Exclusive hadronic cross sections from BABAR and implications for the muon g-2

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- the muon g-2
- the BABAR ISR program
- results from BaBar
  $\pi^+\pi^-(\gamma)$, $K^+K^-(\gamma)$, $\pi^+\pi^-\pi^+\pi^-$, $K^+K^-\pi\pi$, $K_S\,K_L$,
  $K_SK_L\pi^+\pi^-$, $K_SK_S\pi^+\pi^-$
- BABAR data impact on the g-2 prediction

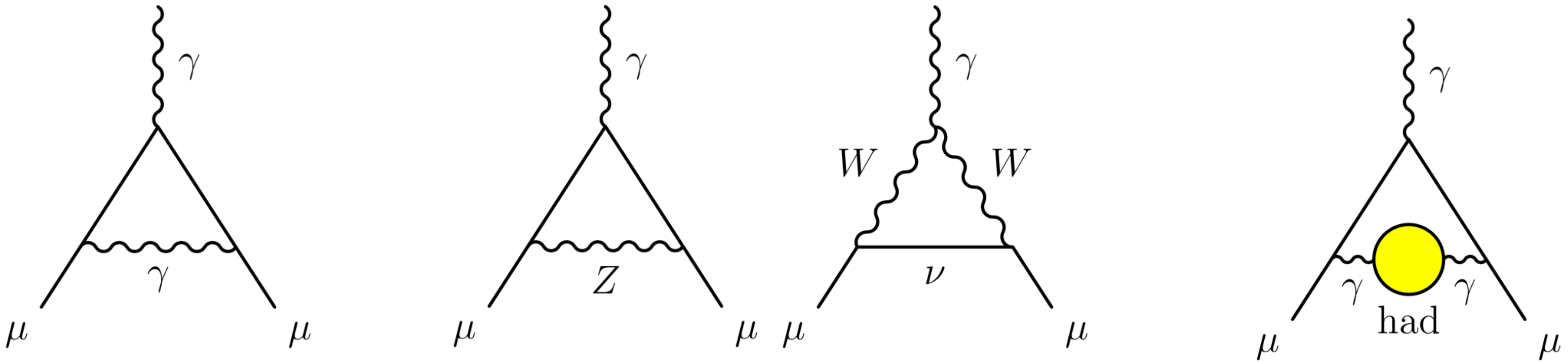
Institute of High Energy Physics
Chinese Academy of Sciences

Muon anomalous magnetic moment a_μ

- Dirac predicts muon magnetic moment: $\vec{M} = g_\mu \frac{e}{2m_\mu}\vec{S}$ with $g_\mu = 2$
- Quantum loop effects lead deviation => **muon anomalous magnetic moment:** $a_\mu \equiv \frac{g_\mu - 2}{2}$
  Can be measured: a_μ^{exp}
  Predicted within SM: a_μ^{SM}
- Deviation in a_μ^{exp} from a_μ^{SM} => **new physics beyond SM!**
- a_μ prediction by SM:

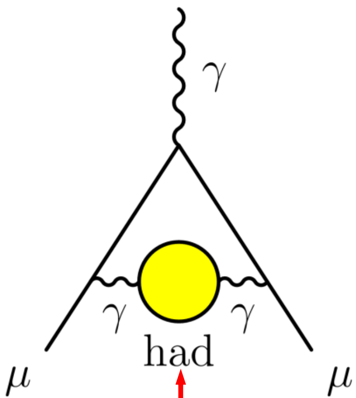
$$a_\mu^{\rm SM} = a_\mu^{\rm QED} + a_\mu^{\rm EW} + a_\mu^{\rm Had}$$

Main uncertainty to a_μ^{SM}



LO(Leading Order) QED LO weak LO hadronic

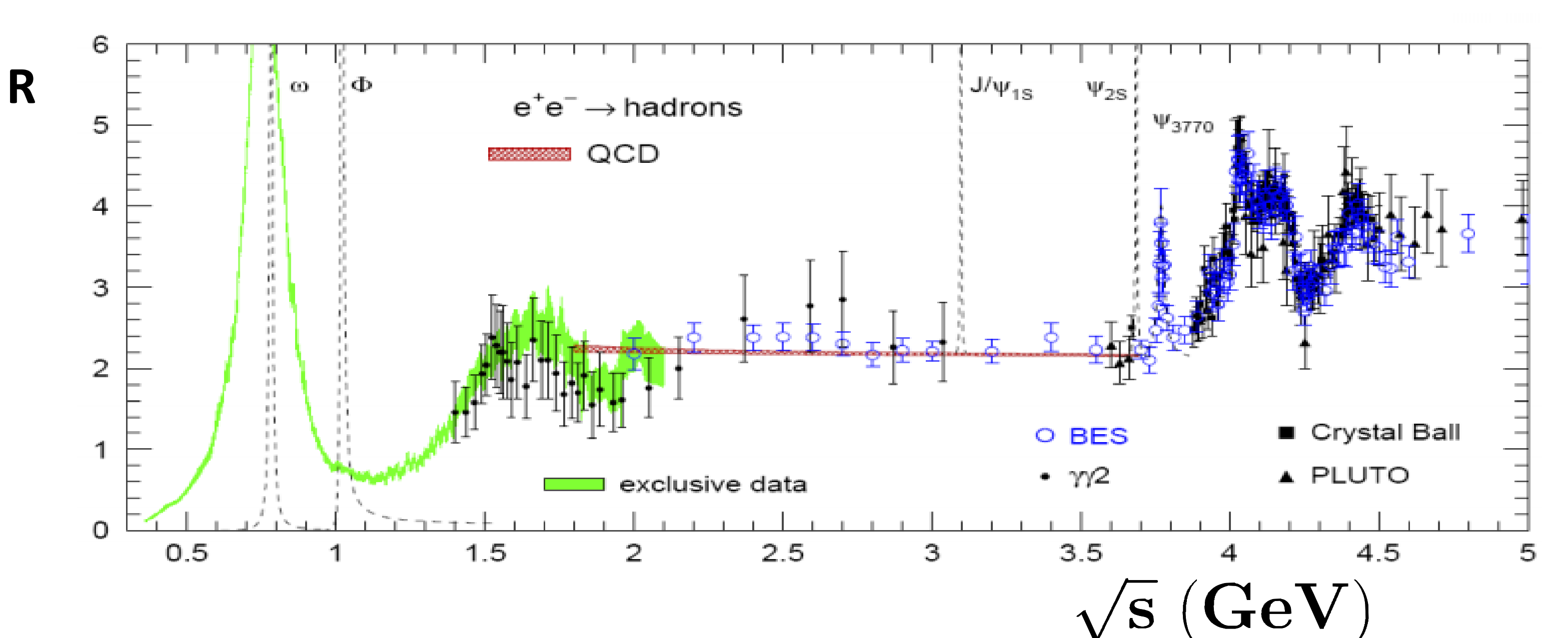
Hadronic Vacuum Polarization and R



Hadronic Vacuum Polarization

Cannot be calculated from QCD (low mass scale), but one can use experimental data on e+e−→hadrons cross section (Bouchiat-Michel 1961 ***J. Phys. Radium 22, 121***)

$$12\pi \,\mathrm{Im}\Pi_{\gamma}(s) = \frac{\sigma^{0}\,[e^{+}e^{-} \rightarrow \mathrm{hadrons}(\gamma)]}{\sigma_{pt}[e^{+}e^{-} \rightarrow \mu^{+}\mu^{-}]} \equiv R(s)$$

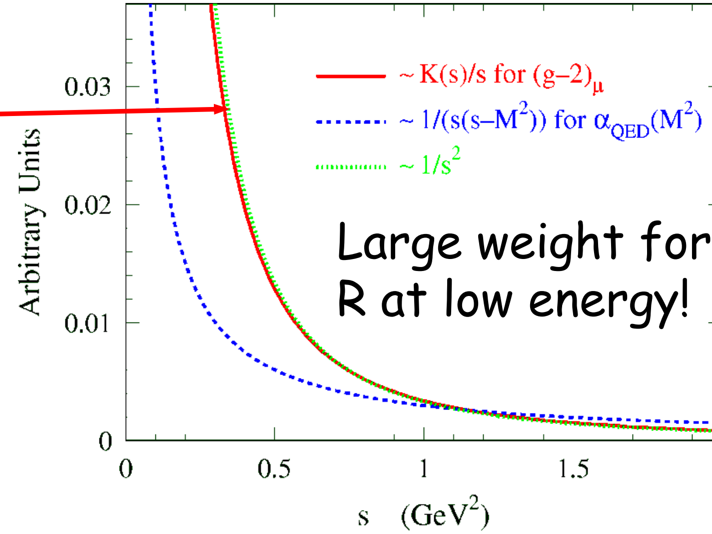


$a_\mu^{had,LO}$ calculation with experimental data

K(s): QED kernel function

$$a_\mu^{had} = \frac{\alpha^2}{3\pi^2}\int_{4m_\pi^2}^{\infty} ds \, \frac{K(s)}{s} \, R(s)$$

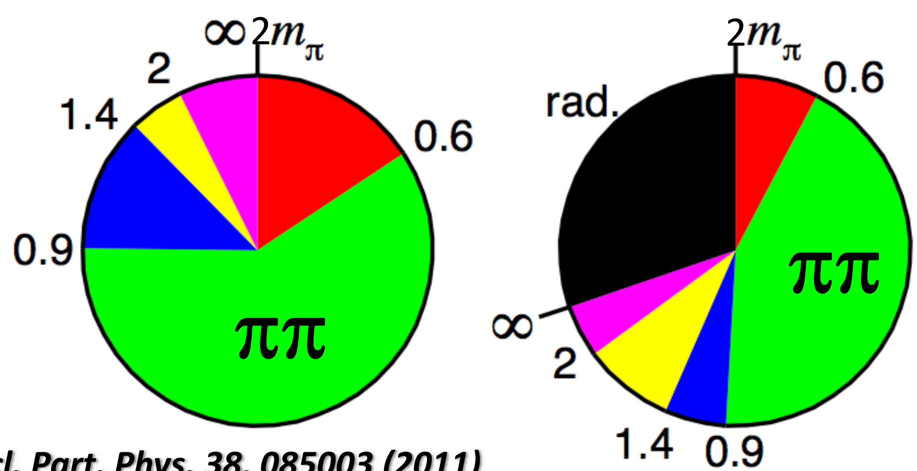
Dispersion relation



Large weight for R at low energy!

value (error)^2

$a_\mu^{had,LO\ VP}$



HLMNT 11

J. Phys. G: Nucl. Part. Phys. 38, 085003 (2011)

➔ Energy region 0.6-0.9 GeV dominates

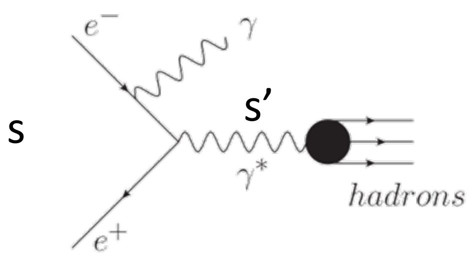
➔ 2π channel contributes more than 70%

Methods for σ(ee→hadrons) measurement at different energies

➤ **Scan method:** taking ee data at different energies

Experiment (detector)	collider	location
OLYA, ND, SND, CMD, CMD-2	VEPP-2M	Novosibirsk, Russia
M3N, DM1, DM2	DCI	Orsay, France
GG2	Adone	Frascati, Italy
CLEO	CESR	Cornell University, USA
BESIII (R scan)	BEPC-II	IHEP, Beijing, China

➤ **ISR method:** taking ee data at a fixed energy but with very high luminosity to be able to extract σ(ee→hadrons) trough ISR process ee→γγ*→γ+hadrons



$$\frac{d^2\sigma_{e^+e^-\to \text{hadrons}\,\gamma_{ISR}}}{d\sqrt{s'}\,d\cos\theta^*_\gamma} = \frac{2\sqrt{s'}}{s} W(s, s', \theta^*_\gamma)\,\sigma_{e^+e^-\to \text{hadrons}}(\sqrt{s'})$$

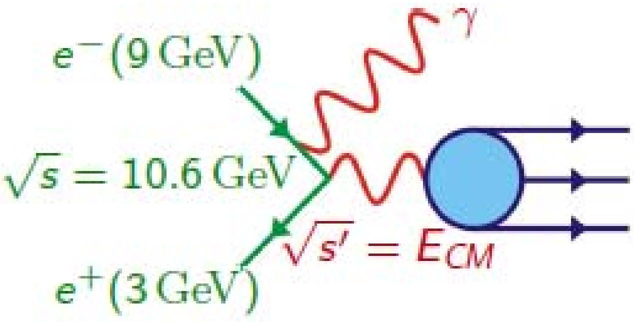
$\sqrt{s'}$: Effective energy in ISR process ($m_{hadrons}$)

$W(s, s', \theta^*_\gamma)$: radiator function

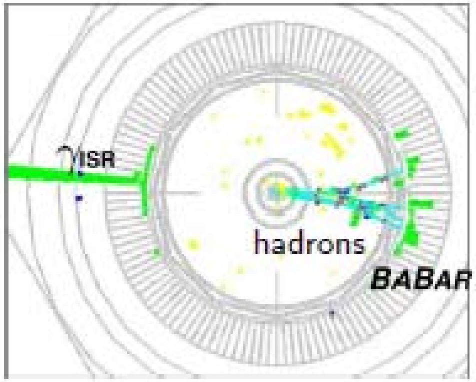
$\sigma_{e^+e^-\to \text{hadrons}}(\sqrt{s'})$: attempt to measure

Experiment (detector)	collider	location
KLOE	DAΦNE (@ϕ)	Frascati, Italy
BaBar	**PEP-II (@Υ(4S))**	**SLAC, USA**
BESIII (using high lumi. ψ(3770) data)	BEPC-II	IHEP, Beijing, China

ISR study at BaBar



- High luminosity
- Large s allows
  high energy ISR photon



- High energy (E^*_γ >3 GeV) photon detected at large angle (LA)
  → defines $\sqrt{s'} = E_{CM}$ and provides strong background rejection
- Event topology: ISR photon back-to-back to hadrons
  → high acceptance for hadrons, strong boost to hadrons
  (measurements from threshold and easier PID)
- Final state can be hadronic or leptonic (QED)
  → $\mu^+\mu^-\gamma(\gamma)$ events used to get ISR luminosity
- Kinematic fit including ISR photon
  → removes multi-hadronic background; improves mass resolution (a few MeV)
- **Continuous measurement from threshold to 3-5 GeV**
  →reduces systematic uncertainties compared to multiple data sets with different colliders and detectors

The BaBar ISR program

- almost complete set of exclusive hadronic e^+e^- annihilation channels up to 2 GeV

- published:

$\pi^+\pi^-$	PRL 2009; PRD 2012
$K^+ K^-$	PRD 2013
$\pi^+\pi^-\pi^0$	PRD 2004
$K^0_S K^{+-} \pi^{-+}$, $K^+ K^- \pi^0$, $K^+ K^- \eta$	PRD 2005; PRD 2008
$2(\pi^+ \pi^-)$, $K^+ K^- \pi^+ \pi^-$, $K^+ K^- 2\pi^0$, $2(K^+ K^-)$	PRD 2007; PRD 2012; PRD 2012
$2(\pi^+ \pi^-) \pi^0$, $2(\pi^+ \pi^-) \eta$, $K^+ K^- \pi^+ \pi^- \pi^0$, $K^+ K^- \pi^+ \pi^- \eta$	PRD 2007
$3(\pi^+ \pi^-)$, $2(\pi^+ \pi^- \pi^0)$, $2(\pi^+ \pi^-) K^+ K^-$	PRD 2006
$\Phi\ f^0(980)$	PRD 2006; PRD 2007
$p \bar{p}$	PRD 2006, PRD 2012
$\Lambda \bar{\Lambda}$, $\Lambda \bar{\Sigma}^0$, $\Sigma^0 \bar{\Sigma}^0$	PRD 2007
$K^0_S K^0_L$, $K^0_S K^0_L \pi^+ \pi^-$, $K^0_S K^0_S \pi^+ \pi^-$	PRD 2014

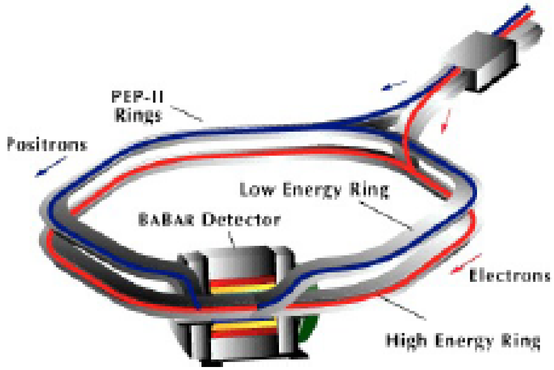
- preliminary: $K^+ K^-$ large Q^2

- in progress: $\pi^+ \pi^- 2\pi^0$, $\pi^+ \pi^- 3\pi^0$, $K^0_S K^{+-} \pi^{-+} \pi^0$, $K^0_S K^{+-} \pi^{-+} \eta$

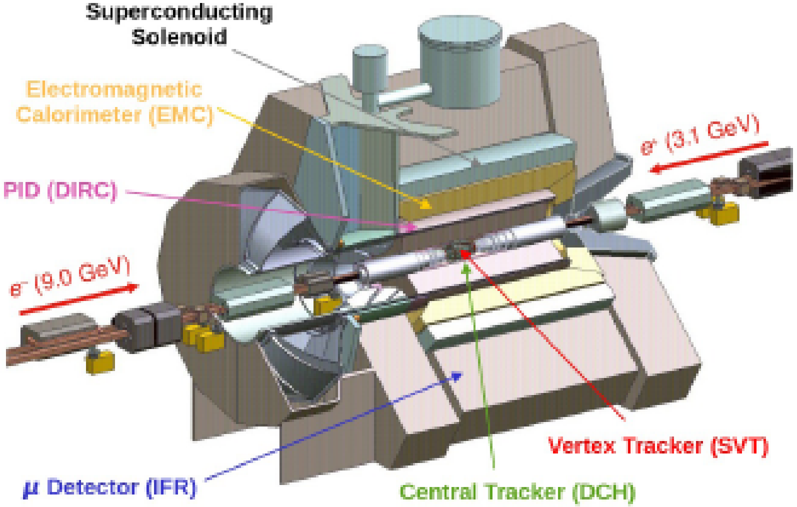
- not covered: $\pi^+ \pi^- 4\pi^0$, $K^0_S K^0_L \pi^0 \pi^0$

PEP-II and the BaBar detector at SLAC

- asymmetric e^+e^--collider:
  9 GeV (e^-) and 3.1 GeV (e^+)
- $\sqrt{s} = 10.58\,\mathrm{GeV} \Rightarrow \Upsilon(4S)$
  $\Rightarrow$ above $B\bar{B}$-threshold



- multi purpose detector
- data taken from 1999 – 2008
- integrated luminosity: 531 fb^{-1}
  on $\Upsilon(4S)$: 454 fb^{-1}
  $\approx 600 \cdot 10^6$ $B\bar{B}$-pairs



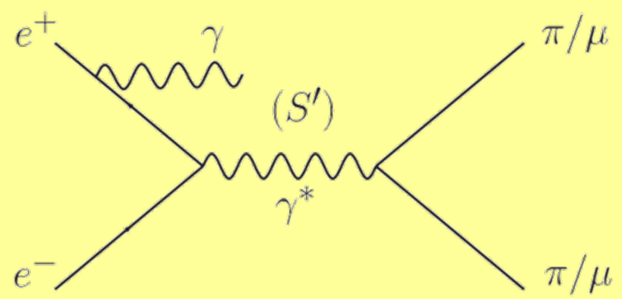
The BaBar ISR method for $\mu\mu\gamma(\gamma)$, $\pi\pi\gamma(\gamma)$, $KK\gamma(\gamma)$

$e^+ e^- \rightarrow \mu^+ \mu^- \gamma\ (\gamma)$ and $\pi^+ \pi^- \gamma\ (\gamma)$, $K^+K^-\ \gamma(\gamma)$ measured simultaneously
Kinematic fits with additional ISR or FSR photon

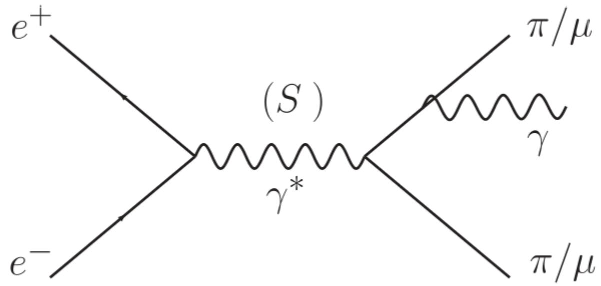
$$x = 2E^*_\gamma/\sqrt{s}$$

$$s' = s(1-x)$$

ISR

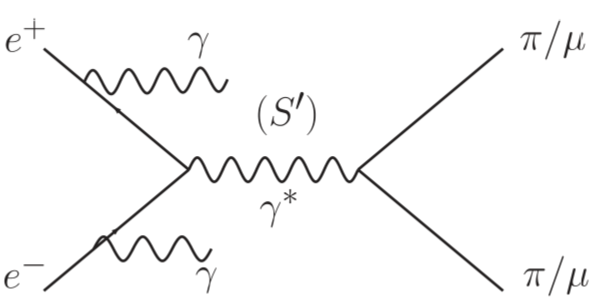


FSR

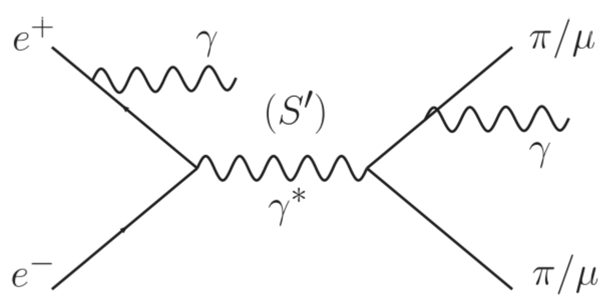


measure ratios
$\pi\pi/\mu\mu$ KK/$\mu\mu$
=>ISR lumi drops out
Systematics cancel

ISR + add. ISR

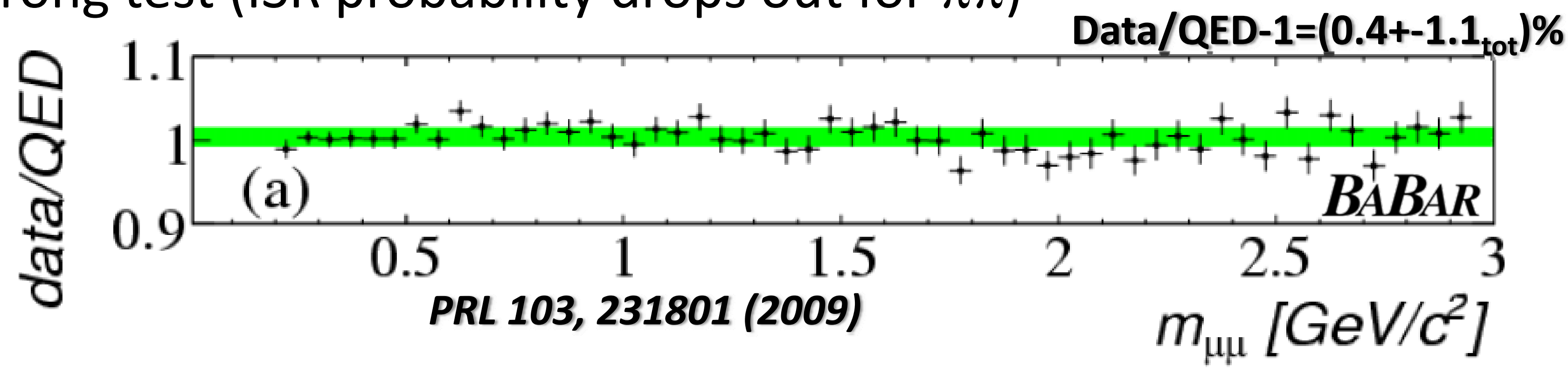


ISR + add. FSR



QED Test with μμγ sample

- absolute comparison of μμ mass spectra in data and in simulation (AfkQed based on EVA)
- simulation corrected for data/MC efficiencies (trigger, tracking, PID)
- AfkQed corrected for incomplete NLO using Phokhara
- strong test (ISR probability drops out for ππ)



$$\frac{\sigma^{data}_{\mu\mu\gamma(\gamma)}}{\sigma^{NLO\ QED}_{\mu\mu\gamma(\gamma)}} = 1 + (4.0 \pm 1.9 \pm 5.5 \pm 9.4)\ 10^{-3}$$

(0.2 – 3 GeV)

BaBar ee luminosity

ISR γ efficiency 3.4 syst.
trigger/tracking/PID 4.0

Cancel in ππ/μμ ratio measurement

LO FSR in $e^+e^- \to \mu^+\mu^-\gamma(\gamma)$ and $\pi^+\pi^-\gamma(\gamma)$

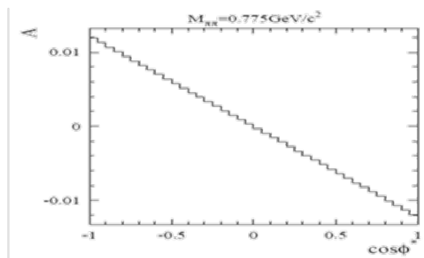
- Should be subtracted
- Theoretical prediction/estimation
  QED for ee→μμγ: reliable
  model dependent estimation for ee→ππγ: very small, big uncertainty
- **Measurement through charge asymmetry** (*talk at Tau Workshop 2014*)

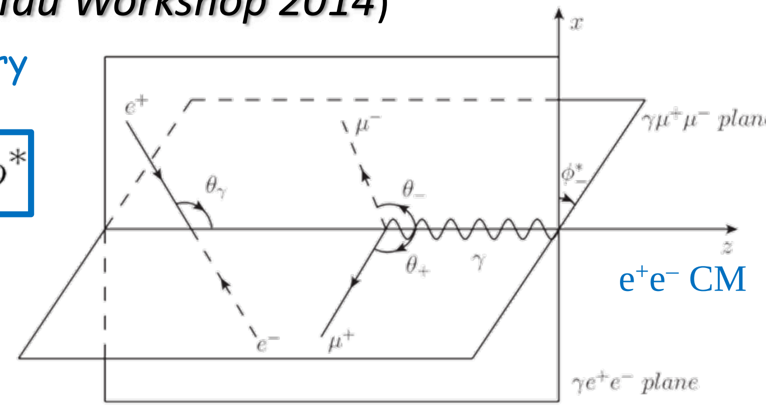
Charge asymmetry:

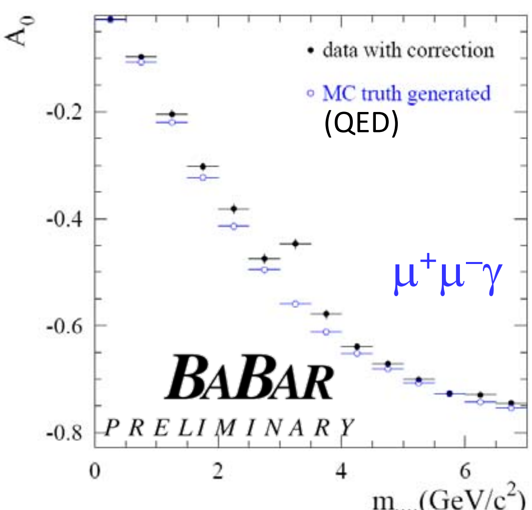
$$A = \frac{|\mathcal{M}|^2 - |\mathcal{M}_{x^+\leftrightarrow x^-}|^2}{|\mathcal{M}|^2 + |\mathcal{M}_{x^+\leftrightarrow x^-}|^2} = \frac{2\mathcal{R}e(\mathcal{M}_{\rm ISR}\mathcal{M}^*_{\rm FSR})}{|\mathcal{M}_{\rm ISR}|^2 + |\mathcal{M}_{\rm FSR}|^2}$$

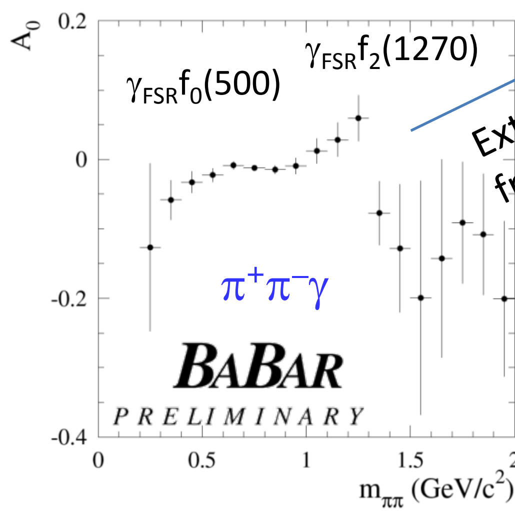
Simple charge asymmetry pattern found:

$$A(\phi^*) = A_0 \cos\phi^*$$

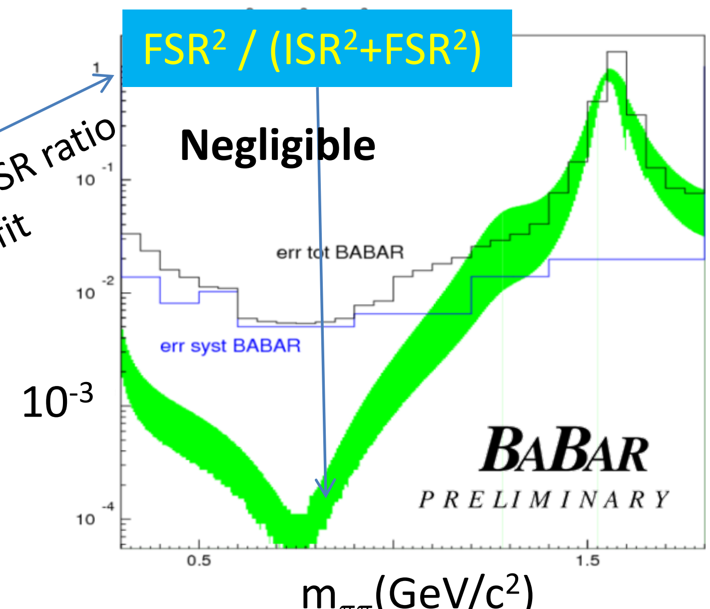








Extract FSR ratio from a fit

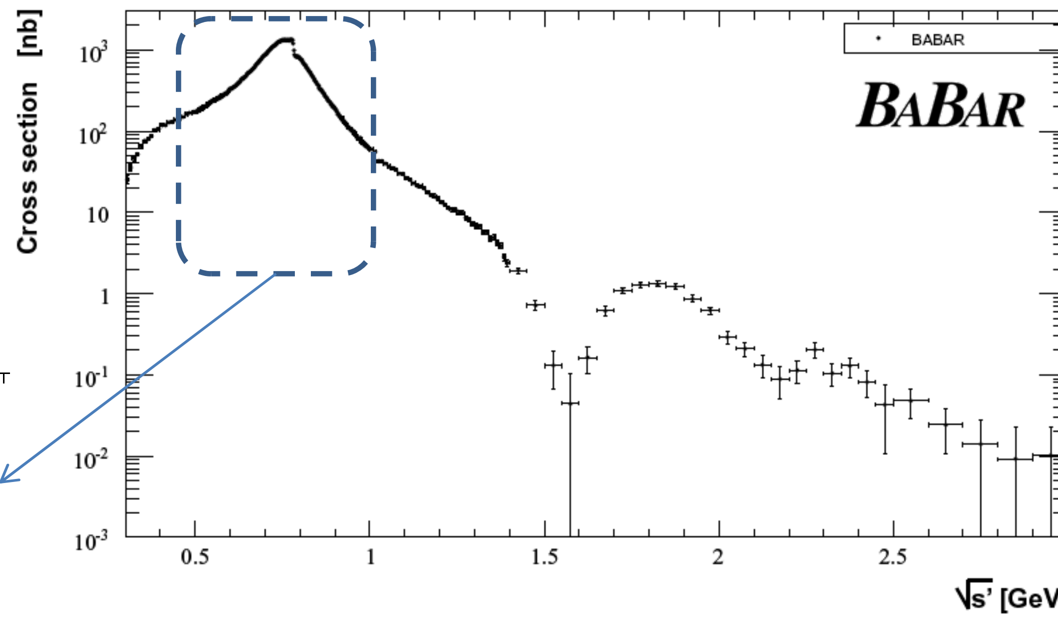


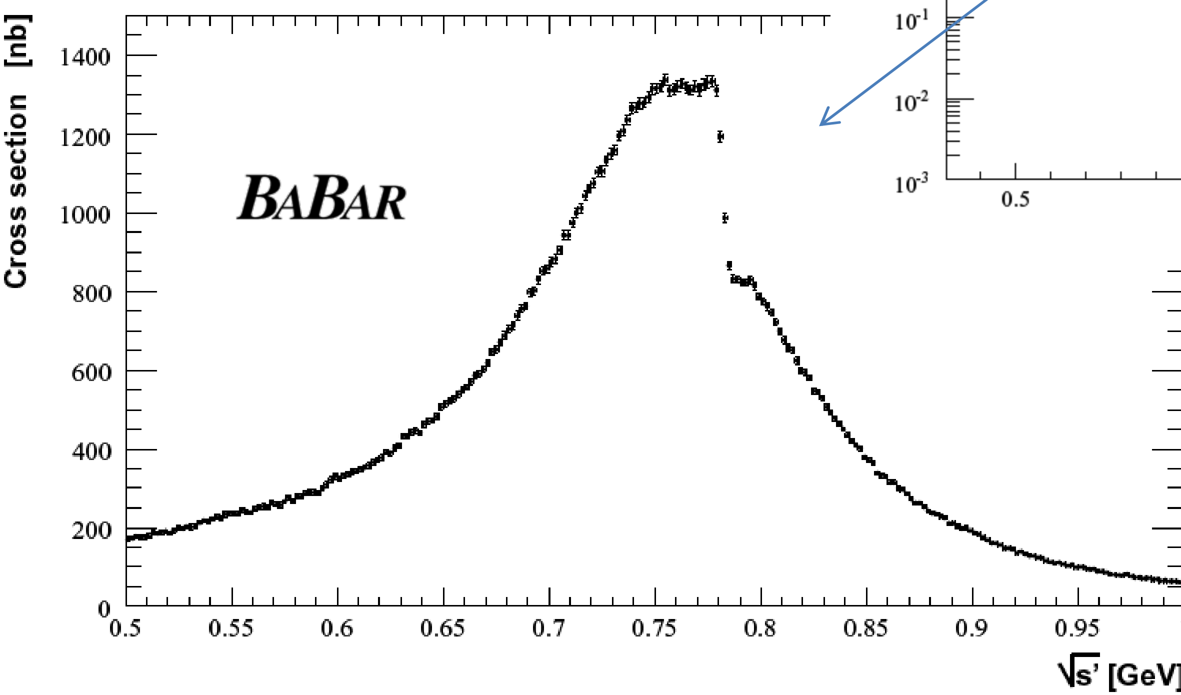
Results on $e^+e^- \to \pi^+\pi^-$ cross sectons

PRL 103, 231801 (2009)

PRD 86, 032013 (2012)

- Dominant by ρ
- $\rho-\omega$ interference
- Radial excitations





Square of pion form factor:

$$|F_\pi|^2(s') = \boxed{\frac{3s'}{\pi\alpha^2(0)\beta_\pi^3}}\,\sigma_{\pi\pi}(s')$$

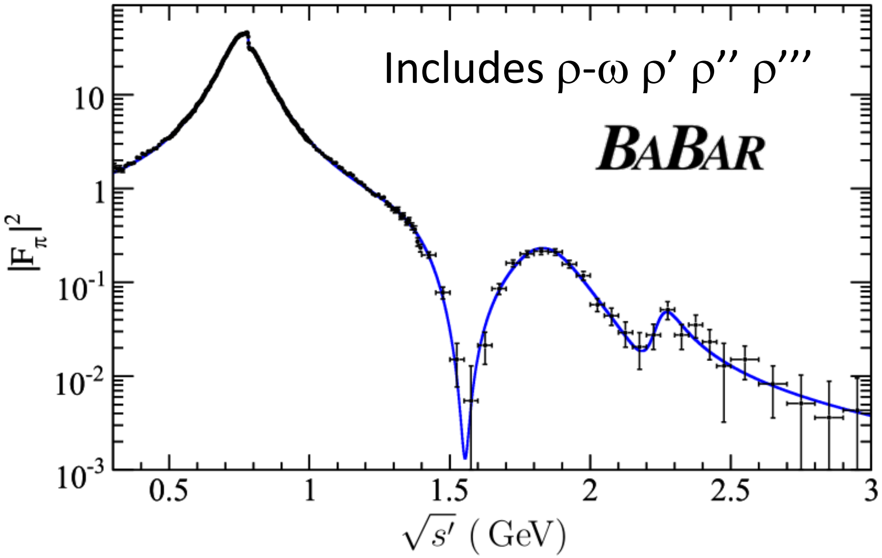
$\downarrow$

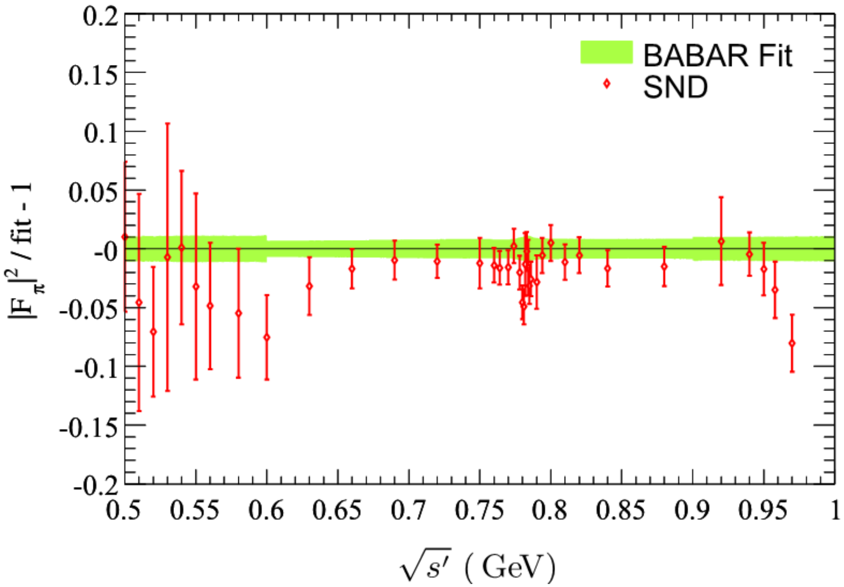
$1/\sigma_{pt\pi}$

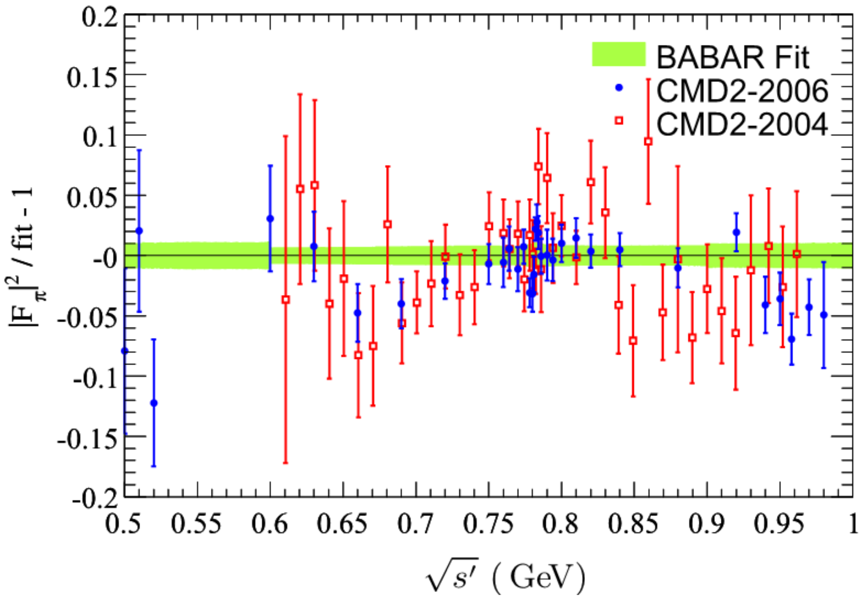
$\sigma_{pt\pi}$: the lowest-order cross section for pointlike spin 0 charged particles

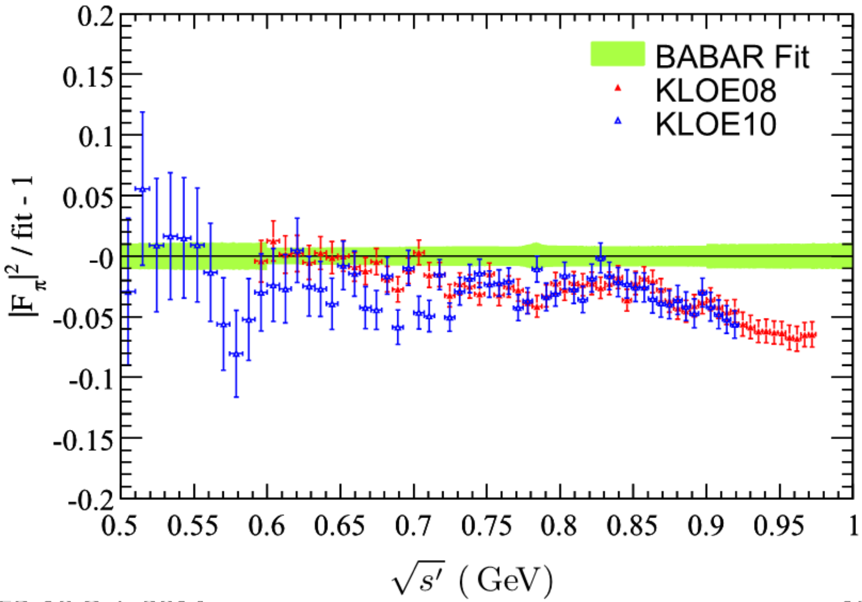
Fit of F_π and Comparison to previous experiments

Fit with vector-dominance model (**VDM**)





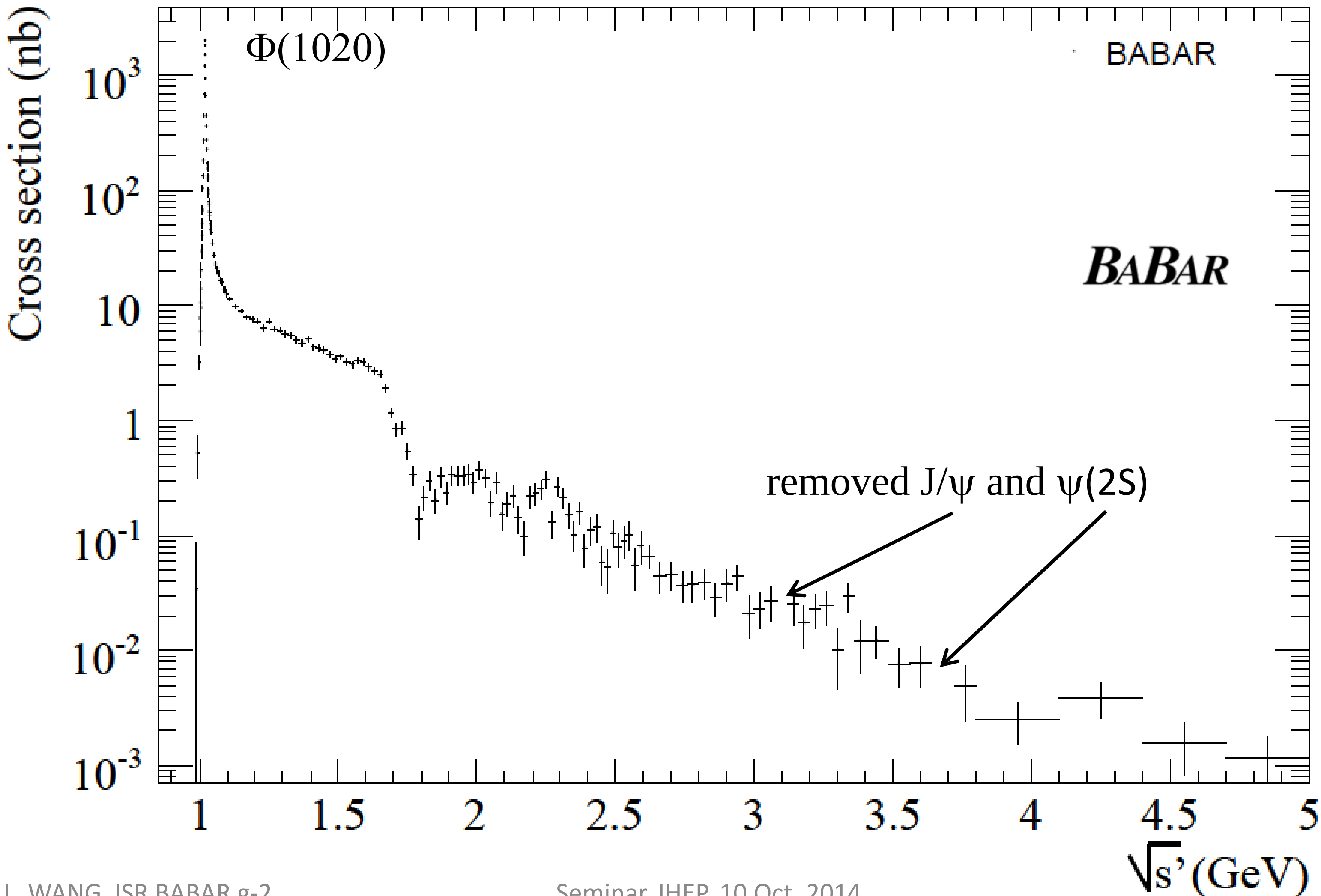




Results on the $e^+e^- \rightarrow K^+K^-$ cross section

- effective ISR luminosity obtained with μμ sample as for ππ cross section
- FSR measured and included

PRD 88, 032013 (2013)

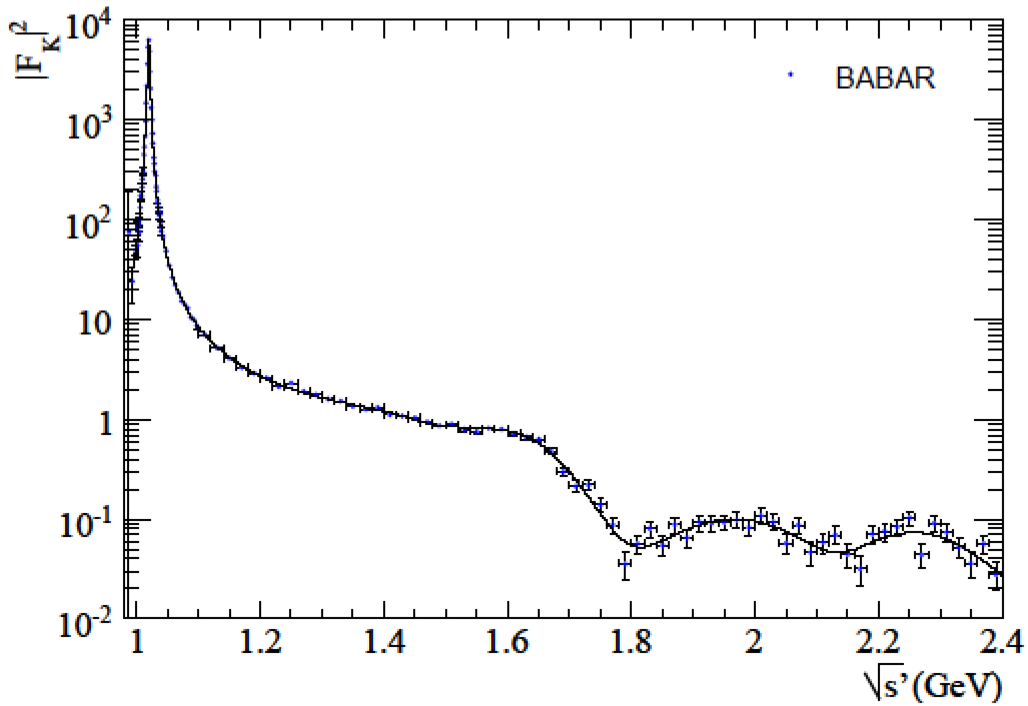


The ϕ parameters ($e^+e^- \to K^+K^-\gamma_{ISR}$)

m_ϕ, Γ_ϕ, and ϕ cross section obtained from a VDM fit of the form factor (Kuehn et al.)

$m_\phi = (1019.51 \pm 0.02 \pm 0.05)$ MeV
$\Gamma_\phi = (4.29 \pm 0.04 \pm 0.07)$ MeV

Good agreement with PDG:
$m_\phi = 1019.455 \pm 0.020$ MeV
$\Gamma_\phi = 4.26 \pm 0.04$ MeV

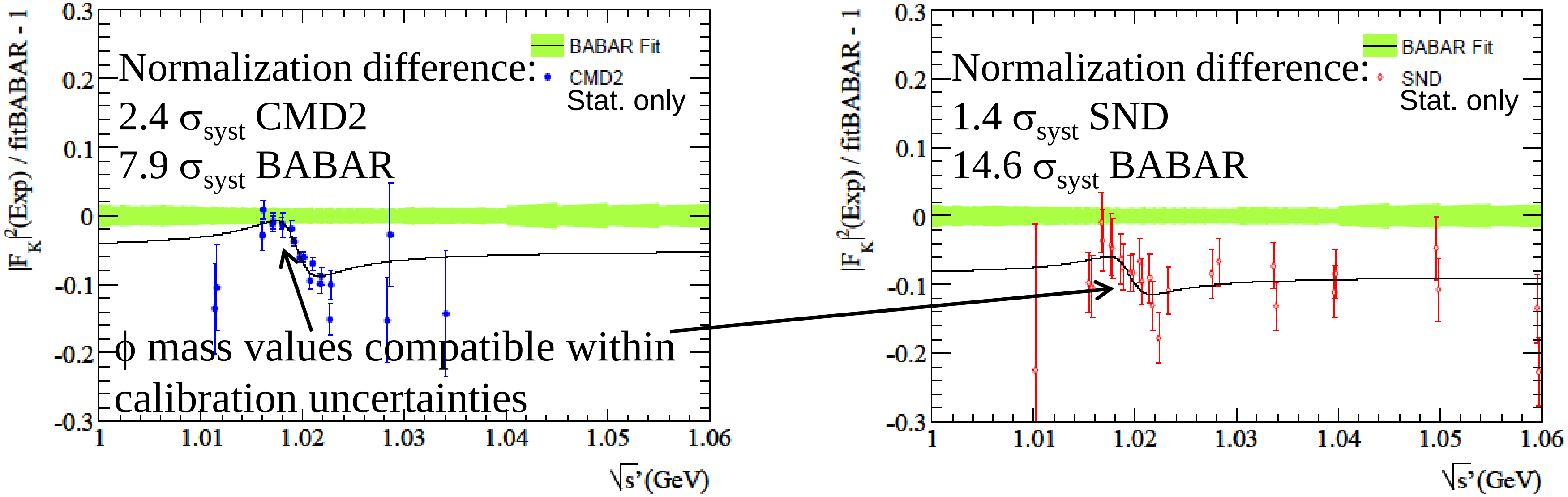


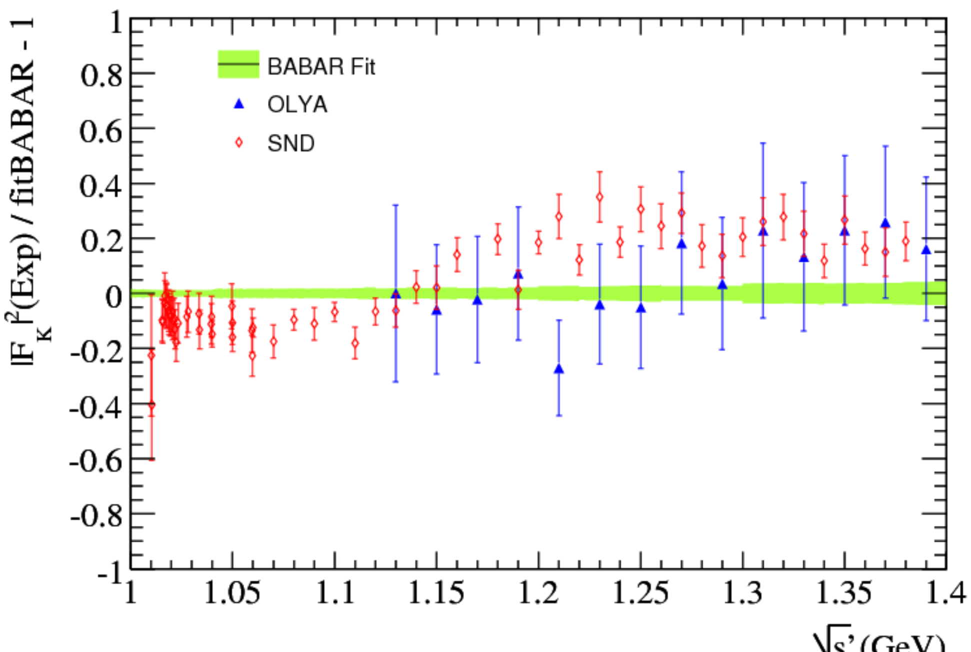
From integrated ϕ peak:

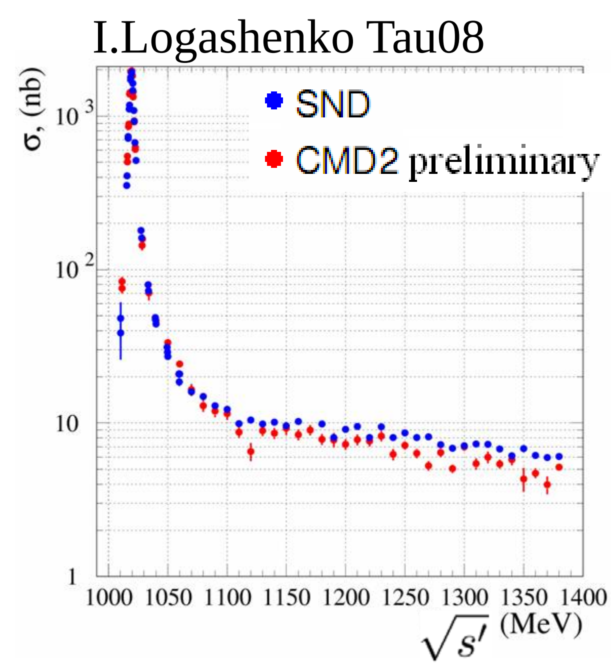
$\Gamma^\phi_{ee} \times B(\phi \to K^+K^-) = (0.6344 \pm 0.0059_{exp} \pm 0.0033_{fit} \pm 0.0015_{cal})$ keV (1.1%)

[CMD2: $(0.605 \pm 0.021 \pm 0.013)$ keV (4.1%)]

Comparison of $|F_K|^2$ to previous experiments







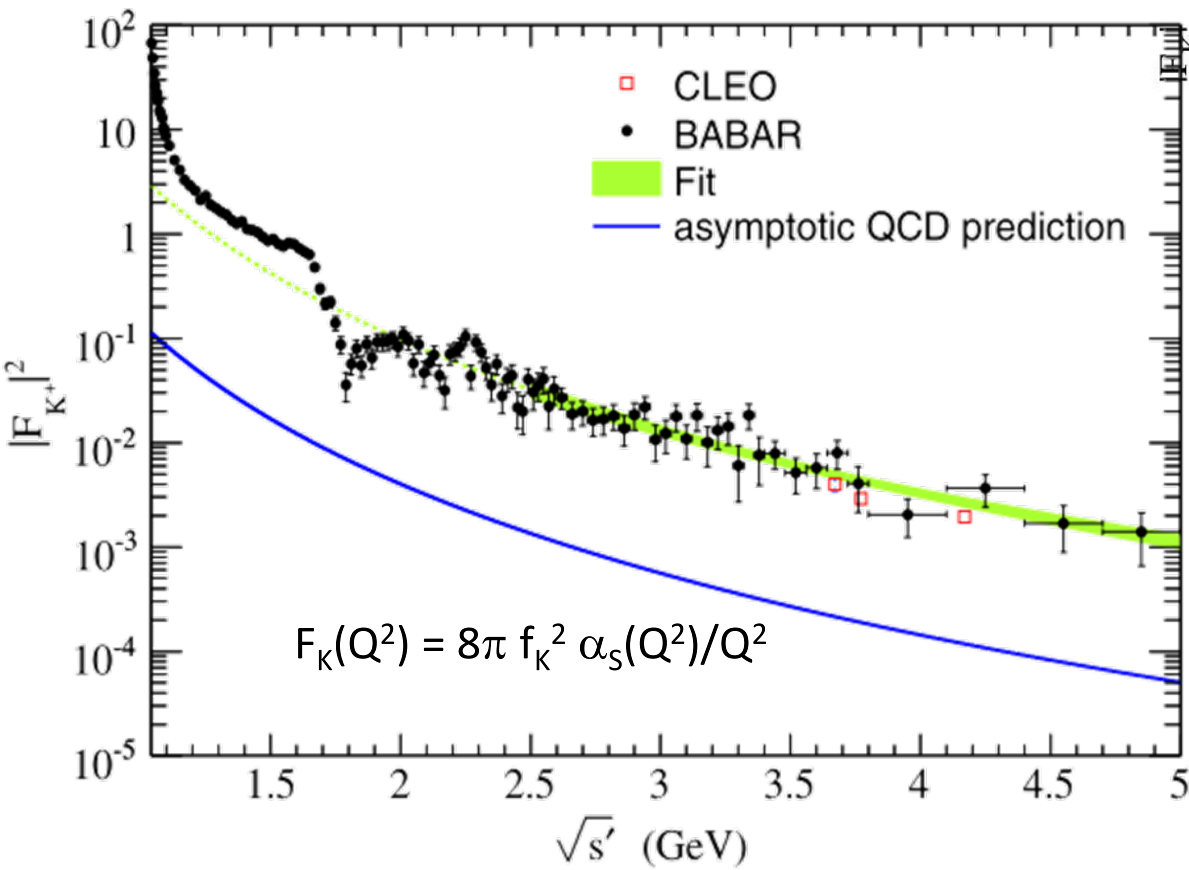
Charged kaon form factor at large Q^2 ($e^+e^- \to K^+K^-\gamma_{ISR}$)

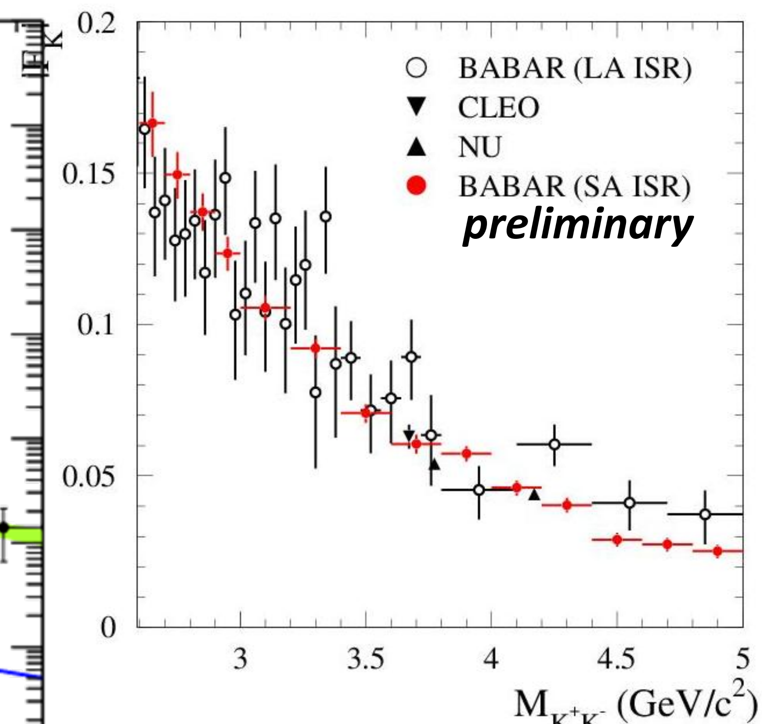
Predictions based on QCD in asymptotic regime (Chernyak, Brodsky-Lepage, Farrar-Jackson)

→ power law $F_K \sim \alpha_S(Q^2)\, Q^{-n}$ with n=2 in good agreement with data (2.5-5 GeV n=2.10 ± 0.23)

→ but data on $|F_K|^2$ a factor ~20 above prediction ! (confirm CLEO measurements)

→ no trend in data up to 25 GeV^2 for approaching the asymptotic QCD prediction

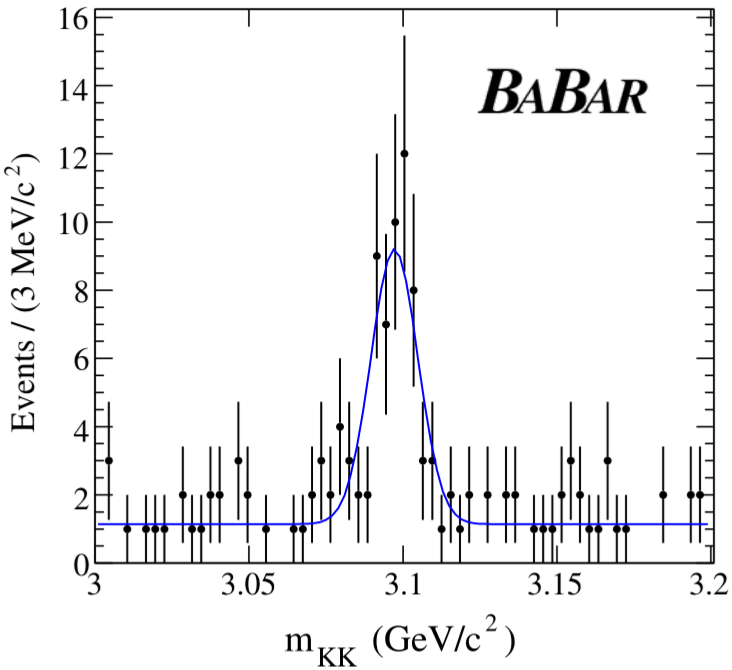


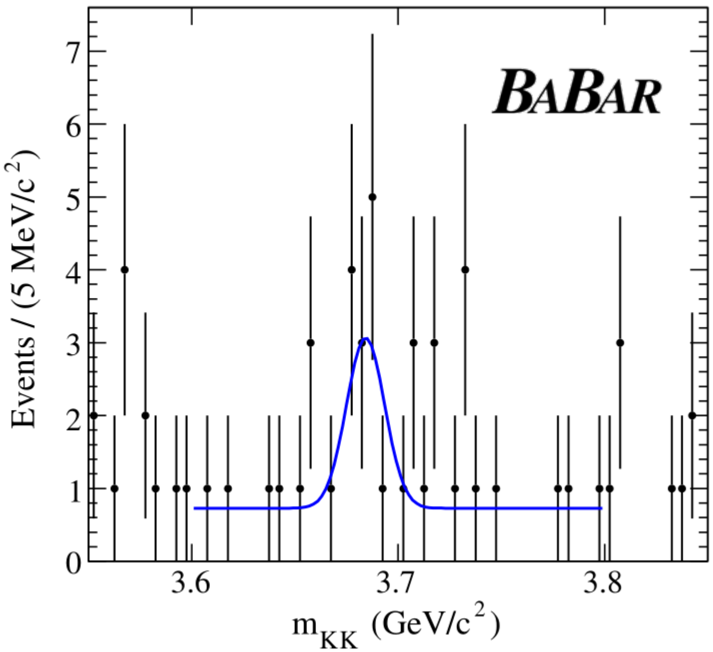


ISR with small-angle photon:
much larger ISR luminosity,
but efficiency only for large masses

$e^+e^- \rightarrow J/\psi, \psi(3686)\gamma_{ISR} \rightarrow K^+K^- \gamma_{ISR}$

An example of charmonium study through ISR





$$\mathcal{B}(J/\psi \rightarrow K^+K^-) = (2.56 \pm 0.44_{\rm exp} \pm 0.07_{\Gamma^{ee}}) \times 10^{-4}$$

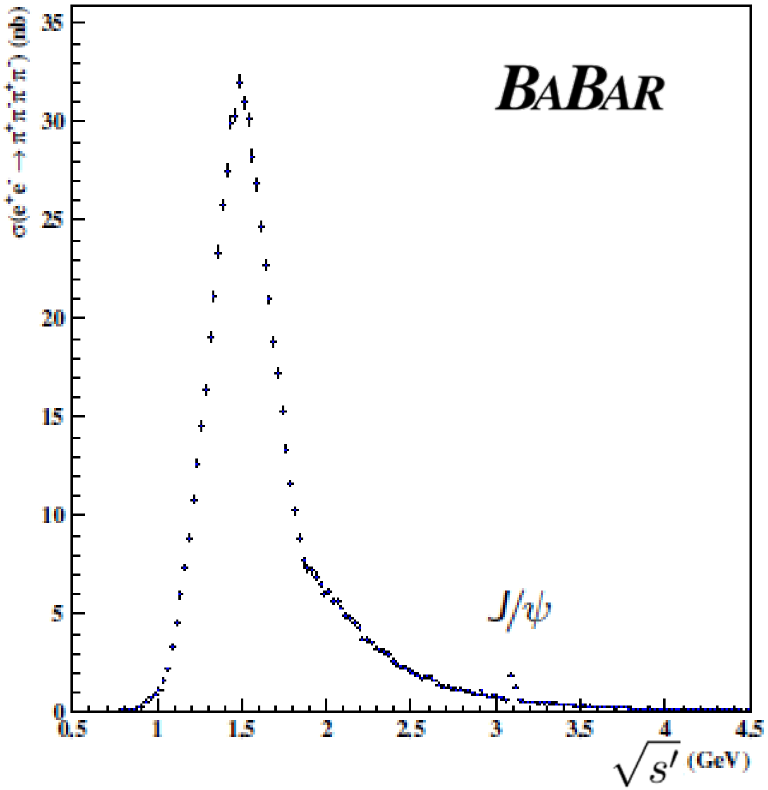
PDG2014: (2.7+-0.17)*10^{-4}

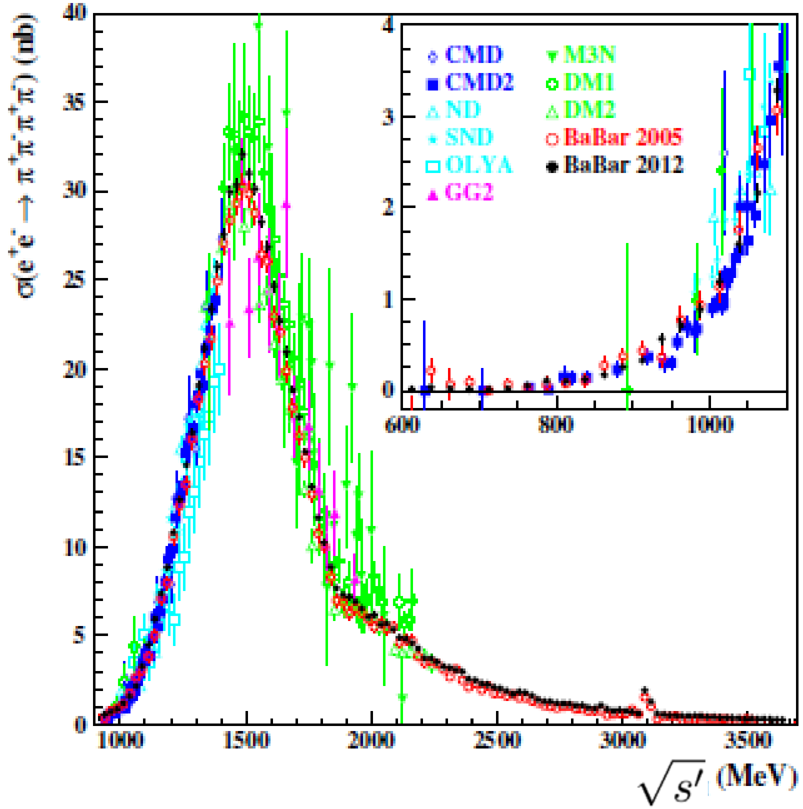
$$\mathcal{B}(\psi(2S) \rightarrow K^+K^-) = (1.50 \pm 0.59_{\rm exp} \pm 0.03_{\Gamma^{ee}}) \times 10^{-4}$$

PDG2014: (0.71+-0.05)*10^{-4}

Results on $\sigma(e^+e^- \to \pi^+\pi^-\pi^+\pi^-)$

PRD 85, 112009 (2012), based on 454 fb^{-1} (earlier publication on 89 fb^{-1})

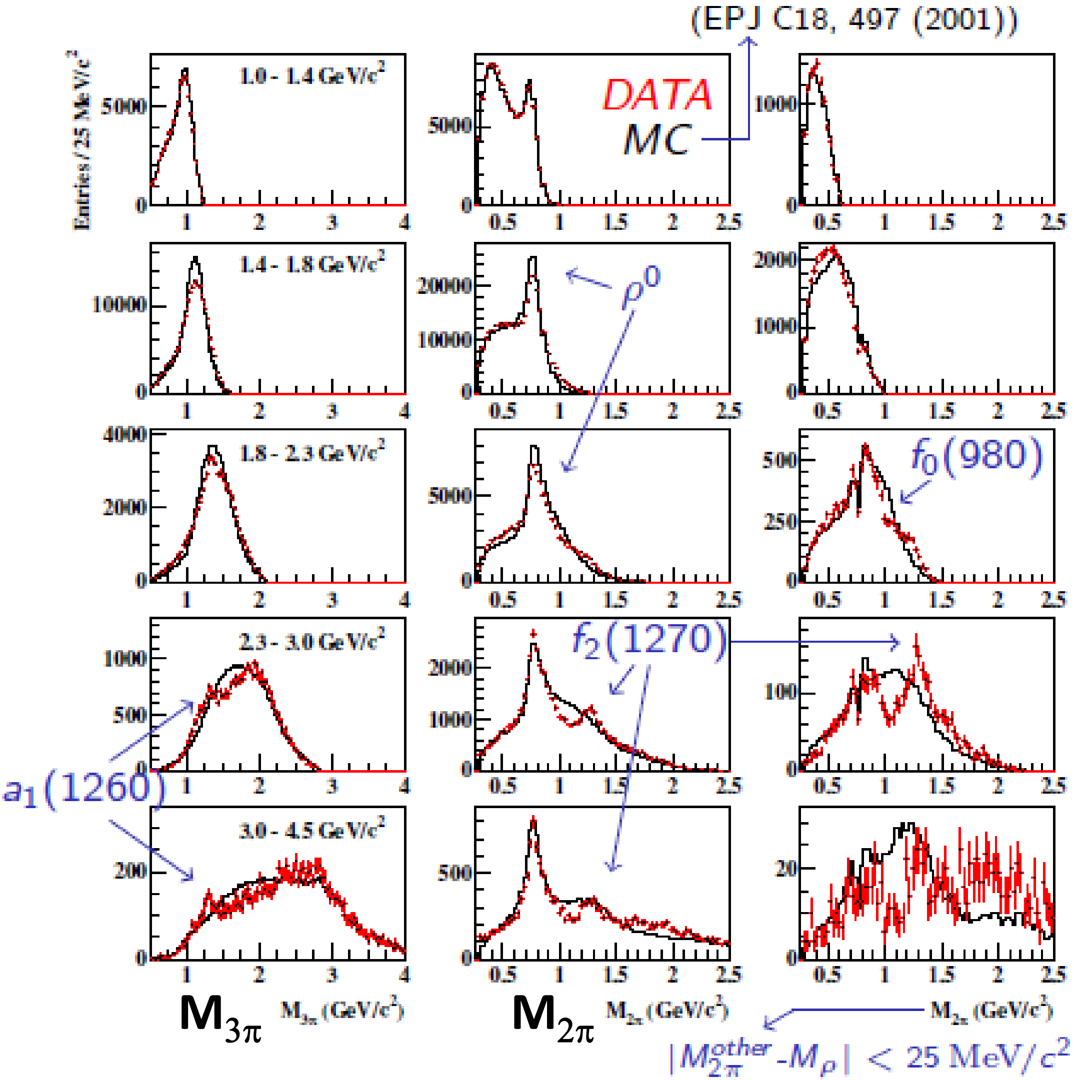




- Systematic uncertainties
  2.4% in peak region (1.1-2.8 GeV)
  11% (0.6-1.1 GeV)
  4% (2.8-4.0 GeV)
- J/ψ visible

- $<$ 1.4 GeV: agreement with previous *BABAR* results, SND and CMD-2 data
- $>$ 1.4 GeV: highest precision (DM2, 20%)

Internal structures in different sqrt(s') energy slides for $e^+e^- \to \pi^+\pi^-\pi^+\pi^-$



First column (4 entries/event):

$a_1(1260)$

Second column (4 entries/event):

strong ρ^0 contribution
e.g. for $M_{4\pi} > 1.4\,\mathrm{GeV}/c^2$:
1/4th of entries in ρ^0 peak
$\rho^0\rho^0$ is forbidden
$\to$ ρ^0 in each event!

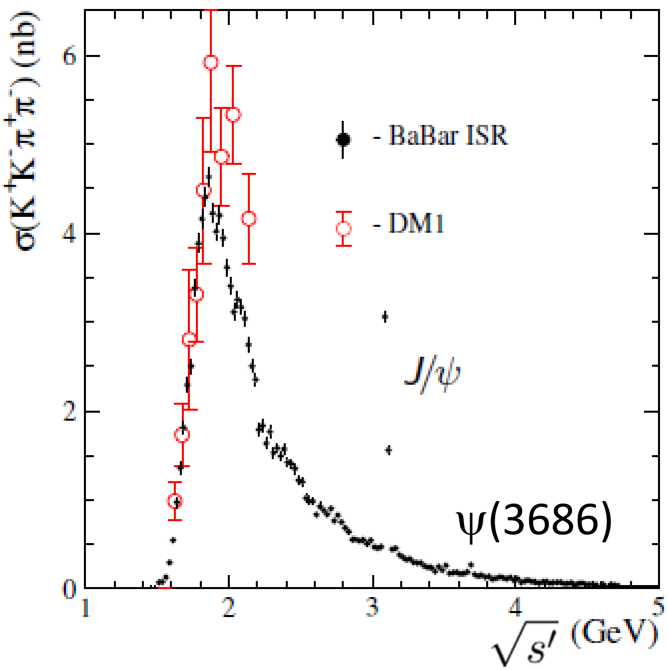
Third column (1 entry/event):

2π lie within ρ^0 mass
$\to$ other $\pi^+\pi^-$'s mass plotted

$f_2(1270)$, $a_1(1260)$, $f_0(980)\ldots$?
$\to$ Partial Wave Analysis needed

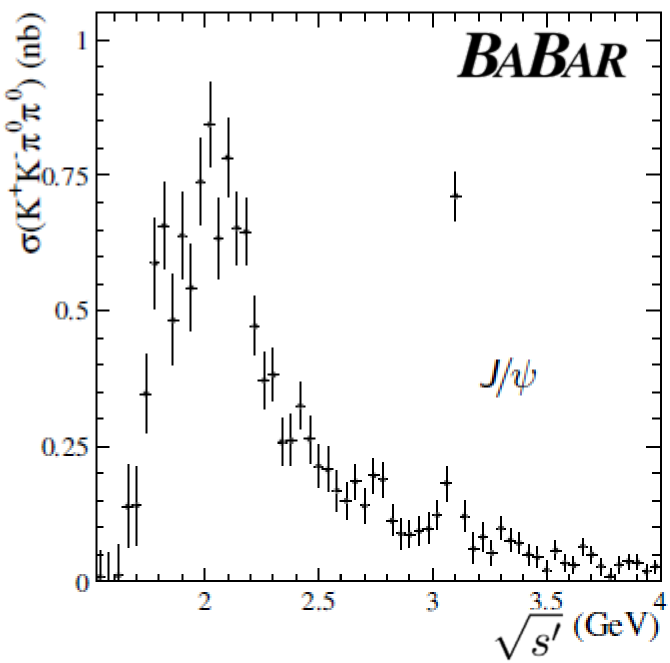
Results on $\sigma(e^+e^- \to K^+ K^- \pi^+\pi^-,\ K^+K^- \pi^0\pi^0)$

PRD 86, 012008 (2012) based on the full BABAR statistics (454 fb^{-1})



- syst. uncertainty: 4 - 11%
- resolution: 4.2 - 5.5 MeV
- J/ψ clearly visible

huge improvement compared to existing data!



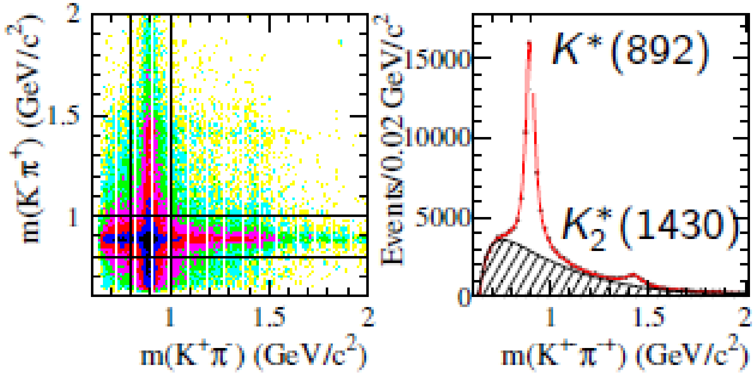
- syst. uncertainty: 7 - 16%
- resolution: 8.8 - 11.2 MeV
- J/ψ clearly visible

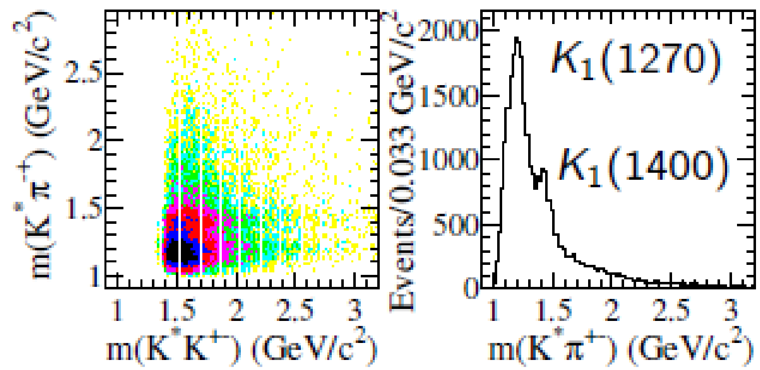
First measurement!

Intermediate resonances for $e^+e^- \to K^+K^-\pi^+\pi^-$, $K^+K^-\pi^0\pi^0$

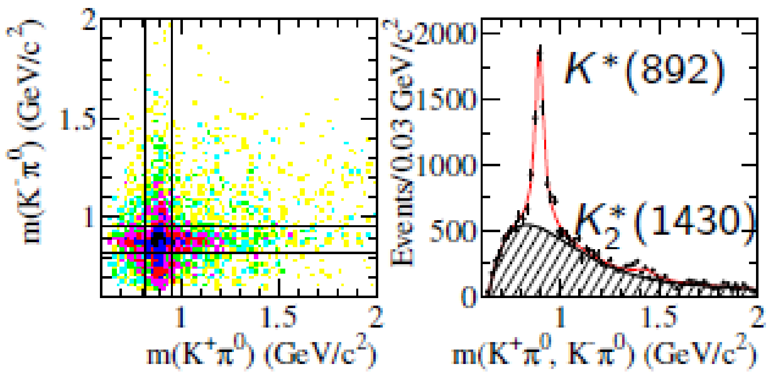
BABAR

$K^+K^-\pi^+\pi^-$



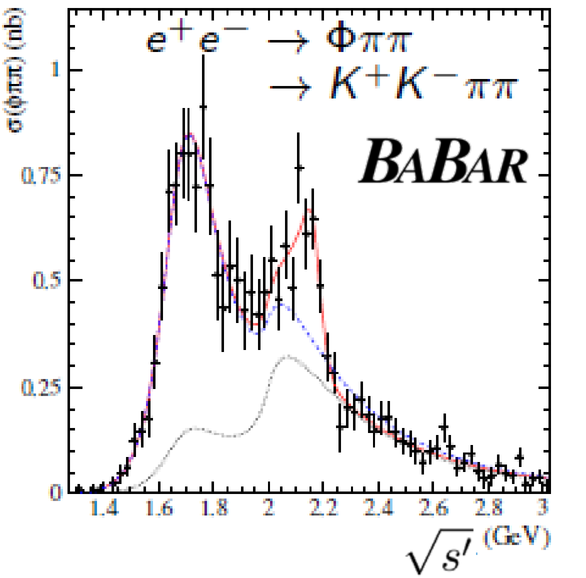


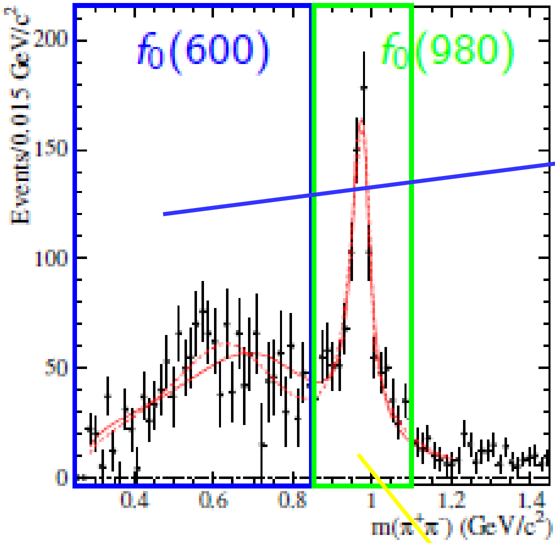
$K^+K^-\pi^0\pi^0$

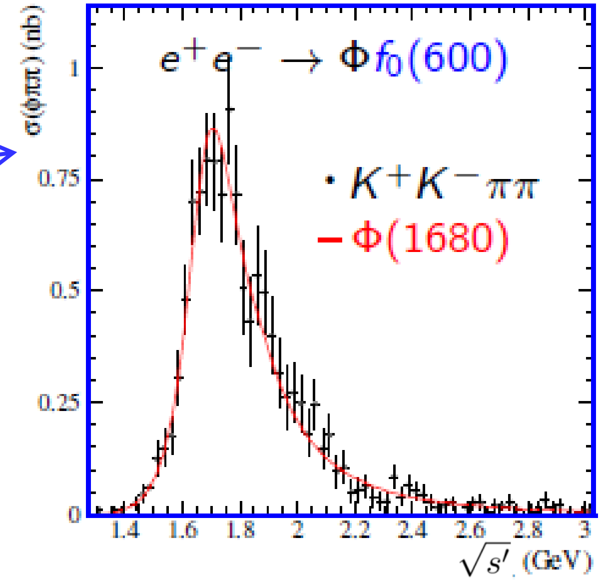


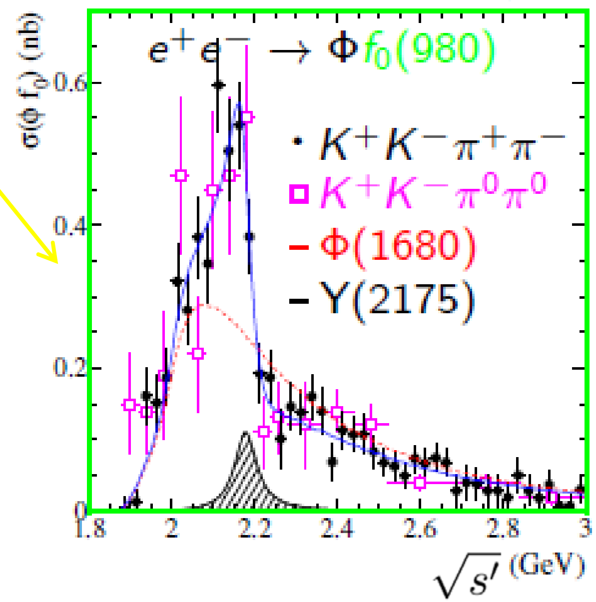
cross section dominated by
$K^*(892)K^\pm\pi^\mp$ final state
$K_1(1270, 1400) \to K^*(892)\pi$ and
$K_1(1270) \to K\rho(770)$ are seen

$\sigma(e^+e^- \to \phi\pi\pi \to K^+K^- \pi\pi)$









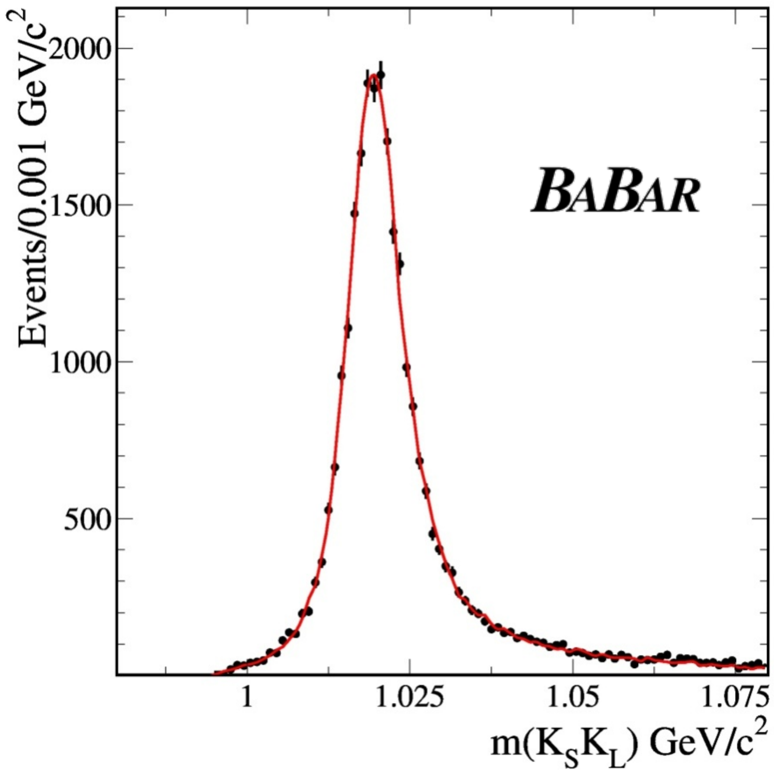
- Requirement: $\phi \to K^+K^-$
- Fit assumes two resonances
- Y(2175) confirmed: $J^{PC} = 1^{--}$
  M=$2176 \pm 14 \pm 4$ MeV/c^2; $\Gamma = 90 \pm 22 \pm 10$ MeV
- Might not be a radial excitation: width too small & should also decay into $\phi f_0(600)$ as for $\phi(1680)$
- Strangeness partner of Y(4260)? Hybrid-candidate?

New results on $e^+e^- \to K_SK_L$: ϕ region

PRD 89, 092002 (2014) based on the full BABAR statistics (454 fb^{-1})

- K_S reconstructed $\pi^+\pi^-$
- K_L direction measured in EM calorimeter
- K_L efficiencies measured using kinematically constrained $\phi \to K_S (K_L)$

$$\Gamma^{\phi}_{ee} \times B(\phi \to K_SK_L) = (0.4200 \pm 0.0033_{stat} \pm 0.0122_{syst} \pm 0.0019_{fit})\ \text{keV} \quad (3.0\%)$$



$m_\phi = (1019.46 \pm 0.04 \pm 0.06)$ MeV

$\Gamma_\phi = (4.21 \pm 0.10 \pm 0.07)$ MeV

Good agreement with PDG:

$m_\phi = 1019.455 \pm 0.020$ MeV

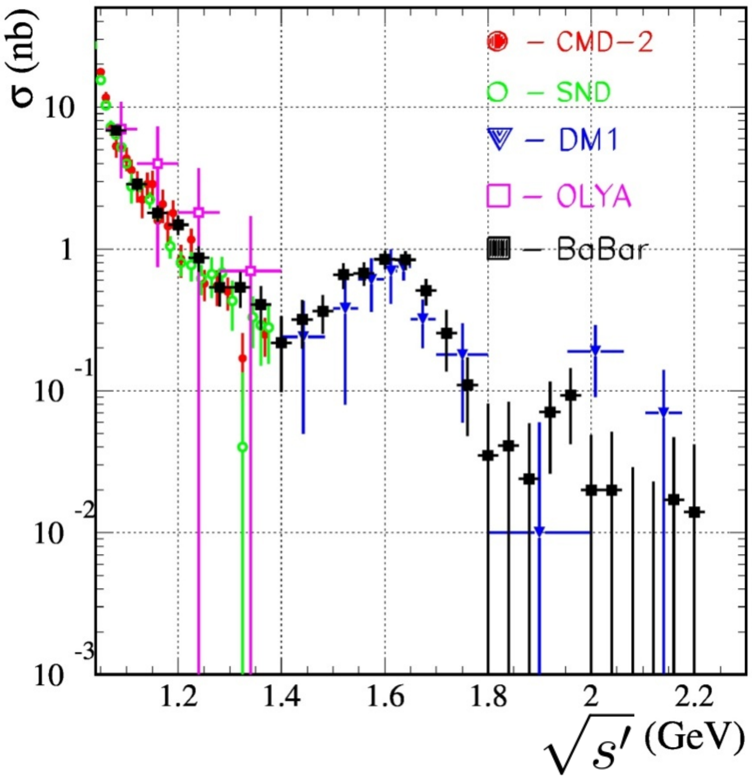
$\Gamma_\phi = 4.26 \pm 0.04$ MeV

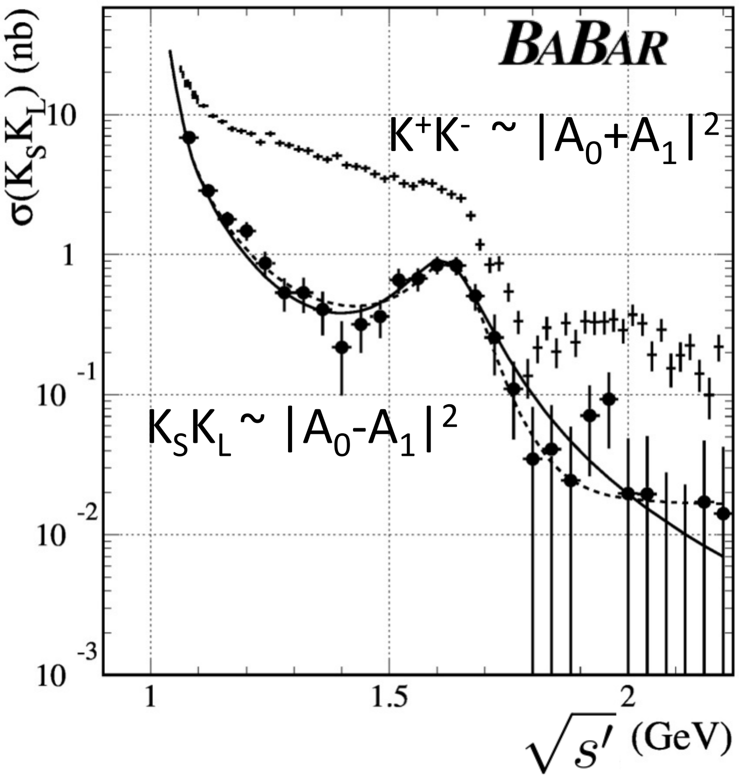
$$\frac{B(\phi \to K_SK_L)}{B(\phi \to K^+K^-)} = 0.662 \pm 0.021 \quad \text{BABAR}$$

[0.68 ± 0.03 CMD-2]

[0.671 ± 0.023 PDG BR av]

New results on $e^+e^- \to K_S K_L$: large Q^2



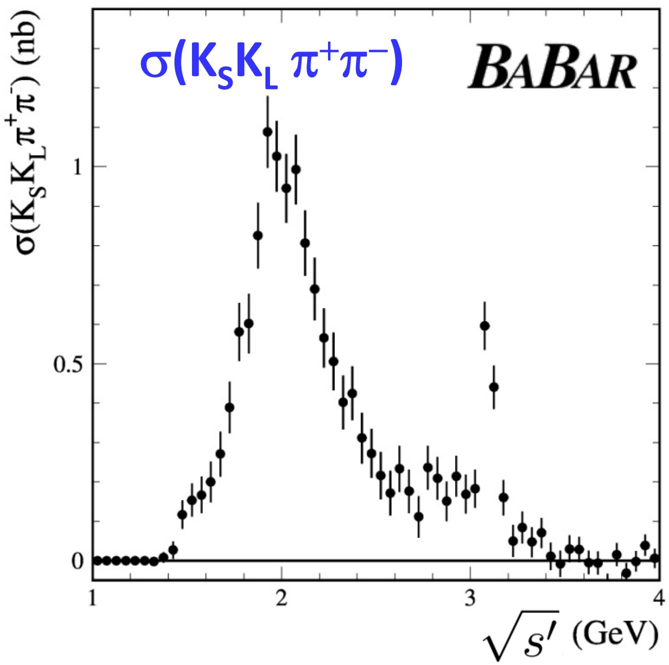


large effect suggestive of $\phi(1680)$

large interference between isoscalor part and isovector part

New results on $e^+e^- \to K_S K_L \pi^+\pi^-$, $K_S K_S \pi^+\pi^-$

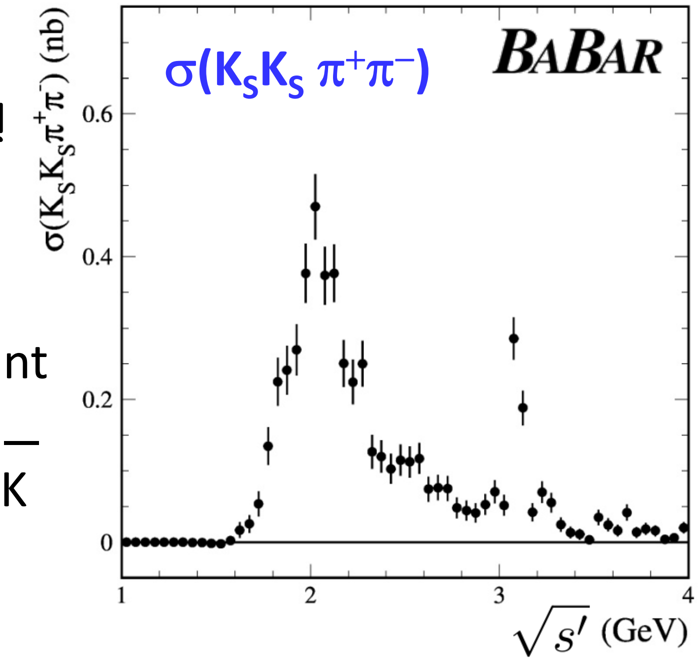
PRD 89, 092002 (2014) based on the full BABAR statistics (454 fb^{-1})

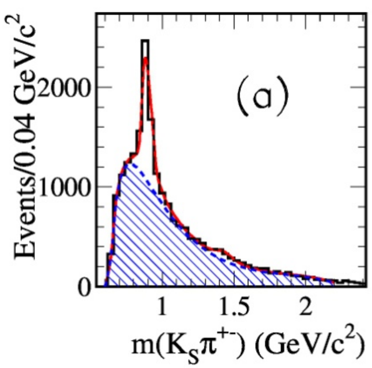


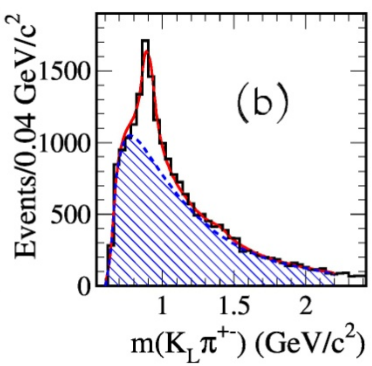
First measurements!

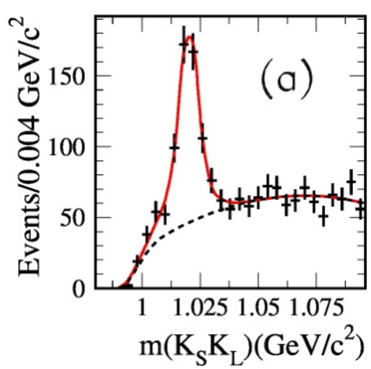
K*(890) K π dominant

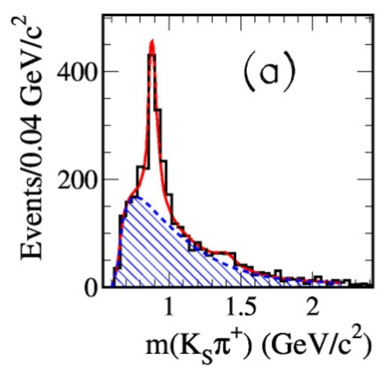
also φ ππ ρ K $\bar{K}$

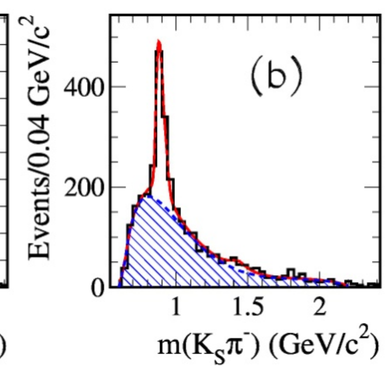










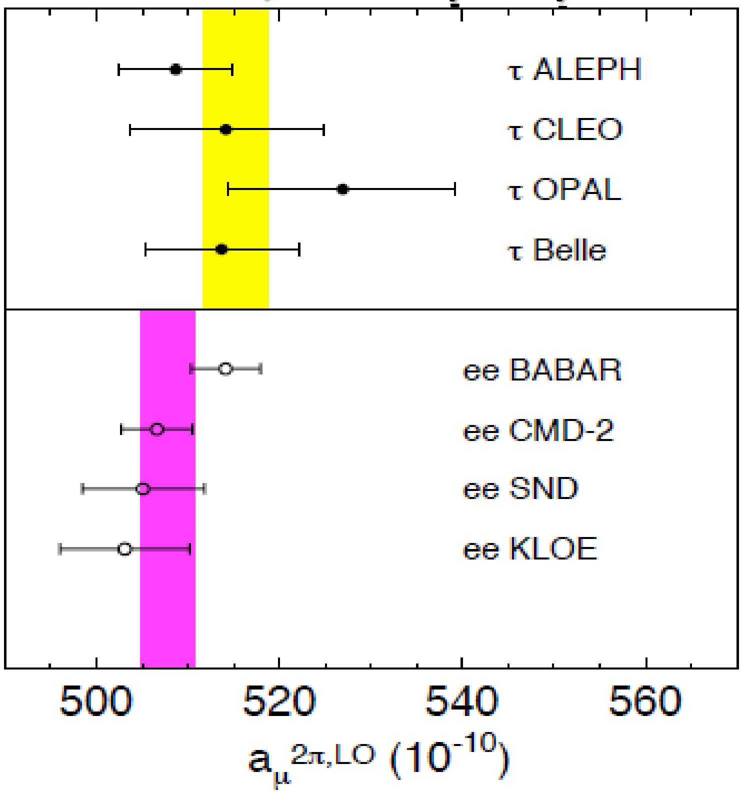


Impact of BABAR data for g-2: $\pi^+\pi^-$

$a_\mu^{2\pi,LO}$ integral from threshold to 1.8 GeV:

- BaBar most precise (with CMD-2)
- BaBar reduces tension between e^+e^- and τ

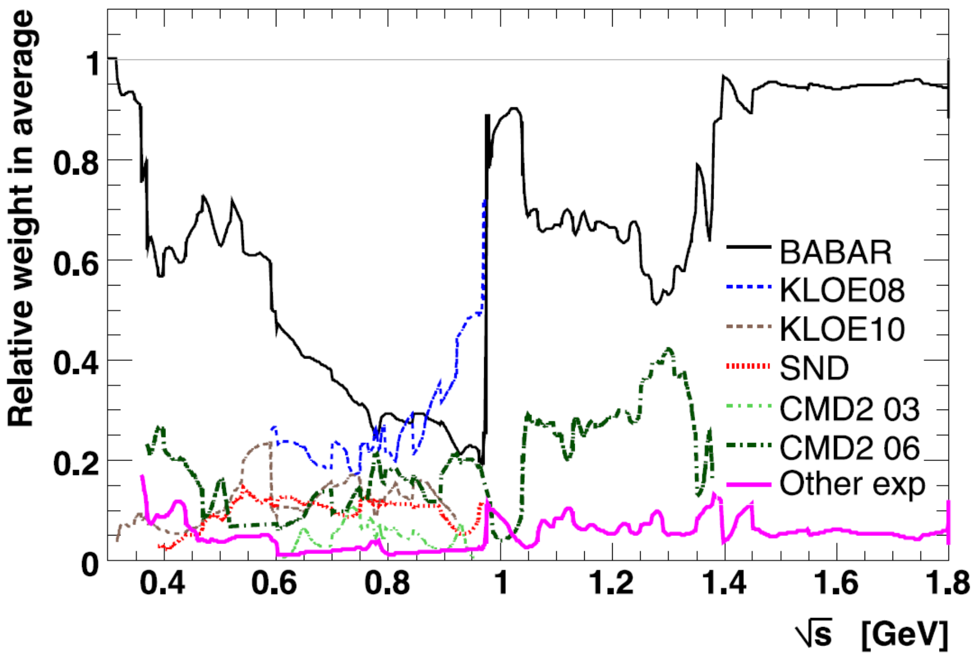
PRD 86, 032013 (2012)



Weights of different experiments in combining their results (DHMZ 2010)

Eur. Phys. J. C (2011) 71: 1515

BABAR dominates everywhere, except between 0.8 and 0.93 GeV where KLOE is the most precise



Impact of BABAR data for g-2: K^+K^- and $2(\pi^+\pi^-)$

$a_\mu^{KK,\,LO}$ [0.98;1.800] GeV = (22.95 ± 0.14 (stat) ± 0.22 (syst)) 10^{-10} (1.1%)

DHMZ 2010: update of all results before BABAR:

$a_\mu^{KK,\,LO}$[0.98;1.8]GeV = (21.63 ± 0.27 (stat) ± 0.68 (syst))10^{-10} (3.4%)

BABAR more precise than previous world average by a factor of 3

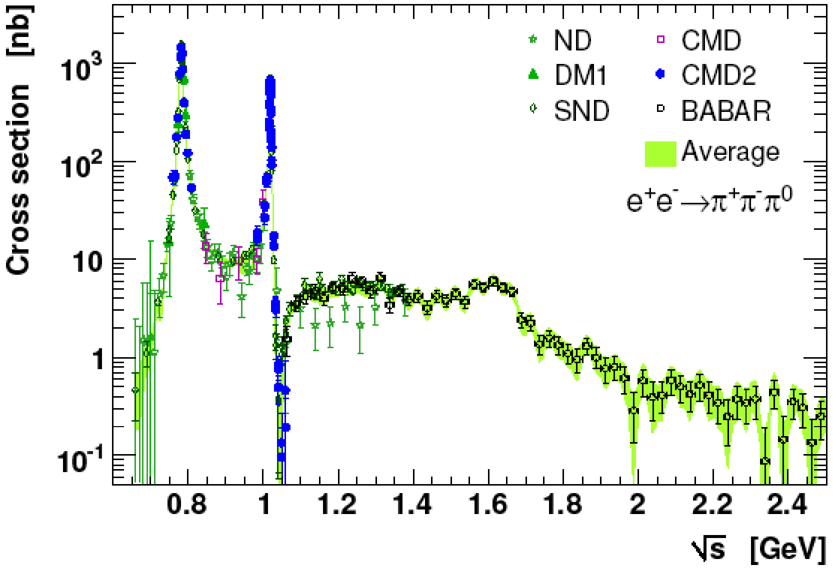
$a_\mu^{4\pi,\,LO}$ [0.98;1.800] GeV = (13.64 ± 0.03 (stat) ± 0.36 (syst)) 10^{-10} (2.6%)

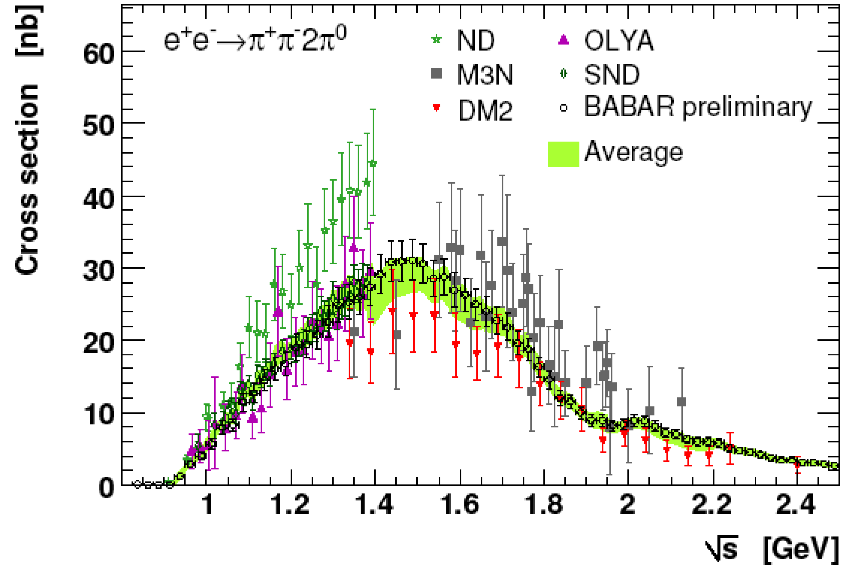
DEHZ 2003: all results but BABAR 2007:

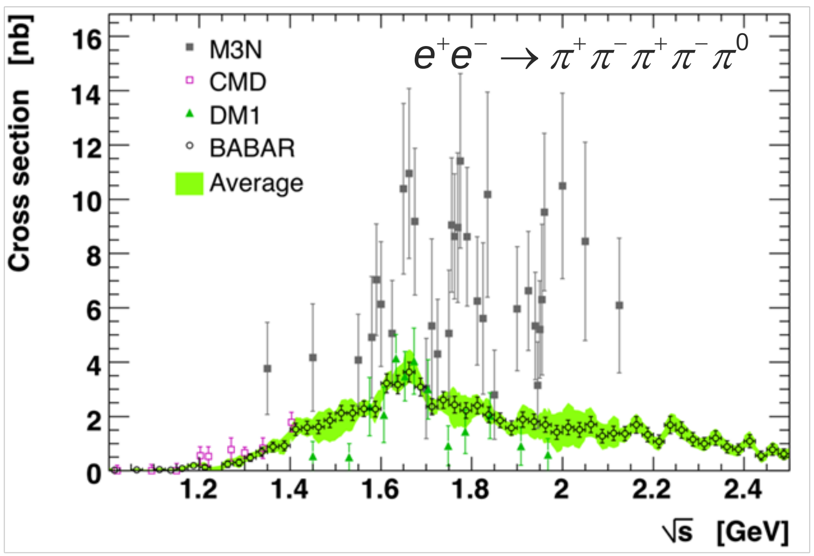
$a_\mu^{4\pi,\,LO}$[0.98;1.8]GeV = (13.95± 0.90 (exp) ± 0.23(rad))10^{-10} (6.7%)

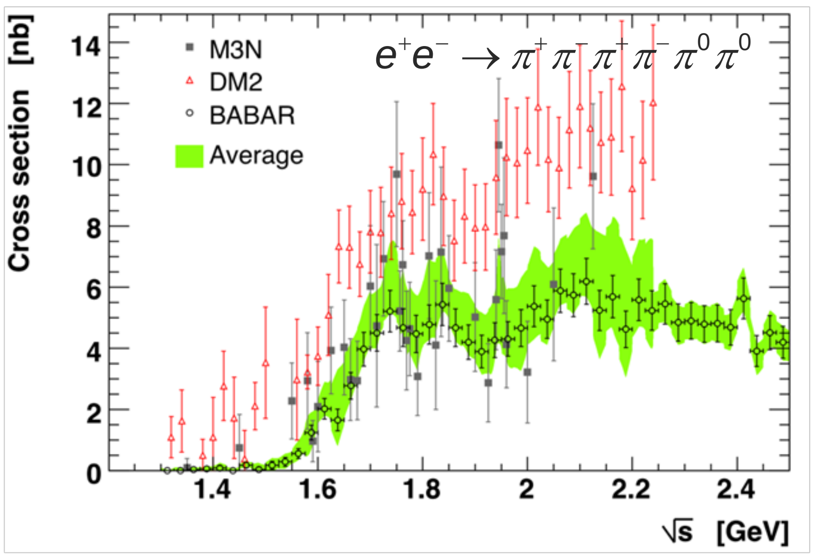
BABAR more precise than previous world average by a factor of 2.6

Impact of BABAR data for g-2: other exemples









Impact of BABAR data for g-2: final-state dynamics

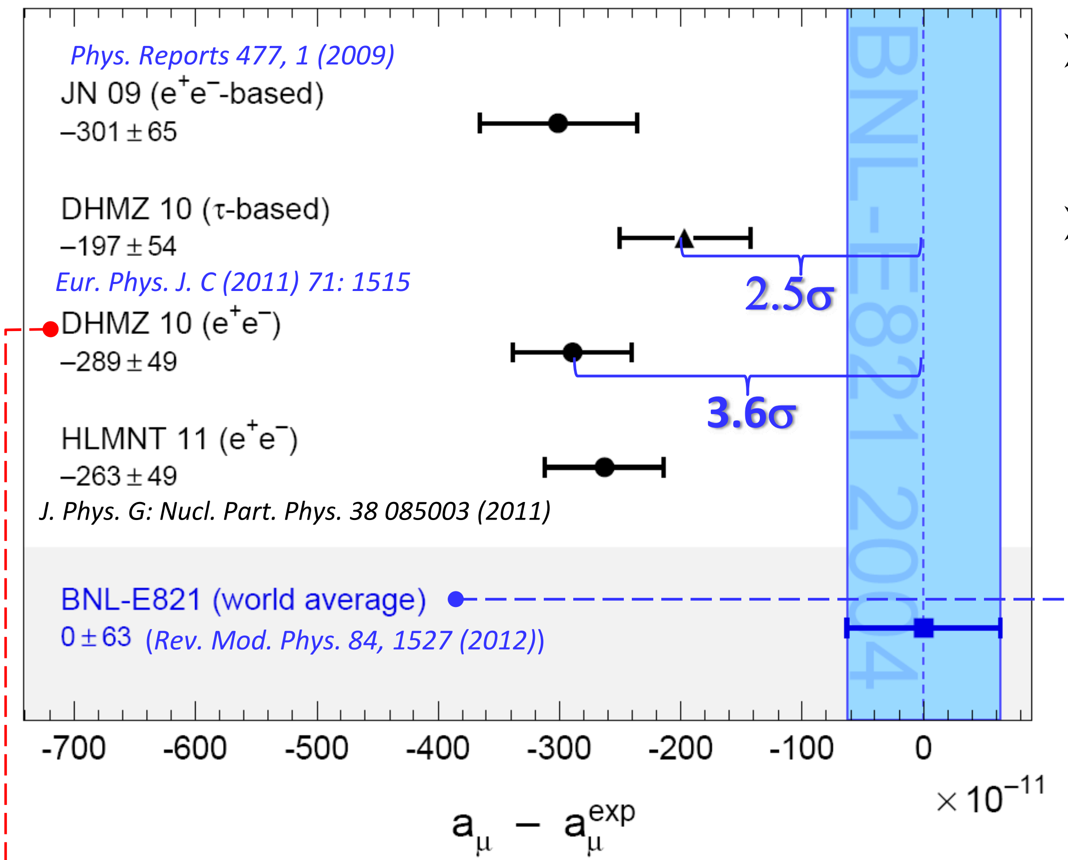
- R measured by summing exclusive cross sections
- Below 1.8 GeV hadron multiplicities up to 6 (all pions) and 4 ($K\bar{K}$ + pions) are adequate. But some channels are very hard to measure: $\pi^+\pi^-4\pi^0$, $K_SK_L\pi^0$, $K_SK_L2\pi^0$
- Isospin symmetry can be used for estimating unmeasured modes: but often only upper bounds are obtained (***Eur. Phys. J. C 27, 497–521 (2003)***)
- Knowledge of final-state dynamics as available in BABAR analyses is essential to provide reliable estimates: (DHMZ 2010 ***Eur. Phys. J. C (2011) 71: 1515***)

- example: contribution to a_μ ($\times10^{-10}$) from $K\bar{K}\pi\pi$

2003: estimate	2.2 ± 1.0
2010: only $K^+K^-\pi^+\pi^-$ and $K^+K^-\pi^0\pi^0$ available with final states	1.35 ± 0.38
2014: $K_SK_L\pi^+\pi^-$ and $K_SK_S\pi^+\pi^-$ available with final states	in progress

Status of a_μ

SM predictions **measurement**



- interesting but inconclusive discrepancy (<5σ)
- future improvement from both sides:
  - measurement: new project at Fermilab & JPARC to improve accuracy by a factor 4
  - prediction: new data from KLOE, VEEP-2000 and BESIII as inputs

$a_\mu^{SM}[e+e-] = (11\,659\,180.4 \pm 4.1 \pm 2.6 \pm 0.2)\ 10^{-10}$

HVP LBL EW (±4.9)

$a_\mu^{exp} = (11\,659\,209.1 \pm 5.4 \pm 3.3)\ 10^{-10}$

Conclusions

- Through the ISR method, BaBar has carried out a consistent and almost complete program to measure precise cross sections for the dominant channels of $e^+e^- \rightarrow$ hadrons from threshold to ~2 GeV. (a few more channels in progress)

- Results presented in this talk: $\pi^+\pi^-$, K^+K^-, $\pi^+\pi^-\pi^+\pi^-$, $K^+K^-\pi^+\pi^-$, $K^+K^-\pi^0\pi^0$, K_SK_L, $K_SK_L\pi^+\pi^-$, and $K_SK_S\pi^+\pi^-$.

- BABAR results have a large impact on the hadronic vacuum polarization (HVP) contribution to the muon g-2.

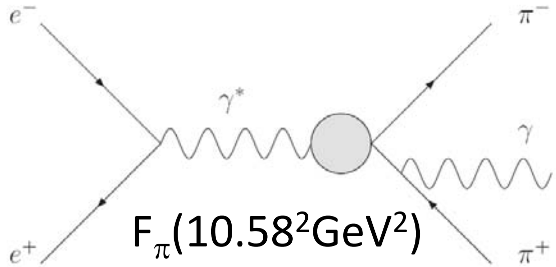
- In addition to HVP there are other applications of these data in progress for QCD tests with finite energy sum rules, complementing similar studies with hadronic τ decays.

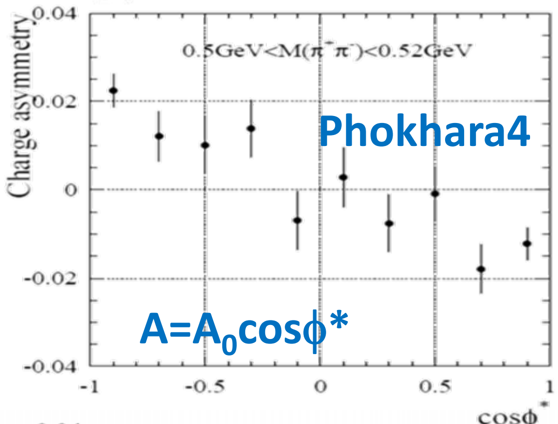
- Also BABAR ISR results provide input to hadron spectroscopy, resonance dynamics and measurement of baryon form factors.

Backup slides

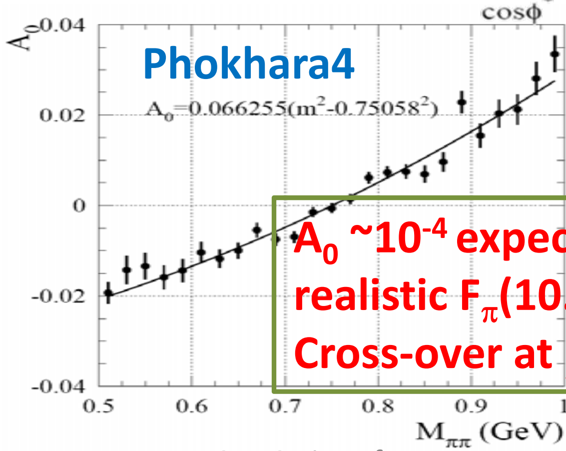
FSR models for $e^+e^- \to \pi^+\pi^-\gamma$

FSR model 1 (point-like pion)



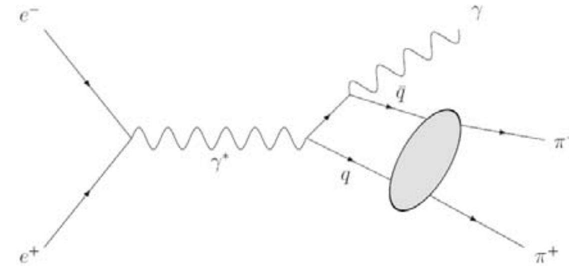


x100

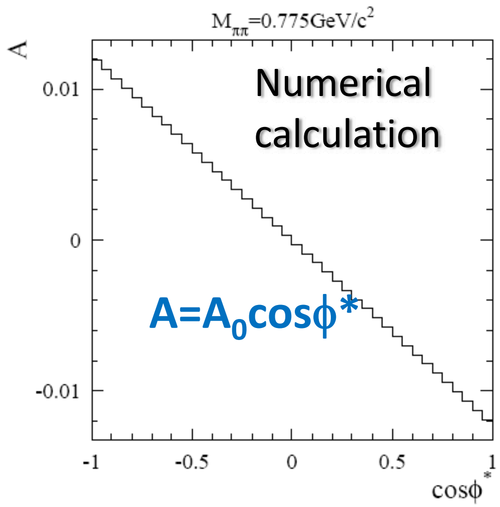


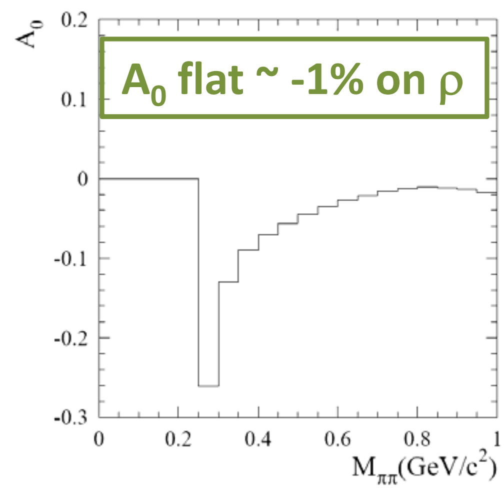
A_0 ~10^{-4} expected with realistic $F_\pi(10.58^2 GeV^2)$
Cross-over at ρ mass

FSR model 2 (FSR from quarks)



Differential cross sections:
Z. Lu and I. Schmidt, PRD 73, 094021 (2006);
Erratum, PRD 75, 099902(E) (2007)
C-even part 2-pion state described by amplitudes $\Phi^+_{u,d}$:
M. Diehl, T. Gousset and B. Pire, PRD 62, 073014 (2000)
S-wave $f_0(600)$ + D-wave $f_2(1270)$





Revised FSR model 2 for $e^+e^-\to\pi^+\pi^-\gamma$

$$\Phi_u^+(z, m_{\pi\pi}^2, \cos\theta^*) = \Phi_d^+(z, m_{\pi\pi}^2, \cos\theta^*)$$

Diehl et al. PRD 62, 073014 (2000)

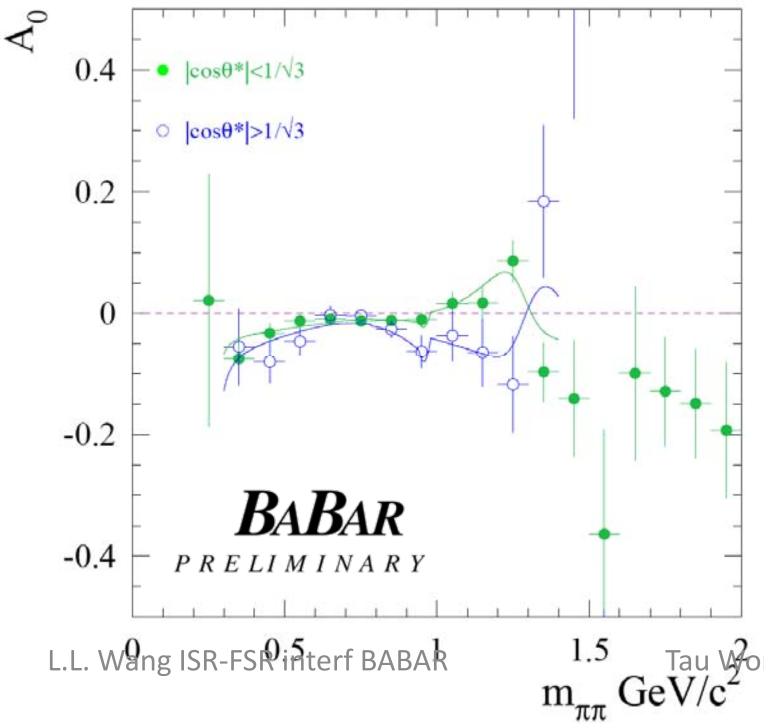
$$= 10z(1-z)(2z-1)R_\pi \left[-\frac{3-\beta^2}{2} e^{i\delta_0(m_{\pi\pi}^2)} + \beta^2 e^{i\delta_2(m_{\pi\pi}^2)} P_2(\cos\theta^*)\right]$$

S-wave ('f_0' phase shift)   D-wave (f_2 helicity 0)

- Fit with our revised model:

$$10z(1-z)(2z-1)\left[c_0 \frac{3-\beta^2}{2} e^{i\delta_0(m_{\pi\pi}^2)} + c_2 \beta^2 \mathrm{BW}_{f_2}(m_{\pi\pi}) P_2(\cos\theta^*)\right]$$

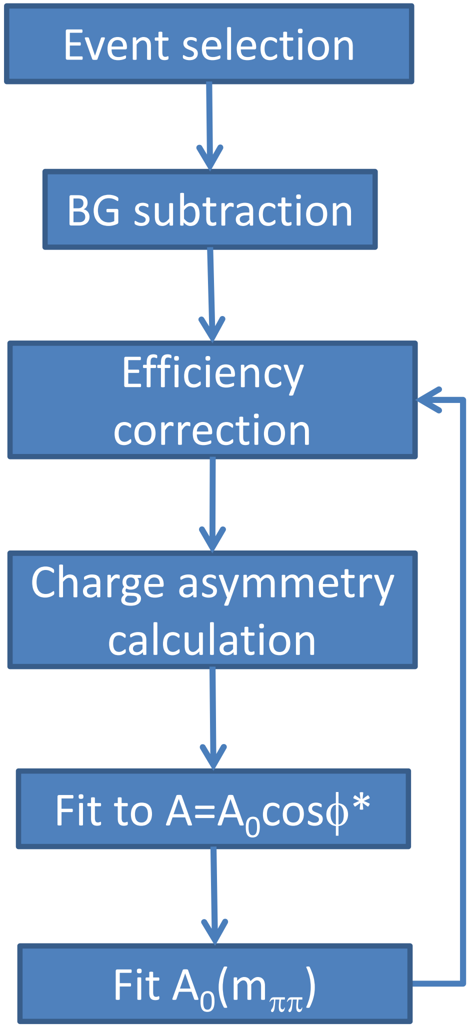
- $P_2(\cos\theta^*) = (3\cos^2\theta^* - 1)/2$ => on f_2: charge asymmetry changes sign at $|\cos\theta^*| = 1/\sqrt{3}\ (\approx 0.58)$



- Analyses are done below and above cosθ*=1/sqrt(3) separately (standard selection for >1/sqrt(3), modified selection for <1/sqrt(3), BG sub, efficiency correction)
- **charge asymmetry has opposite sign around f_2**
- Fit A_0 between 0.3 and 1.4GeV with the revised FSR model

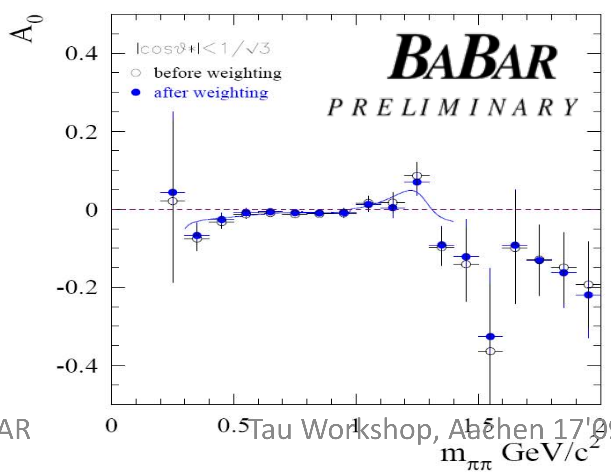
	|cosθ*|<1/sqrt(3)	|cosθ*|>1/sqrt(3)
c_0	-1.13 ± 0.24	-1.59 ± 0.36
c_2	-5.4 ± 1.8	-5.4 ± 3.1

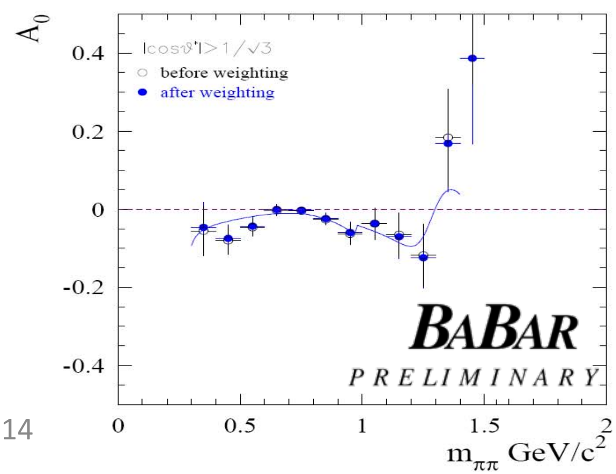
Monte Carlo reweighting and iterations



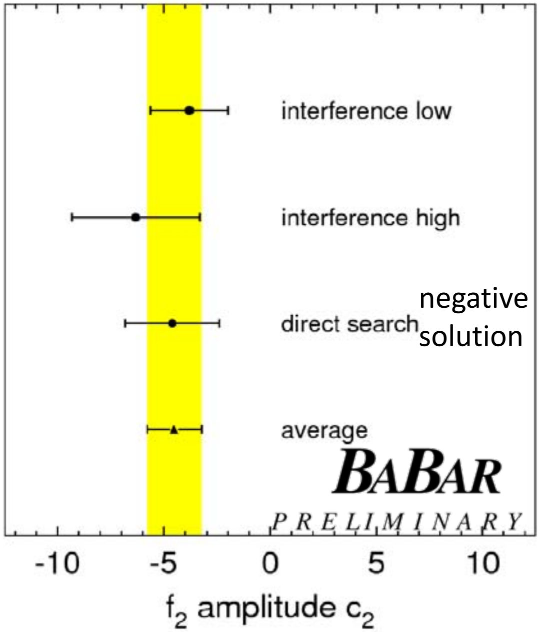
- Measured A_0 could be different from value in MC: extreme case for ee→$\pi\pi\gamma$ where AfkQed has null A_0
  => iterations to recompute efficiencies
- Reweight MC event by event with
  $w=(ISR^2+FSR^2+\text{interference})/ISR^2$
- Fit results stable after 2 iterations

iteration	c_0	c_2
0	-1.27	-5.4
1	-0.866	-4.31
2	-0.959	-4.54
3	-0.93 ± 0.20	-4.48 ± 1.56





Results for $ee \rightarrow f_{0,2}(\pi\pi)\gamma_{FSR}$



- 3 independent consistent c_2 measurements
- average $|c_2| = 4.5 \pm 1.3$
- 3.6σ significance for $e^+e^- \rightarrow f_2 \gamma_{FSR}$

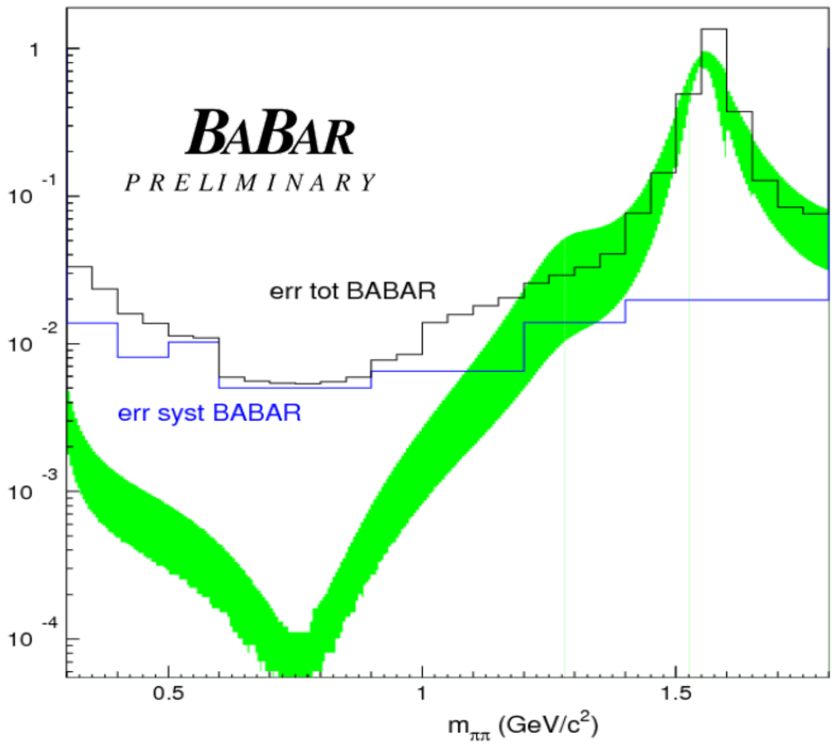
BABAR PRELIMINARY	c_0	c_2
This work	**-0.93 ± 0.20**	**-4.5 ± 1.3**
Diehl et al. 2000	-0.5 ± 0.5	$+0.5 \pm 0.5$
Chernyak (private com .)	–	$|c2| = 2.2 \pm 1.1$

$\Rightarrow$ S wave consistent with Diehl et al., but not the D wave (sign and magnitude)

$\Rightarrow$ f_2 consistent with Chernyak in magnitude

Consequence for $\sigma(e^+e^- \to \pi^+\pi^-)$ and a_μ

$FSR^2 / (ISR^2+FSR^2)$



- **Relative contribution of FSR** estimated with the FSR model 2 using our measured f_0 and f_2 parameters

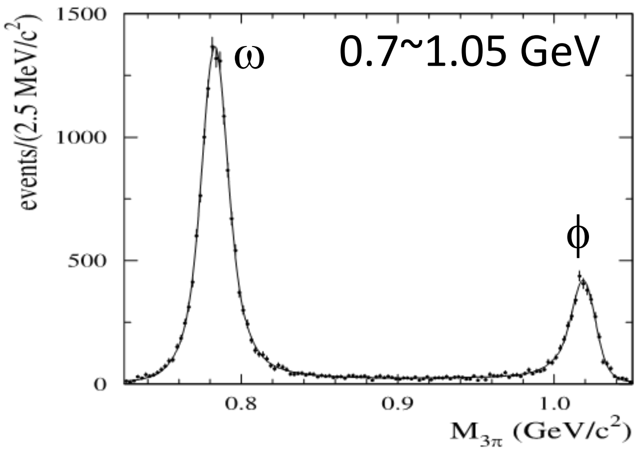
- FSR contribution negligible in ρ mass region
- comparable with the total error of the cross section measurement above 1.2GeV
- Measured F_π between 1.5 and 1.6 GeV very small ~0

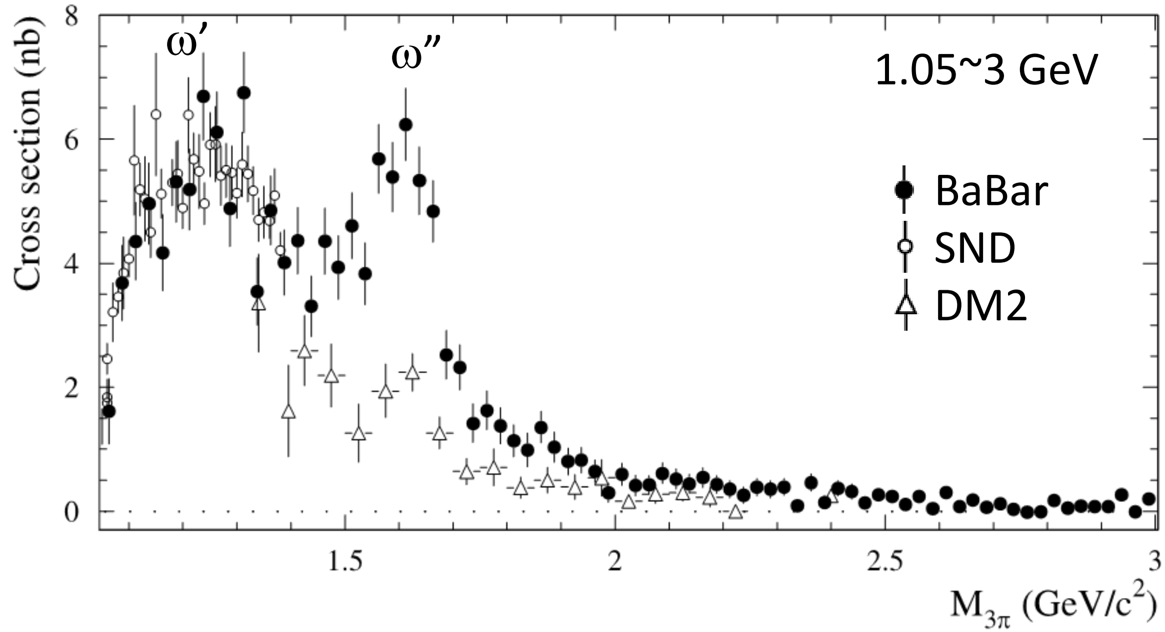
	$\pi\pi$ ($2m_\pi$~1.8GeV)	$\pi\pi$ FSR($2m_\pi$~1.8GeV)
a_μ^{had} (10^{-10})	$514.09 \pm 2.22 \pm 3.11$	0.26 ± 0.12

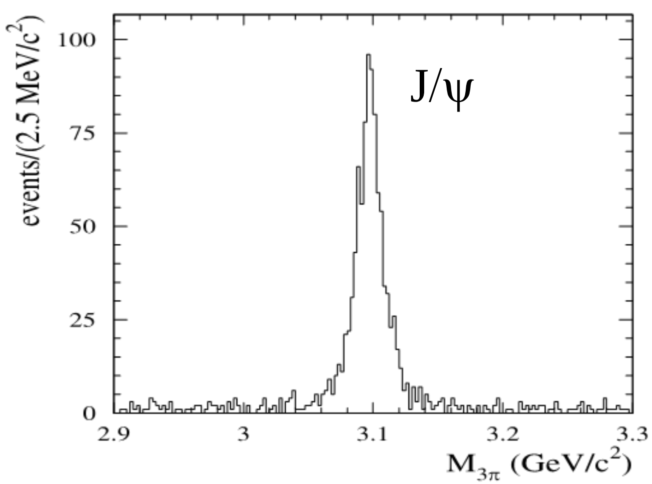
negligible effect

$e^+e^- \to \pi^+\pi^-\pi^0$

published in 2004, based on 89 fb^{-1} data







$$\mathcal{B}(J/\psi \to 3\pi) = (2.18 \pm 0.19)\%$$

PDG2014: (2.11+-0.07)% dominant by BESIII

New result on e+e-→ppbar