

Towards nature of the $X(3872)$ resonance *N.N. Achasov^{1;1)} E.V. Rogozina²¹ Laboratory of Theoretical Physics, Sobolev Institute for Mathematics, 630090, Novosibirsk, Russian Federation² Novosibirsk State University, 630090, Novosibirsk, Russian Federation

Abstract: We construct spectra of decays of the resonance $X(3872)$ with good analytical and unitary properties which allows to define the branching ratio of the $X(3872) \rightarrow D^{*0}\bar{D}^0 + c.c.$ decay studying only one more decay, for example, the $X(3872) \rightarrow \pi^+\pi^- J/\psi(1S)$ decay, and show that our spectra are effective means of selection of models for the resonance $X(3872)$.

Then we discuss the scenario where the $X(3872)$ resonance is the $c\bar{c} = \chi_{c1}(2P)$ charmonium which "sits on" the $D^{*0}\bar{D}^0$ threshold.

We explain the shift of the mass of the $X(3872)$ resonance with respect to the prediction of a potential model for the mass of the $\chi_{c1}(2P)$ charmonium by the contribution of the virtual $D^*\bar{D} + c.c.$ intermediate states into the self energy of the $X(3872)$ resonance. This allows us to estimate the coupling constant of the $X(3872)$ resonance with the $D^{*0}\bar{D}^0$ channel, the branching ratio of the $X(3872) \rightarrow D^{*0}\bar{D}^0 + c.c.$ decay, and the branching ratio of the $X(3872)$ decay into all non- $D^{*0}\bar{D}^0 + c.c.$ states. We predict a significant number of unknown decays of $X(3872)$ via two gluons: $X(3872) \rightarrow gluon\ gluon \rightarrow hadrons$.

Key words: Charmonium, molecule, two-gluon decays

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1 Introduction

The $X(3872)$ resonance became the first in discovery of the resonant structures XYZ ($X(3872)$, $Y(4260)$, $Z_b^+(10610)$, $Z_b^+(10650)$, $Z_c^+(3900)$), the interpretations of which as hadron states assumes existence in them at least pair of heavy and pair of light quarks in this or that form.

Thousands of articles on this subject already were published in spite of the fact that many properties of new resonant structures are not defined yet and not all possible mechanisms of dynamic generation of these structures are studied, in particular, the role of the anomalous Landau thresholds is not studied.

Anyway, this spectroscopy took the central place in physics of hadrons.

Below we give reasons that $X(3872)$, $I^G(J^{PC}) = 0^+(1^{++})$, is the $\chi_{c1}(2P)$ charmonium and suggest a physically clear program of experimental researches for verification of our assumption.

2 How to learn the branching ratio $X(3872) \rightarrow D^{*0}\bar{D}^0 + c.c.$ [1]

The mass spectrum $\pi^+\pi^- J/\psi(1S)$ looks as the ideal Breit-Wigner one in the $X(3872) \rightarrow \pi^+\pi^- J/\psi(1S)$ decay, see Fig. 1 (a).

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1) E-mail: achasov@math.nsc.ru

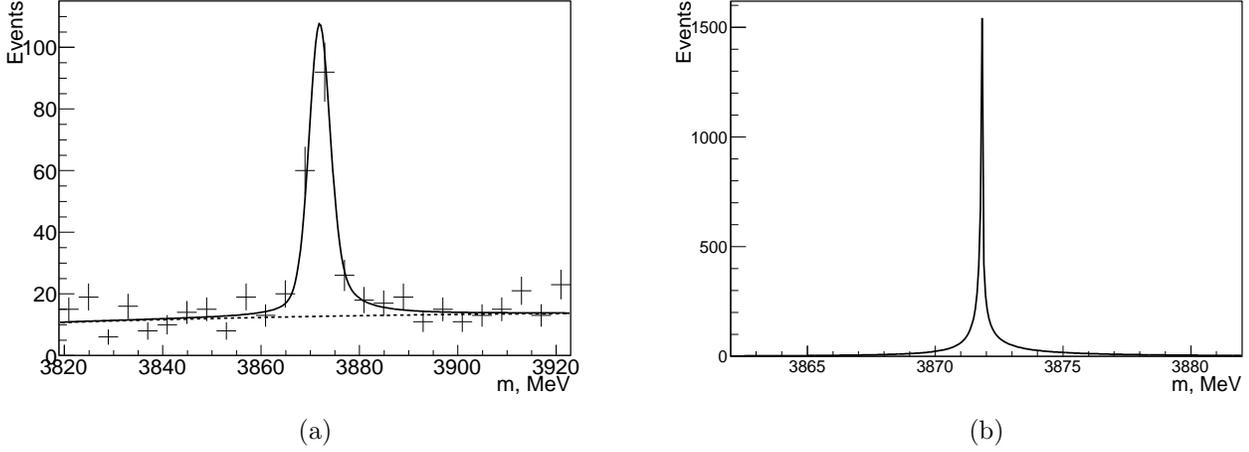


Fig. 1. (a) The Belle data [2] on the invariant $\pi^+\pi^-J/\psi(1S)$ mass (m) distribution. The solid line is our theoretical one with taking into account the Belle energy resolution. The dotted line is second-order polynomial for the incoherent background. (b) Our undressed theoretical line.

The mass spectrum $\pi^+\pi^-\pi^0J/\psi(1S)$ in the $X(3872) \rightarrow \pi^+\pi^-\pi^0J/\psi(1S)$ decay looks in a similar way [3, 4].

The mass spectrum $D^{*0}\bar{D}^0 + c.c.$ in the $X(3872) \rightarrow D^{*0}\bar{D}^0 + c.c.$ decay [5] looks as the typical resonance threshold enhancement, see Fig. 2.

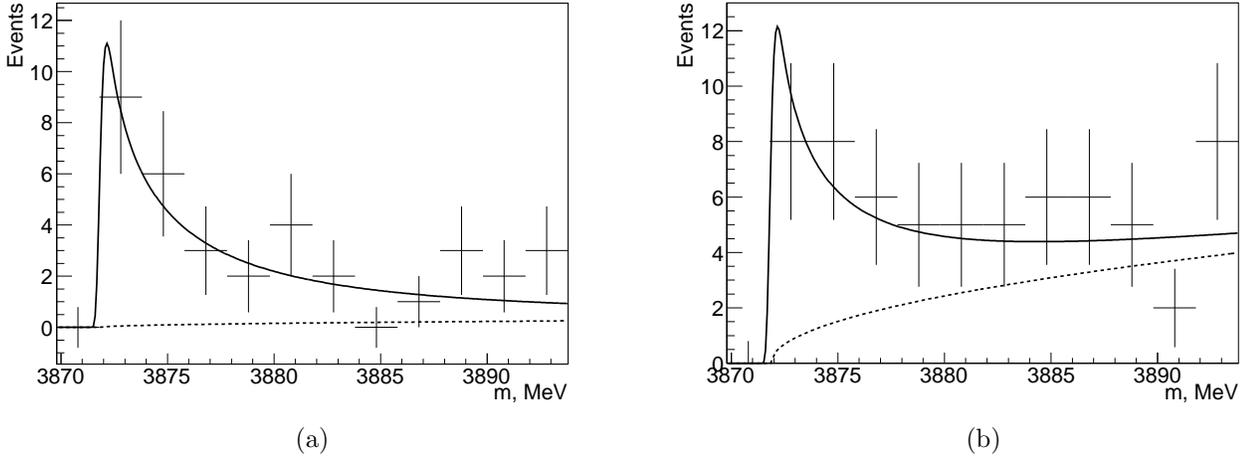


Fig. 2. The Belle data [5] on the invariant $D^{*0}\bar{D}^0 + c.c.$ mass (m) distribution. The solid line is our theoretical one with taking into account the Belle energy resolution. The dotted line is a square root function for the incoherent background. (a) $D^{*0} \rightarrow D^0\pi^0$. (b) $D^{*0} \rightarrow D^0\gamma$.

If structures in the above channels are manifestation of the same resonance, it is possible to define the branching ratio $BR(X(3872) \rightarrow D^{*0}\bar{D}^0 + c.c.)$ treating data of the two above decay channels only.

We believe that the $X(3872)$ is the axial vector, 1^{++} [6, 7]. In this case the S wave dominates in the $X(3872) \rightarrow D^{*0}\bar{D}^0 + c.c.$ decay and hence is described by

the effective Lagrangian

$$L_{XD^*D}(x) = g_A X^\mu \left(D_\mu^0(x) \bar{D}^0(x) + \bar{D}_\mu^0(x) D^0(x) \right). \quad (1)$$

The width of the $X \rightarrow D^{*0}\bar{D}^0 + c.c.$ decay

$$\Gamma(X \rightarrow D^{*0}\bar{D}^0 + c.c., m) = \frac{g_A^2}{8\pi} \frac{\rho(m)}{m} \left(1 + \frac{\mathbf{k}^2}{3m_{D^{*0}}^2} \right), \quad (2)$$

where \mathbf{k} is momenta of D^{*0} (or \bar{D}^0) in the $D^{*0}\bar{D}^0$ center mass system, m is the invariant mass of the $D^{*0}\bar{D}^0$ pair,

$$\rho(m) = \frac{2|\mathbf{k}|}{m} = \frac{\sqrt{(m^2 - m_+^2)(m^2 - m_-^2)}}{m^2}, \quad m_{\pm} = m_{D^{*0}\pm m_{D^0}}.$$

The second term in the right side of Eq. (2) is very small in our energy region and can be neglected. This gives us the opportunity to construct the mass spectra for the $X(3872)$ decays with the good analytical and unitary properties as in the scalar meson case [8, 9].

The mass spectrum in the $D^{*0}\bar{D}^0 + c.c.$ channel

$$\frac{dBR(X \rightarrow D^{*0}\bar{D}^0 + c.c., m)}{dm} = 4 \frac{1}{\pi} \frac{m^2 \Gamma(X \rightarrow D^{*0}\bar{D}^0, m)}{|D_X(m)|^2}. \quad (3)$$

The branching ratio of $X(3872) \rightarrow D^{*0}\bar{D}^0 + c.c.$

$$BR(X \rightarrow D^{*0}\bar{D}^0 + c.c.) = 4 \frac{1}{\pi} \int_{m_+}^{\infty} \frac{m^2 \Gamma(X \rightarrow D^{*0}\bar{D}^0, m)}{|D_X(m)|^2} dm. \quad (4)$$

In others $\{i\}$ (non- $D^{*0}\bar{D}^0$) channels the $X(3872)$ state is seen as a narrow resonance that is why we write the mass spectrum in the i channel in the form

$$\frac{dBR(X \rightarrow i, m)}{dm} = 2 \frac{1}{\pi} \frac{m_X^2 \Gamma_i}{|D_X(m)|^2}, \quad (5)$$

where Γ_i is the width of the $X(3872) \rightarrow i$ decay.

The branching ratio of $X(3872) \rightarrow i$

$$BR(X \rightarrow i) = 2 \frac{1}{\pi} \int_{m_0}^{\infty} \frac{m_X^2 \Gamma_i}{|D_X(m)|^2} dm, \quad (6)$$

where m_0 is the threshold of the i channel.

The inverse propagator $D_X(m)$

$$D_X(m) = m_X^2 - m^2 + Re(\Pi_X(m_X^2)) - \Pi_X(m^2) - im_X \Gamma, \quad (7)$$

where $\Gamma = \Sigma \Gamma_i < 1.2$ MeV is the total width of the $X(3872)$ decay into all non- $(D^{*0}\bar{D}^0 + c.c.)$ channels.

$$\Pi_X(s) = \frac{g_A^2}{8\pi^2} \left(I^{D^{*0}\bar{D}^0}(s) + I^{D^{*+}D^-}(s) \right), \quad m^2 = s. \quad (8)$$

When $m_+ = m_{D^*} + m_D \leq m$,

$$I^{D^*\bar{D}}(m^2) = \frac{(m^2 - m_+^2)}{m^2} \frac{m_-}{m_+} \ln \frac{m_{D^*}}{m_D} + \rho(m) \left[i\pi + \ln \frac{\sqrt{m^2 - m_-^2} - \sqrt{m^2 - m_+^2}}{\sqrt{m^2 - m_-^2} + \sqrt{m^2 - m_+^2}} \right] \quad (9)$$

where $m_- = m_{D^*} - m_D$.

When $m_- \leq m \leq m_+$,

$$I^{D^*\bar{D}}(m) = \frac{(m^2 - m_+^2)}{m^2} \frac{m_-}{m_+} \ln \frac{m_{D^*}}{m_D} - 2|\rho(m)| \arctan \frac{\sqrt{m^2 - m_-^2}}{\sqrt{m_+^2 - m^2}}, \quad (10)$$

where $|\rho(m)| = \sqrt{(m_+^2 - m^2)(m^2 - m_-^2)}/m^2$.

When $m \leq m_-$ and $m^2 \leq 0$,

$$I^{D^*\bar{D}}(m) = \frac{(m^2 - m_+^2)}{m^2} \frac{m_-}{m_+} \ln \frac{m_{D^*}}{m_D} - \rho(m) \ln \frac{\sqrt{m_+^2 - m^2} - \sqrt{m_-^2 - m^2}}{\sqrt{m_+^2 - m^2} + \sqrt{m_-^2 - m^2}}. \quad (11)$$

Our branching ratios satisfy the unitarity

$$1 = BR(X \rightarrow D^{*0}\bar{D}^0 + c.c.) + BR(X \rightarrow D^{*+}\bar{D}^- + c.c.) + \sum_i BR(X \rightarrow i). \quad (12)$$

Fitting the Belle data [2], we take into account the Belle results [2]: $m_X = 3871.84$ MeV $= m_{D^{*0}} + m_{D^0} = m_+$ and $\Gamma_{X(3872)} < 1.2$ MeV 90%CL, that corresponds to $\Gamma < 1.2$ MeV, which controls the width of the $X(3872)$

signal in the $\pi^+\pi^-J/\psi(1S)$ channel and in every non- $(D^{*0}\bar{D}^0 + c.c.)$ channel. The results of our fit are in the Table 1.

Table 1. BR_{seen} is a branching ratio for $m \leq 3891.84$ MeV, Γ in MeV, g_A in GeV.

Γ	$1.2_{-0.4}$	mode	$D^{*0}\bar{D}^0 + c.c.$	$D^{*+}D^- + c.c.$	Others
$g_A^2/8\pi$	1.4_{-1}^{+5}	BR	$0.6_{-0.1}^{+0.02}$	$0.31_{-0.16}^{+0.13}$	$0.1_{-0.1}^{+0.3}$
χ^2/Ndf	45/42	BR_{seen}	$0.3_{-0.2}^{+0.1}$	$0.03_{-0.02}^{+0.004}$	$0.09_{-0.1}^{+0.3}$

Our approach can serve as the guide in selection of theoretical models for the $X(3872)$ resonance. Indeed, if 3871.68 MeV $< M_X < 3871.95$ MeV and

$\Gamma_{X(3872)} = \Gamma < 1.2$ MeV then for $g_A^2/4\pi < 0.2$ GeV² $BR(X \rightarrow D^{*0}\bar{D}^0 + c.c.; m \leq 3891.84$ MeV) < 0.3 . That is, unknown decays of $X(3872)$ into non- $D^{*0}\bar{D}^0$ states

are considerable or dominant.

For example, in Ref. [10] the authors considered $m_X = 3871.68$ MeV, $\Gamma = 1.2$ MeV and $g_{XDD^*} = g_A\sqrt{2} = 2.5$ GeV, that is, $g_A^2/8\pi = 0.1$ GeV². In this case

$BR(X \rightarrow D^{*0}\bar{D}^0 + c.c.) \approx 0.2$, that is, unknown decays $X(3872)$ into non- $(D^{*0}\bar{D}^0 + c.c.)$ states are dominant. For details see Table 2.

Table 2. Branching ratios for the model from Ref. [10]. Γ in MeV, m_X in MeV, g_A in GeV.

m_X	3871.68	mode	$X \rightarrow D^{*0}\bar{D}^0 + c.c.$	$X \rightarrow D^{*+}D^- + c.c.$	$X \rightarrow Others$
Γ	1.2	BR	0.176	0.045	0.779
$g_A^2/8\pi$	0.1	BR_{seen}	0.14	0.011	0.761

3 $X(3872)$, $I^G(J^{PC}) = 0^+(1^{++})$, as the $\chi_{c1}(2P)$ charmonium [11]

Contrary to almost standard opinion that the $X(3872)$ resonance is the $D^{*0}\bar{D}^0 + c.c.$ molecule or the $qc\bar{q}\bar{c}$ four-quark state, we discuss the scenario where the $X(3872)$ resonance is the $c\bar{c} = \chi_{c1}(2P)$ charmonium which "sits on" the $D^{*0}\bar{D}^0$ threshold.

The two dramatic discoveries have generated a stream of the $D^{*0}\bar{D}^0 + D^0\bar{D}^{*0}$ molecular interpretations of the $X(3872)$ resonance.

The mass of the $X(3872)$ resonance is 50 MeV lower than predictions of the most lucky naive potential models for the mass of the $\chi_{c1}(2P)$ resonance,

$$m_X - m_{\chi_{c1}(2P)} = -\Delta \approx -50 \text{ MeV}, \quad (13)$$

and the relation between the branching ratios

$$\begin{aligned} & BR(X \rightarrow \pi^+\pi^-\pi^0 J/\psi(1S)) \\ & \sim BR(X \rightarrow \pi^+\pi^- J/\psi(1S)), \end{aligned} \quad (14)$$

that is interpreted as a strong violation of isotopic symmetry.

But the bounding energy is small, $\epsilon_B < (1 \div 3)$ MeV. That is, the radius of the molecule is large, $r_{X(3872)} > (3 \div 5)$ fm = $(3 \div 5) \cdot 10^{-13}$ cm. As for the charmonium, its radius is less one fermi, $r_{\chi_{c1}(2P)} \approx 0.5$ fm = $0.5 \cdot 10^{-13}$ cm. That is, the molecule volume is 100 \div 1000 times as large as the charmonium volume, $V_{X(3872)}/V_{\chi_{c1}(2P)} > 100 \div 1000$.

How to explain sufficiently abundant inclusive production of the rather extended molecule $X(3872)$ in a hard process $pp \rightarrow X(3872) + anything$ with rapidity in the range 2,5 - 4,5 and transverse momentum in the range 5-20 GeV [12] ? Really,

$$\begin{aligned} & \sigma(pp \rightarrow X(3872) + anything) BR(X(3872) \rightarrow \pi^+\pi^- J/\psi) \\ & = 5.4 \text{ nb} \end{aligned} \quad (15) \text{ where } s < m_+^2, \quad \Lambda^2 \gg m_+^2.$$

and

$$\begin{aligned} & \sigma(pp \rightarrow \psi(2S) + anything) BR(\psi(2S) \rightarrow \pi^+\pi^- J/\psi) \\ & = 38 \text{ nb}. \end{aligned} \quad (16)$$

But, according to Ref. [7]

$$BR(\psi(2S) \rightarrow \pi^+\pi^- J/\psi) = 0.34 \quad (17)$$

while

$$0.023 < BR(X(3872) \rightarrow \pi^+\pi^- J/\psi) < 0.066 \quad (18)$$

according to Ref. [13].

So,

$$0.74 < \frac{\sigma(pp \rightarrow X(3872) + anything)}{\sigma(pp \rightarrow \psi(2S) + anything)} < 2.1. \quad (19)$$

The extended molecule is produced in the hard process as intensively as the compact charmonium. It's a miracle.

As for the problem of the mass shift, Eq. (13), the contribution of the D^-D^{*+} and \bar{D}^0D^{*0} loops, see Fig. 3, into the self energy of the $X(3872)$ resonance, $\Pi_X(s)$, solves it easily.

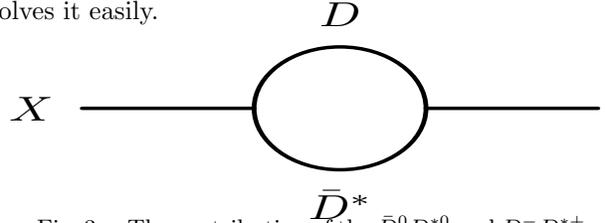


Fig. 3. The contribution of the \bar{D}^0D^{*0} and D^-D^{*+} loops into the self energy of the $X(3872)$ resonance.

Let us calculate $I^{D^*\bar{D}}(s)$ in Eq. (8) with help of a cut-off Λ .

$$\begin{aligned} I^{D^*\bar{D}}(s) &= \int_{m_+^2}^{\Lambda^2} \frac{\sqrt{(s'-m_+^2)(s'-m_-^2)}}{s'(s'-s)} ds' \\ &\approx 2 \ln \frac{2\Lambda}{m_+} - 2 \sqrt{\frac{m_+^2-s}{s}} \arctan \sqrt{\frac{s}{m_+^2-s}}, \end{aligned} \quad (20)$$

The inverse propagator of the $X(3872)$ resonance

$$D_X(s) = m_{\chi_{c1}(2P)}^2 - s - \Pi_X(s) - im_X\Gamma. \quad (21)$$

The renormalization of mass

$$m_{\chi_{c1}(2P)}^2 - m_X^2 - \Pi_X(m_X^2) = 0 \quad (22)$$

results in

$$\Delta(2m_X + \Delta) = \Pi_X(m_X^2) \approx (g_A^2/8\pi^2) 4\ln(2\Lambda/m_+). \quad (23)$$

If $\Delta = m_{\chi_{c1}(2P)} - m_X \approx 50$ MeV, then $g_A^2/8\pi \approx 0.2$ GeV² for $\Lambda = 10$ GeV and $BR(X \rightarrow D^0\bar{D}^{*0} + \bar{D}^0D^{*0}) \approx 0.3$. *

Thus, we expect that a number of unknown mainly two-gluon decays of $X(3872)$ into non- $D^*\bar{D}^0 + c.c.$ states are considerable †. The discovery of these decays would be the strong (if not decisive) confirmation of our scenario.

As for $BR(X \rightarrow \omega J/\psi) \sim BR(X \rightarrow \rho J/\psi)$, Eq. (14), this could be a result of dynamics. In our scenario the $\omega J/\psi$ state is produced via the three gluons, see Fig. 4.

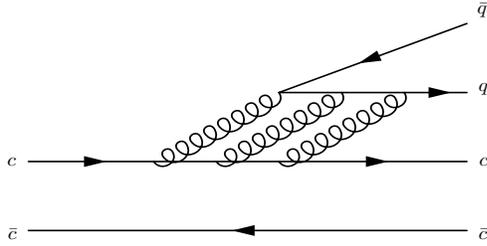


Fig. 4. The three-gluon production of the ω and ρ mesons (the ρ meson via the contribution $\sim m_u - m_d$). All possible permutations of gluons are assumed.

As for the $\rho J/\psi$ state, it is produced both via the one photon, see Fig. 5, and via the three gluons (via the contribution $\sim m_u - m_d$), see Fig. 4.

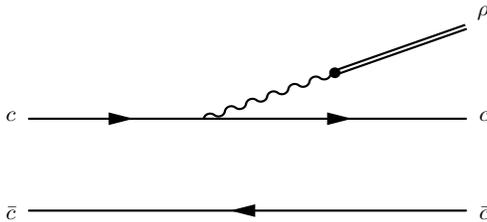


Fig. 5. The one-photon production of the ρ meson. All possible permutations of photon are assumed.

Close to our scenario is an example of the $J/\psi \rightarrow \rho\eta'$ and $J/\psi \rightarrow \omega\eta'$ decays. According to Ref. [7]

$$BR(J/\psi \rightarrow \rho\eta') = (1.05 \pm 0.18) \cdot 10^{-4} \quad \text{and} \\ BR(J/\psi \rightarrow \omega\eta') = (1.82 \pm 0.21) \cdot 10^{-4}. \quad (24)$$

Note that in the $X(3872)$ case the ω meson is produced on its tail ($m_X - m_{J/\psi} = 775$ MeV), while the ρ meson is produced on a half.

It is well known that the physics of charmonium ($c\bar{c}$) and bottomonium ($b\bar{b}$) is similar. Let us compare the already known features of $X(3872)$ with the ones of $\Upsilon_{b1}(2P)$.

Recently, the LHCb Collaboration published a landmark result [14]

$$\frac{BR(X \rightarrow \gamma\psi(2S))}{BR(X \rightarrow \gamma J/\psi)} = C_X \left(\frac{\omega_{\psi(2S)}}{\omega_{J/\psi}} \right)^3 = 2.46 \pm 0.7, \quad (25)$$

where $\omega_{\psi(2S)}$ and $\omega_{J/\psi}$ are the energies of the photons in the $X \rightarrow \gamma\psi(2S)$ and $BR(X \rightarrow \gamma J/\psi)$ decays, respectively.

On the other hand, it is known [7] that

$$\frac{BR(\chi_{b1}(2P) \rightarrow \gamma\Upsilon(2S))}{BR(\chi_{b1}(2P) \rightarrow \gamma\Upsilon(1S))} = C_{\chi_{b1}(2P)} \left(\frac{\omega_{\Upsilon(2S)}}{\omega_{\Upsilon(1S)}} \right)^3 \\ = 2.16 \pm 0.28, \quad (26)$$

where $\omega_{\Upsilon(2S)}$ and $\omega_{\Upsilon(1S)}$ are the energies of the photons in the $\chi_{b1}(2P) \rightarrow \gamma\Upsilon(2S)$ and $\chi_{b1}(2P) \rightarrow \gamma\Upsilon(1S)$ decays, respectively.

Consequently,

$$C_X = 136.78 \pm 38.89 \quad (27)$$

and

$$C_{\chi_{b1}(2P)} = 80 \pm 10.37 \quad (28)$$

as all most lucky versions of the potential model predict for the quarkonia $C_{\chi_{c1}(2P)} \gg 1$ and $C_{\chi_{b1}(2P)} \gg 1$.

According to Ref. [7]

$$BR(\chi_{b1}(2P) \rightarrow \omega\Upsilon(1S)) = (1.63_{\pm 0.34}^{0.4})\%. \quad (29)$$

If the one-photon mechanism dominates in the $X(3872) \rightarrow \rho J/\psi$ decay, see Fig. 5, then one should expect

$$BR(\chi_{b1}(2P) \rightarrow \rho\Upsilon(1S)) \sim (e_b/e_c)^2 \cdot 1.6\% \\ = (1/4) \cdot 1.6\% = 0.4\%, \quad (30)$$

The assumption of the determining role of the $D^\bar{D}^0 + c.c.$ channels in the shift of the mass of the $\chi_{c1}(2P)$ meson is based on the following reasoning. Let us imagine that D and D^* mesons are light, for example, as the K and K^* mesons. Then the width of $X(3872)$ meson is equal 50 MeV for $g_A^2/8\pi = 0.2$ GeV² that much more than the width of its decay into all non- $(D^*\bar{D}^0 + c.c.)$ channels, $\Gamma < 1.2$ MeV. That is, in our case the coupling of the $X(3872)$ meson with the $D^*\bar{D}^0 + c.c.$ channels is rather strong.

†Note that in the $\chi_{c1}(1P)$ case the width of the two-gluon decays equals 0.56 MeV [7] that agrees with $\Gamma < 1.2$ MeV satisfactory.

where e_c and e_b are the charges of the c and b quarks, respectively.

If the three-gluon mechanism (its part $\sim m_u - m_d$) dominates in the $X(3872) \rightarrow \rho J/\psi$ decay, see Fig. 4, then one should expect

$$BR(\chi_{b1}(2P) \rightarrow \rho \Upsilon(1S)) \sim 1.6\%. \quad (31)$$

4 Conclusion

We believe that discovery of a significant number of unknown decays of $X(3872)$ into non- $D^{*0}\bar{D}^0 + c.c.$ states via two gluons and discovery of the $\chi_{b1}(2P) \rightarrow \rho \Upsilon(1S)$ decay could decide destiny of $X(3872)$.

Once more, we discuss the scenario where the $\chi_{c1}(2P)$ charmonium sits on the $D^{*0}\bar{D}^0$ threshold but not a mixing of the giant $D^*\bar{D}$ molecule and the compact $\chi_{c1}(2P)$ charmonium, see, for example, Refs. [15, 16] and references cited therein. Note that the mixing of such states requests the special justification. That is, it is necessary to show that the transition of the giant molecule into the compact charmonium is considerable at insignificant overlapping of their wave functions. Such a transition $\sim \sqrt{V_{\chi_{c1}(2P)}/V_{X(3872)}}$ and a branching ratio of a decay via such a transition $\sim V_{\chi_{c1}(2P)}/V_{X(3872)}$.

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