Measuring the Leading Order Hadronic contribution to the muon g-2 in the space-like region

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Abstract: Recently a novel approach to determine the leading hadronic corrections to the muon g-2 has been proposed. It consists in a measurement of the effective electromagnetic coupling in the space-like region extracted from Bhabha scattering data. The new method may become feasible at flavor factories, leading to an alternative determination, possibly competitive with the accuracy of the present evaluations based on the dispersive approach via time-like data.

Key words: e^+e^- collisions, vacuum polarization, anomalous magnetic moment

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1 Introduction

In a recent paper [1], we explored the possibility to evaluate the leading-order (LO) hadronic contribution to the muon anomalous magnetic moment $a_\mu^{\rm HLO}$, measuring the effective running electromagnetic coupling in the space-like region.

The discrepancy between experiment and the Standard Model (SM) prediction of the muon anomalous magnetic moment a_μ has kept the hadronic corrections under close scrutiny for several years [2–5]: the hadronic uncertainty dominates that of the SM value and is comparable with the experimental one. When the new results from the g-2 experiments at Fermilab and J-PARC will reach the unprecedented precision of 0.14 parts per million (or better) [6–8], the uncertainty of the hadronic corrections will become the main limitation.

An intense research program is under way to improve the evaluation of the leading order (LO) hadronic contribution to a_μ, due to the hadronic vacuum polarization correction to the one-loop diagram [9, 10], as well as the next-to-leading order (NLO) hadronic one. The latter is further divided into the $O(\alpha^3)$ contribution of diagrams containing hadronic vacuum polarization insertions [11], and the leading hadronic light-by-light term, also of $O(\alpha^3)$ [3, 12, 13]. Even the next-to-next-to leading order (NNLO) hadronic contributions have been studied: insertions of hadronic vacuum polarizations were computed in [14], while hadronic light-by-light corrections have been estimated in [15].

The evaluation of the hadronic LO contribution $a_\mu^{\rm HLO}$ involves long-distance QCD for which perturbation theory cannot be employed. However, using analyticity and unitarity, it was shown long ago that this term can be computed via a dispersion integral using the cross section for low-energy hadronic e^+e^- annihilation [16]. At low energy this cross-section is highly fluctuating due to resonances and particle production threshold effects.

An alternative determination of $a_\mu^{\rm HLO}$ can be obtained measuring the effective electromagnetic coupling in the space-like region extracted from Bhabha ($e^+e^- \to e^+e^-$) scattering data, as detailed in Ref. [1]. A method to determine the running of the electromagnetic coupling in small-angle Bhabha scattering was proposed in [17] and applied to LEP data in [18]. As vacuum polarization in the space-like region is a smooth function of the squared momentum transfer, the accuracy of its determination is only limited by the statistics and by the control of the systematics of the experiment. Also, as at flavor factories the Bhabha cross section is strongly enhanced in the forward region, the space-like determination of $a_\mu^{\rm HLO}$ may not be limited by statistics and, although challenging, may become competitive with standard results obtained with the dispersive approach via time-like data.

2 Theoretical framework

The leading-order hadronic contribution to the muon g-2 is given by the well-known formula [5, 16]

$$a_\mu^{\rm HLO} = \frac{\alpha}{\pi^2}\int_0^\infty \frac{ds}{s}\, K(s)\, {\rm Im}\Pi_{\rm had}(s+i\epsilon), \tag{1}$$

where $\Pi_{\rm had}(s)$ is the hadronic part of the photon vacuum polarization, $\epsilon > 0$,

$$K(s) = \int_0^1 dx\, \frac{x^2(1-x)}{x^2+(1-x)(s/m_\mu^2)} \tag{2}$$

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is a positive kernel function and m_μ is the muon mass. As the total cross section for hadron production in low-energy e^+e^- annihilations is related to the imaginary part of $\Pi_{\rm had}(s)$ via the optical theorem, the dispersion integral in Eq. (1) is computed integrating experimental time-like ($s>0$) data up to a certain value of s [3, 19, 20]. The high-energy tail of the integral is calculated using perturbative QCD [21].

Alternatively, if we exchange the x and s integrations in Eq. (1) we obtain [22]

$$a_\mu^{\rm HLO} = \frac{\alpha}{\pi}\int_0^1 dx\,(x-1)\,\overline{\Pi}_{\rm had}[t(x)]\,, \tag{3}$$

where $\overline{\Pi}_{\rm had}(t) = \Pi_{\rm had}(t) - \Pi_{\rm had}(0)$ and

$$t(x) = \frac{x^2 m_\mu^2}{x-1} < 0 \tag{4}$$

is a space-like squared four-momentum. If we invert Eq. (4), we get $x = (1-\beta)\,(t/2m_\mu^2)$, with $\beta = (1-4m_\mu^2/t)^{1/2}$, and from Eq. (3) we obtain

$$a_\mu^{\rm HLO} = \frac{\alpha}{\pi}\int_{-\infty}^0 \overline{\Pi}_{\rm had}(t)\left(\frac{\beta-1}{\beta+1}\right)^2 \frac{dt}{t\beta}. \tag{5}$$

Equation (5) has been used for lattice QCD calculations of $a_\mu^{\rm HLO}$ [23]; while the results are not yet competitive with those obtained with the dispersive approach via time-like data, their errors are expected to decrease significantly in the next few years [24].

The effective fine-structure constant at squared momentum transfer q^2 can be defined by

$$\alpha(q^2) = \frac{\alpha}{1-\Delta\alpha(q^2)}, \tag{6}$$

where $\Delta\alpha(q^2) = -{\rm Re}\overline{\Pi}(q^2)$. The purely leptonic part, $\Delta\alpha_{\rm lep}(q^2)$, can be calculated order-by-order in perturbation theory – it is known up to three loops in QED [25] (and up to four loops in specific q^2 limits [26]). As ${\rm Im}\overline{\Pi}(q^2) = 0$ for negative q^2, Eq. (3) can be rewritten in the form [27]

$$a_\mu^{\rm HLO} = \frac{\alpha}{\pi}\int_0^1 dx\,(1-x)\,\Delta\alpha_{\rm had}[t(x)]\,. \tag{7}$$

Equation (7) can be evaluated by measuring the effective electromagnetic coupling in the space-like region (see also [10]), for instance from Bhabha scattering data.

A few considerations about Eq. (7) are in order here: in Fig. 1 (left) the integrand $(1-x)\Delta\alpha_{\rm had}[t(x)]$ is plotted, using the output of the routine `hadr5n12` [28] for $\Delta\alpha_{\rm had}(t)$. The range $x \in (0,1)$ corresponds to $t \in (-\infty,0)$. The peak of the integrand occurs at $x_{\rm peak} \simeq 0.914$ where $t_{\rm peak} \simeq -0.108$ GeV^2 and $\Delta\alpha_{\rm had}(t_{\rm peak}) \simeq 7.86\times10^{-4}$ (see Fig. 1 (right)). Such relatively low t values can be explored at e^+e^- colliders with $\sqrt{s}$ around or below 10 GeV (the so called "flavor factories").

Depending on s and θ, the integrand of Eq. (7) can be measured in the range $x \in [x_{\rm min}, x_{\rm max}]$, as shown in Fig. 2 (left). Note that to span low x intervals, larger θ ranges are needed as the collider energy decreases. In this respect, $\sqrt{s} \sim 3$ GeV appears to be very convenient, as an x interval $[0.30, 0.98]$ can be measured varying θ between $\sim 2^\circ$ and 28°. It is also worth remarking that data collected at flavor factories, such as DAΦNE (Frascati), VEPP-2000 (Novosibirsk), BEPC-II (Beijing), PEP-II (SLAC) and SuperKEKB (Tsukuba), and possibly at a future high-energy e^+e^- collider, like FCC-*ee* (TLEP) [29] or ILC [30], can help to cover different and complementary x regions. Furthermore, given the smoothness of the integrand, values outside the measured x interval may be interpolated with some theoretical input.

3 $\Delta\alpha_{\rm had}(t)$ from Bhabha scattering data

The hadronic contribution to the running of α in the space-like region, $\Delta\alpha_{\rm had}(t)$, can be extracted comparing Bhabha scattering data to Monte Carlo (MC) predictions. The LO Bhabha cross section receives contributions from t- and s-channel photon exchange amplitudes. At NLO in QED, it is customary to distinguish corrections with an additional virtual photon or the emission of a real photon (photonic NLO) from those originated by the insertion of the vacuum polarization corrections into the LO photon propagator (VP). The latter goes formally beyond NLO when the Dyson resummed photon propagator is employed, which simply amounts to rescaling the α coupling in the LO s- and t-diagrams by the factor $1/(1-\Delta\alpha(q^2))$ (see Eq. (6)). In MC codes, e.g. in `BabaYaga` [31], VP corrections are also applied to photonic NLO diagrams, in order to account for a large part of the effect due to VP insertions in the NLO contributions. Beyond NLO accuracy, MC generators consistently include also the exponentiation of (leading-log) QED corrections to provide a more realistic simulation of the process and to improve the theoretical accuracy. We refer the reader to Ref. [32] for an overview of the status of the most recent MC generators employed at flavor factories. We stress that, given the inclusive nature of the measurements, any contribution to vacuum polarization which is not explicitly subtracted by the MC generator will be part of the extracted $\Delta\alpha(q^2)$. This could be the case, for example, of the contribution of hadronic states including photons (which, although of higher order, are conventionally included in $a_\mu^{\rm HLO}$), and that of W bosons or top quark pairs.

The analytic dependence of the MC Bhabha predic-

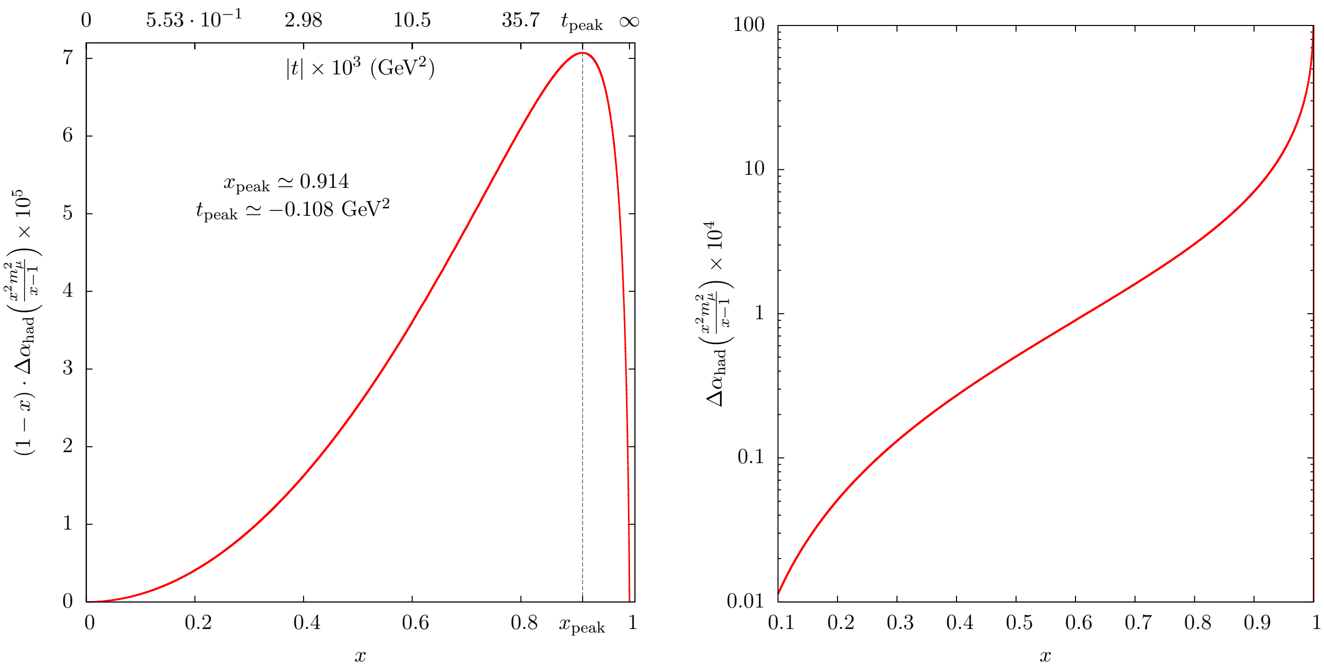

Fig. 1. Left: The integrand $(1-x)\Delta\alpha_{\mathrm{had}}[t(x)] \times 10^5$ as a function of x and t. Right: $\Delta\alpha_{\mathrm{had}}[t(x)] \times 10^4$.

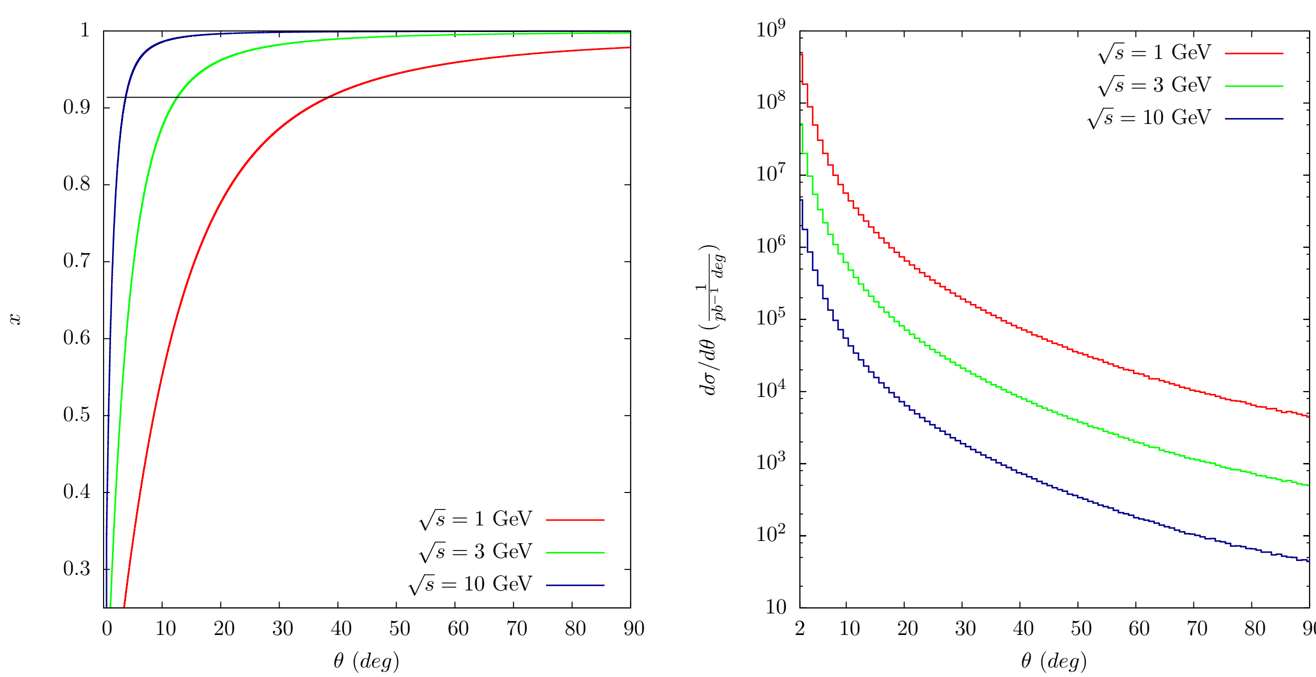

Fig. 2. Left: Ranges of x values as a function of the electron scattering angle θ for three different center-of-mass energies. The horizontal line corresponds to $x = x_{\mathrm{peak}} \simeq 0.914$. Right: Bhabha differential cross section obtained with `BabaYaga` [31] as a function of θ for the same three values of $\sqrt{s}$ in the angular range $2^\circ < \theta < 90^\circ$.

tions on $\alpha(t)$ (and, in turn, on $\Delta\alpha_{\rm had}(t)$) is not trivial, and a numerical procedure has to be devised to extract it from the data. In formulae, we have to find a function $\alpha(t)$ such that

$$\frac{d\sigma}{dt}\Big|_{\rm data} = \frac{d\sigma}{dt}\Big(\alpha(t), \alpha(s)\Big)\Big|_{\rm MC}, \tag{8}$$

where we explicitly kept apart the dependence on the time-like VP $\alpha(s)$ because we are only interested in $\alpha(t)$. Being the Bhabha cross section in the forward region dominated by the t-channel exchange diagram, we checked that the present $\alpha(s)$ uncertainty induces in this region a relative error on the θ distribution of less than $\sim 10^{-4}$ (which is part of the systematic error).

We propose to perform the numerical extraction of $\Delta\alpha_{\rm had}(t)$ from the Bhabha distribution of the t Mandelstam variable. The idea is to let $\alpha(t)$ vary in the MC sample around a reference value and choose, bin by bin in the t distribution, the value that minimizes the difference with data. The procedure is detailed in Ref. [1] and here we only remark that the algorithm does not assume any simple dependence of the cross section on $\alpha(t)$, which can in fact be general, mixing s, t channels and higher order radiative corrections, relevant (or not) in different t domains.

In order to check our procedure, we performed a "pseudo-experiment", generating pseudo-data using the parameterization $\Delta\alpha^{I}_{\rm had}(t)$ of refs. [20, 33] and checking if it can be recovered by inserting in the MC the (independent) parameterization $\Delta\alpha^{II}_{\rm had}(t)$ of Ref. [28] by means of our algorithm. For this exercise, we used the generator `BabaYaga` in its most complete setup.

In Fig. 3, $\Delta\alpha^{\rm extr}_{\rm had}$ is the result extracted with our algorithm: the figure shows that the method is capable of recovering the underlying function $\Delta\alpha_{\rm had}(t)$ inserted into the "data". As the difference between $\Delta\alpha^{I}_{\rm had}$ and $\Delta\alpha^{\rm extr}_{\rm had}$ is hardly visible on an absolute scale, in Fig. 3 all the functions have been divided by $\Delta\alpha^{II}_{\rm had}$ to display better the comparison between $\Delta\alpha^{I}_{\rm had}$ and $\Delta\alpha^{\rm extr}_{\rm had}$.

In order to assess the achievable accuracy on $\Delta\alpha_{\rm had}(t)$ with the proposed method, we remark that the LO contribution to the cross section is quadratic in $\alpha(t)$, thus we have

$$\frac{1}{2}\frac{\delta\sigma}{\sigma} \simeq \frac{\delta\alpha}{\alpha} \simeq \delta\Delta\alpha_{\rm had}, \tag{9}$$

which relates the *absolute* error on $\Delta\alpha_{\rm had}$ with the *relative* error on the Bhabha cross section. From the theoretical point of view, the present accuracy of the MC predictions [32] is at the level of about 0.5‰, which implies that the precision that our method can, at best, set on $\Delta\alpha_{\rm had}(t)$ is $\delta\Delta\alpha_{\rm had}(t) \simeq 2\cdot10^{-4}$. Any further improvement requires the inclusion of the NNLO QED corrections into the MC codes, which is at present not available (although not out of reach) [32].

From the experimental point of view, a measurement of $a_\mu^{\rm HLO}$ from space-like data competitive with the current time-like evaluations would require an $\mathcal{O}(1\%)$ accuracy. Statistical considerations show that a 3% fractional accuracy on the $a_\mu^{\rm HLO}$ integral can be obtained by sampling the integrand in ~ 10 points around the x peak with a fractional accuracy of 10%. Given the value of $\mathcal{O}(10^{-3})$ for $\Delta\alpha_{\rm had}$ at $x = x_{\rm peak}$, this implies that the cross section must be known with relative accuracy of $\sim 2\times10^{-4}$. Such a statistical accuracy, although challenging, can be obtained at flavor factories, as shown in Fig. 2 (right): with an integrated luminosity of $\mathcal{O}(1)$, $\mathcal{O}(10)$, $\mathcal{O}(100)$ fb^{-1} at $\sqrt{s}=1$, 3 and 10 GeV, respectively, the angular region of interest can be covered with a 0.01% statistical accuracy per degree, which must be matched by a similar systematic error.

A fraction of the latter comes from the knowledge of the machine luminosity, which is normalized by calculating a theoretical cross section in principle not depending on $\Delta\alpha_{\rm had}$. We devise two possible options for the normalization process:

1. using the $e^+e^- \to \gamma\gamma$ process, which has no dependence on $\Delta\alpha_{\rm had}$, at least up to NNLO order;

2. using the Bhabha process at $t \sim 10^{-3}$ GeV^2, where the dependence on $\Delta\alpha_{\rm had}$ is of $\mathcal{O}(10^{-5})$ and can be safely neglected.

Both processes have advantages and disadvantages; a dedicated study of the optimal choice will be considered in a future detailed study.

4 Conclusions

We discussed a novel approach to determine the leading hadronic correction to the muon g-2 by measuring the running of $\alpha(t)$ in the space-like region from Bhabha scattering data. Although challenging, we argue that this alternative determination may become feasible with a dedicated experimental and theoretical effort using data collected at present flavor factories and possibly also at a future high-energy e^+e^- collider. The proposed determination can become competitive with the accuracy of the present results obtained with the standard dispersive approach via time-like data.

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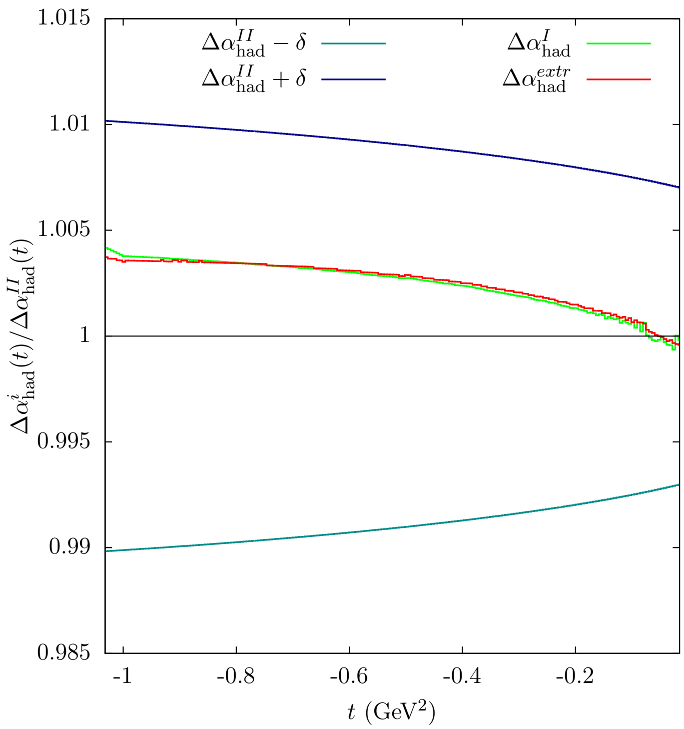

Fig. 3. The extracted function $\Delta\alpha^{\rm extr}_{\rm had}(t)$ compared to the function $\Delta\alpha^{I}_{\rm had}(t)$ used in the pseudo-data (see text). The functions $\Delta\alpha^{II}_{\rm had}(t)\pm\delta(t)$ are shown to display the range spanned by the MC samples. All functions have been divided by $\Delta\alpha^{II}_{\rm had}(t)$. The tiny difference between $\Delta\alpha^{I}_{\rm had}$ and $\Delta\alpha^{\rm extr}_{\rm had}$ is due to the binning discretization.

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