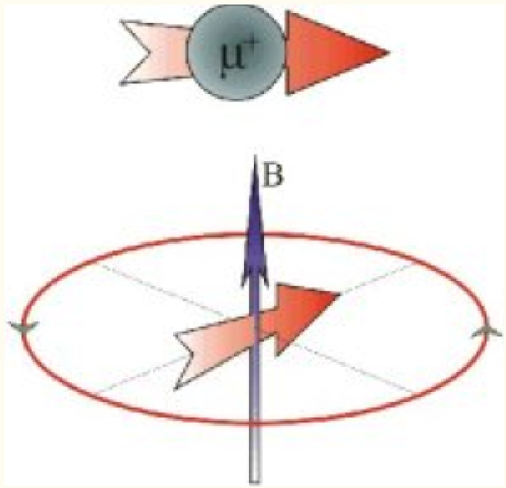
Theoretical review of the muon g-2



International Workshop on e^+e^- collisions from φ to ψ
Hefei
September 24, 2015

赵安杰
Andrzej Czarnecki University of Alberta

with great help from Robert Szafron

Outline

Contributions to the muon *g*-2:

QED

Weak

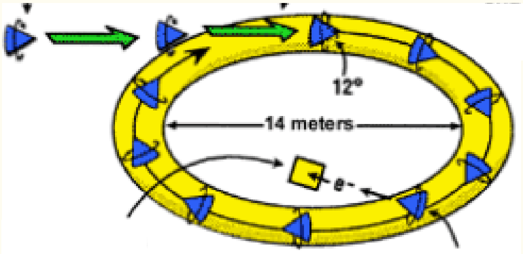
Hadronic
- Vacuum polarization
- Light-by-light scattering

Other effects?

Relation to other observables

The puzzle of the muon magnetic moment

The 3.6 sigma discrepancy persists,



$$a_\mu^{\rm exp} - a_\mu^{\rm SM} = 287(80) \times 10^{-11}$$

PRD 86, 095009 (2012)

This is rather large when compared with other bounds on New Physics:

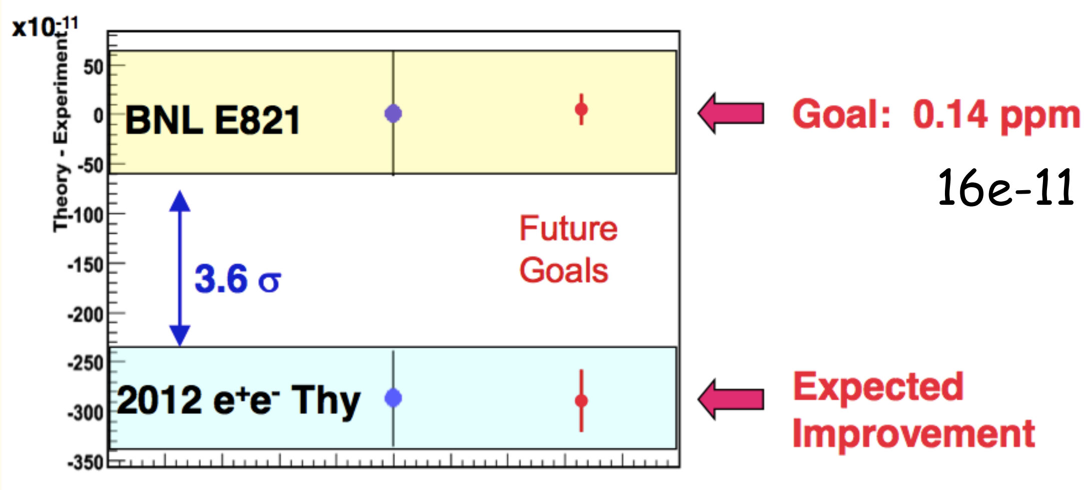
Muon MDM $$d_\mu \sim \frac{e}{2m_\mu} a_\mu^{\rm NP} \sim 3 \cdot 10^{-22}\, e \cdot {\rm cm}$$

Muon-electron transition moment $$|d_{\mu \to e}| < 4 \cdot 10^{-27}\, e \cdot {\rm cm}$$ MEG 2013

Electron EDM $$|d_e| < 8.7 \cdot 10^{-29}\, e \cdot {\rm cm}$$ ACME 2013

Will be probed by Mu2e and COMET (at this meeting: Hai-Bo Li)

How can g_μ-2 be checked?



16e-11

New experiments at Fermilab

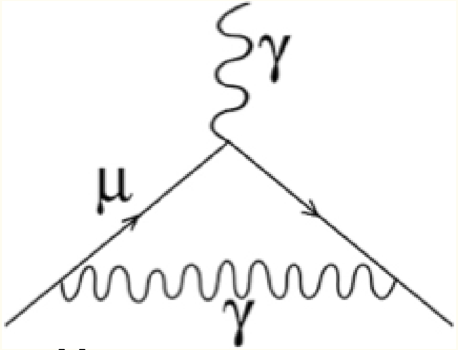
talk by SeungCheon Kim
Also at this meeting:
B. Lee Roberts

and J-PARC (new concept!)

talk by Boris Shwartz

Can we use g_e-2?

QED contribution



One-loop: universal for all leptons

Two- and three-loop: known analytically

Four- and five-loop: about 13k diagrams;

Aoyama, Hayakawa, Kinoshita, Nio: PRL 109, 11808 (2012)

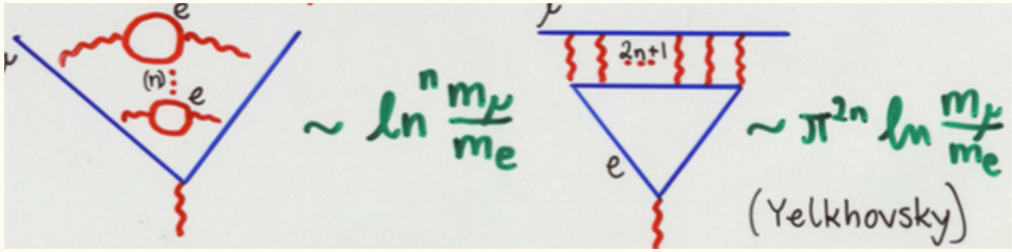
Recent: partial analytical four-loop results!

Kurz, Liu, Marquard, Smirnov, Smirnov, Steinhauser, arXiv:1508.00901

QED contributions

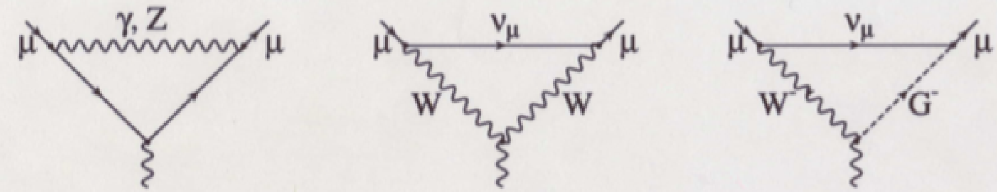
number of loops		$C\left(\frac{\alpha}{\pi}\right)^n$	Contribution x 10^{11}
1	Schwinger, 1948	0.500	116 140 973.318(77)
2	Petermann, 1957 Sommerfield, 1958	0.766	413 217.629(09)
3	S. Laporta E. Remiddi, 1996	24.051	30 141.9024(4)
4	Kinoshita, 1999 Kinoshita, Nio 2004	130.880	381.008(19)
5	Ayoama et al. 2012	753.290	5.094(07)
Total			116 584 718.951(80)

Enhancement specific to the muon *g*-2, absent for the electron *g*-2:



Weak contributions to the muon g-2

One loop:



- $a_\mu^{\rm EW}(\text{1-loop}) \approx \frac{10}{3}\frac{G_\mu}{\sqrt{2}}\frac{m_\mu^2}{16\pi^2} = 195 \times 10^{-11}$

Fujikawa, Lee, Sanda
Jackiw, Weinberg
Altarelli, Cabibbo, Maiani
Bars, Yoshimura
Bardeen, Gastman, Lautrup

- No $\ln \frac{M_{W,Z}^2}{m_\mu^2}$
- Negligible higgs effect

Two loops:

- Large logs Kukhto, Kuraev, Schiller, Silagadze
- Relative to one-loop: $\frac{\alpha}{\pi} \ln \frac{M_{W,Z}^2}{m_\mu^2} \approx 3\%$
- Large coefficient ≈ 7

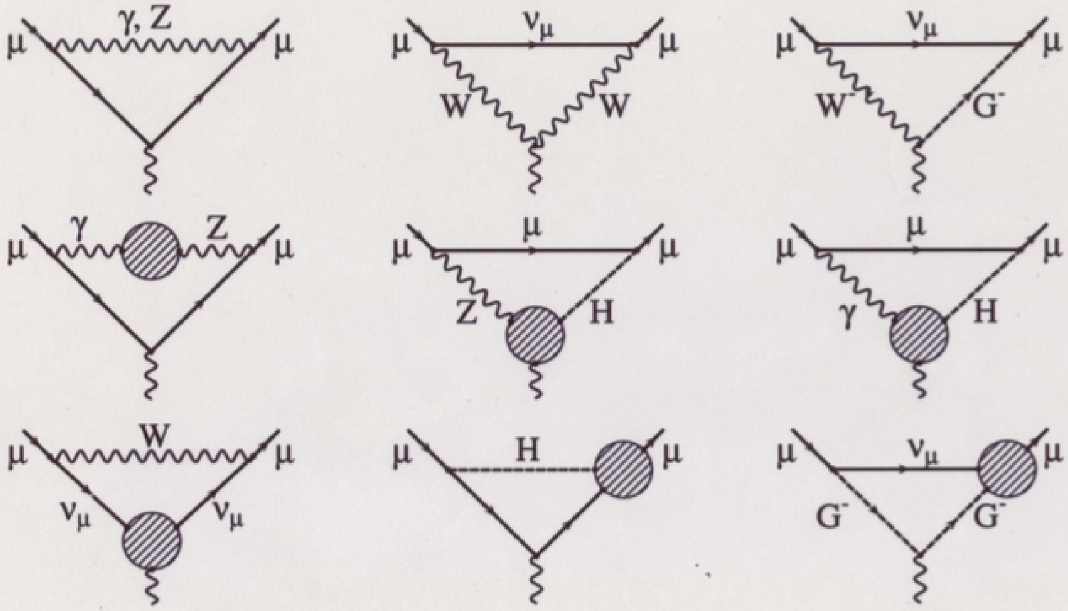
$$a_\mu^{\rm EW}(\text{2-loop}) \approx a_\mu^{\rm EW}(\text{1-loop}) \left[1 + \left(C^{\rm ferm} + C^{\rm bos}\right)\frac{\alpha}{\pi}\right]$$

- Higgs boson important !

After LHC discovery, m_H used by
Gnendinger, Stockinger, Stockinger-Kim
PRD 88 (2013) 053005

Large, universal two-loop electroweak correction

BOSONIC LOOPS: ~ 1600 DIAGRAMS



Logs dominate: $\frac{\Delta a_\mu^{\text{bos}}(\text{2-loop})}{a_\mu^{\text{EW}}(\text{1-loop})} \approx -\frac{107}{30}\frac{\alpha}{\pi}\ln\frac{M_W^2}{m_\mu^2}$ → −23% reduction

(A.C., B. Krause, and W. Marciano, Phys. Rev. Lett. 76 (1996) 3267)

First "full" two-loop EW calculation.

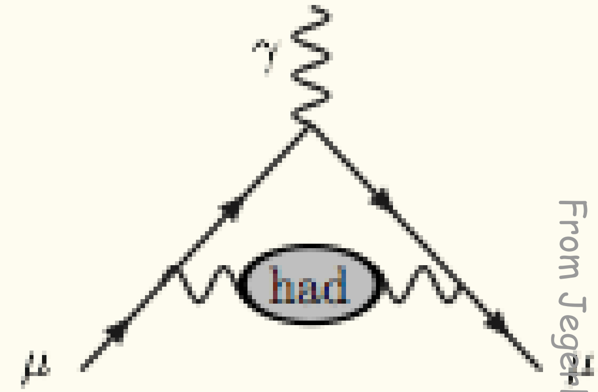
A record QED correction!
Also affects
mu -> e gamma

AC and Jankowski, hep-ph/0106237, PRD65 (2002) 113004

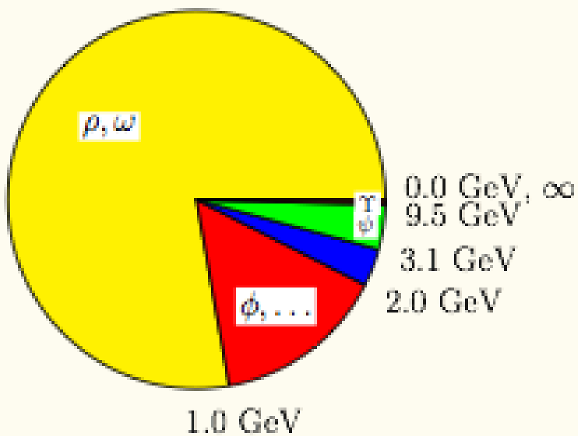
Hadronic vacuum polarization

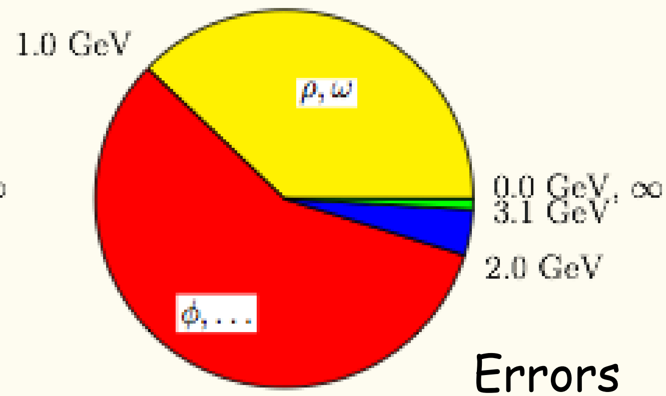
$$a_\mu^{\text{had, LO}} = \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \int_{4m_\pi^2}^{\infty} \mathrm{d}s \frac{R(s)K(s)}{s^2}$$

$$R(s) = \frac{\sigma_{tot}(e^+e^- \to \gamma^* \to hadrons)}{\sigma(e^+e^- \to \gamma^* \to \mu^+\mu^-)}$$



Contributions





Errors

Great importance of the φ to ψ region!

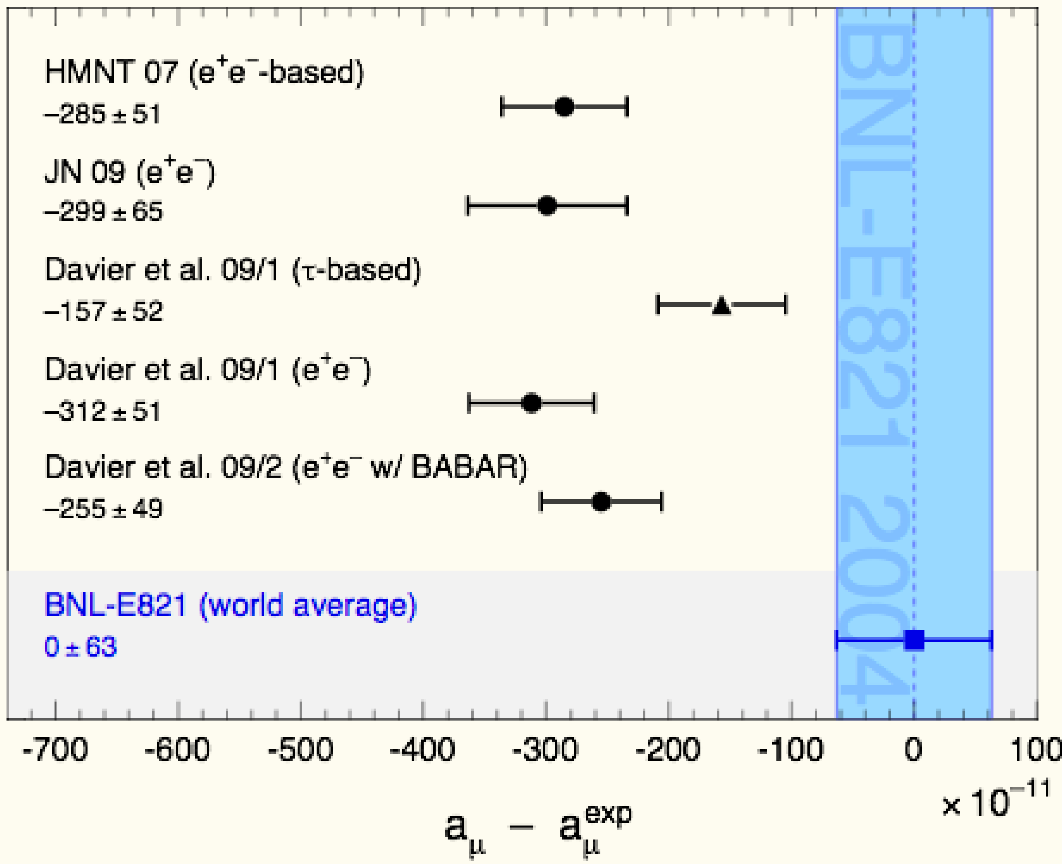
SND: Tatyana Dimova, Konstantin Beloborodov
CMD-3: Gennady Fedotovich
KEDR: Korneliy Todyshev
BaBar: Evgeny Kozyrev
KLOE(2): Veronica De Leo
BESIII: Haiming Ho, Yaqian Wang

From Jegerlehner and Nyffeler, Phys. Rep. 477 (2009) 1

Discrepancy of the prediction and the result

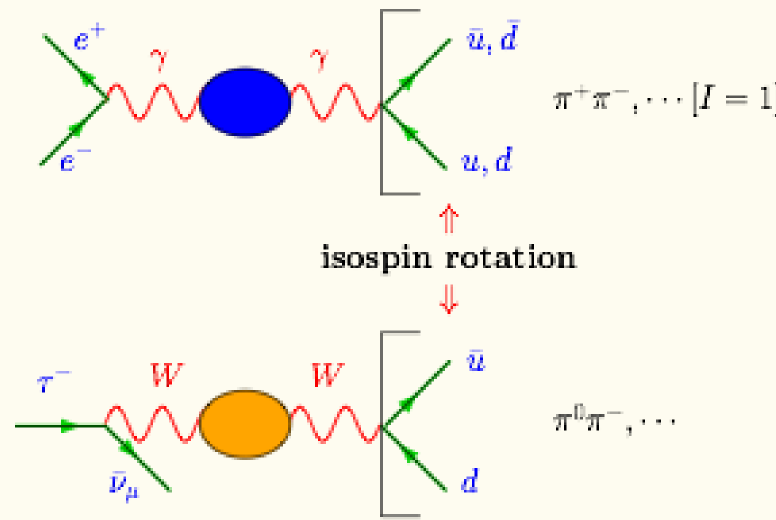
$$a_\mu^{\mathrm{had};LO} = (6949 \pm 43) \times 10^{-11}$$

Hagiwara, Liao, Martin, Nomura, Teubner 2011

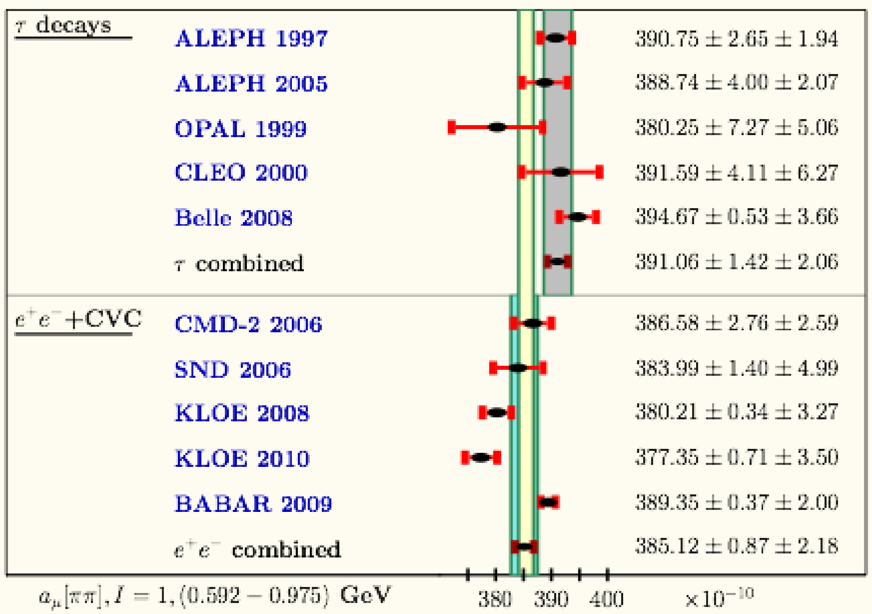


from Davier

Reconciling e^+e^- with τ: isospin breaking



In order to correctly use tau data, isospin-breaking effects must be included: especially gamma-rho mixing
Jegerlehner & Szafron 2011



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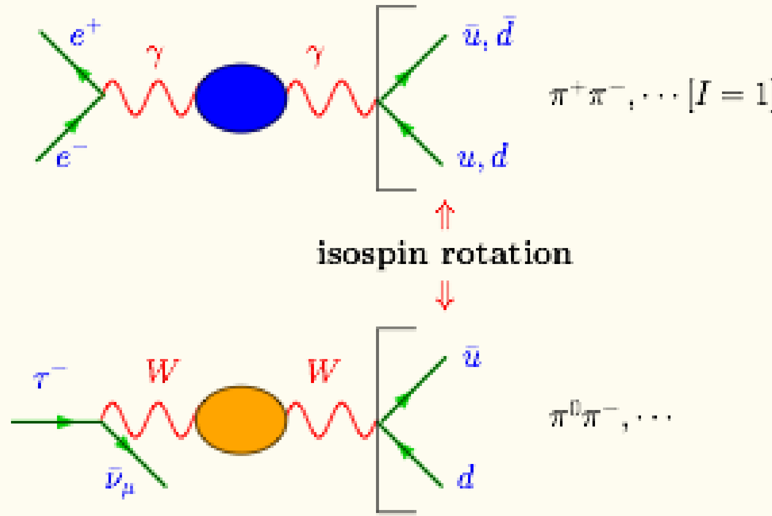

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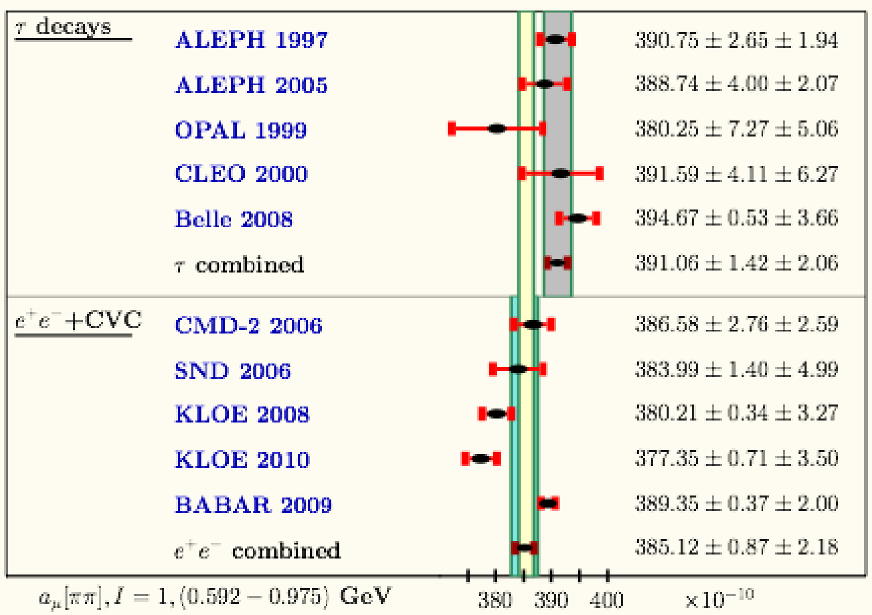
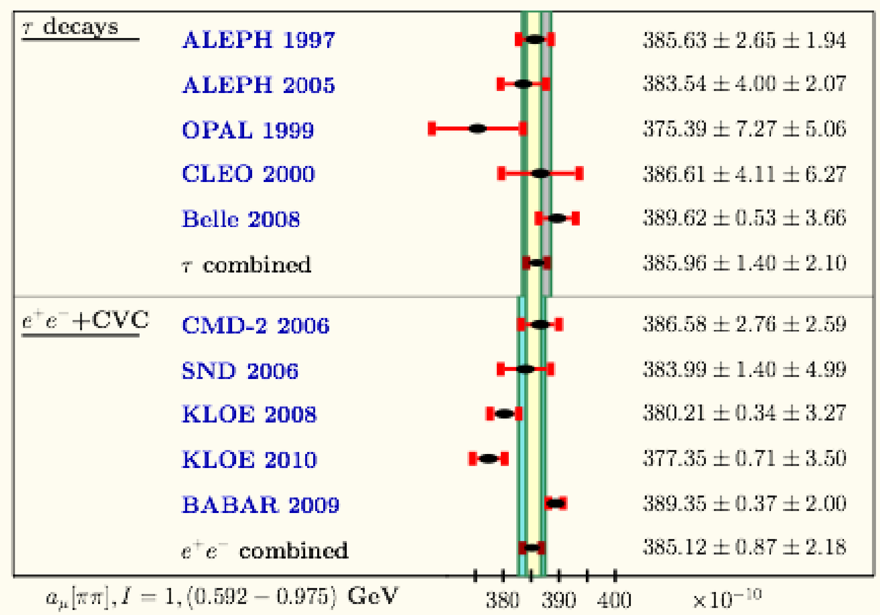
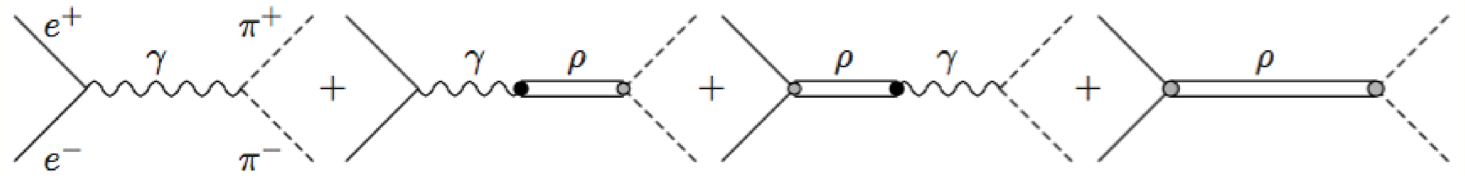




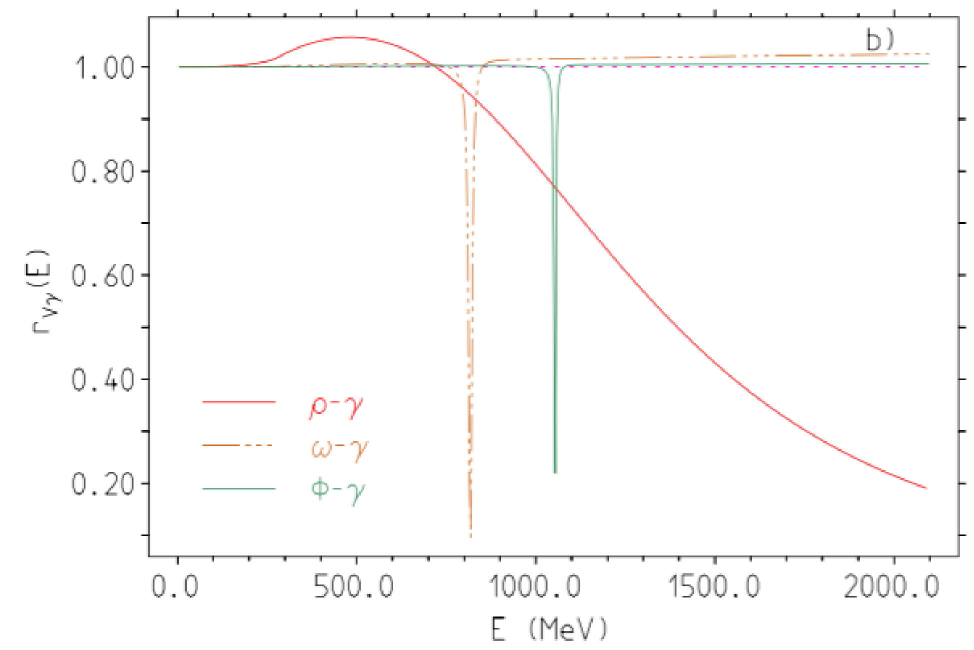
Note the KLOE-BABAR discrepancy; can BESIII resolve it?

rho-gamma mixing



Interference in the neutral channel;
suppressed by M_W in charged;

Tau data must be corrected for
the mixing effect.



Other approaches to the vacuum polarization

Effective lagrangians & global fits

Benayoun, David, DelBuono, Jegerlehner

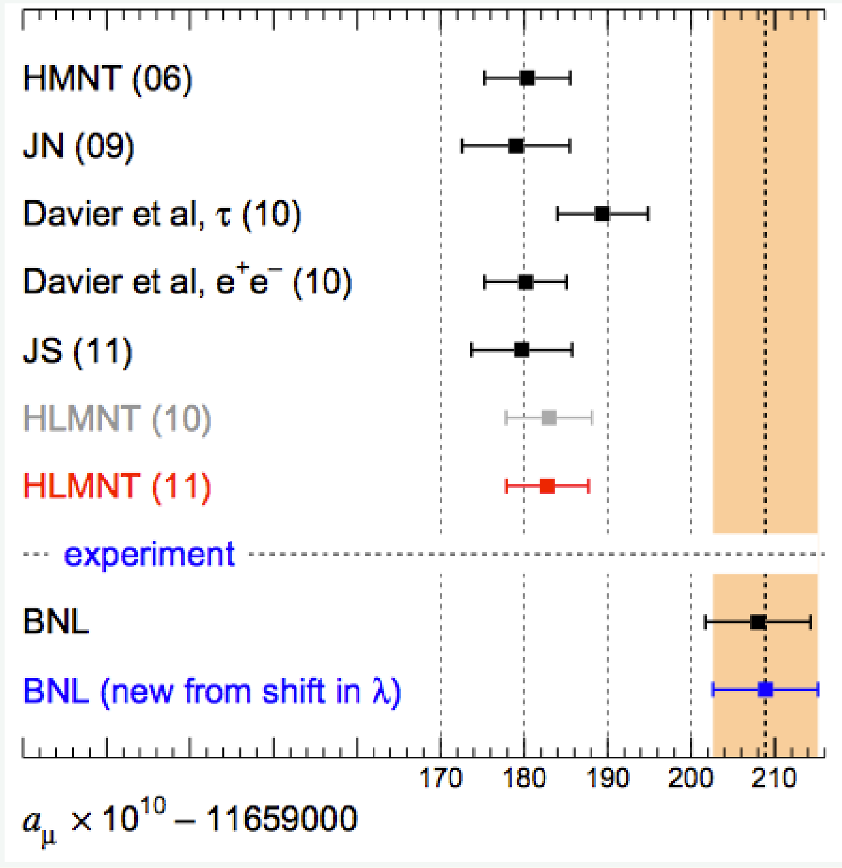
Future possibility: lattice

First results: ETM Collaboration '13

Uncertainties don't seem to be under full control yet

Extraction from Bhabha?

talk by Carlo Carloni Calame



from Nucl.Phys.Proc.Suppl. 218 (2011) 225, Teubner et al.

Hadronic light-by-light

For now, cannot be computed from first principles or connected to measurements.

Model	HLbL contribution x 10^{10}
VMD (Hayakawa)	8.96(1.54)
ENJL (Bijnens)	8.3(3.2)
LMD+V (Knecht)	8.0(4.0)
Q-box (Pivovarov)	14.05
LENJL (Bartos)	10.77(1.68)
(LMD+V)′ (Melnikov)	13.6(0.25)
oLMDV (Nyffeler)	11.6(0.4)
DS (Goecke)	18.8(0.4)
NχQM (Dorokhov)	16.8(1.25)
CχQM (Greynat)	15.0(0.3)

see talks by Alexey Zhevlakov and Rafael Escribano

experimental input: talks by Paolo Gauzzi, Qingnian Xu, Christoph Redmer

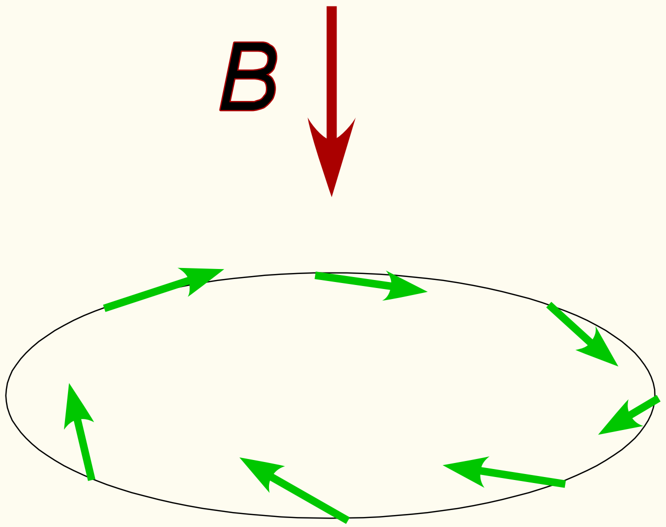
New dispersive approach, model-independent!
Colangelo, Hoferichter, Procura and Stoffer 2014

How do we actually determine g-2?

Measure $$\omega_a = \frac{g-2}{2}\frac{e}{m_\mu}B$$

B from NMR $$\omega_p = \frac{2\mu_p B}{\hbar}$$

$\frac{e}{m_\mu}$ from $$\mu_\mu = g\frac{e\hbar}{4m_\mu}$$

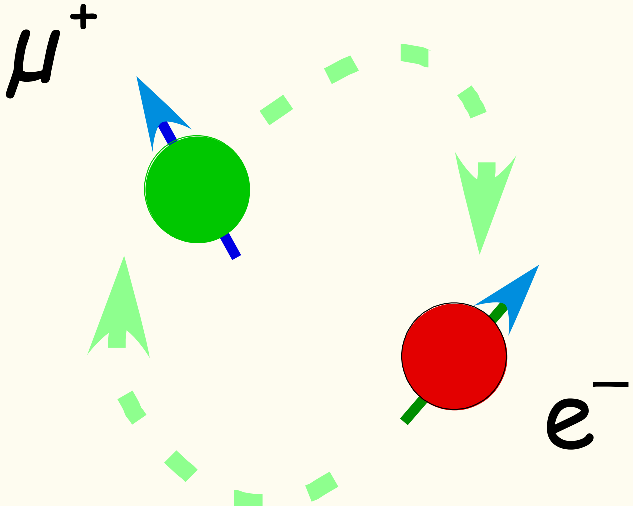


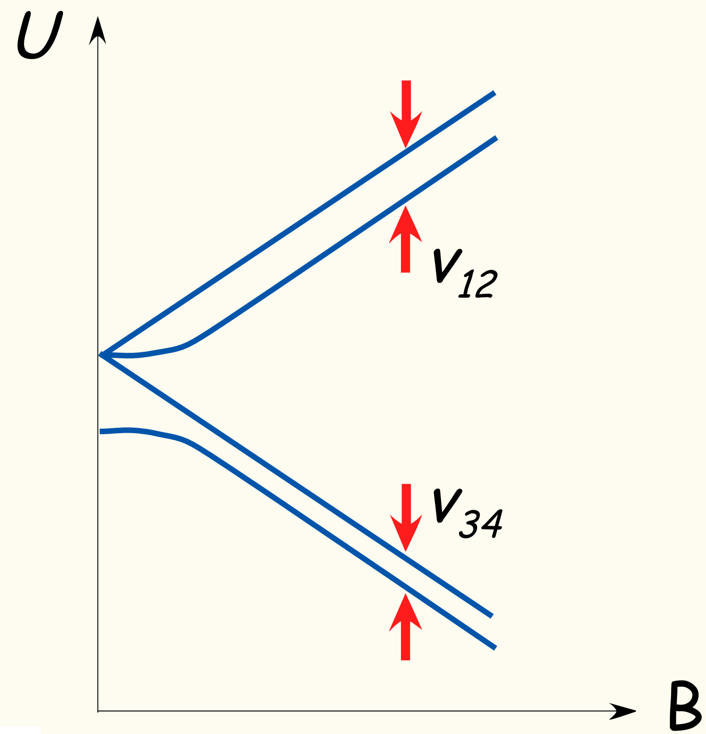
Master formula $$\frac{g-2}{2} = \frac{\omega_\mu/\omega_p}{\mu_\mu/\mu_p - \omega_\mu/\omega_p}$$

Measured by E821

From muonium

Muonium spectrum determines μ_μ/μ_p





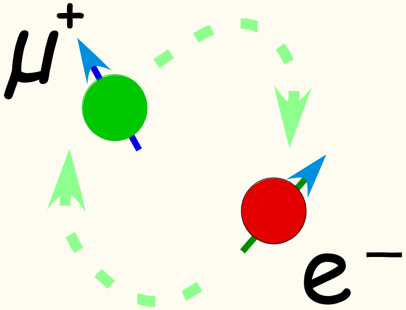
$$\nu_{12}, \nu_{24} \longrightarrow \frac{\mu_\mu}{\mu_p}$$

Measured to relative 120ppb (like $15 \cdot 10^{-11}$ in a_μ)

Will need improvement for the new g-2 results

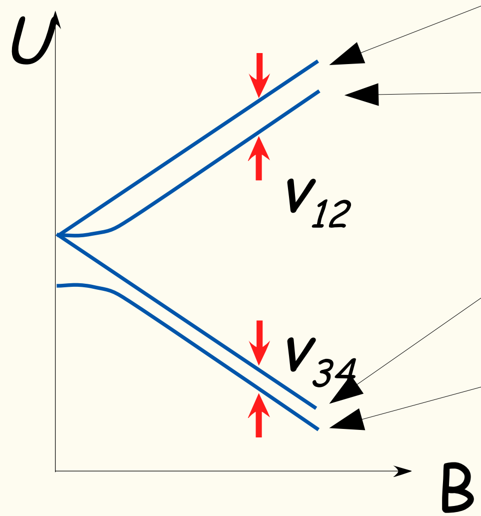
Liu et al., PRL 82, 711 (1999)

How is the muon magnetic moment extracted?



$$H = -(\vec{\mu}_e + \vec{\mu}_\mu)\cdot\vec{B} + c\vec{I}\cdot\vec{J}$$

$$\begin{pmatrix} \frac{c}{2} + k_e - k_\mu & & & \\ & \frac{c}{2} - k_e + k_\mu & & \\ & & -\frac{c}{2} + \surd & \\ & & & -\frac{c}{2} - \surd \end{pmatrix}$$

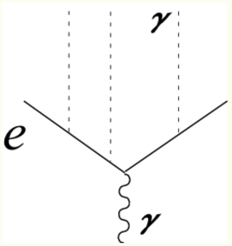


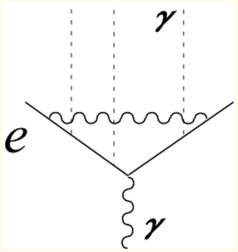
$$\surd = \sqrt{c^2 + (k_e + k_\mu)^2}$$

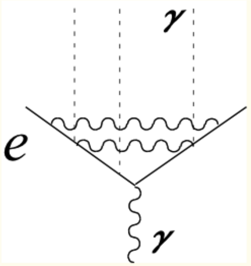
$$c \gg k_e \gg k_\mu$$

Caveat: the magnetic moment in muonium differs from that of a free muon (slightly). Theory input needed!

Binding corrections to g-2







$$g = 2 - \frac{2\,(Z\alpha)^2}{3} - \frac{(Z\alpha)^4}{6} + \ldots$$

$$+ \frac{\alpha}{\pi}\left[1 + \frac{(Z\alpha)^2}{6} + (Z\alpha)^4\,(a_{41}\ln Z\alpha + a_{40}) + \ldots\right]$$

$$+ \left(\frac{\alpha}{\pi}\right)^2\left[-0.65..\left(1 + \frac{(Z\alpha)^2}{6}\right) + (Z\alpha)^4\,(b_{41}\ln Z\alpha + b_{40}) + \ldots\right]$$

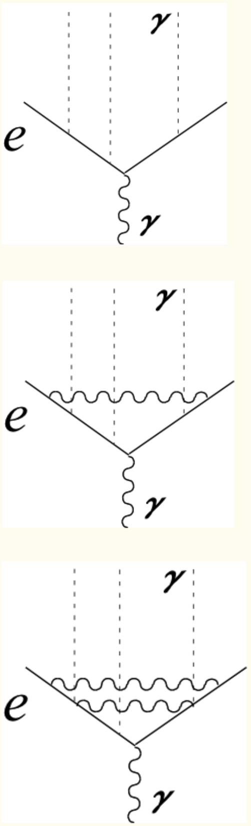
two-loop corrections

$$b_{41} = \frac{28}{9}$$

$$b_{40} = -16.4$$

Pachucki,
AC
Jentschura,
Yerokhin

Binding corrections to g-2



$$g = 2 - \frac{2\,(Z\alpha)^2}{3} - \frac{(Z\alpha)^4}{6} + \ldots$$

$$+\frac{\alpha}{\pi}\left[1 + \frac{(Z\alpha)^2}{6} + (Z\alpha)^4\,(a_{41}\ln Z\alpha + a_{40}) + \ldots\right]$$

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two-loop corrections

$$b_{41} = \frac{28}{9}$$

$$b_{40} = -16.4$$

Pachucki, AC Jentschura, Yerokhin

This has another application: to the determination of the electron mass and, indirectly, of alpha.

How to use g_e-2 to check g_μ-2?

Nature 442, 516 (2006)
PRA 89, 052118 (2014)

Also at this meeting:
Massimo Passera

The second best determination of alpha:
from atomic spectroscopy

$$R_\infty = \frac{m_e c \alpha^2}{2h}$$

Needed precision:

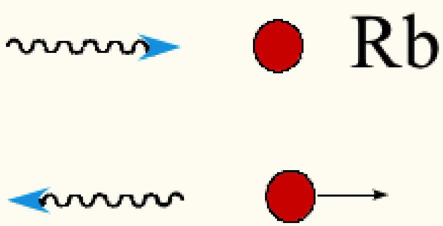
$14 \cdot 10^{-11}$ → $$\alpha^2 = \frac{R_\infty}{2} \cdot \frac{u}{m_e} \cdot \frac{M_X}{u} \cdot \frac{h}{M_X}$$

$7 \cdot 10^{-12}$ (but is it for sure?) → $\frac{R_\infty}{2}$

$8 \cdot 10^{-11}$ NEW Nature 2014 Sturm et al → $\frac{u}{m_e}$

$12 \cdot 10^{-11}$ for Rb (better for He) → $\frac{M_X}{u}$

$124 \cdot 10^{-11}$ improvement needed by factor ~10 → $\frac{h}{M_X}$



gives h/m

$$\alpha(\mathrm{Rb}) = 1/137.035\,999\,049(90) \quad [66 \cdot 10^{-11}]$$

PRL 106, 080801 (2011)

How the binding corrections enter muon g-2

$$\frac{g-2}{2} = \frac{\omega_\mu/\omega_p}{\mu_\mu/\mu_p - \omega_\mu/\omega_p}$$

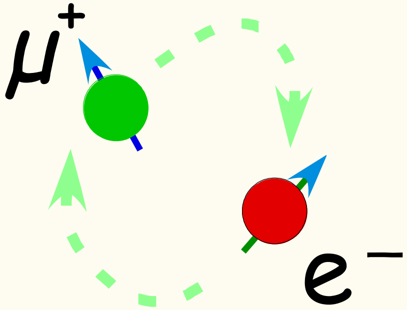
VOLUME 82, NUMBER 4 PHYSICAL REVIEW LETTERS 25 JANUARY 1999

High Precision Measurements of the Ground State Hyperfine Structure Interval of Muonium and of the Muon Magnetic Moment

The gyromagnetic ratios of an electron bound in muonium, g_J, and of a muon in muonium, g'_μ, differ from the free values, g_e and g_μ, by binding corrections [2]

$$g_J = g_e\left(1 - \frac{\alpha^2}{3} + \frac{\alpha^2}{2}\frac{m_e}{m_\mu} + \frac{\alpha^3}{4\pi}\right), \qquad (2)$$

$$g'_\mu = g_\mu\left(1 - \frac{\alpha^2}{3} + \frac{\alpha^2}{2}\frac{m_e}{m_\mu}\right). \qquad (3)$$



Conclusions

QED part well-known. Anticipating analytical results: will improve confidence (even more important for the electron g-2 and alpha)

Hadronic VP: New data needed to reduce errors

Tau data now agree with e+e-

Lattice: part of the future solution?

LbL poorly known; may soon dominate the error; model-independent evaluations needed.

The "other part" of g-2 should be scrutinized.