Radiative μ and τ leptonic decays*

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Abstract: Precise data on radiative leptonic τ decays offer the opportunity to probe the electromagnetic properties of the τ and may allow to determine its anomalous magnetic moment which, in spite of its precise Standard Model prediction, has never been measured. Recently, the branching fractions of the radiative leptonic τ decays ($\tau \to l\nu\bar{\nu}\gamma$, with $l = e, \mu$) were measured by the BaBar collaboration. These precise measurements, with a relative error of about 3%, must be compared with the branching ratios at the next-to-leading order in QED. Indeed the radiative corrections are expected to be of order 10%, for $l = e$, and 3%, for $l = \mu$. Here, we present the prediction of the differential decay rates and branching ratios of the radiative μ and τ leptonic decays in the Standard Model at the next-to-leading order and we compare them with the recent BaBar measurements. Moreover, we report on a dedicated feasibility study for the measurements of the τ anomalous magnetic moment at Belle and Belle II.

Key words: Electroweak radiative corrections, decays of muons, decays of taus, electric and magnetic moments

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1 Introduction

The leptonic decays of the μ and the τ have been one the most powerful tools to study the Lorentz structure of weak interactions. Their precise theoretical formulation in terms of Michel parameters [1–4] places them in a unique position to investigate possible contributions beyond the $V{-}A$ coupling of the Standard Model (SM). Radiative μ and τ leptonic decays, where an inner bremsstrahlung photon is emitted, can be predicted with very high precision and provide an independent probe of the Michel parameters as well as the possibility to extract new combinations like the $\bar{\eta}$ parameter [5–7].

Radiative μ and τ leptonic decays also constitute an important source of background for experiments searching for charged lepton flavour violating decays, such as $\mu^\pm \to e^\pm\gamma$, $\tau^\pm \to l^\pm\gamma$, and even $\mu^\pm \to e^\pm(e^+e^-)$ and $\tau^\pm \to l'^\pm(l^+l^-)$ $(l,l' = e,\mu)$ because of the internal conversion of photons to electron-positron pairs [8–11]

Recently, the BaBar collaboration performed the measurements of the $\tau \to l\gamma\nu\bar{\nu}\,(l = e,\mu)$ branching ratios for a minimum photon energy $\omega_0 = 10$ MeV in the τ rest frame [12]. These measurements, with relative error of about 3%, must be compared with the branching fractions at the next-to-leading order (NLO) in QED. Indeed these radiative corrections, not protected from mass singularities by the Kinoshita-Lee-Nauenberg theorem [13–15], are expected to be of relative order $(\alpha/\pi)\ln(m_l/m_\tau)\ln(\omega_0/m_\tau)$, corresponding to a large 10% correction for $l = e$, and 3% for $l = \mu$. Radiative muon decay was measured long ago in [16]. Preliminary new results were presented recently by the Meg [8] and Pibeta [17] collaborations.

Precise data on radiative τ leptonic decays also offer the opportunity to probe its anomalous magnetic moment ($g{-}2$) and electric dipole moment (EDM). Indeed, the short lifetime of the τ lepton (2.9×10^{-13} s) poses many difficulties for the experimental determination of its dipole moments and it has so far prevented the direct measurement of the $g{-}2$ by means of the τ spin precession in a magnetic field, like in the electron and muon $g{-}2$ experiments [18, 19]. In fact, the present bound on the τ $g{-}2$ is only of $O(10^{-2})$, more than an order of magnitude bigger than the leading contribution $\alpha/(2\pi) \approx 0.001$. Therefore, experiments must attempt the extraction of indirect bounds from τ pair production and decays by comparing sufficiently precise data with the SM predictions.

Here we propose the study of the electromagnetic dipole moments of the τ lepton via radiative leptonic τ decays by means of an effective Lagrangian approach. In Sec. 2 we review the SM predictions at NLO of the differential decay rates and branching ratios of radiative μ and τ leptonic decays. After establishing in Sec. 3 our conventions for the effective Lagrangian, we briefly review the current theoretical and experimental status on the τ $g{-}2$ and EDM in Sec. 4. In Sec. 5 we report the achievable sensitivity to the τ electromagnetic moments at Belle II experiment. We conclude in Sec. 6.

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2 Radiative μ and τ leptonic decays

The SM prediction, at NLO, for the differential rate of the radiative leptonic decays

$$\mu^\pm \to e^\pm \nu \bar{\nu} \gamma, \tag{1}$$
$$\tau^\pm \to l^\pm \nu \bar{\nu} \gamma, \tag{2}$$

with $l = e$ or μ, of a polarized $\mu^\pm$ or $\tau^\pm$ in their rest frame is

$$\frac{d^6\Gamma^\pm(y_0)}{dx\,dy\,d\Omega_l\,d\Omega_\gamma} = \frac{\alpha G_F^2 M^5}{(4\pi)^6} \frac{x\beta_l}{1+\delta_{\rm W}(m_\mu,m_e)} \times \\ \times \left[G \mp x\beta_l\,\hat{n}\cdot\hat{p}_l\,J \mp y\,\hat{n}\cdot\hat{p}_\gamma\,K + xy\beta_l\,\hat{n}\cdot(\hat{p}_l\times\hat{p}_\gamma)\,L \right], \tag{3}$$

where $G_F = 1.166\,378\,7(6) \times 10^{-5}$ GeV^{-2} [20] is the Fermi constant, defined from the muon lifetime, and $\alpha = 1/137.035\,999\,157(33)$ is the fine-structure constant [21, 22]. Calling m and M the masses of the final and initial charged leptons (neutrinos and antineutrinos are considered massless) we define $r = m/M$ and $r_{\rm W} = M/M_{\rm W}$, where $M_{\rm W}$ is the W-boson mass; p and $n = (0,\hat{n})$ are the four-momentum and polarization vector of the initial τ or muon, with $n^2 = -1$ and $n\cdot p = 0$. Also, $x = 2E_l/M$, $y = 2E_\gamma/M$ and $\beta_l \equiv |\vec{p}_l|/E_l = \sqrt{1-4r^2/x^2}$, where $p_l = (E_l, \vec{p}_l)$ and $p_\gamma = (E_\gamma, \vec{p}_\gamma)$ are the four-momenta of the final charged lepton and photon, respectively. The final charged lepton and photon are emitted at solid angles Ω_l and Ω_γ, with normalized three-momenta $\hat{p}_l$ and $\hat{p}_\gamma$, and c is the cosine of the angle between $\hat{p}_l$ and $\hat{p}_\gamma$. The term $\delta_{\rm W}(m_\mu,m_e) = 1.04\times10^{-6}$ is the tree-level correction to muon decay induced by the W-boson propagator [23, 24]. Equation (3) includes the possible emission of an additional soft photon with normalized energy y' lower than the detection threshold $y_0 \ll 1$: $y' < y_0 < y$. The function $G(x,y,c;y_0)$ and, analogously, J and K, are given by

$$G(x,y,c,y_0) = \\ = \frac{4}{3yz^2}\left[g_0(x,y,z) + r_{\rm W}^2\, g_{\rm W}(x,y,z) + \frac{\alpha}{\pi}\, g_{\rm NLO}(x,y,z,y_0) \right], \tag{4}$$

where $z = xy(1-c\beta_l)/2$; the LO function $g_0(x,y,z)$, computed in [25–27], arises from the pure Fermi V–A interaction, whereas $g_{\rm W}(x,y,z)$ is the leading contributions of the W-boson propagator derived in [24]. The NLO term $g_{\rm NLO}(x,y,z,y_0)$ is the sum of the virtual and soft bremsstrahlung contributions calculated in [28] (see also Refs. [29, 30]). The function $L(x,y,z)$, appearing in front of the product $\hat{n}\cdot(\hat{p}_l\times\hat{p}_\gamma)$, does not depend on y_0; it is only induced by the loop corrections and is therefore of $\mathcal{O}(\alpha/\pi)$. The (lengthy) explicit expressions of G, J, K and L are provided in [28]. If the initial $\mu^\pm$ or $\tau^\pm$ are not polarized, Eq. (3) simplifies to

$$\frac{d^3\Gamma(y_0)}{dx\,dc\,dy} = \frac{\alpha G_F^2 M^5}{(4\pi)^6} \frac{8\pi^2 x\beta_l}{1+\delta_{\rm W}(m_\mu,m_e)}\, G(x,y,c,y_0). \tag{5}$$

At the LO, the analytic integration over the allowed kinematic ranges leads, for a minimum photon energy $y_0 = 2\omega_0/M$, to [26, 31]

$$\Gamma_{\rm LO}(y_0) = \frac{G_F^2 M^5}{192\pi^3}\frac{\alpha}{3\pi}\Big\{ 3\,{\rm Li}_2(y_0) - \frac{1}{2}(6+\bar{y}_0^3)\,\bar{y}_0 \ln\bar{y}_0 + \\ + \left(\ln r + \frac{17}{12}\right)(6\ln y_0 + 6\bar{y}_0 + \bar{y}_0^4) - \frac{\pi^2}{2} + \\ + \frac{1}{48}(125 + 45y_0 - 33y_0^2 + 7y_0^3)\,\bar{y}_0 \Big\}, \tag{6}$$

where $\bar{y}_0 = 1 - y_0$ and the dilogarithm is defined by ${\rm Li}_2(z) = -\int_0^z dt\,\frac{\ln(1-t)}{t}$. Terms depending on the mass ratio r have been neglected in the expression for $\Gamma_{\rm LO}(y_0)$, with the obvious exception of the logarithmic contribution which diverges in the limit $r\to0$. However, terms in the integrand $G_{\rm LO}(x,y,c)$ proportional to r^2 were not neglected when performing the integral to obtain (6), as they lead to terms of $\mathcal{O}(1)$ in the integrated result $\Gamma_{\rm LO}(y_0)$. This feature, first noted in [15], is due to the appearance of right-handed electrons and muons in the final states of (1,2) even in the limit $r\to0$, and is a consequence of helicity-flip bremsstrahlung in QED [15, 32–34].

At the NLO, which allows for double photon emission, the branching ratios of the radiative decays (1,2) can be distinguished in two types due to the double real emission:

- The "inclusive" branching ratio, $\mathcal{B}^{\rm Inc}(y_0)$, where in the final state there is at least one photon with energy higher than y_0.
- The "exclusive" branching ratio, $\mathcal{B}^{\rm Exc}(y_0)$, where in the final state there is one, and only one, photon of energy larger than the detection threshold y_0.

It is clear that at the LO the theoretical predictions for these exclusive and inclusive branching ratios coincide – double bremsstrahlung events are simply not considered.

Exclusive and inclusive branching ratios for the radiative decays (1,2) were computed, for a threshold $\omega_0 = y_0(M/2) = 10$ MeV, in Ref. [28] and are reported in Tab. 1. Uncertainties were estimated for uncomputed NNLO corrections, numerical errors, and the experimental errors of the lifetimes. The former were estimated to be $\delta\mathcal{B}_{\rm NLO}^{\rm Exc/Inc} \sim (\alpha/\pi)\ln r \ln(\omega_0/M)\,\mathcal{B}_{\rm NLO}^{\rm Exc/Inc}$. For $\omega_0 = 10$ MeV they are about 10%, 3% and 3% for $\tau\to e\bar{\nu}\nu\gamma$, $\tau\to\mu\bar{\nu}\nu\gamma$ and $\mu\to e\bar{\nu}\nu\gamma$, respectively. They appear with the subscript "N" in Tab. 1. Numerical errors, labeled by the subscript "n", are smaller than those induced by missing radiative corrections. These two kinds of uncertainties were combined to provide the theoretical error of the final $\mathcal{B}^{\rm Exc}$ and $\mathcal{B}^{\rm Inc}$ predictions, labeled by the subscript "th". The uncertainty due to the experimental error of the lifetimes is labeled by the subscript "τ".

The recent measurements by the BABAR collaboration of the branching ratios of the radiative decays $\tau \to l\bar{\nu}\nu\gamma$, with $l = e$ and μ, for a minimum photon energy $\omega_0 = 10$ MeV in the τ rest frame, are [12]:

$$\mathcal{B}_{\rm EXP}\,(\tau \to e\bar{\nu}\nu\gamma) = 1.847\,(15)_{\rm st}\,(52)_{\rm sy} \times 10^{-2}, \qquad (7)$$

$$\mathcal{B}_{\rm EXP}\,(\tau \to \mu\bar{\nu}\nu\gamma) = 3.69\,(3)_{\rm st}\,(10)_{\rm sy} \times 10^{-3}, \qquad (8)$$

where the first error is statistical and the second is systematic. These results are substantially more precise than the previous measurements of the CLEO collaboration [35]. The experimental values in Eqs. (7,8) must be compared with our predictions for the exclusive branching ratios in Tab. 1. For $\tau \to \mu\bar{\nu}\nu\gamma$ decays, the branching ratio measurement and prediction agree within 1.1 standard deviations (1.1σ). On the contrary, the experimental and theoretical values for $\tau \to e\bar{\nu}\nu\gamma$ decays differ by $2.02\,(57) \times 10^{-3}$, i.e. by 3.5σ. This puzzling discrepancy deserves further researches.

The branching ratio of the radiative decay $\mu \to e\bar{\nu}\nu\gamma$ was measured long ago for a minimum photon energy $\omega_0 = 10$ MeV in the μ rest frame [16],

$$\mathcal{B}_{\rm EXP}\,(\mu \to e\bar{\nu}\nu\gamma) = 1.4\,(4) \times 10^{-2}. \qquad (9)$$

This measurement agrees with our theoretical prediction, and new precise results are expected to be published in the near future by the MEG and PIBETA collaborations [8, 17].

	$\tau \to e\bar{\nu}\nu\gamma$	$\tau \to \mu\bar{\nu}\nu\gamma$	$\mu \to e\nu\bar{\nu}\gamma$
$\mathcal{B}_{\rm LO}$	1.834×10^{-2}	3.663×10^{-3}	1.308×10^{-2}
$\mathcal{B}^{\rm Inc}_{\rm NLO}$	$-1.06\,(1)_n\,(10)_N \times 10^{-3}$	$-5.8\,(1)_n\,(2)_N \times 10^{-5}$	$-1.91\,(5)_n\,(6)_N \times 10^{-4}$
$\mathcal{B}^{\rm Exc}_{\rm NLO}$	$-1.89\,(1)_n\,(19)_N \times 10^{-3}$	$-9.1\,(1)_n\,(3)_N \times 10^{-5}$	$-2.25\,(5)_n\,(7)_N \times 10^{-4}$
$\mathcal{B}^{\rm Inc}$	$1.728\,(10)_{\rm th}\,(3)_\tau \times 10^{-2}$	$3.605\,(2)_{\rm th}\,(6)_\tau \times 10^{-3}$	$1.289\,(1)_{\rm th} \times 10^{-2}$
$\mathcal{B}^{\rm Exc}$	$1.645\,(19)_{\rm th}\,(3)_\tau \times 10^{-2}$	$3.572\,(3)_{\rm th}\,(6)_\tau \times 10^{-3}$	$1.286\,(1)_{\rm th} \times 10^{-2}$

Table 1. Branching ratios of radiative μ and τ leptonic decays with minimum photon energy $\omega_0 = 10$ MeV. Inclusive and exclusive ($\mathcal{B}^{\rm Inc/Exc}$) predictions are separated into LO contributions ($\mathcal{B}_{\rm LO}$) and NLO corrections ($\mathcal{B}^{\rm Inc/Exc}_{\rm NLO}$). Uncertainties were estimated for uncomputed NNLO corrections (N), numerical errors (n), and the experimental errors of lifetimes (τ). The first two types of errors were combined to provide the final theoretical uncertainty (th).

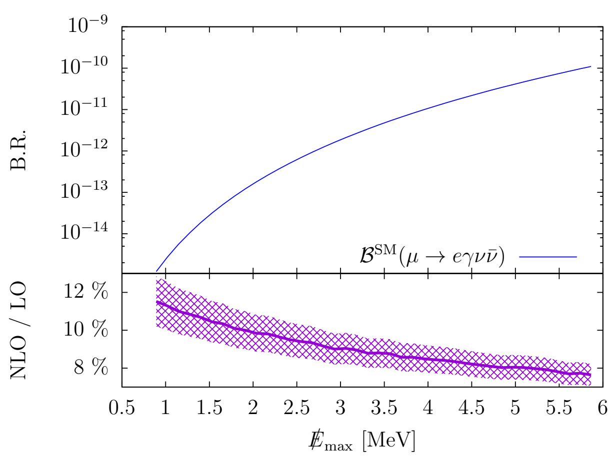

Fig. 1. Top panel: NLO branching ratio of (1) as a function of the invisible energy cut $\not\!\!E_{\rm max}$. Lower panel: the ratio of NLO corrections with respect to the LO branching ratio. The purple band represents the assigned theoretical error due to uncomputed NNLO corrections.

As a further application of our calculation, we studied the impact of the radiative corrections in the phase space region where the neutrino energies (E_ν and $E_{\bar{\nu}}$) are small. As already mentioned in the introduction, this peculiar region is of particular significance for the experiments searching for charged lepton flavour violation. Indeed the SM decays in (1,2) are indistinguishable from the signal, $\mu \to e\gamma$ or $\tau \to l\gamma$, except for the energy carried out by neutrinos. This SM background can be suppressed only via a very precise determination of the final state momenta: the total visible energy of the $e\gamma$ final state (or $l\gamma$) must be as close as possible to the μ mass (or the τ mass).

The upper panel of Fig. 1 shows, for the muon decay (1), the SM prediction at NLO of the branching fraction $\mathcal{B}^{\rm SM}(\not\!\!E_{\rm max})$, defined as the integral of (5) over the phase space region satisfying $\not\!\!E = E_\nu + E_{\bar{\nu}} = m_\mu - E_e - E_\gamma \leq \not\!\!E_{\rm max}$. We note that at NLO also a second soft photon – assumed to be below the detection threshold ω_0 and therefore invisible – can be emitted. We calculated and included these second soft photon effects in $\mathcal{B}^{\rm SM}(\not\!\!E_{\rm max})$ adopting the same technique described in [28] for the numerical evaluation of the exclusive and inclusive branching fractions in Tab. 1.

The lower panel of Fig. 1 presents the ratio of NLO corrections with respect to the LO branching ratio. The relative order of the radiative corrections can be as large as $8-12\%$ for an invisible energy cut $\not{E}_{\rm max}=1-6$ MeV. The purple band represents the theoretical error due to uncomputed NNLO corrections and it is estimated to be $\delta\mathcal{B}^{\rm SM}_{\rm NLO}(\not{E}_{\rm max})=(\alpha/\pi)\ln r\ln\frac{\not{E}_{\rm max}}{m_\mu}\mathcal{B}^{\rm SM}_{\rm NLO}(\not{E}_{\rm max})$.

3 The τ lepton electromagnetic form factors

In the next sections we will report on the possibility to determine the τ dipole moments via radiative leptonic τ decays. We recall that the most general vertex function describing the interaction between initial and final states of an on-shell τ lepton, with four-momenta p and p' respectively, and a photon can be written in the form

$$\Gamma_\mu(q^2)=ie\left\{\gamma_\mu\left[F_{1V}(q^2)+F_{1A}(q^2)\gamma_5\right] +\frac{\sigma_{\mu\nu}}{2m_\tau}q^\nu\left[iF_{2V}(q^2)-F_{2A}(q^2)\gamma_5\right]\right\}, \quad (10)$$

where $e>0$ is the positron charge, m_τ the mass of the τ, $\sigma_{\mu\nu}=i/2[\gamma_\mu,\gamma_\nu]$ and $q=p'-p$ is the ingoing four-momentum of the off-shell photon. The functions $F_{2V}(q^2)$ and $F_{2A}(q^2)$ are related, in the limit $q^2\to 0$, to the measurable quantities $F_{2V}(0)=a_\tau$ and $F_{2A}(0)=d_\tau(2m_\tau/e)$, where a_τ and d_τ are the anomalous magnetic moment and electric dipole moment of the τ, respectively. Deviations of the τ dipole moments from the SM values can be analyzed in the framework of dimension-six gauge-invariant operators. Out of the complete set of 59 independent gauge invariant operators in [36, 37], only two of them can directly contribute to the τ $g-2$ and EDM at tree level (i.e., not through loop effects):

$$Q^{33}_{lW}=\left(\bar{l}_\tau\sigma^{\mu\nu}\tau_R\right)\sigma^I\varphi W^I_{\mu\nu}, \quad (11)$$

$$Q^{33}_{lB}=\left(\bar{l}_\tau\sigma^{\mu\nu}\tau_R\right)\varphi B_{\mu\nu}, \quad (12)$$

where φ and $l_\tau=(\nu_\tau,\tau_L)$ are the Higgs, and the left-handed SU(2) doublets, σ^I the Pauli matrices, $W^I_{\mu\nu}$ and $B_{\mu\nu}$ are the gauge field strength tensors. The leading non-standard effective Lagrangian relevant for our study is therefore given by

$$\mathcal{L}_{\rm eff}=\frac{1}{\Lambda^2}\left[C^{33}_{lW}Q^{33}_{lW}+C^{33}_{lB}Q^{33}_{lB}+{\rm h.c.}\right]. \quad (13)$$

After the electroweak symmetry breaking, the two operators mix and give additional, beyond the SM, contributions to the τ anomalous magnetic moment and EDM:

$$\tilde{a}_\tau=\frac{2m_\tau}{e}\frac{\sqrt{2}v}{\Lambda^2}\,{\rm Re}\left[\cos\theta_{\rm W}C^{33}_{lB}-\sin\theta_{\rm W}C^{33}_{lW}\right], \quad (14)$$

$$\tilde{d}_\tau=\frac{\sqrt{2}v}{\Lambda^2}\,{\rm Im}\left[\cos\theta_{\rm W}C^{33}_{lB}-\sin\theta_{\rm W}C^{33}_{lW}\right], \quad (15)$$

where $v=246$ GeV. Deviations of the τ dipole moments from the SM values could be then determined, possibly down to the level of $O(10^{-3})$, via precise data on τ pair production and τ decays.

4 Experimental determination

The present resolution on the τ anomalous magnetic moment is only of $\mathcal{O}(10^{-2})$ [38], more than an order of magnitude larger than its precise SM prediction [39]

$$a^{\rm SM}_\tau=117\,721\,(5)\times10^{-8}. \quad (16)$$

In fact, while the SM value of a_τ is known with a tiny uncertainty of 5×10^{-8}, the τ short lifetime has so far prevented the determination of a_τ by measuring the τ spin precession in a magnetic field, like in the electron and muon $g-2$ experiments. The present PDG limit on the τ $g-2$ was derived in 2004 by the Delphi collaboration from $e^+e^-\to e^+e^-\tau^+\tau^-$ total cross section measurements at $\sqrt{s}$ between 183 and 208 GeV at LEP2 (the study of a_τ via this channel was proposed in [40]). The measured values of the cross-sections were used to extract limits on the τ $g-2$ by comparing them to the SM values, assuming that possible deviations were due to non-SM contributions to a_τ. The obtained limit at 95% CL is [38]

$$-0.052<\tilde{a}_\tau<0.013, \quad (17)$$

which can be also expressed in the form of central value and error as $\tilde{a}_\tau=-0.018\,(17)$ [38].

The reanalysis of Ref. [41] of various LEP and SLD measurements – mainly of the $e^+e^-\to\tau^+\tau^-$ cross sections – allowed the authors to set the indirect 2σ confidence interval

$$-0.007<\tilde{a}_\tau<0.005, \quad (18)$$

a bound stronger than that in Eq. (17). This analysis assumed $\tilde{d}_\tau=0$. Like earlier ones, it also neglected radiative corrections, but the authors checked that the inclusion of initial-state radiation did not affect significantly their bounds.

The EDM interaction violates the discrete CP symmetry. In the SM, with massless neutrinos, the only source of CP violation is the CKM-phase. Therefore, a fundamental lepton EDM must arise from virtual quarks linked to the lepton through the W boson and also be sensitive to the imaginary part of the $V_{\rm CKM}$ matrix elements. The leading contribution is naïvely expected at the three-loop level, since two-loop diagram is proportional to $|V_{ij}|^2$. The problem was first analyzed in some detail in [42], but it was subsequently shown that the three-loop diagrams also yield a zero EDM contribution in the absence of gluonic corrections to the quark lines [43]. For this reason, lepton EDMs are predicted to be extremely small in the SM, of the $\mathcal{O}(10^{-38}$–$10^{-35})\,e{\cdot}$cm [44], which is far below the current experimental capabilities. Indeed, present experiments can only probe $d_\tau \sim \mathcal{O}(10^{-17})\,e{\cdot}$cm. Also for the electron, $d_e^{\rm exp} < 0.87 \times 10^{-28} e{\cdot}$cm [45] while $d_e^{\rm SM} \sim \mathcal{O}(10^{-38})\,e{\cdot}$cm – it is hard to imagine improvements in the sensitivity by ten orders of magnitude! The present PDG limit on the EDM of the τ lepton at 95% CL is

$$-2.2 < {\rm Re}(\tilde{d}_\tau) < 4.5 \;(10^{-17}\, e\cdot{\rm cm}),$$
$$-2.5 < {\rm Im}(\tilde{d}_\tau) < 0.8 \;(10^{-17}\, e\cdot{\rm cm}); \qquad (19)$$

it was obtained by the Belle collaboration [46] following the analysis of Ref. [47] for the impact of an effective operator for the τ EDM in the process $e^+e^- \to \tau^+\tau^-$.

At the LHC, bounds on the τ dipole moments are expected to be set in τ pair production via Drell-Yan [48, 49] or double photon scattering processes [50]. The best limits achievable in $pp \to \tau^+\tau^-$ are estimated to be comparable with present existing ones if one assumes that the total cross section for τ pair production will be measured at the 14% level. A possible improvement is expected in the $pp \to pp\tau^+\tau^-$ analysis, via the subprocess $\gamma\gamma \to \tau^+\tau^-$, where the Delphi bound (17) might be ameliorated by one order of magnitude [50]. Earlier proposals can be found in [51, 52].

The Belle II experiment at the upcoming high-luminosity B factory SuperKEKB [53] will offer new opportunities to improve the determination of the τ electromagnetic properties. The authors of Ref. [54] proposed to determine the Pauli form factor $F_{2V}(q^2)$ of the τ via $\tau^+\tau^-$ production in e^+e^- collisions at the Υ resonances (Υ(1S), Υ(2S) and Υ(3S)) with a sensitivity of $O(10^{-5})$ or even better. The center-of-mass energy at super B factories is $\sqrt{s} \sim M_{\Upsilon(4S)} \approx 10$ GeV, so that the form factor $F_{2V}(q^2)$ is no longer the anomalous magnetic moment. Furthermore, the contributions to the $e^+e^- \to \tau^+\tau^-$ cross section arise not only from the usual s-channel one-loop vertex corrections, but also from box diagrams, which should be somehow subtracted out. The strategy proposed in [54] to eliminate the contamination from the box diagrams has been to measure the observables on top of the Υ resonances: in this kinematic regime the (non-resonant) box diagrams are numerically negligible and only one-loop corrections to the $\gamma\tau\tau$ vertex are relevant.

However, it is very difficult to resolve the narrow peaks of the $\Upsilon(1S, 2S, 3S)$ in the $\tau^+\tau^-$ decay channel – the $\Upsilon(4S)$ decays almost entirely in $B\bar{B}$ – because of the natural irreducible beam energy spread associated to any e^+e^- synchrotron. If we compare the total width for the Υ resonances ($\Gamma_{\rm tot}^{\Upsilon} \sim 20-50$ keV) with the SuperKEKB beam energy spread $\sigma_{\rm W} = 5.45$ MeV [55], we note that at the Belle II the $\tau^+\tau^-$ events produced with beams at a centre of mass energy $\sqrt{s} \sim M_\Upsilon$ are mostly due to non-resonant interaction. The situation at Belle was similar (the energy spread at KEKB was $\sigma_{\rm W} = 5.24$ MeV [56]). Eventually, the measurement of the $e^+e^- \to \tau^+\tau^-$ cross section on top of the Υ resonances will not eliminate the contamination of the box diagrams.

5 τ dipole moments via $\tau \to l\gamma\nu\bar{\nu}$ decays

The effective Lagrangian (13) generates additional non-standard contributions to the differential decay rate in Eq. (3). For a $\tau^\pm$ they can be summarised in the shifts

$$G \to G + \tilde{a}_\tau\, G_a, \qquad J \to J + \tilde{a}_\tau\, J_a,$$
$$K \to K + \tilde{a}_\tau\, K_a, \qquad L \to L \mp (m_\tau/e)\, \tilde{d}_\tau\, L_d. \qquad (20)$$

Tiny terms of $O(\tilde{a_\tau}^2)$, $O(\tilde{d_\tau}^2)$ and $O(\tilde{a_\tau}\tilde{d_\tau})$ were neglected. Deviations of the τ dipole moments from the SM values can be determined, possibly down to the level of $O(10^{-3})$ comparing the SM prediction for the differential rate in Eq. (3), modified by the terms G_a, J_a, K_a and L_d, with sufficiently precise data.

The possibility to set bounds on $\tilde{a}_\tau$ via the radiative leptonic τ decays in (2) was suggested long ago in Ref. [57]. In that article the authors proposed to take advantage of a radiation zero of the LO differential decay rate in (3) which occurs when, in the τ rest frame, the final lepton l and the photon are back-to-back, and l has maximal energy. Since a non-standard contribution to a_τ spoils this radiation zero, precise measurements of this phase-space region could be used to set bounds on its value. However, our Monte Carlo simulation in Belle experiment conditions shows no significant improvement of the existing limits: the $\tilde{a}_\tau$ upper limit (UL) that can be achieved with the whole Belle statistics, about 0.9×10^9 τ pairs, is only UL$(\tilde{a}_\tau) \simeq 2$ [58, 59].

A more powerful method to extract $\tilde{a}_\tau$ and $\tilde{d}_\tau$ consists in the use of an unbinned maximum likelihood fit of events in the full phase space. In this approach we considered $e^+e^- \to \tau^+\tau^-$ events where both τ leptons decay subsequently into a particular final state: $\tau^\mp$ (signal side) decays to the radiative leptonic mode, the other $\tau^\pm$ (tag side) decays to some well known mode with a large branching fraction. As a tag decay mode we chose $\tau^\pm \to \rho^\pm\nu \to \pi^\pm\pi^0\nu$, which also serves as spin analyser and allows us to be sensitive to the spin dependent part of the differential decay width of the signal decay using effects of spin-spin correlation of the τ leptons [60]. With this technique we analyzed a data sample of $(\ell^\mp\nu\nu\gamma,\ \pi^\pm\pi^0\nu)$ events corresponding to the total amount of data available at Belle and the one planned at the Belle II experiment.

The feasibility study shows that no improvement is expected from Belle data. However the experimental sensitivity on $\tilde{a}_\tau$ at the Belle II experiment, $\sigma_{\tilde{a}} = 0.012$ [58, 59], can already be competitive with Delphi results in (17). On the other hand, the expected sensitivity on the τ EDM, $\sigma_{\tilde{d}} = 6.1\times10^{-17} e\cdot$cm [58, 59], is still worse than the most precise measurement of $\tilde{d}_\tau$ done at Belle in τ pair production [46].

6 Conclusions

We studied at the NLO in the SM the differential decay rates and branching ratios of $\mu \to e\gamma\nu\bar{\nu}$ and $\tau \to l\gamma\nu\bar{\nu}\,(l = \mu, e)$ decays. Our prediction for $l = \mu$ agrees within 1.1σ with the recent BABAR measurements of $\mathcal{B}(\tau \to \mu\gamma\nu\bar{\nu})$, for a minimum photon energy threshold $\omega_0 = 10$ MeV. On the contrary the measurement of $\mathcal{B}(\tau \to e\gamma\nu\bar{\nu})$ differs from our prediction by 3.5σ. This puzzling discrepancy deserves further researches.

Radiative τ leptonic decays can be employed to measure the τ dipole moments at B factories. Deviations of the τ $g-2$ and EDM from the SM predictions can be determined via an effective Lagrangian approach.

Our dedicated feasibility study showed that the measurement of the τ anomalous magnetic moment at Belle II can be already competitive with the current bound from DELPHI experiment, while the expected sensitivity to the τ EDM is still worse than the most precise measurement done at Belle.

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