Status of the Monte Carlo generators for low energy e^+e^- scattering*

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Abstract: In this paper a short review of the existing Monte Carlo generators used for luminosity measurements at meson factories and simulating reactions $e^+e^- \to$ hadrons, $e^+e^- \to$ hadrons + photons or $e^+e^- \to e^+e^-$ + hadrons is presented. I discuss the physical accuracy of the codes with emphasis on QED radiative corrections.

Key words: Monte Carlo generators, radiative corrections, low energy e+e- scattering

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1 Introduction

With the LHC running and no new physics found so far, the role of precise observables like anomalous magnetic moment of the muon $(g-2)_\mu$ is getting more and more important. The quest for precision, initiated many years ago by the community attending the PHIPSI Workshops and on the field of Monte Carlo generators and radiative corrections pursued by Working Group on Radiative Corrections and Monte Carlo Generators for Low Energies (http://www.lnf.infn.it/wg/sighad/) [1], started to pay off. Precise data on hadronic cross sections, with the most important $\sigma(e^+e^- \to \pi^+\pi^-)$, as well as on meson transition form factors, are coming up. More data are analysed by BaBar experiment (http://www-public.slac.stanford.edu/babar/) [2]. New data are coming from BES-III (http://bes3.ihep.ac.cn/index.htm) [3], CMD-3 (http://cmd.inp.nsk.su/?cmd3_main) [4] and SND (https://wwwsnd.inp.nsk.su/) [5]experiments. KLOE2 (http://www.lnf.infn.it/kloe2/) will produce results on $\gamma-\gamma$ form factors [6] making use of their newly installed low angle detectors. We will soon have the new $(g-2)_\mu$ experiment running at Fermilab (http://muon-g-2.fnal.gov/)[7], which is going to improve the already impressive accuracy [8] by about factor 4. In some years from now we could expect also data coming from a completely independent method developed at J-PARC (http://g-2.kek.jp/portal/index.html) [9] and new precise hadronic cross section measurements by BELLE-2 (http://belle2.kek.jp/) [10]. Also on the theory side continuous efforts towards improving of the accuracy of calculations of the $(g-2)_\mu$ are to be acknowledged. A summary and perspectives in this field can be found in a short resume [11] of two very busy workshops held at Mainz last year (April 1-10, 2014 in Mainz, Germany). Concluding their outcome in one sentence one can say that it will be difficult to reach the precision of the new $(g-2)_\mu$ Fermilab experiment, but with new emerging ideas a significant improvement is guarantied.

This paper is a short review of the current status of Monte Carlo generators used at meson factories and scan experiments. The generators are used there for many purposes helping in luminosity measurements, in measuring of the hadronic cross section using scan and radiative return method, and in measuring of meson transition form factors in virtual $\gamma-\gamma$ scattering. In Section **2** the status of radiative corrections is given. It is discussed if one still needs to improve the radiative corrections and/or its tests in view of the improving experimental accuracy. Another accuracy limiting factor, especially for accuracy of efficiencies and acceptance corrections, might be the wrong modelling of the hadron-photon interactions. This issue is discussed in Section **3**. Conclusions are presented in Section **4**.

2 Radiative corrections

The radiative corrections are the most crucial for the physical accuracy of Monte Carlo codes. For low energies the most important, and for most of the applications the only important, corrections are the ones coming from quantum electrodynamics (QED).

2.1 Monte Carlo generators used for luminosity measurements

The experiments use mostly the Bhabha scattering at large angles for luminosity monitoring. As the considered energy is only up to 10-11 GeV the weak corrections are almost negligible and definitively well under control. The Monte Carlo generators used by the experimental groups BabaYaga@NLO [12], BHWIDE [13] and MCGPJ [14] are well established and stable for a long time. The comparisons between them made by various groups (see for example [15]) show that they agree between themselves

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at about 0.1% for the integrated cross sections, but in some corners of the phase space they might disagree up to 1%. This accuracy is adequate for the precision required at the low energy experiments and it looks that right now further progress here is not needed.

The reaction $e^+e^- \to \gamma\gamma$ used for cross checks of the Bhabha luminosity measurements is also generated by BabaYaga@NLO [16], by MCGPJ [17–19] and a BKQED [20]. Here no extensive comparisons were made and for the accuracy one has to rely on the authors estimates. The BKQED has a declared precision of about 1% the MCGPJ 0.2% and BabaYaga@NLO 0.1%.

The reaction $e^+e^- \to \mu^+\mu^-\gamma$ is used as a luminometer in some of the low energy experiments using the radiative return method. The generator PHOKHARA [21] now including complete NLO corrections [22] was compared with KKMC generator. The biggest differences for muon invariant mass distribution were found [24, 25] to be 0.25%, well contained in the PHOKHARA estimated precision [26]. It is difficult to asses an error of the generator AfkQed used by BaBar experiment [27]. The accuracy of the structure function approach used in this generator was discussed in [26]. It is very sensitive to event selections used by an experiment as photons spectra are partly integrated in the generator and its typical value is from a few per mile to a few percent.

2.2 Monte Carlo generators used in scan measurements

For low hadron multiplicities the already mentioned generators: BabaYaga@NLO, KKMC, MCGPJ, and PHOKHARA provide the possibility of generation of the reactions $e^+e^- \to$ hadrons and $e^+e^- \to \mu^+\mu^-$ with the accuracy estimated by authors of the codes at a similar level as discussed in the previous subsection. Yet there are limitations: in BabaYaga@NLO from hadrons only $\pi^+\pi^-$ channel is generated and only initial state corrections (ISR) are included; in KKMC the hadronic final states are modelled with low accuracy; in MCGPJ only $\pi^+\pi^-$ and K^+K^- channels include radiative corrections going beyond ISR and in PHOKHARA also only ISR corrections are included. No systematic comparisons of the codes was performed. A limited sample of comparisons can be found in [25]. It would be desirable to make such comparisons in future and disentangle the radiative correction effects from the modelling of photon-hadron interactions. Due to the lack of final state emission (FSR) modelling in the original generators most of the experiments use PHOTOS [28] as a source of additional FSR emission. With this approach one neglects ISR-FSR interference unless a dedicated 'fine tuning' of the PHOTOS is performed.

Monte Carlo generators used in inclusive measurements have a completely different philosophy as it is impossible to make a dedicated fine tuning (form factors modelling) for all separate multi-hadron final states. Instead, hadronisation models are used like in LUARLW generator [29] or methods based on structure function approach with a combination of Lund model and decay chains based on the measured branching fractions are deployed, like in ZRC generator [30]. Unfortunately no comparisons between these generators are available. It is also difficult to asses an error to the simulated distributions. Another possibility of progress in this direction might be offered by an automatising of the cross section calculations like in Carlomat 3.0 generator [31] if a better modelling of hadron production is provided.

2.3 Monte Carlo generators used for radiative return measurements

For the radiative return measurements essentially only two generators were used: AfkQed and PHOKHARA. At the early stage a precursor of the PHOKHARA generator, EVA generator [32, 33], was used, but its development was abandoned as the PHOKHARA approach provides with much better theoretical accuracy. The structure functions used in EVA [34] are the ones used in AfkQed. The accuracy stated for PHOKHARA in Subsection **2.1** is valid also for hadronic final states as far as the ISR radiative corrections are concerned. In principle the estimated 0.5% might look conservative as the biggest difference with KKMC, which was found, is 0.25%. Yet the tests were performed for inclusive event selection and some event selections might enhance the relative size of higher order corrections neglected in PHOKHARA event generator. So without a detailed dedicated studies the conservative estimate of of 0.5% has to be taken. In [24] the observed difference was guessed to come from third order corrections neglected in PHOKHARA. It is indeed true as shown in Fig. 1. The test was performed using analytic results with no cuts imposed, thus it is only indicative and the event selection might change the output. Yet it confirms the 'guess' from [24]. Relative differences between differential, in the invariant mass of the muon pair (Q^2), cross sections of the reaction $e^+e^- \to \mu^+\mu^-\gamma$ calculated using NLO formulae from [35, 36] (marked as Ber in Fig. 1) and the semi-analytic ones available in KKMC based on [37] are shown in Fig. 1. The semi-analytic KKMC result is expanded and: 1) only NLO terms are kept (marked as KKMC(al) in Fig. 1); 2) up to NNLO terms are kept (marked as KKMC(al2) in Fig. 1); 3) the complete semi-analytic result of KKMC is used (marked as KKMC in Fig. 1). In [38] it was shown that PHOKHARA is numerically equivalent to the semi-analytic results of [35, 36], if one integrates over the whole phase space with the exception of the muon pair invariant mass. Thus the comparison shown in Fig. 1 proves that the bulk of the difference between KKMC and PHOKHARA is coming from the NNLO corrections and that beyond this level the corrections are well below 0.1% level. From Fig. 1 it is also clear that the PHOKHARA generator in its radiative return mode cannot be used close to the nominal energy of the experiment as it lacks the exponentiation.

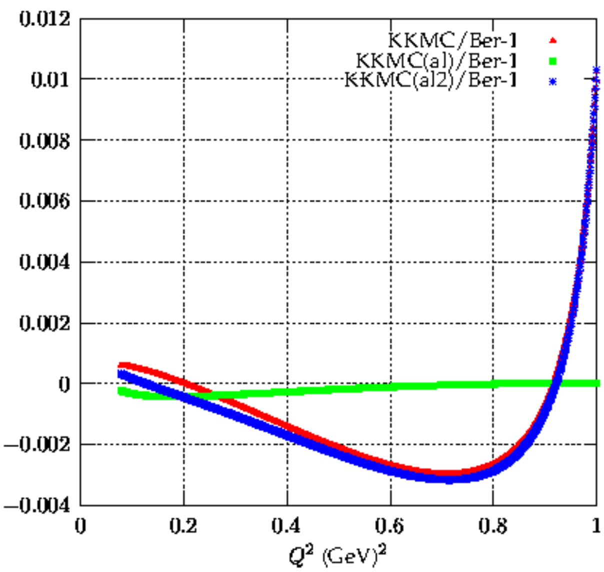

Fig. 1. Numerical comparisons between four analytic results concerning ISR radiative corrections to the reaction $e^+e^- \to \mu^+\mu^-\gamma$. See text for details.

The FSR corrections are included in PHOKHARA only for some of the final states. For the $e^+e^- \to \mu^+\mu^-\gamma$ reaction they are exact at NLO level [22] and the code includes also ISR-FSR interference at the same level. For the $e^+e^- \to \pi^+\pi^-\gamma$ reaction an improved scalar QED was used [39] and supplemented with contributions coming from radiative ϕ decays [40]. Similar approach was adopted for the $e^+e^- \to K^+K^-\gamma$ reaction supplemented with the modeling of the J/ψ and $\psi(2S)$ contributions to this process [41]. The modeling of the final state emission for the reaction $e^+e^- \to \bar{p}p\gamma$ was added to the code recently [42]. For other final states FSR corrections are not included.

2.4 Monte Carlo generators used for reactions $e^+e^- \to e^+e^-$hadrons

Some of the experiments have their own, 'home made', generators not relying on theoretical groups. KLOE was using generator of the reaction $e^+e^- \to e^+e^-\pi^0\pi^0$ [43] relying on equivalent photon approximation (EPA) of the matrix element. This approximation works well if both virtual photons are quasi real and might be wrong by large factors if this is not fulfilled. CLEO was using TwoGam generator by D.Coffman and V.Savinov not well documented in publicly available sources and based on EPA. BELLE was using TREPS [44] again using EPA. Publicly available generators were developed already long time ago. The reaction $e^+e^- \to e^+e^-\pi^+\pi^-$ was generated in [45] were in the QED part there were no approximation, while for the modeling of the $\gamma^*{-}\gamma^* \to \pi^+\pi^-$ amplitude the quark model was used. Other amplitudes contributing to this process were neglected. Aiming for being used at LEP, a GALUGA generator of processes $e^+e^- \to e^+e^-X$ with X being a meson produced in $\gamma^*{-}\gamma^*$ was developed [46]. The modeling of the $\gamma^*{-}\gamma^*{-}X$ part used in this generator is quite involved and will not be discussed here. The EKHARA generator of the process $e^+e^- \to e^+e^-\pi^+\pi^-$ was born [47] as a tool to provide a background for the pion form factor measurement by KLOE. The two photon amplitudes were negligible, as compared to other amplitudes, for the KLOE event selection used in the pion form factor measurement and the generator was not optimised for event selections relevant for the $\gamma^*{-}\gamma^* \to \pi^+\pi^-$ amplitude measurement. Other modes $e^+e^- \to e^+e^-\pi^0$, $e^+e^- \to e^+e^-\eta$ and $e^+e^- \to e^+e^-\eta'$ were added to this generator later [48] aiming for simulation of the $\gamma^* - \gamma^*$ processes. The phase space simulation was adopted from [46] and the modeling of the transition form factors relies now, after the recent update, on the model developed in [49] based on the resonance chiral perturbation theory. All the generators mentioned above do not contain radiative corrections. The only generator containing radiative corrections through structure function method is GGRESRC [50] developed for a single tag experiment and used by BaBar [51]. Unfortunately there exist no other generator containing radiative corrections to allow for independent tests of the code.

3 The importance of modeling of hadron-photon interactions

A modeling of the hadron-photon interactions, as well as the internal structure of the form factors and transition form factors, is crucial for the quality of event generators. Even if the QED radiative corrections are included with care, providing in principle a decent accuracy, a generator can be completely wrong if the hadronic part is modeled losely. This might affect for example acceptance corrections giving wrong extrapolation to the regions not covered by a detector. In this respect a continuous improvement of the generators and feedback experiment-theory is necessary.

4 Conclusions

Existing Monte Carlo generators used for luminosity measurements at meson factories and simulating reactions $e^+e^- \to$ hadrons, $e^+e^- \to$ hadrons + photons or $e^+e^- \to e^+e^-$ + hadrons were reviewed with emphasis on physical accuracy of the codes.

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