The η TFF from space- and time-like exp. data

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Purpose:

To present an analysis of the η (and η') transition form factor in the space and time-like regions at low and intermediate energies in a model-independent way through the use of rational approximants

Motivations:

- To extract the slope and curvature parameters of the TFFs as well as their values at zero and infinity from experimental data

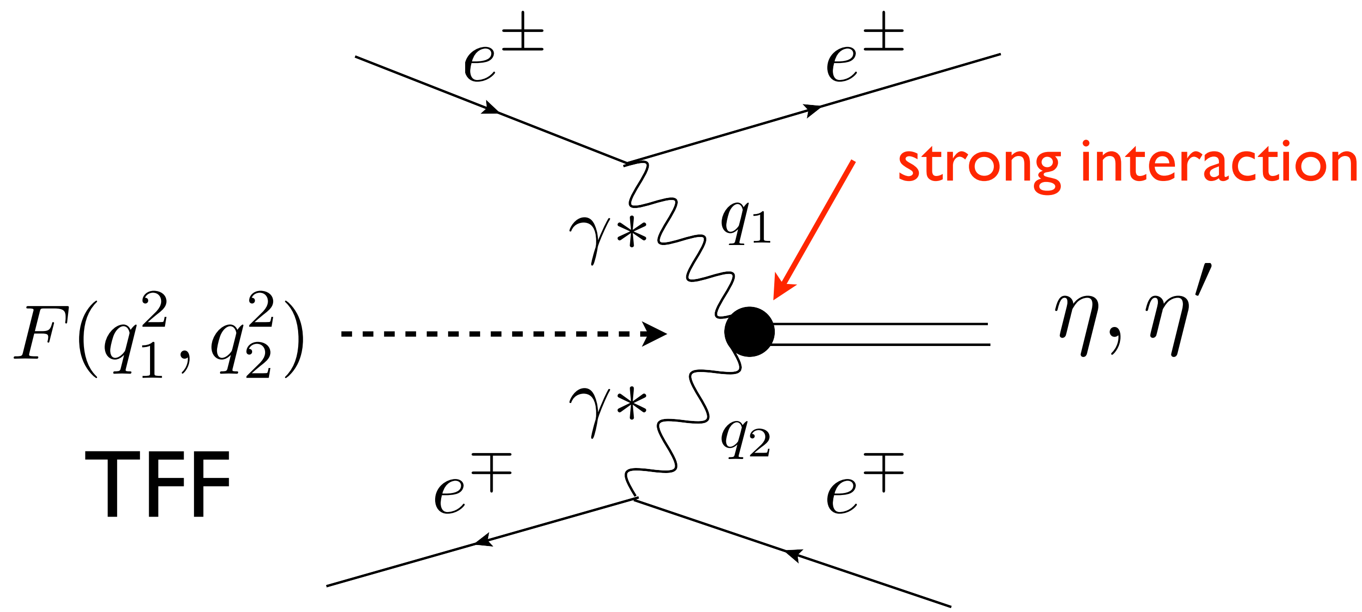
- To discuss the impact of these results on the mixing parameters of the η and η' system and on the determination of the VPγ couplings

Outline:

- *Pseudoscalar transition form factors*
- *Padé approximants*
- *Application to η and η' TFFs*
- *Results*
- *Impact on η-η' mixing parameters*
- *Determination of the VPγ couplings*
- *Conclusions*

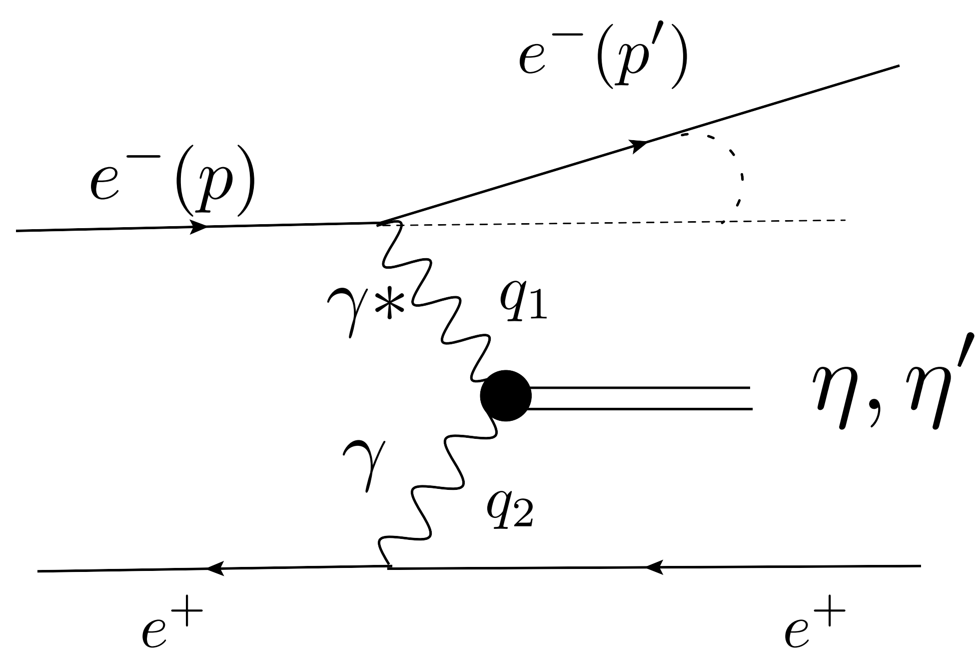
In collab. with P. Masjuan and P. Sánchez-Puertas (Mainz)
Phys. Rev. D89 (2014) 3, 034014 (arXiv:1307.2061 [hep-ph])
Eur. Phys. J. C75 (2015) 9, 414 (arXiv:1504.07742 [hep-ph])

• *Pseudoscalar transition form factors*



not exp. accesible

Single Tag Method



$$F_{P\gamma^*\gamma}(Q^2) \equiv F_{P\gamma^*\gamma^*}(-Q^2, 0)$$

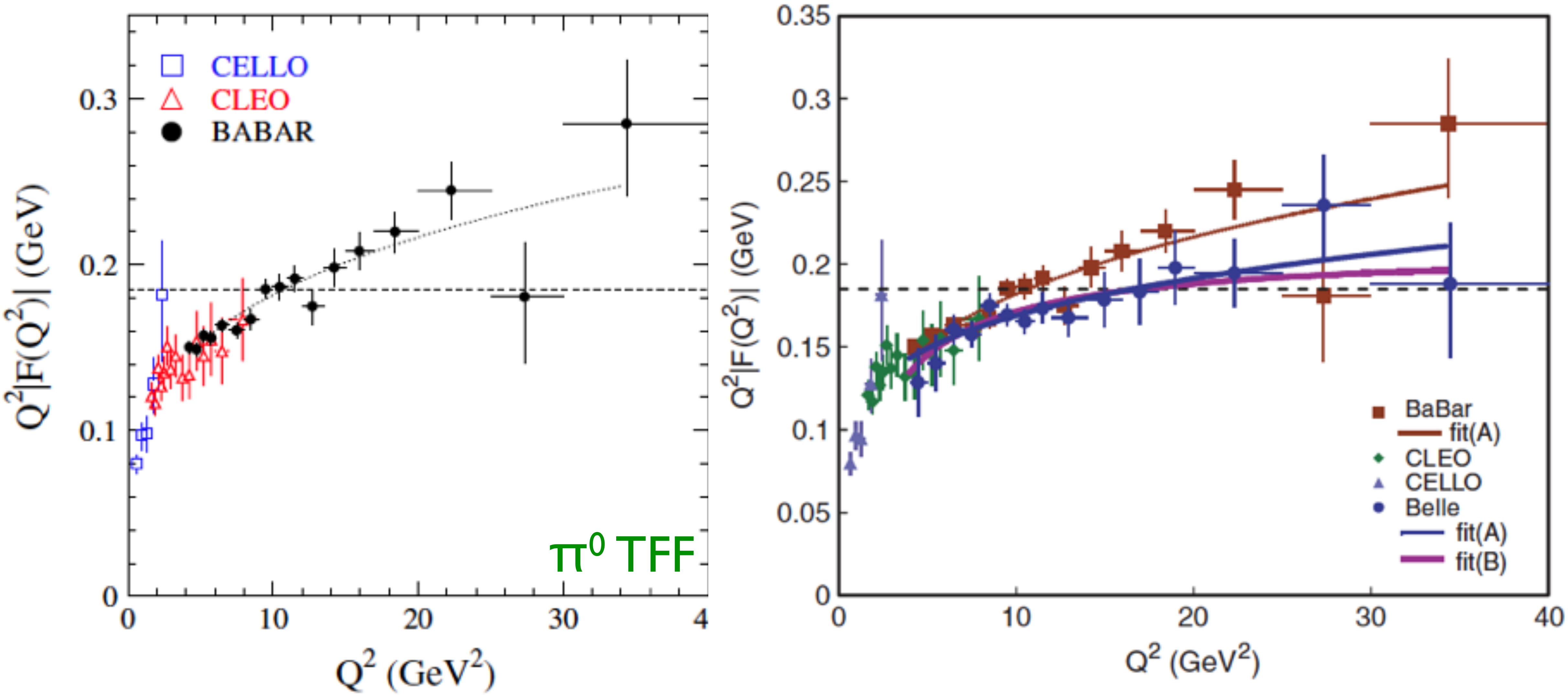
Momentum transfer

- highly virtual photon ⇒ tagged
- quasi-real photon ⇒ untagged

Selection criteria

- 1 e^- detected
- 1 e^+ along beam axis
- Meson full reconstructed

• *Pseudoscalar transition form factors*



S. Uehara et al. (BELLE Collaboration), PRD 86 (2012) 092007

FIG. 22 (color online). The $\gamma\gamma^* \to \pi^0$ transition form factor multiplied by Q^2. The dashed line indicates the asymptotic limit for the form factor. The dotted curve shows the interpolation given by Eq. (9).

B. Aubert et al. (BABAR Collaboration), PRD 80 (2009) 052002

• *Pseudoscalar transition form factors*

@ low-momentum transfer:

slope

$$F_{P\gamma^*\gamma}(Q^2) = F_{P\gamma\gamma}(0)\left(1 - b_P\frac{Q^2}{m_P^2} + c_P\frac{Q^4}{m_P^4} + \cdots\right)$$

curvature

$$|F_{P\gamma\gamma}(0)|^2 = \frac{64\pi}{(4\pi\alpha)^2}\frac{\Gamma(P\to\gamma\gamma)}{m_P^3}$$

exp. decay width

or

$$F_{\pi^0\gamma\gamma}(0) = 1/(4\pi^2 F_\pi)$$

axial anomaly

(not for η and η')

@ large-momentum transfer:

$$F(Q^2) = \int T_H(x,Q^2)\Phi_P(x,\mu_F)\mathrm{d}x$$

$T_H(\gamma^*\gamma \to q\bar{q})$ $\quad$ $\Phi_P(q\bar{q}\to P)$

convolution of perturbative and non-perturbative regimes

@ lowest order in pQCD

$$Q^2F(Q^2) = \frac{\sqrt{2}f_\pi}{3}\int_0^1\frac{dx}{x}\phi_\pi(x,Q^2) + O(\alpha_s) + O\left(\frac{\Lambda^2_{\rm QCD}}{Q^2}\right),$$

$$Q^2F(Q^2) = \sqrt{2}f_\pi$$

- *Padé approximants*

$$Q^2 F_{\eta^{(\prime)}\gamma * \gamma}(Q^2, 0) = a_0 Q^2 + a_1 Q^4 + a_2 Q^6 + \dots$$

$$P^N_M(Q^2) = \frac{T_N(Q^2)}{R_M(Q^2)} = a_0 Q^2 + a_1 Q^4 + a_2 + Q^6 + \dots + \mathcal{O}((Q^2)^{N+M+1})$$

→ simple, systematic and model-independent parametrization of experimental data in the whole energy range (better convergence)

Fitting method: use of different sequences of PAs

- *How many sequences?*
  depends on the analytic structure of the exact function
- *How many elements per sequence?*
  limited by exp. data points and statistical errors

• *Padé approximants*

P. Masjuan, S. Peris and J.J. Sanz-Cillero, PRD 78 (2008) 074028

P. Masjuan, PRD 86 (2012) 094021

How to ascribe a systematic error to the results?

test the method with a model → try different models

• *Log model:* $$F_{\pi^0\gamma^*\gamma}(Q^2) = \frac{M^2}{4\pi^2 f_\pi Q^2}\log\left(1+\frac{Q^2}{M^2}\right),$$

TABLE I. a_0, a_1, and a_2 low-energy coefficients of the log model in Eq. (3), fitted with a $P_1^L(Q^2)$ and its exact values (last column). We also include the prediction for the pole of each $P_1^L(Q^2)$ (s_p) to be compared with the lowest-lying meson in the model.

		P_1^0	P_1^1	P_1^2	P_1^3	P_1^4	P_1^5	$F_{\pi^0\gamma^*\gamma}$ (exact)	
	a_0 (GeV^{-1})	0.2556	0.2694	0.2734	0.2746	0.2751	0.2752	0.2753	
slope	a_1 (GeV^{-3})	0.1290	0.1716	0.1935	0.2051	0.2124	0.2166	0.2294	5.6% of sys. error
curvature	a_2 (GeV^{-5})	0.0651	0.1147	0.1492	0.1725	0.1898	0.2013	0.2549	21% of sys. error
	$\sqrt{s_p}$ (GeV)	1.41	1.22	1.14	1.09	1.05	1.03	0.77	

• *Regge model:*

$$F_{\pi^0\gamma^*\gamma^*}(q_1^2, q_2^2) = \sum_{V_\rho, V_\omega} \frac{F_{V_\rho}(q_1^2)F_{V_\omega}(q_2^2)G_{\pi V_\rho V_\omega}(q_1^2, q_2^2)}{(q_1^2 - M_{V_\rho}^2)(q_2^2 - M_{V_\omega}^2)} + (q_1 \leftrightarrow q_2),$$

slope	a_1 (GeV^{-3})	0.2662	0.3121	0.3338	0.3457	0.3529	0.3571	0.3678	2.9% of sys. error
curvature	a_2 (GeV^{-5})	0.2652	0.3600	0.4244	0.4616	0.4868	0.5030	0.5550	9.4% of sys. error

- *Application to η and η' TFFs*

asymptotic behaviour

To use the P[N,1](Q^2) and P[N,N](Q^2) sequences of PAs

single resonance dominance

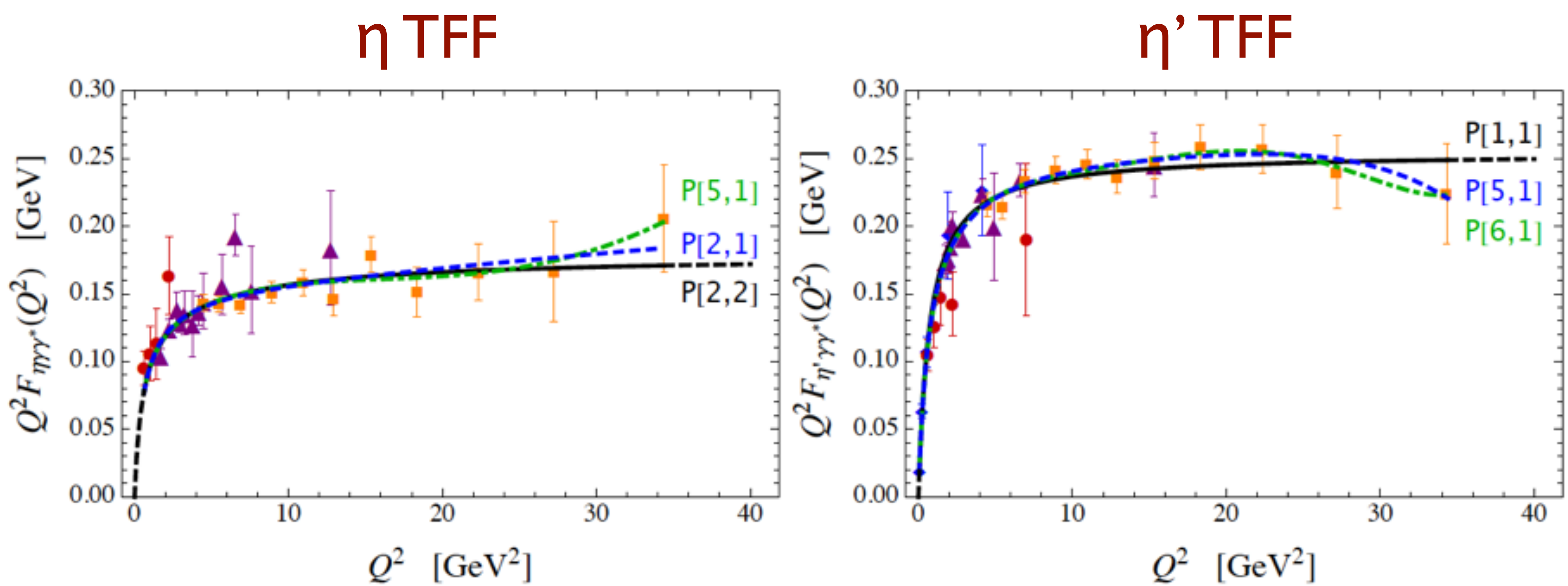


FIG. 1. η- and η'-TFFs best fits (left and right panels reps.). Blue dashed line shows our best $P_1^L(Q^2)$ when the two-photon partial decay width is *not* included in our set of data to be fitted. When the two-photon partial decay width *is* included, dark-green dot-dashed line shows our best $P_1^L(Q^2)$, and black solid line shows our best $P_N^N(Q^2)$. Black dashed lines are the extrapolation of such approximant at $Q^2 = 0$ and at $Q^2 \to \infty$. Data points are from CELLO (red circles) [28], CLEO (purple triangles) [36], L3 (blue diamonds) [31], and *BABAR* (orange squares) [30] Collaborations. See main text for details.

• *Application to η and η' TFFs*

Slope:

η TFF

η' TFF

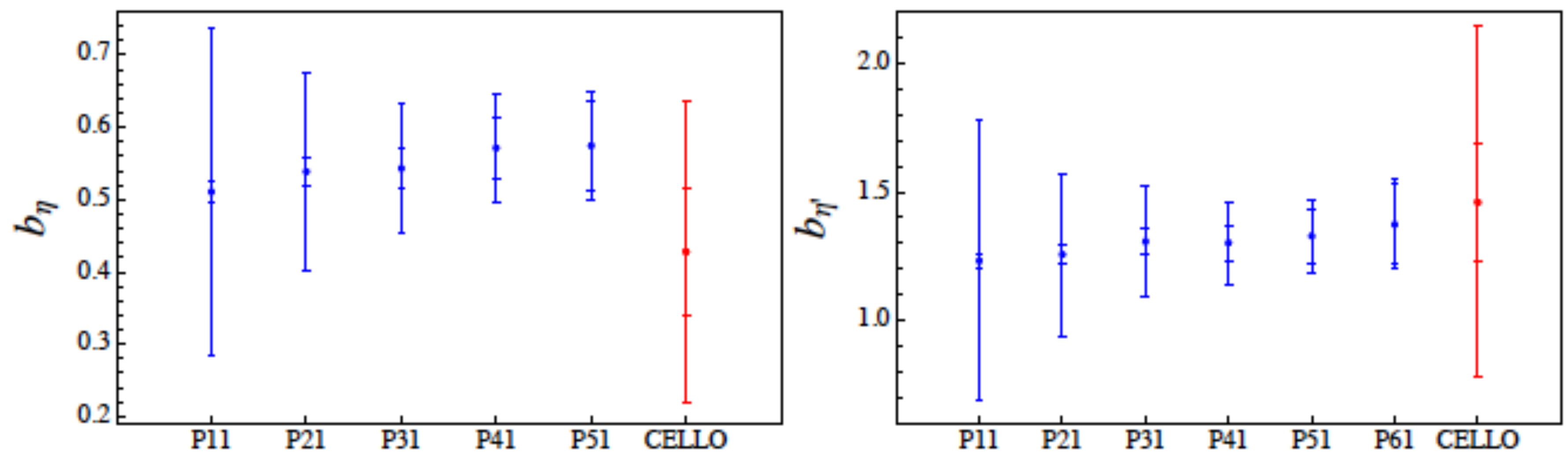


FIG. 2. Slope predictions with the $P_1^L(Q^2)$ up to $L = 5$ and $L = 6$ for the η-TFF and the η'-TFF (left and right panels respectively). The internal band is the statistical error from the fit and the external one is the combination of statistical and systematic errors determined in the previous section.

Curvature:

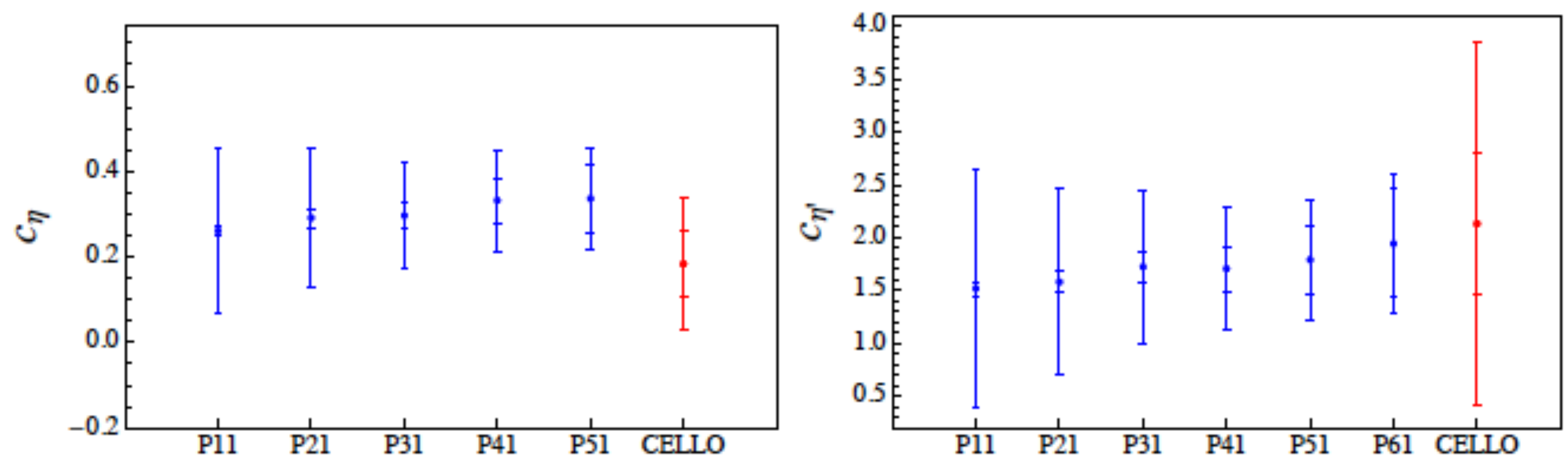


FIG. 3. Curvature predictions with the $P_1^L(Q^2)$ up to $L = 5$ and $L = 6$ for the η-TFF and the η'-TFF (left and right panels respectively). The internal band is the statistical error from the fit and the external one is the combination of statistical and systematic errors determined in the previous section.

- *Results*

Slope and curvature:

$$b_\eta = 0.596(48)_{stat}(33)_{sys}$$
$$c_\eta = 0.362(66)_{stat}(76)_{sys} \times 10^{-3}$$

$$b_{\eta'} = 1.37(16)_{stat}(8)_{sys}$$
$$c_{\eta'} = 1.94(52)_{stat}(41)_{sys} \times 10^{-3}$$

Comparison with other results:

$$F_{P\gamma^*\gamma}(Q^2) = \frac{F_{P\gamma\gamma}(0)}{1+Q^2/\Lambda_P^2}$$

ChPT: b_η=0.51, $b_{\eta'}$=1.47

VMD: b_η=0.53, $b_{\eta'}$=1.33

cQL: b_η=0.51, $b_{\eta'}$=1.30

BL: b_η=0.36, $b_{\eta'}$=2.11

$$\mathcal{F}_{\gamma^*\gamma\mathcal{R}}(Q^2) \sim \frac{1}{4\pi^2 f_\mathcal{R}} \frac{1}{1+(Q^2/8\pi^2 f_\mathcal{R}^2)}$$

CELLO: b_η=0.428(89), $b_{\eta'}$=1.46(23)

CLEO: b_η=0.501(38), $b_{\eta'}$=1.24(8)

Lepton-G: b_η=0.57(12), $b_{\eta'}$=1.6(4)

NA60: b_η=0.585(51)

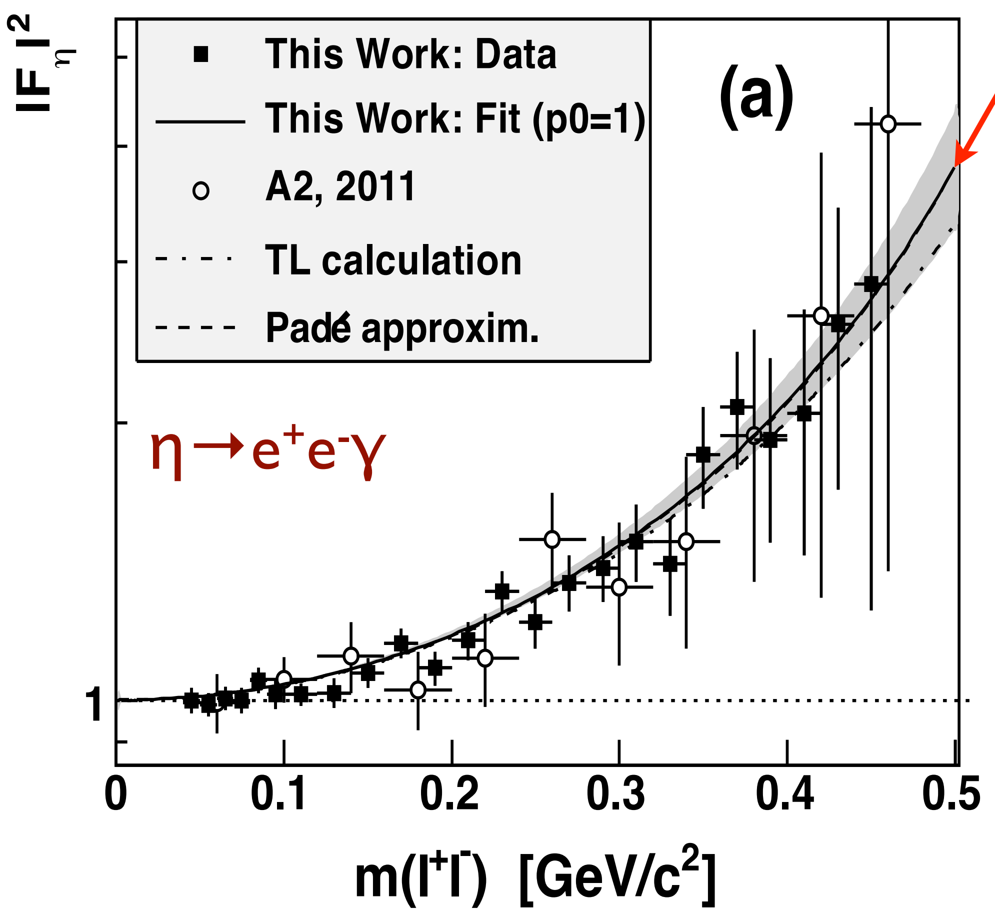
MAMI: b_η=0.58(11), WASA: b_η=0.68(26)

Disp: b_η=0.61(+0.07)(-0.03), $b_{\eta'}$=1.45(+0.17)(-0.12)

$\eta,\eta' \to \gamma^*\gamma$

- *Further applications of this method*

Analysis of time-like processes ($\eta,\eta' \to l^+l^-\gamma$)



Our *prediction* is *behind* the experimental fit!

PREVIOUS RESULTS
$b_\eta = 0.596(48)(33)$,
$c_\eta = 0.362(66)(76)$
Asymptotics $= 0.164(21)$ GeV

Adding MAMI data to our fit

UPDATED RESULTS
(PRELIMINARY RESULTS)
$b_\eta = 0.588(27)(25)$,
$c_\eta = 0.357(38)(61)$
Asymptotics $= 0.174(15)$ GeV

M. Unverzagt et al. (A2 Coll. @MAMI), PRC 89 (2014) 044608

Analysis of π^0, η and η' contributions to HLbL of $(g-2)_\mu$

• *Application to η TFF including time-like data*

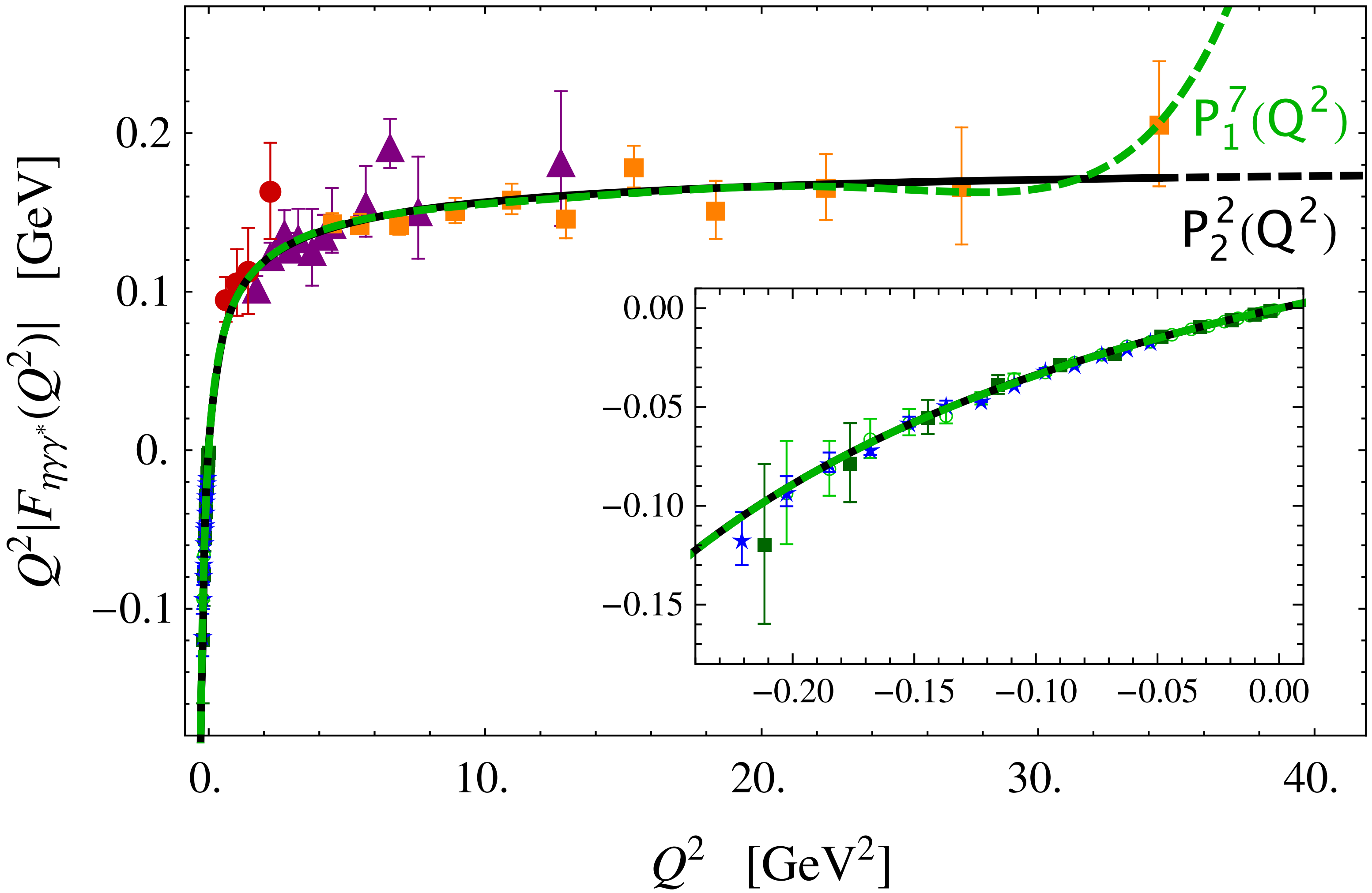
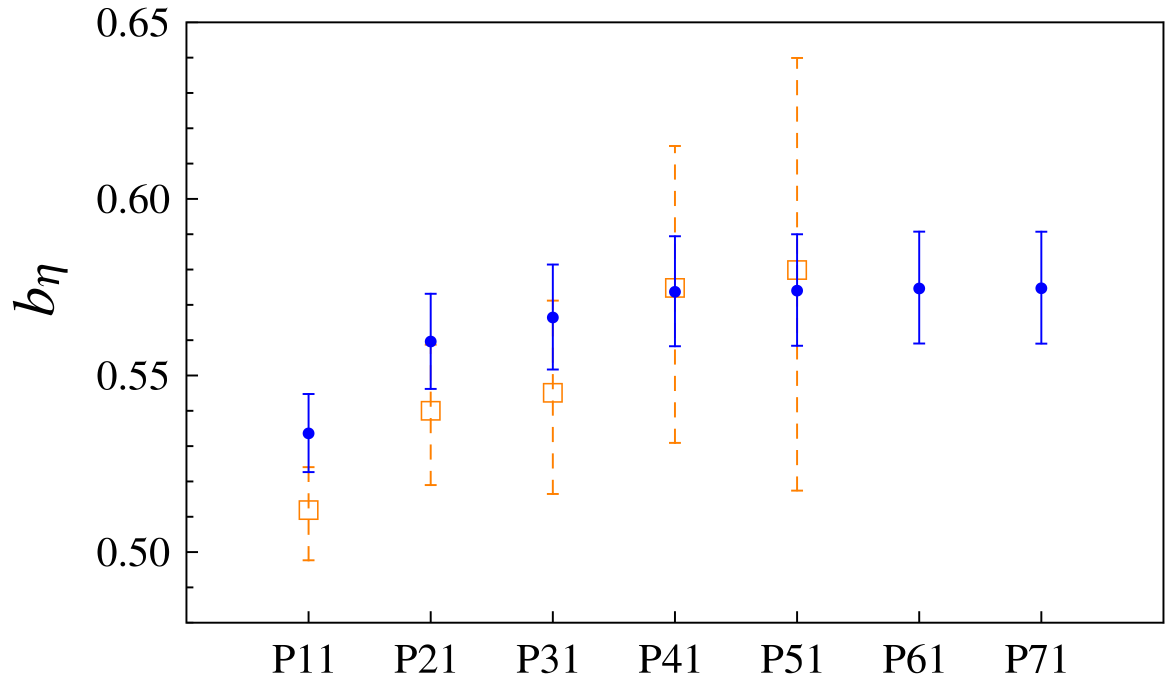


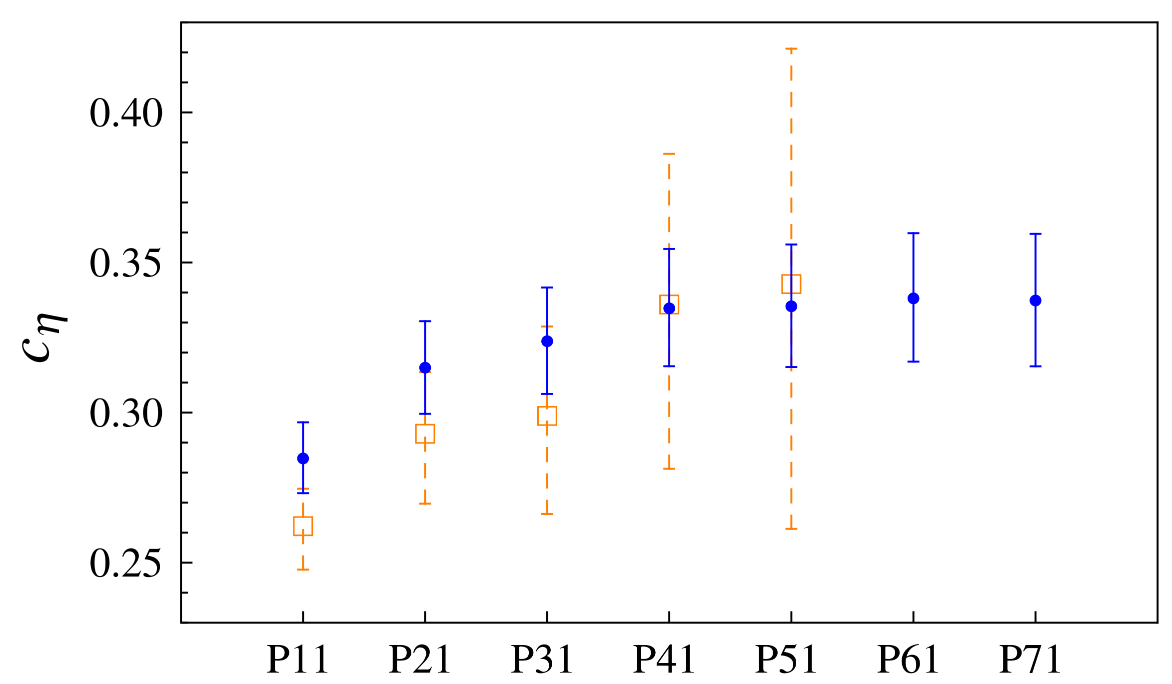
Fig. 1 η-TFF best fits. *Green-dashed line* shows our best $P_1^L(Q^2)$ fit and *black line* our best $P_N^N(Q^2)$ fit. Experimental data points in the space-like region are from CELLO (*red circles*) [9], CLEO (*purple triangles*) [10], and *BABAR* (*orange squares*) [37] Collaborations. Experimental data points in the time-like region are from NA60 (*blue stars*) [11], A2 2011 (*dark-green squares*) [12], and A2 2013 (*empty-green circles*) [14]. The inner plot shows a zoom into the time-like region

• *Application to η TFF including time-like data*

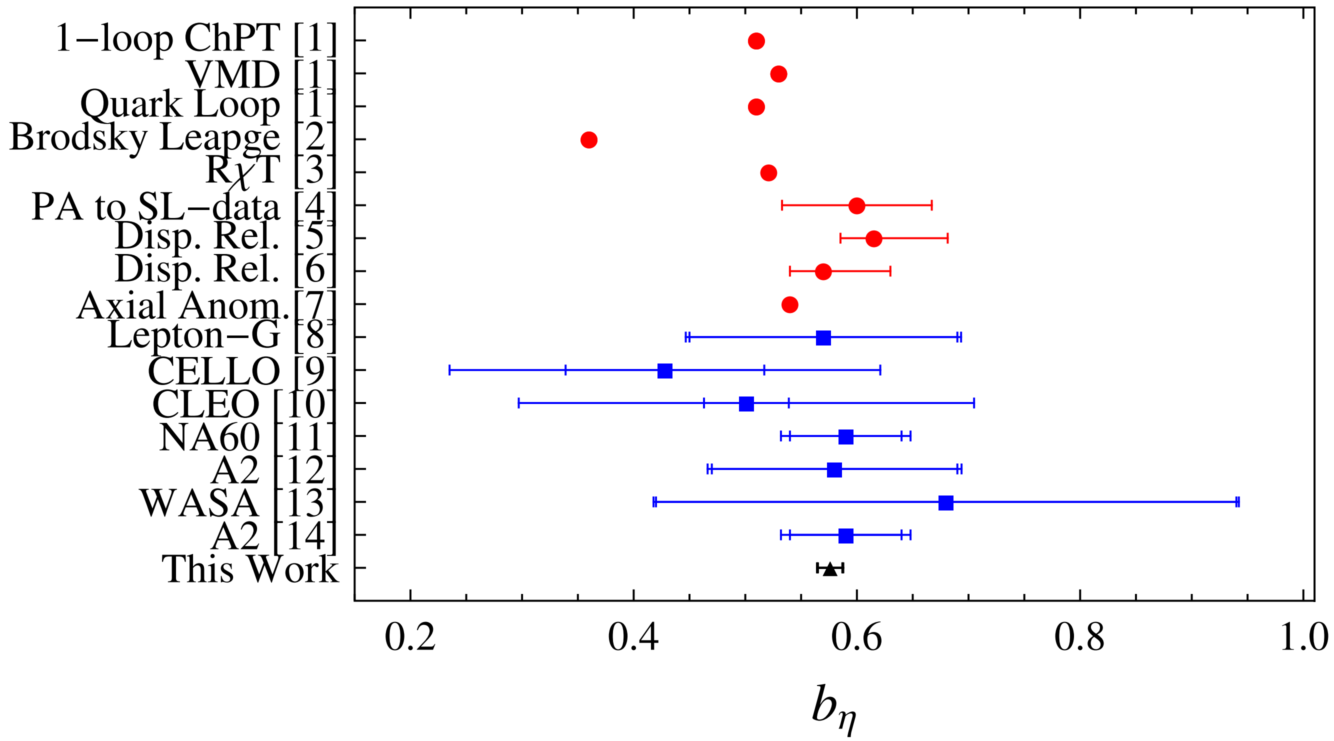
Slope:



Curvature:



Curvature comparison:



Final results:

$$b_\eta = 0.576(11)_{\rm stat}(4)_{\rm sys}$$

$$c_\eta = 0.339(15)_{\rm stat}(5)_{\rm sys}$$

$$d_\eta = 0.200(14)_{\rm stat}(18)_{\rm sys}$$

• Application to η TFF including time-like data

Table 3 Role of the different sets of experimental data in determining slope and asymptotic values of the η TFF

	Data range	$P_1^L(Q^2)$		$P_N^N(Q^2)$		
	(GeV^2)	L	b_η	N	b_η	$\lim_{Q^2\to\infty} Q^2 F_{\eta\gamma^*\gamma}(Q^2)$
CELLO [9]	0.62 to 2.23	2	0.48 (20)	1	0.427 (66)	0.193 (30)
CLEO [10]	1.73 to 12.74	3	0.73 (12)	1	0.522 (19)	0.157 (5)
BABAR [37]	4.47 to 34.38	4	0.53 (9)	1	0.509 (14)	0.162 (3)
CELLO+CLEO [9,10]	0.62 to 12.74	3	0.65 (9)	2	0.704 (87)	0.25 (10)
SL	0.62 to 34.38	5	0.58 (6)	2	0.66 (10)	0.161 (24)
A2-11+A2-13 [12,14]	−0.212 to −0.002	2	0.475 (76)	1	0.551 (40)	0.149 (11)
NA60 [11]	−0.221 to −0.053	3	0.640 (77)	1	0.582 (19)	0.141 (5)
TL	−0.221 to −0.002	3	0.565 (87)	1	0.576 (17)	0.143 (5)
CELLO [9]+TL	−0.221 to 2.23	5	0.531 (39)	2	0.533 (30)	0.203 (58)
CELLO+CLEO [9,10]+TL	−0.221 to 12.74	6	0.567 (22)	1	0.550 (13)	0.152 (3)
A2-11+A2-13 [12,14]+SL	−0.212 to 34.38	7	0.561 (35)	2	0.569 (28)	0.178 (16)
TL+SL	**−0.221 to 34.38**	**7**	**0.575 (16)**	**2**	**0.576 (15)**	**0.177 (15)**

SL refers the space-like data set, i.e., data from CELLO+CLEO+*BABAR* [9,10,37] Collaborations, and *TL* refers to the time-like data set, i.e., data from NA60+A2-11+A2-13 [11,12,14] Collaborations. Bold numbers are our final result. No systematic errors included

- *Results*

η,η'→γγ decay widths (TFFs @ Q^2=0, *space-like data only*):

$$\Gamma^{pred}_{\eta\to\gamma\gamma} = (0.41 \pm 0.18) keV \qquad \Gamma^{pred}_{\eta'\to\gamma\gamma} = (4.21 \pm 0.43) keV$$

$$\Gamma^{PDG}_{\eta\to\gamma\gamma} = (0.51 \pm 0.03) keV \qquad \Gamma^{PDG}_{\eta'\to\gamma\gamma} = (4.34 \pm 0.14) keV$$

Asymptotic values (TFFs @ $Q^2\to\infty$):

disagrees with BABAR @112 GeV (time-like)

$$\lim_{Q^2\to\infty} Q^2 F_{\eta\gamma^*\gamma}(Q^2) = 0.177^{+0.020}_{-0.009}\ \mathrm{GeV}$$

agrees with BABAR @112 GeV (time-like)

$$\lim_{Q^2\to\infty} Q^2 F_{\eta'\gamma^*\gamma}(Q^2) = 0.254(4)\ \mathrm{GeV}$$

→ determination of η-η' mixing parameters

- *Impact on η-η' mixing parameters*

Quark-flavour basis:

large-N$_c$ limit: $\phi_q = \phi_s \equiv \phi$

$$\begin{pmatrix} F_\eta^q & F_\eta^s \\ F_{\eta'}^q & F_{\eta'}^s \end{pmatrix} = \begin{pmatrix} F_q \cos\phi_q & -F_s \sin\phi_s \\ F_q \sin\phi_s & F_s \cos\phi_s \end{pmatrix}$$

pseudoscalar decay constants

Decay widths:

$$F_{\eta\gamma\gamma}(0) = \frac{1}{4\pi^2}\left(\frac{\hat{c}_q F_{\eta'}^s - \hat{c}_s F_{\eta'}^q}{F_{\eta'}^s F_\eta^q - F_{\eta'}^q F_\eta^s}\right) = \frac{1}{4\pi^2}\left(\frac{\hat{c}_q}{F_q}\cos\phi - \frac{\hat{c}_s}{F_s}\sin\phi\right)$$

$$F_{\eta'\gamma\gamma}(0) = \frac{1}{4\pi^2}\left(\frac{\hat{c}_q F_\eta^s - \hat{c}_s F_\eta^q}{F_\eta^s F_{\eta'}^q - F_\eta^q F_{\eta'}^s}\right) = \frac{1}{4\pi^2}\left(\frac{\hat{c}_q}{F_q}\sin\phi + \frac{\hat{c}_s}{F_s}\cos\phi\right)$$

Asymptotic expressions:

$$\lim_{Q^2\to\infty} Q^2 F_{\eta\gamma^*\gamma}(Q^2) = 2(\hat{c}_q F_\eta^q + \hat{c}_s F_\eta^s) = 2(\hat{c}_q F_q \cos\phi - \hat{c}_s F_s \sin\phi)$$

$$\lim_{Q^2\to\infty} Q^2 F_{\eta'\gamma^*\gamma}(Q^2) = 2(\hat{c}_q F_{\eta'}^q + \hat{c}_s F_{\eta'}^s) = 2(\hat{c}_q F_q \sin\phi + \hat{c}_s F_s \cos\phi)$$

- *Impact on η-η' mixing parameters*

Results:

$$\textbf{inputs} : F_{\eta\gamma\gamma}(0),\ F_{\eta'\gamma\gamma}(0),\ \text{asymp}\ \eta$$
$$\Rightarrow\ F_q/F_\pi = 1.07(2),\ F_s/F_\pi = 1.29(16),\ \phi = 38.3(1.6)^\circ$$

$$\textbf{inputs} : F_{\eta\gamma\gamma}(0),\ F_{\eta'\gamma\gamma}(0),\ \text{asymp}\ \eta'$$
$$\Rightarrow\ F_q/F_\pi = 1.06(1),\ F_s/F_\pi = 1.63(8),\ \phi = 41.1(0.8)^\circ$$

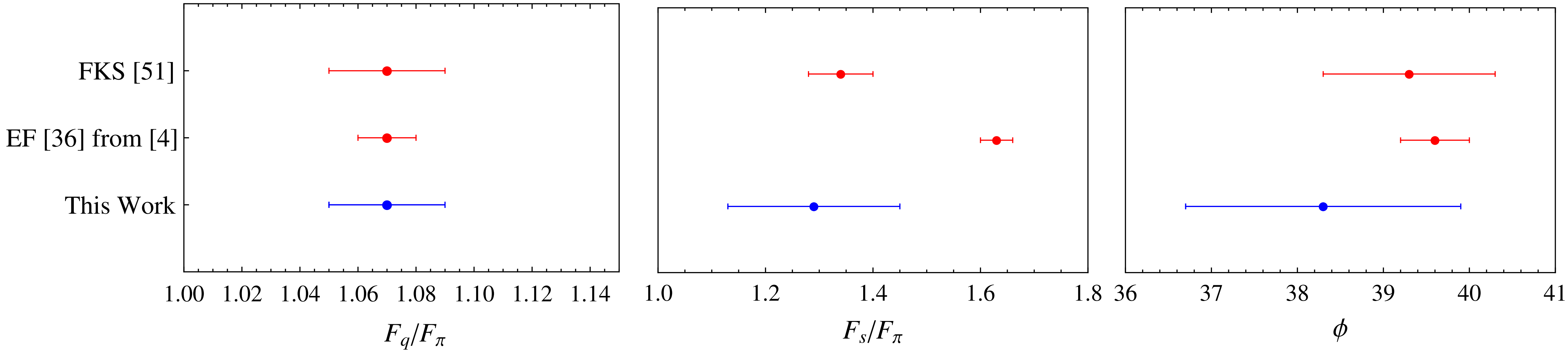


Fig. 5 Mixing parameters of the η–η' system in the flavor basis from different references

- *Determination of the VPγ couplings*

$$\Gamma(V \to e^+e^-) = \frac{4\pi}{3}\alpha^2 \frac{f_V^2}{m_V} c_V^2 \qquad \Gamma(P \to V\gamma) = \frac{\alpha}{8} g_{VP\gamma}^2 \left(\frac{m_P^2 - m_V^2}{m_P}\right)^3$$

$$F_{VP\gamma}(0,0) = \frac{f_V}{m_V} g_{VP\gamma}$$

$$g_{\rho\eta\gamma} = \frac{3m_\rho}{4\pi^2 f_{\rho^0}} \frac{\cos\phi}{\sqrt{2}F_q}, \quad g_{\rho\eta'\gamma} = \frac{3m_\rho}{4\pi^2 f_{\rho^0}} \frac{\sin\phi}{\sqrt{2}F_q},$$

$$g_{\omega\eta\gamma} = \frac{m_\omega}{4\pi^2 f_\omega}\left(\cos\phi_V \frac{\cos\phi}{\sqrt{2}F_q} - 2\sin\phi_V \frac{\sin\phi}{\sqrt{2}F_s}\right),$$

$$g_{\omega\eta'\gamma} = \frac{m_\omega}{4\pi^2 f_\omega}\left(\cos\phi_V \frac{\sin\phi}{\sqrt{2}F_q} + 2\sin\phi_V \frac{\cos\phi}{\sqrt{2}F_s}\right),$$

$$g_{\phi\eta\gamma} = -\frac{m_\phi}{4\pi^2 f_\phi}\left(\sin\phi_V \frac{\cos\phi}{\sqrt{2}F_q} + 2\cos\phi_V \frac{\sin\phi}{\sqrt{2}F_s}\right),$$

$$g_{\phi\eta'\gamma} = -\frac{m_\phi}{4\pi^2 f_\phi}\left(\sin\phi_V \frac{\sin\phi}{\sqrt{2}F_q} - 2\cos\phi_V \frac{\cos\phi}{\sqrt{2}F_s}\right).$$

- *Determination of the VPγ couplings*

Table 4 Summary of VPγ couplings

	Prediction	Experiment
$g_{\rho\eta\gamma}$	1.50 (4)	1.58 (5)
$g_{\rho\eta'\gamma}$	1.18 (5)	1.32 (3)
$g_{\omega\eta\gamma}$	0.57 (2)	0.45 (2)
$g_{\omega\eta'\gamma}$	0.55 (2)	0.43 (2)
$g_{\phi\eta\gamma}$	−0.83 (11)	−0.69 (1)
$g_{\phi\eta'\gamma}$	0.98 (14)	0.72 (1)
$R_{J/\Psi} = \frac{\Gamma(J/\Psi \to \eta'\gamma)}{\Gamma(J/\Psi \to \eta\gamma)}$	4.74 (55)	4.67 (20)

Experimental determinations are from Ref. [42]

$$R_{J/\Psi} = \tan^2(\phi_q)\left(\frac{m_{\eta'}}{m_\eta}\right)^4\left(\frac{M_{J/\Psi}^2 - m_{\eta'}^2}{M_{J/\Psi}^2 - m_\eta^2}\right)^3$$

- *Summary and Conclusions*

We have analyzed the experimental data on the η and η' TFF at low and intermediate energies with a model independent approach based on Padé approximants (extending the analysis for the π^0-TFF) P. Masjuan, PRD 86 (2012) 094021

We have obtained accurate values of the corresponding slope and curvature parameters as well as the values of the TFFs at zero and infinity

We have quantified the impact of these results on the η and η' mixing parameters and the VPγ couplings

We have quantified the effect of including time-like data in the η TFF, thus enhancing the space-like based results

More experimental data would be desirable (BESIII, BELLE?, KLOE, WASA) to further improve this method