad J/\psi \to Pe^+e^-$$

$$P \to V\gamma, \quad V \to P\gamma, \quad P \to \gamma\gamma, \quad P \to \gamma l^+l^-, \quad V \to Pl^+l^-,$$

$$\eta \to \gamma\gamma^*, \quad \eta' \to \gamma\gamma^*, \ \phi \to \eta\gamma^*$$

$$P = \pi, K, \eta, \eta' \ \text{ and } \ V = \rho, K^*, \omega, \phi,$$

106 data points in total

[PDG, CPC '14], BESIII [PRD89,092008(2014)]

[Lepton-G,CELLO,TPC, SND … , on the light flavor sector]

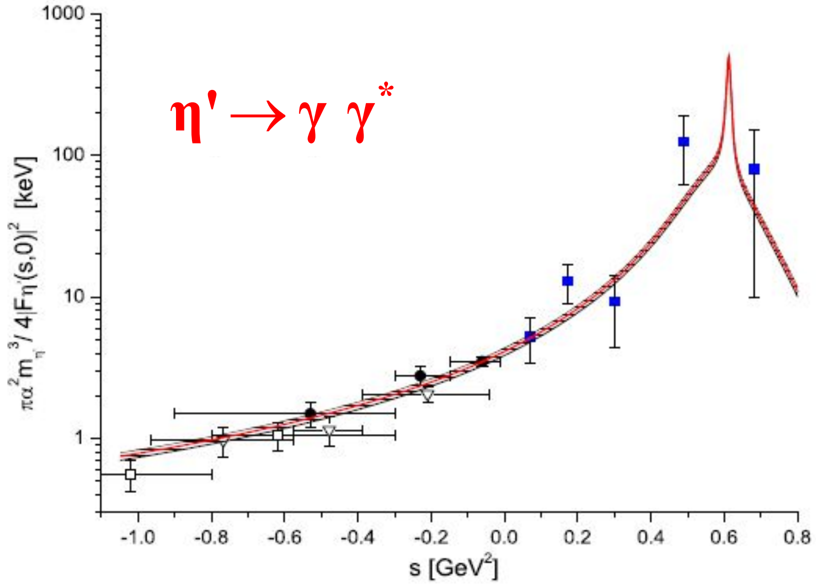
	Fit with J/Psi and light flavor data	Fit with only light flavor data
F_8	$(1.45 \pm 0.04)F_\pi$	$(1.37 \pm 0.07)F_\pi$
F_0	$(1.28 \pm 0.06)F_\pi$	$(1.19 \pm 0.18)F_\pi$
θ_8	$(-26.7 \pm 1.8)^\circ$	$(-21.1 \pm 6.0)^\circ$
θ_0	$(-11.0 \pm 1.0)^\circ$	$(-2.5 \pm 8.2)^\circ$
F_V	134.9 ± 3.2	136.6 ± 3.5
$\tilde{c}_3$	0.0029 ± 0.0006	0.0109 ± 0.0161
$\tilde{d}_2$	0.081 ± 0.006	0.086 ± 0.085
h_1	$(-2.36 \pm 0.13) \times 10^{-5}$	...
h_2	$(-4.73 \pm 1.26) \times 10^{-5}$	...
h_3	$(3.85 \pm 0.45) \times 10^{-6}$	...
g_1	$(-2.92 \pm 0.17) \times 10^{-5}$	...
g_2	$(5.93 \pm 1.04) \times 10^{-4}$	...
r_1	0.44 ± 0.10	...
δ_η	$(39 \pm 44)^\circ$	...
$\delta_{\eta'}$	$(115 \pm 13)^\circ$	...
$\frac{\chi^2}{d.o.f}$	$\frac{96.0}{106-15} = 1.06$	$\frac{64.0}{70-8} = 1.03$

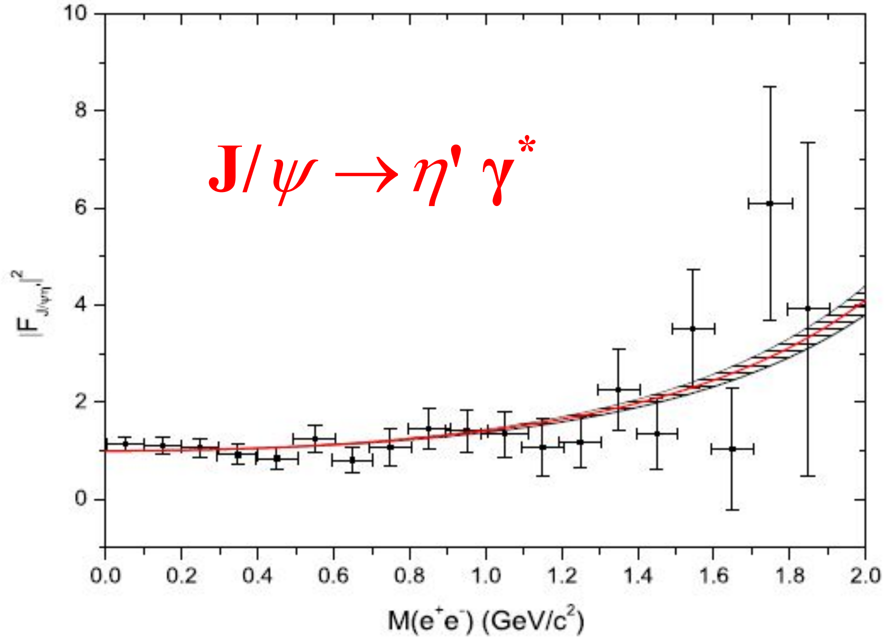
Uncertainties of parameters are clearly decreased when including J/Psi data

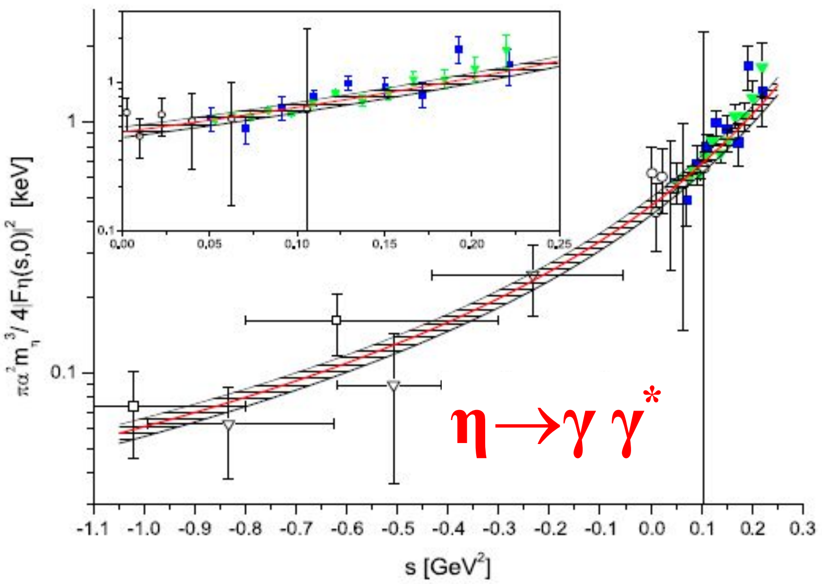
	Experiment	Fit
$\psi \to \rho^0\pi^0$	5.3 ± 0.7	5.6 ± 0.7
$\psi \to \rho\pi$	16.9 ± 1.5	16.4 ± 1.9
$\psi \to \rho^0\eta$	0.193 ± 0.023	0.202 ± 0.047
$\psi \to \rho^0\eta'$	0.105 ± 0.018	0.110 ± 0.035
$\psi \to \omega\pi^0$	0.45 ± 0.05	0.45 ± 0.12
$\psi \to \omega\eta$	1.74 ± 0.20	1.74 ± 0.25
$\psi \to \omega\eta'$	0.182 ± 0.021	0.184 ± 0.040
$\psi \to \phi\eta$	0.75 ± 0.08	0.82 ± 0.11
$\psi \to \phi\eta'$	0.40 ± 0.07	0.38 ± 0.13
$\psi \to K^{*+}K^- + \text{c.c}$	5.12 ± 0.30	4.79 ± 0.51
$\psi \to K^{*0}\bar{K}^0 + \text{c.c.}$	4.39 ± 0.31	4.43 ± 0.38
$\psi \to \pi^0\gamma$	0.0349 ± 0.0032	0.0303 ± 0.0086
$\psi \to \eta\gamma$	1.104 ± 0.034	1.101 ± 0.079
$\psi \to \eta'\gamma$	5.16 ± 0.15	5.22 ± 0.15
$\psi \to \pi^0 e^+ e^-$	$(0.0756 \pm 0.0141) \times 10^{-2}$	$(0.1191 \pm 0.0138) \times 10^{-2}$
$\psi \to \eta e^+ e^-$	$(1.16 \pm 0.09) \times 10^{-2}$	$(1.16 \pm 0.08) \times 10^{-2}$
$\psi \to \eta' e^+ e^-$	$(5.81 \pm 0.35) \times 10^{-2}$	$(5.76 \pm 0.16) \times 10^{-2}$

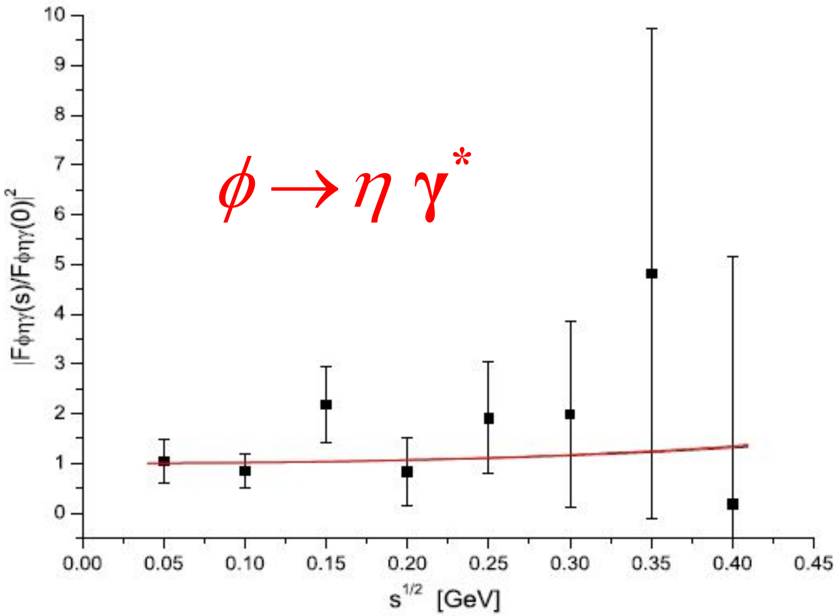
	Experiment	Fit
$\Gamma_{\omega\to\pi\gamma}$	757 ± 28	750 ± 33
$\Gamma_{\rho^0\to\pi^0\gamma}$	89.6 ± 12.6	78.0 ± 3.4
$\Gamma_{K^{*0}\to K^0\gamma}$	116 ± 12	116 ± 5
$\Gamma_{\omega\to\eta\gamma}$	3.91 ± 0.38	5.16 ± 0.41
$\Gamma_{\rho^0\to\eta\gamma}$	44.8 ± 3.5	42.6 ± 3.5
$\Gamma_{\phi\to\eta\gamma}$	55.6 ± 1.6	55.4 ± 3.7
$\Gamma_{\phi\to\eta'\gamma}$	0.265 ± 0.012	0.265 ± 0.027
$\Gamma_{\eta'\to\omega\gamma}$	6.2 ± 1.1	6.2 ± 0.4
$\Gamma_{\eta\to\gamma\gamma}$	0.510 ± 0.026	0.463 ± 0.038
$\Gamma_{\eta'\to\gamma\gamma}$	4.30 ± 0.15	4.13 ± 0.26
$\Gamma_{\eta\to\gamma e^- e^+}$	$(8.8 \pm 1.6) \times 10^{-3}$	$(7.7 \pm 0.6) \times 10^{-3}$
$\Gamma_{\eta\to\gamma\mu^-\mu^+}$	$(0.40 \pm 0.08) \times 10^{-3}$	$(0.36 \pm 0.03) \times 10^{-3}$
$\Gamma_{\eta'\to\gamma\mu^-\mu^+}$	$(2.1 \pm 0.7) \times 10^{-2}$	$(1.6 \pm 0.1) \times 10^{-2}$
$\Gamma_{\omega\to\pi e^- e^+}$	6.54 ± 0.83	6.81 ± 0.30
$\Gamma_{\omega\to\pi\mu^-\mu^+}$	0.82 ± 0.21	0.67 ± 0.03
$\Gamma_{\phi\to\eta e^- e^+}$	0.490 ± 0.048	0.464 ± 0.031

Results for Decay widths


$\eta' \rightarrow \gamma\ \gamma^*$
s [GeV^2]
$\pi\alpha^2 m_{\eta'}^3/4|F_{\eta'}(s,0)|^2$ [keV]


$J/\psi \rightarrow \eta'\ \gamma^*$
M(e^+e^-) (GeV/c^2)
$|F_{J/\psi\eta'}|^2$


$\eta \rightarrow \gamma\ \gamma^*$
s [GeV^2]
$\pi\alpha^2 m_\eta^3/4|F_\eta(s,0)|^2$ [keV]


$\phi \rightarrow \eta\ \gamma^*$
$s^{1/2}$ [GeV]
$|F_{\phi\eta\gamma}(s)/F_{\phi\eta\gamma}(0)|^2$

Strong Interaction K.O. EM Interaction in J/Psi -> V P

Isospin conserved cases	Exp. data	Strong interaction
$\lvert G_{\psi\to\rho^0\pi^0}\rvert$	2.541 ± 0.154	2.933 ± 0.144
$\lvert G_{\psi\to\rho\pi}\rvert$	4.415 ± 0.192	5.080 ± 0.250
$\lvert G_{\psi\to\omega\eta}\rvert$	1.499 ± 0.084	1.628 ± 0.097
$\lvert G_{\psi\to\omega\eta'}\rvert$	0.552 ± 0.031	0.659 ± 0.059
$\lvert G_{\psi\to\phi\eta}\rvert$	1.069 ± 0.056	1.346 ± 0.066
$\lvert G_{\psi\to\phi\eta'}\rvert$	0.910 ± 0.076	1.178 ± 0.126
$\lvert G_{\psi\to K^{*+}K^-}\rvert$	1.860 ± 0.054	2.473 ± 0.089
$\lvert G_{\psi\to K^{*0}\bar{K}^0}\rvert$	1.726 ± 0.060	2.468 ± 0.082

[Chen, ZHG, Zou, PRD'15]

Isospin violated cases	Exp. data	EM interaction
$\lvert G_{\psi\to\rho^0\eta}\rvert$	0.498 ± 0.029	0.510 ± 0.056
$\lvert G_{\psi\to\rho^0\eta'}\rvert$	0.418 ± 0.034	0.429 ± 0.063
$\lvert G_{\psi\to\omega\pi^0}\rvert$	0.722 ± 0.039	0.722 ± 0.091

Consistent with conclusions in [Q.Zhao, G.Li, C.H.Chang, PLB'07]

F_8	$(1.45 \pm 0.04)F_\pi$	$(1.37 \pm 0.07)F_\pi$
F_0	$(1.28 \pm 0.06)F_\pi$	$(1.19 \pm 0.18)F_\pi$
θ_8	$(-26.7 \pm 1.8)^\circ$	$(-21.1 \pm 6.0)^\circ$
θ_0	$(-11.0 \pm 1.0)^\circ$	$(-2.5 \pm 8.2)^\circ$
F_V	134.9 ± 3.2	136.6 ± 3.5
$\tilde{c}_3$	0.0029 ± 0.0006	0.0109 ± 0.0161
$\tilde{d}_2$	0.081 ± 0.006	0.086 ± 0.085
h_1	$(-2.36 \pm 0.13) \times 10^{-5}$	$\cdots$
h_2	$(-4.73 \pm 1.26) \times 10^{-5}$	$\cdots$
h_3	$(3.85 \pm 0.45) \times 10^{-6}$	$\cdots$
g_1	$(-2.92 \pm 0.17) \times 10^{-5}$	$\cdots$
g_2	$(5.93 \pm 1.04) \times 10^{-4}$	$\cdots$
r_1	0.44 ± 0.10	$\cdots$
δ_η	$(39 \pm 44)^\circ$	$\cdots$
$\delta_{\eta'}$	$(115 \pm 13)^\circ$	$\cdots$
$\frac{\chi^2}{d.o.f}$	$\frac{96.0}{106-15} = 1.06$	$\frac{64.0}{70-8} = 1.03$
F_q	$(1.15 \pm 0.04)F_\pi$	$\cdots$
F_s	$(1.56 \pm 0.06)F_\pi$	$\cdots$
ϕ_q	$(34.5 \pm 1.8)^\circ$	$\cdots$

Valuable information for eta-eta' mixing directly extracted from Exp data !

Useful inputs to determine the chiral LECs !

3. Top-down study of the eta-eta' mixing

$$\begin{pmatrix} \eta \\ \eta' \end{pmatrix} = \frac{1}{F} \begin{pmatrix} F_8 \cos\theta_8 & -F_0 \sin\theta_0 \\ F_8 \sin\theta_8 & F_0 \cos\theta_0 \end{pmatrix} \begin{pmatrix} \eta_8 \\ \eta_0 \end{pmatrix}$$

What is the underlying reason to consider such a form ?

Any relation between the four mixing parameters?

Can we calculate them in an underlying theory?

Yes, let's go to the top-down approach !

In this case only the light pseudoscalar mesons are relevant dynamical d.o.f: the regime of Chiral Perturbation Theory

Relevant U(3) ChPT Lagrangian for eta-eta' mixing

δ-expansion: $p^2 \sim m_q \sim 1/N_c \sim \delta$

LO: $\mathcal{O}(\delta^0) = \{\mathcal{O}(N_C p^2)\ ,\ \mathcal{O}(N_C^0 p^0)\}$

$$\mathcal{L}^{(\delta^0)} = \frac{F^2}{4}\langle u_\mu u^\mu \rangle + \frac{F^2}{4}\langle \chi_+ \rangle + \frac{F^2 M_0^2}{12}\left[\ln \det u - \ln \det u^\dagger\right]^2$$

$$\Phi = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta_8 + \frac{1}{\sqrt{3}}\eta_0 & \pi^+ & K^+ \\ \pi^- & \frac{-1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta_8 + \frac{1}{\sqrt{3}}\eta_0 & K^0 \\ K^- & \bar{K}^0 & \frac{-2}{\sqrt{6}}\eta_8 + \frac{1}{\sqrt{3}}\eta_0 \end{pmatrix}$$

$$u = \exp\left(\frac{i\Phi}{\sqrt{2}F}\right)\ ,\quad u_\mu = i u^\dagger \partial_\mu U u^\dagger\ ,\quad \chi_\pm = u^\dagger \chi u^\dagger \pm u \chi^\dagger u$$

Di Vecchia, Veneziano NPB'80 , Rosenzweig, Schechter, Trahern PRD'80 , Witten Ann.Phys.'80

NLO: $\mathcal{O}(\delta) = \{\mathcal{O}(N_C p^4)\ ,\ \mathcal{O}(N_C^0 p^2)\}$

$$\mathcal{L}^{(\delta)} = L_5 \langle u_\mu u^\mu \chi_+ \rangle + \frac{1}{2} L_8 \langle \chi_+ \chi_+ + \chi_- \chi_- \rangle + \frac{F^2 \Lambda_1}{12} \partial^\mu X \partial_\mu X + \frac{F^2 \Lambda_2}{12} X \langle \chi_- \rangle$$

$$X = 2 \log \det u = i\sqrt{6}\eta_0 / F$$

Kaiser, Leutwyler, EPJC'00

NNLO: $\mathcal{O}(\delta^2) = \{\mathcal{O}(N_C^{-1} p^2)\ ,\ \mathcal{O}(N_C^0 p^4)\ ,\ \mathcal{O}(N_c p^6)\}$

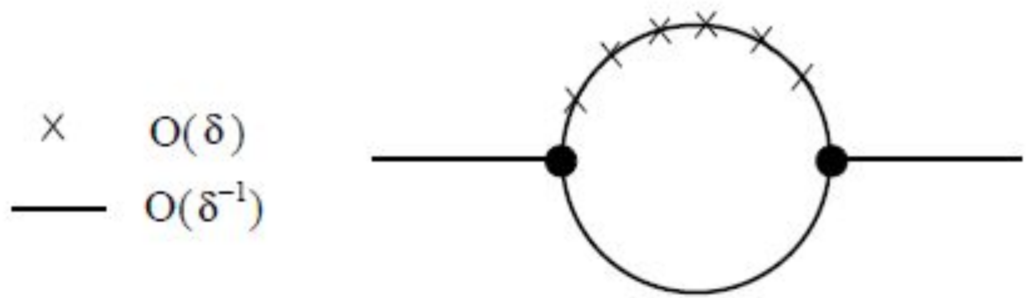
$$\mathcal{L}^{(\delta^2)} = \frac{F^2 v_2^{(2)}}{4} X^2 \langle \chi_+ \rangle + L_4 \langle u^\mu u_\mu \rangle \langle \chi_+ \rangle + L_6 \langle \chi_+ \rangle \langle \chi_+ \rangle + L_7 \langle \chi_- \rangle \langle \chi_- \rangle + L_{18} \langle u_\mu \rangle \langle u^\mu \chi_+ \rangle + L_{25} X \langle \chi_+ \chi_- \rangle + C_{12} \langle h_{\mu\nu} h^{\mu\nu} \chi_+ \rangle + C_{14} \langle u_\mu u^\mu \chi_+ \chi_+ \rangle + C_{17} \langle u_\mu \chi_+ u^\mu \chi_+ \rangle + C_{19} \langle \chi_+ \chi_+ \chi_+ \rangle + C_{31} \langle \chi_- \chi_- \chi_+ \rangle \ .$$

Herrera-Siklody, Latorre, Pascual, Taron NPB'97; Bijnens, Colangelo, Ecker, JHEP'99

Eta-Eta' mixing

Technical Point. For calculating one-loop diagrams

η_0 and η_8: inappropriate bases for loop calculations



Solution: to use $\bar{\eta}$, $\bar{\eta}'$ from LO diagonalization in loops

$$\begin{pmatrix} \bar{\eta} \\ \bar{\eta}' \end{pmatrix} = \begin{pmatrix} c_\theta & -s_\theta \\ s_\theta & c_\theta \end{pmatrix} \cdot \begin{pmatrix} \eta_8 \\ \eta_0 \end{pmatrix}$$

General parameterizations of $\overline{\eta}$, $\overline{\eta}'$ bilinear terms

$$
\begin{aligned}
\mathcal{L} \;=\;& \frac{\delta_1}{2}\,\partial_\mu\partial_\nu\overline{\eta}\partial^\mu\partial^\nu\overline{\eta} + \frac{\delta_2}{2}\,\partial_\mu\partial_\nu\overline{\eta}'\partial^\mu\partial^\nu\overline{\eta}' + \delta_3\,\partial_\mu\partial_\nu\overline{\eta}\partial^\mu\partial^\nu\overline{\eta}' \\
&+\frac{1+\delta_{\overline{\eta}}}{2}\partial_\mu\overline{\eta}\partial^\mu\overline{\eta} + \frac{1+\delta_{\overline{\eta}'}}{2}\partial_\mu\overline{\eta}'\partial^\mu\overline{\eta}' + \delta_k\,\partial_\mu\overline{\eta}\partial^\mu\overline{\eta}' \\
&-\frac{m^2_{\overline{\eta}} + \delta_{m^2_{\overline{\eta}}}}{2}\overline{\eta}\,\overline{\eta} - \frac{m^2_{\overline{\eta}'} + \delta_{m^2_{\overline{\eta}'}}}{2}\overline{\eta}'\overline{\eta}' - \delta_{m^2}\,\overline{\eta}\,\overline{\eta}'\,.
\end{aligned}
$$

δ_i's are calculated up to NNLO in δ-counting U(3) ChPT

LO from $\mathcal{L}^{\delta^0}$: $\quad \delta_i = 0\,,$

NLO from $\mathcal{L}^{\delta^1}$:

$$
\begin{aligned}
\delta_1^{NLO} &= \delta_2^{NLO} = \delta_3^{NLO} = 0\,, \\
\delta_{\overline{\eta}}^{NLO} &= \frac{8L_5}{3F^2}\left[m_\pi^2(-c_\theta^2 - 4\sqrt{2}c_\theta s_\theta + s_\theta^2) + 2m_K^2(2c_\theta^2 + 2\sqrt{2}c_\theta s_\theta + s_\theta^2\right] + s_\theta^2\Lambda_1\,, \\
\delta_{\overline{\eta}'}^{NLO} &= \frac{8L_5}{3F^2}\left[m_\pi^2(c_\theta^2 + 4\sqrt{2}c_\theta s_\theta - s_\theta^2) + 2m_K^2(c_\theta^2 - 2\sqrt{2}c_\theta s_\theta + 2s_\theta^2\right] + c_\theta^2\Lambda_1\,, \\
\delta_k^{NLO} &= -\frac{16L_5}{3F^2}(m_K^2 - m_\pi^2)(\sqrt{2}c_\theta^2 - c_\theta s_\theta - \sqrt{2}s_\theta^2) - c_\theta s_\theta\Lambda_1\,,
\end{aligned}
$$

......

NNLO from $\mathcal{L}^{\delta^2}$ and chiral loops:

$$\delta_1^{NNLO} = \frac{32C_{12}}{3F^2}\left[m_\pi^2(-c_\theta^2 - 4\sqrt{2}c_\theta s_\theta + s_\theta^2) + 2m_K^2(2c_\theta^2 + 2\sqrt{2}c_\theta s_\theta + s_\theta^2\right],$$

$$\begin{aligned}
&\delta_{\overline{\eta}}^{NNLO} = \\
&\frac{8L_4}{F^2}(2m_K^2 + m_\pi^2) + \cdots \\
&+\frac{128L_5L_8}{3F^4}\left[m_\pi^4(-c_\theta^2 - 4\sqrt{2}c_\theta s_\theta + s_\theta^2) + m_K^4(2c_\theta^2 + 2\sqrt{2}c_\theta s_\theta + s_\theta^2)\right] + \cdots \\
&+\frac{16C_{14}}{3F^4}\left[3m_\pi^4 + 4m_K^4(2c_\theta^2 + 2\sqrt{2}c_\theta s_\theta + s_\theta^2) - 4m_K^2 m_\pi^2(2c_\theta^2 + 2\sqrt{2}c_\theta s_\theta + s_\theta^2)\right] + \cdots \\
&+\frac{c_\theta^2}{F^2}A_0(m_\pi^2) + \cdots, \\
&\cdots\cdots
\end{aligned}$$

$$A_0(m^2) = -\frac{m^2}{16\pi^2}\log\frac{m^2}{\mu^2}$$

[X.K.Guo, ZHG, Oller, Sanz-Cillero, JHEP'15]

To obtain physical η and η'

$$
\begin{aligned}
\mathcal{L} \;=\; & \frac{\delta_1}{2}\,\partial_\mu\partial_\nu\overline{\eta}\partial^\mu\partial^\nu\overline{\eta} + \frac{\delta_2}{2}\,\partial_\mu\partial_\nu\overline{\eta}'\partial^\mu\partial^\nu\overline{\eta}' + \delta_3\,\partial_\mu\partial_\nu\overline{\eta}\partial^\mu\partial^\nu\overline{\eta}' \\
& + \frac{1+\delta_{\overline{\eta}}}{2}\partial_\mu\overline{\eta}\partial^\mu\overline{\eta} + \frac{1+\delta_{\overline{\eta}'}}{2}\partial_\mu\overline{\eta}'\partial^\mu\overline{\eta}' + \delta_k\,\partial_\mu\overline{\eta}\partial^\mu\overline{\eta}' \\
& - \frac{m_{\overline{\eta}}^2 + \delta_{m_{\overline{\eta}}^2}}{2}\overline{\eta}\,\overline{\eta} - \frac{m_{\overline{\eta}'}^2 + \delta_{m_{\overline{\eta}'}^2}}{2}\overline{\eta}'\overline{\eta}' - \delta_{m^2}\,\overline{\eta}\,\overline{\eta}'\,.
\end{aligned}
$$

Field redefinitions can eliminate the high derivative terms:

$$
\bar{\eta} \to \bar{\eta} - \frac{\delta_1}{2}\Box\bar{\eta} - \frac{\delta_2}{2}\Box\bar{\eta}' \;\;,\;\; \bar{\eta}' \to \bar{\eta}' - \frac{\delta_2}{2}\Box\bar{\eta} - \frac{\delta_3}{2}\Box\bar{\eta}' \;\;,
$$

with d'Alembert operator $\Box = \partial_\mu\partial^\mu$.

$$\mathcal{L} = \frac{1+\tilde{\tilde{\delta}}_{\overline{\eta}}}{2}\partial_\mu\overline{\eta}\partial^\mu\overline{\eta} + \frac{1+\tilde{\tilde{\delta}}_{\overline{\eta}'}}{2}\partial_\mu\overline{\eta}'\partial^\mu\overline{\eta}' + \tilde{\tilde{\delta}}_k\,\partial_\mu\overline{\eta}\partial^\mu\overline{\eta}'$$
$$-\frac{m^2_{\overline{\eta}}+\tilde{\tilde{\delta}}_{m^2_{\overline{\eta}}}}{2}\overline{\eta}\,\overline{\eta} - \frac{m^2_{\overline{\eta}'}+\tilde{\tilde{\delta}}_{m^2_{\overline{\eta}'}}}{2}\overline{\eta}'\overline{\eta}' - \tilde{\tilde{\delta}}_{m^2}\,\overline{\eta}\,\overline{\eta}'\,.$$

$\longrightarrow$

$$\begin{pmatrix}\eta\\ \eta'\end{pmatrix} = \begin{pmatrix}\cos\theta_\delta & -\sin\theta_\delta\\ \sin\theta_\delta & \cos\theta_\delta\end{pmatrix}\begin{pmatrix}1+\delta_A & \delta_B\\ \delta_B & 1+\delta_C\end{pmatrix}\begin{pmatrix}\overline{\eta}\\ \overline{\eta}'\end{pmatrix}$$
$$= \begin{pmatrix}\cos\theta_\delta & -\sin\theta_\delta\\ \sin\theta_\delta & \cos\theta_\delta\end{pmatrix}\begin{pmatrix}1+\delta_A & \delta_B\\ \delta_B & 1+\delta_C\end{pmatrix}\begin{pmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{pmatrix}\begin{pmatrix}\eta_8\\ \eta_0\end{pmatrix}$$

All coefficients are calculated up to NNLO in δ-counting U(3) ChPT

F_π and F_K are also calculated to the same order.

- We also give the relation to the popular two-mixing angle scheme:

$$\begin{pmatrix} \eta \\ \eta' \end{pmatrix} = \frac{1}{F} \begin{pmatrix} F_8 \cos\theta_8 & -F_0 \sin\theta_0 \\ F_8 \sin\theta_8 & F_0 \cos\theta_0 \end{pmatrix} \begin{pmatrix} \eta_8 \\ \eta_0 \end{pmatrix} .$$

$$F^8_\eta = \cos\theta_8 F_8 \qquad F^8_{\eta'} = \sin\theta_8 F_8$$
$$F^0_\eta = -\sin\theta_0 F_0 \qquad F^0_{\eta'} = \cos\theta_0 F_0$$

$$\langle 0|A^a_\mu|\eta^{(')}\rangle = i p_\mu F^a_{\eta^{(')}}$$

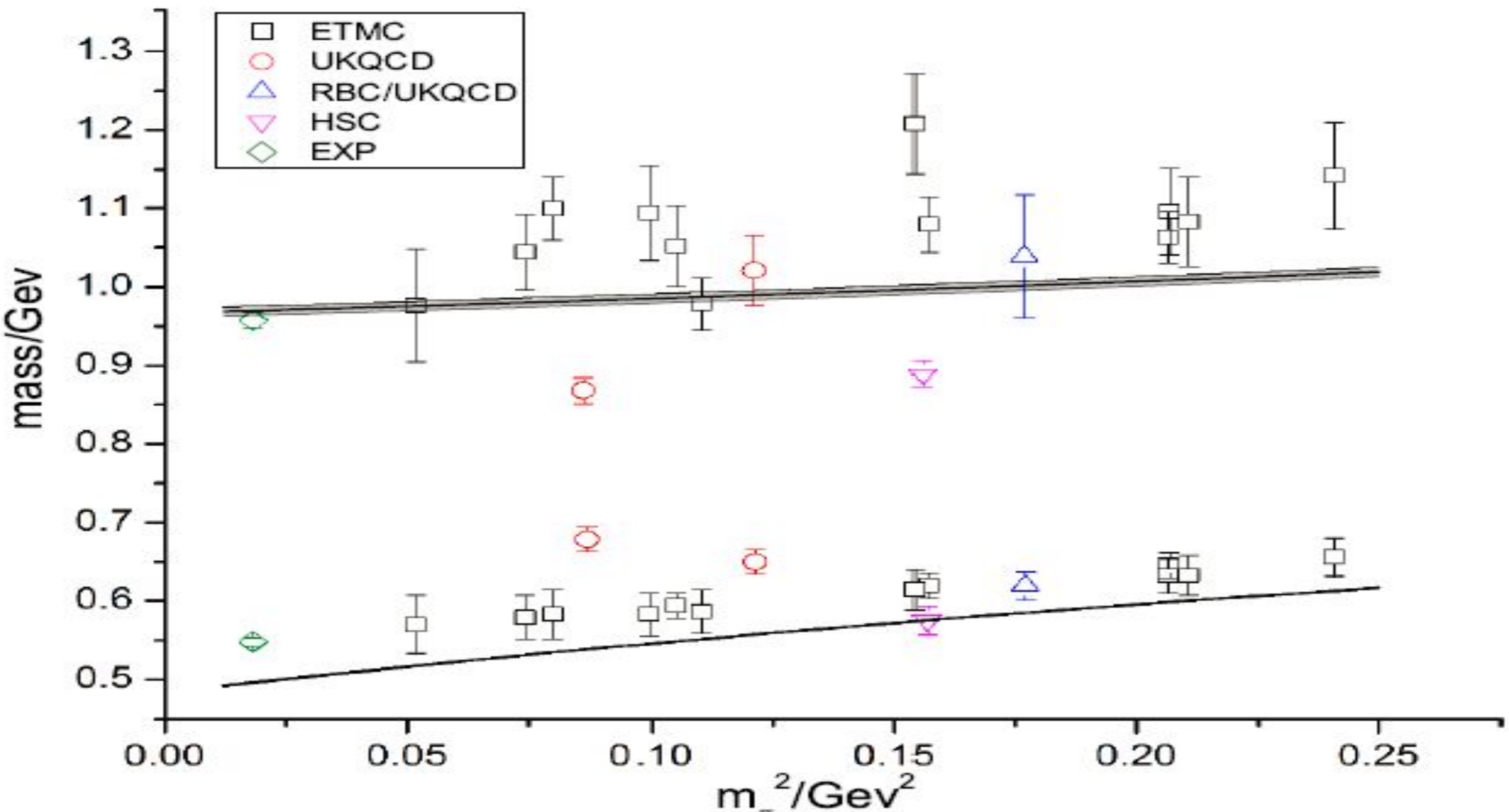
Now the mixing parameters (F_0 F_8 θ_0 θ_8)are calculated in terms of the chiral low energy constants up to NNLO !

Numerical discussions

Leading order masses for η and η'

One free parameter, $m_\eta = 496.4 \pm 1.3$, $m_{\eta'} = 970 \pm 6$ MeV

$M_0 = 836 \pm 8$ MeV $\theta = -18.9^o \pm 0.3^o$



NLO and NNLO Fits

Fitted observables: **[X.KGuo, ZHG, Oller, Sanz-Cillero, JHEP'15]**

1. LQCD + experimental values:
   m_η, $m_{\eta'}$, m_K, F_π, F_K, F_K/F_π
2. Phenomenological values: Y.H.Chen, Z.H.Guo, B.S.Zou, PRD91,014010 F_8, F_0, θ_8, θ_0

- **NLO** Free parameters: L_5, L_8, Λ_1, Λ_2

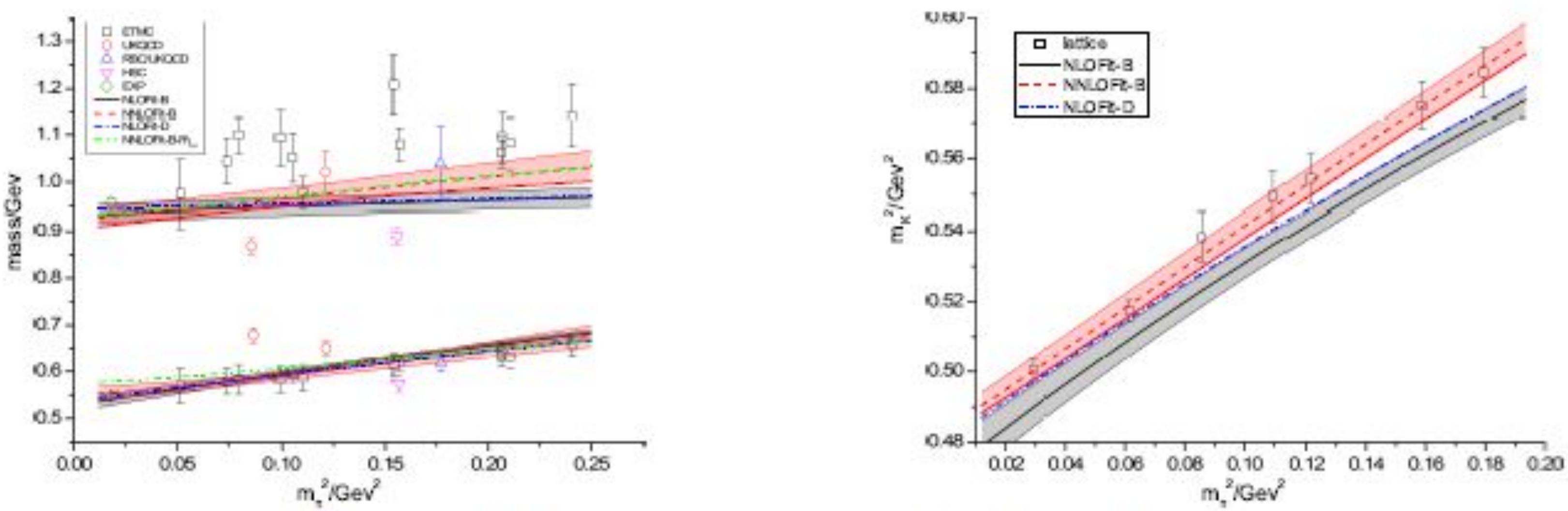


Figure: Left: m_η and $m_{\eta'}$. Right: m_K^2

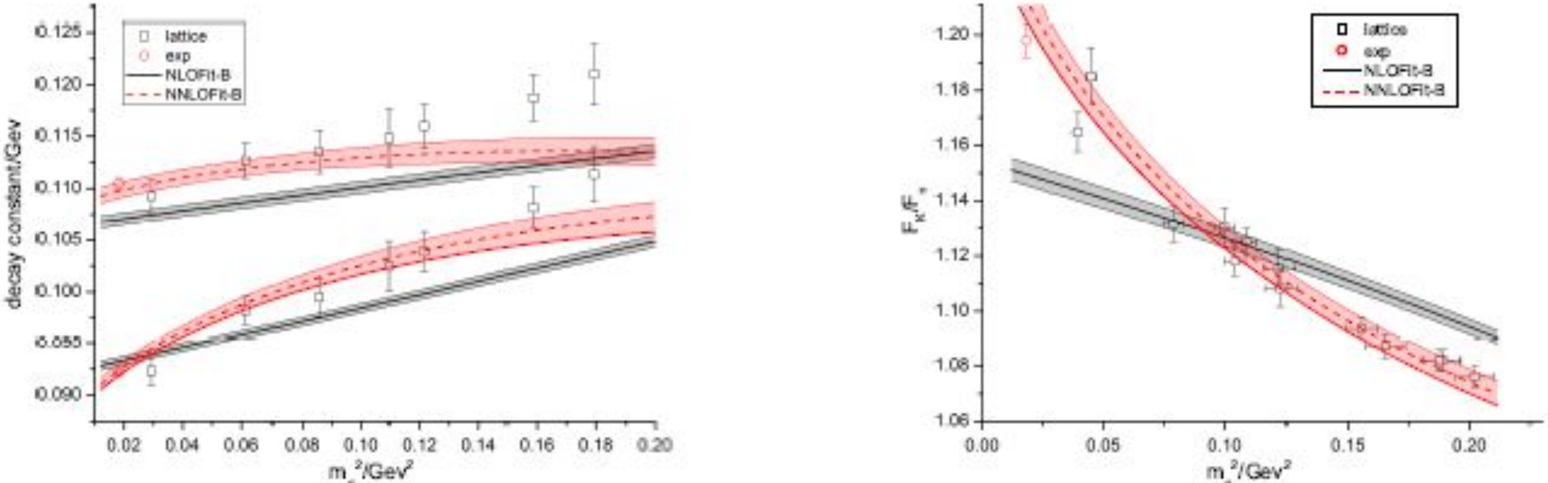


Figure: Left: F_π and F_K. Right: F_K/F_π

	NLOFit-A	NLOFit-B
$\chi^2/(\text{d.o.f})$	481.2/(76-5)	477.7/(76-6)
M_0 (MeV)	835.7*	767.3$\pm$31.5$\pm$32.3
F (MeV)	92.1$\pm$0.2$\pm$0.6	92.1$\pm$0.2$\pm$0.6
$10^3 \times L_5$	1.45$\pm$0.02$\pm$0.30	1.47$\pm$0.02$\pm$0.29
$10^3 \times L_8$	1.00$\pm$0.07$\pm$0.10	1.08$\pm$0.05$\pm$0.04
Λ_1	0.02$\pm$0.05$\pm$0.06	-0.09$\pm$0.08$\pm$0.02
Λ_2	0.25$\pm$0.06$\pm$0.02	0.14$\pm$0.07$\pm$0.03

	Input	NLOFit-A	NLOFit-B
F_0 (MeV)	118.0 $\pm$16.5	104.9$\pm$2.9$\pm$0.3	99.7$\pm$3.6$\pm$1.6
F_8 (MeV)	133.7 $\pm$11.1	113.2$\pm$0.3$\pm$4.4	113.5$\pm$0.3$\pm$4.2
θ_0 (°)	-11.0 $\pm$3.0	-7.2$\pm$2.1$\pm$1.3	-10.6$\pm$2.4$\pm$0.1
θ_8 (°)	-26.7 $\pm$5.4	-21.5$\pm$2.2$\pm$3.9	-25.4$\pm$2.6$\pm$2.3
$m_s/\widehat{m}$	27.5 $\pm$3.0	22.6$\pm$0.8$\pm$0.6	21.9$\pm$0.6$\pm$1.2

Statistical+Systematic errors. Systematic errors are estimated by using F_π in ChPT expressions for each fit

- **NNLO Fits:**
  - m_K^2, F_π, F_K and F_K/F_π are not reproduced at NLO accurately
  - One-loop ChPT contributions start at NNLO
  - Eleven new free parameters: $v_2^{(2)}$, L_4, L_6, L_7, L_{18}, L_{25}, C_{12}, C_{14}, C_{17}, C_{19}, C_{31}
  - Constraints:
    1. $M_0 = 836$ MeV (LO value)
    2. $v_2^{(2)}$ only enters in the combination $M_0^2 + 6v_2^{(2)}(2m_K^2 + m_\pi^2)$, $v_2^{(2)} \to 0$.
    3. $|\Lambda_1| < 0.4$, $|\Lambda_2| < 0.7$ (Natural-value estimates)
    4. L_{18}, $L_{25} \to 0$ (They do not appear in SU(3) ChPT and only affects η-η' mixing)
    5. $C_i \to \alpha C_i^{th}$, with C_i^{th} from Dyson-Schwinger calculations of S. Z. Jiang, Y. Zhang, C. Li and Q. Wang in Phys. Rev. D 81, 014001 (2010) [Fit A], 1502.05087 [hep-ph] [Fit B]
  - Instead of 11 new free parameters $\to$ 4 new free parameter

	NNLO-A	NNLO-B
$\chi^2/(\text{d.o.f})$	212.4/(76-9)	231.9/(76-9)
F (MeV)	81.7±1.5±5.3	80.8±1.6±6.1
$10^3 \times L_5$	0.60±0.11±0.52	0.45±0.12±0.78
$10^3 \times L_8$	0.25±0.07±0.31	0.30±0.06±0.30
Λ_1	-0.003±0.060±0.093	-0.04±0.06±0.13
Λ_2	0.08±0.11±0.20	0.14±0.10±0.40
$10^3 \times L_4$	-0.12±0.06±0.19	-0.09±0.06±0.23
$10^3 \times L_6$	-0.05±0.04±0.02	0.03±0.03±0.02
$10^3 \times L_7$	0.26±0.05±0.06	0.36±0.05±0.12
α	-0.59±0.09±0.18	-0.76±0.08±0.44

[Bijnens, Ecker, '14]

	Input	NNLO-A	NNLO-B
F_0 (MeV)	118.0 ±16.5	108.0±1.5±3.6	109.1±1.3±5.9
F_8 (MeV)	133.7 ±11.1	124.7±1.2±8.7	126.5±1.2±11.8
θ_0 (Degree)	-11.0 ±3.0	-6.8±1.1±2.6	-6.8±0.9±3.7
θ_8 (Degree)	-26.7 ±5.4	-26.8±1.1±0.2	-27.9±1.0±1.4
$m_s/\widehat{m}$	27.5 ±3.0	27.0±0.6±0.4	29.4±0.4±0.6
F_q (MeV)	106.0 ± 11.1*	92.8±1.1±1.2	92.7±1.0±1.0
F_s (MeV)	143.8 ± 16.5*	136.4±1.5±10.0	139.0±1.4±14.9
θ_q (°)	34.5 ± 5.4*	36.4±1.4±0.2	35.8±1.2±0.3
θ_s (°)	36.0 ± 4.2*	37.8±0.9±1.5	37.1±0.8±1.1

Summary and Prospects

• Successful descriptions for the light-meson radiative processes and the J/Psi decays involving eta and eta' are provided by using RChT and effective Lagrangians.

Eta-eta' mixing parameters are directly extracted from various Exp data here.

• The NNLO calculation for eta-eta' mixing is performed and confronted to lattice QCD results.

U(3) ChPT is demonstrated to be a sophisticated tool for chiral extrapolations for pi, K, eta and eta'.

• Continuing Exp and lattice QCD efforts would inspire more works: strong decays of eta and eta', radiative decays with light pseudoscalars, eta and eta' in photoproduction ……

Thank you! 谢谢大家！